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A DICTIONARY

OF

APPLIED PHYSICS

EDITED BY

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IN FIVE VOLUMES

VOL. II

ELECTRICITY

MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON
1922

PRINTED IN GREAT BRITAIN

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CORRIGENDA, VOL. II

From List of Contributors, p. vii, *omit* SCHOFIELD, F. H., M.A.—Tungsten Arc Lamp. (Transferred to Vol. IV.)

Page 1, col. 1, line 9 from foot, ADMITTANCE, *read* the reciprocal of the impedance of an alternating current circuit. It is measured by

$$\left[R^2+\left(L\omega-\frac{1}{K\omega}\right)^2\right]^{-\frac{1}{2}},$$

R, L, and K being the resistance, self-inductance, and capacity of the circuit and ω the pulsatance.

,, 11, col. 2, line 22, *for* $\mp Vv+4v^2$ *read* $\pm 4Vv+4v^2$.

,, 25, col. 2, line 23; page 377, col. 1, line 3 from foot; page 395, col. 1, line 20, and footnote, *for* Hartmann Kämpf *read* Hartmann Kempf.

,, 106, col. 2, line 29, formula (27) *for* $h-\sqrt{h^2-r^2}$ *read* $h+\sqrt{h^2-r^2}$.

,, 108, col. 2, formula (50), *for* $v=I_{max.}\sin(\omega t+\phi)$ *read* $i=\frac{V_{max.}}{Z}\sin(\omega t+\phi)$.

,, 108, col. 2, formula (54), *for* Z *read* Z^2.

,, 108, col. 2, line 7 from foot, *for* minimum *read* maximum.

,, 108, col. 2, line 6 from foot, *for* $-\omega^2L$ *read* $+\omega^2L$.

,, 108, col. 2, line 3 from foot, *for* maximum *read* minimum.

,, 123, col. 1, line 15, *for* $s-c-\frac{(s-a)(s-b)}{c}$ *read* $s-c+\frac{(s-a)(s-b)}{c}$.

,, 383, col. 1, line 8 from foot, *for* of the coils *read* of the secondary coil.

,, 391, col. 1, line 5 from foot, *for* $\log_e\frac{l}{d}$, *read* $\log_e\frac{2l}{d}$.

,, 392, col. 2, last line, *for* reactance *read* impedance.

,, 402, col. 1, line 9 from foot, *for* which reduces to *read* which, when $LS^2K^2\omega^2$ is negligible, reduces to.

,, 589, col. 1, line 26, and page 1096, col. 2, line 5, *for* Móscicki *read* Mošcicki.

,, 591, col. 2, line 5 from foot, *for* Broun *read* Braun.

,, 598, col. 2, lines 19 and 22 from foot, *for* Pierce's *read* Peirce's.

,, 618, col. 1, line 18 from foot, *for* a periodic *read* aperiodic.

,, 680, col. 1, Ref. No. 19, *for* J. L. Eckersley *read* T. L. Eckersley.

,, 708, col. 1, line 6 from foot, *for* michrom *read* microhm.

,, 727, col. 1, line 14 from foot, *for* $-L(di/di)$ *read* $-L(di/dt)$.

,, 781, col. 1, *Fig.* 1, Morse sign for 2 is - - — — —.

,, 948, col. 1, line 2 from foot, *for* conductor *read* condenser.

,, 948, col. 2, line 21, *for* the value *read* the numerical value.

,, 972, col. 1, line 8 from foot, *for* $-\omega^2bG^2$ *read* $+\omega^2bG^2$.

,, 972, col. 1, formula (33), *for* $-\frac{(c-\omega^2K)G^2}{\cdots}$ *read* $+\frac{(c-\omega^2K)G^2}{\cdots}$.

,, 972, col. 2, formula (36), *for* $\omega(L'-L)=-\ \ldots$ *read* $\omega(L'-L)=+\ \ldots$

,, 973, formula (41), *read* $G \doteq 90000(R'-R)/\sigma n$.

,, 1070, col. 1, footnote, *for* Strenstom *read* Stenström.

DICTIONARY OF APPLIED PHYSICS

— A —

"A" Connections: in telephony, the completion of a current between two subscribers on the same exchange. See "Telephony," § (5).

"A"–"B" Connections: in telephony, the completion of a circuit between two subscribers on different exchanges. See "Telephony," § (6).

Abraham, H., work of, on the measurement of "v." See "'v,' Measurement of," § (5).

Absolute Measurements of Electrical Units: the measurement of electrical units in terms of the fundamental units of mass, length, and time. See "Electrical Measurements," § (8) *et seq.*

Summary of results. See "Electrical Measurements," §§ (20), (35), and (37); "Units of Electrical Measurement," § (1).

Absolute Systems of Measurement: systems in which the units of length, mass, and time are taken as fundamental units. See "Electrical Measurements," § (1); "Units of Electrical Measurement," § (1).

Absorption (Dielectric): the absorption of energy by the dielectric when a condenser is first charged and then discharged. See "Capacity and its Measurement," § (8); "Dielectrics," § (4).

Accumulators: cells employed for storing electrical power. See "Switchgear," § (14); "Batteries, Secondary," § (1).

Accuracy Characteristics of Ammeters and Voltmeters. See "Direct Current Indicating Instruments," § (15).

Admittance: the reciprocal of the reactance of an alternating current circuit. It is measured by

$$\left(L\omega - \frac{1}{K\omega}\right)^{-1},$$

L and K being the inductance and capacity of the circuit, ω the pulsatance. See "Inductance, The Measurement of," § (3).

Aerial Cables, Use of, in Telephony. See "Telephony," § (2).

Agnew Dynamometer: a form of heavy current dynamometer, employed for the measurement of current or power. See "Alternating Current Instruments," § (11).

Agnew Galvanometer. See "Vibration Galvanometers," § (17).

Air Condensers (Electric). See "Capacity and its Measurement," § (32).

For radio-telegraphic work and measurement of capacity at low frequencies. See "Radio-frequency Measurements," § (28).

Of variable capacity. See "Capacity and its Measurement," § (32) (ii.).

Alexandersen Alternator, The: a machine for the generation of current of radio frequency. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (5).

Alkali-chlorine Cells. See "Electrolysis, Technical Applications of," § (28).

Alternating Current, measurement of, with vibration galvanometers. See "Vibration Galvanometers," § (42).

Alternating Current Bridges, use of, for capacity measurements. See "Capacity and its Measurement," § (48).

Alternating Current Electrolysis, as shown by the corrosion of underground structures by stray alternating currents. See "Stray Current Electrolysis," § (19).

ALTERNATING CURRENT INSTRUMENTS AND MEASUREMENTS FOR COMMERCIAL FREQUENCIES [1]

(Instruments for acoustic and radio frequencies are not included)

I. Physical Properties made use of

§ (1).—Three properties of the electrified condition of matter are commonly made use of in electrical measuring instruments.

[1] Further details of many of the instruments mentioned in this section, together with references to literature, are given in "Electrical Measurements," *F.A. Laws* (McGraw Hill), 1917.

They are: (1) The electromagnetic forces resulting when currents flow in neighbouring conductors, or when one current is replaced by a constant magnetic field. (2) The electric forces resulting when neighbouring conductors are at different electrical potentials. (3) The heat which is generated in a portion of a circuit carrying a current causing a rise of temperature. Electrical methods, such as change of resistance or thermovoltages, are commonly used as the change-of-temperature detectors.

Voltmeters, ammeters, and wattmeters of the moving-coil or dynamometer type and oscillographs are examples of the first kind. Induction instruments also fall in this class. Electrostatic instruments are examples of the second class. The chief types are voltmeters and wattmeters.

In the third class are various types of thermo-ammeter depending on temperature changes, either of the main conductor carrying the current to be measured, or of a secondary circuit close to it, which is heated by radiation or convection. They are especially valuable for very high frequencies, 10^3 to 10^7 per second.

II. Electromagnetic Instruments

These comprise voltmeters, ammeters, wattmeters, and subsidiary apparatus of special types, such as phase indicators, synchroscopes.

The voltmeters, ammeters, and wattmeters are made either without iron or with iron. In the case of instruments containing iron more than one method of using it is employed.

§ (2) Instruments without Iron. — The common principle employed in electromagnetic instruments without iron is illustrated in *Fig.* 1,

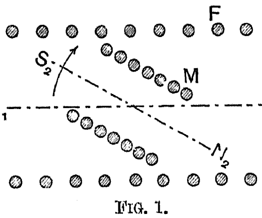


Fig. 1.

in which the interaction of magnetic fields produced by a fixed coil or coils F and by a second movable coil M, often of smaller dimensions, results in a tendency for the movable coil to turn in the direction indicated by the arrow, if the direction of the magnetic fields is as shown.

When the coils have their axis parallel the torque is zero; the system being stable if the fields are in the same direction. If the fields are in opposite directions the system is unstable.

Instruments operated by the interaction of currents in a movable and a fixed coil or coils are conventionally termed dynamometer instruments. They are of two types. Imagine the fixed coil to be divided into two halves. In the one type the current flows in the same direction in the two halves, producing an *axial* field; in the other the current in the two halves of the fixed coil flows in opposite directions, producing a *radial* field. Most makes of commercial instruments are of the axial type. The Kelvin balance is an example of the radial type.

§ (3) Method of Reading. (*a*) *Null Instruments*; (*b*) *Indicating or Pointer Instruments*.—The indications of an instrument may be read by measuring the force required to keep the moving part in equilibrium, which is generally a position of symmetry. Such instruments may be described as null instruments. In many types the moving part is allowed to deflect to a considerable extent against some opposing force, and the deflection is a measure of the quantity required. Such are termed indicating or pointer instruments.

§ (4) Null Instruments.—In the case of an instrument of the axial type the torque is a maximum when the coils are at right angles, assuming that the field produced by the fixed coil is approximately uniform. In such a position the torque is but slightly affected by small displacements. If an opposing torque is applied to the moving part, equal to the magnetic torque, the moving coil may be kept perpendicular to the fixed one. If the opposing torque is measured, as, for example, by the measured deflection of a spring, such deflection becomes a measure of the magnetic torque. The torque, for constant relative position of the coils, is proportional to the product of the intensities of the two fields and therefore to the product of the currents in the coils. If the same current passes through the two coils in series the torque is proportional to the square of the current, so that the current is proportional to the square root of the torque, *i.e.* to the square root of the deflection of the instrument spring.

§ (5) The Siemens Dynamometer.[1]—This instrument (*Fig.* 2), once much used as a secondary standard, especially for alternating currents, works on this principle, the current in the moving part being led into and out of it through mercury cups placed in the axis of rotation. The pointer attached to the spiral spring usually makes one turn for the full reading of the instrument. The formula for calculating the current is $C = K\sqrt{D}$, where D is the rotation of the top end of the spiral spring which is necessary to restore the quadrature position of the two circuits. The displacement of the moving coil is limited to a few degrees each side of the position of balance. The value of K must be determined

[1] *Laws*, p. 77.

by calibration with continuous or alternating currents of known value.

The Kelvin balance and more convenient portable instruments have largely superseded the Siemens instrument; but some of the

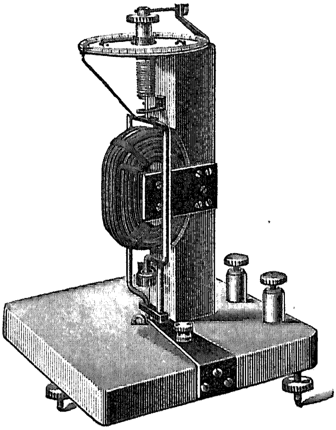


FIG. 2.

most accurate wattmeters have been designed on this principle (Duddell-Mather, Drysdale).[1]

§ (6) KELVIN BALANCE.—This is an electro-magnetic instrument for measuring the current or power in a circuit by the force required to restore equilibrium in a manner similar to the operation of a balance. The currents in the two halves of the fixed coils run in opposite directions. The magnetic field is thus radial. In this instrument a restoring force is applied to the moving part to bring it back to the equilibrium position which it assumes when there is no current passing. The axis of rotation is horizontal, as in the ordinary balance, instead of being vertical, as in most other instruments. The restoring force is the effect of gravity on a moving mass, which may be several grams, and which can be moved along the balance arm till the equilibrium position is attained.

The instrument and a diagram of the circuits are shown in *Figs.* 3, 4.

The larger, upper and lower turns represent the four fixed coils which are commonly wound on slate. The moving coils work between them with sufficient clearance to allow of a few degrees of tilt. The current in the upper and lower coils at one end of the balance are in opposite directions to one another. They therefore produce a "consequent" field between them which is radial, and the current in the moving coil placed in the field causes a vertical force to act on it. The direction of the currents at the other end of the beam is such that not only do the forces add, but also the current in the moving coil is in the opposite direction to what it is in the end first considered. The result is that the instrument is unaffected by a constant external field. Such a field produces equal and opposite forces on the two moving coils. The instrument is therefore astatic.

The current is led into and out of the moving part by a number of fine bronze strips in parallel. These also take the weight, and act as the fulcrum of the balance. Owing to the relatively large surface a very large current can be carried by a very small total cross-section of copper by this method. A similar scheme for leading in the current has been adopted in the standard ampere balance used for realising the value of the ampere, but in this instrument the balance beam is supported on a knife-edge of agate of the usual form.

When used in the manner shown, in which the current goes through all the coils in series, the forces are proportional to the square of

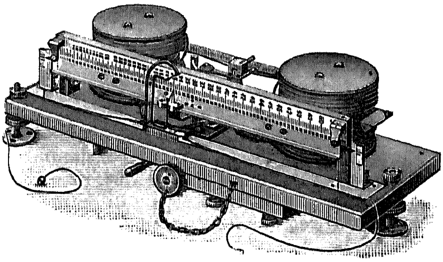


FIG. 3.

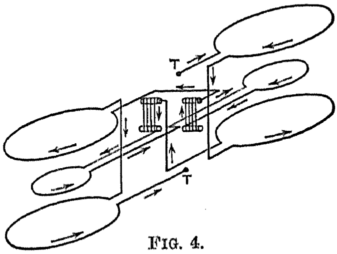


FIG. 4.

the current, so that the current is proportional to the square root of the reading of the position of the sliding weights. To cover the range of which the instrument is capable four different weights are generally provided. By this means in the 100-ampere instrument, for instance, full-scale reading is obtainable at 100, 50, 25, and 12·5 amperes. The instrument is not so sensitive at the lower ranges. The weight for 100 amperes is about 60 grams and the couple about 2000 gram-centimetres.

Like all instruments having conductors of comparatively large cross-section, there is an error due to frequency in the larger sizes;

[1] See § (10).

but up to the 600-ampere size this is negligible for commercial current measurements at ordinary frequencies. The balance can be relied on to about 0·2 per cent at full load and can be calibrated by continuous currents.

This type of instrument may also be used as a wattmeter by sending the main current through one set of coils and a current proportional to the voltage in the circuit through the other. In practice the moving coils are the voltage coils, and are wound of many turns of fine wire with an added resistance in series, the amount of which depends upon the voltage to be dealt with. On account of the large magnetic field that is required the inductance of the moving-coil circuit is very considerable and the current in it lags behind the voltage. The phase displacement may produce serious errors unless allowed for, as will also eddy-current effects, if the fixed-coil circuit is insufficiently stranded. The scale is a "linear" one, not a "square law."

§ (7) Indicating or Pointer Instruments. —If the moving coil is allowed to deflect against a spring or the force of gravity instead of being completely restored by mechanical force, a pointer P, attached to the coil, can be made to indicate the product of the two currents. In this manner a practicable deflection of about 100° is obtainable. A very large number of modern instruments operate on this principle, the products of different manufacturers differing in detail, but all similar as regards fundamental theory.

From the general principles of the interaction of circuits, it will be seen that if the initial position of the moving coil is as shown at A (*Fig.* 5), and the full-scale deflection as

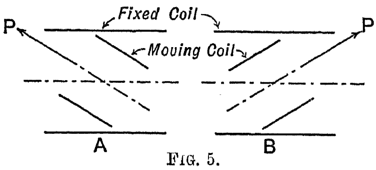


Fig. 5.

shown at B, the scale divisions representing the current in one coil (the current in the other coil remaining constant) will be nearly equal. Similarly if the current measured passes through both coils the scale division will approximately follow a "square law," as in the case of the Siemens dynamometer.

If the fixed coil is divided into two halves separated from each other, as in the Helmholtz galvanometer, an approximation to a uniform field may be obtained midway between them, in which the moving coil moves. This construction facilitates the insertion of the moving coil. Lord Rayleigh has pointed out that over a considerable angular deflection very close approximation to constancy of force can be obtained with circular coils whose radii have the ratio $\sqrt{0{\cdot}3}$ or 0·548.[1]

§ (8) Indicating Dynamometer Instruments. (i.) *Ammeters.*—The general outline of electromagnetic instruments given above shows that a current may be measured by passing it through a fixed and a moving coil in series, and that if the resulting force is opposed by a spring the deflection of the pointer is an approximate measure of the square of the current.

The moving coil is supported either by a metallic strip or by hard pivots working in jewelled cups. Two bronze helical springs usually supply the controlling torque, and are so arranged that any tendency and change of zero which might occur, due to change of temperature, etc., if one spring alone were used, is counteracted by the direction in which the other tends to move.

For small currents, up to about 0·5 ampere, the springs are used also for leading the whole current into and out of the moving coil.

This type of instrument is suitable for currents of all ordinary commercial frequencies and often considerably higher, and in the case of small currents, in which the cross-section of the winding is small, the type is suitable for frequencies of several hundreds or even thousands a second. Above 0·5 ampere the current becomes too large to be carried by the springs, which, if made of sufficient section to carry it without undue heating, would be too stiff for the forces involved.

A fraction of the current, $\frac{1}{2}$ ampere or less, is therefore passed through the moving coil, which is connected to the ends of a resistance, placed in series with the fixed coils; this carries the remainder of the current, following the principle used to a very large extent in continuous current measurements.[2] In addition to correct resistance ratios between the two circuits in parallel, the proper division of current must be also obtained with alternating currents of the frequencies for which the instrument is designed. The relatively fine-wire circuit of the moving coil has a measurable inductance which may be appreciable in instruments designed for high precision. The effect of this inductance in reducing the current through the moving coil, and, in the case of wattmeters, the phase angle introduced, may be compensated by a condenser placed in parallel with the series resistance which is commonly added to such a moving coil, partly to diminish the effect of temperature on the resistance of the circuit, and partly for adjustment purposes.

The compensation is practically perfect for commercial frequencies when $L=Cr^2$, the

[1] Lord Rayleigh, *Phil. Mag.*, Dec. 1886, xxii. 470.
[2] See "Direct Current Indicating Instruments," §§ (1) and (20).

symbols representing the inductance capacity and resistance of the parts of the circuit shown in the diagram (*Fig.* 6).[1]

The moving coil carries the condenser current as well as the current through the resistance r. The current through L and r would naturally diminish with increase of frequency due to the inductance of L. The

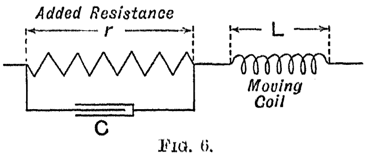


FIG. 6.

current through C and L will increase with frequency. If the values are chosen in accordance with the above equation the compensation is practically perfect both as regards magnitude and phase.

Many of the difficulties of the measurement of large currents are avoided by the use of current transformers. The main current to be measured traverses the primary of a transformer the secondary of which is connected to a low-resistance ammeter. The ratio of the turns is such that the nominal secondary full-load current is 5 amperes, and for industrial ammeters about 400 ampere-turns are used for commercial frequencies. Where accuracy is of importance 1200 ampere-turns are desirable.

(ii.) *Voltmeters.* — The indicating dynamometer voltmeter is practically an ammeter with the circuits arranged to carry about 0·05 ampere or more. For the finest instruments small currents are used, while for large instruments more power and larger currents are required.

In series with the circuit through the moving and fixed coils a sufficient quantity of resistance alloy wire, practically inductionless, is added, depending upon the voltage which it is desired to measure. In the case of the finest apparatus this added resistance is sufficient to render negligible the effects of inductance and temperature for voltages of 100 or more. In the case of voltages of the order of 10 or less the temperature correction may be appreciable, and an adjustable resistance is sometimes added which is set to correspond to the reading of a thermometer which is permanently fixed with its bulb inside the instrument. Compensation for inductance can be carried out as in the case of the ammeter.

For voltages above 500 or 1000 a potential transformer[2] is used to step down the voltage to a safe and easily measured value. Just as 5 amperes has been adopted for current transformers, so 110 volts has been adopted as the standard low-pressure voltage of potential transformers. The primary side is wound to suit the supply voltage, which may be anything from 200 volts to 66,000 or more. Sometimes transformers of a ratio of unity, such as 110 : 110, are used for isolation purposes.

(iii.) *Wattmeters.* — A wattmeter measures the mean value of the instantaneous product of the current and voltage of a circuit.

In the type of instrument under consideration the fixed coil may be similar to that of an ammeter of the same current capacity, while the moving coil with a high resistance in circuit is supplied by the voltage in question.

The instrument may be looked upon constructionally as half-ammeter and half-voltmeter. It has a nearly linear scale and for most technical purposes it is used at approximately constant voltage.

As in the case of ammeters for large currents a 5-ampere instrument is used with current transformers; and for large voltages a potential transformer is used with a secondary voltage of 110. For an account of the forces acting in a wattmeter, see § (20).

§ (9) GENERAL METHOD OF COMMERCIAL MEASUREMENT, USING AMMETERS, VOLTMETERS, AND WATTMETERS.—A complete set of instruments for ordinary commercial measurements consists of a voltmeter reading up to about 120 volts, with perhaps a second range obtained by the addition of an extra resistance to read up to double this value, an ammeter to read 5 amperes, and a wattmeter for 110 volts 5 amperes. All other higher voltages and currents are catered for by the use of potential and current transformers, the voltmeter and pressure circuit of the wattmeter being in parallel, and the ammeter and current circuit of the wattmeter being connected in series to the secondary circuit of the transformer.

Ammeters, voltmeters, and wattmeters of this class are very similar instruments. In the ammeter and wattmeter practically identical fixed coils are used for the main current. The moving-coil circuits in the voltmeter and wattmeter are also very similar, both having high resistances.

In the ammeter the moving coil with little or no added resistance is connected either across the fixed coils or across added resistance in the main circuit.

§ (10) WATTMETERS (DRYSDALE AND DUDDELL-MATHER).—These wattmeters are examples of the type analogous to the Siemens dynamometer ammeter, in which the reading is obtained in terms of the amount of rotation which has to be applied to a helical spring in order to bring the moving coil to its initial position at right angles to the fixed coil.

(i.) *Drysdale Wattmeter.* — The fixed coils carry the main current and are arranged in

[1] E. B. Rosa, *Bull. Bur. Standards*, 1907, iii. 43; *Laws*, p. 313. See "Inductance, Measurement of," § (3).

[2] See "Transformers, Instrument," § (8).

ten sections which can be put in series or parallel or intermediately by a cylindrical commutator, so that the same ampere-turns are obtainable with currents in the ratio of 1, 2, 5, 10. The moving coil is attached to a mica support, and being divided into two parts in which the currents flow in opposite directions, it is astatic to stray fields. The whole wattmeter movement is duplicated, so as to make the instrument available for use on two- or three-phase circuits. The two elements of the meter are placed vertically one above the other, with the moving coils fixed to the same mica support, so that the torques of the two meters are added together. The two elements are placed with their axes of symmetry at right angles to one another,

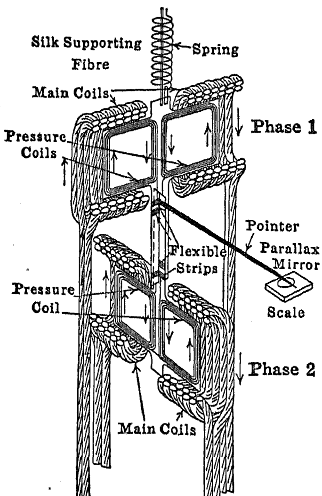


FIG. 7.

so as to ensure the least possible interference of one on the other. No metal other than the windings is used in the construction of the important parts of the instrument in order to avoid the possibility of error due to eddy currents which might be set up in such metal parts (*Figs.* 7, 8).

This wattmeter and the Duddell-Mather instrument are similar in design and operation. The main difference is that, in the latter, astaticism is achieved by having two moving coils in the same plane, one above the other. These are connected in series, and the current flows through them in opposite directions. The fixed coils are duplicated in a similar manner. The Duddell-Mather instrument is only made in the single wattmeter form.

(ii.) *Duddell-Mather Wattmeter.*—The fixed coils are divided into ten sections, which can be connected in series or in parallel or intermediately, so that full-scale deflection can be obtained with currents of the ratio of 1, 2, 5, 10. The instrument is made in seven

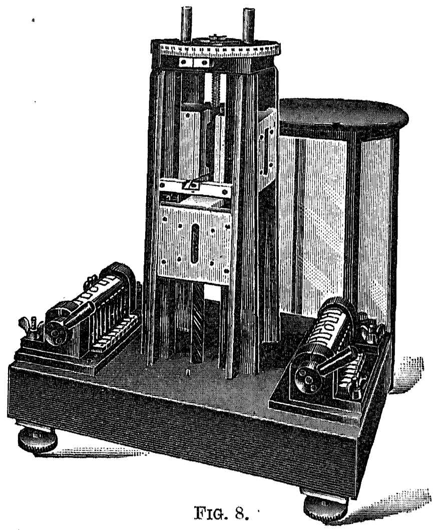


FIG. 8.

sizes, covering altogether a wide range of currents, from 0·1 to 1000 amperes (*Figs.* 9, 10).

Astaticism is obtained by duplicating the various coils and arranging that the currents go in opposite directions in each set.

The current coils are shown at C and the moving coils at *c*. They are fixed to a sheet of mica which is extended below at P, where it acts both as an air damper inside a glass

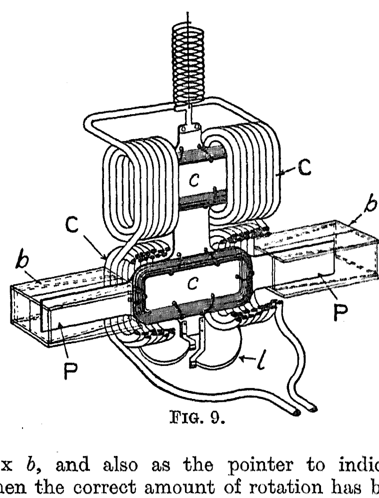


FIG. 9.

box *b*, and also as the pointer to indicate when the correct amount of rotation has been applied to the control spring. This is ensured by making it appear coincident with the fine line marked along the centre of the box when viewed from above. The current is led into the moving coils by fine ligaments *l*.

The voltage used on the pressure circuit is quite low, about 1 volt, and external resistances are added to suit the voltage of the

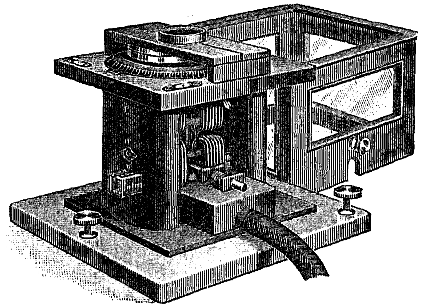


FIG. 10.

circuit. These resistances are made on a loom. The warp, about 8 inches wide, is made of silk threads, and a fine silk-covered resistance wire is used as a weft. In this way a resistance very free from inductance is constructed, which, when mounted zigzag fashion round porcelain rods, enables excellent cooling to be obtained.

For voltages above 5000 it may be desirable to immerse the resistance in oil. They may be made in sections in this manner for circuits up to 100,000 volts.

A valuable feature of this wattmeter is that the pressure coil and resistances are wound with much thicker wire than is sufficient to give full deflection when the current and voltage are in phase, so that if the current and voltage are small, nearly in quadrature, giving only a small deflection, the current in the pressure circuit may be increased up to ten times, if desired, by reducing the added resistance. In this way full deflection may be obtained with a power factor of 0·1. When used on the 100-volt range the resistance of the moving coil is $\frac{1}{10}$ of the whole, and being of copper its change of resistance with temperature may become important.

To determine the effect, if any, of the inductance of the pressure coil, which may be appreciable at low power factors, a special coil and switch is arranged so that an additional inductance equal to that of the coil may be switched into the circuit without altering the resistance. This gives a very valuable method of giving a direct indication of the magnitude of the inductance error when using the instrument at power factors of the order of 0·01 on circuits of 100 to 200 volts. On circuits of several thousand volts the effect would be very small; but in such cases the currents due to distributed capacity, between the various parts of the high added resistances and their surroundings, might cause errors which are difficult to determine. They can be reduced by a somewhat elaborate system of electrostatic shields fed from subsidiary resistance used as a potential divider.[1]

§ (11) AGNEW DYNAMOMETER.[2]—A dynamometer instrument in which the magnetic field, surrounding a cylindrical linear conductor carrying a comparatively large current, acts on a small coil close to it, tending to turn the coil so that its plane becomes perpendicular to the field cutting it.

This type of instrument was specially designed for the measurement of large currents, up to 5000 amperes (*Figs.* 11, 12).

A circular horizontal conductor, which may be tubular for the sake of water cooling, carries the main current. Close to it is suspended a small coil carrying a current which may be derived from the potential points of a

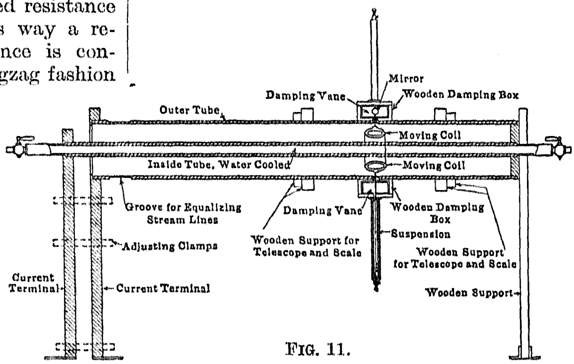


FIG. 11.

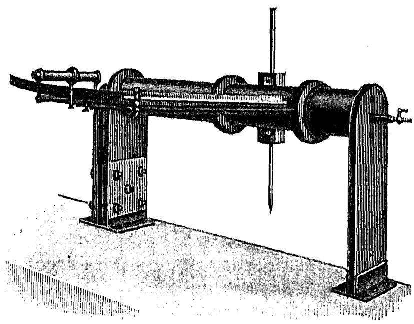


FIG. 12.

resistance through which the main current passes, in which case the instrument operates as a "square law" ammeter; or the moving coil may be fed from a separate circuit, in which case it may be used as a wattmeter.

[1] Orlich and Schultze, *Archiv der Elekt.*, 1912, i. 1.

[2] "A Tubular Electrodynamometer for Heavy Currents," P. G. Agnew, *Bull. Bureau Standards*, 1912, viii. 651; *Laws*, p. 87.

In practice two similar coils are used, one above and one below the tube. They are rigidly connected together without touching the tube, and are wound of insulated silver wire. In the illustration they are shown for clearness nearly at right angles to their working position. A torsion suspension is used. The main return conductor takes the form of a concentric tube enclosing the central conductor, and the moving coils.

It is important that the distance of the small coils from the central axis should remain constant. Provided symmetry is observed in the current distribution the indications of the instrument are independent of the diameter of either of the tubes. The instrument's indications are therefore independent of any symmetrical redistribution of currents in the conductors due to skin effects when alternating currents are used. This is a particularly valuable property of this type of instrument.

§ (12) Irwin Dynamometer Instruments.—These are electrodynamometers of the moving-coil type. The moving coil is placed between two fixed coils in which the currents circulate in opposite directions. They are made as ammeters, voltmeters, and wattmeters. The instrument is similar to the Kelvin balance as regards the magnetic field produced by the fixed coils. The currents circulate in opposite directions, giving rise to a radial field between them. In the Kelvin balance the moving coil tends to move axially in this field, either towards one of the fixed coils or the other according to the direction of the current passing through it.

In the Irwin instrument the moving coil is of the form ᗡD, there being two D-shaped coils so mounted that the force acting on the system is one of rotation and not one of translation.

The force acting on the coil is opposed by a spring in the usual way. As the currents in the effective parts of the coil flow in opposite directions, the instrument is unaffected by stray fields, assuming the intensity of such fields in the neighbourhood of the moving coil is constant.

§ (13) Instruments depending on the Use of Iron.—The force acting on soft iron in a magnetic field is used for operating alternating current instruments, such as voltmeters and ammeters, particularly those for use on switchboards.

They have a considerable advantage over instruments of the same type when used for direct currents in that the hysteresis error is negligible. When used for direct currents there is liable to be a serious difference between the indications when the current is rising and when it is falling. This can be made inappreciable in the case of instruments designed for alternating currents. They can also be made practically free from friction errors, since the rapid pulsation of the torque, combined with the slight vibration commonly present, assists in overcoming friction, the effect of which would otherwise be considerably more appreciable.

(i.) *Attracting Solenoid Type.*—A principle commonly used is that of the "sucking solenoid." The current to be measured is sent through a solenoid into which soft iron, usually in the form of a thin rod, is drawn against the force of a spring or of gravity. The relative size of the scale divisions can be adjusted to a considerable extent by the shape of the parts, by the value of the magnetic flux, and by variation of the effective pull through some device whereby there is an alteration of leverage as the deflection changes.

There are many modifications of this type. One of these is a simple arrangement by Elihu Thomson, in which the spindle of the pointer has an iron disc attached to it centrally, but on the skew. The disc is surrounded by a coil carrying the current and having its axis also inclined to the spindle. On the passage of the current the disc tends to set itself with its plane parallel to the magnetic field of the coil, thus producing a torque on the spindle to which the pointer is attached. The torque is commonly opposed by a spring.

(ii.) *Repulsion Type.*—A second principle made use of is the repulsion of two pieces of soft iron when placed close together in a magnetic field. A bar of iron in a magnetic field collects the flux in space near each end, the total energy being reduced by the concentration of flux in the length of the bar. If two soft-iron bars are placed parallel alongside one another in a magnetic field, they are magnetised and repel one another, since by separation the total energy is diminished. This motion is utilised to measure the strength of the current.

In another pattern of instrument each piece of soft iron is a thin sheet bent so as to form a segment of a circular cylinder; the two segments are concentric, and one can rotate about their common axis, carrying with it the indicating pointer of the instrument. The edges of the sheets are so shaped that when they become magnetised under the action of a current circulating in the coil of the instrument a torque is produced, which turns the movable segment about its axis. This motion is opposed, in some cases by a spring, in others by gravity, and the strength of the current is indicated by the motion of the pointer; the iron is so shaped as to produce as even a scale as possible. Excellent instruments of this type are made.

An example of the use of a moving iron instrument for measuring voltages up to 100,000

is shown in *Fig.* 13, which represents the arrangement of the apparatus at the National Physical Laboratory for generating, regulating, and measuring voltages up to 100,000. The voltmeter, of the iron repulsion type, is designed to have a very even scale for an A.C. instrument, and full deflection is produced by 500 volts. By the addition of resistances of integral multiples of that of the voltmeter, full-scale deflection can be obtained for 1, 2, 5, 10, 20, 40, and 100 kilovolts. The resistance boxes shown in the diagram are each insulated so that either end or the centre of the high voltage winding can be put at earth potential. The total resistance for 100,000 volts is 5 megohms.[1]

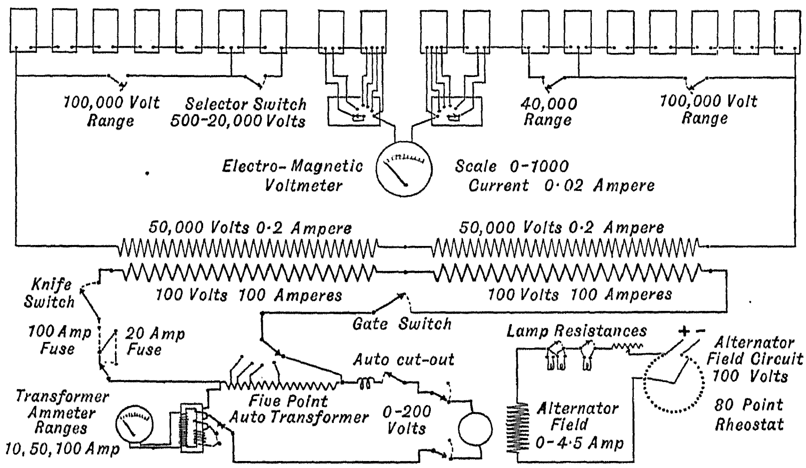


FIG. 13.

III. ELECTROSTATIC INSTRUMENTS[2]

§ (14) FORCES AVAILABLE. — Electrostatic instruments depend for their indications on the mechanical forces which arise when two materials are at a difference of electrical potential.

When two plane surfaces are at unit distance apart they attract or repel each other with a force F equal to $V^2A/8\pi d^2$, where V is the difference of potential in electrostatic units, A the area, and d the distance apart, the effect of the edges being neglected.

The order of magnitude of the forces available in instruments of this type may be illustrated by the following example (*Laws*, p. 245):

Let A be 100 sq. cm.,
V be 100 volts,
d be 1 cm.,

$$V=100 \text{ volts}=100\times10^8 \text{ electromagnetic units}$$
$$=\frac{100\times10^8}{3\times10^{10}} \text{ electrostatic units}$$
$$=\frac{1}{3} \text{ electrostatic unit;}$$
$$F=\frac{V^2A}{8\pi d^2}=\left(\frac{1}{3}\right)^2\frac{100}{8\pi}=\frac{100}{72\pi}=\frac{100}{226}=0{\cdot}442 \text{ dyne,}$$

which is about equal to the weight of 0·45 milligramme.

The forces available for instruments of this nature are therefore very small when ordinary commercial voltages of the order of 100 volts have to be dealt with. For higher voltages considerably more power is available as the forces go up as the square of the voltage. Above a few thousand volts the dimensions of the apparatus have to be increased on account of the electric breakdown of air at high stresses. This breakdown has been detected at the surface of a fine wire surrounded by a concentric metal cylinder, 10 cm. in diameter at 3000 volts difference of potential.

In spite of the rapid diminution of the force available at low voltages, instruments have been developed to read as low as 2 volts for full-scale deflection, using a fine-strip suspension.[3] Portable pivoted instruments giving full-scale deflection with 60 volts are also made.

§ (15) VOLTMETERS.—Practically all instruments of this type are founded on the types of instrument developed by Lord Kelvin.

They are of two general types:

(1) Those in which the movement of the attracted conductor is perpendicular to its surface (attracted disc electrometer).

(2) Those in which the movement is in the

[1] *I.E.E.* xlix. 6.
[2] Kelvin, reprint of *Papers on Electrostatics and Magnetism* (2nd ed.), 1884, p. 283 (Macmillan), a fundamental paper. See Bibliographies, *I.E.E.*, 1912, xlix. 53, and 1913, li. 327.
[3] Addenbrooke, *Electrician*, 1900, xlv. 901.

plane of the moving part (multicellular voltmeter).

The most important instrument depending on the attraction and normal displacement of

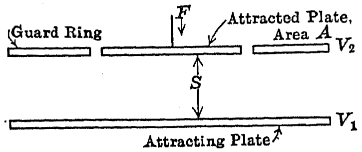


FIG. 14A.

parallel conductors is the Kelvin absolute electrometer. In this instrument the "edge" effect is avoided by a guard plate. The attraction of one plate is measured by the deflection of a spring which has to be calibrated by known weights (*Figs.* 14A, 14B).

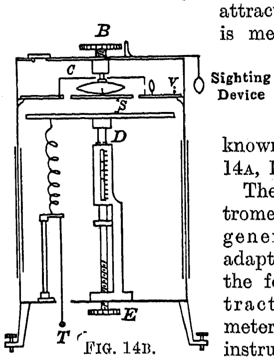


FIG. 14B.

The absolute electrometer is not in general use; but adaptations of it in the form of the attracted disc voltmeter are valuable instruments for the measurement of high voltages. The Kelvin pattern is made in two sizes—one up to 30,000 volts, and a larger one up to 100,000 volts. The latter is about 6 feet high and 3 feet in diameter. The sensitivity may be altered by changing the weight which the disc has to raise when it is attracted.

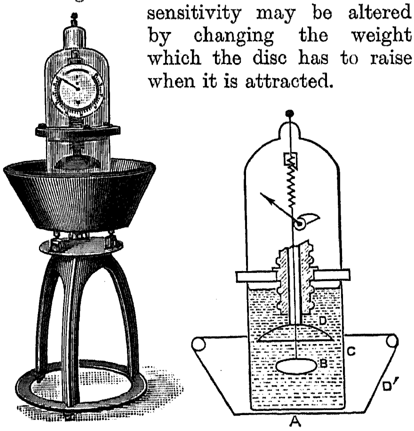


FIG. 15.

Other forms of attracted disc voltmeter for high voltages have been developed by Jona,[1] Siemens and Halske [2] (*Fig.* 15), Abraham.[3]

[1] *Science Abstracts*, 1905, 8B, No. 1390.
[2] *Laws*, p. 258. [3] *C.R.*, 1895, cxx. 726.

§ (16) ELECTROMETER TYPE OF VOLTMETER.—A more convenient form of electrostatic instrument for general purposes is obtained by allowing the attraction of the conductors to cause a rotation of one of them about an axis.

If the axis is vertical the suspension of the moving part may be by a fine strip of bronze, which acts as the conductor to the moving part. The restoring couple is supplied by the twist of the suspending strip. If the torque is sufficient a bifilar suspension may be used. To increase the torque the cellular type was developed in which as many as 10 or more "needles" are used. The needles are of a special dumb-bell shape, and a deflection of about 60° is obtainable. A pointer moving over a scale indicates the voltage applied.

This type of instrument has been used at the National Physical Laboratory for alternating measurements work of the highest accuracy.[4] A mirror is attached to the moving part and the indications are read on a scale 10 feet away and about 15 feet long. The instruments read to about 130 volts; and over the usual working part of the scale, 50 to 120 volts, the scale is divided to hundredths of a volt.

The dividing of the scale is done empirically by applying different voltages in integral steps of half a volt to the instrument from a special potential divider. The potential divider is supplied by continuous current, and the voltmeter forms the link between continuous current and alternating current measurements.

A precision form of this instrument by E. H. Rayner [5] has a very light moving system enabling a bifilar suspension to be used, practically abolishing the elastic fatigue, which is a serious disadvantage, in the case of torsion strip instruments used with this degree of sensitivity.

The needle has been modified in form, enabling a more even scale law to be obtained.

§ (17) AYRTON-MATHER VOLTMETER.—The design of the moving part N is shown in *Fig.* 16. It consists of two portions of a very light cylinder, and is attracted into the space between cylindrical inductors. Special consideration has been given to obtaining a high ratio between torque and inertia. The most sensitive form has a fine metal strip suspension and will give full-scale deflection with about 10 volts. Variable air damping is used. Portable pivoted instruments are made to

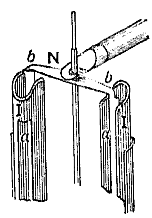


FIG. 16.

[4] Paterson, Rayner, and Kinnes (Bibliography), *Proc. I.E.E.*, 1913, li. 294.
[5] *Proc. I.E.E.*, 1921, lix. 138.

give full-scale deflection for 130 volts and upwards. At higher voltages sufficient power is available for use as a switchboard instrument. All such instruments will give little deflection till a third or a half full-scale voltage is applied. By suitably designing them a relatively even scale can be obtained for the top third of the range.

§ (18) ELECTROMETER INSTRUMENTS. — In the general form of electrometer instrument the moving conductor, termed the needle, is enclosed in a box consisting of four hollow quadrants nearly touching each other; a small central hole is provided for the suspension to pass through.

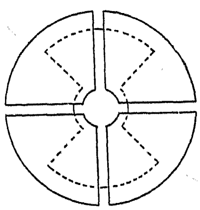


FIG. 17.

The moving conductor is, in plan, usually bounded by circular and radial lines as shown in *Fig.* 17.

The opposite pairs of quadrants are commonly connected together.

The needle is generally charged to a potential differing from that of the quadrants by from 40 to 200 volts. If the quadrants are connected together so as to be at the same potential, there will be no deflection of the needle provided there is complete symmetry. In practice a small deflection will result, which may be reduced to zero by slight adjustment of the instrument, such as by altering the level. If a small deflection remains it will not affect the accuracy of the readings, which must be taken from this "electrical zero" position.

When reading zero the needle should occupy an unsymmetrical position under one of the radial quadrant slits, and the adjustment should be such that its centre line will come opposite one of the slits at half-scale deflection. This will allow 10° to 15° deflection on either side, which, assuming a mirror is used, will give an apparent deflection of 40° to 60°.

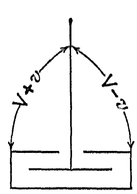


FIG. 18.

When the needle is charged to a potential V relative to the quadrants, they apply to it equal and opposite torques proportional to V^2.

Now suppose that instead of the quadrants both being at a potential V relative to the needle, one is charged to a potential $V+v$ and the other $V-v$, then the one set of quadrants will produce a torque proportional to $(V+v)^2$ and the other $(V-v)^2$ (*Fig.* 18). The resulting torque is $(V+v)^2-(V-v)^2=4Vv$. Assuming V is constant the torque is proportional to the difference of potential $2v$ between the quadrants.

If the instrument is allowed to deflect and the rate of change of capacity of needle and quadrants is linear over the working range, the resulting deflection is proportional to v.

In actual working it is often impracticable to divide up the voltage, $2v$, to be measured into two halves, since this would require a dividing resistance to keep the mean voltage of the needle equal to V (*Fig.* 19).

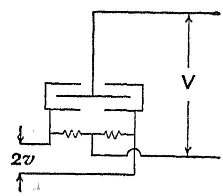


FIG. 19.

The common method of using the instrument is to keep one quadrant at potential V and to apply the potential, $2v$, to be measured, between it and the other quadrant whose potential will be $V \pm 2v$.

The torque in this case will be

$$(V \pm 2v)^2 - V^2 = \pm Vv + 4v^2.$$

The quadratic term is generally small, as v is commonly much less than V. In any case it is of little practical importance, as the instrument has to be calibrated empirically by applying known values of v from a standard cell, or, when the scale law is required to be known accurately at several points, by using a potential divider in the form of a suitable resistance carrying a known current.

As the instrument takes no current other than the very small capacity current which ceases, for continuous potentials, when full deflection is obtained, it has been largely used for physical research, especially in cases where an instrument requiring a continuous supply of power is inadmissible. The Dolezalek form has been much used in this manner.

In some early forms of Kelvin electrometer a comparatively short bifilar suspension is used which requires large voltages to operate it. It was developed at the time that atmospheric electrification was engaging Sir William Thomson's attention, when an instrument for several hundred volts was required.

§ (19) DOLEZALEK ELECTROMETER. — The Dolezalek instrument (*Fig.* 20) is designed for high sensitivity. The needle is made of "silvered" paper or aluminium, and is partly cut away to reduce the inertia without serious reduction of torque. The suspension may be of fine bronze or, where the highest sensitivity is required, silvered quartz or glass can be used, which gives much less control. By this means a sensitivity of 150 mm. can be obtained at 1 metre with 100 volts on the needle and 0·1 volt on the quadrants, using a suspension 0·006 mm. diameter. Suspensions of half this diameter can be used.

§ (20) ELECTROMETER AS A WATTMETER.— It has been shown that the torque on the

electrometer needle is proportional to Vv, where $V+v$ and $V-v$ are the differences of potential between the needle and the two sets of quadrants. This holds of steady voltages.

If the voltage varies the mean torque is proportional to the mean value of the product Vv. If V is proportional to the voltage in the circuit and v to the current, the instrument will indicate the mean power and will act as a wattmeter.

If the voltage varies sufficiently rapidly compared with the time of swing of the instrument the needle will take up a steady

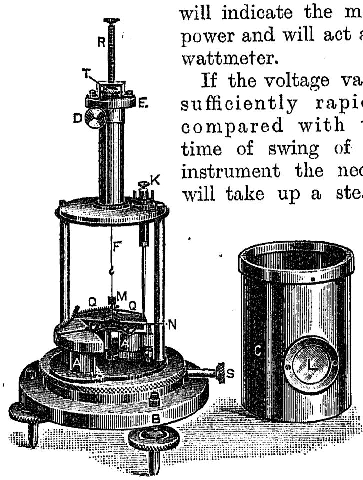


FIG. 20.

position, corresponding to the mean value of the product Vv. The product Vv has here a special significance, as in the general case the variations of V and v are not in phase.

Suppose the voltage between the needle and the mean of the two quadrants is represented by a harmonic function of the time, such as $V \sin \theta$ where $\theta = \omega t$, t being time and ω the pulsatance or radians per second—which is equal to $2\pi f$ where f is the frequency—and that the similar harmonic function $2v \sin (\theta - \alpha)$ represents the difference of potential between the quadrants, then the torque is proportional to the mean value of $2Vv \sin \theta \times \sin (\theta - \alpha)$ or

$$2Vv\,(\sin \theta^2 \cos \alpha - \sin \theta \cos \theta \sin \alpha).$$

The average value of ($\sin \theta \cos \theta$ or $\frac{1}{2} \sin 2\theta$) is zero throughout the range of θ from 0 to π, so the mean torque is proportional to $Vv \sin^2 \theta \cos \alpha$.

The average value of $\sin^2 \theta$ per cycle is $\frac{1}{2}$, thus the mean torque is proportional to $Vv \cos \alpha$. This is independent of frequency, assuming the needle potential has time to follow accurately the potential of supply. The value $v \cos \alpha$ may be looked upon as the component of the vector v in phase with V. The instrument gives no indication of the quadrature component $v \sin \alpha$.

It will be seen that superposed on the torque in a constant direction represented by $\sin^2 \theta \cos \alpha$ there is an alternating torque of double the frequency given by $\frac{1}{2} \sin 2\theta \cos \alpha$. The mean value of each component of the torque depends on α, and when α is 90° the term representing the unidirectional torque vanishes. The instrument reads zero, but the alternating torque to which it does not respond has then its maximum amplitude.

The unidirectional torque proportional to $\sin^2 \theta$ is not simple harmonic, while the oscillating torque of double the frequency is harmonic.

If α is 45° so that $\cos \alpha$ and $\sin \alpha$ are equal, we see that the relative amplitudes of the two torques are in the ratio of the average values of $\sin^2 \theta$ to $\frac{1}{2} \sin 2\theta$ taken between the values 0 and π for 2θ the harmonic component. This ratio is $\pi/2$, since the mean value of $\sin 2\theta$ during a cycle 0 to $\pi/2$ is $2/\pi$ and that of $\sin^2 \theta$ is $\frac{1}{2}$.

If V is proportional to the voltage of a circuit in which the pressure varies harmonically, simply, or complex, and v is proportional to the current in the same circuit, then the instrument indicates the mean value of the product of their in-phase vectors and therefore the power in the circuit.

The forces acting between the coils of the electromagnetic type of wattmeter, §§ (9), (10), (11), are quite similar.

§ (21) POWER MEASUREMENT BY THE ELECTROSTATIC WATTMETER.[1]—The use of the instrument as a wattmeter in this manner has been extensively developed at the National Physical Laboratory.

Among the advantages is the possibility of its very wide application, the same instrument being used for circuits varying from 100 volts to many thousands of volts, with currents from a fraction of a thousandth of an ampere up to several thousand amperes. It is also of the greatest value in the measurement of very small phase angles, fractions of a minute being easily measured in certain types of test.

Readings are made by reflected light on a scale 3 metres away. The scale, horseshoe-shaped, is about 5 metres long. The voltage used on the needle is generally 100, and up to 3 volts between the quadrants. The standard value is 2 volts, represented by 2000 divisions on the scale. To obtain the desired sensitivity the distance between the quadrants is made only 2 mm. Special attention has to be paid to the design and construction of the needle in these circumstances. The difference of potential, proportional to and in phase with the current, is obtained from the ends

[1] Orlich, *Science Abstracts*, 1904, vii. A, No. 159; 1909, xii. A, No. 694; 1909, xii. B, No. 632; Paterson, Rayner, and Kinnes, *I.E.E.*, 1913, li. 294.

of a resistance through which the current passes.

The resistances for the smaller currents, 0·1 ampere to 20 amperes, are made of constantan wire No. 24, double silk covered, twinned and silk covered again. A length corresponding to 2 ohms single, *i.e.* 4 ohms of twin wire, is cut off and forms a very non-inductive go-and-return resistance, capable of carrying half an ampere. The two wires at one end are soldered to the terminal blocks, and the other ends are soldered together, and are connected to another terminal block to form the "mid-point of the current resistance" which is required for making connection to the potential circuit.

For larger currents than 20 to 40 amperes the amount of heat to be dissipated becomes inconveniently great, and a tubular type of resistance is used for currents 50 to 2000 amperes.[1] These are made of manganin tube, the diameter and thickness being chosen so that about 50 cm. length is required to produce a two-volt difference of potential at the rated currents of 50, 100, 200, 500, 1000, and 2000 amperes. The tubes are enamelled inside to prevent corrosion, and are cooled by passing tap water through them; the conductivity of the water is negligibly small compared with that of the tube. A special contrivance is adopted to make the resistances as non-inductive as practicable. The potential leads from the ends of the tube are carried to the centre along a thin metal sheath wrapped on the tube and separated from it by a thin layer of insulation. By this means the effect of the current in the tube in producing an inductive E.M.F. in the potential circuit is reduced to a small fraction of what it would otherwise be, especially in the case of resistances for the larger currents. Except in special cases, where accurate quadrature measurements are involved, the induction error is negligible.

When voltages higher than 100 have to be dealt with, the needle of the wattmeter and of the electrostatic voltmeter, which is generally used in parallel with it, are connected to a potential dividing resistances. These are woven wire resistances as used in the Duddell-Mather wattmeter capable of carrying 0·05 ampere. The connections are arranged so that over one of the sections there is 100 volts drop when the circuit voltage is applied to a suitable part of the resistance.

Some of these resistances are in 5 sections of 20,000 ohms each; each section is capable of carrying 1000 volts. One of the end sections is subdivided into ten subsections, so that each will have 100 volts on it.[2] From the end one connections are made to the wattmeter and voltmeter. Such a method of division permits of a fraction equal to 100 volts being obtained when the supply voltage varies from 100 to 5000 volts in steps of 100 volts. Many other ratios can be obtained when required. For high voltages further similar resistances are put in series. The capacity current required by the instruments is negligible at ordinary frequencies (*Fig.* 21).

In all work of this nature it is essential that the instruments should be at earth potential. It is therefore arranged that the end of the potential dividing resistance and the middle point of the current resistance are connected to earth.

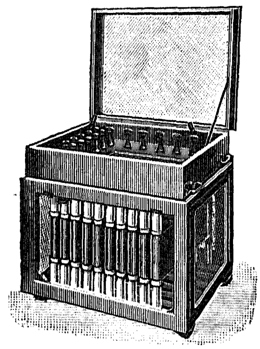


FIG. 21.

One of the main uses of the electrostatic wattmeter is for investigating the errors of other wattmeters and of watt hour meters. For this work an "artificial load" is used. The voltage required is generated by one alternator, a step-up transformer being used if required. The current is generated by a second alternator, generally through a step-down transformer if more than a few amperes are required. The connections are shown in the diagram (*Fig.* 22).

The electrostatic wattmeter is also very valuable for measuring small voltages if a voltage of the same frequency and phase can be applied to the needle, and also for measuring departures from the quadrature of two voltages by similar methods.

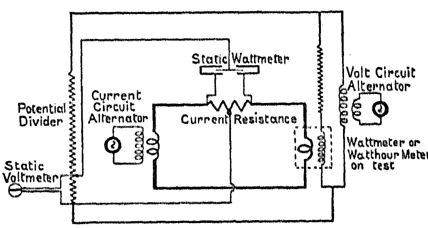


FIG. 22.

§ (22) ELECTROSTATIC WATTMETER ON A REAL LOAD OPERATING AT FULL-CIRCUIT VOLTAGE.—The consideration of the electrometer as a wattmeter has been confined to the case where the voltage between the needle and the average of the two quadrants has been kept constant, as can be done on an *artificial* load

[1] *J.I.E.E.*, 1908-9, xlii. 455. See also "Inductance, Measurement of," § (47).
[2] *J.I.E.E.*, 1913, li. 308.

in which the voltage and current are supplied from separate sources, either from separate machines or by the interposition of a transformer which provides the requisite insulation between the two circuits.

When used to measure the power on a real load the readings are modified by the effect of the power expended in the resistance R used for obtaining the necessary potential difference across the quadrants.[1] The power absorbed in the load is equal to the indication of the wattmeter less half the power ($\frac{1}{2}RI^2$) expended in the resistance R (*Fig.* 23).

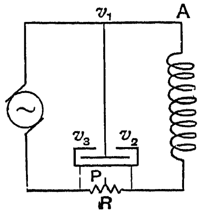


FIG. 23.

Let v_1 be the needle potential, v_2 and v_3 the quadrant potentials, then the force on the needle due to one set of quadrants is $(v_1-v_2)^2$, and to the other $(v_1-v_3)^2$. The resulting torque and deflection will be proportional to

$$(v_1-v_3)^2-(v_1-v_2)^2=2v_1v_2-2v_1v_3+v_3^2-v_2^2,$$

which may be written

$$2(v_2-v_3)\left(v_1-\frac{v_2+v_3}{2}\right).$$

If P is the middle point of the resistance R its potential is $(v_2+v_3)/2$, and $v_1-(v_2+v_3)/2$ is the difference of potential between A and P, v_2-v_3 is the voltage across R; and if R is noninductive $v_2-v_3=Ri$.

The instantaneous torque is proportional to

$$Ri\left(v_1-\frac{v_2+v_3}{2}\right),$$

i.e. to R × the watts expended in the circuit between A and P.

If I is the effective value of the current and W the watts in the circuit, then the deflection of the instrument is proportionate to $R(W+\frac{1}{2}RI^2)$, and $\frac{1}{2}RI^2$ must be subtracted from the apparent power measured by the instrument. When the power factor is low the correction may amount to a large fraction of the apparent power.

§ (23) ELECTROSTATIC WATTMETER ON A REAL LOAD OPERATING AT LESS THAN THE CIRCUIT VOLTAGE.[2]—When the voltage of the circuit is higher than that for which the instrument is designed a suitable fraction of the voltage may be used by exciting the needle from a potential divider, which often takes the form of an accurately divided high resistance capable of withstanding several hundred or thousand volts, the needle being operated at 100 or 200 volts. Let v_1' be the needle voltage, and N the multiplying power of the potential divider (*Fig.* 24).

$$N=\frac{R_1+R_2}{R_2}=\frac{v_1-v_3}{v_1'-v_3},$$

$$\therefore v_1'-v_3=\frac{1}{N}(v_1-v_3).$$

As in the case of a real load without a potential divider, the instantaneous torque is proportional to

$$=(v_2-v_3)\left(v_1'-\frac{v_2+v_3}{2}\right)$$

$$=Ri\left(v_1'-v_3-\frac{v_2-v_3}{2}\right)$$

$$=Ri\left(\frac{1}{N}(v_1-v_3)-\frac{v_2-v_3}{2}\right)$$

$$=\frac{R}{N}i(v_1-v_3)-\frac{R^2i^2}{2}.$$

If w is the value of the instantaneous power in the load

$$i(v_1-v_3)=w+Ri^2.$$

The instantaneous torque is proportional to

$$\frac{R}{N}(w+Ri^2)-\frac{R^2i^2}{2};$$

the mean torque

$$k\theta=\frac{R}{N}(W+RI^2)-\frac{R^2I^2}{2},$$

$$W+RI^2=\frac{N}{R}k\theta+\frac{NRI^2}{2},$$

$$W=\frac{N}{R}k\theta+\frac{N-2}{2}RI^2.$$

The value of $(N/R)k\theta$ is preferably obtained by calibrating the instrument on a real load with a suitable value of R, the needle being excited with the same voltage as that obtained from the potential divider when the instrument is in use.

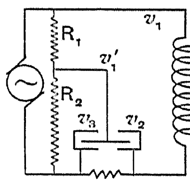


FIG 24.

If the power factor be low, the value of $RI^2(N-2)/2$ may be a large proportion of the total power indicated by the instrument, and to obtain the difference accurately very precise measurements of the current are necessary.

A special and very valuable case occurs when $R_1=R_2$, so that $N=2$ when the correction vanishes, and the necessity for the measurement of RI^2 disappears.

If R_1 is greater than R_2, so that $N>2$ and $(N-2)/2$ is positive, and if the power factor is low, the value of $[(N-2)/2]RI^2$ may exceed the power in the load. In such circumstances the wattmeter deflection will be negative, and

[1] Russell, *Alternating Currents*, 1904, i. 189.
[2] Russell, i. 195.

again an accurate measurement of I becomes necessary, especially for relatively large values of I, as the true power may be only a tenth of that indicated by the wattmeter deflection.

§ (24) ELECTROSTATIC WATTMETER USED FOR MEASUREMENTS OF POWER LOSS IN INSULATING MATERIAL.[1]—The power absorbed by insulating materials when subjected to alternating electric stress is generally greater than would be indicated by the measurement of the current passing under the influence of similar continuous potentials.

In the case of good insulators, such as mica and paraffined paper, the actual power lost under alternating stress may be less than one thousandth of the amount indicated by the product of the voltage and current, but the loss will be many times that calculated from measurements of resistances as measured by direct current.

In less perfect insulators the effect is often still greater and the power expended in the insulation may be sufficient to raise its temperature appreciably. In very many insulating materials used in electrical plant, such as varnished papers and cloths, the effect of increase of temperature is greatly to increase the loss. The heating may therefore become cumulative, in which case failure will occur, often indicated by a rapid rise of temperature, which may supervene after several minutes or hours of a much slower initial rise. A wattmeter will indicate the whole history of the process, and will, under suitable conditions, give a definite proof of inevitable failure, often a considerable time before such failure occurs and, in favourable circumstances, before permanent damage has been caused.[2]

Mr. G. L. Addenbrooke initiated the electrostatic wattmeter method for investigations of this nature, and other workers have developed it. For testing insulation in this manner the electromagnetic type of wattmeter is not often suitable, as the currents are very small. It can, however, be used in special cases, such as for testing long lengths of cable where a large amount of insulating material is tested at once.

By the electrostatic method even a few square inches of insulating material may be sufficient to give a satisfactory measurement. By varying the value of the resistance R, which can be done while the insulation is under test, a wide range of power and current can be dealt with. In this manner it is possible to measure a few hundredths of a watt at 10,000 volts.

The current is measured by an electrostatic voltmeter. If only a low voltage on R is admissible, this can be an instrument reading up to 2 volts. On circuits of several thousand volts a 10-, 60-, or 100-volt instrument may be suitable and is considerably quicker in action.

For work of this nature on insulating materials at low-power factors there is very great advantage in using the electrometer at half the circuit voltage in order to eliminate the correction $RI^2(N-2)/2$.

In the case of voltages of the order of 10,000, 5000 may be used on the instrument with great increase in torque above that possible with 100 volts. Clearances must, however, be considerably increased, and as a result accuracy of manufacture and adjustment becomes of much less importance. When the voltage is about 7000, using a clearance of 2 cm. between the quadrants and a needle with a well-rounded edge, the electric stress begins to ionise the air. The effect becomes appreciable in measurements on condensers at low-power factors.

Above such voltages the whole instrument can be put in mineral oil, which it is possible to refine so as to have a power factor of considerably less than 1 in 1000. By this means 60,000 volts has been used by C. E. Skinner of Pittsburg, between needle and quadrants, and instruments of this type promise to be of great value in the investigation of many technical problems.

§ (25) ERROR DUE TO CAPACITY CURRENTS.—It is of first importance when dealing with circuits of very low-power factor that the potential divider should be true not only in ratio but, what is more important, in phase when the needle is connected to it.

The needle, especially at high voltages, may take an appreciable capacity current, and it is difficult to prevent this upsetting of the phase of the point on the potential divider feeding it. By using a relatively large current in the potential divider the effect may be reduced; but this cannot be carried far without considerable cost and expenditure of a very considerable power.

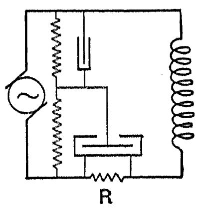


FIG. 25.

A better method would be to put a condenser across the upper part of the potential divider so adjusted as to provide a capacity current approximately equal to that of the wattmeter. This condenser would probably consist of plates in oil (*Fig* 25).

A more convenient way of avoiding such difficulties is to connect the needle to the middle point of the secondary winding of

[1] Miles Walker, *Am. I.E.E.*, 1902, xix. 1035; Rayner, *I.E.E.*, 1912, xlix. 3; Skinner, *J. Franklin Institute*, 1917, p. 667.

[2] Rayner, *I.E.E.*, 1912, xlix. 3.

transformer used to generate the high voltage (*Fig.* 26).

The resistance of the transformer winding, which will be probably of the order of 3000 ohms in the case of a transformer for 100,000 volts, is far less than any practicable external potential divider of the resistance type.

For instance, if a resistance as low as 1 megohm were used as a potential divider for 100,000 volts it would absorb 10 kilowatts.

The phase error produced would be about 300 times that when using the winding of the transformer itself as the potential divider.

At high voltages the capacity current taken by the quadrants which are connected to the

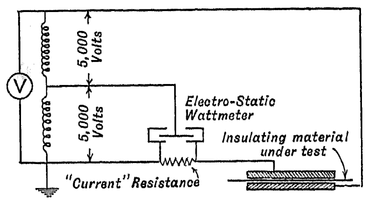


Fig. 26.

load side of the resistance R becomes comparable with the load current itself when testing small amounts of insulating materials. In such cases high values of R would be used and an appreciable deflection may be obtained, even when the high voltage connection to the load is interrupted. Such a deflection will vary as R. It may be allowed for by carrying out the test with different values of R and extrapolating[1] to the value indicated when R = zero.

IV. Power Measurements

§ (26) In Alternating Current Circuits.—Measurements of power in direct current systems are commonly made by ammeters and voltmeters, or, where higher accuracy is required, by a potentiometer.[2]

For alternating power measurements a wattmeter must be used, unless the phase relation between the voltage and current is known.

For commercial work indicating wattmeters are commonly used, with or without instrument transformers according to the voltage and current to be measured. For laboratory and research purposes other types are available.[3]

§ (27) Single-phase Measurements.—The voltage circuit of the wattmeter is supplied direct from the circuit to be measured, or from a step-down transformer in the case of high voltages. In the case of an electrostatic instrument a potential dividing resistance is commonly used. The current passes through the instrument direct, or for currents above 20 to 50 amperes, through a current transformer, which supplies a nominal full-load secondary current of 5 amperes, for which the indicating instrument is wound. In the case of electrostatic wattmeters a resistance is used in the current circuit of a value to give about 1 to 2 volts difference of potential on the quadrants.

Polyphase Power Measurements

§ (28) Two-phase Measurements.—A two-phase circuit is treated as two separate single-phase circuits. A wattmeter is placed in each circuit and the sum of the indications of the two gives the power. For commercial circuits instrument transformers are generally used.

Certain makes of both the portable type (such as some patterns by the Weston Co.) and of the laboratory type (such as the Drysdale instrument) are made with two wattmeter elements operating on the same axis, one above the other. One instrument only is then required, the total power being given by a single reading. Such instruments may be also used on three-phase circuits.

§ (29) Three-phase Measurements.—The measurement of three-phase power is commonly carried out by the two-wattmeter method.

The current circuits of two wattmeters form part of the circuits of two of the three con-

Fig. 27.

ductors while the potential coils are connected to the third conductor (*Fig.* 27).

The method is a special case of the following theorem enunciated by Blondel.[4]

If power be supplied to any system through n conductors in which are severally connected the current coils of n wattmeters, one terminal of the pressure circuit of each wattmeter being connected to its current circuit, and all the free terminals being connected together, then the total power supplied to the system is the algebraic sum of the n wattmeter readings.

If one of the conductors is chosen as the

[1] *Journal I.E.E.*, 1912, xlix. 45.
[2] See "Potentiometer, System of Measurement."
[3] See "Meters for Electrical Supply."

[4] "Measurements of the Energy of Polyphase Currents," A. Blondel, *Proc. International Elec. Congress* (Chicago), 1893, p. 112.

common point, so that the two ends of the potential circuit of this particular wattmeter are connected to the same point, and therefore no current passes through the coil, this wattmeter will read zero, and may therefore be removed. It follows that $(n-1)$ wattmeters will measure the power supplied through n conductors to any type of load, provided that the current in each $(n-1)$ of the conductors traverses the current coil of one of the $(n-1)$ wattmeters, while one end of the potential circuit of each wattmeter is connected to the nth conductor, and the other end to the conductor connected to the current circuit of that wattmeter.

§ (30).—The theory of the measurement by the two-wattmeter method of the power in a three-phase circuit may be deduced as follows.

The general case may be considered as corresponding to a combined star and mesh system, which reduces to either a simple star or mesh circuit by considering the appropriate currents equal to zero [1] (*Fig.* 28).

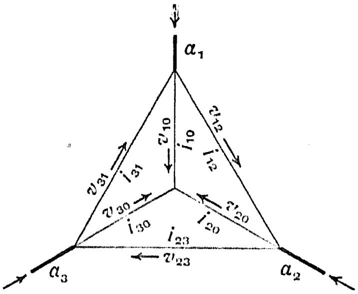


FIG. 28.

Taking instantaneous values for the voltages v and the currents i and a in the different conductors, and the positive directions those shown by the arrows,

$$a_1=i_{12}-i_{31}+i_{10},$$
$$a_2=i_{23}-i_{12}+i_{20},$$
$$a_3=i_{31}-i_{23}+i_{30},$$

so $\quad a_1+a_2+a_3=i_{10}+i_{20}+i_{30}=0.$

The power

$$p=v_{12}i_{12}+v_{23}i_{23}+v_{31}i_{31}+v_{10}i_{10}+v_{20}i_{20}+v_{30}i_{30}.$$

To prove the theorem it is necessary to eliminate one line current, say a_1, and the opposite voltage v_{23}.

$$v_{23}=-v_{31}-v_{12},$$

and $\quad i_{10}=-i_{20}-i_{30},$

so

$$p=v_{12}i_{12}+(-v_{31}-v_{12})i_{23}+v_{31}i_{31}+v_{10}(-i_{20}-i_{30})+v_{20}i_{20}+v_{30}i_{30},$$

$$p=v_{12}i_{12}+(-v_{31}-v_{12})i_{23}+v_{31}i_{31}+(v_{20}-v_{10})i_{20}+(v_{30}-v_{10})i_{30},$$

[1] Russell, *Alternating Currents*, i. 236.

and $\quad v_{10}-v_{20}=v_{12};$

or $\quad v_{20}-v_{10}=v_{21},$

and $\quad v_{30}-v_{10}=v_{31},$

so

$$p=v_{12}i_{12}+(-v_{31}-v_{12})i_{23}+v_{31}i_{31}+v_{21}i_{20}+v_{31}i_{30}.$$

and as $\quad v_{12}=-v_{21},$

$$p=v_{21}(i_{23}-i_{12}+i_{20})+v_{31}(i_{31}-i_{23}+i_{30})$$
$$=v_{21}\cdot a_2+v_{31}\cdot a_3.$$

That is, the instantaneous power is the sum of the products of the current a_2 and the voltage between a_2 and a_1, and of the current in a_3 and the voltage between a_3 and a_1. As a wattmeter measures the mean value of the instantaneous product of the current and voltage of the circuits connected to it, two instruments arranged as in *Fig.* 27 will measure the power whether the load be mesh or star or of any other configuration, and whether balanced or not.

A more general method of considering the theorem is as follows (*Laws*, p. 328).

Suppose a load system is fed by n wires carrying currents of instantaneous value i_1, i_2, i_3, i_n, and the potentials of these are v_1, v_2, v_3, . . . v_n; the instantaneous power is

$$p=i, v, +i_2v_2+i_3v_3+\ .\ .\ .\ +i_mv_n. \quad (1)$$

As a datum for reference of potential, take any point on the system. Let its potential be v_0; then the algebraic sum of the instantaneous currents being zero, we have

$$0=i_1+i_2+i_3+\ .\ .\ +i_n,$$

$$\therefore 0=i_1v_0+i_2v_0+i_3v_0+\ .\ .\ .\ +i_nv_0 \qquad (2)$$

Subtracting (2) from (1)

$$p=i_1(v_1-v_0)+i_2(v_2-v_0)+i_3(v_3-v_0)+\ .\ .\ .\ +i_n(v_n-v_0).$$

The mean value of $i_1(v-v_0)$ will be the indication of a wattmeter through whose current coil i_1 passes, and whose pressure coil is energised by the difference of potential (v_1-v_0). The total power is measured by n such wattmeters. If v_0 is made the same as one of the line potentials v_1, or v_2, or v_3, etc., that wattmeter will read zero and may be removed as mentioned previously. The potential v_0 may also be the common point of all the potential circuits of the n wattmeters, disconnected from any other part of the circuit. The sum of the readings is independent of the values of the resistances of the potential circuits, which may be all different provided each wattmeter is adjusted to read correctly when used alone on a simple single-phase circuit.

§ (31) Phase Relations in Three-phase Circuits.—The phase relation of current and voltage in the wattmeter circuits when measuring three-phase power by the two wattmeter method differ by thirty degrees, when the load is non-inductive.

In one instrument the voltage leads in

front of the current and in the other lags behind it.

A, B, and C (*Fig.* 29) represent the relative directions of the phase angles of the currents, and D, E, F of the voltages. Let the current coils of the wattmeters be placed in B and C. If the rotation of the vectors is counter clock-wise it will be seen that the current B leads on the voltage E and the current C lags behind the voltage in F. The angles are 30° in each case, the power is V.I. cos 30° and V.I. cos (−30°) where V and A are root mean square values. The total power is $W_B + W_C = 2$ V.I. $\sqrt{3}/2 =$ V.I. $\sqrt{3} =$ V.I. × 1·732.

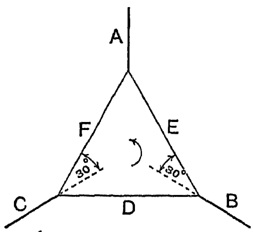


FIG. 29.

If the load be inductive the current vectors lag behind the voltage vectors.

If the lag be 30° the power on the B side wattmeter W_B becomes

$$W_B = \text{V.I.} \cos(30° - 30°) \quad = \text{V.I.}$$

$$W_C = \text{V.I.} \cos(-30° - 30°) = \text{V.I.} \times 0{\cdot}5.$$

$$\text{Total power} = W_B + W_C \quad = \text{V.I.} \times 1{\cdot}5.$$

The indication of wattmeter B has risen to a maximum; its current and voltage are in phase.

If the lag is increased to 60° we have

$$W_B = \text{V.I.} \cos(30° - 60°) \quad = \text{V.I.} \sqrt{3/2}.$$

$$W_C = \text{V.I.} \cos(-30° - 60°) = 0.$$

$$\text{Total power} = W_B + W_C \quad = \text{V.I.}\ {\cdot}866.$$

If the lag is increased to 90°, we have

$$W_B = \text{V.I.} \cos(30° - 90°) \quad = \text{V.I.} \times 0{\cdot}5.$$

$$W_C = \text{V.I.} \cos(-30° - 90°) = -\text{V.I.} \times 0{\cdot}5.$$

$$\text{Total power} = W_B + W_C \quad = 0.$$

The indications of wattmeter A have become negative, and in practice the current in it must be reversed and the reading subtracted. The numerical value of the indications of each wattmeter are the same as that of W_C at 30° lag.

§ (32).—The diagram (*Fig.* 30) shows the readings of the wattmeters on a balanced three-phase load, as the three-phase power factor varies, and can be used to determine the power factor from the measurement of the power by two wattmeters.

A. Power in the meter in which the current is in advance of the voltage.

B. Power in the meter in which the current lags behind the voltage.

The curves, which are sinusoidal, are continuous beyond the limits of the diagram, and are equally applicable to a "leading" load, being symmetrical about the vertical line representing zero angle of lag.

The power factor of a balanced three-phase circuit may be considered as the ratio of the actual power in the circuit to the value which would be obtained if the voltage and current vectors were rotated relatively till the power

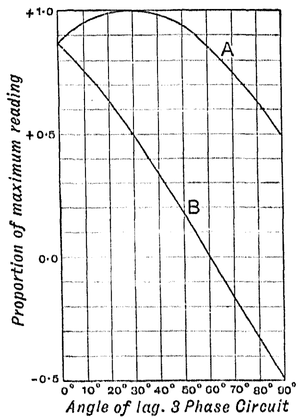


FIG. 30.

was a maximum, *i.e.* until the phase displacement was +30° in one circuit and −30° in the other, when measuring by the two wattmeter method.

§ (33).—The power factor may be determined from the ratio of the readings of the wattmeters.

It is represented by the ratio of

$$\frac{\cos(30° + \phi) + \cos(-30° + \phi)}{\cos 30° + \cos(-30°)}.$$

The relation over the useful range is shown in *Fig.* 31, where the abscissa is the ratio of the wattmeter readings.

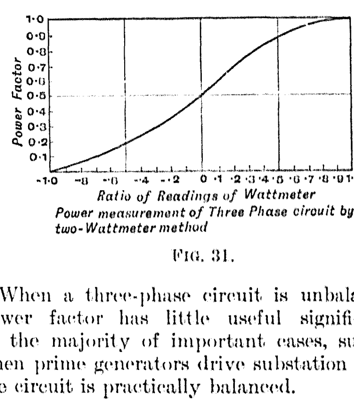


FIG. 31.

When a three-phase circuit is unbalanced, power factor has little useful significance. In the majority of important cases, such as when prime generators drive substation plant, the circuit is practically balanced.

V. Alternating Current Energy Meters

§ (34).—Alternating current integrating power meters of three types have been in general use.

(i.) Commutator meters.
(ii.) Clock meters.
(iii.) Induction meters.

(i.) *Commutator Meters.*—Commutator meters are now very little used, being largely displaced by induction meters. They are very similar to the direct current energy meters,[1] in which an armature with a commutator revolves in a field produced by fixed coils traversed by the current to be measured. The current in the armature is proportional to the voltage of the circuit. Little or no iron is used in the magnetic circuit, so that the fluxes are proportional to the currents in the two circuits and the torque on the armature is proportional to the product of the voltage and current. A retarding torque proportional to the speed is provided by a magnetic brake as in induction meters. Compensation for the phase displacement of the flux due to the inductance of the pressure winding is necessary for accuracy when the power factor is low.

§ (35) (ii.) *Clock Meters.*—The only type in common use is the Aron meter. It depends upon the change of rate of a pendulum which is acted upon by electromagnetic forces, either opposing or assisting the force of gravity. The pendulum has a horizontal coil of fine wire at its lower end, through which a current passes which is proportional to the voltage of the circuit. The current coil is fixed, also horizontally, just below the pendulum. In practice two pendulums are used, in one of which the electromagnetic force assists gravity and in the other it opposes. One pendulum therefore gains and the other loses when compared with the rate when no current is flowing. A mechanical differential gear integrates the difference in rate, which is proportional to the power; the energy is registered on dials in the usual manner. In order to allow for slight errors between the two pendulums, an automatic switch is provided which reverses the currents in the voltage coils periodically, and at the same time alters the train of mechanism so that the differential motion still causes the dials to move in the same direction as before.

The motive power is provided by a spring which is wound up electrically at short intervals. One advantage of this type of meter is that the smallest load will be integrated, whereas other types of meters are not expected to register if the load is appreciably under one per cent of its full value. A disadvantage of the meter is the time required to check its accuracy. There is no spindle rotating with a constant velocity, by the speed of which the accuracy of the meter can be quickly checked even at low loads. This is an essential part of other meters.

§ (36) (iii.) *Induction Watt-hour Meter.*—The induction alternating energy meter[2] is an integrating power meter which is operated by two or more alternating magnetic fields. These fields produce eddy currents in a disc or cylinder of aluminium or copper. The interaction of the currents and the fields produce a resultant tangential force on the disc, tending to rotate it. By suitable design and adjustment the torque may be made closely proportional to the mean value of the power to be measured. A retarding torque proportional to the velocity is provided by a permanent magnet or magnets which produce eddy currents in another part of the same disc. The result is that the speed of rotation is proportional to the power.

This type of meter is manufactured by many firms, the designs differing only in detail. It is used in very large numbers for the measurement of widely different amounts of power, such as several thousand kilowatts in many generating stations to a few watts in private houses.

Its general adoption is largely due to its mechanical simplicity. The operating part is a light disc, usually of aluminium, fixed on an axle, the whole weighing 10 to 30 grams. It is moved by the action of currents induced in it. There are no rubbing contacts to lead current into and out of the moving part. Its simplicity is ideal, and enables a robust meter to be made, which is not likely to be easily deranged. The actual paths of the currents produced in the disc and the interaction of these with the operating magnetic fluxes are among the most complicated matters in electrical engineering.

§ (37) Theory of Induction Meters.—An outline of the theory is as follows:

Let M (*Fig.* 32) be a C-shaped magnet with a small air gap in which a disc of aluminium can rotate without touching the magnet faces. Suppose the magnet is wound with a coil traversed by an alternating current I_1. There will be an alternating flux cutting the disc, in which eddy currents will be generated, represented by the dotted lines in diagram B, as shown by the arrows.

If S_1 and S_1' represent two other magnets similar to one another wound with another circuit carrying a current I_2 so that the flux at any moment is in the same direction in the parts vertically opposite to one another, the positions of which are shown by full circles in diagram B, it will be seen that, as the

[1] See "Meters for D.C. Electricity," Part III.

[2] H. G. Solomon, *Electricity Meters* (Griffin & Co.); *Laws*, p. 473.

circular eddy currents generated by the flux produced by M are in opposite directions where they cut these circles, and the fluxes due to S_1 and S_1' are also in opposite directions at right angles to the disc where they act on

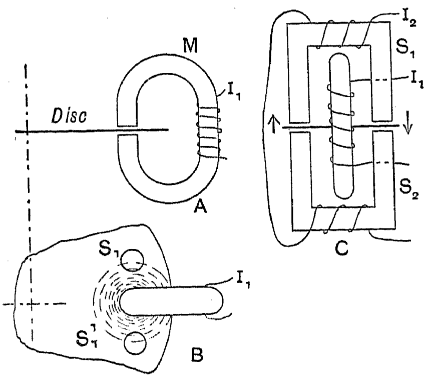


FIG. 32.

these currents, the two systems of fluxes produced by S_1 and S_1' will apply a tangential force to the disc in the same direction, and so tend to make it rotate.

Now the two fluxes due to the current I_2 in S_1 and S_1' also produce eddy currents which are acted on by the flux produced by M, resulting in a second system of forces acting on the disc.

These eddy currents are shown as dotted circles in *Fig.* 33.

The circular currents in the disc go in opposite directions at any instant, so that in the part of their path where they come within the sphere of action of M their radial direction is the same. The two fluxes therefore act in the same direction in producing torque on the disc.

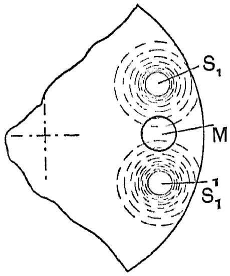


FIG. 33.

This torque, however, is in the *opposite* direction to that produced by the flux due to M just described, which generates the eddy currents in the disc acted on by the flux due to S_1, S_1'. The resultant is the difference of the two torques.

§ (38) PHASE RELATION OF FLUXES AND CURRENTS.—That this is so can be seen by a consideration of the phase relations of the quantities in wattmeters of this type (*Fig.* 34).

The magnet M is commonly wound with several thousand turns of fine copper wire and is connected across the pressure V_1 to be measured. The circuit being highly inductive, the current I_1 and the flux Φ_1 it produces will lag nearly 90°. Actually the angle of the useful flux is made exactly 90° by a special adjustment. The curves *a* and *b* show the relative relation of V_1 and Φ_1.

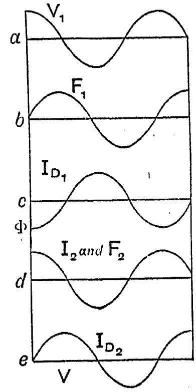


FIG. 34.

Φ_1 produces an E.M.F. E_1 in the disc, lagging 90° behind F_1. This produces a current in the disc I_{D_1} lagging somewhat behind E_1, on account of the self-inductance of the eddy currents.

Neglecting this lag, the phase of the eddy currents I_{D_1} is shown in *c*. The currents are acted on by the flux due to S_1, S_1'. In practice these are supplied with a current proportional and in phase with that in the circuit to be measured.

If the power factor is unity the current I_2 and the flux Φ_2 which it produces are in phase with V_1, as shown at *d*. The interaction of F_2 and I_{D_1} will therefore correspond to their being 180° out of phase, *i.e.* the disc will tend to rotate in the opposite direction to what it would if they were in phase. Now the current in the disc I_{D_2} produced by Φ_2 due to the current I_2 in S_1 and S_2 is in phase with the flux Φ_1. For the phase of I_{D_2} is shown at *e* neglecting the eddy current inductance error as before, 90° behind Φ_2. It is therefore in phase with the flux Φ_1.

The two torque systems are therefore in opposition, and the resulting torque is the difference.

The actual eddy currents in the disc are the vector resultant of the two systems of currents which are in quadrature.

§ (39) QUADRATURE ADJUSTMENT.—The important adjustment is that by which the useful fluxes produced by the two magnet systems are caused to have exact time quadrature when the power factor is unity.

The flux produced by the magnet M is a little short of 90° behind the voltage. An additional flux must therefore be produced in the air gap, the vector resultant of which with that produced by M must have the 90° lag. This additional flux is often produced by the addition of a coil of a few turns on the end of the magnet. The ends of the coil are connected to the end of a variable low resistance. The current and flux in this coil is not very different from 180° lag, so that the resultant with that of the useful portion of the primary

flux may be made to have the desired phase angle of 90°. This adjustment has to be made by trial and error, testing the meter at various power factors at approximately constant current and voltage.

Another method of obtaining quadrature between the two operating fluxes is to compensate the want of quadrature between flux and voltage in the pressure coil by splitting up the series (current) winding into two parallel circuits of different self-inductance. By adjusting the relative amounts of current in the two the phase in one of them can be given a value different from that of the total current which will compensate for the want of quadrature in the pressure windings.

§ (40) Light-load Adjustment.—To compensate for friction at light loads, so as to prevent the meter running unduly slow, the end of the shunt magnet is often surrounded by a closed circuit of aluminium or copper nearly touching the disc. By moving this at right angles to the line joining it to the centre of the disc and keeping it parallel to its surface, the currents in it will cause a time dissymmetry of the useful flux, and so produce a travelling field tending to rotate the disc.

To prevent a slow continuous creep of the disc, if the light-load adjustment is somewhat overdone, a few iron filings are sometimes attached to the disc at one spot, which cause it to stop when they come under the permanent brake magnet. Sometimes a short length of iron wire is fixed to the disc near the axle. This is bent so as to come sufficiently under the influence of the permanent magnet at the nearest point in its rotation to hold the disc still if there is no load on the meter.

§ (41) Full-load Adjustment of Induction Meters.—This is often done by varying the distance from the centre of rotation of the disc at which the permanent brake magnet is fixed, in the gap of which the disc rotates. The permanent magnet is generally fixed opposite the driving magnet so as not to be influenced by an accidental short-circuit current passing through the current coils.

Another method is to vary the effect of a magnetic shunt on the permanent magnet.

The alternating magnetic fluxes from the driving magnets also produce a material braking effect.

§ (42) Accuracy of Induction Meters.—Induction watt-hour meters have a calibration curve of the type shown in *Fig.* 35.

If the meter runs correctly at about 0·1 of full load and 0·5 per cent slow at full load, it is likely to run 0·5 per cent to 1·5 per cent fast at about half load.

The drop in speed at full load is largely due to the increase of series flux cutting the disc as the load increases. This increases the braking action and slows the meter. It may also be due to the falling off in permeability of the series magnets at the higher inductions and therefore of the flux they produce.

(i.) *Power Factor.*—The power-factor characteristic obtained by keeping the full-load

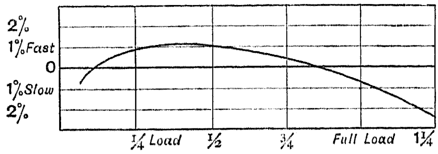


Fig. 35.

current constant and varying the real power, is practically a straight line, and may be given any desired inclination by varying the quadrature adjustment.

(ii.) *Starting Current.*—A meter may be considered satisfactory if it starts with less than 1 per cent of full load.

(iii.) *Speed.*—The full-load speed of meters of this type is commonly 40 to 50 revolutions a minute.

There is theoretically an error due to the speed of rotation of the disc. This enters as a ratio of the angular velocity of the disc to the electrical-phase velocity. The ratio is about 1 or 2 per cent. The effect is further reduced in that the alternating fluxes are kept small compared with that of the permanent magnet.

(iv.) *Counting Mechanism.*—The disc spindle is connected with the dial mechanism by spur or worm gearing. The dials are commonly geared to one another as simply as possible, so that one rotates in the opposite direction to the next, as is commonly done in gas-meters.

The more complicated mechanism required in a dial with cyclometer figures is sometimes found to result in sufficient friction to reduce the speed of the disc, especially at low loads, when a change over from one dial to another is being carried out. In the simplest forms of dial mechanism the friction ought to be negligible.

(v.) *Effect of Temperature on the Torque.*—The driving torque, being due to eddy currents which are dependent on the conductivity of the material of the disc, depends upon the temperature, since the disc is commonly made of aluminium, which varies in resistance about 0·4 per cent per degree. The torque diminishes with rising temperature. Since the braking is also due to similar eddy currents, the retarding torque varies in a manner to compensate for the alteration of driving torque.

This type of meter has a great advantage in this respect over practically all types of

continuous-current meters, with similar braking arrangements. In such continuous-current meters the braking torque depends on the temperature, while the driving torque is not easily given a compensating correction.

(vi.) *Frequency Error.*—The main error caused by change of frequency is due to departure from exact quadrature relation of the shunt and series fluxes. The change is not serious in practice as commercial alternating supply systems keep the frequency very constant.

(vii.) *Potential Error.*—There is a small error due to small changes of voltage. Such a change is due to the useful flux not being exactly proportional to the voltage, to possible slight departure of quadrature adjustment, and to alteration in braking force by change of the shunt flux.

(viii.) *Material of Disc.*—Aluminium is preferable to copper on the score of lightness. It has also been shown that the superior conductivity of copper enhances the effect of self-induction in the eddy currents, and that aluminium is preferable also in this respect. Discs are commonly pressed in order to make the metal "flow" slightly. Such treatment equalises any strains in the metal produced by rolling it into sheet form, and prevents warping. Some makers also corrugate them slightly to give them additional stiffness.

§ (43) Size of Meters.—Meters are commonly made for direct connection to circuits when the voltage is from 100 to 500 volts. The full-load current passing through the meter is generally between $2\frac{1}{2}$ and 50 amperes.

Use of Instrument Transformers.—Above these values the meters are usually supplied through special transformers, commonly known as instrument transformers. Potential transformers are universally made to give 100 to 110 volts on the secondary circuit to which the pressure coil of the meter is connected, whatever the primary voltage may be. Current transformers are likewise made with a 5-ampere circuit for the secondary, and meters are made for 5 amperes for full load to correspond.

In the case of high voltages there is the immense advantage that, by thorough insulation between primary and secondary of the transformers, the meter or other instruments in the secondary circuits are made safe to handle.

A watt-hour meter (or wattmeter) with a range of 100 to 110 volts and 5 amperes thus becomes a universal instrument. Used alone it will measure a load up to 0·5 kilowatt. Connected to a suitable potential transformer for 20,000 volts, and a current transformer for 1000 amperes, it will measure 20,000 kilowatts.

The instrument [1] transformers are therefore very important links in the measurement of power, and much attention has been given to their design and the measurement of their characteristics.

[1] See "Instrument Transformers."

§ (44) Two- and Three-phase Induction Watt-hour Meters.—The measurement of energy in a two-phase circuit or in a three-phase circuit by the two-wattmeter method requires two meters. This is commonly done by a double deck arrangement, driving a spindle and counting mechanism by means of two meter elements one above the other, the discs being fixed to the spindle a few inches apart. The same meter may be used either for two-phase or three-phase measurements, but unless it is accurately adjusted for power-factor error it will not register exactly the same for equal amounts of energy in the two cases on account of the phase relation of potential and current vectors being different in the two cases.

§ (45) Action of Braking Magnets.—In nearly all types of commercial integrating electricity meters for both alternating and continuous currents the main retarding force is produced by the action of a permanent magnet on a rotating disc or drum which moves in its air-gap. The disc or drum is commonly made of aluminium or copper.

This principle of applying a retarding force to a moving mass, by means of the interaction of a magnetic field and the current which it produces in a conductor cutting the field, is largely used for damping the oscillations of many types of electrical and other instruments.

If the dotted circle (*Fig.* 36) represents the section of the magnet at the air-gap, eddy currents will be produced in the part of the disc as it comes into the field of the magnet, as shown at A.

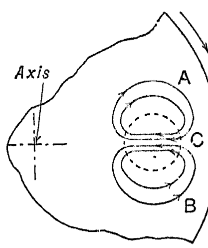


Fig. 36.

As the disc leaves the region of the magnet a similar system of eddy currents is generated at B. These will circulate in the opposite direction, so that the current C, produced by both systems in the part between the poles of the magnet, will be the sum of the two. The action of the field of the magnet on this current will be in the direction tending to oppose the rotation of the disc.

The braking force is proportional to the product of current and field, and, as the current is proportional to the velocity and the field is constant, the braking force is proportional to the velocity. When the conductivity and velocity are high the self-inductance of the currents in the disc may produce a slight distortion, and it has been

shown that aluminium may be preferable to copper in this respect as having a higher resistivity in addition to its lower density. Aluminium is practically always used in the case of induction meters.

Brake magnets are usually rectangular in cross-section, and for a given shape it is more advantageous to arrange the magnet as at E (*Fig.* 37) rather than F, as a greater torque for a given velocity is produced thereby.

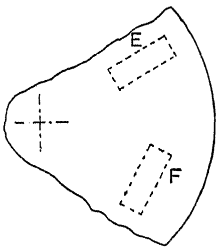


FIG. 37.

As the braking force varies with the linear velocity, the farther the magnet is placed from the centre of the disc the greater is its effect. Many meters have their rate adjusted by such alteration of position. This only holds up to a certain point, as the eddy currents are seriously interfered with when the magnet comes close to the edge of the disc.

VI. Phase Meters, Power-factor Meter, Synchroscopes

§ (46) Phase Meter or Power-factor Meter.—A phase meter is an instrument which indicates the phase angle between the potential and current in an alternating circuit.

As the cosine of the phase angle is equal to the power factor the instrument is commonly graduated in terms of power factor. They are generally termed power-factor meters.

(i.) *Power-factor Meters for Single-phase Circuits.*—The outline of the working of a phase meter is as follows. Two similar fine-wire coils A and B (*Fig.* 38) are fixed together

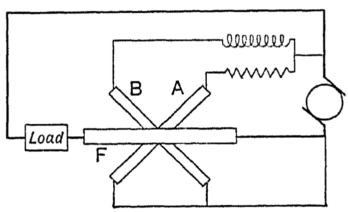


FIG. 38.

nearly at right angles, and are pivoted at their junction, and are placed within a fixed coil F which carries the load current, either directly or through a current transformer.

The coils A and B are fed from the source of supply with a large resistance in series with one (A), and a large inductance in series with the other (B). Suppose the ampere turns in each are the same. We have then two alternating fields, A and B, at right angles to one another, with nearly 90° phase displacement between them. These give rise to a rotating field which is perpendicular to the plane of A at the instant when the voltage of the circuit A is a maximum.

The forces between the fixed and moving coils may be deduced, as in *Fig.* 39, from the theorem of the equivalence of revolving and alternating magnetic fields.

An alternating magnetic field of maximum value 2H and period T may be regarded as equivalent to two equal fields, each of value H, revolving uniformly in opposite directions in the periodic time T. The angle which each revolving field makes with the direction of the alternating field is zero when the numerical value of their resultant is a maximum, *i.e.* 2H.

The rotating field due to the two moving coils may be considered as equivalent to a small magnet rotating in the periodic time T. Suppose this magnet has the direction MN at a given instant, and that at the same instant the field F_a due to the fixed coil is equivalent to the oppositely rotating fields F_1 and F_2, there will be mutual action between F_1 and

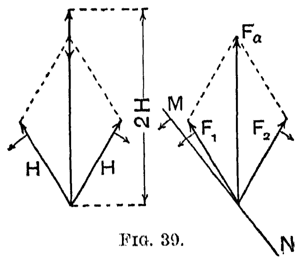


FIG. 39.

MN rotating in the same direction tending to pull them into coincidence.

There will be no resultant couple between MN and the oppositely rotating field F_2 when taken over a complete cycle. The moving coil therefore takes up a position such that the direction of F_1 becomes coincident with that of MN.

The planes of the coils A and F become coincident at unity power factor, and the angular departure from this position is a measure of the phase angle of the circuit.

The crossed coils must be free to turn through practically 360° to cover all possible power-factor conditions. They require the currents to be led into them by fine strips which have no appreciable mechanical control. To prevent them being damaged the motion of the pointer may be limited to one turn by a stop. In practice the power factor of a circuit generally keeps within a range of about 90°.

In the above it has been assumed that the currents in the moving coils are in quadrature. The lag in the inductive coil must necessarily, however, be less than 90°. If the coils are

at right angles an elliptical field will result; but if the coils are set at an angle equal to the phase difference between the currents in the coils a circular field of constant intensity is produced.

Such meters, depending on the inductance of a circuit to produce approximate quadrature, and requiring equality or constancy of ratio of ampere turns, can only be accurate at one frequency.

(ii.) *Power-factor Meter for Three-phase Currents.*—The power factor of a three-phase circuit has little practical meaning unless the circuit is balanced.

In this case use may be made of the current I_A in one line and the potential between this phase and each of the other two phases, v_1, v_2. The two potentials, v_1, v_2, have a time-phase difference of 60° (*Fig.* 40).

The moving coils are fixed at this angle and are fed from AB and AC with a high resistance in series.

We have therefore a rotating field produced by the two alternating fields which have a mechanical angular displacement equal to the time-phase displacement of the currents in the two circuits. The conditions are therefore the same as in the case of the single-phase meter.

The field produced by the fixed-current coil A may be regarded as equivalent to two fields

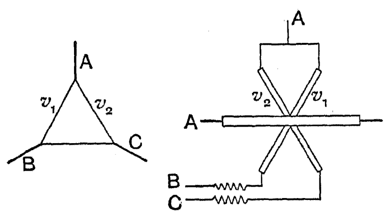


FIG. 40.

equal to $\frac{1}{2}I_A$, rotating in opposite directions; and the rotating field produced by the moving crossed coils "hangs on" to the one of two which rotates in the same direction. The equilibrium position of the coils is therefore a measure of the phase angle exactly as in the case of the meter for single-phase currents.

In the single-phase meter the rotating field has to be obtained by artificial phase displacement of the current in one of the moving coils. In the three-phase meter the phase displacement is ready to hand by using two of the line voltages. Such a three-phase instrument is therefore independent of frequency since no inductances are used. The rotating field may be produced by using three V-connected coils at 120° fed from each of the three phases of the system, instead of the two 60° coils mentioned above.

A balanced load has been assumed, and the phase measurement obtained by using the current in one line only. If the circuit is unbalanced, a sort of mean power factor can be obtained by taking the current in each of the other phases with suitably placed coils and obtaining an average effect. The result is hardly worth the complication in ordinary circumstances.

Power-factor meters and other similar apparatus are worked off instrument transformers as a general rule.

§ (47) GIFFORD POWER-FACTOR INDICATOR.—The Gifford indicator (*Fig.* 41) is not limited

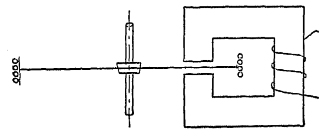


FIG. 41.

in its motion by the necessity of leading in the current to the moving part by metal strips.

The current is generated in the moving part by induction.

A C-shaped laminated electromagnet has a small air-gap in it in which a disc of mica fixed on the axle of the moving part revolves. Round the edge of the disc is wound a thin coil of wire which acts as the secondary circuit of a transformer, of which the winding on the magnet forms the primary. The operating forces are produced in a suitable winding connected in series with the secondary coil.

§ (48) SYNCHROSCOPES, SYNCHRONISERS.—These are apparatus for indicating when two alternating current machines are in phase, so that they can be connected in parallel without disturbance of the system.

The principles used are closely allied to those of the phase meter (*q.v.*). In the phase meter a definite direction in a system of movable crossed coils, rigidly connected together, takes up a position, as regards the current in a fixed third coil, which is equal to the phase angle of the circuit. Suppose the fixed third coil is supplied by a second alternator which is nearly in synchronism with the one supplying the moving coils. The current in the fixed coil will cause the moving coil system to rotate continuously at a speed corresponding to the difference in speed of the two machines, the phase difference passing through 360° in the time that one machine gains or loses one cycle on the other. The direction of rotation is a positive indication as to which is the faster machine. The moving coils of the synchroscope are fed through slip rings instead of fine metal strips as in the phase meter, so that it can rotate continuously. To increase the torque, an electromagnet may be used to supply the fixed field. The indicator is generally arranged so that the pointer points

vertically upwards when the machines are in phase and ready for paralleling.

§ (49) Synchronising Lamps.—Before synchroscopes were developed lamps fed by potential transformers from each machine were commonly used. Machines which require synchronising apparatus are practically always high voltage ones and require step-down transformers to supply measuring apparatus.

If the secondaries of the transformers are connected in series so as to add their voltages when the machines are in phase, the lamp fed by them glows brightly when the machines are in synchronism. If they are in opposition, the point of synchronism is indicated by darkness. Such a method gives no indication as to which machine is the faster.

A scheme of using three synchronising lamps on a three-phase circuit which indicates which machine is the faster is shown diagrammatically in *Fig.* 42.

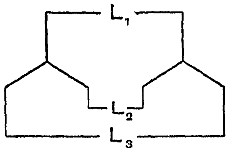


Fig. 42.

L_1, L_2, L_3 represent three lamps connected to the two machines.

At synchronism lamp L_1 is dark and lamps L_2, L_3 glow equally. As synchronism is departed from L_1 begins to glow; but a much more sensitive indicator is the difference in brightness between L_2 and L_3, since one increases and the other diminishes in brightness. The direction of the relative speed of the machines is indicated by whether L_2 or L_3 is growing in brightness as L_1 becomes dark.

§ (50) Weston Synchroscope.—The Weston synchroscope is a combination of a pointer instrument and a synchronising lamp.

The electrical circuit is of a dynamometer type controlled by a spring, which normally keeps the pointer vertical, pointing to synchronism. Suppose the fixed coil of the dynamometer is supplied by one machine through a condenser and the moving coil by another machine through a resistance. If the machines are in phase the two currents are in quadrature and there is no resultant torque on the moving coil. If the machines begin to go out of phase, the pointer will move in one direction or the other according to which machine goes the faster.

The instrument will also indicate synchronism when the machines are 180° out of phase.

To eliminate the ambiguity as regards true synchronism a synchronising lamp is added behind the pointer, which is itself behind a translucent screen and can only be seen when the lamp is glowing so as to throw a shadow of the pointer on the screen. The synchronising lamp only glows when the phase difference is within about 40° of zero. The pointer is out of sight at one end or other of its scale when the phase difference is considerable. The result is that the pointer is seen to move over the scale, which occupies some 90° in the region of synchronism, in such a direction as to indicate whether the incoming machine is fast or slow. The pointer goes backwards over the scale while the lamp is dark, so that the illusion of continuous rotation is produced as long as the machines run at different speeds. The pointer appears to rotate slower and slower as synchronism is approached, and finally stands vertical, fully illuminated, when synchronism is reached.

The instrument avoids the use of rubbing contacts, which have to be of very high quality to prevent appreciable friction and possible errors, which might be a very serious matter in paralleling modern high speed turbo-alternators.

VII. Frequency Meters

§ (51) Hartmann Kämpf Frequency Meter.—This is an instrument for indicating the number of oscillations per second of the current or voltage in an electric circuit. The word wavemeter is commonly used in the case of radio frequencies of a few thousand and upwards. A wavemeter is divided with an inverse time scale.

This (*Fig.* 43) is a quasi-mechanical meter of the vibrating reed type.

A thin strip of steel is supported by a solid block at one end and usually has the free end bent at right angles. Its length and thickness are adjusted till its free period is of the desired value.

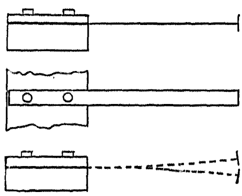


Fig. 43.

When placed on a part of a machine vibrating mechanically with this periodicity, such as a reciprocating engine, the reed will commence to vibrate. The vibration is detected by observing it endways at E.

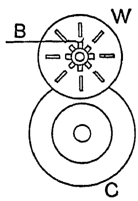
To cover the working range a number of reeds are arranged alongside one another, each adjusted to a definite frequency. The steps may be about 0·5 per cent or 1 per cent. The one closest to the period of the machine will vibrate with the largest amplitude. One or two reeds on each side may be "forced" to vibrate with lesser amplitudes.

As applied to an electrical frequency meter, the reeds are acted upon by an electromagnet through the windings of which a small current passes from the circuit to be measured, with a suitable resistance in series.

The reeds are attracted twice every complete period, so that a reed with a free period of 100 per second vibrates when acted upon by an alternation current of 50 ~ and it is marked 50. The steps are commonly quarter, half, or whole periods between one reed and the next. The final adjustment may be made by adding or scraping off paint or solder from the free end of the reed.

If the reeds are magnetised sufficiently strongly by a permanent magnet the effect of the alternating current magnet will be only to reduce and increase the force acting on the reed and not to completely reverse it. The reed will then only vibrate once instead of twice per period, and a reed with a free period of 100 will now respond to an alternating current of 100 instead of 50.

If such a permanent magnet is arranged to be put in or out of action as desired the range of the instrument is doubled.



§ (52) Weston Frequency Meter.—This meter makes use of the diminution of the current in an inductive circuit with increase of frequency in comparison with the constancy of a current in a simply resistant circuit.

Two crossed coils C_1, C_2 (*Fig.* 44) are connected to inductances L and resistances R as shown. Increase of frequency diminishes the current through C_2 owing to inductances L, and so increases the proportion of the current going through the coil C_1 and resistances R.

The coils C_1, C_2 being at an angle and carrying currents with phase difference give rise to an elliptical rotating field. A piece of soft iron P free to move inside the coils without control

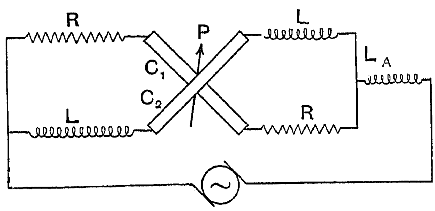


Fig. 44.

sets itself along the major axis of the ellipse. The direction of the axis in space varies with the frequency, so that a pointer attached to P may be made to indicate the frequency. L_A is an added inductance to limit the current taken when used on circuits of ordinary commercial voltages, and also to filter out to a considerable extent any harmonics which may be present.

Above a few hundred volts all such instruments would be operated by an instrument transformer with a secondary voltage of 100 to 110.

§ (53) A. Campbell Frequency Indicator.—A vibrating reed type of indicator in which the effective length of the reed is varied by a sliding clamp similar to the adjustment commonly provided for the regulation of pendulum clocks.

By this means one reed may be made to cover a considerable range of frequency. The motion of the clamp is effected by rack and pinion till the resonant amplitude is obtained; a pointer operated by the pinion mechanism indicates the frequency on a dial.

§ (54) Drysdale Frequency Indicator.—This (*Fig.* 45) is an instrument for determining the speed of shafts, etc.

A geometrical pattern on the end of the shaft whose speed is to be measured is illu-

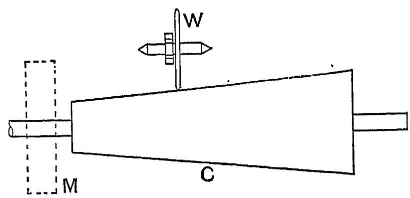


Fig. 45.

minated periodically by the light produced by an instantaneous electrical discharge through a neon tube. The frequency of the discharges is adjusted until the pattern appears stationary. The frequency is then read off from the setting of the mechanism required to produce the stationary effect. Neon is chosen on account of the visual brightness of the light emitted.

A conical brass cylinder C is rotated by a phonic wheel motor M, which is driven by a tuning-fork of known periodicity, say 50 ~.

On the cylinder rests a light brass wheel, which may be of a diameter about equal to the average diameter of the cylinder. This wheel is driven by the cylinder without any appreciable slip and has a small contact-breaker on it, B, which interrupts the primary current in an induction coil. The secondary circuit is connected to the neon tube, which illuminates the pattern on the shaft at every break of the contact-maker.

Suppose the frequency of the fork is 50 and the motor has 8 poles, the cylinder will rotate 50/8 times a second. If the contact-breaker has 8 parts and the jockey wheel is in contact with the cylinder where the diameters are the same, the neon tube will be illuminated $(50/8) \times 8 = 50$ times a second.

If the machine whose speed is being investigated is an 8 pole alternator giving a frequency of 25 ~, its speed will be 25/4 revolutions a second. A pattern on a disc at the end of the shaft repeated 8 times, *i.e.* every

45°, will pass an observer's eye $8 \times (25/4) = 50$ times a second. If the illumination is practically instantaneous, the pattern will appear at rest at the synchronous speed. If the speed of the machine differs slightly from the synchronous speed, the pattern will appear to revolve slowly. It is made to appear stationary by moving the jockey wheel parallel to the axis of the cylinder, increasing or diminishing its speed until the stationary effect is produced. This movement is read on a scale indicating the position of the wheel along the cone. To check the relative speed of the jockey wheel and cylinder, the wheel has radial slits in it, equal in number to the poles of the driving motor. The poles of the driving motor are observed through the slits, and they will appear to be at rest at certain definite relative speeds of wheel and cylinder, which enables the reading of the longitudinal scale to be calibrated at about 5 points.

By attaching a revolution counter to the spindle of the conical cylinder, the frequency of the fork can be obtained by observing the time taken for a few hundred or thousand revolutions.

If only certain definite speeds are required to be determined or kept constant which are not incommensurable with that of the fork, the apparatus is much simplified. The tuning-fork is made to interrupt the primary circuit of the induction coil, and a suitable pattern is drawn to suit the speed of the shaft. If a pattern, repeated n times in 360°, appears at rest, so will one of $2n$, $3n$, etc., but one of $n/2$ will not, if n corresponds to the "fundamental." By having various patterns with "over tones" and "submultiples" the real frequency of an unknown speed can be picked out when there might be some doubt as to whether the speed were a multiple or submultiple of that indicated.

VIII. Alternating Current Instruments. General Methods of Calibration

§ (55).—The indications of instruments for alternating currents are naturally based on the ohm, volt, and ampere. As these units relate primarily to continuous currents, there must be a link in the chain where the change-over from continuous to alternating quantities takes place. This change-over may be where it can be done most conveniently and accurately, which will depend largely on the equipment available.

Instruments used for alternating current measurements may be broadly classified as (1) those which must be calibrated by alternating currents and (2) those which may be calibrated by continuous currents.

Among the former are instruments which depend upon inductances, capacity, or phase displacement for their operation; such as induction ammeters, voltmeters, wattmeters, and watt hour meters, phase and power factor indicators, instrument transformers.

Among the instruments which may be calibrated by continuous currents, though errors frequently of considerable importance may result, are dynamometer type ammeters, voltmeters, wattmeters, electrostatic voltmeters and electrometers, oscillographs.

There are two technical limitations which govern the choice of methods, the one is that it is difficult to measure comparatively low alternating voltages and large alternating currents when high accuracy is required. Instruments for such purposes, which will operate by both alternating and continuous currents, will often have errors difficult to allow for if calibrated by continuous currents, without taking into account possible errors due to inductance, alteration of current distribution, etc., when used on alternating current circuits.

In the measurement of the quantities concerned in continuous current measurement, recourse is almost invariably made to the potentiometer for ascertaining the dimensions of the quantities concerned. The potentiometer avoids the necessity for measuring quantities by highly accurate deflecting instruments. It is equally accurate for measuring a few tenths of a volt as for a thousand volts and for a few millionths of an ampere up to several thousand amperes.

The use of the potentiometer for alternating currents is much more limited, and though it is of great value for special purposes, it cannot displace other apparatus where the highest accuracy is required. In such cases it is necessary to use a deflecting instrument.

The most convenient link, and for many purposes the most accurate, between alternating and continuous currents is the electrostatic voltmeter. Like many instruments for alternating currents the accuracy of reading falls off rapidly below about one-third of the full scale value; and to cover a considerable range two or more instruments may be desirable.

A convenient range for an instrument suitable for a large proportion of work of a technical nature is one which will read from about 20 to 130 volts. This type of instrument has been developed at the National Physical Laboratory as the primary link between continuous and alternating currents. Over the upper half of the range the scale is divided to 0·01 volt, so that at 100 volts readings to 1 in 10,000 can be made, the spacing per volt at 100 volts being about 90 mm. The readings are made by light reflected from a mirror on the instrument, on to the scale about 2½ metres away. A special potential divider enables the scale to be calibrated by continuous potentials at every half-volt. The potential divider consists of a number of

accurately adjusted resistances in series and carries exactly 0·02 ampere. The correctness of the value of this current is ensured by passing it through a resistance whose value in ohms is numerically equal to 50 times the voltage of a Weston cell. A Weston cell will be balanced when placed across this resistance; and a variable resistance in the main circuit is adjusted until this balance is obtained. The correct potential is then established on the potential divider shown in *Fig.* 46, in which A represents the 10 volt steps and B the 0·5 volt steps; C the resistance for balancing

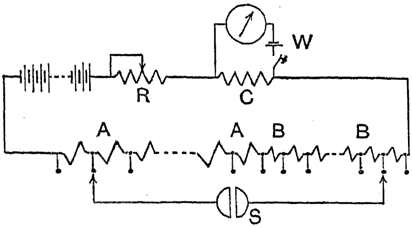


FIG. 46.

the voltage of the Weston cell, and R the adjusting resistance; S is the electrostatic voltmeter.

To eliminate the effect of contact difference of potential, which will generally have a value between a few hundredths and two- or three-tenths of a volt, the mean of the deflections obtained when reversing the polarity of the instrument is taken when using it on alternating circuits.

This type of potential divider was adopted as it avoids all slide wires which are commonly used when continuously variable quantities require to be measured. Slide wires of this nature often give trouble by wear and dirty contacts, and since calibrations half a volt apart suffice, there is considerable advantage in having all resistances in the form of coils, when consistent high accuracy is necessary.

The accuracy of such a potential divider depends upon the correctness and constancy of the ratio of the "cell" resistance to that of the rest of the resistances, especially such as are in common use.

Although the resistance of the part used to balance the Weston cell is practically incommensurable, approximately $50 \times 1{\cdot}0183$ ohms at 20° C., an arrangement of interconnections of the main coils of the potential divider, which are of 500 ohms each and are permanently connected in series, can be made which will give an equivalent resistance so close to such a value that the error may be neglected. The connections shown in *Fig.* 47 represent an equivalent resistance of $50 \times 1{\cdot}018518$, which corresponds to the E.M.F. of the cell at about 15° C. By slight variations of this scheme equivalent resistances corresponding to the E.M.F. of the Weston cell at several other temperatures can be obtained.

For the calibration of voltmeters above 120 or 200 volts other potential dividing resistances are used, a known fraction of the high voltage being applied to the voltmeter. For a large amount of work this fraction is so chosen that the electrostatic instrument indicates 100 volts.

The calibration of low-reading electrostatic voltmeters for use on alternating currents may be carried out in a similar manner by means of known continuous potentials. In instruments for 2 to 10 volts the contact effect is occasionally relatively important, and the difference on reversing may be so large as to make it doubtful whether it is correct to take the mean of the deflections as being the deflection for the same effective alternating potential. In such cases the best way is to calibrate by alternating current, using a resistance with 100 volts on it as a potential divider, and with an accurately calibrated 100-volt electrostatic voltmeter, across the

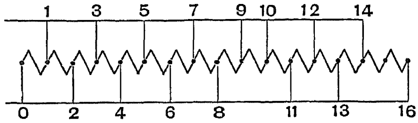


FIG. 47.

whole. The voltmeter to be calibrated is placed across a suitable fraction of the potential divider.

§ (56) DYNAMOMETER VOLTMETERS.—Most of the instruments in common use are intended for circuits of 50 to 100 volts or more. They can be calibrated by comparison with an electrostatic voltmeter, using a potential divider for the latter when necessary.

In the case of low-reading instruments, 20 volts or less, recourse may be had to a step-up transformer. Such a transformer may have its primary and secondary subdivided so that different ratios of the secondary to primary voltage may be obtained. In this way the voltage applied to the voltmeter under test may be transformed up to a voltage suitable for accurate reading on the electrostatic voltmeter.

The transformer, working practically on open circuit, takes a very small current—less than 0·001 ampere is practicable—so that when placed across a standard resistance R_1 carrying a few amperes the shunting effect is quite negligible. If the current in this resistance is accurately measured, the voltage on the resistance is known and so the transformer ratio determined.

The simplest way to determine accurately the value of such a current of a few amperes is to put it through a resistance R_2 capable of

carrying 100 volts or through a combination of such resistances in series or in parallel.

The voltage across the resistance is measured directly on the electrostatic voltmeter, which, as shown in *Fig.* 48, may be arranged so that it can be quickly switched over on to the secondary of the step-up transformer.

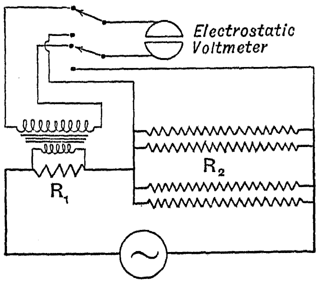


FIG. 48.

§ (57) CALIBRATION OF AMMETERS.—The method just described of using resistances, R_2, capable of carrying 100 volts each, forms a very simple and also the most accurate method of calibrating ammeters up to about 20 amperes.

Such resistances may be composed of two circuits wound on a wooden frame about 70×50 cm. Each circuit is 100 ohms and will carry one ampere without appreciable heating. Looked at endways, if one circuit be wound left-handedly the other circuit is wound right-handedly. The wires are tied closely together in the vertical parts and have therefore very small inductance. The two circuits are intended to be used in parallel, so that the difference of potential of adjacent wires is kept quite small, not more than about four volts; they are double insulated and varnished.

Each frame when supplied with 100 volts has exactly two amperes flowing through it, which may therefore be read to the same accuracy as the electrostatic voltmeter, about 1 part in 10,000.

For other currents such frames may be put in series or in parallel to suit the instrument to be tested. For still smaller currents finer wire and higher resistances, such as those of the Duddell-Mather woven-wire type, can be used.

For calibrating ammeters for larger currents the high-ratio transformer can be used.

For this purpose the current is passed through a resistance designed to give a convenient voltage (2 volts). The high-ratio transformer, calibrated in the manner described above, is placed across this, and the value of the current is indicated by the electrostatic voltmeter connected to the secondary, the transformer ratio being conveniently 1 : 50, giving 100 volts for 2 volts on the primary. The resistances should be without appreciable inductance. Currents up to 2000 amperes can be satisfactorily dealt with by this method.

In the case of currents larger than 2000 amperes instrument transformers are generally used. These can often be calibrated by using a current of say $\frac{1}{2}$ to $\frac{1}{10}$ of the rated current, passing it 2 to 10 times through the transformer so as to give the number of ampere-turns corresponding to full load.

Certain types of ammeters such as current balances and dynamometer type instruments can be calibrated by continuous currents, which are measured by a potentiometer. Precautions against errors due to the earth's and to stray fields must be taken, such as by reversing the current. In the case of instruments for large currents important errors may not be detectable by continuous currents, as the eddy current effect resulting in a different current distribution over the cross-section of the conductor when alternating currents are used may become appreciable.

The error in the Kelvin balance, for instance, at commercial frequencies is inappreciable in the 600 ampere and smaller sizes. They may therefore be calibrated by continuous currents.

In commercial measurements many difficulties are avoided by using special current transformers, the ratio of the turns being such that 5 amperes circulates in the secondary for full-rated current in the primary. Such instruments are made for primary currents of 5 amperes up to 30,000 amperes.

The accurate measurement of alternating *current* is, from a commercial point of view, not nearly so important as the measurement of *power*. Knowledge of the value of current alone does not give a measure of the power in a circuit, unless the phase relations between the current and voltage in the circuit are known.

The alternating current wattmeter is therefore a more important instrument than the ammeter.

§ (58) CALIBRATION OF WATTMETERS.—Practically all commercial wattmeters of the best quality are of the ironless dynamometer type. For small currents and for voltages of the order of 100 they can be calibrated by continuous currents, which should be reversed to detect and allow for the effect of the earth's or any stray field. The induction type is also used, chiefly for switchboard purposes, and must be calibrated by alternating currents.

The very great advantage, and in many cases the practical necessity, of using instrument transformers has led to the universal practice of designing wattmeters for use with 100 to 150 volts on the pressure circuit and five amperes for full-load rating in the current circuit. The instrument must be tested by

alternating currents to determine the error, if any, due to inductance in the pressure circuit or eddy currents in the metal work, which should be quite negligible in a properly designed instrument.

The five-ampere winding is of so small a cross-section that no error due to alteration of current distribution in it will be produced. Having once proved a given instrument by alternating currents, continuous currents may be used for routine calibrations.

When used with instrument transformers it is desirable to have the whole set of apparatus tested *en bloc* as one unit. The instrument transformers will retain their characteristics, unless subjected to serious short-circuits or, in the case of current transformers, to accidental opening of the secondary circuits when on load.

The best quality of both potential and current transformers for portable use are made with series-parallel connection, so that such ratios as 3300 to 110 and 6600 to 110 can be obtained with the same potential transformer, and 10, 20, 40 and similar ratios obtained for the primary currents of current transformers. Except for possible small errors due to capacity, in potential transformers for high voltages such as 20,000, the correction for ratio and phase angle for one ratio may be taken as applying to other ratios of the same instrument.

The calibration of instruments such as power factor meters is commonly done in a similar manner to wattmeters by alternating current.

§ (59) Alternating Current Potentiometer.—The potentiometer, which is the universal instrument by which currents are determined in terms of a difference of potential between the end of a known resistance, has been developed by C. V. Drysdale for the measurement of alternating potentials.

The main circuit of the potentiometer must be supplied with alternating current of exactly the same frequency as that in the circuit to be measured, which means in practice that they must be supplied from the same source.

Further, the phase relation between the circuit to be measured and that supplying the potentiometer may have any value from 0° to 360°. In order to obtain a balance, as indicated by the detecting instrument, it must be possible to give to the potentiometer current any phase displacement relative to that of the potential supplying it. This is accomplished by supplying it through a phase-shifting transformer which consists of a stator wound with a circuit (*Fig.* 49), which produces a rotating field in a closely-fitting rotor. The rotor has a winding on it which supplies the potentiometer current. A rotation of the rotor of the phase-shifting transformer through 360 has the effect of producing a phase shift of the same value, and a pointer on the rotor axis indicates the phase angle on a suitably divided scale.

The "balance" is obtained by successive approximation of the usual adjustment of the potentiometer contacts (dial and slide wire reading) and of the phase-shifting transformer until the indicator shows no deflection.

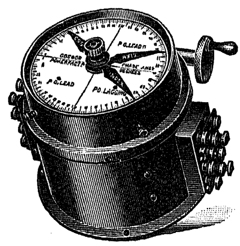


Fig. 49.

The indicator for low frequencies is a vibration galvanometer which must be closely tuned to resonate to the frequency of the circuit. For higher frequencies a telephone may be used.

The main potentiometer current must be kept at some known constant value. This is done by switching it over on to a continuous current circuit, which is adjusted until a balance is obtained at the proper setting, when a Weston cell is connected up in the usual manner. The reading of a sensitive dynamometer type ammeter in the main circuit is noted. This ammeter must read correctly with alternating and continuous currents. The potentiometer is then thrown on to the A.C. supply, which is adjusted so that the ammeter reads the same value. Arrangements are provided for reversing the ammeter in order to eliminate the effect of stray fields.

The phase-shifting transformer is usually supplied by single-phase current. To obtain the necessary field distribution a split-phase scheme is used, part of the excitation being provided by a circuit containing a condenser and a resistance in series. These are adjusted until the A.C. current in the potentiometer circuit is as nearly constant as possible, when the phase-shifting transformer is rotated to any position. As it is necessary to have the split-phase circuit somewhere near the resonating point, it must be adjusted for changes of frequency. As the vibration galvanometer has also to be adjusted for such changes, it is necessary to have a source of a very steady frequency and voltage for satisfactory working.

§ (60) Cathode Ray Oscillograph.—An apparatus for delineating the instantaneous value of the current or voltage in a circuit, by the deflection which a magnetic or electric field produces on a fine cathode stream passing through the field. The cathode stream [1] is generated by the influence of a high continuous-current voltage on a gas in a glass or similar tube at low pressure. The cathode

[1] See article "Piezo Electricity."

may be cold, or heated as in the thermionic valve.

The cathode ray oscillograph suggested by Braun [1] enables records of wave-forms up to many hundred thousands a second to be recorded by means of the most recent developments of this type of instrument.

A stream of cathode rays R is emitted by a cold or hot cathode K in glass tube (*Fig.* 50).

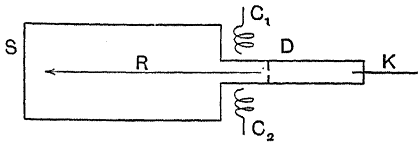


FIG. 50.

The stream falls on a metal diaphragm D with a small hole in it, and passes on to a sensitive screen S at the end of the tube, where its presence is indicated by a phosphorescent glow.

A divided coil, C_1 and C_2, has its axis transverse to the tube, so that a current in it produces a magnetic field across the path of the charged particles, deflecting them sideways at right angles to the field.

An alternating current in the coil C_1, C_2 produces the effect of a line of light on the screen, which, when observed in a rotating mirror, gives the usual effect of a periodic curve. The deflection of the ray can be made accurately proportional to the current in the coil.

The instrument therefore acts as an oscillograph, with the great advantage of having no inertia errors, at least up to a frequency of some millions a second.

A valuable feature is that a second magnetic field can be applied at any desired angle to the axis of C_1, C_2, so that combined effect of two fields can be produced. This can be extended to more than two fields. If in the case of the use of two fields at right angles one of them is given a value proportional to time,[2] the effect of stationary waves can be produced in a manner exactly analogous to the oscillating mirror mentioned in the description of the electromagnetic oscillograph.

The cathode should be excited by a steady potential of 10,000 to 20,000 volts. The higher the potential and the vacuum the faster the motion of the particles forming the stream, the finer the trace on the screen, and the less they are deflected by a given magnetic field. A high vacuum is necessary, which is not a difficult matter, even in large apparatus with modern "molecular" air-pumps.

Recent developments, however, have enabled much lower pressures to be used.[3]

[1] *Ann. der Phys.*, 1897, lx. 552.
[2] Zenneck, *Ann. der Phys.*, 1899, lxix. 838.
[3] *Physical Review*, March 1921, p. 420.

In early forms of this type of apparatus it was difficult to obtain photographs on account of the low intensity of the light after passing through the end of the tube.

By placing the photographic plate inside the tube, and allowing the rays to fall on it without such absorption, this disadvantage disappears, and records can be obtained of wave-forms up to a few million a second.

The diaphragm is commonly earthed, and it is important to have the coils C_1, C_2 at earth potential or to insert an earthed screen between them and the tube. Otherwise they will deflect the rays by the electric field.

§ (61) ELECTROSTATIC OPERATION OF CATHODE RAY TUBES. — In addition to using a magnetic field to deflect the cathode stream it is also possible to use an electric field.

For this purpose the stream is made to pass

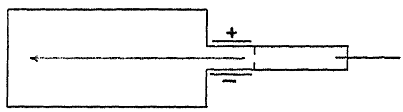


FIG. 51.

between two metal plates, which may be inside or outside the tube (*Fig.* 51).

These plates are connected to the conductors whose difference of potential is to be investigated. The negatively charged particles of the cathode stream are deflected towards the positively charged plate.

Two sets of plates at right angles may be used, or one set of plates producing an electric field and a set of coils producing a magnetic field.

§ (62) DUFOUR OSCILLOGRAPH.—Dufour [4] has added subsidiary details to the cathode ray oscillograph for facilitating the study of alter-

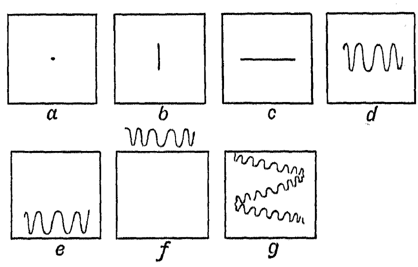


FIG. 52.

nating currents of frequencies up to a million and more a second.

Fig. 52, *a* represents the end view of the cathode stream, impinging on a screen or plate. If an alternating current is sent through a set of deflecting coils which have their axis horizontal, the ray will be drawn out into a band *b*.

[4] *Journal de Physique*, Nov. 1920, p. 146.

Suppose a second set of coils at right angles to the first has an alternating current in it, this alone will give the effect c. If b and c act together, and c is about 5 to 20 times as slow as b, the effect d will be produced if the exposure is of suitable duration. If the frequency b is of the order of 10^5 and that of c 10^4, it will be difficult to arrange a short enough exposure not to go over the same part of the plate many times and so produce the effect of general fog. To avoid this the ray is made to work near one edge e by the adjustment of a permanent magnet, if necessary. It is then pulled back across the field by a continuous-current electromagnet, so as to be near where the top of the photographic plate will be, or beyond it as at f.

After the plate is put in position the exciting circuit of the electromagnet is broken more or less rapidly according to the circumstances. As the field dies down the ray sweeps across the plate in a compound zigzag line g, often giving the appearance of a spiral wound round a cylinder. The value of the transverse frequency c will be commonly known, as it will often be obtained from an oscillating valve or arc with suitable inductance and capacity in circuit, and a wave-meter can be used to determine its frequency. The photograph will enable the frequency and value b and its over-tones to be measured. The instrument can be calibrated by continuous potentials.

§ (63) Ryan Oscillograph.[1]—This is a cathode ray oscillograph with the circuits arranged so as to give a power diagram.

The cathode ray describes a closed curve analogous to that of a piston engine indicator,

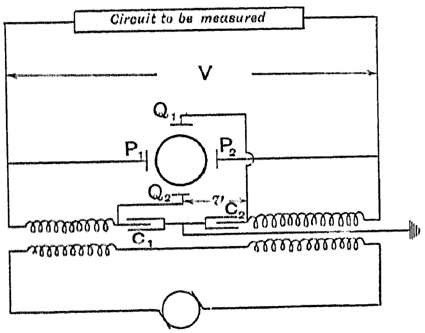


Fig. 53.

the area of which gives the energy in the circuit per cycle.

The same type of apparatus has been adapted by J. P. Minton[2] to the investigation of the energy absorbed by insulating materials under alternating electric stress.

The method devised by Ryan is to apply to two opposite plates, P_1 and P_2, the potential of the circuit V. In the case of a circuit of inconveniently high voltage some fraction of it is obtained by inserting series condensers in the circuit to form a potential divider.

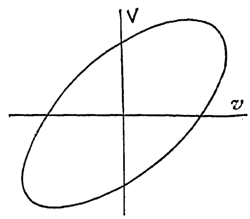


Fig. 54.

To the two plates Q_1, Q_2 (*Fig.* 53) at right angles to P_1, P_2 a potential v is applied by connecting them to two condensers c_1, c_2, which have their common terminal connected to earth.

The current through those condensers is the current i in the circuit to be measured.

The instrument describes the curve connecting V and v (*Fig.* 54). If $c_1 = c_2 = 2c$, then $c(dv/dt) = i$.

An elemental area of the curve $dA = V \, . \, dv$.

Thus $$dA = V\frac{i}{c}dt,$$

$$A = K\int V \, . \, i \, . \, dt.$$

Since $\int V \, . \, i \, . \, dt$ represents work, the area of the curve is equal to the work done per cycle.

The instrument has been specially applied to determining the power lost in high-voltage transmission lines, which is largely due to partial breakdown of the air at high electric stress.

§ (64) Technical Application of Cathode Ray Oscillograph.—The cathode ray oscillograph may be used to investigate transient phenomena which can be made to manifest themselves as changes of voltage or current.

An interesting development has been the measurement of pressure.

For this purpose the piezoelectric properties of such crystals as tourmaline or Rochelle salt can be used. When subjected to mechanical pressure in certain directions they develop a difference of polarity. If the parts at which difference of polarity is produced are connected metallically to the plates of the tube the spot of light will be deflected. To obtain the time-history of the pressure a magnetic or electric field may be applied so as to produce a deflection at right angles corresponding to some known function of the time, as in the Dufour oscillograph. From the curve described by the cathode stream the pressure-time relation can be deduced. It has been used in this manner to determine the intensity of pressure of under-water explosions.

E. H. R.

[1] H. J. Ryan, *Trans. Am. Inst. El. Eng.*, 1911, xxx. 1089. See also "Alternating Current Wave Forms," § 4 (vi.).

[2] J. P. Minton, *Trans. Am. Inst. El. Eng.*, 1915, xxxiv. 1627.

ALTERNATING CURRENT WAVE FORMS, THE DELINEATION OF

§ (1) Point by Point Methods.—The method of determining the wave shape of an alternating P.D. which was first used, depended on the determination of each point of the P.D. curve. It was first employed by M. Joubert[1] and is generally called the Joubert point by point method. The alternator whose wave form it is desired to find is fitted with a contact maker which closes the circuit between the machine and a voltmeter, at a definite point on the wave. If the alternator is not accessible, a synchronous motor may be used to drive the contact maker.[2] The voltmeter shows the P.D. existing at the terminals of the alternator at the instant at which the contact is made. An electrostatic voltmeter is the most suitable instrument to use, but when one is not available an ordinary moving coil voltmeter may be employed, shunted by a condenser. The condenser gradually collects a charge proportional to the P.D. existing at the instant of closing circuit, and then discharges through the voltmeter. If the capacity of the condenser be suitably chosen, the voltmeter may be made to indicate to a high degree of accuracy the voltage actually existing at the time of closing circuit. It is evident that the voltage found is the average value of the P.D. at the instant of closing circuit of a large number of succeeding waves, and unless conditions in the circuit are steady the method of recording is not of very much use. A great deal also depends, in this method, on the form of contact maker that is used. If reasonably accurate records are to be obtained, the time of contact must be very short. The contact should not last more than $\frac{1}{50}$th of the duration of a half period, hence the time of contact allowable is only $\frac{1}{5000}$th of a second for a 50-cycle frequency.

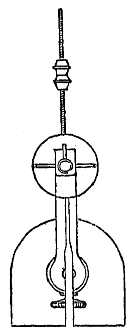
The construction of a contact maker, therefore, that will act with certainty is not an easy matter. The one which has been found most successful by the author consists of two pieces of watch-spring which are periodically short-circuited by a revolving brass plate. Having determined the P.D. at one point on the curve, the contact maker is moved round through a known distance, which can be measured on a scale attached to the alternator, and a series of records obtained of the P.D. of the machine, for different positions. The delineation of current may be effected by a similar arrangement. The voltage available for operating the shunted millivoltmeter, or whatever instrument may be used for the purpose of recording, is very much less than it is when wave shapes of potential difference are being measured, hence contact and thermo E.M.F.'s are apt to give trouble, and the readings of the instrument made much less steady. A great many experiments were made by Mr. Duddell and the author using a very sensitive electrostatic voltmeter of the quadrant type with the needle charged by a battery of small cells. The advantage of the electrostatic type of instrument for recording is that it does not require the use of the auxiliary condenser, and the quickness with which the instrument reaches its steady reading is much greater. Even under the most favourable conditions this method of delineating wave shapes is of no practical use except when circuit conditions are steady.

§ (2) The Ondograph.—In order to record wave forms more quickly, an instrument called the Ondograph was devised by M. Hospitalier.[3] In this apparatus the wave form is registered on a sheet of paper by a pen similar to that used in the ordinary barograph; it is an improvement on the original Joubert point by point method in that the point of contact is varied continuously by a gearing attached to the synchronous motor which is used to drive the contact maker.

The apparatus is shown in perspective and diagrammatically in *Fig.* 1. A synchronous motor A drives, on the one side, a contact maker D, and, on the other, a recording drum C through a series of gear wheels B. The recording instrument is shown at E, the needle of which, F, traces on the drum the wave shape which it is desired to find. The motor is driven directly from the alternating supply. The terminals H, H′ are connected to a condenser which serves the same purpose as that used in the point by point method, with a resistance in series with it of sufficient magnitude to prevent serious sparking. The terminals I and I′ are connected to the source of supply. When the contact maker D rotates, the series of connections is as follows. One terminal of the supply I is connected to the condenser H′ and to the recording instrument; the other terminal I′ is connected to the terminal of the contact maker *d* and so to the revolving cylinder D. When D is connected to *d* by the thin plate which projects from the surface of the cylinder, the condenser C is charged to a potential difference equal to that which exists at the instant at which the circuit is closed. At this time the brush *d″* rests on an insulating part of the cylinder D, but immediately afterwards *d″* makes connection with the cylinder, while *d* is insulated, with the result that the condenser C is discharged by the circuit H*d′d″*EH′C′ through

[1] Joubert, *Journal de Physique*, 1880.
[2] Fleming, *Electrician*, xxxiv. 460.
[3] *Journ. Inst. El. Eng.*, 1903–4, xxxiii. 75.

the instrument, producing a deflection corresponding with the instantaneous value of the voltage. The synchronous motor train B is arranged so that when A has made 1000 turns D makes 999 turns, and C has made $\frac{1}{3}$rd of a revolution. Thus during 1000 revolutions the contact maker will have made one revolution less than the synchronous motor, which will correspond with one complete alternation of E.M.F. in the circuit. The complete revolution of the drum, therefore, will correspond with three complete waves. The galvanometer E is of the ordinary moving coil type with special arrangements for setting the zero of the instrument and balancing the pointer. Some records obtained with this apparatus are shown in *Fig.* 2.



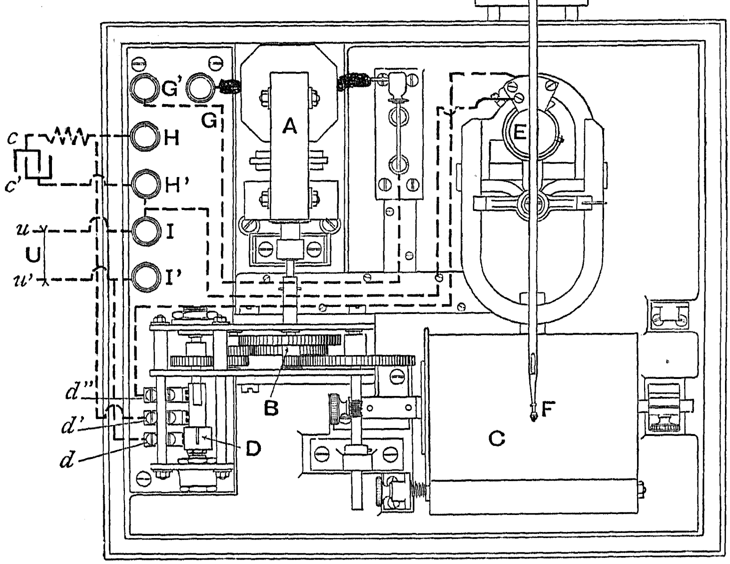


FIG. 1.

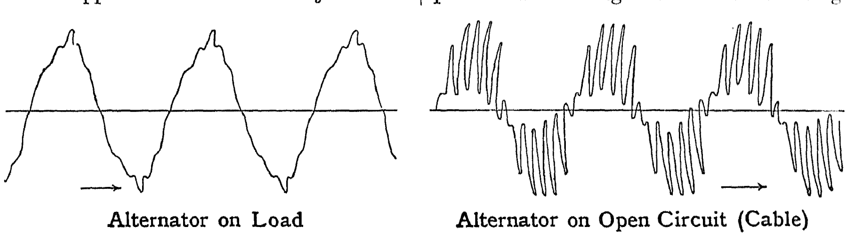


FIG. 2.

§ (3) EARLY HISTORY OF OSCILLOGRAPHS.—Instruments like the ondograph are obviously limited in their application, as they are restricted to making records of periodic phenomena which persist over several seconds; they cannot record transient phenomena. Attempts have been made, therefore, to design an instrument which would record instantaneously the current which was passing through it. Such an instrument was subsequently called an Oscillograph. The earliest type of apparatus proposed consisted of a telephone diaphragm, to which some arrangement was attached, by which its motion could be recorded. One of the first attempts of this kind was made by Frölich.[1] He attached either a plane or concave mirror to the diaphragm of an ordinary telephone through which was passed the alternating current to be recorded. The mirror was not attached to the centre of the diaphragm, but eccentrically, so that when the diaphragm was bent by the action of the current the mirror was tilted slightly. The light from an arc lamp was reflected from the mirror and focussed, so as to form a bright spot either on a moving photographic plate or a revolving mirror. In order to give opposite deflections of the mirror for opposite directions of flow of current, the telephone must be fitted with a permanent magnet, the strength of which is either weakened or strengthened by the current passing round

[1] Q. Frölich, *Elekt. Zeits.* viii. 210, x. 65 and 345.

the coil of the telephone. If the change in strength of the attraction of the diaphragm by the magnet is relatively small, the alteration in deflection of the diaphragm will be approximately proportional to the magnitude of the current passing.

There is, of course, one obvious disadvantage of this method of recording: the diaphragm of a telephone is relatively massive and undamped, which makes it distort the shape of any current curve it attempts to record.

Professor Elihu Thomson [1] modified this arrangement by introducing a system of magnifying levers between the diaphragm and the mirror, thus rendering a very small motion of the diaphragm sufficient to produce a record; he also used a second coil in his telephone instead of the permanent magnet. This coil carried a continuous current, and the other the alternating current which it was wished to record, the object of this arrangement being to ensure a stricter proportionality between current and deflection than is possible when the action of the instrument depends on the weakening and strengthening of a permanent magnet. Another arrangement used by M. Guyau [2] for observing the motion of a telephone diaphragm is to attach a plane silvered mirror to it, and place in front of this mirror a fixed reference mirror. Interference fringes are formed, by a monochromatic source, by interference between the beams reflected from these mirrors, and as the diaphragm moves these fringes alter. To observe their motion they are focussed on a horizontal slit behind which is a sensitised film driven by clockwork, and moving vertically. The fringes so photographed trace out curves corresponding with the motion of the diaphragm.

Other suggestions have been made, such as the use of the phonograph, for recording the motions of the diaphragm, the actual record being made subsequently by running the wax cylinder through a specially arranged receiver, with a pointer attached to the pin of the phonograph, which in turn is connected with a system of magnifying levers to produce the deflection of a mirror.

Among various suggestions that have been made for constructing oscillographs may be mentioned one of Professor Nichols, who constructed an instrument which consisted of a fine jet of mercury [3] through which the alternating current that it was wished to record was sent. This jet was passed between the poles of a powerful magnet and a shadow of the jet as it passed the magnet was thrown, by a beam of light, on a moving photographic film. The deflection of the jet is approximately proportional to the instantaneous value of the current. Another suggestion, worked out in some detail by M. Pionchon [4] and Professor Crehore,[5] is to make use of the property of a magnetic field of rotating the plane of polarisation of a beam of polarised light, which is passed through a solution of carbon bisulphide. If the coil producing the magnetic field carries an alternating current, the plane of polarisation will be rotated in one direction or the other according to the direction of the current passing. In the actual arrangement the light from a powerful source is first passed through a polarising Nicol prism, then in succession through a quartz plate, a solenoid containing a glass tube full of carbon bisulphide, an analysing Nicol and a slit; the light emerging from the slit is split up, either by a prism or diffraction grating, into a spectrum. The effect of the quartz plate is to produce a rotation of the plane of polarisation of the light which depends on the wavelength. The plane of polarisation of the blue light in the spectrum is rotated through a smaller angle than the red, and so on. When, therefore, no current is flowing through the coil, there will be a dark patch on the spectrum, which may be moved backwards and forwards by turning the analysing Nicol. By adjusting the position of the analysing Nicol, the dark band may be set near the middle of the spectrum. If now an alternating current is sent through the solenoid a rotation of the plane of polarisation is produced on the light as it passes through the carbon bisulphide in the solenoid, and the dark patch on the spectrum will move backwards and forwards across the spectrum in a direction depending on the direction of the current, and by an amount very nearly proportional to the strength of the current.

If, therefore, a photographic plate is moved so as to make a trace of the motion of this dark patch, a record will be obtained of the variation in strength and direction of the current flowing through the coil. This arrangement has many obvious disadvantages. As the dark patch is of considerable size, no very clear definition can be obtained on the photographic plate. The only advantage of this method is that the pointer consists of a beam of light which, of course, is weightless, so that the apparatus can be employed just as easily to measure a very high frequency current as it can one of the ordinary working frequency. It has not been developed, however, on a practical scale.

Another method of tracing out a wave form has been devised by Professor Janet.[6] This is a chemical method, and depends for its action on the fact that if an electric

[1] Thomson, *La Lumière élect.* xxvii. 339.
[2] Guyau, *Comptes Rendus*, clvi. 777.
[3] *Am. Acad. Proc.* xlii. 57.
[4] Pionchon, *Comptes Rendus*, cxx. 872.
[5] Crehore, *Phys. Rev.* ii. 122, iii. 63.
[6] Janet, *Comptes Rendus*, cxviii. 862, and cxix. 58, 217, 399.

current be passed through a paper soaked in a mixed solution of ferrocyanide of potassium and nitrate of ammonia, a coloured mark will be made on the paper, the intensity of which depends on the strength of the current. An improvement on this arrangement is to use a number of styles connected to a number of batteries arranged in series. The alternating P.D., which it is required to record, is connected between the drum and the middle style. At any instant there is a style for which the algebraic sum of the E.M.F.'s between it and the cylinder is approximately zero. By rotating the drum curves would be formed by the gaps in the series of parallel straight lines formed by the styles on the paper.

It has also been suggested that the capillary electrometer might be used for recording alternating wave forms.[1] The mass of the moving meniscus in the instrument is very small and its motion can readily be photographed; it responds very quickly to any varying force acting on it. This apparatus, however, has not been used to any appreciable extent for this purpose. Another method, originally suggested by F. Braun,[2] was to use as a recorder a pencil of cathode rays. These are deflected by a magnetic or electrostatic field, and if a coil carrying the alternating current or a pair of electrodes connected to the source of P.D. is so arranged as to influence these rays, the deflection produced will be proportional to the current which is passing through the coil. These rays may fall on a fluorescent screen contained within the tube, and produce a bright spot whose motion can be observed in a rotating mirror, or the moving spot may be recorded on a photographic plate. This method has lately been developed for high frequency measurement and will be described more fully.

§ (4) Modern Forms of Oscillographs.—Of the forms of oscillograph mentioned above there are two types, both originally suggested by Monsieur Blondel,[3] which have become practical instruments of great value. The first of these is that in which the record is obtained by the deflection of a pair of tightly stretched wires or strips arranged in a magnetic field.

(i.) *Duddell Oscillograph.*—Monsieur Blondel produced such an instrument, but the form of apparatus devised originally by Mr. Duddell[4] has been most generally employed in this country. In the original form, an electromagnet with poles N, S and with a small air-gap was made (see *Fig.* 3), with a pair of phosphor bronze strips stretched between them; these were passed round a pulley. They were tightly stretched by a spring balance attached to the pulley P. The ends of the strips were connected to two terminals. The alternating current passed up one of them and down the other. Thus the two strips are displaced in opposite directions, their deflection being observed by means of a mirror attached at their centres. The essentials of such an instrument are: (1) very short periodic time compared with the periods of the wave forms being recorded; (2) critical damping, *i.e.* the motion ceases to be oscillatory when the strips are deflected; (3) negligible self-induction; and (4) sufficient sensibility.

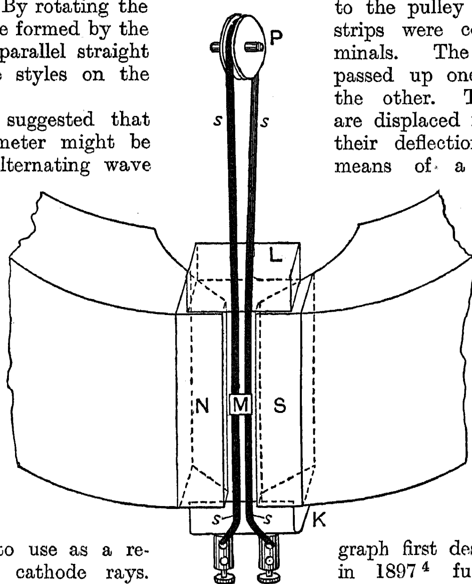


Fig. 3.

The Duddell oscillograph first described by Mr. Duddell in 1897[4] fulfilled all these conditions, the agreement between the curve traced out by it and the curve obtained by a point by point method being almost perfect (see *Fig.* 4). In a later form the vibrating system was designed so as to obtain a periodic time of less than 1/10,000 of a second. This was done by using strips of phosphor bronze stretched nearly to their ultimate tensile strength and using a very small mirror to indicate the deflection. In the first design of this instrument the greatest difficulty lay in securing critical damping.[5] In the Duddell instrument this was effected by using short strips and immersing the whole oscillograph system in an oil bath, the oil being of a very viscous kind. The oil was contained by a chamber of which the sides are formed by the pole pieces, the back by a brass plate, and the front by a lens.

In the later types the magnet was arranged with very narrow bridge pieces, placed so as

[1] Burch, *Electrician*, xxxvii.
[2] Braun, *Wied. Ann.*, 1897, lx. 552.
[3] Blondel, *La Lum. élect.* xli. 401.
[4] Duddell, *Electrician*, xxxix. 636, and *Journ. Inst. El. Eng.* xxviii. 1.
[5] See "Galvanometer," § (10) (iii.).

to make each strip vibrate in a little cell of its own. By these methods, critical damping was secured, and with critical damping the

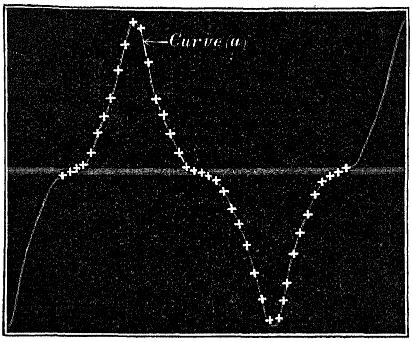


FIG. 4.

moving strip reaches its steady deflection in the shortest possible time.

The beam of light reflected from the mirror may be received on a screen or photographic plate, the instantaneous value of the current being proportional to the displacement of the spot of light. This spot moves backwards and forwards across the plate with great rapidity and gives the impression of a line of light. To obtain the wave form, the photographic plate must be moved in a direction at right angles to that of the line formed by the vibrating spot, or another mirror may be introduced in the path of the beam and this mirror rotated or oscillated so that it moves the beam uniformly (*i.e.* proportionally to the time) in a direction at right angles to the plane of vibration due to the varying current. The spot of light will then depict on a stationary screen the curve of variation of current with time. If the variations are periodic and the second mirror is synchronised, the spot of light traces out the wave form over and over again, and if it is thrown on a screen of ground glass the appearance is that of a continuous wave or line. Small fuses mounted in glass tubes are used to protect the strips from injury in case of excessive current.

The most recent form of the oscillograph is shown in *Fig.* 5; the same instrument

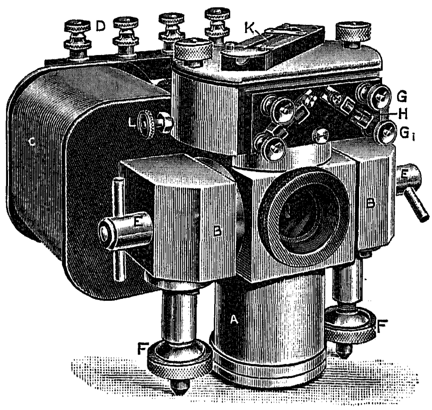


FIG. 5.

with permanent instead of electromagnets is shown in section in *Fig.* 6. A is the oil bath

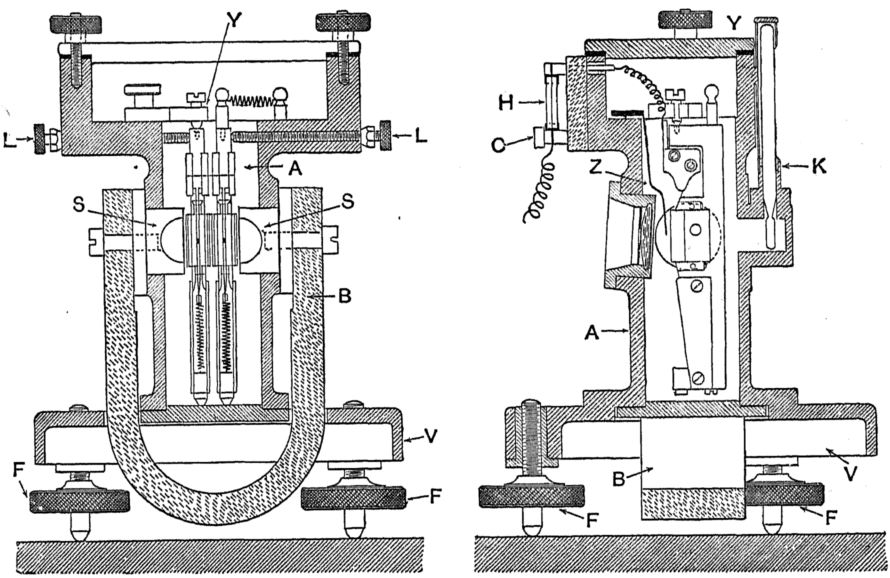


FIG. 6.

in which the two vibrators are fixed. The oil bath is formed of a brass box and is held

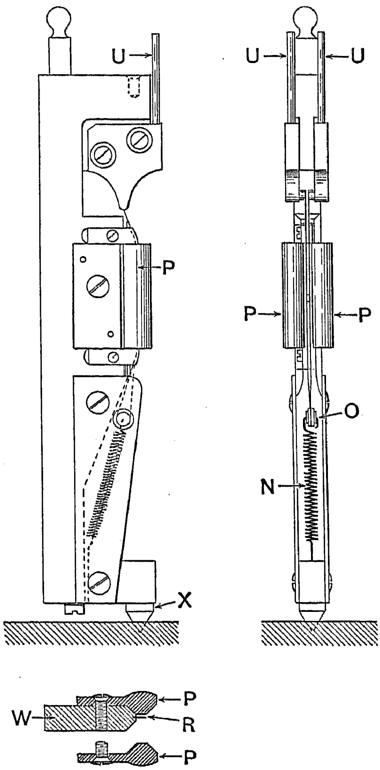


FIG. 7.

in position between the magnet poles by brass hand bolts EE (*Fig.* 5). The connections from one of the two vibrators are brought out to two terminals G and G_1. The fuse for protecting the vibrator is shown on the face of the terminal plate, and is arranged inside a glass tube fitted with brass caps which fit into two connecting clips. A thermometer is used for observing the temperature of the oil, this being the quantity on which depends the effective damping of the instrument. The vibrator itself is shown in *Fig.* 7. The brass frame W supports two soft iron pole pieces P. Between them is a long narrow groove divided into two parts by a thin soft iron partition R which runs up the centre. The current is led in by a brass wire U, passes from an insulated brass plate to the strip, which is passed over an ivory guide block down one of the narrow grooves, then over a second guide block and round the ivory pulley O and so back to the other terminal U; the spring N is used to adjust the tension on the strips. Half-way up the groove the centre partition is cut away so as to allow a mirror to be attached to the strips.

(ii.) *Recording Oscillographs.*—The simplest form of recording camera is that using a falling plate; in this arrangement a photographic plate is allowed to fall freely down a light tight slide,[1] past a horizontal slit through which the beam of light reflected from the oscillograph mirror passes. The normal speed of the plate is 400 cm. per second as it passes the slit. The speed of course gradually increases as the plate falls, so that the time scale is not quite uniform, but if the plate falls from a height of 3 or 4 feet, the variation is not enough to produce any sensible alteration in the wave shape. For recording transient phenomena, a modified cinematograph camera can be employed, in which the image of the wave is recorded on a revolving film, similar to that used in the ordinary cinematograph. When it is desired to observe the wave shape without taking a photograph, a revolving mirror, usually four-sided, is arranged inside the box containing the outfit, and is driven round, either by hand or by a small motor. If the oscillating spot of light is observed in this mirror, it is spread out into a curve which represents the wave shape.

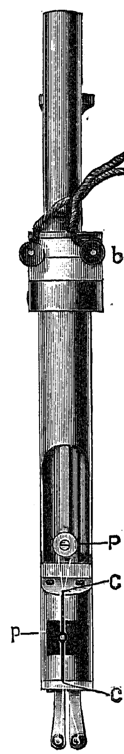


FIG. 8.

If it is desired to make tracings of the wave or to project them on to a screen, the beam of light, after it has been reflected from the oscillograph mirrors, falls on a vibrating mirror which gives it a deflection proportional to time in a direction at right angles to that produced by the oscillograph. This mirror is vibrated by a cam on the shaft of a synchronous motor, and makes a motion forward during a period corresponding to three half alternations of the current; during the next half period the mirror springs back to its original position, but in order to prevent the spot from showing on the screen during this motion, a shutter is arranged on the shaft which cuts off the light during this half period. The motor is of the attracted iron type, the rotor carrying four pieces of iron which are attracted by the alternating magnetisation of the iron horseshoes which carry the coils. The motor may be started

[1] See "Radio-frequency Measurements," § (4) (vi.).

up either by hand or by a contact maker on the shaft which makes and breaks contact to the magnet coils, and so enables the motor to be run up to speed.

(iii.) *Blondel Oscillograph.*—The oscillograph which has been constructed by M. Blondel is very similar in general design. M. Blondel uses a separate vibrator to carry the oscillating strips, *i.e.* a mounting for the strips which can be removed *in toto* from the instrument, in order to facilitate repairs. The form of his vibrator is drum which carries the photographic films is shown at E; the lens which focusses the beam on to the film is shown at F_{60}. The oscillating mirror, which gives the time motion for visual observation, is shown at M; the lens F_{150} focusses the light from this mirror on to a ground glass screen at the top of the cover. The lens F_{150} and the mirror M are carried on an arm *ch* which can be operated by the lever *m* when the instrument is to be used for photographic recording.

Another form of oscillograph developed by

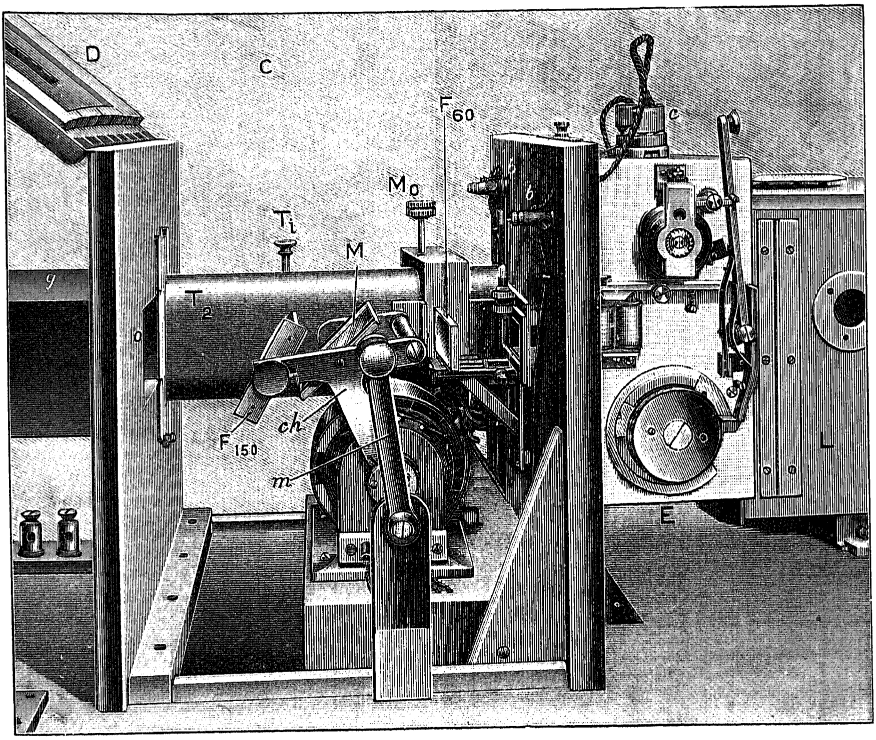


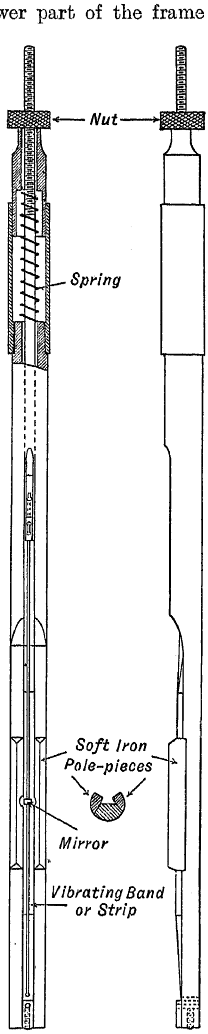
FIG. 9.

shown in *Fig.* 8. Instead of flat strips he uses round wires, as originally proposed by him in 1901, the advantage of the round wire being that a much narrower air-gap can be used in the magnet, and so a more powerful field obtained than if the flat strips, as used by Duddell, are employed. The damping is effected by oil, and the whole vibrator is immersed in an oil bath.

The most striking feature of the instrument, as compared with the Duddell, is the very powerful magnet. It is made with three vibrators so as to enable three records of wave form to be made simultaneously. The form of photographic recorder used by M. Blondel is shown in *Fig.* 9. The box containing the M. Blondel is one depending on the deflection of a band of iron strip stretched tightly in a strong magnetic field, its width being along the lines of force (see *Fig.* 10).

The tension of the band may be adjusted so as to have as short a period as 1/40,000 second. A powerful permanent magnet provides the necessary controlling force on the strip. The deflecting force due to the alternating current is provided by two small coils, with their axes at right angles to the direction of the magnetic field due to the permanent magnet. If a current is sent through these coils the field in the gap is correspondingly distorted and the band takes up a position with its width along the resultant field. A

small mirror is attached to the centre of the band which indicates its motion. The entire lower part of the frame carrying the band is put into a tube filled with castor oil. This gives the necessary damping; the front of the tube is fitted with a lens, just as in the Duddell oscillograph, and prevents distortion of the spot of light in its passage through the tube. The optical system



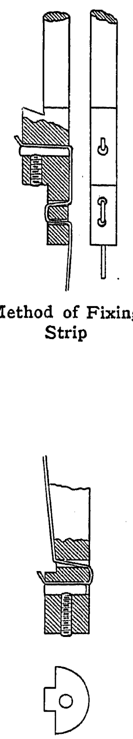


FIG. 10.—Details of Blondel Oscillograph.

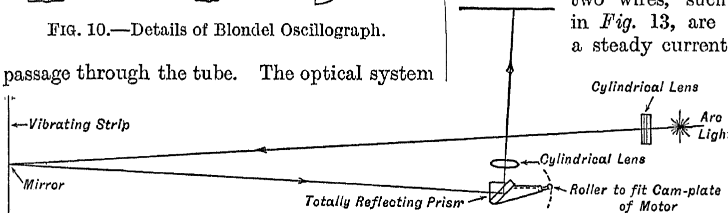


FIG. 11.—Arrangement for recording Vibrations of Oscillograph.

of this oscillograph is shown in *Fig.* 11, and is similar to that used in the Duddell instrument.

(iv.) *General Electric Company Oscillograph.*—A special type of oscillograph designed for use in works testing has been constructed by the General Electric Company of Schenectady. The form of this instrument is shown in *Fig.* 12. The vibrator consists of a separate unit which can be completely removed from the oil as in the later forms of the Duddell and Blondel oscillographs. Each vibrator is in a separate cell, and is placed between the poles of separate magnets, the arrangement for the three vibrators being shown in the figure. The tension on the strips is adjusted by the tension screw TS operating through a small spring balance. The vibrator terminals are shown at VT and the magnet coil terminals at MP. The tangent screw W is used

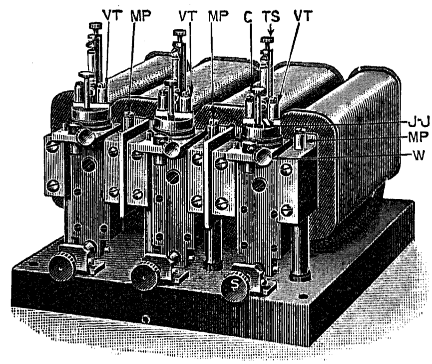


FIG. 12.

for adjusting the zero of the instrument. To obtain records with this instrument a special combined photographic drum and tracing attachment is used. The film roll is driven by a small synchronous motor, which is started automatically just before the shutter opens; the mechanism is so arranged that the shutter remains open for one complete revolution of the drum.

(v.) *Irwin Oscillograph.*—Another form of oscillograph has been suggested by Mr. J. T. Irwin,[1] and has been developed into a practical instrument. It depends on the fact that if two wires, such as CD and EF in *Fig.* 13, are so arranged that a steady current I_b flows through them from a battery B, in the direction indicated by the arrows *b*, *b*, and if there is superimposed on this a current I_a flowing from C to F in the direction indicated by the arrows *a*, *a*, the difference in the heating of the strips CD and EF will be proportional to the current

[1] *Proc. Inst. E.E.* xxxix. 617.

I_a. This may easily be seen algebraically. The heating of the strip CD is proportional to $(I_a - I_b)^2$ and that of EF to $(I_a + I_b)^2$, the difference between these quantities equals $4I_aI_b$, and, if I_b remains constant, is proportional, therefore, to I_a. The wires sag when the current passes, and if a mirror is attached to the mid points of CD and EF the deflection of the mirror will be proportional to the current I_a. The practical difficulty is to make certain that the two strips CD and EF and the sag in them are exactly the same. This is clearly very difficult to ensure when one considers that the motion, in some of these instruments, corresponding with sensible deflections is as small as 1/10,000 of an inch. The arrangement adopted by Mr. Irwin is shown in *Fig.* 14. The two wires C, D, *Fig.* 14, A, are each of the form shown in *Fig.* 14, B—*i.e.* they are passed round an ivory pulley which serves to give the necessary tension to the wires through the spring S. The wires are connected together by threads as shown in *Fig.* 14, C. If there is no current passing through C′C, D′D, or EE′, F′F, and if the wires are exactly similar at the same temperature, then their sag will be the same in each case. If a current is sent through the 4 portions in series then there will be an increase in length of all 4 wires, but no deflection. If the wire CC′, D′D is heated more than the wire EE′, F′F, the mirror M will be tilted by an amount which depends on the difference in the rate of heating of the two wires. With this arrangement, however, the objection is that the rate at which the wires will heat up is very slow. Mr. Irwin states that the instrument will not indicate accurately for frequencies much above 5 per second, even when the strips are immersed in oil so as to increase the rate of cooling and so make the instrument reach its steady deflection more rapidly. To make

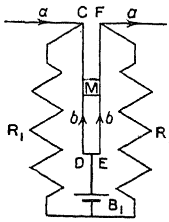
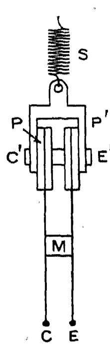
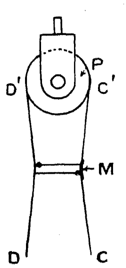


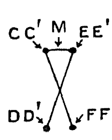
FIG. 13.



A



B



C

FIG. 14.

it practical for ordinary frequencies, the resistance R_4 (see *Fig.* 15) is shunted by a condenser K. The resistance of the instrument itself is low compared with R_4, being about 1/200 of it, for a voltage of 100 across AB. In this case the difference in temperature of the two wires of the instrument at any instant is proportional to the difference of potential across AB, *Fig.* 15. This may be shown mathematically as follows:

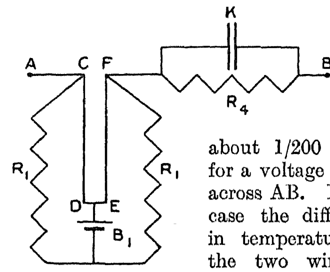


FIG. 15.

The temperature rise of the wires when heat is supplied to them depends on their mass and specific heat and on the rate at which they cool.

Thus $\qquad ms\,\dot{t}_1 = w - bt_1,$

where m is the mass of the wire,

s its specific heat,

$\dot{t}_1$ = rate of increase of temperature with time,

w = power supplied to wires,

bt_1 = heat lost per second by radiation, etc.

Writing $ms = a$ the equation becomes

$$a\dot{t}_1 + bt_1 = w = (I - i)^2 r$$

where I is the steady current and i is the varying current in one wire.

In the same way, for the other wire

$$a\dot{t}_2 + bt_2 = (I + i)^2 r,$$

where t_2 and $\dot{t}_2$ are the temperature and rate of increase of temperature for the second wire. Thus if

$$\theta = t_2 - t_1,$$

we have

$$a\dot{\theta} + b\theta = 4I\,ir.$$

Now the current i flowing through a condenser K shunted by a resistance R is given by

$$i = \left(\frac{V}{R} + \dot{V}K\right),$$

where V and $\dot{V}$ are the potential difference and rate of change of P.D. respectively.

Hence

$$a\dot{\theta} + b\theta = 4I\,ir\left(\dot{V}K + \frac{V}{R}\right).$$

If, therefore, a/b equals KR, θ is proportional to V, and the difference in temperature of the two strips will be proportional to the P.D. on the circuit. The deflection of a mirror attached to them, provided it is light, and the system has a sufficiently short periodic time, will, therefore, also be proportional to the P.D.

To record current, the instrument may be shunted by an inductance of suitable value and thus made to give deflections proportional to the current passing in the circuit. The form

adopted is shown in *Fig.* 16, the arrangement of the vibrating wires being similar to that in *Fig.* 14. The screws at the top of the instrument are used to adjust the tension on the strips, but this does not alter the natural period so much as varying the amount by which the wires are tied back in attaching the mirrors. In a particular case quoted by the constructor, the natural period was altered from 1/3,200 to 1/2,900 of a second by reducing the tension from 10 oz. to 6·6 oz. The natural period of the strips is, of course, rather longer when the strips are immersed in oil than when they are in air. It is possible to obtain critical damping with an oil slightly more viscous than paraffin, so that the damping difficulty does not appear to be so serious as in the other forms of oscillographs which have been described.

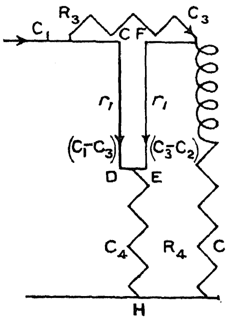


FIG. 16.

It has been shown that the indication of the hot - wire instrument depends on the product of the two currents I and *i*. If one of these currents, which Mr. Irwin has called the polarising current, instead of being constant, is proportional to the pressure which is applied to the circuit, and the varying current is the alternating current which is being observed, or a fraction of it, it is clear that the difference in heating of the two strips on which depends the deflection of the instrument, will be proportional to the power that is being expended in the circuit. When this oscillograph, therefore, is suitably connected it may be used to record variations in the power expended in a circuit. In order that the instrument may work satisfactorily, the resistances must be adjusted so that the power used in the instrument is a comparatively small fraction of the power being used in the circuit. Mr. Irwin has obtained some power diagrams which approximate very closely to those obtained by calculation from the pressure and current curves.

(vi.) *Cathode Ray Oscillograph.*—An oscillograph depending for its action on the deflection of a beam of cathode rays has been developed by Ryan.[1] The form of tube used by him is shown in *Fig.* 17. A cone of cathode rays *cr* is emitted by the disc-shaped negative electrode, it falls on an aluminium diaphragm *dd* in the centre of which is a small aperture, it then passes between the two plates Q′Q″, and falls on the fluorescent screen S. The rays are "focussed" by a coil MMF which produces a strong magnetic field along the axis of the cone. This has the effect of concentrating the beam and giving a smaller spot on the screen than is obtained without it. Ryan states that with the standard Muller-Uri-Ryan tube No. 2671 about 2500 ampere turns are required on this coil. The coil should be mounted in the first place axially to the beam, but should be capable of adjustment in any direction so that the resultant of its own field and the earth's field can be made to have the proper relation to the direction of the ray. The action of this magnetic field is explained by Ryan as causing the electrons in the cathode ray beam to move in spirals instead of allowing them to diverge, and the "focussing" is not therefore very accurate. The cathode ray discharge may be produced by an influence machine, or by a "diode" used in conjunction with a small high-tension transformer. After a time the cathode ray tube becomes "hard," as is found in X-ray work, and to soften it, that is, to reduce the vacuum, a small platinum tube *pt* is sealed into the cathode ray tube at one end, the other end, projecting outwards, is closed. Hydrogen may be passed through the tube by heating it in an alcohol flame, and the tube may be adjusted by having a spark-gap shunting it, so as to maintain such a vacuum that the discharge just passes through the tube, instead of across the spark-gap. In order to make a trace of the curve outlined on the screen *ss*, the whole tube may be put in a box and the screen observed through a smoked glass on which its shape may be

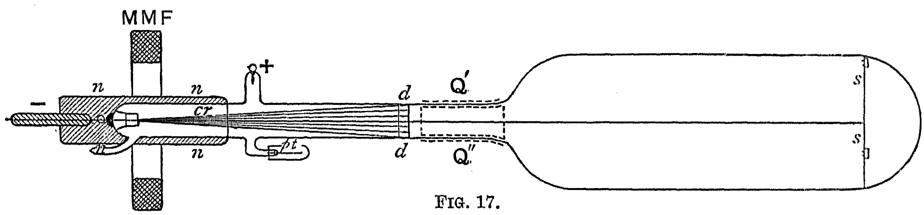


FIG. 17.

[1] Ryan, *Trans. A.I.E.E.* xxii. 539; *Proc. A.I.E.E.*, 1911, p. 532. See also "Piezo Electricity" and "Cathode Ray Manometer," Vol. I.

scratched. A better plan is to photograph the screen.

The arrangement shown in *Fig.* 17 is

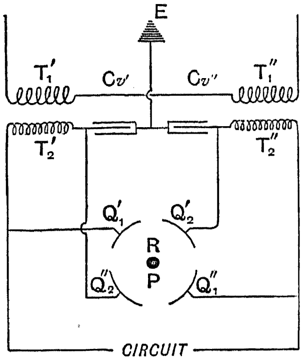


FIG. 18.

intended for taking oscillograph records of power. In the arrangement described two

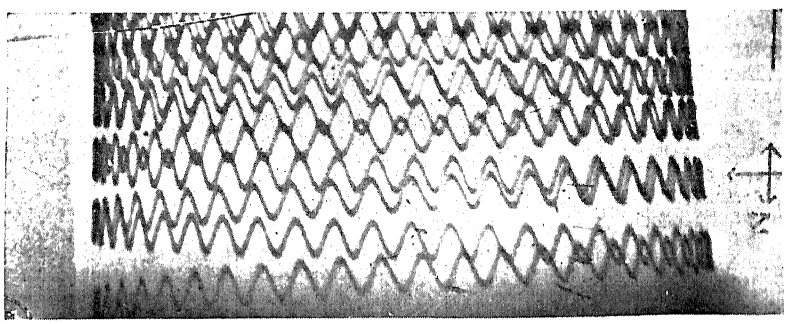


FIG. 19.

pairs of quadrants are shown, which are connected as shown in *Fig.* 18. The pressure between Q'_1 and Q''_1 will be equal to v, the potential difference applied to the circuit, and will produce a displacement y (say); the potential difference between Q'_2 and Q''_2 will be proportional to the current which is passing but will be 90° out of phase with it.

If x is the displacement of the cathode ray beam (which is proportional to the potential difference between the ends of the condenser $C_{v'}$ and $C_{v''}$) the current i which is passing may be written $c(dx/dt)$, where c is a constant depending on the capacity of the condenser and the size of the tube. The displacement of the pointer in the x direction, therefore, will be given by $dx=(idt/c)$, and the area of the curve formed on the screen by the cathode ray pointer, which will be equal to $\int_0^T ydx$, will be proportional to $\int_0^T vidt$, *i.e.* to the energy which is being expended in the circuit.

Not only can this instrument be used to delineate power curves, but, if the pointer is controlled, either by a magnetic field varying according to a sine law, or in some known way, it may be used to record wave shapes. One of the latest developments in the application of this form of apparatus has been the delineation of the wave shape of very high-frequency currents. The full details of this apparatus have not yet been published,[1] but the general scheme is evident from the oscillograms of an alternating current of 200,000 ~ per second that are given in *Fig.* 19. The "time" motion of the cathode ray beam is given by a magnetic field or electrostatic field of low frequency, while the current whose wave shape is required, produces a magnetic or electrostatic field in a direction at right angles to the time motion. For high-frequency wave delineation the cathode ray vacuum tube is by far the most promising apparatus that has been developed.

E. W. M.

ALTERNATORS, Description of (Alternating Current Generators). See "Dynamo Electric Machinery," § (12).

Design of. See "Dynamo Electric Machinery," § (4).

For supplying current to alternating current bridges. See "Inductance, The Measurement of," § (6).

High Frequency. For the production of continuous waves for wireless telegraphy. See "Wireless Telegraphy," § (17) (ii.).

AMALGAMATION of the zinc plate in an electric cell, as a protection from local action. See "Batteries, Primary," § (7).

[1] See "Cathode Ray Manometer," Vol. I.

AMMETERS or AMPERE METERS: instruments for the measurement of electric currents. See "Switchgear," § (26).

Alternating Current, Calibration of. See "Alternating Current Instruments," § (57).

Damping of. See "Direct Current Indicating Instruments," § (5).

Direct Current. See *ibid.* § (1), etc.

Dynamometer, Indicating. See "Alternating Current Instruments," § (8).

AMPERE: the unit of electrical current on the practical C.G.S. system of units,

1 ampere $=10^{-1}$ C.G.S. units of current.

See "Units of Electrical Measurement," § (21); "Electrical Measurements, Systems of," § (23).

International: the practical unit of electric current adopted by international agreement. One international ampere deposits per second ·00111800 gramme of silver from a solution of nitrate of silver in water. See "Electrical Measurements, Systems of," § (40); "Units of Electrical Measurement," § (31).

AMPERE HOUR METERS. See "Meters for D.C. Electricity," Part I.; "Alternating Current Instruments," § (34).

AMPERE TURNS: the product of the number of turns in a circuit and the current in amperes which is circulating in it. If the circuit take the form of a long solenoid the magnetic intensity at any point within it is equal to $4\pi/10 \times$ ampere turns per unit length. See "Dynamo-electric Machinery," § (1); "Electromagnetic Theory," § (13).

AMPLIFIERS: arrangements of thermionic valves for magnifying very minute currents.

Design of, for general laboratory measurements. See "Thermionic Valve, its Use in Radio Measurements," § (6).

Thermionic, Various Types of. See "Thermionic Valves," §§ (9), (12).

ANION: a term used in electrolysis to denote the constituent of the electrolyte which migrates towards the anode. See "Electrolysis and Electrolytic Conduction," § (1).

ANODE: a term used in electrolysis to denote the metallic conductor at which the current enters the electrolyte. See "Electrolysis and Electrolytic Conduction," § (1).

ANTENNA: The earthed. See "Wireless Telegraphy," § (12).

Calculation and measurement of capacity of. See "Radio-frequency Measurements," § (31).

ANTENNA ADJUSTMENT: tuning by inductance and by condensers. See "Wireless Telegraphy," § (13).

ANTENNA EFFECT: a source of error in direction-finders for radio-telegraphy. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (12).

ARC LAMPS

§ (1) CHARACTERISTICS OF THE ARC.—The *Electric Arc* is used for illumination because its high temperature offers one of the most efficient means of generating useful light rays.

The name "arc" is given to a stream of hot gases carrying an electric current across a gap between two electrodes. The arc is usually started by first bringing the electrodes into contact and then slowly separating them to the required distance when the stream of vapour is generated and maintained by the evaporation of one or both of the electrodes by the action of the current.

The temperature of an arc stream is assumed to be the same as the boiling point or the vapour point of the negative electrode. Carbon is generally used for the electrodes because it has the highest vapour temperature, and because it passes directly from the solid to vapour (*i.e.* without becoming liquid) and therefore retains its shape up to the moment of transition.

The characteristics of the arc are greatly modified by the composition and physical properties of the electrodes, also by the current density and the distance between the electrodes. All these variables render the action complicated, and, because of its relative simplicity, it is advisable to approach the subject by a study of the pure carbon arc.

Previous to the modern development of the high efficiency "flame" arc the arc between pure carbon electrodes was usually employed, and its characteristics have been very fully recorded, notably by Mrs. Ayrton in her book *The Electric Arc.*

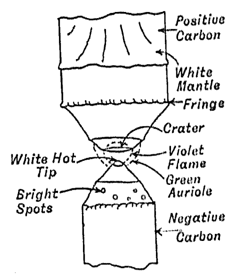


FIG. 1.

Fig. 1 (reproduced from *Electric Lamps* by Maurice Solomon) represents a typical direct-current open-type arc burning between carbon electrodes. The positive electrode is 18 mm. diameter and the negative 12 mm., these proportions giving even rates of consumption of the two carbons. The current through the arc is 10 amperes and the potential difference between the electrodes 40 volts. The vertical distance from the lowest point on the positive electrode to the top of the negative is 2 mm. The actual length of arc is greater

than 2 mm. because a hollow forms opposite the point of the negative. This hollow or crater in the positive electrode has a curvature of which the point of the negative is roughly the centre.

Between the surface of the crater and the point of the negative is an inverted cone of hot gases which carry the current from one carbon to the other.

The cone of gases in a pure carbon arc gives relatively little light, but an intense light is given by the surfaces that are being evaporated by the action of the current, viz., the crater of the positive carbon and the extreme point of the negative carbon. These two surfaces are of about equal intrinsic brilliancy, but the crater has much the larger area and is therefore the source of practically all the light.

Surrounding the cone of current-conducting vapour is a sheath of vapour which has escaped from the cone and has ceased to be part of the electrical circuit. This sheath resembles an ordinary flame. It gives practically no light.

The cone of conducting vapour is not constant in resistance and of itself it is unstable, but it can be kept constant by suitably varying the voltage across the electrodes, also by varying the length of the arc. The first is done by connecting a resistance in series with the arc, and the length of arc is controlled by an automatic mechanism which is fitted to each lamp.

This mechanism usually consists of an electromagnet carrying the arc current and having its armature arranged to pull the carbons farther apart when the current is above normal and retrace its movement when the opposite conditions prevail. It may have another magnet with a high-resistance winding connected as a shunt to the arc and with its armature arranged to pull the carbons nearer together when the voltage across the carbons

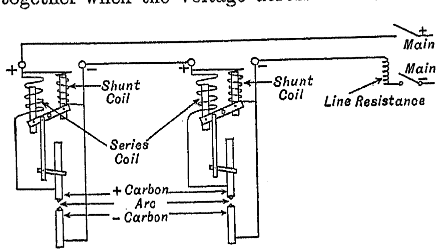


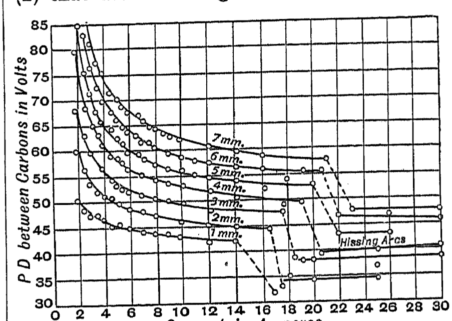
FIG. 2.

is above normal. Arc lamp mechanisms are described in more detail farther on in this article. The details of a typical arc lamp circuit are shown in *Fig.* 2.

The three zones in the arc, namely, the crater, the current-conducting vapour, and the bright point of the negative, have each their particular electrical characteristics. The crater of a pure carbon arc has a potential drop of 30 volts on, or very close to, its surface. The bright point of the negative electrode has a drop close to its surface but only of about 5 volts. Both these voltages are nearly independent of the amount of current passing. The areas of these two terminal regions vary directly as the current. If the current is increased quickly, the bright patch will be enlarged proportionally and will overlap the rim of the crater. If the current decreases the bright patch will cease to cover the whole of the crater, but in each case the crater ultimately burns to a diameter equal to the new diameter of the bright patch.

The cone of conducting gases also varies in area in such a manner that it is always approximately proportional to the current passing, but the voltage drop per unit of length *decreases* as the total current increases.

Summing up, it will be seen (1) that the terminal regions do not affect the value of the current, *i.e.* they cannot prevent it varying, and (2) that the cone of gases causes an unstable



P.D. and Current for Different Lengths of Arc.
Carbons: Positive 11mm.; negative 9mm.

FIG. 3.

condition. For instance, the current through an arc of fixed length and constant voltage will not remain constant at any value. If such an arc is above a critical length the current dies down and the arc goes out, and if it is below that critical length the current increases indefinitely. A line resistance is therefore connected in series with the arc, so that should the current increase the resulting rise of voltage across the resistance causes a decrease of voltage across the arc equal to the natural decrease of voltage along the cone of gases.

Mrs. Ayrton gives several sets of curves showing the relations between current and voltage and length of arc. *Fig.* 3 is one which illustrates the unstable feature very clearly, that is the fall of voltage which occurs when the current is increased and the length of arc is kept constant. The length of arc is

measured in the way described above and the full length of the vapour column is greater by a distance equal to the depth of the crater. For the present, attention is called to the inverse current-voltage characteristic for readings below 16 amperes. The sharp drop which occurs with higher currents, *i.e.* when the arc hisses, is of somewhat different character and will be dealt with later on.

§ (2) Arcs in Series or in Parallel.—Each series of lamps requires its separate steadying device, because if two series were connected in parallel and if they run on a common line resistance the current would grow in one series and die out in the other.

The mechanism which adjusts the length of arc does to some extent steady the arc and long series of lamps are easier to control than short series, because the variation in volts caused by a variation in any one arc is only a comparatively small part of the whole voltage drop and can be counteracted by the mechanism.

Ten " open type " arc lamps in series with 16 per cent of line resistance will burn more steadily than two lamps in series with 20 per cent. The arcs in both cases take about 40 volts each. " Enclosed " arc lamps having 70-75 volts across each arc require a larger proportion of line resistance, 20-30 per cent, because the long arc has a larger inverse-voltage factor. The effect would be more marked if the arc were not enclosed and so shielded from draughts.

A choke coil on an A.C. circuit and the self-induction in the open coil armature of the Brush Arc Lighter D.C. generator (a machine used in the early days of arc lighting for series constant-current lamps) act in the same way as a line resistance and do not absorb energy. The modern steadying device, which depends on the demagnetising of the field of the generator, is not quite so satisfactory because it does not answer instantly, but it has many advantages to set against that.

§ (3) Conditions of Steadiness. (i.) *The Continuity of the Arc.*—Another important characteristic which adds to the difficulty of maintaining a steady arc is the fact that if the current is interrupted for only a very short time it is impossible to restart the arc except by bringing the carbons together or by applying sufficient voltage to jump the gap. It is best explained by considering what goes on with an alternating current arc. This arc dies out at each half cycle and the current does not start to flow in the reverse direction until the voltage has risen sufficiently to jump the gap. The proportion of the cycle which must elapse before this critical voltage is reached depends on the temperature of the gases left in the gap, hence low periodicities will cause a greater lag, because the gases have more time to cool off before the voltage rises sufficiently. Again, electrodes having low vapour temperatures and low voltage per unit length of arc stream will not restart at all. Thus it is impossible to maintain an alternating current arc with such electrodes. In this respect pure carbon should make the best electrode because it has the highest vapour temperature, but in practice a carbon cored with a mixture of soft carbon and potassium silicate gives the steadiest arc with alternating current.

The explanation of this depends on various properties of the arc such as the resistances of the gases with and without the coring materials, the rates at which they cool, and the extent to which evaporation is continued during the break from one half cycle to the next.

Since the temperature of the carbon is above the vapour point of the other materials a fairly constant supply of gas is maintained. The noise with cored carbons is more musical than with uncored carbons—the crackling sound typical of the hissing D.C. arc is absent, but this may only show that the arc follows the same path at each half cycle and not that there is any marked difference in the continuity of evaporation.

When current ceases to flow at the end of each half cycle the full instantaneous E.M.F. of the circuit is then impressed on the electrodes, consequently the larger the steadying device (line resistance or choke coil) the earlier is the instant when there is sufficient pressure to jump the arc gap. A choke coil gives better results than a line resistance for two reasons : (1) it takes less energy than a resistance, and (2) it makes the arc current lag behind the E.M.F. of the supply, consequently when current ceases to flow through the arc the line E.M.F. has already reversed and has risen to an extent depending on the time during which the current lagged behind the E.M.F. With sufficient choke voltage it is possible to make the current restart immediately and so bring the " power factor " of the arc itself very close to unity. A.C. arcs with a low " power factor " (*i.e.* long idle period) are less efficient than when there is no idle period. " Power factor " of the arc itself must be considered apart from the overall power factor. When the latter is low, the former is high.

(ii.) *The Path of the Arc.*—Having provided for the continuity of the arc, either A.C. or D.C., the next thing is to fix its path and after that to supply it with a stream of vapour unchanging in quantity and composition. Position of the stream is influenced by draughts, by magnetic fields (including that of the arc itself), and by variation of the materials (physical and chemical) round the margin of the crater, or at any other point on the positive electrode to which the arc may take a new path through the hot gases which surround it. As regards the negative electrode the arc can

slide over it, but cannot jump from one point to another.

Even if a conductor more negative in potential than the negative electrode is introduced into the arc stream near the negative end, the arc will not at once transfer to it. A cold carbon of small diameter connected in this way may be passed across and through the arc stream without causing the arc to transfer to it. If the point is held in the arc stream until it becomes white-hot then the arc may transfer to it, but even this is probably due to a momentary disturbance of the original arc and to a static spark jumping from the point of the third carbon, which, according to the present theory, starts a new arc.

At the positive electrode, however, the arc stream can and does choose the path of least resistance if it is not otherwise influenced by draughts or magnetic fields.

Positive carbons are usually cored with materials which have been found to tend to keep the arc central on the carbon. The action appears to depend on these materials throwing off a stream of vapour of lower resistance than that which comes from the body of the carbon. The lower resistance may result from the physical condition of the core or from its chemical composition. For instance, carbon in a powdered form, loosely held together, will not have the heat-conducting property of the solid carbon body, and consequently will more quickly rise in temperature when exposed to the heat of the arc. As regards chemical composition it may be assumed that practically all substances, including those used for colour radiation, give off vapours having a lower resistance than carbon vapour.

Disturbance may occur through changes in the condition of the carbon body—for instance, the binding material may be burnt away by contact with the air and leave a layer of finely divided carbon on the surface. If the arc should leave its central position and so touch this layer of dust, then the resistance of the resulting vapour is less than the original stream, and the arc shifts bodily on to the layer of dust and generally takes a circular path round the cone before returning to the core. Somewhat similar disturbances are caused by layers of material which have condensed on the cone or the sides of the positive carbon. In open-type arcs the carbon passes away as CO_2, but some of the materials used for coring as well as some impurities from the body of the carbon condense and settle down as dust after they leave the arc stream. These layers of dust will lead the arc up the sides of the carbon, assisted by draughts and by magnetic effects, until the arc is so lengthened that it goes out or until the carbons are brought together by the controlling mechanism and a new arc thus started in the correct position.

Impurities in the body of the carbon are evaporated when the arc reaches them, and as they are generally of materials which give a much lower resistance stream than carbon the result is a sudden increase of current.

The foregoing are all disturbances due to variations of materials. Another set of disturbances can be said to be due to errors of position, that is, the positions of the various surfaces of the positive carbon in relation to each other and to the point of the negative carbon. The ideal conditions for steady burning are: no effects from draughts or magnetic fields, carbons exactly on a common axis, a true formation of the ends of the carbons and an arc stream of constant resistance and having its greatest conductivity at its centre and on the axis of the carbons.

(iii.) *Diameter of the Carbons.*—There are practical limits to the diameter of the carbon for a given current.

The arc will only remain central so long as the shell of the carbon is burnt away as quickly as the core. With open-type lamps the greater part of the shell just burns away and the resulting gases do not get into the arc stream, but if this burning does not take place fast enough the crater becomes deeper and its edge gets nearer to the point of the negative carbon, thus offering a path of less resistance, and the arc wholly or partly leaves the core until it has burnt away the over-prominent rim of the crater. It is necessary to avoid this wandering because the effect on the light is noticeable, and also the arc may touch a fringe of loosened or of deposited material as explained above.

There are also limits to the reduction of the diameter of the carbon for a given current. In practice it is the quick rate of consumption and the great length that would be required, but beyond that is the necessity of maintaining sufficient area at the end of the carbon to prevent the arc overlapping, and this is particularly so with a pure carbon arc. Overlapping in this instance may cause the arc to hiss.

In the foregoing it has been assumed in every instance that owing to its higher temperature the centre of the arc stream has less resistance than any other path open for the current. This supposition has also been made for the terminal regions, but it is probably not true of the positive terminal region in some instances. This region has the greatest voltage drop per unit of length, and the drop is greater the higher the vapour temperature of the electrode; this is highest with pure carbon. If the surface of the carbon becomes irregular, and if one of the projecting points is bathed in the hot gases which immediately surround the arc stream, the voltage between that part of the stream near the terminal region and the point on the positive carbon may be sufficient to cause a discharge through the

gases; the arc shifts bodily to this new position. It will be understood that this discharge occurs between two parts of the positive terminal region, and does not pass direct from the negative electrode.

For picture projection work it is necessary to displace the negative carbon from the axis of the positive so that the negative does not throw a shadow on to the picture. The axes of the two carbons may be kept parallel or may be placed at an angle to each other. In both cases the ends of the carbons assume shapes which are not symmetrical with the general line of the arc stream, and there is consequently a tendency for the arc to shift to the "high" points. This is usually counterbalanced by a magnetic field arranged to bend the arc stream, so that it starts in a direction normal to the cone of the negative and ends as a normal to the crater of the positive carbon.

§ (4) The Hissing Arc.—Under certain conditions the positive end of the arc stream will travel over the carbon at a high speed—too quickly for all its movements to be followed by the eye. There is a distinct falling-off in light, and a loud noise which has given it the name "hissing arc."

An ordinary carbon arc burning in air will always hiss when the current density in the positive carbon exceeds the critical value for that particular carbon. Hissing will begin below this value with a short arc. It may also start if a sudden increase of current causes the arc stream to expand beyond the rim of the crater, or if one of the carbons is moved sideways or the arc is blown sideways.

When hissing starts there is always a sudden fall in the arc voltage, and this fall practically all occurs near the positive crater region. This drop is shown by the dotted parts of the curves already given in *Fig.* 3. Mrs. Ayrton gives the results of a long series of tests on hissing arcs. Her conclusions are that the hissing and the fall of potential are due to the oxygen of the air getting to the crater surface, because an arc will not hiss if surrounded with nitrogen or carbon dioxide. The explanation suggested for the quick movements is that air first combines with the carbon on part of the crater surface, then the products of combustion temporarily shield the surface but afterwards disperse, and a fresh supply of air rushing in the action is repeated.

As an alternative theory the present writer suggests that the skin of unoxidised gases immediately surrounding the arc stream near the positive crater is disturbed when the arc grows in area beyond the rim of the crater and coming in contact with the air undergoes a change which reduces its resistance and so *attracts* the arc. The arc then continues its movement as the layer of gas on its forward side is changed in its turn. This presumes an orderly movement, and it would account for the humming which precedes the hissing when the current is slowly increased. The hissing may be started when air rushes in to fill the vacuum caused by the gases cooling in the rear of the moving arc. A strong rush may catch up the arc and, acting on its rear surface, may reduce the resistance at that point, and by making the direction of travel erratic cause a hissing instead of a humming sound. The foregoing all refers to arcs with ordinary carbons. Flame-cored carbons work at much higher densities before hissing, and this fits in with the theory now put forward because flame arcs require less voltage drop per unit of length and work at lower temperatures, and therefore are not so easily short-circuited by the surrounding gases. The theory that the crater region of a pure carbon arc may be short-circuited by the surrounding gases will account for many of the difficulties experienced with pure carbon arcs and explain why coring with substances other than carbon overcomes these difficulties.

§ (5) Effect of Magnetic Fields.—Magnetic fields acting on the arc are of three classes: (i.) From external sources, (ii.) from the current-carrying parts of the lamp, and (iii.) from the current in the carbons and in the arc itself.

The effect of a magnetic field on the arc is to force the arc in the direction which will cause the greatest increase in the number of lines of magnetic force linked with the circuit of which the arc forms part.

Generally speaking, the carbon arc works best when not influenced by any magnetic field, but one is often used to counteract the effect of draught and also for bending the arc stream to alter the direction the crater faces as already explained. Pure solid carbon arcs do not work satisfactorily with a curved arc stream, but cored carbons work well enough if the arc is long, say one requiring 60 volts or more. Flame carbons work well with a curved arc stream at as low a voltage as 38 volts.

With regard to the three classes of field, (i.) the external fields do not often cause trouble, but it is advisable to bear the question in mind both when designing and when erecting lamps—particularly lamps with long arcs such as those used for photographic work. Alternating current arcs are only influenced by A.C. fields in synchronism. The effect of the Earth's field is distinctly noticeable on D.C. lamps. In north latitude its vertical component causes a horizontal arc to curve to the left, looking from positive to negative. The effect is seen in the type of flame lamp which has converging carbons with the arc at the lower ends, and lamps with long iron frames or cases show it more than others. It is usually counterbalanced by making one pole of the blow magnet (used for bending the arc downwards) stronger than the other or by the stray field from one of the operating

solenoids. Lamps for use south of the equator require the opposite effect.

The horizontal component of the Earth's magnetism cannot be counteracted permanently unless the lamp is always hung one way round. It causes a vertical arc to curve towards the east if the upper carbon is positive. Its intensity varies with the latitude but its direction is constant.

When lamps are hung close together the field from the operating solenoid in one lamp may act on the arc of another lamp, although it will not affect the arc of its own lamp if the arc is vertical, because the field is vertical in that region and therefore in line with the arc stream.

(ii.) *Fields from Parts of the Lamp carrying Current.*—These include those from operating solenoids and magnets, which, however, can usually be so placed that the resulting field is in line with the arc stream and therefore does not influence it. Parts carrying the main current near the arc itself are not so easily dealt with. When possible the current should be split and carried equally on each side of the arc. This is not always practical in a searchlight, and with currents of 100-200 amperes the field is relatively strong. If the carbons are horizontal and the connections to the holders come from below there will result a strong field which curves the arc upwards. There will also be the up-draught caused by the heat of the arc. It is usual to counteract these forces either by an electromagnet (or a solenoid with or without an iron core), or by so placing a piece of iron that it attracts the arc by virtue of the magnetic field generated by the current in the carbons and in the arc itself.

The usual form for the last mentioned is a "C"-shaped piece of iron concentric with arc and with the gap at the top.

With heavy current searchlights, say 100 amperes and upwards, it is possible to get the necessary control by passing the main current through one or more conductors placed parallel with the arc. If the current passes in the same direction as through the arc the conductor attracts the arc and *vice versa.* Better results are obtained by repulsion because, as the arc retreats from the conductor, it passes into a weaker field and the forces thus tend to balance. On the other hand, if the arc is attracted towards a conductor it enters a stronger field and so tends to move nearer to the conductor. The same is true of pieces of iron arranged to attract the arc, such as the "C"-shaped iron mentioned above.

(iii.) *Magnetic Fields produced by the Current in the Carbons and in the Arc itself.*—These act in the same way as those of Class (ii.) for any definite position of the arc, but the arc may take up a new position and so change the conditions. A full consideration of this point will explain why small-diameter, coppered-core, negative carbons were developed for searchlights and cinema projector work and why they give so much better results than the earlier types.

First consider an arc, say of 100 amperes, with carbons on a common axis and the arc for the time being on this axis. If the conductors leading to the carbons are so arranged that they do not produce a field near the arc or that their field is in some way counteracted, then there will only be the usual circular field round the carbons and the arc typical of a long straight conductor.

Now consider what happens if the arc moves bodily sideways, say to the edge of the carbon, but still keeping its axis parallel with that of the carbons. The current will still flow evenly along the carbon most of the way, but near the end it will be crowded over to the point from which the arc starts, and thus for a short distance the mean path of the current is not axial and the current at this point will therefore have a magnetic effect on the arc. The result is a curved arc stream, the amount of curvature depending on the distance the arc is displaced from the axis. Large diameter carbons will show this disturbance much more than those of small diameter. When the arc has been curved in this way each part of the arc will then have a magnetic influence on the remainder, thus tending to enlarge the loop, but at a position beyond the tangent to the mean path in the carbon a balance is obtained between the two forces —the current in the carbon now acting as a

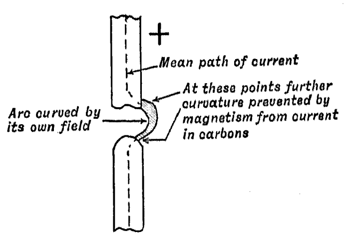


FIG. 4.

restraining force to prevent further curvature (see *Fig.* 4).

The modern negative carbon for searchlights and cinema projectors is made small in diameter and has also special means for keeping the current-path central. It is provided with a high conductivity core of hard carbon and the core is heavily coated with copper. This core is virtually the electrode proper—the use of the outer shell is chiefly to support the core and to protect it from oxidisation. The positive carbon is also made small in

diameter, the current density being increased until it is as near the hissing point as is practical.

The magnetic forces acting on the arc have to be considered in conjunction with draughts due to the heat of the arc. With horizontal carbons the forces are very similar in their effects when both are acting in an upward direction. With vertical carbons the draughts do not play a part until the arc has first been displaced by other forces, but then the effect may become serious, particularly with long arcs such as those used for photographic work. An arc for this purpose is 2 in. or more in length, and is enclosed in a relatively small globe, but sufficient air remains in the globe to cause considerable convection currents. If the up-draught gets full control of the arc stream the latter takes a shape like the figure 7, the horizontal limb starting from the side of the positive carbon. The magnetic effect of the current in the positive carbon restrains the arc from over-curvature in the manner already explained, but this force is greatest next to the carbon. The up-draught acts on the horizontal limb of the 7 and the two forces together are the cause of the elbow in the arc stream (see *Fig.* 5). In practice

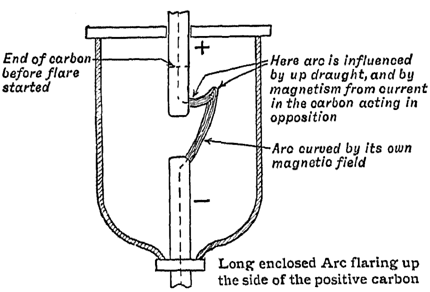


Fig. 5.

successful operation is attained by using small diameter carbons of good quality—if there is nothing to cause the initial disturbance the up-draught does not get a chance to displace the arc.

§ (6) Best Forms of Arc for Projectors.—In the foregoing it has been assumed that an arc stream maintained on the axis of the carbons gives the best results for projectors. This is true of a pure carbon arc because it will not burn steadily unless the stream is maintained on that line. When flame carbons are used the current density in the positive carbon may be increased considerably without causing hissing, and then the greatest brilliancy for that type of carbon is obtained by forcing the negative stream away from the positive, so that there are then two streams distinctly visible, the true electrical discharge coming from the negative, and a stream of evaporated material coming from the positive. These streams cross each other and so complete the electrical circuit. The negative carbon is set at an angle to the positive and below it so that its shadow does not fall on the mirror. Magnetic and draught effects are such that the two streams meet near the positive carbon but well above a line joining the point of the negative and the centre of the positive crater (see *Fig.* 6). The positive stream is considerably diverted above the axis of the positive carbon and passes so close to the upper lip

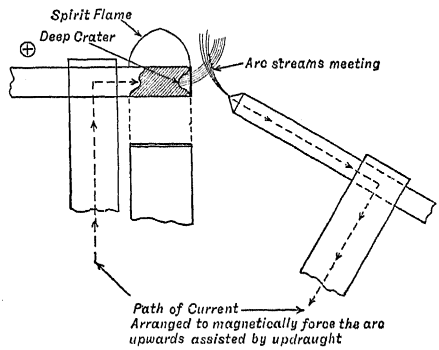


Fig. 6.

of the crater that the arc is inclined to transfer itself to the lip.

The Beck Arc.—The crater formation is preserved by rotating the positive carbon slowly, and in the "Beck" arc this action is supplemented by surrounding the carbon (but not the arc stream) with a flame of methylated spirit. If the spirit flame plays on the arc stream the latter makes a whistling noise and becomes very unsteady.

The action of the spirit flame on the positive crater appears to be twofold. It preserves the edge of the crater by preventing air getting to it and it reduces the conductivity of the vapours in that region; possibly this is also because the air is kept away. The crater becomes very deep because the arc cannot transfer itself to the lip. At the bottom of the crater there is an area of the greatest brilliancy that has been obtained so far. However, there are several objectional features to set against it. For instance, flame carbons generate fumes which may condense on the mirror, and they also give a long luminous flame, the image of which is inverted and thrown on the foreground.

It has not been determined how many of the special features are necessary to get this increased brilliancy, but it is certain that if, other things being unaltered, the negative stream is allowed to impinge straight into the crater, the voltage across the carbons drops and the brilliancy decreases very considerably.

§ (7) Flame Arcs.—A flame arc is more efficient in illumination than a pure carbon arc because the former gives "coloured" radiation, *i.e.* gives a larger proportion of the wave-lengths to which the eye is sensitive. The light from the crater of a pure carbon arc gives "black body" radiation corresponding to its temperature, which is the vapourising temperature of carbon. All wave-lengths are given in the visible range, and for a considerable range on either side of it, so that a large proportion of the energy is wasted in invisible radiation or in wave-lengths to which the eye is not very sensitive. The carbon arc is converted into a flame arc for coloured radiation by feeding into the arc stream materials which radiate a larger proportion of useful wave-lengths, notably calcium salts, which as gases heated to the temperature of the arc stream give off a preponderance of yellow rays, the colour to which the eye is most sensitive. Such materials are usually introduced into the core of the carbon. They are insulators when in the solid state, but become conductors when evaporated, and in that way they modify the electrical condition of the arc stream besides modifying the light.

The electrodes themselves, as well as the means of controlling the arc, have been developed mostly by experiment, and it has been only during the last ten years that theory has assisted to any extent, notably in the selection of materials to give coloured radiation and the method of carrying them into the arc stream.

In a paper on Yellow Flame Arcs read before the I.E.E. in 1912 Mr. Solomon gave the following figures:

	Watts per Candle Power.	
	Mean Spherical.	Lower Hemispherical.
1. Converging carbon flame lamps	0·276-0·313	0·145-0·159
2. Enclosed flame lamp	0·458	0·374
3. Open type with Blondel carbons (end on)	0·178	0·095

Other things being equal, the efficiency of a yellow flame lamp increases with the proportion of flame materials in the arc. When these are above a certain proportion they cause flickering.

Class 1 contains a high proportion, but the carbons are of small diameter and therefore work steadily. Class 3 contains a larger proportion, the carbons, however, are relatively large in diameter, and the steadiness not so good as in No. 1.

The effective proportion of flame material in the arc No. 2 was low, moreover the wattage of this arc was 357·5 as against 435-450 for No. 1, and 396 for No. 3. The efficiency, therefore, was reduced in consequence. Since that date considerable improvements have been made in carbons for enclosed flame lamps. See "Carbons for Arcs, the Manufacture of."

§ (8) Characteristics of Arcs in Air, in Inert Gases, and in Vacuum. *Enclosed Arcs and Economisers.*—Arcs burning in vacuum or inert gases are of no use for illumination, mainly because it is necessary to carry away from the arc all the material which is evaporated. If any of it condenses on the carbon unsteadiness results. Even in air a short arc will "mushroom"—that is, carbon will build up on the negative, destroy its symmetrical shape, and cause unsteadiness, besides blocking the light. However, in "enclosed" lamps and lamps fitted with economisers advantage is taken of gases less active than pure air. In enclosed lamps the oxygen becomes exhausted by combustion of the carbon, also the heat in the enclosure rarefies the gases. The inlet for fresh air and the size of the globe are adjusted until the burning away of the carbon is just fast enough to keep the ends clean. The carbons become practically flat on the burning ends and the arc is continually moving about burning each part in turn—the area of the carbon being much greater than the area of the arc.

When flame carbons are used in enclosed lamps it is necessary to deposit the fumes on surfaces other than the enclosing globe. This is done in several ways, but all depend on convection currents carrying the gases into additional chambers and over large cool surfaces on which the fumes are deposited, the gases returning to the globe after they have been clarified. It is necessary to keep the globe hot to prevent condensation on its surface, and therefore it is generally surrounded by another globe which keeps the cool outer air away from the inner globe.

Economisers retard the consumption of the carbon by restricting the circulation of air in the neighbourhood of the arc. They are principally used for open-type flame arcs, both those with vertical carbons end on and those with converging carbons. They are usually bell-shaped and the carbon (or carbons) pass through close-fitting holes at the top of the bell, the arc being located in the mouth of the bell. The air in the bell is rarefied by heat and is also robbed of oxygen to a certain extent, so that the higher the arc is in the bell the slower is the burning of the carbon which is above it. With end-on carbons the bell therefore counteracts uneven burning of the top and bottom carbons and keeps the arc fixed (focussed). The same is true with converging carbons—if one carbon burns slower than the other its point stands lower down

and, coming in contact with less heated and denser air, is burnt away faster. This property of the economiser is essential to the success of the converging carbon lamp. Without it the points of the carbons would not remain level with each other—the smallest difference in their rates of burning would soon put them right out of balance because in unrestricted draughts the lower point burns slower than the higher one. Even if the points remained level the unrestricted draughts would make the arc very unsteady.

Some economisers are made of fireclay and some of iron—the latter are used for magazine lamps; they collect a layer of white deposit from the arc gases and reflect as much light as the fireclay type. The iron bell is also used as a shield from a strong magnetic field located above a slotted horizontal partition in the bell. This field is used to blow out the arc which is formed at the carbon grips when the stumps of carbon are ejected. The field also extinguishes the arc on the last pair of stumps for which there is no means of ejection, and it is used for lamps with one pair of carbons for this latter purpose.

§ (9) Automatic Arc-controlling and Carbon-feeding Mechanism.—The function of the automatic mechanism is to start the arc and to maintain it at the desired current and voltage, also to feed the electrodes as they are consumed.

The mechanisms may be divided broadly into two classes—(i.) those which strike an arc of definite length and afterwards feed the carbons step by step, and (ii.) mechanisms with a floating adjustment.

(i.) *Step by Step Mechanism.*—In this class the usual arrangement consists of a series magnet which separates the carbons to strike the arc by moving over a fixed distance, and a shunt-controlled mechanism which feeds the carbons when the voltage of the arc exceeds a certain value. Some of the large searchlight lamps are built on this principle—the moving parts being too heavy to control with a floating adjustment. With the non-floating mechanism there is no retrace—that is to say, if through some irregularity in the supply voltage or in the electrodes the voltage across the carbons rises momentarily the carbons will feed until the voltage drops. After this feed has occurred the cause of the disturbance may be removed, in which case the carbons would need to be pulled apart to a certain extent to bring the arc back to its normal condition, but as there is no retrace in the feed mechanism the arc has to burn at the reduced length until some of the carbon has been consumed and normal length of arc regained in that way.

The general effect on the arc of a mechanism which has no retrace is to reduce the average voltage and increase the average current passing through the arc when the conditions of the circuit are unstable from any cause. This characteristic has its good points. The increased current causes an increased drop in the line resistance and thus a natural increase of the steadying effect given by the line resistance. Therefore, with a mechanism having no retrace the lamp will burn at a higher current and lower voltage across the arc when the circuit conditions are unsteady, whilst with everything under the best conditions the average voltage of the arc will approximate the voltage to which the feeding mechanism is set. An unsatisfactory feature of this mechanism is its inability to bring the carbons nearer to each other quickly when conditions are unstable, thus allowing the arc to go out frequently when it might have been maintained.

Owing to the unstable nature of the arc, it is advisable to counteract the variations in the composition of the electrodes and any effects due to the arc wandering over the surface of the carbons, also those which occur due to the supply voltage not being constant, and to other lamps in the circuit not functioning properly.

(ii.) *Floating Mechanism.*—With a floating mechanism any reduction of the arc current immediately brings the carbons nearer to each other. It may also cause a feed in the same way as with a non-floating mechanism, but when the circuit conditions return to normal the mechanism retraces the carbons all or part of the way and brings the conditions in the arc to normal. Generally speaking, it is not advisable to have a floating mechanism adjusted to a straight line characteristic. It gives better results if it makes the lamp take a little more current when the mechanism is above the feeding point. This gives the floating mechanism a somewhat similar characteristic to that of the non-floating mechanism, namely, an unsteady circuit tends to make the lamp take more current and thus increase the steadying effect.

The same may be true of the dash-pot arrangement which is used for preventing quick movements of the mechanism. It is advisable to have a valve in the dash-pot which retards movements which separate the carbons and allows quick motion in bringing the carbons together.

§ (10) Forces used for controlling the Mechanisms.—With mechanisms of the non-retracing type electromagnets are generally used. It is of very little consequence whether the pull of the magnet is constant over the length of the movement, because it is only necessary to move the mechanism over a definite distance and then to arrest it.

Again, as regards the shunt-controlled feed gear the usual thing is to release a detent controlling a train of wheels and a retarding

fan, and this detent can be operated practically as well with a magnet as with a solenoid.

Another type of feed gear for this class of lamp consists of a shunt magnet and vibrating armature on the same principle as the ordinary electric bell, the vibration of the armature being used to operate a ratchet-wheel step by step and so feed the carbons. The armature is controlled by a spring which is adjusted so that the armature begins to vibrate when the feeding voltage is reached and ceases to vibrate when the voltage drops below the critical amount.

In mechanisms of the floating type solenoids give better results because the pull is more even, also there is very little side pull, and, if the solenoids *lift* the cores, a minimum of friction on the pivots. Magnets have been used for operating floating mechanisms, but it is necessary to shape the pole pieces and arrange the armature so that it has an oblique approach, so as to give an approximately constant pull over the working range. All this, however, means very close adjustment and strong construction; because if the parts become worn or displaced the characteristic of the pull is altered altogether.

The controlling windings for the various types of lamp are as follows:

(i.) *Lamps in Parallel.*—For lamps to burn in parallel a series-wound solenoid is sufficient. The lamp then takes an approximately constant current, the voltage of the arc being controlled by the voltage of the supply and the amount of line resistance in the circuit—that is to say, if a lamp is burning on a 100-volt circuit and the solenoid is wound to balance a gravity or spring controlled mechanism when 6 amperes are passing through the solenoid, and if the line resistance is 5 ohms, the result will be an arc of 70 volts. If now the supply voltage is increased to 110 without other conditions being altered, the arc voltage will go up to 80, because the current will remain constant and the absorption in the line resistance is therefore constant, hence the extra voltage has to be absorbed in the arc.

Series windings have also been used with fair success for short series of what are known as "shuntless" lamps, but in that case the solenoid is constructed to depart considerably from a straight-line law. The system will be best explained by considering what would happen if a number of ammeters, say 4, are connected in series, the ammeters being identical in construction and calibration. If 5·5 amperes pass through these meters all the needles will stand at a certain position on the dials, and if the current is increased to 6 amperes they will all move a definite distance and take up a 6-ampere position, and so on to another position if the current is increased to 6·5 amperes.

Now imagine that the distance between the 5·5 ampere and the 6·5 ampere position is equal to the length of a 6·5 ampere arc of say 70 volts, then imagine the "ammeters" are strong enough to operate 4 lamp mechanisms attached to the points of the needles so that when the four needles reach the 5·5 ampere positions they engage the clutches of the carbons and when moved further up 4 arcs are struck, the arcs being in series with the ammeter windings. With such an arrangement the four lamps will strike up and burn at 6·5 amperes until the carbons have been consumed to some extent. Then as the current drops the needles will sink back gradually, until when the mechanisms have retraced their movements half-way all the arcs will be burning at 6 amperes and, providing the carbons have been consumed evenly in the four lamps, the four arcs will still be of equal length although slightly longer than they were when taking 6·5 amperes.

This action goes on until the mechanisms further retrace their paths and get near to the 5·5 ampere positions where the clutches originally gripped the carbons. At this position, however, a further consumption of the carbons and a further reduction of the current would make the clutches lose their grips of the carbons, and as this may occur in one and not happen in the other three, it would be impossible to ensure an even feeding of the carbons, therefore a mechanism is added which extinguishes all the arcs just before the clutches would be opened say at the 5·75 ampere position. This arc-extinguishing mechanism may consist of an additional magnet capable of arresting the mechanism even when say only 50 per cent of the normal current is passing, and fitted with a pin and slot device which is free at the 5·75 ampere position and upwards, or it may be that the solenoid core is made in two parts, also with a pin and slot device, or again it may be simply a mechanical stop which temporarily arrests the mechanism on the downward stroke when it falls to the 5·75 ampere position.

It is essential that the arcs are extinguished and all pairs of carbons brought together, so that when each clutch takes a fresh grip of its carbons all arcs are restarted evenly and the sequence of motions repeated.

With enclosed lamps, for which type this shuntless mechanism was designed, the sequence of operation need not occur more than once an hour, and the momentary blink due to the resetting action is not objectionable at this long interval.

(ii.) *Lamps in Series.*—Series enclosed arc lamps have been controlled by the expansion of a stretched wire which carries the main current. When an excess current passes it allows the carbons to be separated either by a spring or counterweights, and when the current decreases the wire cools, contracts, and pulls the carbons nearer until a balance is attained. The action is the same as if the above-mentioned "ammeters" were hot-wire meters instead of electromagnetic. The heating and cooling of the strip is slow, so no dash-pot is necessary, also no restarting gear is necessary because the movements are so

sluggish that the arcs go out much more frequently than is required to stop individual feeding; in fact, that is where the system fails —too frequent extinctions and long periods of darkness whilst the wires are cooling down sufficiently to open the clutches.

Arc lamps fitted with shunt control only have been made for working in series, but they give very poor current regulation except when there is a large proportion of line resistance. On a typical circuit, say two open-type arcs on 110 volt with 42 volts on each arc, assuming the shunt control follows a straight-line law and has only a 5 per cent lag due to friction, then the current will vary in proportion to the variation of voltage across the line resistance. Five per cent of the arc voltages is 4·2, so the current will drop as 30·2 : 26 or 14 per cent.

With two 40-volt arcs on 100 volts the current will drop 17 per cent. Variations of the supply voltage have a similar effect. An increase from 110 to 115 volts will increase the current as 26 : 31 = 19 per cent, and in the second circuit an increase from 100 to 105 volts as 20 : 25 = 25 per cent.

The pull of the shunt solenoid or magnet is counterbalanced by gravity or a spring, therefore the carbons separate when the circuit is switched off, and are pulled into contact when switched on, afterwards separating until the rise of voltage across the arc and across the shunt circuit increases the pull enough to cause a floating balance.

(iii.) *The Differential Control.*—A differential control for a floating adjustment combines the best features of the simple series- and shunt-wound types, and the great majority of modern arc lamps are controlled in this way. The typical arrangement consists of two solenoid cores attached to the opposite ends of a lever, one being pulled upwards by a solenoid carrying the arc current and the other is pulled upwards by another solenoid wound with fine wire and connected as a shunt to the arc (see *Fig.* 2). The solenoid carrying the arc current is usually the stronger. The carbons are brought together by gravity when the lamp is switched off, and the difference of power between the two solenoids is used for balancing this gravity effect. A common relation between these forces is main coil 3 balancing the shunt coil 2 and gravity 1, but when the length of arc is adjusted by moving the points of the carbons in a line at right angles to the direction of feed, as in converging-type flame lamps, the proportion of gravity effect may be reduced and the coils made equal in strength or nearly so.

The characteristic of a lamp with shunt and series coils equal in strength is such that current and voltage across the arc rise and fall together and equally as the line resistance or the line pressure is varied.

When the forces are respectively 3, 2, and 1 as explained above, then a 2 per cent increase of current will cause approximately a 3 per cent increase of the voltage across the arc.

In another type of differential control the shunt coil is placed below the series coil, the one iron core serving for the two coils. This arrangement works at its best when the two coils magnetise the core in the same direction. With coils magnetising in opposite directions one coil will repel the core when it is mainly in the other coil.

A third type of differential control consists in putting the shunt and series windings on the same bobbin, winding one over the other and connecting them in opposition. Regulation is obtained by the shunt weakening the pull of the series, but it must not be possible for the shunt to become the stronger, else it will separate the carbons when its proper function is to allow them to be brought together. This arrangement will work well enough for say 2 lamps in series, where the full circuit voltage on one of the shunt coils would not be strong enough to lift the iron core, but even then the shunt has to be made relatively weak and that gives poor balancing of the two arcs.

(iv.) *Magnetic Control.*—Magnetic deflection of the arc has been tried as a means of maintaining the current at the desired value, but because it disturbs the natural alignment of the arc it tends to cause unsteady burning. However, the current in the converging carbon flame arc is momentarily controlled by the blow-magnet to some extent, but that is because the blow-magnet is instantaneous in its action. At the most it only smooths out the ripples which the floating regulation cannot deal with because it is too sluggish.

§ (11) Types of Control Mechanism.—There is an endless variety of mechanisms for operating arc lamps, but the differences are mostly in detail. They can be classified by the type of feeding gear used, and generally success depends on the efficiency of the feeding gear. A differentially controlled floating regulation can fully control the arc whilst it retains a hold on the carbons. As the carbon burns away the mechanism returns to the point where it first gripped the carbon, or some part attached to it, and there, at what is called the feeding-point, it must be possible for the carbon to slip through the grips. In open-type lamps, on account of the short arc and the rapid consumption of the carbons, the feed must be so gradual that it does not cause a blink in the light.

A train of wheels with a retarding fan or an escapement controlled by a detent gives

excellent results, but needs more care than a clutch or a brake.

(i.) *Clutches and Brakes.*—These are generally applied in such manner that the grip increases as the resistance to movement increases. This is a precaution against slipping when the lamp is pumping.

With only 16 to 20 per cent of line resistance and the self-induction of the series coils to limit it, the instantaneous current may be four times the normal and at a time when there is no voltage on the shunt coil. Dash-pot action cannot be increased because it would retard recovery after a feed, hence the clutch or brake must be very positive in its action.

Clutches are very simple, the simplest of all being the washer clutch, which is a plain washer having a hole very slightly larger than the rod it grips. When lifted at a point near the rim it tilts and grips the rod. The heavier the rod the harder it is gripped. On returning to the feeding-point the lower end meets a stop which reduces the angle of tilt and allows the rod to slip. The strength of the grip depends on the ratio between the horizontal distance from rod to lifting-point and the vertical distance between the two gripping-points. Therefore when the hole wears, or if a smaller rod is used, the tilt is increased and the grip weakens.

The above-mentioned ratio must not be less than 4 to 1, and this is true of most clutches and brake gear.

Improvements on the washer clutch consist in extending the leverage of the lifting-point and in applying a spring both to assist in gripping and to bring the clutch back to the gripping angle quickly when the lamp is pumping. With a suitable spring the clutch may even gain on the rod, *i.e.* the rod is jerked upwards by the rush of current and when the mechanism is checked by the dash-pot the rod travels through the clutch, but is caught directly it begins to fall.

Developments of the washer clutch consist in a long sleeve through which the rod slides and a horizontal lifting lever pivoted to the sleeve. A short extension of this lever just above the pivot engages with the rod and grips it. The leverage can be kept constant independent of wear.

Some brake wheels are operated by what is virtually a clutch acting on the inner and outer surfaces of the rim of the wheel or between a central boss and one of these surfaces.

Flexible band brakes are used in two ways. The two ends may be attached to a lever at points which compare with the two gripping-points of the washer clutch, in which case the grip is proportional to the resistance it meets, or one end may be connected directly to the operating gear and the free end of the band attached to a spring, which pays out on the "strike" and takes up the slack on the return movement until the feeding-point is reached, when the band is loosened to let the wheel slip.

(ii.) *Feed Control.*—Fineness of feed depends on the nature of the gripping surfaces and on the rods or brake rims being parallel and true. Rubber grips give a creeping feed but lose their resilience, also they wear and alter the feeding-point. Metal chains used as band brakes give the best metal to metal feed. Metal clutches on parallel rods do not give a creeping feed, but will do so if the rod is made taper. Most excellent results are obtained with a rod having not more than $\frac{1}{8}$ per cent taper. Feed rods with a number of $\frac{1}{8}$ per cent slopes and steps cause a severe blink once every five hours, but during those intervals the feed is so good that the rod has to be marked to show its movement. Such slopes necessitate not more than a 15 per cent increase in the range of the striking gear, *i.e.* the feeding-point varies 15 per cent of the total movement of the clutch.

Rotary motors are used to control the feed in some lamps. The motor may be wound differentially and so control all movements of the carbon, but they are usually too slow when shortening the arc, and let it go out when a quicker movement would keep it going. In a modern searchlight burning flame carbons, a shunt-wound motor is used for feeding only. The field cores work at high density so that speed varies with the voltage, and this automatically keeps the arc volts constant.

Reciprocating motors (an elaboration of the vibrating armature already mentioned) are used for magazine flame lamps for feeding the converging carbons. Arc adjustment is done by swinging one of the carbons laterally. The motor is stopped and started by variation of voltage (in which case the fall of the core does the work) or by an arrangement similar to the "feeding-point" control of a clutch. In the latter arrangement, when the carbons have been swung towards each other to the desired limit, a shunt make-and-break gear is released and continues acting until the feed allows the carbons to be swung apart again. The make-and-break gear controls a shunt-wound solenoid whose core operates the feeding gear. The carbons are of small diameter, but are gripped near their lower ends and current is led through the grips. Therefore it has only a short distance to go along the carbons, and it is not necessary to copper them or use metal cores.

The magazines hold up to twelve pairs of carbons, each 15 inches long, giving 120 to 150 hours' burning and the option of retrimming at any time without waste. The small diameter carbons give the steadiest flame arc

obtainable and average seven candles per watt over the ordinary lighting angles.

The carbons are either forced down by two dredger chains with "buckets" engaging the top ends of the carbon or by two fingers which are carried up and down by a cross bar. The cross bar is racked up and down by a reciprocating rack rod engaging reversible pawls, a second (fixed) rack being used to support the bar during the idle part of the stroke.

Each pair of carbons pushes down the burning stumps of the previous pair and finally ejects them. There is an interval of darkness not exceeding two seconds in the latest type of lamp. This is attained by gripping the carbons at a point not more than ¼ in. above their highest apex and allowing a long swing inwards from the feeding-point, so that the new carbons first touch high up in the bell and the arc is struck whilst they are feeding down to the normal burning position.

In another type two sets of magazines are used and carbons fed alternately from each set, the arc changing over from one to the other without an interval of darkness.

In all converging carbon flame lamps, and particularly in magazine lamps, there is a chance that some of the deposit on the economiser will become dislodged and get wedged between the carbons and insulate them. In the rack-fed lamp mentioned above, this trouble is overcome by oscillating one of the magazines, at right angles to the line of the arc, each time the carbon feeds. This rubs the carbon points together and dislodges anything that gets between them.

A very simple type of feed control which has been used for converging carbon flame lamps works on the same principle as the candle feed in a carriage lamp. The burning end of the negative carbon rests on a metal abutment, and as it burns away the part resting on the abutment crumples up and so lowers the carbon. The positive carbon may be supported from the negative so that the two are lowered together. The abutment cannot be used on the positive carbon or for alternating current because of danger of the arc transferring to it.

Unsatisfactory features are the distortion of the end of the negative and the excessive condensation of flame material on it, both of which cause the arc to wander.

§ (12) PROTECTIVE DEVICES.—Lamps working more than two in series require their shunt coils protected against the chance of the full voltage of the circuit being maintained on one lamp. This will happen if one lamp has no carbons or if they are held apart by some mechanical fault. The carbons in the other lamps are brought together and allow the full voltage to get through to the faulty lamp. The usual protection is an automatic switch which connects a resistance across the carbons of the faulty lamp. The switch may be operated by the differential floating regulator gear, being closed when the mechanism falls well below the feeding-point. Alternatively an extra shunt magnet has been used to close the switch when the arc voltage rises say 50 per cent above normal, but this has the disadvantage that the auto switch is open at the instant of switching on the series of lamps, so that full voltage may get across a faulty lamp for the short time taken to close the automatic switch. All insulation is better protected by a switch controlled by shunt and series solenoids differentially arranged, the series being the stronger so that the auto switch is closed by gravity when the main switch is opened. When the current is again switched on the equivalent resistance is already in circuit, and remains in circuit until sufficient current passes through the carbons and the series solenoid to open the auto switch.

A. E. A.

ARMATURE: that part of a dynamo machine in which the E.M.F. of the machine arising from electromagnetic induction is generated. In D.C. machines it is usually the rotating part, in alternators it is generally stationary. See "Dynamo Electric Machinery," §§ (4), (9).

ARMATURE REACTION: the effect on the magnetic field of a dynamo machine due to the current carried by the armature. See "Dynamo Electric Machinery," § (5).

ARMATURES, MAGNETO:
- Cores of. See "Magneto, The High-tension," § (11) (ii.).
- Windings of. See "Magneto, The High-tension," § (11) (iii.).

ARNÒ'S ELECTROMETER METHOD: a null method of capacity measurement. See "Capacity and its Measurement," § (39).

ARTICULATION, TELEPHONIC, TESTS OF: tests of the capability of a telephone system for transmitting fundamental speech sounds. See "Telephony," § (11).

ASTATIC COILS: coils so wound that their currents produce no external magnetic field, and that a uniform alternating magnetic field induces no voltage in them.
- Use of, for inductance standards. See "Inductance, The Measurement of," § (58).

ATMOSPHERE, CONDUCTING LAYERS IN, SITUATION OF. See "Magnetism, Theories of Terrestrial and Solar," § (24).

ATMOSPHERICS: irregular disturbances picked up by the receiving apparatus in wireless telegraphy. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (10).

— B —

BATTERIES, PRIMARY

§ (1) HISTORICAL INTRODUCTION.—The primary cell in its simplest form arose from a chance observation of Galvani, who noticed that recently skinned frogs' legs, hung by a copper wire to an iron balcony, were convulsed whenever they touched the iron. It was found that these movements could be reproduced by connecting the nerves and muscles by a piece of metal, and Galvani therefore supposed that a separation of positive and negative electricity occurred at the junction of nerves and muscles.[1] This idea was disproved by Volta, who showed that movements could equally well be produced by connecting two parts of the same muscle by an arc of two metals instead of one—for example, by iron and copper—the important point being that a junction of dissimilar metals should form part of the circuit. This condition was, of course, fulfilled in Galvani's original observation. Thus Volta introduced his contact theory, according to which an electric force arises on placing two dissimilar metals in contact; and in support of this theory he constructed in 1799, and described in 1800, the form of dry battery known as Volta's pile, consisting of discs of zinc, wet cloth, and copper piled upon one another in that order, so that a disc of cloth was always encountered in passing, say, upwards from zinc to copper, but not in passing from copper to zinc. By this means a large number of cells were obtained in series, with a high total E.M.F., but having the disadvantage of high internal resistance.

Thus the terms "galvanic electricity" and "voltaic electricity" arose and denoted what may be called low potential electricity as opposed to the high potential, frictional, or electrostatic electricity which had so far been available.

It was a comparatively short step to replace the wet cloth of Volta's pile by free electrolyte. This was done by Volta[2] in his "crown of cups," which consisted of a series of cups containing salt water in which stood zinc

[1] Volta, *Roy. Soc. Phil. Trans.*, 1793, Part i. 10.
[2] *Roy. Soc. Phil. Trans.*, 1800, Part ii. 403.

and copper plates, the zinc of each cell being connected to the copper of the next, so as to form a number of simple cells in series.

In 1801 Davy tried acid electrolytes, but they do not appear to have come readily into favour. Progress then became slow. An important advance was made in 1830, for Kemp, followed by Sturgeon, then introduced amalgamation of the zinc when used in acid solutions.

Polarisation also received serious consideration. Probably Becquerel was the first to have a true understanding of this subject, for in 1829 [1] he described cells of the Daniell type, but they were not of a very practical nature. It was in 1836 that Daniell [2] described his well-known cell, and three years later Grove [3] described the cell with which his name is still associated.

with the tops projecting. The plates are then found to be at different potentials, so that the cell thus formed has an E.M.F.

The value of the E.M.F. depends largely on the materials or metals used for the plates. When two metals, such as zinc and copper, are placed in contact it is found by electrostatic tests that the zinc is at a higher potential than the copper, and zinc is therefore said to be electro-positive to copper. By this and other means all the chemical elements can be arranged in a series, known as the electro-chemical series, such that each element is electro-positive to all those that follow it and electro-negative to those above it. The following series (reading down each column in succession) is given by G. Gore, but it should be noted that the exact order may vary slightly according to the conditions.

Electro-chemical Series

Electro-positive

Caesium	Magnesium	Nickel	Hydrogen	Rhodium	Selenium
Rubidium	Aluminium	Thallium	Mercury	Platinum	Phosphorus
Potassium	Chromium	Indium	Silver	Osmium	Sulphur
Sodium	Manganese	Lead	Antimony	Silicon	Iodine
Lithium	Zinc	Cadmium	Tellurium	Carbon	Bromine
Barium	Gallium	Tin	Palladium	Boron	Chlorine
Strontium	Iron	Bismuth	Gold	Nitrogen	Oxygen
Calcium	Cobalt	Copper	Iridium	Arsenic	Fluorine

Electro-negative

The Smee cell appeared in 1840. In the same year J. T. Cooper modified the Grove cell by substituting carbon plates for the platinum, but this cell is usually known by the name of Bunsen, to whom we owe the popular bichromate cell.

No further progress appears to have been made until 1868, when the Leclanché cell appeared. This cell is remarkable, not only on account of the extent to which it has been applied, but also on account of the dry cell, which is now used to the extent of many millions per annum, and which is simply a modified form of Leclanché cell so far as fundamental principles are concerned.

I. The Simple Voltaic Element and its Defects

An appreciation of the physics underlying primary cells is most easily gained by first considering the simplest form of cell and the defects from which it suffers.

§ (2) Fundamental Principles. — A primary cell may be defined as a device for the direct transformation of chemical energy into electrical energy. In its simplest form it is obtained by standing two plates of dissimilar conducting material in an electrolyte,

[1] *Ann. de Chim. et de Phys.*, 1829, xli. 5.
[2] *Roy. Soc. Phil. Trans.*, 1836, Part i. 109.
[3] *Phil. Mag.*, 1839, 3rd Series, xv. 287.

It has been found that if a high E.M.F. is desired, the elements used for the plates should be as far apart on the electro-chemical series as possible. Thus zinc may be used on the one hand and copper on the other, but it is better to use platinum or carbon rather than copper, because these conductors are further away on the series. Some metals are not very suitable owing to special reasons. For example, iron is seldom employed, because oxides of iron are liable to be thrown down and are troublesome.

The terminals on the plates for connection to the external circuit are called the poles. The current is found to flow in the external circuit from the electro-negative plate. Consequently the terminal on the electro-negative plate (*e.g.* carbon) is the positive pole and the terminal on the electro-positive plate (*e.g.* zinc) is the negative pole.

It is the electro-positive plate that tends to pass into solution in the electrolyte, and that usually supplies most of the energy by combination with the negative ion of the electrolyte.

A reasonably high E.M.F. is naturally regarded as one of the essentials of a commercial primary cell. A second essential is that chemical action in the cell should only take place (at least to a serious extent) when the external circuit is closed. Unfortunately this condition rules out some of the most

electro-positive elements when an aqueous solution is used as an electrolyte. For example, potassium and sodium cannot be used, as they decompose water vigorously.

The E.M.F. also depends on the electrolyte. Solutions of salts may be used, but dilute acids have a lower resistivity and generally give higher E.M.F.

A cell consisting of two plates standing in an electrolyte is often termed a "simple voltaic element." Such a cell is shown dia-

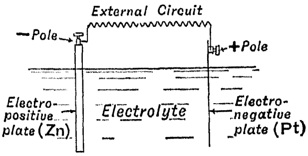


FIG. 1.

grammatically in *Fig.* 1, the plates being platinum and zinc.

§ (3) ACTION OF A SIMPLE CELL.—We may now consider what takes place in this simple cell when it supplies a current. Suppose that the electrolyte is dilute sulphuric acid, and that the plates are platinum and zinc; then the current in the external circuit flows from the platinum to the zinc, but in the electrolyte it passes from the zinc to the platinum, as indicated in *Fig.* 2. When a current flows through an electrolyte by means of electrodes, electrolysis necessarily takes place, and thus the electro-chemical changes which ensue in a cell when it supplies a current are subject to the laws of electrolysis. It follows, therefore, that the positive ion (in the present instance hydrogen) will be deposited on the platinum (the electro-negative plate) since the current flows from the zinc to the platinum.

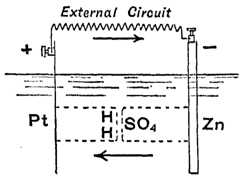


FIG. 2.

The negative ion, or acid radicle (SO_4), will be deposited on the zinc plate and will be exactly equivalent in amount to the hydrogen. Moreover, it will proceed to combine with the zinc, forming zinc sulphate. It may be said, as a general statement, that the electro-positive element of a cell is the one that is attacked, and hydrogen or an equivalent metal ion is deposited on the electro-negative element.

The electro-positive element is commonly called the negative plate (corresponding to its negative pole) and the electro-negative element is frequently called the positive plate; and we shall now adopt this nomenclature, though these terms are not strictly correct.

The above action is represented in *Fig.* 2. It is not intended to give the impression that the SO_4 deposited on the zinc and the hydrogen deposited on the platinum belong to the same molecule of acid; they certainly do not, but it is convenient to represent the main reaction in the simplest manner.

§ (4) POLARISATION.—Now, although the SO_4 combines with the zinc, the hydrogen has no such effect on the platinum. If the hydrogen were freely given off there would be little objection thereto, but unfortunately it remains on the surface of the platinum to a considerable extent. This has the effect of changing the character of the platinum plate, so that it is more in the nature of a hydrogen plate, and since hydrogen is much more electro-positive than platinum the E.M.F. becomes correspondingly reduced. This change is known by the name of polarisation. It gradually passes off when the current is interrupted, because the hydrogen diffuses into the electrolyte. *Polarisation* may be defined as a temporary reduction in E.M.F., due to an alteration of the plates or of the electrolyte brought about by the action of the cell. Polarisation may also occur at the zinc plate, but as it is chiefly noticeable at the electro-negative plate, the term generally has reference to that plate.

Polarisation is one of the difficulties encountered in all primary cells. It is particularly noticeable in the simple cell above described, because in such a cell no means are adopted for its elimination, and it is for this reason that the simple cell is generally of no value in practical work. The only simple cells that have proved commercially useful are the cell due to Smee and a recent cell due to Féry (described later). The former consists of zinc and platinum standing in dilute sulphuric acid, just like the cell we have been considering, but there is this fundamental difference, that the platinum plate, instead of having an ordinary bright surface, is platinised—that is, it has received an electrolytic deposit of platinum upon it. The effect of this is that the surface, instead of being smooth, is very finely roughened, and consequently the hydrogen is much more easily evolved. It appears that any finely roughened surface is effective in this way. For example, specially prepared carbon has been so used and has proved effective. The great advantage of the Smee cell is its simplicity, but it has the considerable disadvantage of a low E.M.F., under 0·5 volt.

§ (5) OVERVOLTAGE.—The extent of the polarisation for different electro-negative plates depends upon what is termed "overvoltage."[1] Theoretically, the evolution of hydrogen in electrolysis should take place when the applied electric pressure exceeds a certain value. The

[1] See "Electrolysis, Technical Applications of."

minimum value, with suitable electrodes, is about 1·1 volts, and if the pressure required is found to be higher with a given pair of electrodes the excess pressure beyond 1·1 volts is termed the overvoltage for the combination. It is found that platinised platinum as a cathode has the lowest overvoltage among the elements commonly available. On the other hand, some metals, such as mercury, have a very high overvoltage. Thus the extent to which polarisation may interfere with the action of a simple cell depends considerably upon the electro-negative element that is used.

II. Local Action and Amalgamation

§ (6) Local Action.—When impure zinc is placed in dilute acids it dissolves rapidly. If the zinc is pure the action is slow, but pure zinc is too costly for use in batteries. Consequently the ordinary commercial zinc is used, and if the electrolyte is an acid the solution takes place irrespective of whether the cell is supplying a current unless some preventive means are adopted. This solvent action, which thus serves no useful purpose, is known as *local action*, and is due to the impurities forming local voltaic circuits.

This will be understood on reference to *Fig.* 3. Here Z represents a rod of zinc in dilute sulphuric acid. Suppose N to be a metallic impurity, such as lead. Since lead is electro-negative to zinc we have a small short-circuited cell consisting of the particle of lead in contact with the zinc, both being in contact with the acid. Consequently local currents will flow from the zinc through the acid to the lead, and hydrogen will be evolved at N. The same thing will happen at every point where there is an electro-negative impurity on the surface, and consequently the zinc dissolves rapidly.

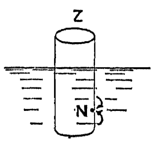


Fig. 3.

Local action is most evident in acid electrolytes, but it also takes place in others, though generally so slowly as to be comparatively unimportant.

§ (7) Amalgamation.—The remedy that is always adopted is the amalgamation of the zinc. This is readily effected by bringing the zinc plate into contact with mercury in dilute acid and rubbing the plate so as to spread the film until a perfectly bright surface is obtained throughout. The zinc is then found to be free from attack by local action even in an acid electrolyte, but the protective effect wears off in time.

This protective action has not been satisfactorily explained. On theoretical principles it might be expected that such treatment would be ineffective, because mercury is markedly electro-negative to zinc, and thus it would be thought that the zinc in contact with mercury would be readily dissolved through local action. The mercury may possibly act as a filter so far as the ordinary impurities are concerned, only permitting the zinc to pass through, and thus the mercury would remain effective until it became too thin, but this does not explain the lack of local action by the mercury itself.

Overvoltage might be supposed to afford an explanation because the overvoltage of mercury is high. Thus hydrogen would not be evolved readily from a mercury surface. But against this explanation is the fact that the overvoltage of lead is almost equally high and lead is one of the most common impurities of commercial zinc. As hydrogen is evolved readily from the lead impurities it should not experience much greater difficulty in the case of mercury.

It is found that mercury assumes the electro-positive position of zinc by addition of the latter with remarkable readiness. Hockin and Taylor [1] found that even one-millionth part of zinc was sufficient to cause this change.

III. Depolarisation and Internal Resistance

In what follows we shall deal only with polarisation at the positive plate, as it is there much more important, and the term usually refers to that plate. What is required is the more or less complete removal of the polarising ion, which in practice is hydrogen.

§ (8) Depolarisation by Substitution.—The most effective method of depolarisation is to substitute a harmless ion in place of the harmful hydrogen ion. This is the method adopted in the Daniell cell. In this cell the zinc and sulphuric acid are contained in a porous pot; the latter stands in an outer jar containing a copper plate in a solution of copper sulphate. The effect of this arrangement will be appreciated on reference to *Fig.* 4. The left-hand part of the diagram shows

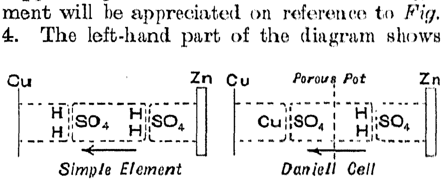


Fig. 4.

the action of a simple element consisting of zinc and copper standing in dilute sulphuric acid. When the circuit is closed the current flows through the electrolyte in the direction of the arrow, the SO_4 ion attacks the zinc, and the hydrogen ions are deposited on the copper, where they form gaseous hydrogen molecules. On the right-hand side of the diagram the

[1] *Soc. Tel. Eng. J.*, 1879, viii. 282.

corresponding action of the Daniell cell is indicated. Here the current in the sulphuric acid is carried by the SO_4 and H ions as before, but when the current passes to the copper sulphate it is then carried by the SO_4 and Cu ions, and it is copper that is deposited upon the electro-negative plate. In other words, copper has been substituted for hydrogen, and there is no polarisation, because copper is deposited upon copper. This is the most effective means of depolarisation that can be adopted, but it is not generally available, because it necessitates using an electrolyte containing, as the positive ion, the metal of which the electro-negative plate is formed.

§ (9) Depolarisation by Oxidation.—Much the most usual method is to remove the hydrogen by oxidation. For this purpose various oxidising agents have been used, and they may be classified broadly as (1) solid, (2) liquid, and (3) gaseous.

In regard to solid depolarisers, it may be said that the choice is somewhat circumscribed, partly because solid oxides do not always give up their oxygen with sufficient rapidity, and partly because of their low conductivity. For example, manganese peroxide is used extensively in Leclanché and dry cells, but its action is slow, and since it is practically non-conducting it must be mixed with a considerable proportion of carbon or graphite to obtain the necessary conductivity. Thus the liberated hydrogen is not necessarily in contact with the oxidising agent, and the net result is that a depolariser made up with manganese peroxide is not very effective, though for many purposes it is sufficient for the intermittent use to which such cells are usually applied.

Cupric oxide is an oxidising agent which is sometimes employed. This is more effective than manganese peroxide, because it is sufficiently conducting to be formed into plates without the admixture of some conductor; moreover, it gives up its oxygen readily. Thus it is found to be quite an effective depolariser, but unfortunately, owing to other properties, its use is very restricted.

Solid depolarisers also have the disadvantage that the reducing action is confined to the outer layers. In that way they may be inefficient, because only a small proportion of the depolariser is used up by the time the cell becomes exhausted for practical work unless the material is finely divided.

Liquid depolarisers have the general advantage that, as a rule, a gas acts more readily upon liquids than upon solids at ordinary temperatures. On the other hand, the whole of a liquid suffers through any change, on account of the diffusion of those parts which have been reduced, and therefore all the liquid gradually loses its effectiveness; whereas in the case of a solid the parts not immediately exposed form a sort of reserve which are attacked later. Thus, when a liquid depolariser has been diluted beyond a certain point by the products formed as the result of reduction, the depolarisation becomes continuously less effective, and this deterioration is generally more marked than in the case of solid depolarisers. It must be understood, however, that depolarisers, both solid and liquid, vary considerably in their effectiveness, and, therefore, no very general statement can be laid down.

Sometimes the liquid depolariser is simply mixed with the acid, as in the bichromate cell. In other cases it is necessary to separate the depolariser from the acid by means of a porous pot. This is so in the Grove cell, which consists of a sheet of platinum standing in nitric acid, which is separated by a porous pot from the dilute sulphuric acid surrounding the zinc. The effect of the nitric acid acting as a depolariser may be represented by *Fig.* 5. The hydrogen ions are deposited upon the platinum from the nitric acid, and there form hydrogen gas, which reacts chemically with the nitric acid, or more probably it acts in the nascent state. The nitric acid deteriorates in this process through dilution by the water and nitrogen oxides so formed. A porous pot is necessary, because the nitric acid must be concentrated to be effective, and if this concentrated acid were allowed to come in contact with the zinc, local action would take place, even if the zinc were amalgamated.

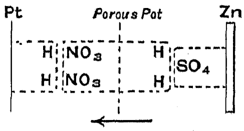


Fig. 5.

Gaseous depolarisers need not be seriously considered. Atmospheric oxygen has the advantage of being inexhaustible, and can be used free of cost, but it necessitates the exposure of the positive plate to the atmosphere, and this usually cannot be accomplished at all easily. The only cells in which this method seems to have been applied usefully to a limited extent are the "R. and R." dry cell, by Rylander and Rudolphs of Sweden, and Féry's cell. Jungner has also worked in this direction.

Depolarisation by physical means has also been attempted. The finely roughened surface of the platinum plate in a Smee cell may be looked upon as a physical method. Again, attempts have been made to give a certain motion to the positive plates of a cell, so that continually varying portions are exposed to the air, any deposited hydrogen being thus removed by the oxygen on the surface of the plate, but such a method is too complicated to be of commercial value.

§ (10) Internal Resistance.—When a cell supplies current to an external circuit the P.D. is lower than the E.M.F. on account of the internal resistance of the cell. Representing the resistance of the external circuit by R, and the internal resistance by r, the current I is given by $I=E/(R+r)$, in which E is the E.M.F. of the cell. Equally, if the P.D. is represented by V, then $V=E-Ir$.

Assuming there is no polarisation, and that the internal resistance is constant, then the value of the current would be determined for a given E.M.F. simply by the external resistance. According to experiments by Carhart,[1] however, the internal resistance decreases as the current supplied by the cell increases. But there is some doubt whether this apparent variation may not be due to polarisation. This latter view has been supported by K. E. Guthe.[2]

Generally speaking, polarisation is not absent, and therefore, at the moment when the external circuit is broken, the E.M.F. does not return at once to the value which it had before the circuit was closed. In time the original value is regained, but the recovery is more or less gradual according to the effectiveness of the depolarisation. For this reason it is difficult to determine accurately the value of the internal resistance by voltmeter readings on opening the external circuit. A somewhat more reliable result is obtained at the moment of closing the circuit.

IV. Theories of the Voltaic Cell

In the space available for the present article it is only possible to glance very briefly at the theories which have arisen in connection with the voltaic cell.

§ (11) The Chemical and Contact Theories.—The contact force, or E.M.F. of contact, when two dissimilar metals are brought together is considerable. For example, in the case of zinc/copper it has been found to be about 0·75-0·85 volt. In the absence of the law of conservation of energy, it was therefore not surprising that Volta should attribute the E.M.F. of a voltaic cell to the contact force of the two elements forming the plates. Volta's method of observation consisted in forming a condenser of two plates of the particular metals, which were then metallically connected for an instant. On separating the plates the leaves of a gold-leaf electroscope connected to one of them diverged, giving a measure of the charge and, consequently, of the force giving rise thereto. As the result of this work Volta enunciated what is known as Volta's law, namely, that if a conductor is made up of a number of metals so that there are several junctions, the contact forces are additive. For example, if the conductor consists of copper at one end and zinc at the other, with a number of metals, M_1, M_2, M_3, between them, as in *Fig.* 6, the contact force between the ends is given by $Cu/M_1+M_1/M_2+M_2/M_3+M_3/Zn$, where M_1/M_2 is the contact force between M_1 and M_2, etc. Also, since there is no E.M.F. in a closed circuit of this kind so long as all parts are at the same temperature and varying magnetic fields are absent, it follows that the contact force of such a series is simply equal to the contact force of the two end metals, for, if the total E.M.F. is zero, we have $Cu/M_1+M_1/M_2+M_2/M_3+M_3/Zn+Zn/Cu=0$. But $Cu/Zn=-Zn/Cu$, which demonstrates the statement.

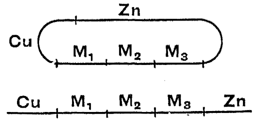


Fig. 6.

Other physicists soon discovered that chemical action invariably occurs when a cell generates a current, and thus a very prolonged controversy sprang up between the followers of Volta and those who held that the E.M.F. was purely chemical.

§ (12) Difficulties in investigating Contact Force.—There is considerable difficulty, however, in the quantitative investigation of contact force and in determining its cause with any certainty. For example, Sir Oliver Lodge[3] has expressed the view that when two metals are in contact they are at the same potential, but differently charged, forming practically an air battery, the charges resulting from oxidation. Thus each metal may be regarded as possessing a certain E.M.F. proportional to the heat evolved on oxidation, and the contact force would thus be equal to the differences of these two E.M.F.'s. In this way the contact force of zinc/copper would be about one volt. Experimentally, the contact force in this case has been found to be 0·71 volt by Pellat, 0·75 volt by Ayrton, and 0·85 volt by Clifton. This is sufficient to show that the results by different observers vary considerably and that they are only roughly of the same order as the figure derived from oxidation.

A brief examination of the work in this subject is sufficient to afford ample explanation of the variable results so often obtained. All such measurements really depend upon the charge of a very small condenser. Consequently the quantity of electricity involved in the measurement is extremely small, and the quantity of oxygen or other gas so involved is infinitesimal. It is not easy, therefore, to say whether the contact force is being measured between two metals or between their oxides,

[1] *Phys. Rev.*, 1894–95, ii. 392.
[2] *Phys. Rev.*, 1898, vii. 193.
[3] *Brit. Assoc. Report.*, 1884, p. 464.

for a film of oxide is not necessarily detected easily or removed with ease when detected.

Bottomley[1] found that the contact force of zinc/copper remained unchanged when the air pressure was reduced to 2½ millionths of an atmosphere, and also when the air was replaced by hydrogen, but such an experiment does not necessarily prove that contact force is independent of the oxygen in the air. An experiment of greater interest is that by Brown,[2] who found that when air was replaced by hydrogen sulphide the contact force of iron/copper was reversed. In this case we are dealing with a gas which forms sulphide films with great ease, and, therefore, some marked change would be expected, but it would not be reasonable to suggest that the contact force so measured was that of iron/copper in hydrogen sulphide gas. De Broglie[3] has found that the greater part of the contact force vanishes on drying the atmosphere. This would point to voltaic action.

Not only is contact force varied by chemical action, but it may vary also with the physical state of the metals, such as polishing. Experiments of this kind have been carried out by J. R. Erskine-Murray[4] and by N. Hesehus.[5] In considering such experiments, however, it must be borne in mind that chemical action of the air or other gas on such surfaces is not easily excluded, and that gas films, in one form or another, are often very difficult to remove.

Q. Maiorana[6] has found that contact force is greatly reduced by low temperatures, obtained by dropping liquid air on the junction. This again may possibly be regarded as supporting the idea that the force is due to the voltaic action of moisture.

This brief account is sufficient to indicate the extreme difficulty in devising satisfactory experimental methods in this subject so that a definite theory may be formulated. So far as the voltaic cell is concerned, contact force is not now regarded as the most important constituent of the E.M.F., because, as will be seen later, the transformations of energy take place at other points.

§ (13) Seat of the E.M.F.—There has been almost as keen a controversy over the precise location of the E.M.F. in a voltaic cell as there was over the rival theories of contact force and chemical action. At first sight it would not be unnatural to suggest as the seat of the E.M.F. either the metallic junction (of, say, zinc to copper) or the area of contact of the zinc with the electrolyte, according to the point of view. There is at once a difficulty, however, in accepting the first alternative, because, as already stated, the contact force is distinctly smaller than the E.M.F. of the cell. There are also the contact forces of the metal/liquid junctions, but these are much smaller, and Ayrton and Perry[7] showed that the E.M.F. is equal to the sum of all the contact forces in the cell.

In considering this part of the subject it becomes necessary to define exactly what is meant by a seat of E.M.F., as otherwise a discussion may mean very little. If the term is restricted to any point in a circuit where energy is transformed, then we have a definition which appears reasonable, provided that such points can be found. In many electrical circuits no such point or points could be suggested, as, for example, in the electromagnetic production of E.M.F. Accepting this definition, however, it is found that when a current flows across a copper/zinc junction heat is absorbed or evolved according to the direction of the current, thus indicating the presence of an E.M.F., but this is quite small, corresponding with the well-known Peltier effect.

The view is sometimes taken that the E.M.F. is due to the sum of the contact forces at the junctions, and that the current is maintained by chemical action.

If, on the other hand, it is considered that the E.M.F. arises where the chemical action takes place, then there is no legitimate reason to restrict attention to the zinc/electrolyte contact alone. Voltaic action differs from ordinary chemical action in that it takes place in equivalent amounts at both the plates of the cell and not merely at a single surface. Consequently both the zinc/electrolyte and copper/electrolyte contacts (in the case of a zinc and copper cell) should be considered.

§ (14) Thermo-chemical Considerations.—Considerable light is thrown upon the production of E.M.F. by considering the thermochemical changes which take place. But before we pass to this subject it may be well to state generally the essential conditions which must be fulfilled by the constituents of a voltaic cell. They may be stated to be as follows:

(i.) At least three elements are necessary.

(ii.) One of these must be a conductor of the first class (*i.e.* not decomposed by the passage of a current), one must be of the second class (*i.e.* an electrolyte), and the third may be of either class.

(iii.) Two of these constituents must be capable of chemical interaction. It must be borne in mind, however, that this interaction need (and indeed should) only take place when the circuit is closed. Ordinary chemical action is quite a different matter.

[1] *Brit. Assoc. Report*, 1885, p. 901.
[2] *Phil. Mag.*, 5th Series, 1878, vi. 142.
[3] *Comptes Rendus*, 1911, clii. 696.
[4] *Roy. Soc. Proc.*, 1898, lxiii. 113.
[5] *Journ. Russk. Chimicesk.*, 1902.
[6] *Nuovo Cimento*, 1900, xii. 196.
[7] *Roy. Soc. Proc.*, 1878, xxvii. 196.

(iv.) Finally, it follows from the above that the liberation of an ion is necessary; or, in other words, separation of the electro-positive from the electro-negative ion must take place.

All commercial cells consist of two different conductors of the first class together with one or more electrolytes.

Now, although the chemical reaction taking place in any cell is split up so that half of it occurs at one plate and half at the other, yet from the energy point of view the chemical action may be considered as a whole. It is a common observation that when chemical changes take place heat is either absorbed (endothermic reactions) or evolved (exothermic reactions). Voltaic cells depend generally upon exothermic reactions.

Assuming that the reaction only takes place when the circuit is closed, then the whole of the heat due to the chemical reaction, and which would otherwise be liberated, should appear as electrical energy. For example, if zinc sulphate is the compound formed by the action of a cell, then in the production of, say, one gramme of zinc sulphate the cell should generate a quantity of electrical energy equivalent to the heat evolved in the formation of one gramme of zinc sulphate. Since the electrical energy is given by the product of the quantity of electricity into the E.M.F. under which it flows, it follows that for a given quantity of electricity the E.M.F. will vary according to the heat that is evolved.

It remains to choose a suitable unit of weight of the compound under consideration, and here the laws of electrolysis come to our assistance. We know that if a quantity of electricity q liberates one gramme of hydrogen in the electrolysis of a solution of, say, HCl, it will decompose one gramme molecule of HCl. Equally this quantity q will decompose a gramme molecule of any other salt of which the molecule contains only a single atom of a monovalent metal, such as NaCl. If the metal is divalent, or if the molecule contains two atoms of a monovalent metal, such as $ZnCl_2$ or Na_2SO_4, the gramme molecule requires $2q$ of electricity for its decomposition; and so on for higher valencies. Consequently, if we take q, the quantity of electricity which liberates one gramme of hydrogen, as our unit of electricity, then the gramme molecule of the compound will be a convenient unit of weight.

Assuming that all the heat of the reaction is converted into electrical energy and that the E.M.F. depends only upon this heat, then we may equate the electrical energy to the heat H_c in the chemical reaction, expressed as work, or $qE = H_c$, or, if the heat is expressed in calories, $qE = JH$, in which E is the E.M.F., H is the heat of formation in calories of a gramme molecule of the compound that is formed in the case of a monovalent metal (since q is the quantity of electricity necessary for the liberation of *one* gramme of hydrogen), and J is the mechanical equivalent of heat. Since q is 9654 C.G.S. units (96,540 coulombs) and J is 42×10^6 ergs, we have

$$E = \frac{42 \times 10^6}{9654} H \text{ in C.G.S. units,}$$

or

$$E = \frac{4 \cdot 2}{96{,}540} H \text{ volts}$$

$$= \frac{H}{23{,}000} \text{ volts (monovalent metal).} \quad (1)$$

Usually the metal is divalent, and in that case q becomes $2q$, the final result being

$$E = \frac{H}{46{,}000} \text{ volts (divalent metal).} \quad (2)$$

This equation is due to Kelvin.

Helmholtz saw the possibility of the E.M.F. being dependent on thermal E.M.F.'s in the cell in addition to the heat of reaction, and that consequently the simple equation of Kelvin might be incorrect. This proved to be the case, and we shall now consider briefly by an approximate proof the modification that is necessary.

If the E.M.F. is due not merely to the chemical heat H_c but also to heat of a purely thermo-electric character, which we may call H_t, then we must write $qE = H_c + H_t$ in place of our original expression. In order to find a value for H_t we may put a theoretically reversible cell through a sort of Carnot cycle. A reversible cell is one in which the chemical reactions are completely reversed by passing a reverse current, or charging current, through the cell. This condition is very approximately fulfilled by a Daniell cell containing zinc sulphate and copper sulphate solutions. Let such a cell, having an E.M.F. E at the absolute temperature T, be put through the following cycle of operations: (1) raise its temperature slightly above T, say to $T + dT$, in which case its E.M.F. will generally change to $E + dE$; (2) allow it to generate q coulombs of electricity at this temperature, giving electric energy equal to $q(E + dE)$; (3) allow the cell to cool down to the original temperature T; and (4) pass q coulombs of electricity in the reverse direction (*i.e.* in opposition to the E.M.F.), the electrical energy in this case being equal to qE.

The cell is now in its original state. The heat that was required to raise it to the temperature $T + dT$ has been recovered upon cooling. The chemical changes which gave rise to the current at the higher temperature have been reversed by the charging current at the lower temperature, the cell being by hypothesis reversible. It is true that any I^2R loss due to the current flowing through the resistance of the cell is not reversible, but this may be made negligibly small by making the

current I very small. As a net result of these operations, however, a certain amount of energy has nevertheless been gained, for the energy given out at the higher temperature was $q(E+dE)$ and that returned at the lower temperature was qE. Consequently there has been a gain of qdE, and this can be supplied continuously by repeating the operation. This energy has not been supplied by the chemical reactions, because these have been reversed; it must, therefore, be due to the term H_t.

Here it is necessary to remember that if one body is at absolute temperature T and a second body at temperature $T+dT$, work can be done by the flow of heat H from the hotter to the cooler body. But only a fraction of this heat H can be transformed into work, the amount of work so obtainable being given not by H but by HdT/T.

Thus the energy obtainable from H_t in the cycle of operations is not equivalent to H_t but to $H_t dT/T$. We may therefore write

$$H_t\frac{dT}{T}=qdE,$$

or

$$H_t=qT\frac{dE}{dT}.$$

Making use of this value of H_t we can now write

$$E=H_c+T\frac{dE}{dT},$$

or numerically, as before,

$$E=\frac{H}{46,000}+T\frac{dE}{dT}\text{ volts (divalent metal).} \quad (3)$$

This is the Helmholtz equation, and it shows that the simpler Kelvin equation holds only if the temperature coefficient of a cell is zero.

Some interesting conclusions can be drawn from this equation.

If the E.M.F. rises with rise of temperature, then dE/dT is positive and the E.M.F. is greater than that due to the heat H. Since the additional electrical energy must be derived from some source, this further supply of heat is derived from the surroundings. In such a case the cell cools when it generates a current so as to absorb the required heat. Since the amount of heat in question is never large, this cooling can only be observed when the current is small; otherwise the effect is masked by the heat resulting from the internal loss in the cell due to internal resistance.

If the E.M.F. falls with rise of temperature, then dE/dT is negative and the E.M.F. is smaller than that due to the heat H. The cell then heats up on generating a current, irrespective of the internal loss.

The application of these principles will be rendered clear by taking two examples. Suppose we have a simple cell consisting of zinc and platinum in dilute sulphuric acid. The body that is formed is zinc sulphate. The heat of formation of a gramme molecule of zinc sulphate, using zinc and dilute sulphuric acid, is 37,730 calories, and therefore the E.M.F. is

$$\frac{37,730}{46,000}=0{\cdot}82\text{ volt,}$$

assuming that the temperature coefficient is zero.

In the case of the Daniell cell we have not only the formation of zinc sulphate but the decomposition of copper sulphate. Now the formation of copper sulphate is endothermic, absorbing heat. Consequently its decomposition results in the evolution of heat, and this heat is added to that resulting from the heat of formation of the zinc sulphate. Thus we have

Heat of formation of zinc sulphate . .	37,730
Heat of decomposition of copper sulphate	12,400
Total heat	50,130

In the Daniell cell the temperature coefficient is negligible, and therefore the E.M.F. is given simply by

$$\frac{50,130}{46,000}=1{\cdot}09\text{ volts.}$$

This calculated value is very close to that which is actually observed, and which varies somewhat with the strength of the solutions.

It will be noticed that the use of the copper sulphate in the Daniell cell not only maintains the E.M.F. through efficient depolarisation, but increases it considerably beyond that of the simple cell in which the copper sulphate is absent.

As an instance in which the temperature coefficient is considerable we may take what is known as Grove's gas cell, or the oxygen-hydrogen cell. If two strips of platinised platinum stand in dilute sulphuric acid, and so that the exposed part of one plate is surrounded by hydrogen and the exposed part of the other plate is surrounded by oxygen (the gases being contained in inverted cylinders dipping into the acid), the combination is found to act like a cell of which the plates are hydrogen and oxygen respectively. The active part is the surface of the platinum near the surface of the electrolyte, and here no doubt the gases are absorbed, so that when the cell is allowed to generate a current they combine voltaically to form water. Now the heat of formation of a gramme molecule of water is 68,300, and therefore, if the temperature coefficient were negligible, we should expect the E.M.F. to be $68,300/46,000=1{\cdot}48$ volts (the figure 46,000 being used because, although hydrogen is monovalent, there are two hydrogen atoms in the molecule of water). But Smale [1] found

[1] *Zeits. physik. Chem.*, 1894, xlv. 577.

the temperature coefficient to be negative, equal to 0·00141 volt per degree C. Therefore we have, at 17° C. (or 290° absolute),

$$E = \frac{68{,}300}{46{,}000} - 290 \times 0{\cdot}00141,$$
$$= 1{\cdot}48 - 0{\cdot}41,$$
$$= 1{\cdot}07 \text{ volts.}$$

This is about the value observed experimentally and differs seriously from that due to the heat of formation of water alone.

Although the application of thermo-chemical data in this way appears simple, it is often difficult through lack of knowledge of the exact chemical conditions and reactions. Moreover, such data are often of doubtful accuracy and may have been obtained with materials which are not precisely similar to those taking part in voltaic changes.

From the Helmholtz equation it will be realised that it is impossible to obtain a voltaic cell having a high E.M.F. (such as 10 volts), because the heat of formation of most compounds that can be utilised is equivalent to about 2 volts or less.

§ (15) The Osmotic Pressure Theory.—Thermo-chemical data give an insight into the action of a cell from the point of view of energy, but if it is desired rather to look into the mechanism of voltaic action we may turn to the Osmotic Pressure Theory. This depends upon the idea of "electrolytic solution pressure" as put forward by Nernst. Neither theory excludes the other.

A case analogous to solution pressure, commonly accepted, is the evaporation of a liquid into an atmosphere above it in a closed vessel. The evaporation continues by means of molecules leaving the liquid until the stage is reached when as many molecules return from the atmosphere into the liquid as leave the liquid and pass into the atmosphere. This is the state of equilibrium, and it arises when the partial pressure of the vapour is equal to the vapour pressure of the liquid. Similarly, if a solid is in contact with a solvent, we may consider that, when the solution is saturated, as many molecules return from the solvent to the solid as escape from the solid into the solution; or, if we may regard the solid as having a solution pressure, we may say that equilibrium occurs when the osmotic pressure of the solute becomes equal to the solution pressure of the solid.

In this connection it must be remembered that osmotic pressure is a measure of the number of molecules in solution in unit volume, at least in the case of dilute solutions. In such solutions the laws of Avogadro, Boyle, and Charles all apply as they do in gases, and the constant which occurs in those laws is the same for solutions as for gases. The investigations of van't Hoff led to the striking conclusion that the osmotic pressure exerted by the molecules of a given weight of substance in solution is equal to the pressure which an equal weight of the substance would exert at the same temperature if it occupied a volume in the gaseous state equal to that of the solution.

The analogy of evaporation of a liquid into a gas and the passing of a solid into solution thus becomes very marked. But, in order to account for the E.M.F. between a metal and any electrolyte in which it is immersed, Nernst went a step further and introduced the idea of *electrolytic* solution pressure of the metals as distinct from ordinary solution pressure, to which reference has just been made. Electrolytic solution pressure is taken to be a measure of the tendency of a metal to pass into free ions when placed in an electrolyte. Suppose that such free ions are formed. They will be positively charged, and the electrolyte containing them will therefore also be positively charged; but since both kinds of electricity must be formed simultaneously, it follows that the metal will be negatively charged and will therefore attract the positive ions back towards itself again. As soon as the attraction of the metal for the positive ions is equal to the electrolytic solution pressure, equilibrium is established and no more ions can pass into solution. A very few ions are sufficient to bring about this equilibrium, because the ionic charges are large.

If for any metal the electrolytic solution pressure be P and the osmotic partial pressure of the metallic ions already in solution be represented by p, then three cases are possible according as $P > p$, or $P = p$, or $P < p$. If $P > p$, more metal ions pass into solution and the metal becomes negatively electrified until equilibrium is produced; but the E.M.F. is smaller than it would be if no ions were already present, because there is already an osmotic pressure to oppose the electrolytic solution pressure. If $P = p$, nothing happens; in other words, the metal does not become charged and there is no potential difference. Lastly, if $P < p$, some ions are precipitated upon the metal and give up their charge, so that the metal becomes positively charged, the solution being negative.

We thus have what is called a "double layer" at the surface of separation between the metal and the electrolyte, the metal being, say, negatively electrified, and close to it a layer of positive ions. Here the ions may be looked upon as being under a pressure P equal to the electrolytic solution pressure, whereas any ions in the body of the electrolyte are under a pressure p equal to the osmotic partial pressure of these ions. If a current flows, due to the E.M.F., the ions pass from the pressure P to the pressure p, and in this change a certain

amount of work must be done by the ions. This we can proceed to calculate and thus arrive at a value for the E.M.F.

First let us take the ordinary case of work done by an expanding gas. Suppose a certain quantity of gas is held in a cylinder by a piston, the length of the cylinder up to the piston being x and the area of the piston (or cross-section of the cylinder) being A. Let the pressure of the gas be p and its volume v. Then the total pressure outward on the piston is pA. If the gas causes the piston to move outwards through a small distance dx, then the work done by the gas in this movement is $pAdx$. But since Adx is the variation in volume, or dv, this expression becomes pdv, and the work done in a change of volume from v_1 to v_2 is thus $\int_{v_1}^{v_2} pdv$. Further, in gases we have the relation $pv = \mathrm{RT}$ (R being a constant and T the absolute temperature), so that the

$$\text{work done} = \mathrm{RT}\int_{v_1}^{v_2} \frac{dv}{v},$$

$$= \mathrm{RT} \log_e \frac{v_2}{v_1} = \mathrm{RT} \log_e \frac{\mathrm{P}_1}{\mathrm{P}_2}.$$

It becomes necessary to define R by taking a definite quantity of gas, and here again, for this particular purpose, the gramme molecule is a convenient unit. Now the gramme molecule of a gas occupies a volume of 22·38 litres at normal temperature and pressure, and if this is taken as a basis the value of R, expressed as heat, becomes 2 calories.

In the case of an electrode in an electrolyte, when a current flows the ions pass from the electrolytic solution pressure P to the osmotic partial pressure p, and we may apply the result just obtained to evaluate the work done by the ions. On the other hand, if the metal is monovalent, 96,540 coulombs will be required to decompose one gramme molecule, and if E (in volts) is the value of the single E.M.F. of the metal/liquid junction, the work done is also equal to 96,540E, so that we may write

$$96{,}540\mathrm{E} = \mathrm{RT} \log_e \frac{\mathrm{P}}{p},$$

if R is suitably expressed. As already stated, R is equal to 2 calories, or to $2 \times 42 \times 10^6$ C.G.S. units of work (the mechanical equivalent of heat being 42×10^6 ergs). Since a coulomb is one tenth of the C.G.S. unit, and a volt is 10^8 C.G.S. units, we have

$$\mathrm{E} = \frac{2 \times 42 \times 10^6}{96{,}540 \times 10^{-1} \times 10^8} \mathrm{T} \log_e \frac{\mathrm{P}}{p} \text{ volts}$$

$$= 0{\cdot}000087\mathrm{T} \log_e \frac{\mathrm{P}}{p} \text{ volts}$$

$$= 0{\cdot}0002\mathrm{T} \log_{10} \frac{\mathrm{P}}{p} \text{ volts},$$

or, generally,

$$\mathrm{E} = \frac{0{\cdot}0002}{v} \mathrm{T} \log_{10} \frac{\mathrm{P}}{p} \text{ volts}, \quad . \quad . \quad (4)$$

in which v is the valency of the metal, T is the absolute temperature (expressed in degrees centigrade), P is the electrolytic solution pressure, and p is the osmotic partial pressure of the ion in solution. This is the Nernst equation of E.M.F.

If the temperature is 15° C., T is 288 and the equation becomes

$$\mathrm{E} = 0{\cdot}0575 \log_{10} \frac{\mathrm{P}}{p} \text{ volts}, \quad . \quad . \quad (5)$$

for a univalent metal. If the valency is v, then

$$\mathrm{E} = \frac{0{\cdot}0575}{v} \log_{10} \frac{\mathrm{P}}{p} \text{ volts}. \quad . \quad . \quad (6)$$

This expression applies to a single plate or electrode. Such a single E.M.F. cannot be measured by ordinary means. For this purpose the dropping mercury electrode has been developed, and by this method the value of the E.M.F. between zinc and normal zinc sulphate solution has been deduced as 0·52 volt. If the zinc sulphate is sufficiently dilute to regard it as completely ionised, then the osmotic partial pressure p of the metal ions due to a gramme molecule of the salt in a litre of solution is 22·38 atmospheres. These values of E and p may now be substituted in the expression for E, and the value of P, the electrolytic solution pressure, may be calculated. The figures so obtained are often enormous or very small. Thus the electrolytic solution pressure of zinc is $2{\cdot}7 \times 10^{19}$ atmospheres, but that of lead is $1{\cdot}1 \times 10^{-3}$ atmospheres.

In every ordinary cell we are concerned with two such expressions as the one just given, one for each plate. Thus, in the case of a reversible cell, of the Daniell type, if we neglect the small E.M.F.'s at the metal/metal and liquid/liquid junctions, we have two metal/liquid junctions contributing to the E.M.F. of the cell. At both these junctions the metal ions tend to pass into solution, but this can only occur at one of the plates, deposition occurring at the other. Thus one plate will be positive and the other negative in respect to the solution. The E.M.F. at 15° C. will be the difference of the two potentials and will be given by

$$\mathrm{E} = \frac{0{\cdot}0575}{v}\left(\log_{10} \frac{\mathrm{P}}{p} - \log_{10} \frac{\mathrm{P}'}{p'}\right) \text{volts},$$

assuming that both the metals are of the same valency. In the simple case where the two solutions have equal concentrations we have $p = p'$ and the formula becomes

$$\mathrm{E} = \frac{0{\cdot}0575}{v} \log_{10} \frac{\mathrm{P}}{\mathrm{P}'} \text{ volts}. \quad . \quad . \quad (7)$$

As an example we may take the Daniell cell with zinc and copper plates. Here $v = 2$, $\mathrm{P} = 2{\cdot}7 \times 10^{19}$ atmospheres for zinc, and $\mathrm{P}' = 4{\cdot}8 \times 10^{-20}$ for copper. Hence, substituting in (7),

$$\mathrm{E} = \frac{0{\cdot}0575}{2} \log_{10} \left(\frac{2{\cdot}7}{4{\cdot}8} \times 10^{39}\right) \text{ volts}$$

$$= 0{\cdot}02875 \times 38{\cdot}7502 \text{ volts}$$

$$= 1{\cdot}11 \text{ volts}.$$

This is approximately the value observed.

It follows from equation (7) that so long as

the two solutions are of equal ionic concentrations the E.M.F. is independent of their strength. This is found to be approximately the case.

Conversely, we may have two solutions of different ionic concentrations but with both plates of the same metal. We then have $p = p'$ and the formula becomes

$$E = \frac{0{\cdot}0575}{n} \log_{10} \frac{p'}{p} \text{ volts.} \quad . \quad . \quad (8)$$

Such cells are termed concentration cells. For example, we may have the cell, silver/(N/100) silver-nitrate-solution/(N/10) silver-nitrate-solution/silver. The E.M.F. is then given by

$$E = \frac{0{\cdot}0575}{1} \log \frac{100}{10},$$
$$= 0{\cdot}0575 \text{ volt,}$$

assuming that the ionisation is complete in both the solutions. Actually the E.M.F. is affected also by the ionic velocities, and the exact expression is more complicated than that just given. The rate at which cells of this kind can generate electrical energy is small, because they depend upon diffusion instead of upon chemical energy, and thus the currents so obtainable are small compared with those from the more usual type of cell, even when regard is had to the low E.M.F.

V. Examples of Practical Cells

§ (16) One-fluid Cells.—The simplest type of cell is that in which only a single electrolyte is employed, and, as simplicity is desirable, many such cells have been developed. The so-called "simple cells" belong to this type, but they are not commonly used, as the E.M.F. is low and the depolarisation is generally poor.

(i.) *The Bichromate Cell.*—With regard to depolarisers, there is a choice between liquids and solids, as already mentioned. The simplest form of one-fluid cell with a liquid depolariser is the Bichromate cell. The electrolyte here consists of dilute sulphuric acid with a bichromate or chromic acid dissolved therein. One plate is of carbon and the other of zinc, which, owing to the electrolyte being acid, must be amalgamated. The E.M.F. is 1·9-2·0 volts. Potassium bichromate is often used as the depolariser; sodium bichromate, however, not only gives a better result, but is much more easily soluble and gives less trouble in the formation of alum crystals, which are liable to form if the cell is used over a considerable period and evaporation is allowed to take place. A still more easy depolariser to use is chromic acid, as it is very quickly dissolved and a smaller weight is required. A suitable formula for general use is as follows:

Water	1 litre	35 fluid oz.
Sodium bichromate .	70 grammes	2·5 oz.
Sulphuric acid (conc.)	100 c.c.	3·5 fluid oz.

The sodium bichromate may be replaced by about 45 grammes or 1·8 oz. of chromic acid in the respective formulae. The sulphuric acid may be added to the water, and the depolariser dissolved therein subsequently, or the acid may be added to the solution of the depolariser.

The depolarisation is not very satisfactory in such cells; it falls off rather rapidly owing to the decomposition of the depolariser and its dilution, not merely with its own products but with the zinc sulphate formed by the action of the cell. A considerably better result is obtained by introducing a porous pot, as seen in *Fig.* 7. The zinc is placed in the porous pot which alone contains the depolarising solution, sometimes mixed with a little acid or zinc sulphate to reduce the internal resistance. When the cell is not in use the zinc should be removed if the electrolyte is acid. If a porous pot is used it should contain a little mercury to help in maintaining the amalgamation of the zinc. Porous pots should be kept in water when out of use, to prevent crystallisation of salts in their pores and consequent splitting.

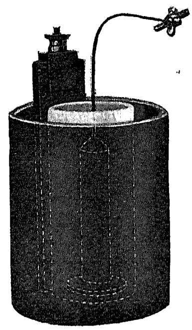


Fig. 7.

Constant polarisation can only be secured by removing and replenishing the depolariser as rapidly as it is used. This method has been successfully applied in the Benkö cell, which has been described in detail by the present writer.[1]

(ii.) *The Lalande Cell.*—When we come to solid depolarisers we find that two main types have been developed. One of these depends upon the use of cupric oxide. This is a convenient depolariser, for it can be made up in the form of slabs fitted into a framework of copper, thus forming the positive plate. The necessary conductivity is given to this plate by reducing the surface. The electrolyte is a solution of caustic soda (one part of soda to three of water by weight), and therefore the zinc does not require amalgamation, as appreciable local action is absent except at the surface of the electrolyte, where it slowly takes effect. Very good results are given by these cells, the depolarisation being excellent and permitting a considerable current, but the E.M.F. is low. At first this may be 1·0 or 1·1 volts, but the effective value is only about 0·75 volt.

This form of cell was originally devised by De Lalande, but has been modified in detail by others, particularly by Edison. Such cells remain effective a considerable time without attention and have therefore been used on a

[1] *Inst. El. Eng. J.*, 1911, xlvi. 741.

considerable scale for track-signalling on railways.

(iii.) *The Leclanché Cell.*—The most notable example of a single-fluid cell with solid depolarisers is the one that was introduced by Leclanché in 1868. In this case the positive consists of a carbon surrounded by manganese peroxide, or some equivalent combination, and the electrolyte is a solution of ammonium chloride (2 oz. to 4 oz. to the pint). Since the electrolyte is neutral or alkaline, local action is negligible, but the zincs are usually amalgamated. A certain amount of local action slowly takes place, more particularly (as is always the case) near the surface of the electrolyte. The E.M.F. initially is about 1·5 volts, but as the depolarisation is not very perfect this value falls somewhat, and when on closed circuit for any length of time the value may be more nearly 1 volt. For this reason Leclanché cells are employed only for intermittent work, and generally where the current required is comparatively small. They can be left without attention and therefore have become very popular in such applications as electric bells.

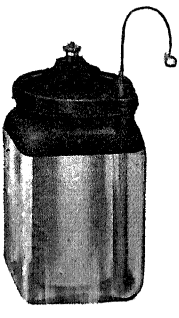


FIG. 8.

The reaction may be roughly described by saying that the chlorine of the ammonium chloride attacks the zinc and that the NH_4 ion passes to the manganese peroxide. Here it splits up into NH_3 and hydrogen, the latter reducing the MnO_2 to Mn_2O_3.

The original and most usual form of Leclanché cell is that shown in *Fig.* 8. Here the manganese peroxide is mixed with broken gas carbon and is held against the carbon plate by being packed into a porous pot. The latter is sealed up with pitch in which there is a small glass vent tube to allow the liberated ammonia to escape. The effectiveness of the depolarisation must depend to no small extent on the intimate mixing of the carbon and the manganese peroxide, because the hydrogen will be deposited upon the carbon (owing to its high conductivity) and must find its way to the manganese peroxide before its oxidation can result. For this reason it has become customary to use both manganese peroxide and carbon in the powdered form. The British Post Office specify that the manganese ore shall pass through a sieve having 40 meshes to the inch, but shall remain on a sieve having 60 meshes to the inch. It is found that sharp crystalline grains of the ore are more effective than rounded grains.

The objection to this construction of the cell is that the porous pot introduces resistance and that the cell is open, so that evaporation takes place. For the former reason, agglomerate blocks, made up of carbon and manganese peroxide held together by some binding material, have been used. These are generally held against the carbon plate, one on each side, by india-rubber bands. In this way the porous pot becomes unnecessary, but the depolarisation is not so satisfactory as with a porous pot properly charged. By adding graphite the agglomerate block may be used to take the place of the carbon. This method is used in the Leclanché-Barbier cell, made by the Silvertown Co. (*Fig.* 9), which has the advantage of being closed and therefore free from evaporation.

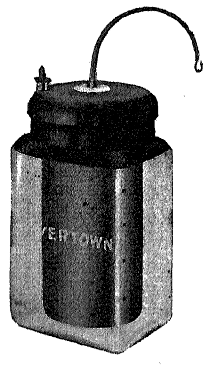


FIG. 9.

The disadvantages of the porous pot have been overcome by using what are known as "sack elements." It may be said that these are obtained by replacing the porous pot by canvas. The carbon is surrounded by the depolarising mixture, which is then bound round tightly in canvas. A suitable top and bottom are fitted as seen in *Fig.* 10, which shows an element by Siemens Brothers & Co. Such elements have been found to be very satisfactory, and cells on this principle have been made in comparatively large sizes so as to be capable of giving much larger currents. The Post Office form of Leclanché cell with sack element, as made by Siemens Brothers & Co., is shown in *Fig.* 11.

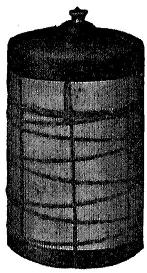


FIG. 10.

Some other depolarising materials have been used to a limited extent in single-fluid cells. Of these, mention may be made of lead peroxide, silver chloride, and acid mercuric sulphate.

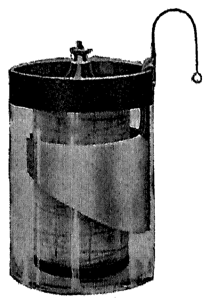


FIG. 11.

(iv.) *Cells with Gaseous Depolarisation.*—Attempts have been made to use atmospheric oxygen as the depolariser, but the difficulty is to bring it into action. The latest cell of this kind is that due to Prof. C. Féry. In this cell the zinc, instead of being of the usual vertical type, is a horizontal plate at the bottom of the cell. If a vertical zinc passing through the surface of the electrolyte is employed in the usual way it becomes attacked at this point by the atmospheric oxygen in the electrolyte, and basic salts are thus formed, which are troublesome. This is avoided by placing the zinc at the bottom of the cell. Any crystals that are formed remain in the lower half of the cell, and the upper part of the solution remains as a store-house of dissolved oxygen. The carbon is in the form of a vertical cylinder with a large surface, and depolarisation is effected by the oxygen in solution coming in contact with the carbon. Such a cell is capable of maintaining a current through 50 ohms for months, the P.D. falling from, say, 0·87 to 0·81 volt.

§ (17) TWO-FLUID CELLS.—In cells of the two-fluid type a liquid depolariser is used and is of such a nature that it cannot be mixed with the electrolyte employed to attack the zinc. The depolarisation is usually very effective, but porous pots are often troublesome.

(i.) *The Daniell Cell.*—There are various modifications of Daniell cell, the simplest consisting of a jar containing a porous pot. In the latter is placed the zinc, and in the space between the porous pot and the outer jar is placed a cylindrical sheet of copper. The zinc stands in dilute sulphuric acid, or preferably in a solution of zinc sulphate, and the copper plate in a saturated solution of copper sulphate. The E.M.F. varies from 1·07 to 1·14 volts, the value depending on the densities of the solutions. The reactions involved have already been described.

The sulphuric acid may be of the strength obtained by mixing one volume of concentrated acid with 10 volumes of water. The zinc sulphate may vary considerably in concentration. If a quarter of a pound is dissolved in a pint of water this will generally be sufficient. The copper sulphate, on the other hand, should be concentrated, and therefore crystals of the salt are often placed in the cell; also the area of the copper plate should be as large as possible. By the nature of the cell the internal resistance is rather high.

It is essential that the copper sulphate should not reach the zinc, as it would then be immediately deposited thereon and local action would ensue. With the arrangement just described diffusion is inevitable after a time and therefore the cell cannot be left for long periods, particularly if not in use.

This defect is somewhat remedied in the gravity form (*Fig.* 12). Here the copper plate is placed at the bottom of the jar, with crystals upon it, and the zinc plate is suspended above. In a cell of this kind diffusion is not so marked, provided the cell is kept in use. Also the porous pot is avoided; but the cell is not so portable, for any shaking tends to increase the diffusion of the copper sulphate. Kelvin's tray battery, which is a form of gravity Daniell, has been largely used in submarine telegraphy. In other modifications, such as that by Siemens and Halske, Minotto's and Meidinger's, various methods have been adopted with a view to greater permanence, at the same time eliminating the porous pot.

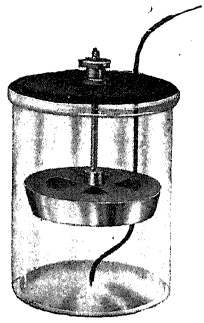


FIG. 12.

(ii.) *The Grove Cell.*—In this cell the depolariser is strong nitric acid. The form usually adopted is illustrated in *Fig.* 13, and the principle of the cell has been already described. The positive plate, which is a sheet of platinum, is placed in a flat narrow porous pot holding the nitric acid. The zinc, which stands in dilute sulphuric acid, is in the form of a heavy casting extending round both sides of the porous pot, so that the internal resistance is low. The E.M.F. is 1·9 to 2 volts, and thus such cells are capable of giving considerable currents provided they are required only for, say, one day. An objection to nitric acid is that nitric and nitrous oxides are formed, giving rise to corrosive fumes.

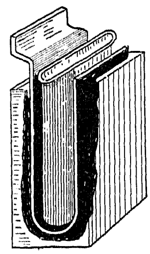


FIG. 13.

(iii.) *The Bunsen Cell.*—Owing to the high cost of platinum, a carbon rod or plate has been used as the positive, thus converting the Grove cell into what is known as the Bunsen. The cell generally takes the form of a cylindrical porous pot which stands in an outer jar. The porous pot contains the nitric acid and a square rod of carbon, and the zinc takes a cylindrical form surrounding the porous pot. Otherwise, what has been said in regard to the Grove cell applies equally to the Bunsen.

§ (18) DRY CELLS.—The term "dry cell" is strictly a misnomer, as no cell can generate current if it is really dry; but the term is conveniently used to denote cells in which the electrolyte is in the form of a paste or otherwise held so that there is nothing to be spilt if

the cell is inverted. Such cells are now used in enormous quantities.

(i.) *Typical Construction.*—So far, dry cells have been based upon the principles of the Leclanché cell, *i.e.* they consist of zinc and carbon plates with manganese peroxide as the depolariser, ammonium chloride being the electrolyte held in some convenient manner. The construction commonly adopted is shown in *Fig.* 14. The container is made of zinc and this is used as the zinc plate. At the bottom of the container is placed a layer of insulation, such as cardboard. The carbon is surrounded by a mass of black depolarising paste, and between this and the zinc container is a white paste which may be regarded as the electrolyte. Above the depolariser and the white paste is a layer of sawdust or similar porous material, and above this the cell is completed with a bituminous seal provided with a vent tube through which any gases may escape. The E.M.F. is about 1·5 volts, as in the Leclanché cell.

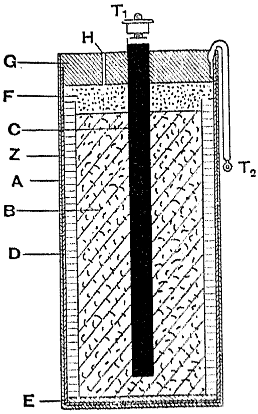


FIG. 14.

A, cardboard case; B, depolarising paste; C, carbon; D, white paste; E, insulating layer; F, sawdust; G, bituminous seal; T_1, positive terminal; T_2, negative terminal; Z, zinc; H, vent.

The exact compositions used are regarded as trade secrets, but the depolarising paste is stated to be of the following character:

Manganese peroxide	10 lb.
Carbon or graphite, or both . . .	10 lb.
Sal-ammoniac	2 lb.
Zinc chloride	1 lb.

Sufficient water is added to make this mixture into a suitable paste. The zinc chloride is necessary to prevent the contents of the cell becoming too dry and it also appears to prevent local action and takes up free ammonia. This paste is rammed into a mould round the carbon, the two being then removed and placed in the container. The white paste is then run in and allowed to set. The latter may be made up of flour, plaster of Paris, sal-ammoniac, and zinc chloride in suitable proportions, with water to form a paste. Sometimes a gelatinous paste is used.

Since the depolarising paste contains electrolyte, the white paste need only be thick enough to keep the black paste from touching the zinc container. The process can be carried a step further by abolishing the white paste, and substituting for it a layer of blotting-paper. This course is often adopted in America. It has the advantage of reducing the internal resistance to a minimum, but the paper is more fragile from the point of view of preventing the depolariser from touching the zinc.

The carbon with the surrounding depolarising paste is often made up in the form of a sack element, in which case a gelatinous electrolyte is used. This is particularly the case in small flash-light cells.

(ii.) *Desiccated Cells.*—Dry cells, although very convenient, suffer from the defect that they dry up in course of time, particularly if not in use. This may be due partly to evaporation and partly to the "setting" of the ingredients. Thus the internal resistance, which may be very low initially, slowly increases. Sometimes, also, gases are formed which cannot escape, or expansion takes place from other causes, and the cell bursts.

In order to eliminate the deterioration which may take place on merely keeping such cells in stock, cells have been developed which may be termed "desiccated cells," as they are truly dry and will not generate a current until water has been added to them.

In these cells the positive element is generally made up in the sack form, and this is placed in the container along with a sufficient supply of sal-ammoniac crystals. Thus the cell becomes practically a Leclanché wet cell on adding the necessary water. In the cell made by the General Electric Co., however, a hollow perforated carbon is used, surrounded by a dry depolarising mixture together with sal-ammoniac crystals, the mixture being kept away from the zinc by absorbent paper. The cell is rendered active by adding water through the carbon.

§ (19) STANDARD CELLS.—Standard cells are of the greatest importance in electrical measurements and much research has been carried out in this direction.

(i.) *The Weston or Cadmium Cell.*—This cell, which is due to E. Weston of the United States, was recommended as a standard of electrical pressure by the International Conference on Electrical Units and Standards which met in London in 1908. The construction of the cell, which can only be very briefly stated here, will be understood by reference to *Fig.* 15. It is set up in a glass vessel consisting of two tubes closed at one end and connected by a cross tube, giving what is called the H-form. The positive electrode is mercury and the negative electrode is a cadmium amalgam containing 12·5 per cent of cadmium. Above the mercury is a layer of paste made by mixing mercurous sulphate with powdered crystals of cadmium sulphate and a saturated aqueous solution of

cadmium sulphate. A layer of cadmium sulphate crystals is then introduced into each limb, and finally the cell is filled to a convenient level with a saturated solution of cadmium

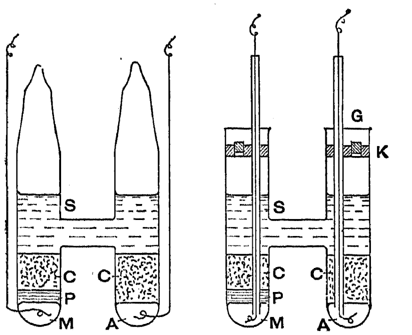


FIG. 15.

M, mercury; A, amalgam; P, paste; C, cadmium sulphate crystals; S, saturated solution of cadmium sulphate; K, cork; G, marine glue.

sulphate. Connection to the mercury and amalgam can be made by means of platinum wires sealed into the glass as shown on the left, in which case the tops of the tubes may be sealed off by heating in the flame of a blowpipe; or platinum wires protected by glass tubes can be used as shown in the right-hand figure, in which case the cell is sealed by means of corks and marine glue. The preparation of the materials must be carried out with great care if the highest accuracy is desired. The methods to be followed will be found in various papers.[1]

The E.M.F. of the cell (as agreed by the National Physical Laboratory, the Bureau of Standards at Washington, and the Reichsanstalt) is 1·0183 volts at 20° C. The variation of E.M.F. with temperature between 0° C and 40° C. follows the formula (recommended by the International Conference in 1908):

$$E_t = E_{20} - 0{\cdot}0000406(t-20) - 0{\cdot}00000095(t-20)^2 + 0{\cdot}00000001(t-20)^3.$$

Thus the temperature coefficient is very small and may generally be neglected. Such cells can be made to a high degree of accuracy. For example, it was found that the mean of 13 Weston cells made by H. L. Bronson and A. N. Shaw differed from the mean of those at the Bureau of Standards by less than 4 micro-volts and from the mean of those at the National Physical Laboratory by less than 5 micro-volts.

(ii.) *The Clark Cell.*—This cell, which was due to Latimer Clark, was the first successful standard of electrical pressure. It is obtained by substituting zinc sulphate for cadmium sulphate in the Weston cell. If made in a similar manner and with suitable precautions in the preparation of the zinc sulphate very satisfactory results are obtained. The E.M.F. is 1·433 volts at 15° C. The great objection to the cell is its high temperature-coefficient. According to W. Jaeger and K. Kahle the variation of E.M.F. with temperature is given by the following equation:

$$E_t = E_{15} - 0{\cdot}00119(t-15) - 0{\cdot}000007(t-15)^2.$$

This means that the variation per degree C. is 0·0012 volt, which is a serious amount, whereas the corresponding figure for the Weston cell is only 0·00004 volt. W. R. C.

[1] See Reports of the Electrical Standards Committee of the B.A., 1907, etc. The more important work is summarised in *Primary Batteries* by W. R. Cooper. See also "Electrical Measurements," § (45).

BATTERIES, SECONDARY

§ (1) INTRODUCTION.—There is no essential electro-chemical difference between the secondary cell and the primary cell when either is used as a generator of electrical energy. In the case of the primary cell, however, the interacting bodies must be mostly thrown away and replaced after a certain discharge has taken place if an efficient result is desired. On the other hand, the interacting bodies in a secondary cell are not replaced, but are brought back to their original condition by passing a current in the reverse direction to that of the current on discharge. A secondary cell may thus be defined as a voltaic cell which is reversible; that is, a cell in which the chemical changes can be reversed by the action of a reverse current.

Secondary cells are sometimes called "accumulators" or "storage cells," but these names are misnomers. Such cells do not accumulate or store electrical energy like a condenser; they are more in the nature of converters, for they change electrical energy into chemical energy and *vice versa.*

Many primary cells are more or less reversible, and some of them have therefore been used as secondary cells. As an example of a cell which is irreversible we may take the simple cell consisting of zinc and platinum in dilute sulphuric acid. In its discharge this cell gives zinc sulphate at the zinc plate (which salt passes into solution) and hydrogen on the platinum plate, this gas being slowly evolved. Although a reverse current may re-deposit some of the dissolved zinc it can never replace the hydrogen that has been lost, and therefore such a cell is useless as a secondary cell, being irreversible. A Daniell cell, on the other hand, is fairly reversible; the zinc will be re-deposited and the copper will pass into solution as copper sulphate when a reverse current is passed, so that the original state is reproduced. But the conditions must be suitable for these changes to take place with regularity; also

diffusion leads to deterioration, so that the Daniell cell has not been seriously used as a secondary cell. Many other primary cells, such as Reynier's zinc/sulphuric-acid/lead-peroxide cell, Sutton's copper/sulphuric-acid/lead-peroxide cell, and the Waddell-Entz zinc/potassium-hydrate/copper-oxide cell, have been tried, but none of them have proved to be commercially successful.

An excellent example of a reversible cell is the well-known gas cell due to Sir W. Grove. If two inverted tubes, with platinum electrodes (preferably platinised), stand in dilute sulphuric acid, and are filled with the acid, the passage of a current will decompose the water so that hydrogen will collect in the cathode tube and oxygen in the anode tube as indicated in *Fig.* 1. If, now, the cell is closed through a

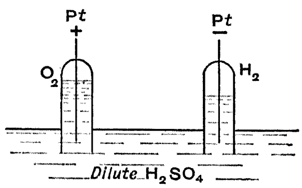


FIG. 1.

resistance it will generate a current until the hydrogen and oxygen have all re-combined to form water, and thus the initial condition will be completely reproduced. The electrodes act as though they consisted of oxygen and hydrogen. Although this cell is in many ways ideal, it suffers from the fact that the action depends upon the area of contact of the platinum with the electrolyte just at the surface of the latter, and consequently the rate at which the combination takes place is low; in other words, the current obtainable is small compared with that

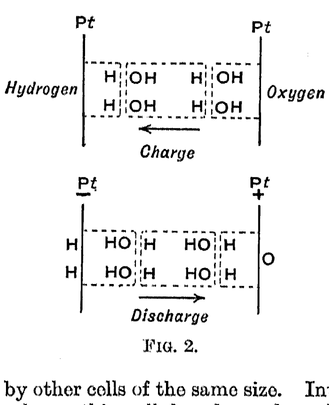


FIG. 2.

given by other cells of the same size. Interesting work on this cell has been done by K. Siegl.[1] The reactions of the gas cell are shown diagrammatically in *Fig.* 2. Here the water is

[1] *Elekt. Zeits.*, 1913, xxxiv. 1317.

looked upon as conducting. The electrolysis, or charging current, causes liberation of hydrogen at the cathode and two hydroxyls at the anode, but the latter combine to give water and oxygen.

Commercial secondary cells are of two general types, namely, (1) the lead cell, and (2) the much more recent iron/nickel cell. The former is acid and the latter is alkaline.

A. Lead Cell

I. General Principles

§ (2) Planté's Work.—The lead cell arose from the work of Gaston Planté on the polarisation of metals in electrolysis. Planté noticed that any voltameter, after the passage of a current, would act as a generator, no matter what the metal of the electrodes, so long as the cathode was polarised, but that lead was particularly active in this respect. He then proceeded to increase the capacity. Lead differs from some metals in that if this metal is used for both electrodes, the hydrogen evolved at the cathode produces no effect upon the lead plate so used, beyond cleaning it; on the other hand, the oxygen evolved at the anode forms a colloidal film of lead peroxide. This film soon stops the oxidising action. Planté found that if the voltameter is then allowed to stand for a sufficient time, or if the cell is short-circuited, the peroxide becomes converted into sulphate, after which the process can be repeated, when not only will the lead sulphate be converted into lead peroxide but more peroxide will be formed than existed previously, because the lead plate can be further attacked until it becomes again protected. By repeating the process a sufficient number of times an adherent and crystalline coating of lead peroxide is obtained of considerable capacity.

The capacity of the other electrode, however, which has been subjected merely to the action of hydrogen, is still negligible, not being appreciably attacked by the acid on discharge. But if a peroxidised plate is made a cathode (that is, if the peroxide upon it is not merely converted into sulphate but the current is still continued in the same direction so as to obtain further reduction) the lead sulphate becomes wholly reduced to lead in a finely divided or spongy form. This lead can then be acted upon by the acid voltaically so that when it is opposed to a peroxidised plate in dilute sulphuric acid a cell is formed capable of supplying a considerable current. When a discharge thus takes place both the spongy lead and the lead peroxide are converted into lead sulphate. A charging current brings back the lead sulphate to spongy lead on the one plate and to lead peroxide on the other, so that the original state is reproduced.

II. Theory of the Lead Cell

§ (3) The Elements.—It has been shown in the article on "Primary Batteries" that the two metals in a cell should be selected so as to be far apart in the electro-chemical series if a high E.M.F. is desired, and thus it might be thought that the use of the same metal, lead, for both the plates would lead to a very poor result. It must be borne in mind, however, that the active material is lead on only one plate, being lead peroxide on the other. It is a well-known fact that if a metal is combined with a very electro-negative body such as oxygen, the resulting compound is electro-negative to the original metal, and the higher the percentage of oxygen the more electro-negative is the compound. Lead peroxide has a high percentage of oxygen, and consequently this compound is very electro-negative to lead, with the result that if lead and lead peroxide are used as the plates of a cell a high E.M.F. is obtained. In this case the lead of the peroxide plate merely acts as a support for the active material.

The lead peroxide plate is commonly called the positive, though electro-negative in character. Similarly the lead plate is usually called the negative, though electro-positive in character. Whatever the method of manufacture, the lead cell is simply a lead/sulphuric-acid/lead-peroxide combination. On discharge, the lead and lead-peroxide are both more or less converted into lead sulphate, whilst on charging the reverse changes take place.

§ (4) The Double Sulphate Theory.—It is rather remarkable that the discharge should result in the same compound at both plates, and this fact has given rise to much controversy. Generally electrolytic reactions give different products at the two electrodes because the ions are different, and therefore the view has often been taken that the formation of lead sulphate on the positive plate could merely be a secondary reaction and could not contribute to the E.M.F., more particularly as it does not depend upon the ion. According to the supporters of the "double sulphate theory," on the other hand, the formation of this sulphate is to be regarded rather as a primary reaction, contributing directly to the E.M.F., and this view is now very generally accepted. The theory is due to Gladstone and Tribe.

The electro-chemical changes that take place can be shown most readily by considering the effect of the ions H and SO_4 of sulphuric acid upon the individual plates.

On discharge, the SO_4 ion attacks the lead of the negative plate, forming lead sulphate. The hydrogen at the positive plate finds a ready depolariser in the lead peroxide, and the reduced oxide is converted into lead sulphate by the sulphuric acid.

On charging, the lead sulphate on the negative is reduced to spongy lead, sulphuric acid being re-formed. This is quite a simple reaction. At the positive plate the lead sulphate is attacked by the SO_4 ion in the presence of water (the acid being always dilute) with the result that the sulphate is converted into lead peroxide and sulphuric acid is re-formed.

These changes are shown in the following equations:

Discharge.

Negative plate . . . $Pb + SO_4 = PbSO_4$.

Positive plate $PbO_2 + H_2 + H_2SO_4 = PbSO_4 + 2H_2O$.

Charge.

Negative plate . . $PbSO_4 + H_2 = Pb + H_2SO_4$

Positive plate $PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4$.

Or on both plates:

$$Pb + PbO_2 + 2H_2SO_4 \rightleftarrows 2PbSO_4 + 2H_2O.$$

On combining the changes that take place at the two plates, bearing in mind that H_2 and SO_4 are together equivalent to H_2SO_4, we obtain the final equation—a chemical equation as distinct from one that is electro-chemical—for the discharge. It will be noticed that if the terms of the pair of equations for the charge be similarly collected together, the molecule of H_2SO_4 which appears on both sides being omitted, the same final equation is obtained, but written backwards. Consequently this equation is conveniently written with arrows indicating that it may be read either way, forwards for discharge and backwards for charge.

§ (5) Experimental Evidence.—We can here only briefly consider the evidence upon which this theory rests.[1]

First, there is the question of the actual bodies involved. Chemical analysis in such cases is often difficult of application. Moreover, when bodies are formed electrolytically it is conceivable that they may differ somewhat from apparently the same bodies formed by ordinary chemical means. The measurement of E.M.F., however, affords a method of detecting differences. There has not been so much question about the spongy lead on the negative; it is rather the lead peroxide on the positive that has given rise to controversy, as it was felt that some other oxide might be involved, or an hydrated form of the peroxide. It is found, however, that a plate made up with chemically prepared PbO_2 gives the same E.M.F. as that

[1] A concise account of the double sulphate theory and others will be found in *The Theory of the Lead Accumulator*, by F. Dolezalek.

observed with the positive plate of a lead cell formed in the ordinary way. The peroxide might be hydrated, but Strecker [1] found that the E.M.F. due to a positive plate is unaffected after the plate has been heated to 170° C., at which temperature the hydrate, if present, would be decomposed. Analysis of the active material at various stages of charge and discharge have been given by Ayrton, Lamb, and Smith.[2] These various investigations are sufficient to prove that PbO_2 is the active material on the positive plate.

It will be realised that if the equations are true, and are due to the actions of the ions as distinct from ordinary chemical changes, the various bodies formed or decomposed must be formed or decomposed in proportion to the quantity of electricity that passes on charge or discharge. Here the lead sulphate is the only body about which there has been much discussion. The formation of sulphate on the negative plate has been found on analysis to agree satisfactorily with that required by the theory. But on the positive plate the agreement (for example, in the work of Mugden [3]) has not been so satisfactory. If the change from a reduced oxide to sulphate is a secondary reaction, agreement would not be expected. On the other hand, a definite divergence from the theoretical figure could hardly be expected, because the conversion to sulphate must be proceeding continually so long as a lower oxide is present. Here then the evidence is somewhat weak, and it is necessary to fall back on thermo-chemical considerations, which will be given later.

The other changes shown in the equations affect the electrolyte, and consist in the formation of water and the using up of H_2SO_4 during discharge and the converse reactions on charge. These changes are found to be strictly proportional to the quantity of electricity that has passed, and since they affect the strength of the sulphuric acid it follows that the variation of the specific gravity of the electrolyte affords a measure of the quantity of electricity that passes. For this reason the specific gravity is the most simple means of determining the state of a cell in regard to charge and discharge. This variation in the amount of H_2SO_4 is often taken as a measure of the amount of $PbSO_4$ formed on the plates, and correctly so, but it throws no light upon the question whether some of the $PbSO_4$ on the positive plate is formed by a secondary reaction.

A final test of the assumed reactions is to ascertain whether the E.M.F. calculated from thermo-chemical data is in agreement with the observed figure, as explained in the article on "Primary Batteries." The E.M.F. is given by the equation

$$E = \frac{H}{46{,}000} + T\frac{dE}{dT},$$

in which H is the heat in calories of the assumed reactions and TdE/dT is the temperature coefficient of the E.M.F. multiplied by the absolute temperature in degrees C. In this case the heats of formation of lead sulphate from lead and from lead peroxide cannot be found directly, so that indirect methods must be adopted. Both Streintz [4] and Tscheltzow [5] have made determinations of this kind. Taking Tscheltzow's results, the calculation may be set out briefly as follows:

Heat of reactions, taking *very* dilute acid	88,800 cal.
Less heat of dilution for acid of sp. gr. 1·150	1,600 „
Heat of reactions . .	87,200 cal.
E.M.F. (omitting $TdE/dT = \frac{87{,}200}{46{,}000} =$	1·89 volts.
Add TdE/dT ($T=290$, $dE/dT=0{\cdot}4 \times 10^{-3}$) = ...	0·12 „
Corrected E.M.F. . . .	2·01 volts.

The first figure given above is true for only very dilute acid, and as sulphuric acid gives out heat on dilution it is necessary to make the correction here shown if the figures are to apply to acid of a specific gravity of 1·150, which is nearer that used in practice. The value of the E.M.F. based on the work of Streintz is somewhat lower, being 1·96 volts. The observed E.M.F. is 1·99 to 2·01 volts, so that the calculated values must be regarded as reasonably satisfactory evidence in support of the reactions assumed in the double sulphate theory. Perhaps the only serious objection that can be taken to this part of the evidence is that many thermo-chemical data are far from accurate.

An interesting light is thrown on the equation of the reactions taking place in the lead cell by the consideration of reversibility. It has been pointed out that the equation may be read in either direction. In other words, the reactions are completely reversible, and therefore it follows that the E.M.F. on charge should be the same as that on discharge, at least when the plates are in the normally charged condition. This has been shown to be the case by Dolezalek.[6] The method followed is indicated in *Fig.* 3. If the potential difference is measured when a low charging current is flowing, a certain value will be obtained which will be in excess of the E.M.F. on account of the internal resistance of the cell, other disturbing causes

[1] *Elekt. Zeits.*, 1891, p. 435.
[2] *J. Inst. El. Eng.*, 1890, xix. 660.
[3] *Zeits. Elektrochemie*, 1899, vi. 309.

[4] *Wied. Ann.*, 1894, lv. 698.
[5] *Comptes Rendus*, 1885, c. 1458.
[6] *Ann. Phys. Chem.*, 1898, lxv. 894, and his *Theory of the Lead Accumulator.*

being practically absent when the value of the current is sufficiently low. If the current is gradually reduced the potential difference falls,

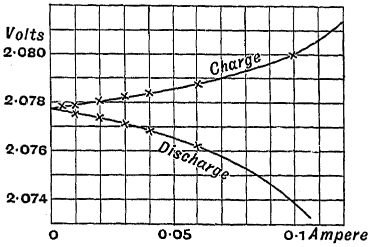


FIG. 3.

becoming more and more nearly equal to the E.M.F. Such observations are shown plotted in *Fig.* 3, giving a curve marked "charge." If this curve is produced until it cuts the axis, a point will be found corresponding to no current, and which may thus be regarded as the limiting value when the charging current is zero, or as the E.M.F. on charge. Similarly the E.M.F. on discharge can be found. If the two curves meet on the axis, as in fact they do, then it may be said that the E.M.F. on charge is the same as that on discharge and that the cell is truly reversible. In actual working, as will be seen later, there are certain variations of the E.M.F. on charge and discharge, but the above holds good for any given normal state of the plates.

§ (6) FÉRY'S THEORY.—Of the alternative theories, reference need be made only to the work of Professor C. Féry,[1] as this forms the most recent attack upon the double-sulphate theory. Féry holds the view that the oxide on the positive plate is Pb_2O_5 instead of PbO_2. This contention is based upon observations that (1) the colour of the active material is much darker than that of normal PbO_2; (2) the active material, when placed in a stream of dry hydrogen, heats up and gives off water until the colour changes to that of ordinary PbO_2; (3) the active material when used as one plate of a cell having zinc as the other plate in dilute sulphuric acid gives an E.M.F. of 2·4 volts, whereas PbO_2 gives 0·7 volt (contrary to the observations of Streintz), and a discharge is obtainable at 2·4 volts, followed by one at 0·7 volt; (4) quantitative analytical measurements indicate a composition approximating to the formula Pb_2O_5. The conclusion is reached that the discharge changes the positive plate from Pb_2O_5 to PbO_2. With regard to the negative, Féry suggests on the ground of colour (which does not appear white on discharge as would be expected from the formation of $PbSO_4$), and from quantitative experiments, that the active lead becomes Pb_2SO_4 on discharge instead of $PbSO_4$. Thermo-chemical support for these conclusions is lacking, as the necessary data are not available. There is other evidence beyond that here briefly given, but it may be said that much more confirmatory evidence will be necessary before this theory can be accepted in place of the double-sulphate theory.

[1] *Société Française des Electriciens, Bulletin,* 1919, ix. 85; *Journ. de Physique,* 1917, v. 187; and 1919, viii. 161.

III. METHODS OF FORMATION

There are three general methods of manufacturing the active surface of plates.

§ (7) THE ORIGINAL PLANTÉ METHOD.—Reference has already been made to the original method adopted by Planté. This gives an excellent type of plate, but the great disadvantage of the method is that it is very slow, particularly in the later stages as the thickness of the lead peroxide increases. Short-circuiting of the cell at intervals is necessary, as already explained, and the number of times that the operation of charging and short-circuiting must be carried through is large. Consequently the process is costly in electric energy and in the capital that is locked up during the period of manufacture.

The process of converting the lead (or oxides in other processes) into active material is termed "forming" the plate. The plates would generally be formed by using dummy plates as the cathodes. In the Planté process only positives are directly produced. Negatives are obtained by taking fully formed positives and reversing the current (*i.e.* using these positives as cathodes), so that the lead peroxide is not only reduced to sulphate, but the reducing process is continued until the active material is finally reduced wholly to spongy lead. The plate is then in the proper state to be used as a negative. Owing to its high cost the original Planté process is no longer employed.

§ (8) THE FAURE, OR PASTED, PROCESS.—In order to eliminate these difficulties Camille Faure introduced the use of lead oxides pasted on to a lead grid, so that some of the necessary work should thus be performed chemically, only the remaining oxidation or reduction being left for the current to perform. This method led to a great saving in time, and is therefore still in common use. There is the further advantage that both positive and negative plates can be formed at the same time. For the positives red lead or Pb_3O_4 is used, since this oxide has a high proportion of oxygen and it is to be subjected to further oxidation. For the negatives, on the other hand, a much lower oxide is desirable, because it is to be subjected to reduction, and therefore litharge or PbO is used. In either case the oxide is made up into a paste with dilute sulphuric acid, and this is applied to lead grids which act as the

mechanical support. The plates so made are allowed to dry and harden, and when they are sufficiently "seasoned" the positives are made anodes and the negatives cathodes in an electrolytic bath with dilute sulphuric acid as the electrolyte, and a suitable current is passed for a considerable time until the Pb_3O_4 is oxidised to PbO_2 and the PbO is reduced to spongy lead. Sometimes the plates are formed with dummies as the other electrodes.

§ (9) THE QUICK FORMATION PLANTÉ PROCESS.—Although the pasted type of plate can be made much more quickly than a Planté plate it is not so strong, the active material being more delicate. A further process has therefore been developed in which some of the corrosion of the lead plate is carried out by chemical or electro-chemical means much more rapidly. For example, if lead is boiled in very dilute nitric acid it becomes oxidised, and this coating of oxide may then be further peroxidised electrolytically. The more usual method, however, is to carry out the two operations together in an electrolytic bath by adding what are termed "quick formation re-agents." As examples of such bodies, nitrates, acetates and chlorates may be mentioned. These re-agents, under the influence of the current, attack the lead, probably forming somewhat unstable lead compounds, which are at once converted by the further action of the current into PbO_2. Thus a good Planté formation is obtained in a comparatively short time.

IV. GENERAL CHARACTERISTICS OF PLATES

§ (10) PLANTÉ PLATES.—Planté negatives are not generally used, as their advantages are not compensated by the additional weight of lead that is entailed. It is otherwise in the case of positives, and consequently Planté positives are employed extensively in stationary batteries where weight is unimportant. Cast lead is commonly used for plates, as this is more readily attacked in the process of formation than rolled lead. Since the capacity depends upon the active area many types of plate have been developed with a view to making the area as great as possible per unit of apparent or nominal surface (*i.e.* the area disregarding indentations), but the type now usually adopted is that which is cast with a large number of grooves, as shown in *Fig.* 4. There is a centre core at A (which is sometimes eaten away during the formation), and there are transverse ribs B at intervals, which give the necessary strength whether the centre core remains or is removed. By using carefully made gun-metal moulds the actual surface may thus become eight to ten times the nominal surface. As many as 40 ribs per inch have been cast in this way, but 30 ribs may be regarded as a more practical limit. A certain thickness of the rib must be left for formation, and if the ribs are very close together the grooves become so filled up with active material that the free circulation of the acid is impeded.

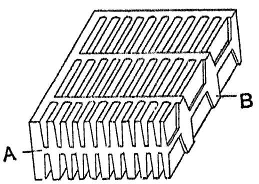


FIG. 4.

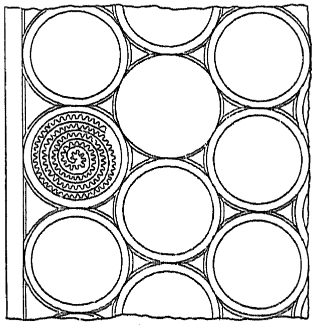


FIG. 5.

One exception to this type of plate has survived and should be mentioned here, namely, the Chloride positive. This is shown in *Fig.* 5. It consists of a grid of antimonial lead having a number of circular perforations. If lead contains a small percentage of antimony its strength is much increased and it is unaffected in the forming bath. The necessary soft lead is in the form of special strip, as seen in *Fig.* 6. This is rolled into spirals, which are fitted into the circular openings in the grid under pressure. The plates are then formed, the formation taking place only on the soft lead spirals.

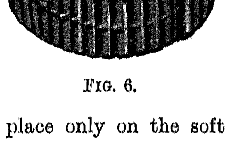


FIG. 6.

§ (11) PASTED PLATES.—The fundamental difficulty in the pasted type of plate is that the paste tends to fall away from the grid, and therefore all grids are made with a view to retaining the paste as completely as possible. Much ingenuity has been displayed in this direction. Broadly, such grids may be classed as (1) those in which the ribs are thicker outside than inside, and (2) those in which the ribs

are thicker inside than on the outside (see *Fig.* 7). Since the positive active material tends to expand, and the negative active material tends to contract, the first type is often used for positives and the second type for negatives, the idea being that the expansion or contraction, as the case may be, will serve to maintain

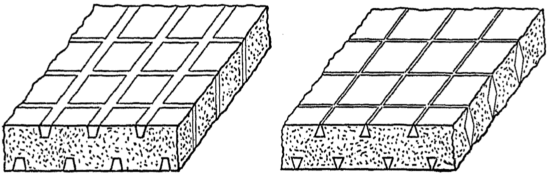


FIG. 7.

contact with the ribs. The grids are always cast from antimonial lead, as it is essential, apart from questions of strength, that they should not undergo formation. From the nature of the construction it follows that pasted plates are much lighter than Planté plates for a given capacity, and they can be made very light for special purposes.

§ (12) SEMI-PLANTÉ POSITIVE PLATES.—What may be described as "Semi-Planté" positive plates are made by taking a thick plate of soft lead, and making grooves or shelves in it such that these will readily hold a paste. Such a plate is shown in *Fig.* 8. They are only given a short formation, and initially they act as pasted plates. In the process of use, however, the supporting lead is acted upon; and, although the capacity of the paste may fall off, further peroxide is formed on the surface of the lead, so that the plate eventually becomes more of a Planté than a pasted plate.

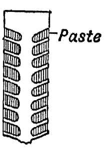


FIG. 8.

§ (13) CHARACTER OF THE ACTIVE MATERIAL.—Difficulties arise in the design, manufacture, and working of plates owing to the inherent properties of the active materials. It may be well, therefore, at this juncture to say something about the often conflicting conditions that have to be met.

From the very nature of the problem it is essential that the active materials should be porous, because the capacity of a plate in ampere-hours depends upon the conversion of a solid material into lead sulphate and the converse change upon charging. Since chemical changes in solids are generally restricted to the surface layers, at least when such changes have to be complete in a few hours, it follows that anything like complete conversion can only take place if the material is very porous. In practice it is improbable that more than about half the active material undergoes the chemical changes, and even so it is difficult to provide for free access for the required quantity of electrolyte.

There is a further consideration which renders high porosity a necessity. Gladstone and Hibbert [1] showed that the E.M.F. of a cell depended on the strength of the sulphuric acid. Their results are given in the form of a curve in *Fig.* 9, from which it will be seen that a change in percentage strength from 20 per cent to 40 per cent entails a rise of E.M.F. from 2·0 volts to 2·1 volts. Now, during the discharge of a cell the acid is being used up and thus becomes diluted within the pores of the active material. The higher the rate of discharge the more will the acid become weakened in this way, and it can only regain its normal strength by diffusion of the stronger acid from the body of the cell. This process leads to a temporary lowering of the E.M.F. on discharge. Similarly, in charging a cell sulphuric acid is set free within the pores of the active material

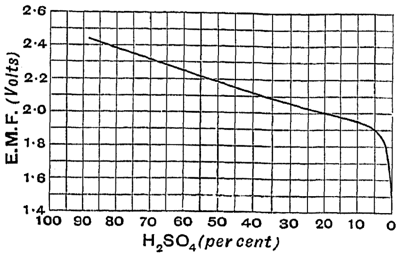


FIG. 9.

and results in the electrolyte being locally strengthened. This again causes the E.M.F. to be increased above the normal value until the strength of acid is reduced by diffusion. It is therefore desirable that the porosity should be sufficient to permit this diffusion to take place readily; otherwise the working of the cell can only be inefficient.

Thus both the spongy lead and the lead peroxide must be porous, but of the two the spongy lead is the more porous and the more yielding. This difference is of some importance, because lead sulphate is more voluminous than either the lead or lead peroxide from which it is formed; consequently the porosity on both plates diminishes on discharge and stresses are set up owing to the desire to expand. Owing to the more yielding character of the spongy lead, the stresses set up in negative plates are less serious than those set up in positive plates. In a positive plate, on the other hand, if there is much more chemical action on the one side than on the other, the stresses on the one side may be sufficient to

[1] *J. Inst. El. Eng.*, 1892, xxi. 412.

cause buckling of the plate, there being a lack of balancing stress on the other side. On this account a cell always contains one more negative plate than positive, so that both sides of every positive plate may be worked equally in charging and discharging. Buckling is practically limited to positive plates and may be caused by over-discharge, resulting in unequal action and more expansion than can be taken up.

Although porosity is of the utmost importance, it is essential that the plate as a whole should have sufficient mechanical strength to ensure a reasonable life. Many methods have been used to secure high porosity, but this has often been attained at the expense of the necessary strength, with the result that a high capacity has been obtained for a given weight but with a poor life, the active material becoming detached from the support or otherwise losing much of its value. For this reason the porous pot form of cell has been attempted more than once, but such cells have always failed to give ultimate satisfaction.

Another point to which considerable attention must be given, more particularly in large cells, is that the lead grids should have sufficient cross-section to carry the current without undue voltage drop. For efficient working and good life it is very necessary that the chemical action should take place evenly over the whole surface of the plate. Now the specific resistance of lead is about 13 times that of copper. It will be seen, therefore, that a serious drop in voltage for a given current is much more readily obtained with lead than with copper conductors, and that insufficient lead in the grids will lead to the chemical action taking place near the lug where the current is collected from the plate in preference to the more distant parts.

§ (14) Box Negatives.—Plates lose their capacity through the porosity of the active material becoming less and through loss of contact with the grid. Also an actual shedding of the active material occurs as the cell is used. As already mentioned, the spongy lead is more yielding than lead peroxide, and is liable to contract. For these reasons, what is known as the box type of negative has been introduced. As seen in *Fig.* 10, this consists of a series of small boxes having a top and bottom of perforated sheet lead. The boxes are made in two halves, and for this purpose a perforated sheet of lead is placed in a special mould into which molten antimonial lead is run. This becomes fused to the sheet, and the mould is so made that ribs are thus formed at intervals, running at right angles to one another, making a series of small trays as the final result. If two such trays are placed with the ribs against each other, a small box is formed having a perforated top and bottom. Actually the ribs of one set of trays are made with projecting pins at intervals, and these fit into corresponding holes in the other set of ribs, so that the two can be easily fixed together. Before doing so some unformed paste is placed in each box, and thus when the trays are fitted together we have a plate in which the paste is contained within perforated lead enclosures. This is now formed electrolytically in the usual way. The advantage of this form of construction is that the active material is well supported, and consequently a paste can be used giving very porous spongy lead.

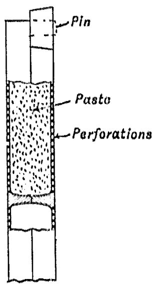
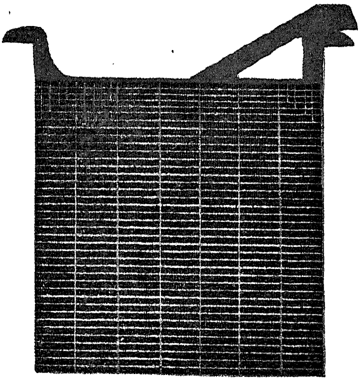


Fig. 10.—Part Section.

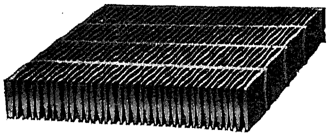
There still remains the difficulty of contraction and resulting loss of contact with the lead support, and to provide against this defect it is usual to add a certain proportion of an "expander." An expander is an inert material which slowly expands under the action of the cell and causes the active material to swell somewhat in the course of time, thus maintaining satisfactory contact between the spongy lead and the support. Secrecy is maintained as to the actual materials employed, but it may be mentioned that alumina has been used for this purpose. Expanders may be used in the ordinary pasted type of negative, but in that case care must be taken to avoid too large a proportion; otherwise the spongy lead becomes forced out, as the grid provides no means of yielding such as is inherent in the perforated lead plate of the box negative.

§ (15) Summary of Characteristics of Plates.—It may be said that the Planté positive is more robust than the pasted positive, but there is not the corresponding advantage in the Planté negative. Consequently it is usual in stationary batteries, where weight is of little consequence, to use Planté positives and pasted negatives. Planté positives must be thick to allow sufficient metal for further formation in the course of time through the effect of continued charge and discharge. Thus the positives tend to maintain their capacity. Negative plates, on the other hand, require much less metal, as this is not attacked, but they tend to lose their capacity through loss of porosity and through loss of contact. The negative active material is more porous and more yielding than the positive. The positive active material is more delicate, less yielding, and more subject to stresses which may cause buckling. Negative plates have greater capacity than positives, owing to the greater porosity of the active material. Typical plates by

Pritchett & Gold and Electrical Power Storage Co., Ltd., are shown in *Figs.* 11 and 12.

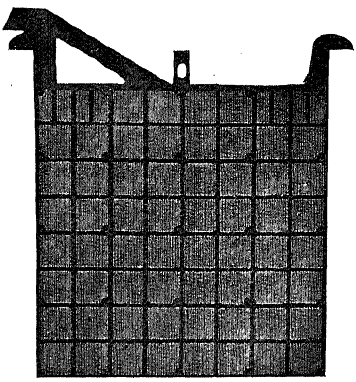


Complete Positive.

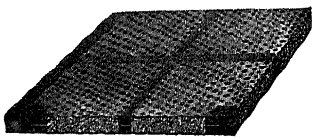


Enlarged Section.

FIG. 11.



Complete Negative.



Enlarged Section.

FIG. 12.

V. DETAILS OF CELLS

The details here given refer to the larger types of cells used in stationary work.

§ (16) BOXES AND SECTIONS.—First, there is the container or box. The material used for this purpose depends upon the size of the cell. Glass boxes are commonly used for the smaller sizes, and have the advantage that inspection of the plates is facilitated by being able to see through the spaces between the plates. For large cells glass becomes unsuitable, and then antimonial lead boxes may be used, or, more usually, wooden boxes lead lined. Ebonite and celluloid are only used for small portable cells.

A number of plates are used in parallel to obtain the necessary capacity. A set of positive plates or of negative plates for a cell is termed a positive or negative section. The plates are usually cast with lugs which, in the case of glass boxes, rest on the top edges of the box, the plates being thus suspended. Although in small cells plates sometimes rest on blocks of wood or porcelain at the bottom of the box, suspension from the top is much preferable, because this leaves a free space in which detached active material may collect without touching the plates. If this touches the plates it provides an internal leakage path, the material being more or less conducting, and the cell then fails to retain its charge satisfactorily. In the case of lead-lined boxes two vertical sheets of glass are placed in the box, one on each side, and the lugs rest on these above the lead lining. This is shown in *Fig.* 13, illustrating a battery by the Hart Accumulator Co., Ltd. The lugs of the positive plates are welded, or "burnt" on as it is termed, to a lead terminal bar, thus forming the positive section. Similarly, the negative plates are connected together, forming the negative section.

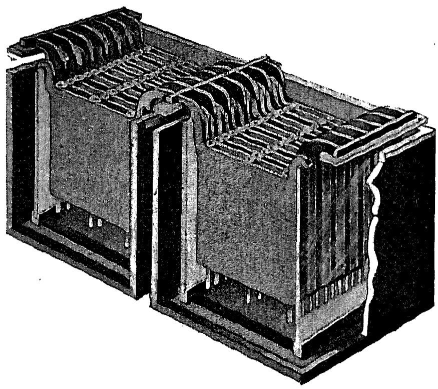


FIG. 13.

When cells are used in series as a battery the positive bar of one cell is connected to the negative bar of the next, the method of connection depending on the size of the cells. In the case of the smaller sizes the lugs of the terminal bars may be bolted together, brass bolts with lead-covered heads and nuts being

preferably used for this purpose. But if the cells are large the connections are made by lead burning. Up to a certain size the ends of the bars may be connected together, but in the largest cells the individual plates are placed in position in the cells, and the lugs of positive and negative plates in adjacent cells are burnt on to a common connecting bar between them, so as to reduce the resistance of the connection as far as possible. The general arrangement will be followed from *Fig.* 13.

§ (17) SEPARATORS.—It is desirable to place separators between the plates to prevent accidental contact and short-circuits. Ebonite is sometimes used for this purpose in small portable cells. In large cells the distance between the plates is usually about ½ inch, and glass tubes standing vertically on the floor of the boxes have been extensively used as separators. The objection to this method is that detached active material may sometimes form a bridge between the plates, more particularly when the plates are not quite true, and where consequently the distance between plates may be reduced considerably here and there. This difficulty has been overcome by the use of the wood board separator, which consists of a thin sheet of wood about the same size as the plates. *Fig.* 14 shows a separator

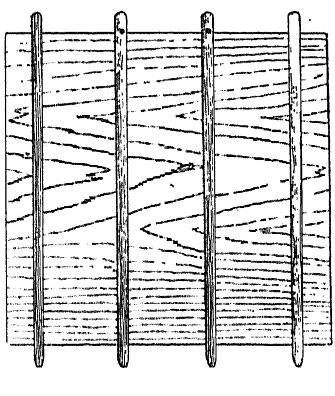


FIG. 14.

of this kind as used by Pritchett & Gold and E.P.S. Co., Ltd. It is carried by wooden dowels and is suspended from the tops of the plates, or is supported by the dowels resting on the bottom of the box. Thus internal short-circuits are almost impossible. A soft wood is used in making these separators, and it is subjected to special treatment, so that such bodies as acetates, which would be harmful to the plates, are removed. Such separators do not materially increase the internal resistance.

The tops of the plates in open cells are generally covered by loose sheets of glass. These are termed "spray arresters," and serve to arrest the acid spray which is given off at the end of a charge when a good deal of gas is being evolved through the decomposition of the electrolyte.

§ (18) ELECTROLYTE.—The electrolyte is dilute sulphuric acid, of specific gravity 1·200-1·215 when the cell is fully charged. On discharge the specific gravity falls to 1·170. These results refer to stationary cells. Pure "Brimstone Acid" should be used (*i.e.* acid made from sulphur), as acid made from pyrites may contain harmful impurities. Water is required at intervals for supplying the loss due to evaporation and gassing, and for this purpose distilled water should be used. Ordinary water leads to the gradual introduction of salts which may be harmful. Rain water may be used, but in towns this contains ammonia, and the accumulation of this impurity, beyond a certain figure, may lead to loss of capacity in the negatives and to abnormal growth in the positives. For the same reason condensed steam from steam pipes is not desirable.

§ (19) EFFECT OF IMPURITIES ON THE PLATES.—The effect of metallic impurities on the plates depends upon whether the impurities are electro-positive or electro-negative to the plate in question. There are four cases to be considered, but only two of them are important, namely, where the impurity is electro-positive to lead peroxide, or electro-negative to lead. These two cases will be followed from *Fig.* 15.

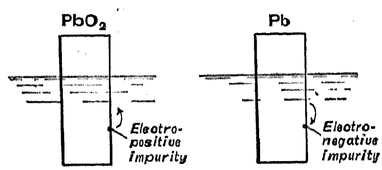


FIG. 15.

The effect is simply one of local action (as described in the article on "Primary Batteries"). On the positive plate, if the impurity is electro-positive to lead peroxide, a local current will flow as shown, the impurity will dissolve, the surrounding lead peroxide will be reduced by the resulting hydrogen, and lead sulphate will be formed. In the case of the negative plate, if the impurity is electro-negative to lead, the local current will be in the opposite direction, hydrogen being deposited on the impurity, and the surrounding lead will be converted into lead sulphate. In both cases it may be said that the plates become more or less discharged. Arsenic, copper, lead, iron, and zinc are electro-positive to lead peroxide; arsenic, copper, and platinum are electro-negative to lead. Platinum is particularly harmful. Many impurities

may find their way into the cell by the electrolyte, as already stated, and some may be introduced in the lead. As a commercial metal, lead is generally fairly pure, but it may contain copper, iron, arsenic, zinc, etc. Of these, iron and copper are the most harmful.

Impurities may also be introduced by the positive plates if manufactured by a quick formation process, and if the quick formation reagent is not thoroughly washed out. If such a reagent is introduced it gives rise to further formation and abnormal growth of the positives.

VI. Capacity, Efficiency, and Working

§ (20) Capacity.—The capacity of a cell is expressed in ampere-hours, and is the number of ampere-hours which it gives on discharge. If a cell is discharged at a constant current such that the commercial discharge is complete in, say, 10 hours, it is said to be discharged at the 10-hour rate; if in 5 hours, at the 5-hour rate, and so on. Before capacity can be defined it becomes necessary to decide when the discharge must be considered to be at an end. Experience has shown that there is a point beyond which it is undesirable to go, and this point is given by the potential difference at the terminals of the cell. This, of course, varies with the current; and therefore the value of the current, as given by the discharge rate, must be taken into account in fixing any limit. The limits fixed by the Tudor Company are as follows:

10-hour rate	1·83 volts per cell		
5 ,,	1·82	,,	,,
3 ,,	1·80	,,	,,
2 ,,	1·78	,,	,,
1 ,,	1·75	,,	,,

Notwithstanding the fact that the voltage limit is lower as the current is increased, the capacity falls off as the rate of discharge is raised. The figures obtained in practice are somewhat as follows, taking the capacity at the 9-hour rate as 100:

Rate of Discharge . .	9 hr.	6 hr.	3 hr.	1 hr.
Capacity	100	90	73	50

At the 1-hour rate the capacity is only about half that at the 9 or 10-hour rate.

Curves of discharge at four rates are given in *Fig.* 16. The E.M.F. cannot be taken as an

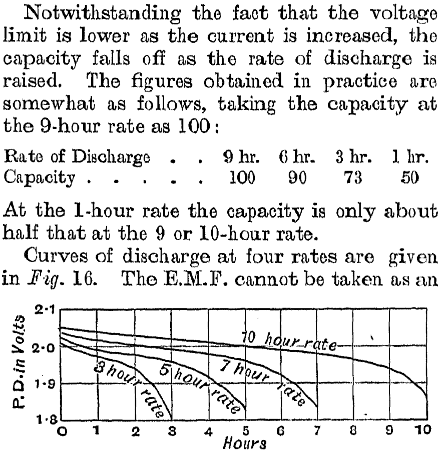


Fig. 16.

indication of the state of a cell, because this varies but little when a cell is not giving current. Immediately after charge the E.M.F. may be as high as 2·4 or even 2·6 volts, but this rapidly falls to a value between 2·05 and 2·1 volts.

§ (21) Efficiency.—The efficiency of a cell can be defined in two ways: (1) Quantity, or ampere-hour, efficiency, which is the ratio of the ampere-hours obtained on discharge to the ampere-hours required on charge; (2) Energy, or watt-hour, efficiency, which is the ratio of the watt-hours obtained on discharge to the watt-hours required on charge. Losses are due to (1) internal resistance, which, however, is small; (2) polarisation, which is comparatively unimportant; (3) low E.M.F. on discharge and high E.M.F. on charge due to changes in acid concentration in the pores of the active material; (4) gassing at the end of a charge. Curves of charge and discharge are given in *Fig.* 17 for one particular rate. It

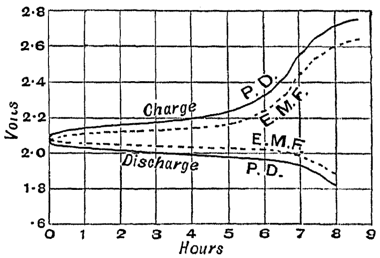


Fig. 17.

will be noticed that the variation of the P.D. is greater than that of the E.M.F. The effect of the first three losses mentioned above is to make the P.D. on charge considerably higher than the P.D. on discharge, and this is where the main energy loss is found. As the rate is raised this difference becomes greater and the efficiency becomes lower.

In testing for efficiency, the maker's limits are taken for the end of the discharge. The corresponding point of cut-off on the charge curve is taken where the curve begins to rise rapidly, *i.e.* where gassing begins to be serious, or where the current is beginning to decompose the electrolyte instead of doing useful work. Constant current is used in both charge and discharge. It is necessary to repeat the tests under identical conditions until the same results repeat themselves, because the results depend upon the previous history of the cell, and if this precaution is not adopted it is possible to obtain a figure for quantity efficiency above 100 per cent.

A distinction must be drawn between efficiency determined in the laboratory and efficiency under working conditions. In the laboratory the quantity efficiency should be about 98 per cent, but in practice this becomes 90-95 per cent. This lower figure is due to the

fact that considerable gassing is necessary if cells are to be kept in condition. Similarly, in the laboratory the energy efficiency is about 85 per cent, which in practice drops to about 75 per cent.

§ (22) WORKING.—In ordinary use it is essential that cells should not be left in the discharged condition, nor discharged below the prescribed voltage limits, because under these conditions the normal lead sulphate changes to a form which is not easily reduced, and the plates are then said to be "sulphated." The precise nature of this change has not been discovered.

The usual indications by which the state of a cell is judged are the specific gravity of the acid and the potential difference. The specific gravity shows a certain lag due to diffusion from the pores of the active material. In practice it is essential that both plates should gas at the end of a charge and that this gassing should continue for half an hour or more, say once a fortnight.

A very useful means of determining whether the positive or negative is at fault is to use a stick of cadmium as a third electrode. This method renders it possible to study the behaviour of both plates separately and compare the results with those of cells which are known to be normal.

VII. SPECIAL CELLS

§ (23) CELLS FOR AUTOMOBILE WORK.—If cells are to be used for driving automobiles or for aeroplane work the reduction of weight becomes of great importance. This is secured in two ways: (1) by reducing the weight of the plates, and (2) by reducing the volume of the electrolyte. The weight of plates is reduced by using pasted plates for both positives and negatives and by making them as thin as possible consistent with the desired life. Plates have been made as thin as $\frac{1}{16}$ inch, but for use on automobiles they must be somewhat thicker.

It is, of course, essential that the normal quantity of H_2SO_4 should be retained, but the water may be considerably reduced with a corresponding saving in weight. This means that the range of specific gravity over which a cell works must be increased. The normal reactions do not take place if the specific gravity is below 1·100 or thereabouts; and if it is above 1·300 the acid acts directly upon the negative plates to a serious extent, giving so-called self-discharge. A range of, say, 1·140-1·280 is practicable.

By adopting these measures much lighter cells may be made. Their effectiveness from this point of view is best expressed in terms of "specific output," or as watt-hours output per lb. of complete cell for a stated rate. The best that can be expected is about 14 watt-hours per lb. at the 5-hour rate, but this figure falls off during the life of the cell somewhat seriously.

§ (24) THE "IRONCLAD EXIDE" CELL.—A special type of cell has been developed by the Chloride Co., and is termed the "Ironclad Exide" cell to emphasise its strength, not as an indication that iron is an element in its construction. The essential feature is the positive, which is made up of an ebonite cylinder with a lead core, the space between being packed with the positive active material, as shown in *Fig.* 18. The ebonite cylinder has a large number of fine saw cuts on each side, so that it acts like a porous pot, permitting the chemical actions to take place, but keeping the delicate active material in position. A positive plate is made by fitting a number of these elements into a lead frame, each end of the lead core being burnt to the frame. The negative is an Exide pasted plate of the usual type. Satisfactory results have been obtained with this cell, which is largely used in automobiles, being more robust than the usual type.

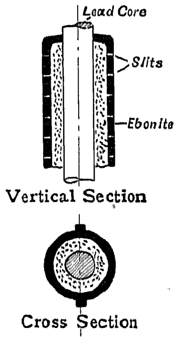


FIG. 18.

§ (25) THE "BIPOL" CELL.—This cell, which is due to H. Leitner and W. H. Exley, is of interest, as it introduces a novel principle. The plates are built up on a wooden frame, the wood being recessed, as seen in *Fig.* 19. In each recess is a lead spiral, and the recesses are filled with paste under pressure. Thus the paste is retained by the wooden structure, but the spirals form, as it were, the conducting grid. Also the paste on each side of the centre web is keyed by having holes through the web. Further, the paste in the grooves is alternately positive and negative, and the spirals of alternate grooves are connected together. The result is that every plate is half-positive and half-negative. The positive portions act with the adjacent negative portions, and as the distance between the two is quite small a low internal resistance is secured. Also very even action is ensured throughout the plate, because

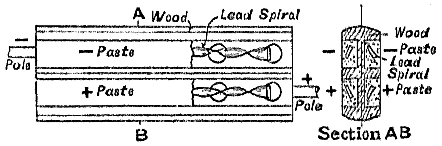


FIG. 19.

every positive portion has a negative portion near to it.

B. The Iron-nickel or Edison Cell

§ (26) *The Iron-nickel or Edison Cell.*—The iron-nickel cell has not been examined theoretically to anything like the same extent as the lead cell. The name of Edison has been chiefly associated therewith, but E. W. Jungner has also worked for years successfully at this subject. The cell has been developed with a view to a high specific output.

Taking the main constituents, it may be said that the charged cell consists of a positive having an oxide of nickel as the active material, a negative with iron as the active material, and an electrolyte of potassium hydrate dissolved in water. The exact formula of the nickel oxide does not appear to have been completely established, but as an approximation we may suppose that it is NiO_2, or the hydrated form $Ni(OH)_4$. The action is more readily seen if the hydroxides are taken, and it may then be represented by *Fig.* 20. On discharge the OH ions of the KOH go to the negative, so that the iron becomes oxidised, and the K ions go to the $Ni(OH)_4$, reducing it to $Ni(OH)_2$. On charge the converse change takes place. As a chemical equation this may be written

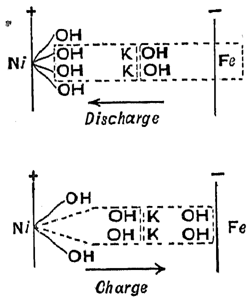


FIG. 20.

$$Ni(OH)_4 + KOH + Fe \rightleftarrows Ni(OH)_2 + KOH + Fe(OH)_2.$$

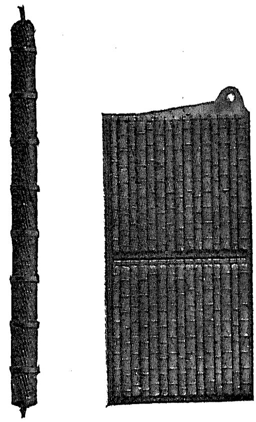
From these considerations an important fact at once becomes evident, namely, that the electrolyte as a whole remains unchanged; it acts merely as a vehicle for the transfer of OH from one plate to the other, and consequently the charge and discharge are not accompanied by any change in specific gravity of the electrolyte.

§ (27) Construction of the Edison Cell.—In practice the electrolyte consists of a 21 per cent solution of potassium hydrate, to which is added a certain percentage of lithium hydrate. The action of the latter is not understood, but it has the effect of materially increasing the capacity. This point has been investigated by L. C. Turnock.[1] It is stated that the amount normally used is 50 grammes per litre.

[1] *Am. Electrochem. Soc. Trans.*, 1917, xxxii. 405.

All the metal parts of the Edison cell are made of sheet steel, nickel plated, and as nickel has a tendency to peel off, the sheets are heated up to a high temperature so as to ensure combination between the steel and the nickel.

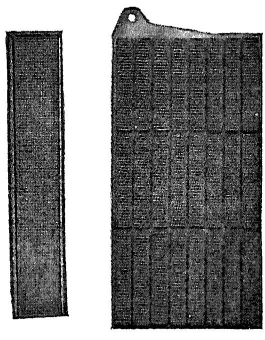
The positive plates consist of 30 tubes held in a nickelled steel frame. The tubes themselves are made from very finely perforated strip of nickelled steel, which is spirally wound and reinforced with steel rings at intervals. The tubes are about four inches long and a quarter of an inch in diameter. They are filled with nickel hydroxide. This material, however, is a rather poor conductor, and consequently thin layers of flake nickel are inserted at intervals. There are over 300 of these double layers, highly compressed, in each tube. The arrangement will be understood from *Fig.* 21.



Tube. Positive Plate.

FIG. 21.

The negative plate is somewhat similar, but in place of tubes 24 flat pockets are used, as seen in *Fig.* 22. The latter are made from finely perforated nickelled steel strip and are filled with specially prepared iron oxide. Here again the conductivity of the oxide is not sufficiently good, and it is therefore improved by incorporating a little mercury. When the pockets are filled they are corrugated, which has the effect of increasing the rigidity and of forcing the metal into close contact with the contents.



Pocket. Negative Plate.

FIG. 22.

Two steel terminal posts are used, having transverse bolts, and upon these are bolted together as many positive and negative plates respectively as are required. The plates of like sign are separated by washers on these

bolts, and the plates in the two sections are kept apart from those of opposite sign by ebonite strip separators. Two end-insulators are provided, so that the whole forms a unit, as seen in *Fig.* 23. This is now inserted in a nickelled steel container. The terminal posts are brought through insulating glands in the cover, and the latter is welded into position. A spring valve is fitted between the terminals, as seen in *Fig.* 24, so that any gas generated can escape, and water or electrolyte can be added. An extremely mechanical arrangement is thus obtained. If cells are connected up to form a battery they are usually mounted in trays so as to keep the containers (being metal) from touching one another.

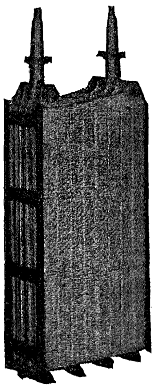


FIG. 23.

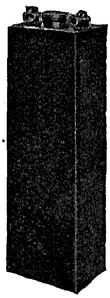


FIG. 24.

The plates are formed by giving the cell a prolonged charge.

§ (28) CHARACTERISTICS.—The E.M.F. of the Edison cell is about 1·4 volt. Upon discharge the P.D. drops continuously and the discharge is considered complete at the 5-hour rate when the P.D. reaches 1 volt. The average P.D. at this rate is about 1·2 volt. From *Fig.* 25 it will be seen that the drop is more continuous and a larger percentage of the initial P.D. than in the case of the lead cell. Charging at constant current is carried out with the same value of the current as in discharging at the 5-hour rate, but the charge is continued for 7 hours. The fall, after the initial rise, which is noticeable in the charge curve is due to the cell warming up, with consequent fall in the resistance of the electrolyte. At the end of the charge the P.D. becomes steady at about 1·84 volt per cell. Since there is no variation in the specific gravity of the electrolyte the state of the cell is judged by the value of the P.D.

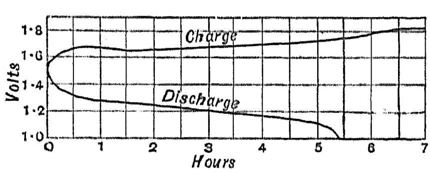


FIG. 25.

The quantity efficiency at the normal 5-hour rate as here described varies from 75 to 80 per cent, and the energy efficiency varies from 55 to 60 per cent. The low efficiency is largely due to decomposition of the electrolyte, which is noticeable even at the beginning of a charge and is very marked at the end. The specific output may be 15 watt-hours per lb., or higher for large-size cells, and the figure is maintained, as the capacity tends to increase when the cell is in use, and this continues for a much longer period than in the case of the lead cell. On the other hand, the watt-hours per cubic foot of space occupied are lower. The internal resistance is higher, potassium hydrate not being such a good conductor as sulphuric acid.

The variation of capacity with rate of discharge is comparatively small, and therefore the number of ampere-hours taken from a cell may be taken as an approximate measure of the extent of discharge under varying conditions.

The electrolyte is kept from carbonating by the container being practically air-tight. Owing to the decomposition during charging it is necessary to add distilled water frequently, but a special device has been developed to facilitate this operation.

The cell is particularly robust, both mechanically and electrically. It may be left in the undischarged state, short-circuited, and charged at high rates. These features, coupled with the high specific output, have caused the Edison cell to be adopted largely for driving electric vehicles.

W. R. C.

BATTERY SWITCH: a switch used to control a number of accumulators. See "Switchgear," § (15).

BECK ARC: a high-intensity arc with the positive carbon rotated and surrounded with methylated spirit vapour. See "Arc Lamps," § (6).

BECQUEREL EFFECT: a change in the electrode potential caused by illuminating the electrode surface. See "Photo-electricity," § (4).

BELLINI-TOSI SYSTEM: an arrangement of receiving apparatus which permits of the determination of the direction from which signals come. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (11).

"BIPOL" CELL: a special type of lead cell, due to Messrs. H. Leitner and W. H. Exley, having a low internal resistance and very even action throughout the plate. See "Batteries, Secondary," § (25).

— C —

CABLES, INSULATED ELECTRIC

Although it is generally recognised that the physical problems connected with the design and manufacture of electric cables are fundamentally bound up with chemical considerations, the interdependence of electrical properties on these relations is not so clearly appreciated.

For example, with regard to conductor materials, the pure metals are better conductors than alloys; and when, by alloying, better physical properties are sought than a given pure metal affords, it is invariably found that such improvements can only be attained at the sacrifice of conductivity.

Again, in dielectric materials, chemical composition (assuming great purity and stability), in conjunction with physical structure, largely determines the degree of excellence of their electrical qualities. Being complex bodies, however, these relations are less direct and simple than in the case of metals; and, as in the familiar case of vulcanised rubber, the chemical composition—as determined by the mechanism of the vulcanising reaction—may be beneficial from both physical and electrical standpoints or advantageous to the one and deleterious as regards the other.

The simple case of conductor materials has been fairly exhausted in the course of production of commercial conductors fulfilling the maximum mechanical requirements compatible with high conductivity, but an enormous amount of work remains to be done in the highly (chemically and physically) complex substances capable of being used as cable dielectrics.

§ (1) Conductors.—With regard to conductor materials commercially admissible for insulated wires and cables, copper occupies the premier position. Its high conductivity not only allows it to be rated at a high current density for a given voltage drop—in other words, the loss incurred in transmitting a given amount of power through it is small compared with that entailed by the use of other commercial metals—but also tends to economise the costly dielectric and protective coverings by reason of their being applied at a radius which is smaller than in the case of a metal of lower conductivity. For purely conductive purposes its only serious rival is aluminium, which, however, requires to have a diameter 28 per cent (or 64 per cent sectional area) in excess of that of copper for equal resistance or a given voltage drop.

This, from the competitive point of view, is largely counterbalanced by its lower specific gravity—2·71 against 8·89 for copper—and, assuming conductivity alone is concerned, the choice largely resolves itself into a matter of relative market prices, although the costs of other components of the complete cable have a bearing on the matter.

There are, however, a number of pros and cons in the case from other standpoints to

which space only permits brief reference. Some of these are of a technical character, while others relate to cable design, and others again to cost considerations entailed by the larger diameter of aluminium cable for a given duty. Technical considerations relate chiefly to the jointing of conductors, which is difficult in the case of aluminium on account of its great affinity for oxygen, which renders soldering—an easy and efficient process in the case of copper—almost impossible, recourse having usually to be made to mechanical joints. The highly electro-positive relation of aluminium to most of the common metals also necessitates careful protection of junctions with them from air and moisture. With regard to design the larger diameter of aluminium is in some cases not a disadvantage, *e.g.* where limitations of dielectric stress [1] demand a conductor radius which is greater than current-carrying capacity requires.

The cost considerations referred to concern cases where the diameters of ducts, or the size of joint boxes, feeder pillars, and fittings are appreciably increased by the use of aluminium.

It is comparatively seldom that tensile strength considerations are predominant in insulated cables, this being, in large cables, sufficiently provided by steel wire armouring. In small cables, where this necessity arises, steel wire is used, either stranded up with the copper conductors or in the form of a suspension strand embodied in the cable, otherwise than as a conductor.

The conductivity of steel wire is, however, of some considerable importance apart from such special cases, because even when its primary function is the mechanical protection of a cable, it should, for reasons of safety and efficient maintenance, form part of a complete earthed circuit. (According to the Home Office Regulations regarding the installation and use of electricity in mines, under the Coal Mines Act of 1913, the effective area of the metallic sheath of any cable has to be equal to 50 per cent of that of the largest conductor contained therein.) For the same reason the conductivity of lead as used for cable sheathings is of interest. Moreover, it is sometimes, in the case of certain concentric systems in which the outer conductor is earthed, used to supplement the latter by carrying a proportion of the working current.

The average specific resistance of steel wire of the kind generally used for armouring purposes is about 7·75 times, and that of lead about 12 times, that of annealed copper.

Phosphor bronze is sometimes used as an insulated conductor, when considerable strength, combined with high conductivity, is required, but on the whole shows little, if any, general economy as compared with hard-drawn electrolytically refined copper.

	Specific Resistance, Microhms per cm. cube at 0° C.	Temp. Coeff., Values of a.*	Specific Gravity.	Melting-point ° C.	Average Tensile Strength, lbs. per sq. inch.	Average Elastic Limit, lbs. per sq. inch.	Coeff. of Expansion per ° C.
Copper	$1{\cdot}561\times10^{-6}$ †	0·00428	8·9	1084	60,000	45,850	$1{\cdot}7\times10^{-5}$
Aluminium	$2{\cdot}655\times10^{-6}$ †	0·00435	2·7	657	29,800	20,860	$2{\cdot}2\times10^{-5}$
Steel	$12{\cdot}1\times10^{-6}$	..	7·7	1350	..	..	$1{\cdot}2\times10^{-5}$
Lead	$18{\cdot}63\times10^{-6}$	..	11·37	327·4	..	..	$2{\cdot}924\times10^{-5}$
Phosphor Bronze ("French telegraph" quality)	$5{\cdot}151\times10^{-6}$	..	..	..	105,000	..	..
Zinc	$5{\cdot}571\times10^{-6}$ †	0·00406	7·2	420	17,920	..	$3{\cdot}0\times10^{-5}$
Magnesium	$4{\cdot}355\times10^{-6}$ †	0·00381	1·7	630	..	..	$3{\cdot}2\times10^{-5}$
Magnesium-Aluminium Alloy (90% Mg. 10% Al.)	$5{\cdot}571\times10^{-6}$	..	1·8	620	..	..	..

* $R_t = R_0(1+at)$. Where R_0 = Res at 0° C.
R_t = „ t° C.

† Fleming and Dewar, *Phil. Mag.*, Sept. 1893.

Zinc has been used in Germany during the recent European War as an emergency substitute for copper, and while there is no likelihood of its continued use under conditions even approximately normal (its specific resistance being about three and a half times and its strength only about a third of that of copper, and its low melting-point—420° C.—practically precluding soldering of joints), it is interesting to note that it was for the first time commercially produced in wire form by extrusion of the metal in the form of rods which were afterwards drawn down in the usual way.

Magnesium and the magnesium-aluminium alloys are somewhat attractive from the point of view of conductivity for weight, but there are considerable difficulties in the way of their use in insulated cables.

The table above shows some of the properties of the metals referred to.

[1] See "Dielectrics," §§ (2-4).

§ (2) Dielectrics.[1]—The general suitability of a dielectric material for a given purpose clearly depends on its electrical, chemical, and physical properties.

The interdependence of these properties—as mentioned at the commencement of this article—is not clearly understood or appreciated.

Moreover, it is exceedingly difficult to indicate in a brief and comprehensive manner their relations.

Naturally, electrical properties are of primary importance, and these are affected by chemical composition and physical structure, these in turn being interrelated.

As an example of the former, hydrocarbons have vastly better electrical properties than carbohydrates or proteins, *e.g.* the phosphoprotein, casein.

As an instance of the latter, rubber, vulcanised in the ordinary way by the agency of sulphur and heat, may have the mechanism of its vulcanising reaction varied by the introduction of small quantities of different metallic oxides, with the effect that its physical structure will also vary; corresponding differences will be found in its electrical properties.

Chemical properties affect the durability of the dielectric material and its general stability under working conditions, normal and abnormal.

Physical properties are closely allied to chemical properties, in that they are largely dependent on chemical composition, which (in conjunction with manipulative treatment) determines physical structure.

The mechanical requirements of the conditions of manufacture and usage of an electric cable determine the minimum radial thickness of dielectric, and this is clearly dependent on the physical limitations of the dielectric material.

In practice it is almost invariably the case that when the requirements regarding dielectric strength are low, *i.e.* in all low-pressure cables, the necessary electrical properties are amply provided when the mechanical requirements are fulfilled.

The fundamental importance of the physical properties of dielectric materials is therefore obvious.

It must be premised that commercial considerations have a considerable over-all bearing on the subject of dielectric materials, and that, in what follows, only permanent dielectrics are under discussion, as distinct from "coverings" permeable to moisture, such as cotton coverings used for dry indoor work, *e.g.* bell, telephone, telegraph, and analogous work at very low voltages, and usually under intermittent conditions of use.

The resultant effect of all the ordinary considerations which enter into the matter is that present-day dielectric materials are practically confined to gutta-percha, vulcanised rubber, vulcanised bitumen, paper, and varnished cambric.

[1] See also "Dielectrics."

Neither of these approximates to an ideal material, and hence their applications are more or less confined to different spheres of usefulness, or their disposition in the cable has to be varied in accordance with their physical or other limitations.

Commercially it is much more feasible to aim at the lower ideal of a reasonably perfect material (combined with suitable methods of utilisation and disposal) for one general set of conditions of use than to strive after an ideal material which would fulfil all conditions.

General consideration of the properties of the above-mentioned materials reveals the first broad distinction, viz. hygroscopic as distinct from water-resisting or non-hygroscopic material.

Gutta-percha is the best representative of the latter class, whereas paper and varnished cambric (or, in fact, any fibrous materials) are hygroscopic to an extent which, under practical conditions, requires the protection of a homogeneous waterproof covering, such as lead sheathing.

Thus the detailed construction of the various types of cable depends on the dielectric material employed.

A further broad distinction is that, as conditions of manufacture, transport, and installation (apart from special requirements of use) demand that cables should be more or less flexible, so that they may at least be coiled and uncoiled without injury, the dielectric materials have either to possess natural flexibility, as in the case of gutta-percha, indiarubber or vulcanised bitumen, or be so subdivided and applied (*i.e.* cut into strips and spirally wound around the conductor, as in the case of paper or varnished cambric) as to compensate by such disposition for lack of that physical property.

Moreover, it is practically essential from the manufacturing point of view that the materials which possess inherent flexibility should be capable of being reduced to a suitable state of plasticity in order that they may be rolled into strips or shaped and extruded in cylindrical form around and enclosing the conductor. For manufacturing reasons also they must possess considerable cohesion and—at least when warm or when freshly cut—natural adhesion under mechanical pressure. Fortunately they do, at some stage or in some condition which can be taken advantage of in manufacture, possess such physical properties. Unvulcanised rubber strips, for example, can be butt welded in the cold by pressing freshly cut edges together and hammering in a manner comparable with welding white-hot iron strips.

With regard to physical properties demanded by conditions of use, these flexible non-hygroscopic materials require to possess resilience or toughness, or a modicum of each, in order to resist deformation.

Vulcanised rubber possesses a maximum of the former property, gutta-percha a maximum of the latter but a minimum of the former, especially at slightly elevated temperatures, such as 80° to 100° C.

Vulcanised bitumen is in an intermediate position, being tough and showing little resilience at 15°-20° C. (even approaching a hard brittle state at a few degrees below the freezing-point of water), but at 50°-70° C. becomes very resilient and loses its toughness, its behaviour under mechanical stress being then roughly comparable with that of a gelatinous body.

Paper and woven fabrics, such as form the foundation of varnished cambric (and of the treated tapes used for the protection or reinforcement of the non-hygroscopic dielectric materials), possess certain flexibility. In paper this is comparatively small, the flexibility of the dielectric as a whole being attained by the relative movement of the convolutions, *i.e.* by their sliding on each other; in varnished cambric, largely by the relative movement of warp and weft of the woven fabric; and in tape coverings partly by this and partly by distortion of the underlying compressible dielectric material.

In connection with the case of varnished cambric, it is worthy of note that the varnish films are invariably less extensible than the fabric and are therefore very liable to be broken up by undue extension of the latter, with considerable detriment to the electrical properties of the whole dielectric.

It is necessary, in order to be able to form a clear view of the relation of properties to uses of dielectric materials, to take into account some of their chemical characteristics. For instance, gutta-percha is liable to oxidation when exposed to air and light, and as above-ground conditions entail such exposure as well as risk of moderately elevated temperatures—which as already noted are physically detrimental to it—it is fortunate that its particularly good non-hygroscopic qualities permit its excellent electrical properties to be taken advantage of under conditions which do not entail its exposure to the above-mentioned deleterious factors, *i.e.* for under water-purposes, such as submarine telegraph work.

Pure unvulcanised rubber, such as produced from the latex of *Hevea Brasiliensis*, having a low resin content, and preserved from enzyme action, etc., by smoke coagulation, is a very stable material under ordinary atmospheric conditions. Its physical structure is such, however, that it will absorb appreciable quantities of water (as received from Para in laminated ball or "biscuit" form, it contains 16-20 per cent by weight of moisture). Its use as a dielectric material is therefore limited. When compounded and vulcanised it is rendered non-absorbent. Like gutta-percha it is susceptible to the action of sunlight, a remarkable feature being that, while the ultra-violet rays have a detrimental action, certain rays corresponding to the blue-green portion of the spectrum have a strengthening action on newly formed sheets of pure rubber.[1]

The chemical stability of vulcanised rubber is variable, primarily because the production of "soft rubber" goods—under which category rubber dielectrics fall—is dependent on the arresting at a suitable stage of a chemical reaction which is incomplete. Ebonite might for the sake of illustration be regarded as the "saturated" product of such a reaction, although such a description is not complete without reference to the proportion of vulcanising ingredients.

In addition to this the quality and proportion of the raw rubber, the skilful design of the rubber "mixing," and its suitable vulcanisation are important factors in determining its rate of natural deterioration and its susceptibility to atmospheric and other deteriorating influences.

Among the latter an important source of deterioration is the catalytic "oxygen carrying" effect of copper conductors on rubber in contact with them, which happens when the tin coating on them is weak or damaged.

This is, of course, primarily a superficial action, just as is the oxidation of the external surface of rubber by exposure to air at high temperatures, or the chemical action of acid fumes, etc., and although the gradual penetration of such action is slow, it must be remembered that the attack on both sides of such a comparatively thin sheet of rubber as an ordinary thickness of dielectric represents is a severe condition.

Distinct from this, however, is the kind of autoxidation or deterioration of the mass which may arise either from the so-called "after vulcanisation" effects or from bad design of the rubber compound.

The latter may be taken to include the use of organic adulterants, or unskilful proportioning of the ordinary legitimate inorganic compounding ingredients and vulcanising agents.

The chemistry of this matter is rather complicated and somewhat obscure, and cannot be adequately referred to here. The mass deterioration above mentioned, however, may be regarded as resulting in weakening of the physical structure due to chemical

[1] Patent No. 23727/10 Beaver and Claremont.

decomposition or rearrangement of the original molecular state.

The "after vulcanisation" effects above mentioned are not usually very clearly distinguishable from those more palpably due to the use of unsuitable materials, but in so far as they are more liable to occur when an excess of uncombined sulphur is present in the rubber (some free sulphur always exists) their cause may be regarded as closely related to bad composition design.

In general, all vulcanised rubber compounds are more or less susceptible to this mass deterioration, which is commonly accepted as a natural occurrence.

Given moderately good design and suitable vulcanisation treatment, however, it is generally greater in low-grade rubber compounds, *i.e.* containing comparatively small percentages—say 25-35 per cent—of rubber, than in those containing 50-70 per cent, the former obviously consisting, from the physical aspect, of a more extended network of rubber substance—holding a large amount of more or less amorphous matter—than the latter; in which, moreover, the proportion of non-rubber substance is small enough to be largely composed of materials which to some extent enter into the vulcanising reaction, resulting in a more than proportionately homogeneous mass.

Apart from chemical analysis—interpretation of the results of which is difficult, even in expert hands—the susceptibility to these forms of deterioration is judged in ordinary commercial practice by large buyers, by means of standard physical tests consisting of heat treatment in air and in steam.

Vulcanised bitumen is remarkably inert to ordinary atmospheric influences and to acids. It is, however, somewhat susceptible to the action of alkalies, although in the ordinary cases of exposure—of which the water of coal mines in this country may be taken as typical—such action is slight and superficial. If, however, as the result of leakage from an incipient fault, alkaline matter is produced by electrolytic decomposition of water at the virtual negative electrode of the leakage circuit, the action is such as to convert the material into a structureless mass of the consistency of clay or mud.

Another type of action, in which leakage current is a factor, first investigated and described by the author,[1] results in the conversion of the normal resilient non-meltable rubber-like state to a condition resembling that of soft pitch, possessing no resilience and readily liquefiable by heat.

Apart from these actions which arise from unsound electrical conditions, however, the material is, as stated above, remarkably inert, and is practically free from the mass deterioration to which vulcanised rubber is more or less naturally susceptible, the above mentioned actions being quite local.

Paper is normally an exceedingly durable material, but may vary in this respect according to the character of the fibres of which it is composed and the methods of preparing them for the paper-making process.

The fibrous composition may vary from pure cellulose, represented by bleached cotton, to mechanical wood, the cellulose content of which is of the order of 45 per cent to 62 per cent, depending on the botanical origin of the wood.

Chemical wood, prepared by the sulphite process, is sufficiently freed from non-cellulose substances (lignone) by the treatment, to have a cellulose content of 80-90 per cent, although this partakes to some extent of the character of oxycellulose, which has less resistance to oxidation and other chemical changes than pure cellulose.

Given careful manufacture, however, and freedom from residual chemicals, very strong and durable paper may be made from chemical wood, especially if only a moderately drastic chemical treatment is employed and a comparatively low yield of cellulose is acceptable for the sake of a more permanent product.

The difference between the chemical stability of pure cellulose (cotton) or pectocellulose (flax, hemp, etc.) and ligno-cellulose (mechanical wood) is demonstrated by the fact that cotton and flax papers exist which have been exposed to atmospheric influences for hundreds of years without as much deterioration as may occur to a mechanical wood paper (newspapers, for example) in a few months.

The strong durable papers used in cable manufacture have usually been made from the pectocellulose group of fibres, manilla, flax, hemp, etc., but at the present time the shortage caused by the recent European war—large areas now devastated having formerly produced the major portion of the world's supply of hemp and flax—is causing advantage to be taken of the improved products of the chemical wood pulp industry, and mixtures of this class of fibre with manilla and hemp are being largely employed by European cable makers.

The following diagram (*Fig.* 1) illustrates the relative durabilities of similarly made papers of different composition, as indicated by their breaking strengths after various intervals of time when heated at 120° C. *in vacuo*. This temperature was chosen in order to obtain the comparative results within a reasonable time, and not as representing practical conditions. The vacuum—averaging twenty-eight inches of mercury—was maintained in order to prevent undue oxidation

[1] *I.E.E. Journal*, liii., Paper on "Cables."

and to ensure uniform conditions as regards humidity.

In the varnished cambric type of dielectric material, given a pure varnish, fully oxidised

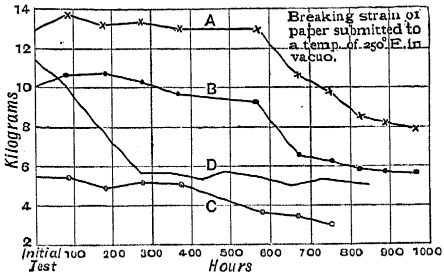


FIG. 1.

Curve *A* represents a pure manilla paper:
,, *B* a mixture of manilla with 50-60 per cent of chemical wood:
,, *C* a pure flax paper: and
,, *D* a paper consisting solely of chemical wood.

to the stage represented by "linoxyn," the product is very durable and resistant to ordinary atmospheric influences, even at the highest temperatures which are permissible for the fabric constituting its foundation. It is, however, susceptible to "superoxidation"[1] when exposed to nascent ozone which may be produced in close proximity to it under conditions of electric stress sufficient to cause static discharges on its surfaces or even in air gaps in the dielectric mass.

The consequent decomposition of the varnish film not only destroys its physical structure, but produces strongly acid bodies which attack and weaken the fabric.

§ (3) CONDITIONS OF USE OF INSULATED CABLES.—These may be divided into (*a*) external and (*b*) internal conditions.

The former chiefly relate to considerations of ambient air temperature, moisture conditions, chemical influences, and so forth. Such conditions or influences are generally countered in various ways, *e.g.* moisture, by impervious sheathings; corrosion, by resistant coverings, etc., and do not necessarily affect the question of the type of dielectric which can be used for a given duty.

The latter (internal conditions) bear more directly on this question, and a brief consideration shows that these conditions differ widely according to the purpose for which a cable is used.

With regard, for example, to heat produced internally, it will be recognised that this is negligible in certain cases for two or three reasons.

Firstly, in telegraph, telephone, and signalling cables not only is the use intermittent, but the current is usually of such low order of average magnitude that the minimum size of conductor which practical conditions render permissible is large compared with the duty.

Secondly, for a continuous small load—as in the case of electric light wires—considerations of minimum size, together with required conditions of voltage drop, keep the current density and the temperature rise low in circumstances where the former could, from the heat dissipation point of view (viz. in small cables), be higher than in larger cables.

Thirdly, in large cables where the current density is fairly high, but still limited by considerations of voltage drop.

In the case of power (feeder) cables, however, not only may voltage drop considerations be comparatively negligible, but in addition to the heat produced by I^2R losses in the conductor, there may be, in very high-pressure cables, additional heating in the cable due to losses in the dielectric.

There is a direct connection in any given dielectric material between the latter and another important internal condition, viz. the electric stress in the dielectric. This only comes into consideration in the case of cables for use at working pressures exceeding a few thousand volts, mechanical considerations (as already indicated) preponderating in the determination of dielectric thicknesses at low pressures, thus keeping dielectric stresses at low values. Heating from this source is therefore a matter of the electrical design of the cable.

§ (4) DESIGN.—In low-pressure cables the dielectric thickness increases with the size of the conductor, but where electric stress considerations predominate, the thickness may decrease as the size of the conductor increases above a critical point which depends on the relation between the working pressure and the maximum permissible stress in the dielectric.[2]

As is well known, in the case of a homogeneous dielectric surrounding a cylindrical conductor the maximum stress is at the surface of the conductor.

If S is the maximum permissible stress in kilovolts per centimetre, E is the working pressure in kilovolts, r is the radius over conductor in centimetres, R is the radius over dielectric in centimetres, and ϵ is the base of Napierian logarithms $=2\cdot7183$, then we find that $S=E/r\log_\epsilon(R/r)$.

It follows that $$\frac{R}{r}=\epsilon^{\frac{E}{Sr}},$$

and that for given values of E and S, R will be a minimum when $E/Sr=1$, *i.e.* when $r=E/S$; in which case $R=r\epsilon=2\cdot7183r$.

[1] Beaver, *Jour. I.E.E.*, 1911, xlvii. 530. Discussion on Fleming and Johnson's Paper on "Chemical Action in the Windings of High-voltage Machines."

[2] See "Dielectrics," § (7).

This ratio for the smallest cross-section of cable for given values of E and S is exemplified in the following diagram (*Fig.* 2), in which various values of r/R are plotted against SR/E, and from which it will be seen that the smallest ordinate has a value of $r/R = \cdot 37$ or $R/r = 2\cdot 7183$.

The permissible value of working stress in a dielectric depends on the margin of safety desired, and on the degree of dielectric heating which is allowable. The dielectric strength of impregnated paper varies in practice between 200 and 300 kilovolts per cm., and the margin of safety is not usually less than 6 or 7. In high-pressure cables considerations of dimensions and weight per length (and consequently of cost) demand that dielectric stresses should be pressed to the highest practicable limit,[1] but it has to be borne in mind that while dielectric losses vary generally as the square of the voltage, they also increase rapidly (above a critical point) with temperature, so

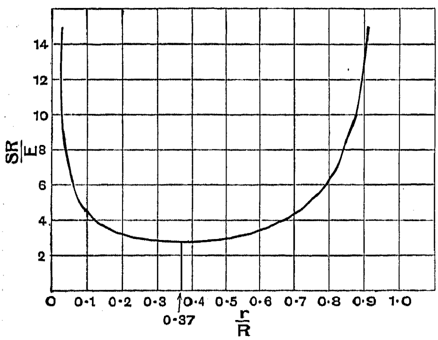


FIG. 2.

that the dielectric heating, superimposed on that due to the I^2R losses in the conductor, is liable to become cumulative, with destructive effects.

The magnitude of the dielectric loss depends, of course, on the power factor of the insulating material.

The following diagram, *Fig.* 3, illustrating the general order of variation of power factor with temperature, is based on average figures given by Still,[2] which agree closely with values determined by Clark and Shanklin[3] at 60 cycles.

Taking the broadest view of design, the cable designer has to primarily consider, in a given case :

(*a*) The requirements entailed by the conditions of use, both internal and external.

(*b*) The intrinsic properties of the available dielectric materials.

(*c*) The compatibility of (*b*) with (*a*).

[1] For data regarding these relations, see the author's Paper on "Cables," *l.c. ante.*

[2] *Electric Transmission of Energy*, p. 224.

[3] *A.I.E.E. Journal*, xxxviii. No. 6, p. 663.

From the foregoing notes regarding properties of dielectric materials, and conditions of use of insulated cables, it scarcely needs

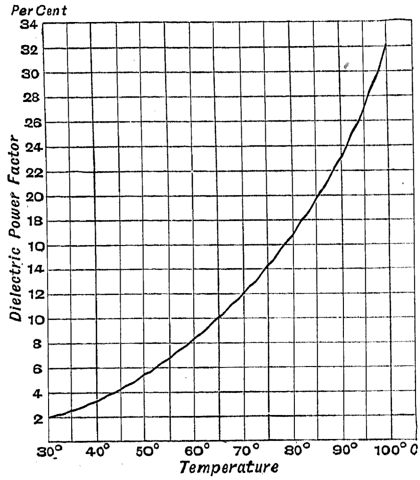


FIG. 3.

pointing out—to take one or two examples—that gutta-percha is never usable for work which entails either internal or external heating, and that on account of its susceptibility to heat and light, and on the other hand its high specific insulation resistance, low specific inductive capacity, and imperviousness to water, its ideal sphere of usefulness is for submarine telegraph work; that vulcanised rubber is best employed where non-hygroscopic properties and moderate resistance to heat is required, and for purposes where its great flexibility, combined with the resistance to mechanical damage, which its strength and resilience affords, is demanded by the conditions of installation and use.

In transmission and distribution cables generally, considerations of cost alone preclude the use of rubber insulation ; and as it happens that the cheaper fibrous (hygroscopic) dielectrics — of which impregnated paper is practically the only type now used for such purposes—have not only superior durability, but also high dielectric strength, low specific inductive capacity and low power factor, the provision of a waterproof sheath, usually lead, is all that is necessary to complete a very efficient type of cable which can be employed almost anywhere in fixed positions, *i.e.* except where portability is a condition of use.

Vulcanised bitumen shares, to a limited extent, as has been already noted, the physical properties of rubber and the commercial advantages of paper insulation ; and although, through lack of appreciation of its limitations and the susceptibilities (previously mentioned)

to certain local deteriorating influences caused by unsound electrical conditions, its employment for ordinary feeder and distributor networks has not been an unalloyed success, its non-hygroscopic character and general chemical stability, together with the advantage in weight which the absence of a metal sheath gives it, renders it a valuable dielectric material for special purposes such as mining work.

Varnished cambric, like vulcanised bitumen, has suffered somewhat in reputation and esteem through non-recognition of its limitations.

Its use, in the early days of electric lighting (in America), without lead sheathing, revealed the fact that its supposed non-hygroscopic qualities were of an insufficient order for ordinary use in underground conduits, and it is not now used for such purposes without being provided with a waterproof sheath.

When so constructed, however, it has no electrical nor commercial advantages over a corresponding paper-insulated, lead-sheathed cable. Its freedom from hygroscopic properties, although falling far short of that of the non-fibrous materials, is of a sufficiently high order to render it suitable for many indoor and protected situations, with the advantage in its favour of not requiring cumbersome and expensive end boxes. Such use will probably be found in the near future in generating and sub-station cabling work, which alone would provide a very wide field for it.

Apart from the pros and cons which determine the type of dielectric, the designer has to consider the construction, proportioning, and assembly of the component parts of cables. These are chiefly dictated by electrical considerations, though with due regard to mechanical requirements.

The former entail not only provision for adequate conduction and insulation, but also for the minimising of losses due to inductive effects between the insulated conductors comprising an alternating current circuit, and between them and metal sheathings or other earthed conductors.

This is accomplished by symmetrical assembly and arrangement of the working conductors in relation to one another and to sheathings, etc.

Thus, in a cable to carry single-phase current, the two conductors are arranged either in twin form with the cores laid up spirally together, or in concentric form with one conductor completely surrounding the other. In a 3-phase cable, the three conductors are laid up so that they occupy positions 120° apart.

In either of these cases it will be observed that the condition of complete symmetry is fulfilled.

The most delicate case in which inductive effects due to lack of symmetry come into consideration is that of long distance telephone cables, where the "balance" may be appreciably affected by such a small factor as unequal tension during the twinning operation of the wires comprising a pair, whereby they are unequally disposed around their virtual axis. This results in a liability to interference with adjacent circuits in the same cable, permitting "cross talk" effects to become evident.

Similar conditions regarding geometric symmetry of assembly are demanded by mechanical considerations, although they may be complied with—for D.C. cables—without necessarily attaining electrostatic or electromagnetic symmetry, for example in the case of the triple concentric form of cable often used for D.C. 3-wire distribution.

Purely mechanical considerations — other than that of simple tensile strength—are so often taken for granted in connection with cables that one or two other aspects of mechanical design may be briefly referred to at this juncture.

Taking first the conductor, it is a *sine qua non* that a certain degree of flexibility is required in it, and the purpose for which a cable is intended determines whether such flexibility is simply in the nature of some degree of pliability enabling it to survive bending a few times without becoming kinked or distorted, or whether—as in the case of a portable or trailing cable—provision has to be made for almost indefinitely repeated bending.

Solid wires and ordinary stranded circular and shaped stranded conductors will usually comply with the first-mentioned requirements to a sufficient extent for ordinary non-portable purposes, while on the other hand extreme pliability can be obtained by using very fine wires.

For portable purposes, however, it is practically essential to employ the multiple strand principle, in which a number of stranded "multiples" are taken (in place of the single wires of the ordinary strand) and these stranded together. In addition to the general flexibility of the whole conductor, each member is then flexible in itself, and in addition has a certain lateral stiffness, so that when such members (and their component wires) are relatively displaced by bending the complete cable, each will return to its original position when the cable is straightened out, and this can be repeated indefinitely.

The assembly of the wires forming any stranded conductor must, of course, be in spiral formation, and — within limits — the shorter the pitch or "lay" of the spiral the greater the flexibility, because the degree

of lateral movement of an individual wire for a given radius of bending of the whole is thereby lessened. At the same time, the shorter the lay the greater the ohmic resistance of the conductor, the rate of increase of the latter—at lengths of lay usually employed in stranding conductors—corresponding to the steep part of the hyperbolic curve representing the relation between increase in length and pitch of a spiral.

Another feature of importance in connection with the relation between lateral displacement and lay of a stranded conductor is that the latter should bear a fairly constant relation to the pitch diameter in all layers. Otherwise, for a given radius of bend applied to the whole, some wires bear more tensile stress than others, and consequently do not retract so completely when the bend is removed. Under these conditions, on repeated bending, local kinking and breaking of a proportion of the wires occurs, and the whole stranded conductor becomes distorted.

A general mechanical requirement in the assembly and building up of a cable is that, when finished, the resultant of all the torsional effects due to the spiral application of the component parts should be practically zero. This is of particular importance where, in installing a cable (*e.g.* in a pit shaft), considerable lengths are free to twist, *i.e.* to relieve any unbalanced torsional stress in them while the cable is suspended prior to fixing. Such cables are almost invariably armoured with steel wires, and it will be obvious that such a tendency will be much more acute in a cable armoured with one layer of wires than in a double armoured cable in which the two layers are applied in opposite directions.

§(5) Rating.—The current-carrying capacity of cables is limited by heating effects, and is consequently dependent on heat dissipation facilities.

Rating has only been more or less tentatively standardised up to the present by such authorities as the Institution of Electrical Engineers, the American Institute of Electrical Engineers, and the Verband Deutscher Electrotechniker, and as the basis of evolution of the Rules of each of these bodies varies (*e.g.* maximum permissible temperatures), little complete fundamental data are available. Moreover, as will be shown later, the work of various investigators has only touched the fringe of the subject and is difficult to co-ordinate.

The reason for this appears to be partly because of the almost infinite variety of sizes and assembly of conductors, types, and qualities of dielectric and covering materials, and methods of laying and installing the finished cables, and partly because the total amount of experimental work has been comparatively small and the results obtained therefrom are not readily convertible into independent physical constants from which reliable formulae can be constructed and applied to specific cases.

In general, regarding a cable as a hollow cylinder having an inner diameter (d) represented by the diameter of the conductor, and an outer diameter (D) represented by the diameter over the dielectric, its thermal conductivity (Z) in watts per cm. length per degree C., according to text-books, will be [1]

$$Z_1 = \frac{2\pi k}{\log_\epsilon \frac{D}{d}},$$

where k is the specific thermal conductivity of the dielectric material in watts per cu. cm. per degree C., and $\log_\epsilon$ represents Napierian logarithms.

Its surface thermal conductivity (emissivity) per cm. length of cable per degree C. will be

$$Z_2 = \pi D h,$$

where h is the emissivity in watts per sq. cm. per degree C. The total thermal conductivity of these two thermal conductors, if regarded as analogous to electrical conductors in series, will be

$$Z_3 = \frac{Z_1 Z_2}{Z_1 + Z_2}.$$

If values of k and h are known, the current-carrying capacity will be derivable from

$$I^2R = Z_3(T - T_a),$$

where T is the maximum permissible temperature, and T_a is the temperature of the surrounding air or other medium.

The earliest investigation on the heating effects of electric current appears to have been carried out by Joh. Müller on bare wires in 1849.[2] Müller deduced from his results that the temperature rise varied as the 1·5 power of the current. (More recent investigators have found the exponent to be of the order of 1·25 to 1·3.)

The subject appears to have received no further attention until Professor George Forbes, in 1882, published his results "On the thickness of wires required to carry different electric currents without overheating." [3]

The earliest data of value relating to insulated cables were due to the work of Kennelly, in America, in 1893.[4] The scope was, at that stage of development of the electrical industry, naturally much more limited than at the present time, and the principal results directly applicable to cable work were consequently those relating to rubber insulated wires and cables run in wooden casing.

Fisher, also in America, published the

[1] See "Heat, Conduction of," Vol. I.

[2] *Bericht über die neuesten Fortschritte der Physik*, Band I.

[3] *The Electrician*, 1882.

[4] "Carrying Capacity of Electric Cables submerged, buried, or suspended in Air," *The Electrical World*, 1893, xxii.

results of a large number of tests in 1905, and a year later prepared a table of current ratings.

Then followed—between 1905 and 1912—a number of investigations, both in this country and America, bearing indirectly on the subject, such as those mentioned below.[1]

Professor Porter's Paper (see footnote) is worthy of note, referring as it does to a little appreciated possibility in connection with the dissipation of heat, viz. that of obtaining a set of conditions (thermal conductivity and radius of coating) in which a covering on a wire carrying current may have the apparently paradoxical effect of cooling it as compared with a similar uncoated wire carrying the same current.

The general fact that coating a body with insulating material may in some circumstances have the effect of keeping it cool was fairly well known to physicists, references to it being made by Lord Kelvin [2] and also by Professor Forbes [3] in 1884.

Experimental proof appears to have been first made on coated wires by Professor Porter.

In Melsom and Booth's Paper [4] on "The Heating of Cables with Current," which was the most important and, so far as modern cables were concerned, the first work bearing directly on the rating of cables, it was shown that the critical point in this respect would be reached when D—the diameter—equalled $2k/h$, after which any increase in the thickness of the insulation would tend to keep the conductor warm instead of cooling it. Dr. Russell made a valuable contribution on this point in the discussion on this Paper.

Melsom and Booth's earlier work, carried out in the National Physical Laboratory with the object of forming a basis of revision for the I.E.E. Wiring Rules, dealt chiefly with the types of wire and cable (rubber and paper insulated) used in interior wiring work. In a later Paper [5] the question of buried cables is dealt with.

They developed the following formula for the relation between current density and copper temperature—

$$i=\frac{4}{d}\sqrt{\frac{T-T_a}{1+qT}}\sqrt{\frac{Dk}{0\cdot48\rho\left(D\log_e\frac{D}{d}+2\frac{k}{h}\right)}},$$

where i is the current density in amperes per sq. cm., ρ the specific resistance of copper at 0° C., and q is the temperature coefficient for copper—they found that the factor $(D\log_e(D/d)+2(k/h))$ was practically constant for a wide range of sizes of cable. k is the thermal conductivity and h the emissivity of the cable. For a definite temperature rise under uniform conditions the formula could therefore be written

$$i=K\left(\frac{D}{S}\right)^n,$$

where S is the sectional area of the conductor, K a constant depending on the system of units adopted and the amount of the temperature rise under consideration, and n a constant.

The authors give the following table showing the values of these constants where i represents current density in amperes per sq. cm., S is the total cross-section of the conductors ("either single concentric or twin") in sq. cms., and D is in centimetres.

For temperature rise of

Type of Cable.	11·1° C.	16·7° C.	27·7° C.
Rubber covered in air . .	$i=101\left(\frac{D}{S}\right)^{0\cdot616}$	$i=126\left(\frac{D}{S}\right)^{0\cdot616}$	..
Rubber covered in casing .	$i=91\left(\frac{D}{S}\right)^{0\cdot66}$	$i=106\left(\frac{D}{S}\right)^{0\cdot66}$	..
Lead covered in air . . .	$i=110\left(\frac{D}{S}\right)^{0\cdot50}$	$i=137\left(\frac{D}{S}\right)^{0\cdot50}$	$i=175\left(\frac{D}{S}\right)^{0\cdot50}$

Following Melsom and Booth's valuable work, two or three important contributions to the subject of rating of cables were made, between 1913 and 1916, in America.[5]

In Japan, Matsumoto[6] obtained valuable results from tests on paper-insulated lead-covered cables laid in sand inside a large iron cylinder.

All these investigators dealt with underground ("buried") cables, those in America directing their attention principally to duct lines—the drawn-in system being the chief method of laying according to American practice—while Matsumoto's work bore on

[1] Rayner, "Temperature Experiments at the National Physical Laboratory," *Journal I.E.E.*, 1905; Lees, "Thermal Conductivity of Insulating Materials," *Phil. Trans.*, 1905, xxiv.A; A. B. Field, "Rise of Temperature due to Eddy Currents in Conductors," *Journal of the A.I.E.E.*, 1905, xxiv.; Searle, "Thermal Conductivity of Rubber," *Cambridge Phil. Soc.*, 1907, xiv.; Russell, "Dielectric Strength of Insulating Materials," *Journal I.E.E.*, 1907, xl.; Kennelly, "Heating of Copper Wires by Current," *Jour. A.I.E.E.*, 1907, xxvi., and "Convection of Heat from Wires," *Jour. A.I.E.E.*, 1909, xxviii.; Bacon, "Testing of Heat Insulating Materials," *Engineering*, September 16, 1910; Porter, "On the Lagging of Pipes and Wires," *Phil. Mag.*, 1910, xx.; Symons and Walker, "The Heat Paths of Electrical Machinery," *Journal I.E.E.*, 1912, xlviii.

[2] Paper entitled "On the Efficiency of Clothing for maintaining Temperature," read before the Roy. Soc. of Edinburgh. Abstr. *Nature*, 1884, xxix. 567.

[3] Paper "On the Relation which ought to subsist between the Strength of an Electric Current and the Diameter of Conductors, to prevent Overheating," *Jour. I.E.E.*, 1884, xiii.

[4] *Jour. I.E.E.*, 1911, xlvii.

[5] See *Journal I.E.E.*, 1921, lix. 181; also "Thermal Effects in Cables."

[5] Atkinson and Fisher, "Current Rating of Electric Cables," *Journal A.I.E.E.*, 1913, xxxii.; Dushman, "The Rating of Cables carrying Current," *Journal A.I.E.E.*, 1913, xxxii.; Powell, "The Temperature Rises of Insulating Lead-covered Cables," *Journal A.I.E.E.*, 1916, xxxv.

[6] Report No. 24 of Electro-Technical Laboratory, Tokio, 1916.

conditions more closely related to the "direct laid" system.

Each appears to have done some experimental work, and then endeavoured to convert the results into physical constants and to construct formulae for general application. Experience shows, however, that such expansion is, for the reasons previously referred to, of little practical use.

The materials and their relative disposition in cables are so complex and varied, and their surroundings in actual use are so diverse, that the possibility of treating the subject on similar lines to any ordinary engineering problem is very remote.

For example, taking the simplest case of single-conductor paper-insulated lead-sheathed cable through which a given current is being passed, and in which consequently a given amount of energy is being converted into heat per unit length, the thermal conductivities of the dielectric and sheathing materials and the emissivity of the latter may be accurately known, but the actual means of heat dissipation may be either by conduction or radiation or partly by each, according to whether the cable is embedded in solid material, slung in the air, or drawn into a tube or duct.

Under the first-named condition the heat gradient in the cable itself will be affected by the thermal conductivity of the embedding material, and if this be, for example, the bitumen filling of a cable trough, it will be further affected by the thermal conductivity of the material of which the trough is made, which may vary from wood or earthenware (materials of low conductivity) to asphalt or iron; and still further by the character of the soil, which may vary from the dry brick-like "laterite" soil largely met with in India, to wet sand.

Under the second-mentioned condition, *i.e.* slung in the air, the dissipation of heat is by radiation (into which convection may enter appreciably) which depends largely on the emissivity of the sheathing material, the temperature gradient in the cable being fixed by the relation between this and the thermal conductivity of the dielectric and its sheathings or coverings. It should also be noted that emissivity varies with temperature, and further that the emissivity of a given material varies according to its condition, *e.g.* that of a blackened lead sheath is greater than when the sheath is new and bright.

Under the third general set of conditions represented by drawn-in systems, the material of which the duct is composed (which may vary from fibre to metal) and that in which it is set (which may vary from concrete to clay or sand) will affect the conduction of heat, while the relation between cable and duct diameters will largely determine the radiation component in the total dissipation.

The juxtaposition of other cables, and warm areas in the cable route will also have an appreciable effect on the permissible rating of a given cable.

The form of cable, *i.e.* whether single, twin or multicore, and the presence or absence of dielectric heating will also affect the total temperature of the cable.

Fig. 4[1] illustrates in a simple manner a few of these variations as determined by the writer some years ago.

It will be noted that the curves all relate to one size of single-conductor cable, insulated with various dielectric materials, and laid or fixed in various ways, and to one current density.

Even under these elementary conditions the variations due to the use of different dielectric materials and methods of laying or fixing are very marked, and without labouring the point it is fairly evident that in the present state of knowledge the calculation of carrying capacity without experimental checking on an ample scale is likely to give unreliable results.

A striking illustration of the necessity for reliable data is furnished by the fact that according to the Rules of the A.I.E.E. (stipulating a reduction of 1° C. from the maximum of 85° C. for paper-insulated cable for each 1000 volts of working pressure) the current density decreases as the operating voltage increases, whereas in fact the reverse is permissible, at least for pressures up to 30,000 volts. A formula of the Melsom and Booth type would tend to indicate this, and practical results show that although the necessary correction factors would modify, they would not reverse this tendency.

The Rules of the V.D.E. also indicate slightly lower ratings for cables for working pressures above 3000 volts than for lower operating voltages.

The I.E.E. give no data in this connection.

A fair amount of work, as yet unpublished, has been and is being done on the heating produced in extra high tension cables by the combined effect of I^2R and dielectric losses, which, as pointed out earlier in this article ("Conditions of use") may become cumulative under certain conditions.

The decided upward turn of the loss/temperature curve (previously mentioned) which occurs in most makes of paper dielectric at a more or less elevated temperature, appears however to be generally above the maximum temperature ordinarily permissible in high pressure cables under working conditions.

[1] Beaver. Discussion on Melsom and Booth's Paper, "The Heating of Cables with Current," *l.c. ante.*

Bang and Louis[1] have investigated the effect of dielectric losses on the permissible current capacity of sets of cables in duct lines, but apart from their Paper the literature of the subject is very scanty.[2]

Clearly, the increase of temperature due to dielectric losses above that due to 1^2R losses referred to, but also because of the limiting effect on the current rating.

The extra heating due to dielectric losses in present day 3-core paper-insulated cables for 30-40 kilovolts working is of the general order of 10-12 per cent of that due to 1^2R losses.

It is clearly, however, dependent on the

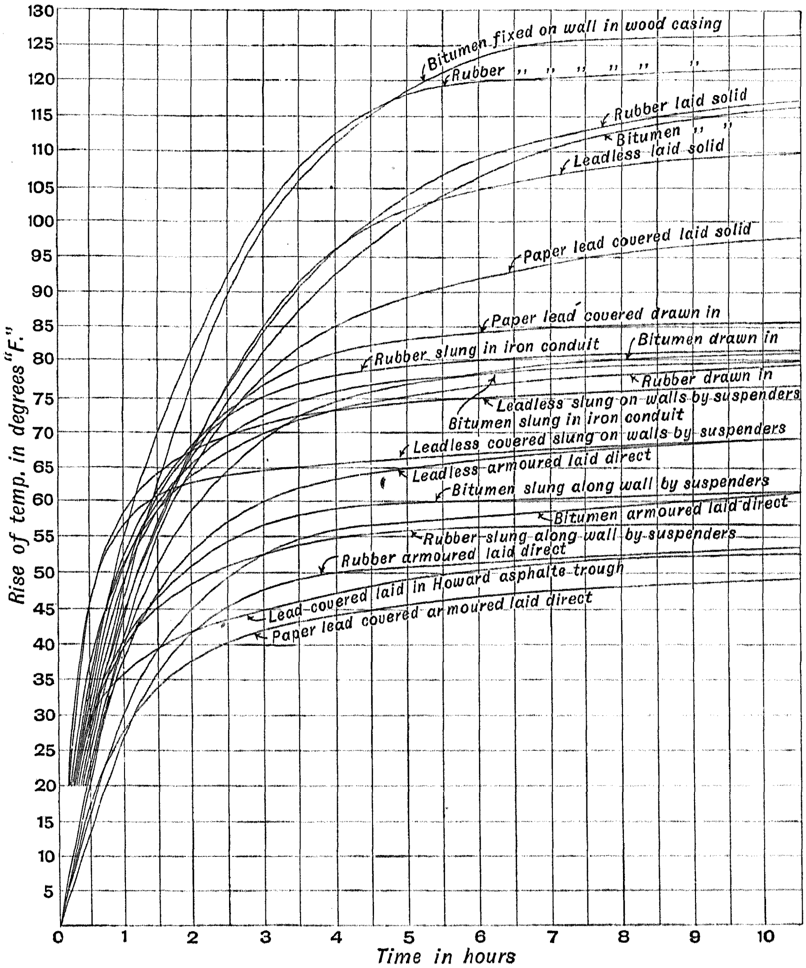


FIG. 4.—Temperature Rise in Four Types of 0·5 sq. in. Single Cables laid in various ways. C.D. 1500 amps. per sq. in.

in extra high pressure cables is of great importance, not only on account of the risk of approaching the cumulative effect above actual values of maximum stress in the dielectric, and on the current density in the conductors, the relation between these factors

[1] "The Influence of Dielectric Losses on the Rating of H. T. Underground Cables," *Jour. A.I.E.E.*, 1917, xxxvi.

[2] Since this article was prepared for the Press, a succinctly written article by Mr. Ralph W. Atkinson has appeared in the *Journal of the A.I.E.E.*, Sept. 1920, p. 831, on "The Current Carrying Capacity of Lead Covered Cables," in which the author gives certain tables of fundamental data and methods of arriving at carrying capacities therefrom by calculation.

The article refers solely to the duct (drawn-in) system of laying, and chiefly to 3-core cables of the paper or varnished cambric-insulated lead-covered type, but is nevertheless a valuable contribution to the literature of the subject, in that it crystallises an important part thereof into a practically usable form.

bringing into view the economic question as to the advantage of reducing dimensions (and therefore the capital cost of the cable) by forcing up the dielectric stress, or conversely reducing the latter so that the current density may be raised as high as possible, the latter tending to reduce the "annual charges" of distribution.

With regard to the effect on rating of the assembly of conductors in a cable, in a recent revision of the Wiring Rules of the I.E.E. (dated October 1919) the following constants are added for use as multipliers for the currents given in the Table for single cables:

Conc.	0·93
3-core	0·88
4-core	0·82

Theoretically these figures should be approximately 0·85, 0·76, and 0·62, and as the values tend to decrease with increasing size of conductors, average practical conditions would probably be met by rounding off these figures to 0·8, 0·7, and 0·6 respectively.

Powell (*l.c. ante*), after working out—on the above-mentioned A.I.E.E. basis—current values for single paper-insulated lead-covered cables for 750, 5000, and 15,000 volts, drawn into ducts, gives the following table of factors connecting the current carrying capacity of multiple conductor cables with that of single conductor cables "having the same total thickness of insulation."

No. of Conductors.	Type of Cable.	Multiply One Conductor Capacity by
		per cent.
2	Flat	87
2	Round	80
2	Concentric	75
3	Round	70
3	Oval Sector	77
3	Clover-leaf Sector	80
4	Round	67

The V.D.E. Rules give ratings for several types of cable (laid direct in the ground) under two headings, viz. for working pressures up to 3 kilovolts and from 3 kilovolts to 10 kilovolts, from which the following ratios may be roughly deduced:

Type of Cable.	Ratio to Carrying Capacity of Single Cable.	
	Working Pressures up to 3 K.V.	Working Pressures 3 to 10 K.V.
	per cent.	per cent.
Twin . . .	70	65
3-core . . .	61·5	59
4-core . . .	57	54
Concentric . .	70	. .
Triple Conc. .	57	. .

It will be quite apparent, however, that any such calculated values may be appreciably fouled by manufacturing variations. For example, a large heavily insulated 3- or 4-core cable is—in the nature of things—liable to be less compactly made than a smaller lightly insulated one of the same type, so that the same effective value of thermal conductivity is not attained, thus varying the temperature gradient in the cable, and consequently the relative sheath temperature; and further, as the emissivity varies with temperature, the total dissipation may be appreciably different.

Multiplying factors for different methods of laying might similarly be experimentally determined under approximately standardised conditions. For example, it can easily be deduced from the foregoing diagram that for a given temperature rise a paper-insulated cable laid solid will only carry about 70 per cent, or when slung in air, about 84 per cent, of the current it will carry when armoured and laid direct in the ground.

Clearly an enormous amount of work remains to be done in order to afford the necessary experimental support to mathematical treatment before the rating of cables can be put on a scientific basis.

Given this support, probably the soundest method of evolving fairly complete formulae would follow Matsumoto's theory, which considers the isothermal lines surrounding a cable as a group of eccentric cylinders, and to add, where necessary, emissivity and other factors—for example, pertaining to the type of cable, the assembly of conductors, and the various methods of laying or fixing them—and in such manner produce empirical formulae, similar to those of Melsom and Booth. It should then be only necessary to insert dimensions and thermal values of the various materials involved in a given case to determine the temperature rise for any value of current.

§ (6) Deterioration of Cables and Palliative Measures.—If this be broadly defined as such deterioration as will permit the electrical failure of a cable by mechanical rupture or weakening of its dielectric or (in the case of hygroscopic insulating materials) of its protective sheathing, permitting the penetration of moisture, it will be readily appreciated that it may be brought about by a large variety of agencies.

These may be classified under two headings, viz.:

(*a*) Natural causes.

(*b*) Abnormal conditions encountered in use.

Some of these causes have necessarily been touched upon in considering the chemical characteristics of dielectric materials in the early part of this article.

With regard to (*a*) it has already been

noted that under normal conditions the dielectric and sheathing materials used in cable manufacture are very inert, although it is well known that vulcanised rubber has an appreciable "natural" rate of deterioration depending on its quality and the skill with which it is designed and manufactured.

This deterioration (in the case of vulcanised rubber) is in the nature of a physical weakening which may be conveniently visualised as the physical breakdown of a network of cohesive material consequent on change in molecular constitution.

Whether the resultant condition is in the nature of hardening or softening depends on the nature of this change, which is in turn dependent on the initial molecular composition of the vulcanised rubber, which again, as mentioned in the early part of this article, is largely dependent on whether its non-rubber ingredients have participated in the vulcanising reaction or not.

Some degree of protection is afforded against this "natural" deterioration (or deterioration under *normal* conditions of exposure) by the application of tapes, braids, and preservative compounds, which tend to exclude air and light. Such exclusion, however, is only partial, as evidenced by the difference between the life of gutta-percha (which, as already noted, is particularly liable to oxidation by exposure to air) when immersed in water and when simply protected from atmospheric influences by these more or less pervious coverings.

The forms of deterioration which fall under category (*b*) pertaining to conditions which may arise in practical use, might perhaps more accurately be defined as more or less local happenings conducive to the production of faults. They are very numerous and complex; and the design of preventive measures depends largely on accurate diagnosis of the causes and processes of development of troubles, and knowledge of practical working conditions.

It will readily be perceived that diagnosis of the primary causes of deterioration is liable to be rendered very difficult in some cases by reason of effects of faults being superposed on the evidence bearing on the conditions leading thereto. For example, a physical or mechanical cause may lead to a fault which may produce electro-chemical (or even purely chemical) effects, due to the action of substances formed electrolytically in the vicinity, as referred to later, which may almost completely mask the original cause.

A few forms of deterioration due to abnormal conditions may with advantage be briefly noted, because with the rapidly increasing use of electric power the variety of conditions of use also becomes multiplied.

Regarding, for instance, the physical effects of temperature on the chief components of an insulated cable, we have as the result of exposure to extreme cold a stiffening effect or loss of resilience in the case of rubber, a tendency to brittleness in the case of vulcanised bitumen, and in impregnated paper a tendency for the laminations to adhere instead of sliding freely over each other.

Rubber which has been frozen exhibits a reluctance to resume its normal resilience on being simply warmed, although it readily regains its normal state on being stretched at normal temperature. In other words, it differs from other materials (in conjunction with some of which it may be used) in regard to the relation between temperature and physical condition, *i.e.* having a lag in this respect which other materials have not.

Generally there is some risk in sharply bending any form of cable when its temperature is very low, although the composition design usually allows for a fair margin of safety in this respect. This risk applies chiefly to installation conditions.

On the other hand, heat—which applies more generally to working conditions—accelerates the natural deterioration of rubber, weakens the resistance of vulcanised bitumen to mechanical stresses, and reduces the viscosity of the impregnating media of paper dielectrics, causing in some cases a tendency for it to become displaced.

Alternations of heating and cooling naturally cause expansion and contraction in cables, and although, as might be expected in view of the spiral construction of most of the component parts of a cable, a considerable proportion of this is accommodated transversely, the more rigid paper-insulated lead-covered type of cable frequently suffers on account of the linear movement. Particularly is this the case where such cables are laid in ducts, because of the comparative freedom which this method of laying affords for movement in a longitudinal direction.

Although the resistance to movement in such cases is small, however, it is not in practice uniformly distributed, *i.e.* so that the movement takes place equally on either side of the centre between two free ends, for example, in a duct length. It therefore sometimes happens that a cable may be expanded largely in one direction (*e.g.* into a manhole at one end of a duct), and subsequent contraction may fail to bring about a corresponding return movement, so that a gradually increasing tensile stress may be set up by successive cycles of expansion and contraction, ultimately resulting in a fracture of the lead sheath. This "creeping" effect is often particularly marked in sloping ducts where cables are fairly heavily loaded. Pallia-

tive measures in the way of anchoring appear to be useless in such cases, and "stepping" the ducts, so that each section between manholes is horizontal, is probably the only satisfactory solution of this trouble.

A feature of the fractures in lead sheaths in such cases is the local crystallisation of the metal, evidently as a result of the mechanical stress. Crystallisation of lead is also producible by pounding or vibration effects and in some obscure cases by chemical attack. It is, however, very difficult to experimentally reproduce such effects, probably because some particular structure of the original lead constitutes a predisposing cause. Further study of this important matter is required. Meantime preventive measures as regards mechanical production of crystallisation relate chiefly to means of obviating the direct transmission of shocks and vibrations to the cable.

So far as underground cables are concerned, methods of laying may be varied to protect them from various deteriorating influences. Effects of movement, *e.g.* pounding by heavy traffic, will be better withstood by an armoured cable laid direct than by a cable laid solid in troughing, because the troughing is comparatively weak at joints between sections, and the movement due to pounding or intermittent stress becomes concentrated thereat.

Armouring is, of course, the best mechanical protection against blows, abrasion, or crushing forces, and in the form of a close sheath of wires affords tensile strength. On the other hand, solid laying in bitumen-filled troughs gives the best chemical protection.

In the case of wires and cables above ground, protection against chemical fumes, etc., has generally to be provided by special coverings and by their treatment with special compounds according to the properties of the chemicals against which protection is desired. In some cases which are neither very severe nor complex, such as in battery rooms where sulphuric acid spray alone has to be considered, a tough acid-resisting rubber sheath, the smooth surface of which can be periodically wiped clean, affords the best protection. Such sheaths can also be made of special rubber compound which resists the action of petrol and oils to a remarkable extent.

The action of ozone (produced by high-tension static discharges) on rubber dielectrics exposed thereto at cable ends trimmed of their coverings to prevent surface leakage, furnishes a remarkable instance of an abstruse kind of deterioration, which often appears in the form of splits or cracks. The writer has demonstrated that this only occurs when the rubber is under tension, as on the outer periphery of a bend, and not when the cable end is straight and therefore free from tensile stress at any part of its surface. In the former case the defect may occur within a few minutes of the exposure of the rubber to the gas, whereas practically no deterioration occurs—as judged by the effect of heat tests on the rubber—in the latter after long exposure.

A point of interest in this connection is that vulcanised bitumen is quite immune from this peculiar effect, but admixture of a very small percentage of rubber with it destroys this immunity. The cause of this phenomenon has not been determined.

Apart from direct chemical action, the likelihood of which is usually known from the character of the surroundings of a cable, electro-chemical action on metal sheathings of cables, caused by vagrant currents in presence of electrolysable media in contact with or in close proximity to a cable, is not only generally more severe, but also more insidious in character; because practically any kind of moisture which is likely to be in contact with a cable sheath will be a good electrolyte, and the paths of stray currents cannot—in detail—easily be foretold.

These vagrant currents may originate from the cable itself (or from a neighbouring cable) by way of leakage over trims at joints or ends, or from incipient faults, or more commonly as return currents from tramway or railway circuits. The magnitude and direction of the latter can, by systematic measurement of potential differences between cable sheaths and earth at various parts of a cable route, be more or less accurately determined; but the former, being of accidental origin, are more difficult to control.

Experience shows that solid laying in bitumen-filled troughs is not a preventive against electrolytic action on the cable, in fact under some circumstances it may aggravate it, because the virtual electrode area is very restricted, and therefore the current density is likely to be high and the corrosion effects correspondingly severe. Again, for similar reasons, metal-sheathed wiring in damp situations in buildings, particularly if run in wood casing, is liable to be badly attacked unless special precautions are taken. These precautions consist in metallically bonding all sheathings together, and earthing them at the source of supply. Intermediate earth plates may be necessary in some cases where the electrolytic survey indicates that current would enter or leave the sheath, so that it can do so by a metallic path. Care has to be taken, however, to avoid inviting vagrant currents on to the cable sheaths.

In principle, the whole matter amounts to short circuiting all the potential electrolytic paths so that any current flowing from or to sheathings passes along metallic conductors instead of by electrolytic paths.

This general principle, first enunciated by the writer many years ago, in opposition to the principle then being largely supported of segregating the sheaths of adjacent lengths of cable, together with the Board of Trade restrictions bearing on maximum earth return voltages, has rendered electrolytic troubles on large networks in this country almost negligible, so far as vagrant currents from external sources (such as electric tramways) are concerned. It has also permitted the free use of metal-sheathed wiring in buildings, which for many years was under an inexplicable cloud owing to troubles which were—though usually attributed to other causes—due to electrolytic action arising from leakage currents from the wires and cables themselves. At the present time the strictest attention is paid to continuity bonding and earthing in metal-sheathed wiring systems; in fact the chief features of most of the special systems now in vogue bear on the matter of efficient and permanent bonding.

The character of the effects of corrosion of lead coverings varies considerably according to the rate at which the action proceeds, and the nature of the electrolyte. A slow or intermittent action may ultimately produce a white powdery deposit of carbonate of lead, due to the formation of a lower oxide and its exposure to moist air, while a rapid action may cause the complete conversion of the entire thickness of lead into a higher oxide, not readily convertible into carbonate, and only rendered noticeable by its reddish colour. The symptoms are not always clear, because in practice effects which are characteristic of actions occurring at anode and cathode in a simple electrolytic cell are frequently mixed up, in close proximity to cable sheaths, owing to the presence of a heterogeneous mass of conducting substances and electrolytes. For this reason it is sometimes difficult to determine from examination whether a direct current has passed to or from an affected sheath. On the other hand, in the case of electrolytic action due to leakage in an alternating current cable, the effects are often less complicated because of the rectifying effect of the electrolyte.

The phenomenon of electric endosmose often plays a conspicuous part in the development of electrolytic cable faults.

When the lead sheath of a paper-insulated cable has been pierced, or penetrated by corrosion, or an incipient fault in a non-metallic sheathed cable has reached a sufficiently low resistance value and a fault circuit is established, this effect causes moisture to be driven from the positive to the negative conductor, with the result that not only is the resistance to earth of the latter reduced by the local accumulation of moisture, but the products of electrolytic action also tend to be concentrated thereat. Many cases have been known of the electrolytic formation of metallic sodium and potassium in close proximity to the virtual negative electrode—the negative cable conductor—surrounded of course by salts of metals. The strongly alkaline character of the moisture in such cases is frequently responsible for direct chemical attack.

It is common knowledge among engineers in charge of direct current networks that failures on negative conductors are much more frequent than on positive conductors; in fact in cases where faults have been known to be in the course of development, a final breakdown has often been postponed by changing the polarity of the affected cables.

When passages exist in a negative cable—even of a capillary order—water is forced along them for considerable distances and up to appreciable pressures by the endosmotic effect. In rubber cables, for instance, blisters full of water are sometimes found to be produced, and on cutting a badly affected cable, water will sometimes spurt for a considerable distance.

It need hardly be pointed out that conditions such as these, as well as others which have been touched upon, should never arise, and would not exist under adequate conditions of maintenance.

The question of a standard of maintenance is a very difficult one, and in spite of its importance its due consideration has lagged behind the advances which have been made in manufacture during the last decade or two. The difficulty appertains chiefly to distributing networks, and may be broadly regarded as due to two chief causes: firstly, the practical impossibility of adequately testing all parts independently of trims and connected apparatus because of the enormous amount of preparatory work entailed in eliminating surface leakage; and secondly, even if the first was rendered possible, of obtaining tests accurately representing the intrinsic values of the insulation resistance of the cable dielectrics, because in a large proportion of cases they are surrounded by other partially insulating materials. For example, in the cases of non-metallic sheathed cables laid in bitumen-filled wooden troughs—a common condition—a test between conductor and earth reveals little regarding the state of the cable dielectric, because any flaw therein is in series with a path of high resistance through the filling and troughing material to earth.

A brief reference may here be made to an incidental though sometimes serious effect arising from or developing out of the conditions last mentioned. When the resistance of such a fault path becomes low enough for an

appreciable current to flow, local heating ensues, and considerable volumes of combustible gas are formed by the volatilisation of the bituminous, trough-filling, and cable-covering materials. Many cases have occurred in which gases generated in this manner have found a way into adjacent feeder pillars, cellars, etc., and have caused violent explosions.

Such high-resistance fault circuits cannot exist for long in metal-sheathed cables, and consequently the explosion risk is less in their case. It is not uncommon, however, for faults in lead-covered cables to repeatedly "clear" themselves by fusing the lead away round a burn out, thus reconverting the fault

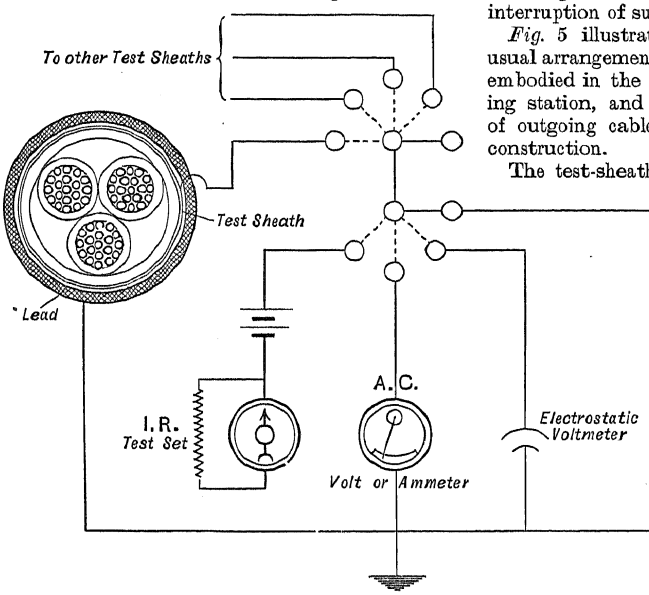


FIG. 5.

into a high-resistance one by increasing the length of its path.

It will therefore be apparent that even in the case of a lead-covered cable a considerable amount of damage may occur before the fault ultimately declares itself.

These considerations, as well as the great general discrepancy between maintenance of cables and that of other component parts of an electric power-supply equipment, led the writer some years ago to devise a simple arrangement [1] whereby a constant watch could be kept on the state of metal-sheathed cables without interruption of supply.

It consists in inserting between the outer boundary of the insulation proper and the metal sheath, an auxiliary conductor in the form of a thin spiral copper tape enveloping all the conductors and their insulation, and insulated from the metal sheath by a thin layer of hygroscopic material, of sufficient thickness to withstand the maximum ill usage to which the cable may be subjected during installation, etc.

The cardinal principle of this construction is that this auxiliary conductor is bound to intercept injurious effects—such as penetration of moisture—proceeding from outside the cable earth, and also leakage from the main conductors to earth. If, therefore, it is connected to suitable observation and testing instruments, indications may be obtained at any desired moment of any abnormal condition arising in the cable, and this without interruption of supply.

Fig. 5 illustrates diagrammatically the usual arrangement of a "detective" panel, embodied in the switchboard of a generating station, and serving for any number of outgoing cables of this "test sheath" construction.

The test-sheath conductor of each cable is connected to separate insulated terminals on the panel, and the selector switch is manipulated to connect any one of them to the test switch, through which they are then connected to observation and testing instruments, such as those shown in the diagram. The electrostatic voltmeter first gives an indication of the existence of any appreciable leakage from the main conductors to the test sheath, and the second instrument affords facility for determining the degree of such leakage. If these first two steps in the test indicate normal conditions in the interior of the cable, the third step is taken of testing the insulation resistance of the hygroscopic dielectric between the test-sheath conductor and the lead covering of the cable, this affording a definite indication of the integrity or otherwise of the waterproof metal sheath of the cable. (It may here be noted that the primary source of failure of a cable is almost invariably extraneous, very few cases ever arising in practice of breakdown due to internal causes.)

The value of this insulation resistance at 15° C. is from 30 to 60 megohms per mile, depending on the diameter of the cable. Lower values are actually obtained under working conditions owing to the warming up of the cable under load, but such variations are easily

[1] Patent No. 22355/12, Beaver and Claremont.

diagnosed, and are not such as to give false indications. In fact, if logged day by day, they give additional interesting information regarding variations in temperature between different cables, and at different periods, *e.g.* during a day or week.

Generally, it will be seen that the system ensures immediate detection of the first phases of development of a fault, and the importance of this in preventing secondary deteriorating effects, such as the electrolysis and other forms above mentioned, can hardly be overrated, especially as little is definitely known as to the time element in the process of fault development in the case of ordinary cables.

In detail, in the event of a fault being indicated on a given cable, not only can an approximate idea be obtained of its degree and probable character (this being largely a matter of intelligent interpretation of test results), but its rate of development can be watched, and a basis formed for decision as to whether prompt action (*i.e.* switching out the cable) is necessary, or whether the location and repair may be deferred to a convenient time. In the latter case the test-sheath conductor is of great value in fault-localising operations (being of known resistance and in intimate contact with the fault), and exact location is rendered possible while the cable is still alive.

Repeated instances have occurred on E.H.T. power systems where this procedure has enabled repair gear to be sent to the located spot at the week end, or some other selected period of light load, thus saving much valuable time and inconvenience.

An important system of automatic protection is also based on this test-sheath design, so that it may be utilised for protective as well as detective functions. This, however, is outside the present scope. (A general description appeared in the *Electrician* of July 5, 1918.)

The difference between the maintenance facilities afforded by this—or any equivalent—system, and the comparative absence thereof represented by previous practice, is obvious; it will also be clear that the method described tends to raise the level of cable maintenance to a plane which is comparable with that of the station plant, or at any rate to one which is more worthy of the important part played by cables in the general scheme of electricity supply.

C. J. B.

Cables, Underground, for telegraph circuits. See "Telegraph, The Electric," § (14).

Thermal and other properties of. See "Thermal Effects in Cables."

Calcium prepared by Electrolysis. See "Electrolysis, Technical Applications of," § (38).

Calibration of Alternating Current Instruments. See "Alternating Current Instruments," § (55).

Calibration Factor (of tubes for mercury resistance standards): a correcting factor arising from the non-uniformity of cross-section.

Values of, for the standards of various countries. See "Electrical Measurements," § (39).

Campbell's Bridge, for the measurement of capacity. See "Capacity and its Measurement," § (61).

For measuring the effective permeability and total losses in iron. See "Magnetic Measurements and Properties of Materials," § (62).

Campbell's Sifter Method, for the comparison of capacity and mutual inductance. See "Capacity and its Measurement," § (56).

Campbell Standard of Mutual Inductance: a primary electrical standard. See "Inductance, The Measurement of," § (54).

Campbell Vibration Galvanometer. See "Vibration Galvanometers," § (10).

Capacitance: the effective capacity of an alternating current circuit. See "Inductance, The Measurement of," § (3).

Capacities (Electrical), comparison of. See "Capacity and its Measurement," § (44).

Capacities in Series and Parallel, formulae for. See "Capacity and its Measurement," § (3).

Capacity or Capacitance: the ratio of the charge on a conductor to the change of potential to which the charge gives rise. In the case of a condenser, the ratio of the charge to the potential difference between the plates. It is measured in farads or fractions of a farad, *e.g.* a microfarad. See "Units of Electrical Measurement," § (17); "Capacity and its Measurement," § (1).

Formulas for the calculation of. See "Capacity and its Measurement," § (7).

Capacity: Comparison of, with resistance. See "Capacity and its Measurement," § (38).

Determination of, in terms of self-inductance. See *ibid.* § (63).

Measurement of, at radio frequencies. See "Radio-frequency Measurements," § (29); "Capacity and its Measurement," § (64).

CAPACITY, ELECTRICAL, AND ITS MEASUREMENT

I. Introductory and Theoretical

§ (1) Definitions.—There is a tendency to replace the term "electrical capacity" by the word "capacitance," which takes its

proper place among the analogous terms resistance, inductance, reactance, admittance, etc., according to Heaviside's system, which has already been so helpful in improving the clearness of electric terminology.

It seems best here to begin with a definition referring to a simple particular case and then to pass to more general conditions.[1]

Let A and B be two conductors infinitely distant from all other conductors, and let B be kept at zero potential while A receives a charge. If q be the charge necessary to bring A to potential v, then

$$q=Kv, \quad . \quad . \quad . \quad . \quad (1)$$

and K is called the capacity (or capacitance) of A.

A quantity of electricity $-q$ will at the same time be induced on B. If B had been kept at potential v_1 instead of zero, while A is brought up to potential v_2, then

$$q=K(v_2-v_1). \quad . \quad . \quad . \quad (2)$$

In equation (1) the values may all be in absolute C.G.S. units (abcoulombs, abfarads, and abvolts). When practical units[2] are used, if q is in coulombs and v in volts, K will be in farads. But K is more usually expressed in *microfarads* (μF), in which case the equation becomes

$$q=10^{-6}Kv. \quad . \quad . \quad . \quad (3)$$

Small capacities are often expressed in millimicrofarads ($m\mu$F) or in micromicrofarads ($\mu\mu$F). For micromicrofarads the shorter term picofarads may be used.

In calculating capacities from dimensions the results are obtained in the electrostatic system of units (abstatfarads). To reduce to microfarads these results have to be divided[3] by 900,000, and we have the following relations:

$$1 \text{ micromicrofarad} = 0{\cdot}90 \text{ abstatfarad}, \quad (4)$$
$$1 \text{ abstatfarad} = 1{\cdot}11\ \mu\mu\text{F}.$$

The electrostatic unit is often written cm. as its dimensions are those of a length.

Caution.—Certain writers mix electromagnetic and electrostatic units in the same formulas; for example, $\lambda_{cm}=4\sqrt{L_{cm}C_{cm}}$. This practice seems quite indefensible and can only lead to confusion.

§ (2) Potential Energy of Charge.—The work done (in *ergs*) in charging a body with quantity Q (to potential V)

$$=\tfrac{1}{2}KV^2=\tfrac{1}{2}QV=\tfrac{1}{2}\frac{Q^2}{K}, \quad . \quad . \quad (5)$$

where Q, V, and K are in absolute units.

If K is in microfarads, Q in coulombs, and V in volts, the work done, in *joules*,

$$=\tfrac{1}{2}KV^2\times 10^{-6}=\tfrac{1}{2}QV=\tfrac{1}{2}\frac{Q^2}{10^{-6}K}. \quad . \quad (6)$$

[1] A. Russell, *Alternating Currents*, vol. i. chap. i.
[2] See "Units of Electrical Measurement," §§ (3), (25).
[3] See "'v,' the Ratio of the Electrical Units," § (6).

This is, of course, also the amount of work given out when the body is discharged.

§ (3) Capacities in Parallel and Series.—If K_1, K_2, K_3 . . . are a number of capacities all connected in parallel, the total resulting capacity is

$$K=K_1+K_2+K_3+\ .\ .\ . \quad . \quad (7)$$

If they are all in series, the capacity of the circuit is

$$K=\frac{1}{1/K_1+1/K_2+1/K_3+\ .\ .\ .}. \quad . \quad (8)$$

When there are only two of them, this reduces to

$$K=\frac{K_1K_2}{K_1+K_2}. \quad . \quad . \quad . \quad (9)$$

For when the capacities are in parallel the potentials are the same for each and the total charge is the sum of the charges. Thus

$$Q=K(V-V'),$$
$$Q_1=K_1(V-V'), \quad Q_2=K_2(V-V'), \ .\ .\ .\ \text{etc.}$$

But $\quad Q=Q_1+Q_2+\ .\ .\ .\ Q_n.$

Hence

$$K(V-V')=\{K_1+K_2+\ .\ .\ .\ \}(V-V'),$$

or $\quad K=K_1+K_2+\ .\ .\ .\ K_n.$

When the capacities are in series the charge on each is the same while the resulting potential difference is the sum of those for each. Thus

$$V_1-V_2=\frac{Q}{K_1}, \quad V_2-V_3=\frac{Q}{K_2}, \ .\ .\ .\ \text{etc.}$$

Hence

$$\frac{Q}{K}=V_1-V_n=Q\left\{\frac{1}{K_1}+\frac{1}{K_2}\ .\ .\ .\ \frac{1}{K_n}\right\},$$

or

$$\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}+\ .\ .\ .\ \frac{1}{K_n}.$$

§ (4) Maxwell's Equations for a System of Conductors.—In many practical problems in which the effects of capacitance occur, the simple definition given by equations (1) and (2) is not sufficiently general. The general case of a system of conductors of any form has been treated by Maxwell[4] as follows. Let $A_1, A_2, A_3, \ldots A_n$ be n conductors in a dielectric medium whose properties remain constant; let $q_1, q_2, q_3, \ldots q_n$ be their charges, and $v_1, v_2, v_3, \ldots v_n$ their potentials. Then Maxwell shows that for a state of equilibrium the charges are determined by a set of n linear equations,

$$\left.\begin{aligned} q_1&=K_{11}v_1+K_{12}v_2+K_{13}v_3+\ .\ .\ .\ +K_{1n}v_n \\ q_2&=K_{21}v_1+K_{22}v_2+K_{23}v_3+\ .\ .\ .\ +K_{2n}v_n \\ &\ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ . \end{aligned}\right\}, \quad (10)$$

where $K_{11}, K_{12}\ .\ .\ .$ depend only on the dimensions and positions of the conductors and on the inductive capacity (or dielectric constant) of the medium.

[4] J. Clerk Maxwell, *Electricity and Magnetism*, 2nd ed., vol. i. § 87.

The coefficients K_{11}, K_{22}, K_{33} . . . are called the *capacities* of the respective conductors, while K_{12}, K_{13} . . . are called the *coefficients of statical induction* and are always negative.

As Orlich [1] points out, it is more convenient for practical purposes to put the equations into the form

$$\left.\begin{aligned} q_1 &= c_1v_1 + c_{12}(v_1 - v_2) + c_{13}(v_1 - v_3) + \ . \ . \ . \\ q_2 &= c_2v_2 + c_{21}(v_2 - v_1) + c_{23}(v_3 - v_2) + \ . \ . \ . \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{aligned}\right\}, \quad (11)$$

where
$$\left.\begin{aligned} K_{11} &= c_1 + c_{12} + c_{13} + \ . \ . \ . \\ K_{22} &= c_2 + c_{21} + c_{23} + \ . \ . \ . \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{aligned}\right\}, \quad . \quad (12)$$

and
$$K_{12} = -c_{12}, \quad K_{21} = -c_{21} \ . \ . \ . \quad (13)$$

As Maxwell has shown that $K_{12} = K_{21}$, $K_{13} = K_{31}$ . . . , there are in all n capacities and $\frac{1}{2}n(n-1)$ coefficients c_{12}, c_{13} . . . The c coefficients are called by Orlich *component capacities* (Teilkapazitäten).

We have also $c_{12} = c_{21}$, and so on. . . . (14)

The general expression for the energy (in ergs) to charge the system is now

$$\tfrac{1}{2}(K_{11}v_1^2 + K_{22}v_2^2 + \ . \ . \ . \ + K_{12}v_1v_2 + K_{13}v_1v_3 + \ . \ . \ .). \quad (15)$$

The use of the component capacities is illustrated in *Figs.* 1 and 2. In *Fig.* 1 are

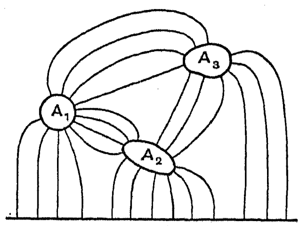


Fig. 1.—System of Three Conductors.

shown three conductors, A_1, A_2, and A_3, and each of them has a field of lines of (electrostatic) force reaching to the earth and to each of its neighbours. The three sets of lines of force from any one of the conductors correspond to the three components of the total charge on that conductor, and the whole system may be represented, as in *Fig.* 2, by the six simple condensers c_1, c_2, c_3, c_{12}, c_{13}, and c_{23}. The potential differences and charges then follow equation (11).

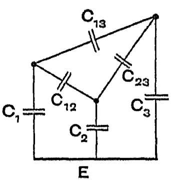


Fig. 2.—Equivalent Simple Condensers.

[1] E. Orlich, *Kapazität und Induktivität*, 1909, p. 20, § 10 (Vieweg u. Sohn).

§ (5) Condensers.—A *condenser* usually means an arrangement of two conductors such that the greater part of the capacity effect is between the conductors and very little of it to earth or other conductors. In a *simple condenser* the whole effect is between the plates. In diagrams a simple condenser is commonly indicated by —||— or —⊂⊃—.

With condensers of capacitances of 0·1 μF and higher the earth capacities seldom cause much error, but for any condenser whose bulk is large in relation to the capacitance the earth capacities may have to be taken into account. For small condensers the safest plan is to construct the condenser with an outer metallic screen connected to one of the sets of conductors. In use the terminal connected to the screen should be practically at earth potential.

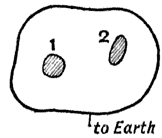


Fig. 3.

There are, however, important practical cases in which it is impossible to ignore the earth capacities. The simplest case of this kind is an open condenser, which may be represented as in *Fig.* 3 by two conductors, 1 and 2, enclosed in a conducting sheath which is earthed. It is equivalent to three simple condensers as shown in *Fig.* 4. Orlich defines the *working capacity* as for the condition $q_1 = -q_2$. When this condition holds, we have from equation (11)

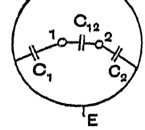


Fig. 4.

$$c_1v_1 + c_2v_2 = 0, \quad . \quad . \quad . \quad (16)$$

and

$$\text{Working Capacity} = \frac{q_1}{v_1 - v_2} = c_{12} + \frac{c_1c_2}{c_1 + c_2}. \quad (17)$$

This case applies to an ordinary double plate condenser in which the distance between the plates is not extremely small compared to the length or breadth of the plates. It also applies to a two-core cable in which the two conductors are open or enclosed in a conducting sheath. As Orlich shows, then, if the conductors are symmetrically arranged, $c_1 = c_2$; and if in use

$$v_1 = \frac{v}{2} = -v_2,$$

then the working capacitance

$$\frac{q_1}{v} = c_{12} + \frac{c_1}{2}, \quad . \quad . \quad . \quad (18)$$

where v is the voltage between the conductors 1 and 2.[2]

[2] For discussion of three-core and other more complicated cables see A. Russell, *Alternating Currents*, vol. i. chaps. iv. and v.

§ (6) INDUCTIVE CAPACITY.—The insulating medium between the two conducting systems in a condenser is called the *dielectric.* If K_0 is the capacitance when this medium is a vacuum and K the value when the dielectric is some substance, then

$$K = \kappa K_0,$$

where κ is constant for the given substance (temperature and other conditions being kept constant). It is called the *inductive capacity* of the substance or the *permittivity* (Heaviside); it is also called the *dielectric constant* or the *dielectric coefficient* of the substance. It is taken as unity for a vacuum. As will be seen later, the inductive capacity of air at atmospheric pressure is also very nearly unity, and may be taken as unity for nearly all practical purposes.

The capacitance of a condenser depends only on the geometrical dimensions and positions of the conductors and the inductive capacity of the dielectric. Its value for air as dielectric can be calculated from the geometrical configuration of the conductors; when the dielectric is some other substance, it is only necessary to multiply the result by the inductive capacity of that substance.

In the following section are given a number of formulas for the calculation of the capacities of condensers of various simple geometrical forms. Unless where otherwise stated the values are for *air dielectric* and are expressed in electrostatic units (*abstatfarads*), the dimensions being in *centimeters.* To reduce to microfarads the values must be divided by 900,000.

§ (7) FORMULAS FOR CALCULATION OF CAPACITANCE. (i.) *Concentric Spheres,* radii r_1 and r_2.—

$$K = \frac{r_2 r_1}{r_2 - r_1} \text{ (abstatfarads)}. \quad . \quad . \quad (19)$$

This may also be written

$$K = \frac{\sqrt{S_1 S_2}}{4\pi b}, \quad . \quad . \quad . \quad (20)$$

where S_1 and S_2 are the surfaces of the spheres and b is the axial distance between their surfaces.

Single Sphere.—By making r_2 infinite in equation (19) we obtain for a single sphere of radius r (far from surrounding bodies)

$$K = r. \quad . \quad . \quad . \quad . \quad (21)$$

For example, a sphere of 1 cm. radius would have capacitance of 1 abstatfarad or about 1·111 $\mu\mu$F. If it were hung near the centre of an empty room 4M × 4M × 4M, this value would be correct to within about 5 in 1000. (A small sphere sometimes forms a convenient rough standard of small capacitance.)

Note. The concentric spheres and the single sphere are almost the only cases in which an exact formula is available. In most other cases the mathematical difficulties due to edges and ends render the formulas only approximate.

(ii.) *Two Spheres,*[1] radii r_1 and r_2 at relatively great distance a apart.—

$$K = \frac{a r_1 r_2}{a(r_1 + r_2) - 2r_1 r_2}. \quad . \quad . \quad (22)$$

(iii.) *Two Equal Spheres*[2] close to one another.—Let the radius of each be r and let the distance between the nearest points be x. Then

$$K \doteq \frac{r}{2}\left(1 + \frac{x}{6r}\right)\left(1{\cdot}2704 + \tfrac{1}{2}\log_\epsilon \frac{r}{x} + \frac{x}{18r}\right). \quad (23)$$

For more elaborate formulas for spheres see Russell (*loc. cit.*).

(iv.) *Two Concentric Circular Cylinders,* radii: r_1 inner, r_2 outer; length l.—When the length is so great (relatively to r_2) that the end effects may be neglected,

$$K = \frac{l}{2 \log_\epsilon (r_2/r_1)}. \quad . \quad . \quad (24)$$

When the distance between the surfaces is very small compared with the radii, *i.e.* $(r_2 - r_1) \ll r_2$,

$$K \doteq \frac{l r_1}{2(r_2 - r_1)}, \quad . \quad . \quad . \quad (25)$$

$$\doteq \frac{S}{4\pi b}, \quad . \quad . \quad . \quad . \quad (26)$$

where S is the surface of one cylinder and b is the axial distance between the two surfaces.

(v.) *Single Circular Wire,*[3] radius r, length l (great compared with r).—If the wire is parallel to the earth at height h,

$$K = \frac{l}{2 \log_\epsilon \dfrac{h - \sqrt{h^2 - r^2}}{r}}. \quad . \quad . \quad (27)$$

(vi.) *Two Long Parallel Horizontal Wires,*[3] each of radius r, length l, at height h and distance apart b.—The capacity between them is

$$K = \frac{l}{4 \log_\epsilon (2h/r) - 2 \log_\epsilon (1 + 4h^2/b^2)}, \quad (28)$$

where h and b are large compared with r.

For a great height, making h infinite, the formula becomes

$$K = \frac{l}{4 \log_\epsilon (b/r)}. \quad . \quad . \quad (29)$$

(vii.) *Two Parallel (Equal) Plates,* area (of one) S, distance apart b.—If the edge action and earth capacitances are neglected the approximate formula is

$$K \doteq \frac{S}{4\pi b} \text{ abstatfarads}. \quad . \quad . \quad (30)$$

[1] W. H. Eccles, *Handbook of Wireless Telegraphy and Telephony*, p. 53.
[2] A. Russell, *Roy. Soc. Proc.* A, 1909, lxxxii. 524.
[3] A. Russell, *Alternating Currents*, vol. i. chap. v., where other cases are also investigated.

If the dielectric between the plates has a dielectric constant κ,

$$K = \frac{\kappa S}{4\pi b \times 900{,}000} \text{ microfarads}, \quad (31)$$

$$\doteq \frac{\kappa S}{113 \cdot 1 b} \mu\mu F. \quad (32)$$

Condensers are commonly constructed of a number of conducting plates, with the alternate ones connected in parallel forming the two conducting systems of the condenser. If there are in all N metal plates, there will be only N − 1 spaces between, and hence

$$K \doteq \frac{(N-1)S}{4\pi b} \text{ abstatfarads}. \quad (33)$$

In most practical condensers these equations are sufficiently accurate to give values near enough for the purposes of design.

Pair of Parallel Plates with a Number of Different Dielectrics in Parallel Layers between.—If the layers have thicknesses b_1, b_2, b_3 . . . and dielectric constants κ_1, κ_2, κ_3 . . ., then

$$K \doteq \frac{S}{4\pi(b_1/\kappa_1 + b_2/\kappa_2 + b_3/\kappa_3 + \ldots)}. \quad (34)$$

(viii.) *Plates at Small Angle.*[1]—If two equal plates of width a are at a small angle θ (radians) to one another with the edges (a) parallel to the line of intersection of their planes, then

$$K \doteq \frac{a}{2\pi\theta} \log_\epsilon \left(\frac{r_2}{r_1}\right), \quad (35)$$

where r_1 and r_2 are the respective distances of the parallel edges from this line.

(ix.) *Single circular plate*, radius r, thickness negligible.

$$K = \frac{2r}{\pi}. \quad (36)$$

(x.) *Two circular plates*, radii r, thickness t small compared with r, distance apart b (also small compared with r).

This is a case where, for accuracy, the capacities to earth should be taken into account.[2] By Orlich's method, as in § (5), when both plates are insulated we have

$$K = c_{12} + \frac{c_1 c_2}{c_1 + c_2}. \quad (37)$$

Neglecting t for the parts c_1 and c_2, we have by equation (36)

$$c_1 = c_2 \doteq \frac{r}{\pi}.$$

When one plate is earthed,

$$K_1 = c_{12} + c_1 = c_{12} + \frac{r}{\pi}. \quad (38)$$

[1] L. E. Dodd, *Phys. Rev.*, 1917, ix. 96.

[2] R. Jaeger, *Dissertation*, Berlin, 1917, and *Ann. d. Physik*, 1917, liii. 409.

Kirchhoff[3] gives the complete formula *for this latter case* as follows:

$$K_1 = \frac{r^2}{4b} + \frac{r}{4\pi}\left[\log_\epsilon \frac{16\pi(b+t)r}{b^2} + \frac{t}{b}\log_\epsilon \frac{b+t}{t} + 1\right]. \quad (39)$$

Hence

$$c_{12} = \frac{r^2}{4b} + \frac{r}{4\pi}\left[\log_\epsilon \frac{16\pi(b+t)r}{b^2} + \frac{t}{b}\log_\epsilon \frac{b+t}{t} - 3\right], \quad (40)$$

and, therefore, by equation (37), when both plates are insulated,

$$K = \frac{r^2}{4b} + \frac{r}{4\pi}\left[\log_\epsilon \frac{16\pi(b+t)r}{b^2} + \frac{t}{b}\log_\epsilon \frac{b+t}{t} - 1\right]. \quad (41)$$

Guard Ring.—In general in a plate condenser the electric field is almost perfectly uniform except near the edges of the plates. In order to maintain the uniformity of the field at the edges (thus rendering the use of the simple formula of equation (30) much more accurate), Lord Kelvin introduced the device of a guard ring. This is shown in section in *Fig.* 5. The disc P is surrounded by a flat ring GG of the same thickness, concentric with P and separated from it by a narrow channel. The outer diameter of the guard ring GG is equal to that of the opposing plate Q. In use P and the guard ring are brought to the same potential, but the charge on P can be determined independently of that on the ring. As the channel has appreciable width, a small correction has to be applied for it. Let r be the diameter of P, and let w, the width of the channel, be small compared with r and b, the distance between the plates. Then,[4] for the capacitance between P and Q,

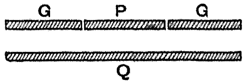


FIG. 5.—Plate Condenser with Guard Ring.

$$K \doteq \frac{r^2}{4b} + \frac{1}{4} \cdot \frac{rw}{b + 0 \cdot 22w}\left(1 + \frac{w}{2r}\right). \quad (42)$$

Another formula is given by Kirchhoff (*Abhandlungen*, p. 117).[5] The ratio of r to b may have any desired value, so long as the radial width of the guard ring is at least 4 or 5 times b.

In a similar way cylindrical guard rings can be applied to the ends of a cylindrical condenser.

§ (8) Residual Charge and Absorption.—In discussing the behaviour of condensers in circuits where they are charged and discharged, it is convenient to confine our attention to the charge that goes in or out at one terminal. Thus when we say that a condenser receives

[3] G. Kirchhoff, *Gesamm. Abhandl.* p. 112, and *Berlin Akad. Monatsberichte*, 1877, p. 144.

[4] J. Clerk Maxwell, *Electricity and Magnetism*, 2nd ed. vol. i. § 201.

[5] See F. Kohlrausch, *Lehrbuch d. prakt. Physik*, 2nd ed., 1905, p. 565.

a charge Q, we mean that a charge $-Q$ has at the same time entered by the other terminal. This is equivalent to a quantity $+Q$ passing in at one terminal and out at the other.

If a condenser receives a charge Q and is then discharged by short-circuiting the terminals, in general the total quantity that passes out is less than Q, and hence a certain part of the energy of the charge Q is spent in the condenser. Usually the greater part of the discharge takes place very quickly, but a certain portion is temporarily bound in the condenser and is slowly discharged if the short-circuit is maintained. This portion is called *residual charge*, and the general effect is described as *absorption*. If the short-circuit (which keeps the terminal potential difference zero) is broken after the main discharge, the terminal potential difference will slowly rise to a small amount owing to the residual charge being gradually set free. In extreme cases continued discharge may be observed even for days.

A *perfect condenser* is one without absorption or leakage, and for many purposes a good air condenser with high insulation gives an extremely near approximation to this ideal. Condensers with solid or liquid dielectrics all show absorption in varying degree depending on the nature and condition of the dielectric substance.[1]

§ (9) Condenser in Alternating Current Circuit.—Let v and i respectively be the instantaneous values of sine-wave alternating voltage and current of pulsatance ω, where $\omega = 2\pi \times$ frequency n, and let V and I be the corresponding effective values. Then

$$v = V_{\max.} \sin \omega t. \quad . \quad . \quad (43)$$

If the voltage v is applied to the terminals of a perfect condenser of capacitance K, and if q is the quantity in the condenser at any moment, then

$$q = Kv \quad . \quad . \quad . \quad . \quad (44)$$

and
$$i = \frac{dq}{dt} = K\frac{dv}{dt} = K\omega V_{\max.} \cos \omega t$$
$$= K\omega V_{\max.} \sin\left(\omega t + \frac{\pi}{2}\right). \quad (45)$$

Hence

$$I = \omega KV, \text{ (amperes, volts, farads).} \quad (46)$$

[In symbolic notation we write

$$v = \frac{1}{j\omega K} i = -\frac{j}{\omega K} i, \quad . \quad . \quad . \quad (47)$$

where $j = \sqrt{-1}$.]

Table I. gives approximate values of the current taken by various condensers at 100 volts for frequencies of 100, 1000, and 1,000,000 ∿ per second respectively.

[1] For discussion of theories of absorption see F. W. Grover, *Bureau of Standards Bulletin*, 1912, vii. 518.

Table I

Condenser Currents at 100 Volts

K.	n=100.	1000.	1,000,000.
μF.	Milliamps.	Milliamps.	Amps.
0·001	0·063	0·63	0·63
0·1	6·28	62·8	62·8
1·0	62·8	628	628

§ (10) Condenser in Series with Inductive Coil.—Let the circuit AB have capacitance K in series with resistance R and self-inductance L, as in *Fig.* 6. Let Z be the impedance and z the symbolic impedance;

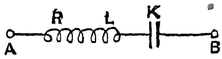


Fig. 6.

then
$$z = R + j\left(\omega L - \frac{1}{\omega K}\right). \quad . \quad . \quad (48)$$

After a steady state is reached,

$$v = zi = \left[R + j\left(\omega L - \frac{1}{\omega K}\right)\right]i \quad . \quad (49)$$

or
$$v = I_{\max.} \sin(\omega t + \phi), \quad . \quad . \quad (50)$$

where
$$\tan\phi = \frac{(L\omega - 1/\omega K)}{R}. \quad . \quad . \quad (51)$$

Also

$$V = ZI = I\sqrt{R^2 + \left(L - \frac{1}{\omega^2 K}\right)^2 \omega^2}, \quad (52)$$

when $\omega^2 LK = 1$, $\phi = 0$, and $Z = R$.

The circuit is then said to be electrically tuned or in *resonance* for pulsatance ω (or frequency n); and the current I is then a maximum for given R, L, V, and ω.

§ (11) Condenser in Parallel with Inductive Coil.—If a condenser K is in parallel with an inductive circuit (R, L) as in *Fig.* 7, then $1/j\omega K$ and $(R + j\omega L)$ are in parallel, and hence

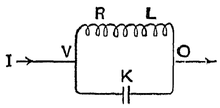


Fig. 7.

$$z = \frac{(R + j\omega L)/j\omega K}{R + j\omega L + 1/j\omega K}$$
$$= \frac{R + j\omega L}{1 - \omega^2 LK + j\omega RK} \quad . \quad . \quad (53)$$

and
$$Z = \frac{R^2 + \omega^2 L^2}{(1 - \omega^2 LK)^2 + \omega^2 R^2 K^2}. \quad . \quad (54)$$

When K is adjusted to make Z a minimum,

$$K = \frac{1}{R^2/L - \omega^2 L}$$

or
$$\omega^2 LK = 1 - \frac{R^2 K}{L} \quad . \quad . \quad (55)$$

Then we have *current resonance* (Orlich); and the current I is a maximum for given R, L, V, and ω.

Also
$$Z = \frac{L}{KR} = R + \frac{\omega^2 L^2}{R}. \quad . \quad . \quad (56)$$

§ (12) Power Loss in Condensers.—As pointed out in § (8), a condenser which shows absorption does not when discharged return the whole quantity which was put into it in charging. Let a condenser be taken through a complete cycle of charge and discharge, and let q, the instantaneous value of the quantity in the condenser, be plotted against v, the instantaneous value of the terminal voltage. The area of the resulting closed curve will be zero in the case of a perfect condenser, but if the condenser has absorption the area ($\int v dq$) of the curve will represent the energy lost in the condenser in one cycle. The result will be in joules if the quantity and voltage are expressed in coulombs and volts. If the cycle is periodically repeated n times per second, the power lost in the condenser will then be $n \times$ (area of curve) in watts. This loss of power is partly due to *dielectric hysteresis*, and has some analogy to the loss due to magnetic hysteresis when iron is magnetised and demagnetised. Since the bound charge takes time to free itself, the loss per cycle usually varies with the frequency n.

An additional amount of power is in general lost due to actual leakage current through the dielectric or across bad insulation at the terminals. Sometimes there is appreciable loss due to the resistance of internal wires and connections or of the plates themselves.

§ (13) Power Loss with Alternating Current.—If alternating voltage V is applied to the terminals of a perfect condenser, by equation (45) the resulting current I will be equal to KωV and its phase will be exactly 90° in front of V (leading). But if the condenser has internal power loss, ϕ the angle of lead will be less than 90° and the *power factor* $\cos\phi$ will not be zero. Then, if W is the power spent in the condenser,

$$W = VI\cos\phi = VI\sin\theta, \quad (57)$$

where $(\pi/2 - \phi) = \theta$, which is called the *phase displacement.*

§ (14) Equivalent Circuit.—For most purposes a condenser with power losses of any kind (dielectric hysteresis, leakance, etc.) may, for a given frequency, be taken as equivalent to a perfect condenser either (A) in series or (B) in parallel with a non-inductive resistance, as the resistance in either case can be so chosen as to waste the same amount of power as the absorptive condenser and thus also give the same phase angle.

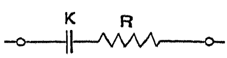


Fig. 8.

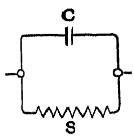


Fig. 9.

In the two systems (A) and (B) let the circuits shown in *Figs.* 8 and 9 be equivalent to the same condenser for pulsatance $\omega = 2\pi n$.

System A.—*Series Resistance.*

If z is the vector impedance,

$$z = R - \frac{j}{\omega K} \quad (58)$$

and

$$\frac{V}{I} = Z = \left(R^2 + \frac{1}{\omega^2 K^2}\right)^{\frac{1}{2}} \quad (59)$$

Here the effective resistance $=R$ and the reactance $= -1/\omega K$.

Thus power spent $= W = RI^2$. (60)

Also $W = VI\cos\phi$,

where

$$\tan\phi = \frac{1}{\omega RK} \quad (61)$$

and

$$\tan\theta = \omega RK. \quad (62)$$

When θ is small, $\theta \doteq \omega RK$ radians, and the *power factor* $\cos\phi \doteq \tan\theta \doteq \omega RK$.

System B.—*Parallel Resistance.*

We have

$$z = \frac{S/j\omega C}{S + 1/j\omega C} = \frac{S}{1 + j\omega SC}$$

or

$$z = \frac{S - j\omega S^2 C}{1 + \omega^2 S^2 C^2}. \quad (63)$$

Thus

$$\frac{V}{I} = Z = \frac{S}{\sqrt{1 + \omega^2 S^2 C^2}}. \quad (64)$$

Here the effective resistance

$$R' = \frac{S}{1 + \omega^2 S^2 C^2}, \quad (65)$$

and the reactance

$$X = -\frac{\omega S^2 C}{1 + \omega^2 S^2 C^2}. \quad (66)$$

Very often $\omega^2 S^2 C^2$ is very large compared with 1, and then we have $R' \doteq 1/\omega^2 SC^2$ and $X \doteq -1/\omega C$.

Also

$$W = \frac{V^2}{S} \quad (67)$$

or $= VI\cos\phi$,

where

$$\tan\phi = \omega SC. \quad (68)$$

When θ is small, $\theta \doteq 1/\omega SK$ radians, and the *power factor* $\cos\phi \doteq \tan\theta \doteq 1/\omega SK$.

Comparison of Systems A *and* B.—We can pass from one equivalent system to the other by means of the following relations:

$$\omega KR = \frac{1}{\omega SC}, \quad (69)$$

$$R = \frac{S}{1 + \omega^2 S^2 C^2}, \quad (70)$$

and

$$K = C\left(1 + \frac{1}{\omega^2 S^2 C^2}\right). \quad (71)$$

$$S = R\left(1 + \frac{1}{\omega^2 K^2 R^2}\right) \quad (72)$$

and

$$C = \frac{K}{1 + \omega^2 K^2 R^2}. \quad (73)$$

In good condensers the power factor ωKR is a small fraction, and in this case

$$\mathrm{K} \doteqdot \mathrm{C} \quad . \quad . \quad . \quad . \quad (74)$$

and
$$\mathrm{R} \doteqdot \frac{1}{\omega^2 \mathrm{SC}^2}. \quad . \quad . \quad . \quad (75)$$

§ (15) Discharge with Inductive Circuit.—The discharge of a condenser through an inductive circuit has very important practical applications. The complete mathematical theory was given by Lord Kelvin [1] many years ago.

Let a (perfect) condenser of capacitance K, charged to a potential difference V_0, have its terminals, at time $t=0$, connected to a circuit of resistance R and self-inductance L. A current will begin to flow out of the positively charged terminal. Let i be the value of this current at time t, and let q and v be the corresponding values of the quantity in the condenser and the terminal potential difference respectively. Let Q be the initial charge, where $\mathrm{Q}=\mathrm{KV}_0$.

Then
$$v=\mathrm{R}i+\mathrm{L}\frac{di}{dt}, \quad . \quad . \quad . \quad (76)$$

$$q=\mathrm{K}v, \quad . \quad . \quad . \quad (77)$$

and
$$i=-\frac{dq}{dt}. \quad . \quad . \quad . \quad (78)$$

These three equations are sufficient to determine q, i, or v in terms of the time t.

Eliminating v and i we have

$$\frac{d^2q}{dt^2}+\frac{r}{\mathrm{L}}\cdot\frac{dq}{dt}+\frac{1}{\mathrm{KL}}q=0, \quad . \quad . \quad (79)$$

the differential equation by which q can be determined in terms of t.

The complete solution of (79) is

$$q=\mathrm{Q}\epsilon^{-bt}\left[\frac{\epsilon^{at}+\epsilon^{-at}}{2}+\frac{b}{a}\cdot\frac{\epsilon^{at}-\epsilon^{-at}}{2}\right], \quad (80)$$

where
$$b=\frac{\mathrm{R}}{2\mathrm{L}} \quad . \quad . \quad . \quad (81)$$

and
$$a=\sqrt{\frac{\mathrm{R}^2}{4\mathrm{L}^2}-\frac{1}{\mathrm{LK}}}. \quad . \quad . \quad (82)$$

The nature of the discharge depends upon the value of a, which may be real, zero, or imaginary. Thus three cases arise.

Case 1.—If $\mathrm{R}^2 > 4\mathrm{L/K}$, then a is real.

In this case equation (80) holds as it stands. It may be written

$$q=\mathrm{Q}\epsilon^{-bt}\left(\cosh at+\frac{b}{a}\sinh at\right). \quad . \quad (83)$$

Since $a^2-b^2=-1/\mathrm{LK}$, and $i=-dq/dt$,

$$i=\mathrm{Q}\left(\frac{a^2-b^2}{a}\right)\epsilon^{-bt}\cdot\frac{\epsilon^{at}-\epsilon^{-at}}{2} \quad . \quad (84)$$

$$=\frac{\mathrm{V}_0}{\mathrm{L}a}\epsilon^{-bt}\sinh at\,; \quad . \quad . \quad . \quad . \quad (85)$$

also
$$v=\frac{q}{\mathrm{K}}=\mathrm{V}_0\epsilon^{-bt}\left[\frac{\epsilon^{at}+\epsilon^{-at}}{2}+\frac{b}{a}\cdot\frac{\epsilon^{at}-\epsilon^{-at}}{2}\right]. \quad (86)$$

Fig. 10 shows examples of the curves connecting q, v, and i with the time t.

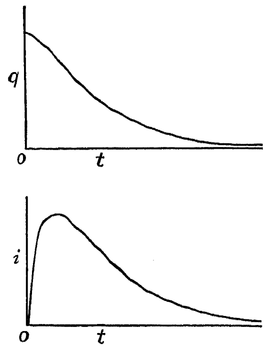


Fig. 10.—Non-oscillatory Discharge.

The current begins at zero, rises to a maximum value

$$a\sqrt{\mathrm{LK}}\left(\frac{b+a}{b-a}\right)^{-b/2a}$$

when
$$t=\frac{1}{2a}\log_\epsilon\left(\frac{b+a}{b-a}\right),$$

and then falls asymptotically to zero at infinite time.

Case 2.—If $\mathrm{R}^2=4\mathrm{L/K}$, then $a=0$.

Thus equation (80) becomes

$$q=\mathrm{Q}\epsilon^{-bt}(1+bt)\,; \quad . \quad . \quad (87)$$

also
$$i=\frac{\mathrm{V}}{\mathrm{L}}t\epsilon^{-bt}, \quad . \quad . \quad . \quad (88)$$

and
$$v=\mathrm{V}_0\epsilon^{-bt}(1+bt). \quad . \quad . \quad (89)$$

The curves for q, v, and i are very similar to those of *Fig.* 10. The discharge is aperiodic (just not oscillatory). The current begins at zero, rises to a maximum value $2\mathrm{V/R}\epsilon$, when $t=1/b$, and then falls asymptotically to zero at $t=\infty$.

Case 3.—If $\mathrm{R}^2 < 4\mathrm{L/K}$, then a is imaginary. Let $a=j\omega$, where $j=\sqrt{-1}$.

The equations then are

$$q=\mathrm{Q}\epsilon^{-bt}\left(\cos\omega t+\frac{b}{\omega}\sin\omega t\right), \quad . \quad (90)$$

$$i=\frac{\mathrm{V}_0}{\mathrm{L}\omega}\epsilon^{-bt}\sin\omega t, \quad . \quad . \quad (91)$$

and
$$v=\mathrm{V}_0\epsilon^{-bt}\left(\cos\omega t+\frac{b}{\omega}\sin\omega t\right)$$

$$=\frac{\mathrm{V}_0}{\omega\sqrt{\mathrm{LC}}}\epsilon^{-bt}\sin(\omega t+\phi), \quad . \quad . \quad (92)$$

where
$$\tan\phi=\frac{\omega}{b}.$$

[1] William Thomson, *Phil. Mag.*, 1853, v. 393, and *Mathematical and Physical Papers*, i. 540.

The curves are of the type shown in *Fig.* 11. At successive equal intervals of time they touch curves of the form $y = \pm A\epsilon^{-bt}$ above and below the axis of t.

The discharge is now oscillatory, the condenser becoming recharged at regular intervals with alternately negative and positive charges of gradually decreasing amount. The action is analogous to dynamic vibration with damping, potential energy being transformed into kinetic energy twice in each cycle, with continuous loss of energy until it is all dissipated. The loss of electrical power is here due to the resistance R, in which the energy is ultimately transformed into heat. The damping of the oscillations is determined by b (*i.e.* R/2L), which is called the *damping coefficient* or the *coefficient of decay*.

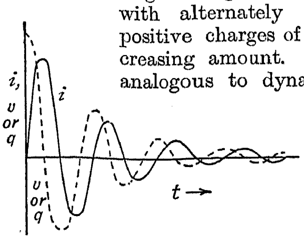


FIG. 11.—Oscillatory Discharge (large damping).

The oscillation *frequency* n is equal to $\omega/2\pi$, and T the *complete period* $=1/n$.

If δ is the *logarithmic decrement* for a complete period

$$\delta = \log_\epsilon\left(\frac{i_0}{i_T}\right) \text{ by definition;}$$

hence
$$\delta = \frac{b}{n}. \quad . \quad . \quad . \quad (93)$$

Equations (91) and (92) show that if the direction of the discharge is taken positive (as here) the current lags behind the voltage by the angle ϕ, which is nearly 90° when b is very small.

If K and R are given, the frequency is 0 when $L = KR^2/4$; then for increase of L it increases to a maximum for $L = KR^2/2$, after which it slowly decreases to 0 as L becomes infinite.

By suitable variation of K and L an enormous range of frequencies can be actually obtained, reaching from a few alternations per minute up to at least 5×10^{10} ∾ per second. The system is of fundamental importance in Radio Telegraphy. For further information see the articles [1] on that subject.

§ (16) CHARGE AND DISCHARGE WITH NON-INDUCTIVE CIRCUIT.—(i.) If a voltage V_0 is applied, at time $t=0$, to a circuit consisting of a condenser K in series with a resistance R, then

$$i = \frac{V_0}{R}\epsilon^{-\frac{t}{KR}}, \quad . \quad . \quad . \quad . \quad (94)$$

where $+i$ is the current going in;

also
$$q = KV_0\left(1 - \epsilon^{-\frac{t}{KR}}\right) \quad . \quad . \quad (95)$$

[1] "Radio Frequency, Measurements at," and "Wireless Telegraphy."

(ii.) If a condenser K charged to voltage V_0 is connected across a resistance R at time t_0, the *discharge current* i is given by equation (94); also

$$q = KV_0\epsilon^{-\frac{t}{KR}}. \quad . \quad . \quad . \quad (96)$$

In both cases the current follows a curve of the kind shown in *Fig.* 12, beginning with a maximum value V_0/R and falling asymptotically to 0 at $t = \infty$. The fraction of its initial value to which the current falls in time t depends on KR, which is called the *time constant* of the circuit. Thus the time constant is the time the current takes to fall to $1/\epsilon$th part of its initial value. The capacitance K is here in farads if R is in ohms.

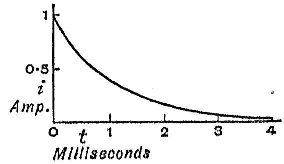


FIG. 12.—Charge or Discharge with Resistance only.

In most cases occurring in practice, except where the resistance R is very large or the condenser has absorption, the charge or discharge is practically complete in a small fraction of a second, as the following example will show. Let a 1 μF condenser charged to 1000 volts be discharged through a resistance of 1000 ohms. Here the time constant $KR = 10^{-3}$ and the initial current is 1 ampere. In 0·001 sec. the current has fallen to 0·37 ampere, in 0·01 sec. to 0·000045 ampere, and in 0·1 sec. it is less than 10^{-56} ampere. The curve in *Fig.* 12 has been drawn for this case.

II. DIELECTRIC CONSTANT

§ (17) MEASUREMENT OF DIELECTRIC CONSTANT.—The inductive capacity or dielectric constant has been defined in § (6). For most insulating materials its value lies between 1 and 10, but in some cases values as high as 80 to 90 occur. Various methods have been used for its measurement, and for pure materials the results obtained by different observers are in tolerably good agreement. Unfortunately, however, the dielectrics, such as mica, glass, or ebonite, which are most useful in practice are usually of variable or uncertain composition, and hence definite values cannot easily be assigned to them. The variations of the dielectric constant with frequency, temperature, or pressure are also of importance.

Many of the methods of measurement involve the determination of capacitance, the actual methods for which will be described in Part IV. For solid and liquid materials tested by comparison with air, it is quite sufficient to take the dielectric constant of

air as 1, but in the case of gases whose inductive capacities are all near unity it is necessary to take for air the more exact value $1{\cdot}0005_8$. The comparative results must be multiplied by this figure.

A. Solids. (i.) *Plate Condenser Method.*—The most usual method is to test the solid in the form of a thin sheet with parallel faces forming the dielectric in a plate condenser, the area of the sheet being much greater than that of the condenser plates. Sometimes these plates are of tinfoil stuck to or pressed against the dielectric sheet, and sometimes of solid metal. In the latter case allowance must be made for the small air spaces due to imperfect contact. For porous materials like paper or silk a correction has also to be made for the internal air spaces, if the result is to apply to the actual solid material. If a guard ring is not used, Kirchhoff's formula for the capacity of two circular plates is applicable, equation (40) giving the air capacity. The intercapacity c_{12} with the given material is got from the observed capacity by subtracting R/π or $R/2\pi$ according as one plate is earthed or both insulated during the measurement.[1]

(ii.) *Equivalent Displacement.*—A more accurate method[2] uses a special air condenser in which the plates are kept accurately parallel while the distance between them can be varied. The plates are set at a certain distance apart and the capacitance measured or merely balanced; the sheet of material is then introduced and the distance between the plates is increased until the balance is restored or the capacitance has the same value as before. If δ = the change of distance, and b = thickness of sheet, then the dielectric constant κ is given by

$$\kappa = \frac{b}{b-\delta}. \qquad . \quad . \quad . \quad (97)$$

This method appears to be more accurate than (i.).

(iii.) *Fusible Materials.*—When the material can be melted it is sometimes possible to pour it into a condenser such as is used in testing liquids (iv.) and allow it to solidify in position. When it contracts much on cooling trouble may arise from air spaces left between the material and the metal surfaces of the condenser.

B. Fluids. (iv.) *Condenser Methods.*—For fluids the condenser can be of such a form that one of the conductors is almost entirely screened from earth by the other; for example, two concentric cylinders. The capacitance of such a condenser is first tested in air (with the outer cylinder earthed), and a further test made with the whole immersed in the fluid to be examined. Nernst's[3] system of condenser consists of a deep metal trough with flat bottom and cylindrical sides forming one conductor, while the other conductor consists of a metal disc which can be supported horizontally inside the trough at various distances from the bottom. After a test with the trough empty, it is filled with the liquid and another test made without altering the position of the disc. Turner[4] used a condenser of this form for very accurate determinations of the dielectric constant of a number of very pure liquids. The disc was supported by a metal stem passing through an insulating cover of the trough. He found that the most accurate method was to test the capacitance with the vessel full and empty, first with the disc and stem in position and then with the stem alone.

In the more modern tests high-frequency currents generated by triode valves are used.[5]

(v.) *By Electric Attraction (Dynamical Effects).*—If two very small spheres at a distance d from one another in air (or rather in a vacuum) have charges of $+q$ and $-q$ abcoulombs respectively, the force of attraction F between them will be q^2/d^2. But if they are in a medium of dielectric constant κ the force will be $q^2/\kappa d^2$. If their intercapacity in air is C and their potential difference V, then

$$q = \mathrm{CV},$$

and $$\mathrm{F} = \frac{\mathrm{C^2V^2}}{d^2}.$$

For the other medium, if the potential difference is still V,

$$q_1 = \kappa \mathrm{CV},$$

and hence $$\mathrm{F}_1 = \frac{\kappa^2\mathrm{C^2V^2}}{\kappa d^2} = \kappa \mathrm{F}.$$

Similarly,[6] if any two conductors are maintained at a given difference of potential the force between them is proportional to the dielectric constant of the medium. Thus the dielectric constant can be determined by measuring the attraction between two conductors, for a given difference of potential, in air and in the other medium respectively.

Lefèvre[7] used two plates and measured the forces by the help of an ordinary balance.

Silow[8] employed a special quadrant electrometer (used idiostatically) which could be filled with the dielectric to be tested. The needle was suspended by a fine wire, which a later observer[9] replaced by a silvered quartz fibre. Alternating potential difference is used to avoid trouble due to polarisation.

(vi.) *By Measurement of Electrical Wave-lengths.*—If electrical waves (Hertzian) are set

[1] R. Jaeger, *Ann. d. Physik*, 1917, liii. 409.
[2] A. Winkelmann, *Wied. Ann.*, 1889, xxxviii. 161.
[3] W. Nernst, *Zeits. phys. Chem.*, 1894, xiv. 622.
[4] B. B. Turner, *Zeits. phys. Chem.*, 1900, xxxv. 385.
[5] H. Joachim, *Ann. d. Physik*, 1919, lx. 570.
[6] See Maxwell, *Electricity and Magnetism*, 2nd ed. vol. i. § 124.
[7] J. Lefèvre, *L'Éclairage él.*, 1895, x. 262.
[8] Silow, *Pogg. Ann.*, 1875, clvi. 389.
[9] J. F. Smale, *Wied. Ann.*, 1896, lvii. 215.

up in a dielectric medium by oscillations of a given frequency, the wave-length is proportional to $1/\sqrt{\kappa}$.[1] Thus the dielectric constant κ can be determined by comparing the wave-length λ_0 in the dielectric with λ the wave-length in air. Then $\kappa = (\lambda_0/\lambda_1)^2$.

Stationary waves are set up along a pair of parallel wires (Lecher System) by means of some source giving oscillations of suitable frequency (n of the order of 2×10^8). For example, the source may be a Blondlot oscillator excited by an induction coil with zinc or cadmium spark gap and Tesla transformer.[2] The wires should be about 1 cm. apart and have their far ends connected. The positions of two wire bridges across the wires are adjusted until an indicator half-way between the bridges shows maximum potential difference. Then the distance between the bridges is half the wave-length. The voltage indicator may be a vacuum tube, a bolometer, a hot-wire voltmeter, or other instrument suitable for such high frequencies. An experiment is first made with the wire system in air, giving λ. Then the wires are immersed in the dielectric liquid contained in a long trough and λ_0 similarly determined. Drude gives a table of corrections to be applied for the limited size of the trough.

§ (18) Values of Dielectric Constant.—In Tables II. and III. are given values of the dielectric constant for a number of solids, liquids, and gases. Where the temperature is not stated it may be assumed to be from 15° to 20° C. For the solids the test frequency may be taken as of the order 1000 ∿ per sec., and for the liquids and gases as 10^6 ∿ per sec. or higher.

Table II

Solids

Substance.	Dielectric Constant κ.	Temperature ° C.
Balata	2·9 to 3·4	15
Calc spar ‖ *	7·6	..
Calc spar ⊥	8·5	..
Celluloid	18·6	..
Cellulose (solid) (dried at 105° C.)	6·6	15
Cellulose triacetate (dried at 105° C.)	3·9	15
Ebonite	2·3 to 3·2	..
Fluor spar	6·8	..
Glass	4 to 10	..
Gutta-percha	2·5 to 3·2	15
Ice	94	−2

* Faces of sheet ‖ (parallel) and ⊥ (perpendicular) to optic axis.

Table II—*continued*

Substance.	Dielectric Constant κ.	Temperature ° C.
India-rubber (pure Pará)	1·7 to 2·6	15
India-rubber (vulcanised)	2·7 to 2·9	..
Marble (paraffined)	8·4	..
Mica,† Bengal ruby	4·2	9
Mica, Canadian	2·9	9
Paraffin wax	1·7 to 2·3	..
Paraffin wax— C_xH_{2x+2} (x=20 to 27)	2·12	15
Melting-point 54° C.	2·0	80
Porcelain	4·4 to 6·8	..
Quartz ‖	4·7	..
Quartz ⊥	4·6	..
Quartz (fused)	3·5 to 3·8	..
Rock salt	5·8	..
Rosin	2·55	..
Shellac	2·8 to 3·7	..
Silk (density 1·51)	4·6 to 4·9	..
Selenium	6·13	16
Sulphur	2·9 to 4·6	..
Tourmaline ‖	6·5	..
Tourmaline ⊥	7·1	..

† According to E. Wilson and T. Mitchell, *Electrician*, 1905, liv. 880. Other observers have obtained higher values of the order of 6 to 8.

Table III

Liquids and Gases

Substance.	Dielectric Constant κ.	Temperature ° C.	Temperature Coefficient. Per cent per ° C.
Liquids—			
Acetone	21	18	..
Amyl alcohol	16·0	20	..
Aniline	7·30	18	−0·35
Benzol	2·29	18	−0·07
Bromine	3·2	..	..
Bromoform	4·6	..	..
Carbon dioxide (liquid)	1·58	0	..
	1·32	30	..
Carbon disulphide	2·5	..	−0·09
„ tetrachloride	2·25	18	..
Chlorides—			
Phosphorous trichloride	3·72	18	..
Arsenic trichloride	12·6	17	..
Silicon tetrachloride	2·56	15	..
Chloroform	5·2	18	..
Ethyl acetate	6·5	..	..
„ alcohol	26·8	15	..
„ chloride	10·90	18	..
„ ether	4·37	18	−0·46
Ethylene chloride	10·8	..	−0·5
Hydrides—			
Ammonia (NH_3)	4·0	−90	..
„ (max.)	25·4	−77	−0·4
„	15·5	+20·5	..

[1] J. Clerk Maxwell, *Electricity and Magnetism*, vol. ii. chap. xx.
[2] E. Lecher, *Wied. Ann.*, 1890, xli. 850; P. Drude, *Wied. Ann.*, 1897, lxi. 466, and *Ann. d. Physik*, 1902, viii. 336.

TABLE III—*continued*

Substance.	Dielectric Constant κ.	Temperature °C.	Temperature Coefficient. Per cent per °C.
Hydrides—*continued*			
Phosphine (PH_3)	2·6	−50	..
	2·88	+15	..
Stibine (SbH_3)	2·58	−50	−0·4
	1·81	+15	..
Arsine (AsH_3)	2·58	−50	..
	2·05	+15	..
Nitrobenzol . .	36·5	18	−0·5
Nitrogen tetroxide	2·56	15	..
Oil, castor. . .	4·8	15	−0·4
„ colza . . .	3·1	..	..
„ olive . . .	3·1	20	−0·36
Petroleum (water white) . . .	2·2	..	..
Petroleum Spirit (Hexane) . .	1·9	..	..
Toluol . . .	2·3	15	−0·09
Turpentine . .	2·2	..	..
Water	81	18	−0·45
Xylol (ortho) . .	2·6	..	..
,, (meta) . .	2·38	18	−0·05
,, (para) . .	2·2	..	..
Gases—*	κ. at 1 Atmo. Pressure.		
Air	1·000580	0	..
Carbon dioxide .	1·000989	0	..
Helium † . . .	1·000074	0	..
Hydrogen . . .	1·000282	0	..
Nitrogen . . .	1·000606	0	..
Nitrous oxide N_2O	1·001129	0	..
Oxygen . . .	1·000547	0	..
Hydrogen (liquid)	1·21	−253	..
Nitrogen „ .	1·42	−205	..
Oxygen „ .	1·46	−253	..

* H. Rohmann, *Ann. d. Physik*, 1911, xxxiv. 979.
† Hochheim, *Deutsch. Phys. Gesell. Verh.*, 1908, x. 446.

§ (19) VARIATION WITH TEMPERATURE.—The temperature coefficients for the more commonly used solids are small (except for very low frequencies). According to Mattenklodt,[1] mica shows no change (to within 1 in 10,000) for a rise of temperature of 30° C., and the capacity shows constancy with variation of voltage up to 600,000 volts per cm.

For ebonite and paraffin wax Pellat and Sacerdote [2] found the following temperature coefficients :

	a, per cent per °C.	Range.
Paraffin wax . . .	−0·036	11 to 32° C.
,, ,,	−0·056	11 to 83
Ebonite	+0·088	10 to 20

According to Schmidt,[3] for sulphur $a = +0·1$ per cent per ° C.

[1] E. Mattenklodt, *Ann. d. Physik*, 1908, xxvii. 359.
[2] H. Pellat and P. Sacerdote, *Comptes Rendus*, 1898, cxxvii. 544.
[3] F. Schmidt, *Ann. d. Physik*, 1914, xliv. 329.

As will be seen from Table II., many of the liquids show higher temperature coefficients than these solids. Dewar and Fleming [4] investigated the dielectric constants of a number of (frozen) liquids down to a temperature of −185° C. They found that many substances which, like water or ethyl alcohol, show abnormally high dielectric constants, give quite small values at the very low temperatures. For example, ice at −185° C. gives $\kappa = 2·4$ to 2·9, and nitrobenzol descends from 32 at 15° C. to 2·6 at −185° C. (These values are for frequencies of the order of about 100 ∿ per sec.)

At very slow frequencies [5] (*e.g.* below 1 ∿ per sec.) the effect of change of temperature is usually very marked.

§ (20) VARIATION OF DIELECTRIC CONSTANT WITH FREQUENCY.—Many substances (solids or liquids) show little change in dielectric constant for change of frequency over a range from 100 ∿ per sec. up to very high values. For example, R. Jaeger [6] tested ebonite, india-rubber, quartz, and a number of kinds of glass, etc., at various frequencies from 250 up to 3×10^7 ∿ per sec. (corresponding to wave-lengths $1·2 \times 10^6$ down to 10 metres) and in general there was at most a variation of 1 or 2 per cent throughout the whole range.

At very low frequencies most substances, however, show considerable increase in dielectric constant (see Curie and Compan, *loc. cit.*).

§ (21) EFFECT OF MOISTURE.—Substances which absorb moisture show large variation in dielectric constant according to the amount of moisture present.

In general also the variations with temperature and frequency increase with increase of moisture.

An example of the large variation with frequency for celluloid and paraffin paper at 19° C. (from the experiments of Thomas [7]) is given by the curves of *Fig.* 13 for low frequencies.

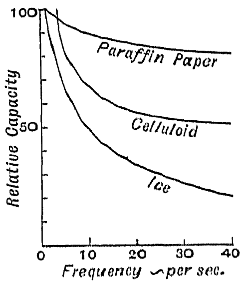


FIG. 13.—Variation of Apparent Dielectric Constant with Frequency (Thomas).

The effect extends to the higher frequencies; from 250 to 2000 ∿ per sec. the apparent dielectric constant of the celluloid fell 7 per cent and that of the paraffin paper 20 per cent. A similar change with frequency is shown by ice (*Fig.* 13).

[4] J. Dewar and J. A. Fleming, *Roy. Soc. Proc.*, 1897, lxi. 2, 299, 316, 358, and 368.
[5] J. Curie and P. Compan, *Comptes Rendus*, 1902, cxxxiv. 1295.
[6] R. Jaeger, *Ann. d. Physik*, 1917, liii. 409.
[7] Phillips Thomas, *Frankl. Inst. J.*, 1913, clxxvi. 283.

Many of the fibrous and other insulating materials used in electrical engineering absorb moisture appreciably. Paper is one of the most important of these, and it is also one of the most absorbent. In the manufacture of air-space telephone cables the paper is dried very thoroughly and sealed up hermetically in a lead tube. As it is desirable to keep the capacity as small as possible the paper strips are wrapped round the numerous wires in such a way as to keep them apart with the smallest possible amount of solid supporting material. The advantage of thorough drying in reducing the dielectric constant and increasing the insulation resistivity is illustrated by the following values [1] obtained for solid (colloidal) cellulose (Table IV.). The resistivities were measured at 200 volts (direct current) after one minute's electrification.

TABLE IV

Material.	Condition.	Dielectric Constant at 15° C.	Resistivity at 25° C. Megohm-cm.
Solid cellulose	Air dry	14	4
	Dried at 105° C.	7·3	1500 million
Cellulose triacetate	Air dry	4·7	200 million
	Dried at 105° C.	3·9	9000 million

In Table IV. are also given, by way of contrast, corresponding values for cellulose acetate, which is largely used as a tough insulating varnish for thin wires. It will be noticed that in its ordinary condition (air dry) it is a good insulator, and that the drying has a comparatively small effect on it. In the case of cellulose (of which paper is a fibrous and porous form) it would appear that a certain portion of the associated moisture can never be got rid of by any process of drying without absolute decomposition of the material. Thus in well paraffined paper or paraffined cotton-covered wire the moisture effects still show themselves.

Gutta-percha, when kept in water, goes on absorbing water for years, but all the while its insulation resistivity is *rising*. It shows, however, considerable change of dielectric constant with frequency [2] (*e.g.* a fall of 8 per cent for 200 up to 2000 ∿ per sec.).

§ (22) VARIATION OF DIELECTRIC CONSTANT WITH PRESSURE.—Ortvay [3] determined the dielectric constants of a number of liquids at varying pressures from 0 up to 500 kgm. per sq. cm. He found that they all obeyed the law $\kappa_p = \kappa_1(1 + \alpha p - \beta p^2)$, where p is the pressure and α and β are positive. The dielectric constant increases as the pressure is raised, but the rate of increase becomes less at the higher pressures.

The behaviour of gases at different pressures has been investigated by Occhialini and Bodareu [4] and others. The following are typical values for air:

p	1	149	334 atmos.
κ	1·000585	1·0839	1·1691.

The values for air practically follow the Mossotti Clausius formula,[5]

$$\frac{\kappa-1}{\kappa+2}\cdot\frac{1}{d}=\text{const.}, \quad . \quad . \quad (98)$$

where d is the density. Nitrogen and hydrogen also obey this law up to pressures of 100 atmos.

§ (23) DIELECTRIC CONSTANT AND OPTICAL PROPERTIES.—According to Maxwell's [6] electromagnetic theory of light the dielectric constant should be equal to the square of the refractive index μ (for the same wave-length λ). As the two quantities cannot be measured at the same wave-length, it is usual to employ Cauchy's formula,

$$\mu = A + \frac{B}{\lambda^2}, \quad . \quad . \quad . \quad (99)$$

or one of the more modern formulae connecting refractive index and wave-length, in order to find by extrapolation the value of μ for wave-lengths much longer than those reached in the infra-red spectrum. Maxwell's law is found to hold for certain gases and for hydrocarbon oils, but not for animal or vegetable oils.

Rubens [7] determined optically the reflecting power (R) of a number of solid and liquid dielectric substances for wave-lengths up to about 0·3 mm. (313 microns). He found that, as the wave-length was gradually raised to this value, R for most of the solids first fell and then rose to a maximum, after which it fell asymptotically towards a steady value. For all the liquids except castor oil (which showed R nearly constant) R followed almost a straight-line law of increase from 30 up to 300 microns wave-length. Rubens also determined the

[1] A. Campbell, *Roy. Soc. Proc.* A, 1906, lxxviii. 196.

[2] K. W. Wagner, *Archiv f. Elektrotech.*, 1914, iii. 67; and (for radio frequencies) G. E. Bairsto, *Roy. Soc. Proc.*, 1920, xcvi. 363.

[3] R. Ortvay, *Ann. d. Physik*, 1911, xxxix. 1.

[4] A. Occhialini and E. Bodareu, *Nuovo Cimento*, 1913, vi. 15; K. Tangl, *Ann. d. Physik*, 1907, xxiii. 559.

[5] See F. Hasenohrl, *Wien. Ber.*, 1896, cv. pt. ii. 460, and D. Negreano, *Comptes Rendus*, 1899, cxxviii. 814, who attributes formula (98) to Lorentz (1880).

[6] Maxwell, *Electricity and Magnetism*, 2nd ed., 1881, § 788.

[7] H. Rubens, *Deutsch. Phys. Gesell. Verh.*, 1915, xvi. 315.

dielectric constants (κ) for the same set of substances by the Lecher method using waves of 10 to 30 metres wave-length. By Fresnel's formula the relation between R and κ should be

$$R = 100\left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1}\right) \quad . \quad . \quad . \quad (100)$$

He found that for the solids the values of R_∞ obtained from this formula were in good agreement with the limits approached in the optical experiments, thus indicating that in the unexplored range from $\lambda = 0{\cdot}3$ mm. to $\lambda = 10{,}000$ mm. there is probably no anomalous dispersion. R. Jaeger (*loc. cit.*) tested the same samples of the solid materials and found that in general their dielectric constants were practically constant down to moderate frequencies.

On the other hand, the results found by Rubens for the liquid dielectrics do not show agreement with formula (99) and indicate the presence of anomalous dispersion.

§ (24) Dielectric Constant of Glass.—The various kinds of glass which are manufactured for optical purposes possess such diversity in composition that it is not surprising that their dielectric constants should include a long range of values. Some typical examples (from Jaeger) are given in Table V., the approximate chemical composition in each case being indicated in columns 2 and 3. All the glasses except the Uviol Crown also contained 1 per cent or less of As_2O_3.

Table V

Various Glasses

Designation.	Chief Constituents (each over 10 per cent).	Constituents under 10 per cent.	Dielectric Constant (for $\lambda=10$ m.).
Uviol Crown. U.V. 3199	SiO_2 B_2O_3, K_2O	ZnO	5·61
Fluor Crown. O. 7185	SiO_3, B_2O_3 K_2O, F, Al_2O_3	..	5·89
Phosphate Crown. S. 367	P_2O_3, K_2O	Al_2O_3, MgO, B_2O_3	6·43
Soda Lime . .	SiO_2, CaO, N_2O	..	7·18
Ordinary Flint. O. 2051	SiO_2, PbO	K_2O, Na_2O	7·46
Heaviest Barium Crown. O. 1993	BaO, SiO_2	ZnO, B_2O_3, Al_2O_3	8·41
Heaviest Silicate Flint	PbO, SiO_2	..	17·70

Gray and Dobbie[1] investigated the dielectric constants of a number of glasses (of known analysis) for quick time of charge both at room temperature and at temperatures of the order of 130° C., also determining the resistivities at temperatures from about 70° up to 140° C. Although the fall in resistivity were enormous over this range of temperature, the increase of dielectric constant was in most cases not large, being of the order of 4 to 15 per cent.

[1] A. Gray and J. J. Dobbie, *Roy. Soc. Proc.*, 1900, lxvii. 197.

III. Construction of Condensers

§ (25) Condensers may be classified according to the purposes for which they are designed, as primary standards, secondary standards, and common condensers. It is more convenient, however, in describing the various types, to arrange them according to the dielectric used in their construction. For good condensers the desirable qualities are permanence, constancy of capacitance, small change with temperature or frequency, small power loss with alternating current, and capability of standing considerable applied voltage. For several reasons compactness (relatively small bulk) is also desirable, and hence in most cases the dielectric substance must be capable of easy subdivision into thin sheets. Few readily obtained materials fulfil the desired conditions, and for present-day practice the list need only include *mica, paraffin-waxed paper, shellacked paper, ebonite, glass, air,* and *oil.* Electrolytic condensers form a class by themselves and will be discussed separately.

§ (26) Dielectric Strength.—The insulating material should be able to bear as high an applied voltage as possible; this property is measured by the *dielectric strength*, which is expressed in volts per cm. of thickness necessary to cause breakdown of the material by sparking through it. The dielectric strength is not constant for various thicknesses of sheet, but decreases with increase of thickness. In Table VI. are given approximate dielectric strengths of the most important materials used in making condensers. They are mostly taken from T. Gray's[2] results, which were obtained with slightly convex discs as electrodes. As most of the materials may vary very considerably in composition, the values in the table must only be taken as indicating the order of magnitude to be expected for a given material.

[2] T. Gray, *Phys. Rev.*, 1898, vii. 199.

TABLE VI

DIELECTRIC STRENGTHS

Material.	Thickness. mm.	Dielectric Strength. Volts per cm.
Air (at normal pressure and temperature)	0·2	57 500
	0·6	49 200
	1	43 600
	6	32 700
	10	29 800
Ebonite . . .	1	500 000
Glass (density 2·5)	1	285 000
	5	183 000
Mica	0·01	2 000 000
	0·1	1 150 000
	0·2	950 000
	1·0	610 000
Paraffin oil . .	..	60 000-100 000
Paraffin wax . .	..	130 000-270 000
Paraffin - waxed paper	0·1	400 000-600 000
Vaseline . . .	..	91 000

§ (27) MICA CONDENSERS. (i.) *Methods of Construction.*—For condensers of the order of 1 μF mica is pre-eminently the best dielectric material. By reason of its natural cleavage it can be readily split into very thin sheets, and its mechanical properties are good, for its resistance to crushing is very great. Also, as is seen from Table VI., its dielectric strength is very high. Even though very thin sheets can be made, a large number are required for a condenser of the order of 1 microfarad. One sheet of 25 sq. cm. area and 0·05 mm. thick gives a capacity of about 0·002 μF, and hence for 1 μF about 500 mica sheets of this size would be required. The actual sheets should have considerably greater area to allow sufficient edges for insulation. For the best standard condensers the sheets must be of well-selected clear ruby mica. These are assembled with alternate sheets of tinfoil in hot melted paraffin wax, the most scrupulous care being taken to avoid any traces of moisture. The operator must have an exceptionally dry skin, for if the high temperature of the process causes any perspiration in the hands, traces of moisture, salt, etc., get into the wax and lower the quality of the condenser. Paraffin wax of melting-point about 63° C. is suitable. In chemical constitution it should consist of only a small number of different paraffins. The pile of sheets is compressed while hot and a certain amount of the paraffin is squeezed out. The behaviour of the condenser afterwards depends on the amount of paraffin wax allowed to remain. The temperature coefficient of the capacitance is usually about −0·02 per cent per degree C.; but, as Curtis[1] showed, by applying sufficient pressure it can often be reduced nearly to zero and sometimes even made positive. Some good authorities, however, state that condensers so treated are very liable to break down, and that it is very desirable to leave a greater proportion of paraffin if good constancy is to be ensured.

[1] H. L. Curtis, *Bureau of Standards Bull.*, 1910, vi. 147.

The two alternate sets of tinfoil sheets are soldered to a pair of flexible leads of low resistance, and as soon as possible the condenser is sealed up permanently in an air-tight case. If this precaution is neglected, the gradual absorption of moisture by the paraffin wax in time spoils the condenser.

Instead of using sheets of tinfoil, sometimes the conducting surfaces are formed by chemically silvering opposite sides of the mica sheets. Condensers made in this way, however, have been found to show slight inconstancy in capacity, which has been traced to uncertainty of contact between patches of the silver with the mica.

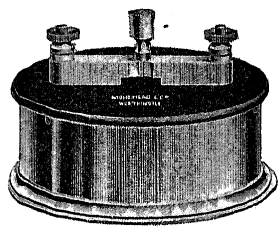


FIG. 14.
Muirhead 1 μF Condenser.

Mica condensers are mounted in a variety of ways. *Fig.* 14 shows one of the most usual forms for a single-valued condenser. When a number of condenser sections are mounted together, a very common plan is to connect them in series to plug blocks as shown in *Fig.* 15. By this system the sections can each be used alone, or they can be connected as desired in series or in parallel to the long blocks which carry the terminals. For example, with the plugs A B in the positions

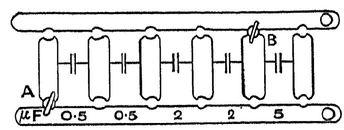


FIG. 15.—Parallel Series Connections for Condenser.

shown the capacitance would be 0·5, 0·5, 2, and 2, all in series, *i.e.* 0·2 μF.

A much more elaborate and convenient arrangement shown in *Fig.* 16 has the sections in decades with 10-fingered switches to put any desired number of the sections in parallel.

Note.—A word of warning is here necessary with regard to the marking of standard condensers. Many condensers in use at the present time marked in microfarads are really in the old B.A. microfarads, which are larger than international microfarads, the relation between these units being

$$1 \text{ B.A. } \mu\text{F} = 1{\cdot}0134 \text{ International } \mu\text{F}.$$

(ii.) *Importance of Steady Temperature.*—Condensers of the type described above, having

their plates embedded in a considerable mass of paraffin wax, do not quickly follow the room temperature when it varies rapidly. For the most accurate work it is best to keep the condenser in an enclosure at constant temperature for several hours before using it. When a condenser is tested at the National Physical Laboratory it is kept in a constant temperature room for 24 hours before the test is made.

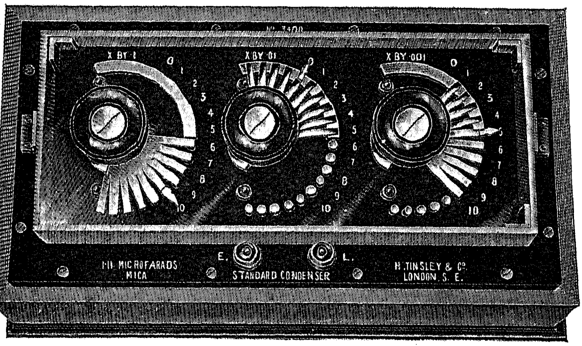


FIG. 16.—Tinsley Subdivided Condenser.

(iii.) *Condensers for High Voltage.*—If a condenser is to stand high voltage, it should not be built with very thick mica sheets, but should consist of a number of equal sections (with mica of normal thickness) connected permanently in series. If there are m equal sections, then each of them will only have to stand $1/m$th of the total voltage. The capacity of each is of course m times the capacity required.

§ (28) PARAFFIN PAPER CONDENSERS.—Mica condensers of large capacitance are very expensive, for the mica is a costly material and large sheets of it are not available. For many purposes paraffin paper condensers can be used instead. In their manufacture the paper must be extremely well dried before paraffining, and the same precautions must be taken to exclude moisture as already described for mica condensers. Only very good paper should be used, free from all surface loading or added mineral matter. A good working thickness is about 0·025 mm. To avoid the danger of breakdown due to pin-holes or other weak spots it is best to use at least two or three sheets together between each pair of tinfoils.

A high-class subdivided paraffin paper condenser having a total capacitance of 20 μF is illustrated in *Fig.* 17.

Mansbridge Type (with tinfoiled paper).—In modern telephone work (since 1900) an immense number of condensers are used. For instance, in 1907 the total world output of condensers, chiefly for this purpose, represented a capacitance of about 5,000,000 microfarads (*i.e.* 5 farads). The method used for standard condensers of assembling the sheets singly by hand was too slow and costly to meet the large demand and much quicker systems of manufacture have been developed. In 1899 Sir John Gavey introduced to the British Post Office the system of forming the condensers by winding together a number of strips of paper and tinfoil. About the same time Mansbridge [1] made two great improvements in the process: (1) the condenser is assembled without paraffin wax, then desiccated in a vacuum oven, and while hot impregnated with melted paraffin, being allowed to cool under pressure in a screw press; (2) instead of tinfoil, paper metallised or coated with a very thin layer of tin is used for the conducting strips. To coat the paper the tin in the form of an impalpable powder, precipitated chemically or electrolytically, is made into a mud with water and a little size to form the adhesive, and then applied evenly to the paper. After drying the paper is passed through heated heavy steel rollers, which compress the tin coating into a lustrous coherent film. The conductivity of this film is only about 25 per cent of that of pure tin, but is sufficient for its purpose. The usual thickness employed is about 0·0025 mm., with about 2 mgm. of tin per sq. cm. A width of 10 cm. will carry a current up to about 4 amperes. Condensers made with it are usually "self-sealing"; if a breakdown occurs at a weak spot the fault burns itself out almost instantly and the insulation is restored. To get rid of pin-holes and other weak spots, before use the paper insulation is subjected to a potential difference of 100 to 200 volts by being passed over a metal roller connected through a resistance to one terminal of a supply circuit, while the film side is connected to the other terminal. In this way a great many of the defective spots are rendered

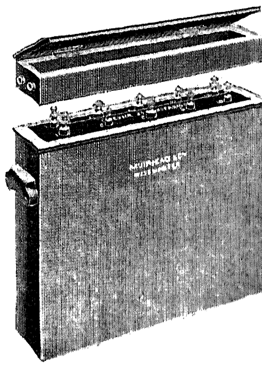


FIG. 17.—Paraffin Paper Condenser (Muirhead).

[1] G. F. Mansbridge, British Patent 19451, 1920, and *J. Inst. El. Eng.*, 1908, xli. 535.

safe. In making a condenser two foiled strips are wound up with two strips of plain paper, and the connections are made by two strips of very thin copper foil laid across the opposite foiled surfaces about the middle of the winding in order to reduce the total internal resistance. The condenser is finally placed in a metal case with semi-plastic paraffin wax, and the opening through which the leads pass out is sealed with a composition which does not contract on setting, consisting of gutta-percha 20 parts, rosin 14, stearine pitch 6, and Stockholm tar 14 parts.

In telephone work the normal capacitance is 2 μF. A foiled paper condenser of this capacitance can be got into a space of 12 cm. × 4·2 cm. × 4·2 cm., and will weigh about 280 grams including the tinplate case.

Mansbridge remarks that for checking the spark in the primary circuit of an induction coil a condenser with low insulation is best. Ordinary paper without paraffin wax has sometimes been used in condensers for this purpose.

§ (29) Shellacked Paper Condensers.—Condensers are sometimes constructed with paper impregnated with shellac or other similar substance. In Meirowsky's type the lacquered paper is passed through hot rollers and wound on a mandrel under great pressure, sheets of tinfoil with projecting tag ends being placed in at suitable intervals as the winding proceeds. The resulting tube, when drawn off the mandrel, is very hard and strong, and the tinfoil sheets are very thoroughly embedded in it. A protecting case is not found necessary. By suitably choosing the thickness allowed between the tinfoil sheets, single sections can be constructed to work at any desired voltage from 500 up to 30,000 volts. A 2 μF condenser of this type to work at 1000 volts (and 50 ∿ per sec.) weighs 6 kgm.

§ (30) Ebonite Condensers.—Ebonite is used as dielectric in certain types of condenser. When in good condition it has high insulating properties, and it can be united very firmly to metal conductors by being "cured on" in manufacture. In the Fleming cymometer [1] (for the determination of high-frequency wave-lengths) the adjustable condenser consists of a long brass tube sheathed with thin ebonite over which another tube slides, thus giving a long range of capacitance.

As another example may be mentioned the adjustable condensers supplied by the Marconi Wireless Telegraph Company, in which very thin sheets of ebonite are arranged between metal plates, giving a comparatively large capacity in small bulk.

[1] J. A. Fleming, *Phil. Mag.*, 1905, ix. 758, and *Phys. Soc. Proc.*, 1905, xix. 603.

The great objection to ebonite thus used is that it is very liable to alter its properties with time. As the deterioration is worst at the surface, it is accentuated by the use of very thin sheets. Its dielectric loss is relatively high.

§ (31) Glass Condensers.—Although not in general suitable for use as standards, condensers with glass dielectric are of distinct value in a number of practical applications.

(i.) *The Leyden Jar.*—The Leyden jar, invented in 1745, is still a convenient condenser for certain purposes. The jar should be made of English flint glass free from bubbles or flaws, and the surfaces of the glass not covered with tinfoil should be coated with shellac varnish and thoroughly dried off. Both inner and outer connections to the tinfoil should make good contact, for a loose connection causes sparking and this is ruinous to the jar. According to Fleming [2] the so-called pint size has usually a capacity of about 0·0015 μF, and the gallon size about 0·003 μF. They will generally stand a potential difference of 20,000 volts. When used with high-frequency currents at high voltages brush discharge occurs at edges of the tinfoil, which increases as the voltage is raised. It acts as an increase of effective plate area and causes an increase of capacity at the higher voltages.

Fig. 18.—Mościcki Glass Condenser (single section).

(ii.) *Mościcki Glass Condensers.*—Mościcki [3] has introduced improved Leyden jar condensers of the form shown in *Fig.* 18, consisting of a long glass tubular flask with a special channelled porcelain or ebonite insulator at the top. In one type the outer conductor is an electrolyte, and another type has both the inner and outer coatings formed by chemically deposited silver strengthened by subsequent electroplating. The inventor found that the tendency to break down is much greatest at the edges of the coatings, and accordingly he protected against breakdown by thickening the glass at the neck of the flask where the edges come. One of the second type, with a height of 120 cm. and surface 200 sq. cm., has a capacitance of 0·01 μF and will work continuously at 10,000 volts (at 50 ∿ per sec.). To obtain larger capacities sets of the tubes are mounted together.

(iii.) *Condensers with Glass Plates in Oil.*—

[2] J. A. Fleming, *Principles of Electric Wave Telegraphy*, 1st ed. p. 59.

[3] I. Mościcki, *Acad. Sci. Cracovie Bull.*, 1904, i. 42; also *Elekt. Zeits.*, 1904, and *Éclairage él.*, 1904, xli. 14.

When exposed to air not artificially dried the surface of glass always absorbs moisture and tends to form a leakage path for electrical discharges. This difficulty is got over by immersing the glass in insulating oil or even by means of a thick layer of vaseline spread over the surface. Oil-immersed glass condensers with metal plates are much used for high voltages at high frequencies. In the type used at the National Physical Laboratory the metal plates are of aluminium 1 mm. thick with all the edges and corners very thoroughly rounded. The plates are cut with lugs as shown in *Fig.* 19, the lugs of alternate plates being at opposite sides. These lugs are clamped together in two sets to leads connected to the terminals. The bundle of plates with sheets of glass between is wrapped with two narrow strips of press-spahn and tightly bound over these with a number of turns of thin tinned copper wire fastened by soldering. The whole is immersed in special paraffin oil (of high boiling-point) which has been dried by many hours' heating above 100° C. Care must be taken to leave no air bubbles between the metal plates and the glass sheets.

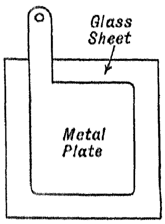


FIG. 19.—Metal Plate and Glass Sheet of Oil-Immersed Condenser.

§ (32) AIR CONDENSERS. — As a dielectric for use in condensers air is superior to solid or liquid substances on account of its freedom from power losses due to leakage and dielectric hysteresis. On the other hand, its dielectric strength and inductive capacity are both lower, for which reason air condensers are relatively very bulky for a given capacitance. There is another very important consideration which prevents the making of very compact air condensers; when the plates are very close to one another the dust particles in the air form dust streaks between the plates, and these leakage paths spoil the good insulation of the condenser. To ensure that this leakage effect shall not occur, the safest way is to lay down the rule that in an air condenser the distance between the surfaces of the conductors should never be less than 2·5 or even 3 mm. At smaller distances there is always risk of leakage due to dust streaks. Such leakage if present can, however, be got rid of by very thorough drying of the air in the condenser, which is best done by means of metallic sodium.

In a condenser with solid dielectric the plates are usually held in position by the dielectric itself, but in an air or oil condenser they must be rigidly supported and kept apart by insulators of solid dielectric. Unfortunately there are very few highly insulating materials that are sufficiently strong and permanent in form to be used for this purpose, since a very small change in the relative position of the plates usually makes a considerable change in the capacitance. Ebonite tends to yield under stress, particularly in warm climates. For the most permanent standards fused quartz or amberite appear to be almost the only suitable insulators. Amberite is reconstructed amber, being made by applying great pressure to amber shavings (wet with ether), which causes them to cohere into a translucent solid. It is an extremely good insulator, and when polished shows very little surface leakage. It is rather brittle, and requires particular care in mechanical working. Fused quartz is harder and has to be ground to shape and then polished. To get the best insulation it should be washed with strong acid after the polishing.

For the purposes of description it will be convenient to consider air condensers in three classes: (i.) Fixed air condensers, (ii.) variable air condensers, and (iii.) condensers with air at high pressure.

(i.) *Fixed Air Condensers.*—The most accurate air condensers are *primary standards* which can be calculated from their dimensions. Forms that have been used for this purpose are either concentric spheres, concentric cylinders with guard rings, or parallel plates similarly guarded. In the determination of "v," the ratio of the electromagnetic to the electrostatic unit of electrical quantity,[1] condensers of these three types have been used by various experimenters. In Rosa and Dorsey's paper[2] will be found fully illustrated descriptions of each of the types. Condensers of this kind are necessarily of very small capacitance, the maximum in the paper just quoted being about 150 $\mu\mu$F.

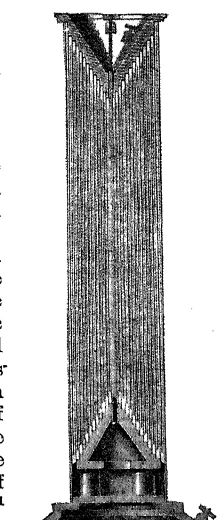


FIG. 20.

For *secondary standards* much higher values are required, and these are obtained by using a large number of parallel plates or concentric cylinders. An illustration of an early example of the cylindrical type is given in *Fig.* 20.

[1] See article on "Electrical Standards."
[2] E. B. Rosa and N. E. Dorsey, *Bureau of Standards Bull.*, 1907, iii. 439.

It was designed by Glazebrook and Muirhead for the Committee of the British Association.[1] In it the two sets of brass cylinders are mounted on stepped brass cones at the top and bottom, the insulation being afforded by three short thick cylinders of ebonite supporting the lower cone. The internal air is kept dry by means of a dish of strong sulphuric acid placed beside these supports. The capacitance is about 0·024 μF.

About the same time Lord Kelvin[2] introduced a portable air condenser with 45 parallel plates, the air-gap distance being 3 mm. The internal air required to be kept artificially dry as the insulating supports were of glass.

More recently improved designs of these two types of air condenser have been brought out by other experimenters. Giebe[3] constructed condensers of both types, but found the plate type superior to that with concentric cylinders. In *Fig.* 21 is shown one of his plate condensers with the case removed. It has a capacity of 0·01 μF, and is built up of seventy-one round plates of magnalium (20 cm. diameter, 1 mm. thick), each of the two sets being mounted with distance pieces on four rods passing through clearance holes in the other set. (Magnalium is a very light alloy of magnesium and aluminium, which is harder than aluminium and easier to work.) The air-gaps are 2 mm., and the solid insulation consists of short amber pillars. The insulation resistance is of the order of 10^9 megohms. When charged to a potential difference of 120 volts, the loss of charge in eight days was less than 5 per cent. The temperature coefficient is about +3 parts in 100,000 per degree C.

FIG. 21.—Giebe Air Condenser (case removed).

Following Giebe's type, Schering and Schmidt[4] constructed a series of air condensers from 0·001 μF upwards, arranged in such a way that, when they are placed one over the top of another in a pile, they are connected in parallel and their capacities add correctly. In *Fig.* 22 is shown a condenser of this additive type made by R. W. Paul. The insulation material used is amberite.

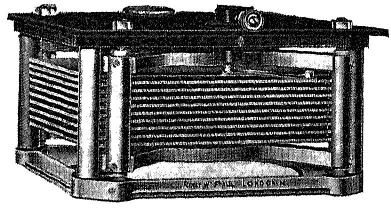


FIG. 22.—Additive Air Condenser (R. W. Paul).

(ii.) *Variable Air Condensers.* (*a*) *Ordinary Type.*—The commonest type of variable air condenser (said to have been invented by Korda in 1893)[5] consists of a set of metal plates fixed parallel to one another on an axle, by which they can be turned so as to lie to a less or greater extent in the air-gaps between a set of parallel fixed plates. The axle carries a pointer which moves over a divided scale, or else a circular scale which is turned past a fixed pointer. The system is shown in plan in *Fig.* 23, in which c is the axis about which the movable plates turn. As the overlapping area a is increased the capacitance rises, its *increase* being approximately proportional to a, and hence also to the change in angular position.

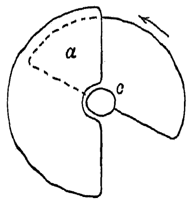


FIG. 23.—System of Variable Air Condenser (in plan).

Fig. 24 shows the working parts of a condenser with a turning scale (divided in

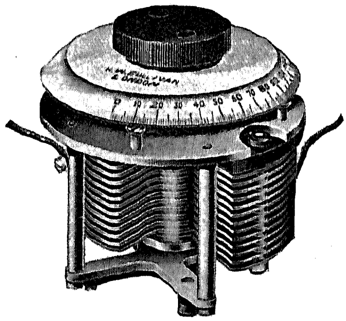


FIG. 24.—Working Parts of Sullivan Air Condenser.

[1] *B.A. Report*, Leeds, 1890, p. 102; and *Electrician*, 1890, xxv. 616.
[2] Lord Kelvin, *Roy. Soc. Proc.*, 1892-93, lii. 6.
[3] E. Giebe, *Zeits. Instrumentenk.*, 1909, xxix. 269.
[4] H. Schering and R. Schmidt, *Zeits. Instrumentenk.*, 1912, xxxii. 253.
[5] This type was first used in radio-telegraphy by A. Köpsel (*Dingler's Polytech. Journal*, 1904, xiv. 319).

degrees); in this design the axle has a lower bearing carried by a metal spider rigidly connected to the upper metal plate. This is a much better system than the more common one in which an upper ebonite plate carries the fixed plates and a single bearing for the axle. To ensure constancy of capacitance the bearings should be very well fitted, and the plates should be of good thickness. In making the plates a suitable material (*e.g.* brass) should be used, and great care should be taken to avoid sag due to elastic fatigue. The corners and edges should all be rounded off.

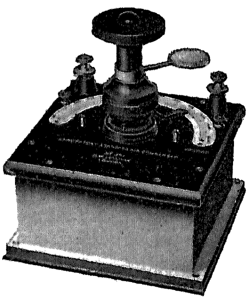


FIG. 25.—Sullivan Variable Air Condenser.

In *Fig.* 25 is shown a Sullivan standard air condenser with fixed scale and moving pointer. The scale reads directly (as all such scales should) in micromicrofarads. A high handle is provided in order to avoid slight errors which sometimes arise when the hand is brought too near the working parts of the condenser. Inside the case there is a metal screen which can be connected when desired to either of the terminals. In general it is best to calibrate the condenser with the screen connected to the movable plates.

The two systems of plates are usually assembled by means of metal rods over which are slipped tubular distance pieces. A more rigid design is that of G. Seibt, in which each set of plates is formed from a single casting, great accuracy in the air-gap spacing being secured by a special system of turning the castings true.

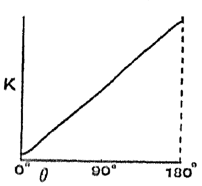


FIG. 26.—Calibration Curve of Turning Plate Air Condenser.

In a well-designed air condenser with semicircular plates the calibration curve, showing how the capacity K varies with the angle θ through which the movable plates have been turned from an initial symmetrical position, is of the form shown in *Fig.* 26. At $\theta=0$ there is a small initial capacity, and as θ is increased, the curve very soon becomes practically a straight line up to nearly 180°, near which the slope usually falls off slightly. Thus over the greater part of the range $K=c+a\theta$, and when the scale is marked to read directly the divisions are uniformly spaced except near the beginning and end of the range.

(*b*) *Square Law Condenser.*—For special purposes the plates of this type of condenser are sometimes shaped so as to give calibration curves of other kinds. For example, for a wave-meter in which the wave-length λ is proportional to the square root of the capacity K, a condenser designed to give K proportional to θ^2 will give an evenly divided scale for λ. Duddell[1] constructed a condenser having this property. The fixed and moving plates have the outlines shown in *Fig.* 27, and one edge of the moving plates follows the polar curve

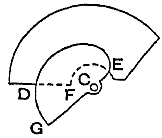


FIG. 27.—Plates of Duddell Square Law Condenser.

$$r^2=4a\theta+b^2, \qquad (101)$$

where the pole is C the centre of the axis of rotation, $r=\text{CD}$, θ(in radius)$=\text{DCE}$, and $b\doteq$ CE the radius of the inner edges of the fixed plates. Then

$$\text{Area DFE}=a\theta^2.$$

Duddell found that when $b=\text{CE}$, with a 1 mm. air-gap and CG about 8·5 cm., the capacitance K in $\mu\mu$F is given by

$$K=\alpha+\beta\theta+\gamma\theta^2,$$

where $\beta\doteq 0{\cdot}25\gamma$. But when b was made equal to 0·87CE, the term in θ was small compared with $\gamma\theta^2$. The range of this square law condenser is short compared with that of the ordinary type.[2]

Occasionally other types of variable air condensers are met with. In some of these one set of concentric cylinders slides within another set; in others the fixed and turning plates of the common type are replaced by two sets of half-cylinders. Here, as in other instruments, rotating systems appear to be handier and more compact than systems involving guided motion in a straight line.

§ (33) EFFECTS OF CAPACITY TO EARTH.—As has already been remarked, when a condenser has small capacitance, it should be enclosed in a conducting screen preferably connected to the moving plates. Unscreened condensers, however, are very often met with, and their readings are frequently affected by errors due to capacity to earth or surrounding objects. Particular care is necessary when the total capacity is below 100 $\mu\mu$F. Orlich's method (§ (5)) is applicable in all cases, by making three measurements (for any given reading). Calling the two sets of plates 1 and 2 and the earth *e* respectively,

[1] W. Duddell, *Inst. El. Eng. J.*, 1914, lii. 275.
[2] See also "Radio-Frequency Measurements."

let the capacitances found be as follows between P and Q

P.	Q.	Capacitance.
1	2 earthed	a
2	1 earthed	b
1 and 2 in parallel	Earth	c

Then $c_{12}+c_{1e}=a,$

$c_{12}+c_{2e}=b,$

$c_{1e}+c_{2e}=c.$

Hence $c_{1e}=s-b,$

$c_{2e}=s-a,$

and $c_{12}=s-c,$

where $s=\frac{1}{2}(a+b+c).$

Thus the "working capacitance"

$$=c_{12}+\frac{c_{1e}c_{2e}}{c_{1e}+c_{2e}}=s-c-\frac{(s-a)(s-b)}{c}. \quad (102)$$

§ (34) Compressed Gas Condensers.—The use of a gas at high pressure as dielectric in condensers was first suggested by Jervis-Smith,[1] and condensers of this type, using compressed air or carbon dioxide gas, were introduced later by Fessenden.[2] The plates in these were discs with an air-gap of 2 to 3 mm. Wien used concentric cylinders with similar distance apart. This type of condenser shows very small dielectric loss even at high voltages, and is specially suitable for radio frequency work. At the higher pressures there is very little gain in dielectric constant (§ (22)), but the dielectric strength is greatly increased, as has been shown by Wolff[3] and (for higher pressures) by Watson. It is roughly proportional to the pressure, and is nearly 300,000 volts per cm. for a pressure of 10 atmospheres, which is at least as high as that of glass. In a well constructed condenser of this kind the air pressure falls very slowly; for example, an initial pressure of 14 atmospheres may fall only 10 per cent in a year.

§ (35) Oil Condensers.—Condensers with oil dielectric are in general similar in construction to air condensers, but with an oil-tight containing vessel. A good oil to use is paraffin oil of high boiling-point; it should be dried by heating for hours above 100° C. before being filled into the container, which should then be closed to prevent absorption of moisture. It has the advantage over air of giving more than double the capacitance for the same size of condenser, but it has slight dielectric loss with alternating currents. There is little change of capacity with frequency.

[1] F. Jervis-Smith, *Nature*, 1893, xlviii. 64.

[2] R. A. Fessenden, *Electrician*, 1905, lv. 795; also M. Wien, *Ann. der Physik*, 1909, xxix. 679.

[3] M. Wolff, *Ann. der Physik*, 1903, xi. 570; and E. A. Watson, *Inst. El. Eng. J.*, 1909, xliii. 113.

Owing to its much greater inductive capacity (4·8) castor oil has been sometimes used in condensers, but it has not proved a success, for, as found by Campbell and Dye,[4] although the dielectric loss per cycle is small at low frequencies, it becomes abnormally high at radio frequencies, even in spite of very thorough drying. The power factor at 800 ∿ per sec. was found to be 0·0005, but at 10^6 ∿ per sec. it was 0·038. The temperature coefficient (−0·4 per cent per degree C.) is also too high. Petroleum behaves far better in all these respects.

It may be remarked here that great care should be taken to have very good connection (whether by flexible conductor or rubbing contact) between the moving parts and terminals in variable condensers, as even a small series resistance sometimes will spoil the otherwise low power factor. Badly insulating ebonite tops are also sometimes responsible for the same effect.

§ (36) Electrolytic Condensers. — Condensers of very large capacity can be constructed[5] without difficulty of metal plates immersed in suitable electrolyte. The metal used must belong to the class known as "valve metals," such as aluminium, magnesium, or tantalum, which have the property, when made the anode in certain electrolytes, of forming on their surface a thin, porous, adhesive layer separated from the actual metal by a thinner layer of gas having an extremely high resistance, which, even with a very small current, allows a high voltage to be maintained. If the applied (direct current) voltage is raised above a certain maximum, which depends on the nature of the electrolyte used, the gas layer breaks down and numerous discharges with sparking occur. For example, for aluminium in solution of sodium sulphate the maximum is 40 volts, whereas in solution of ammonium phosphate it is 460 volts. With aluminium one of the best electrolytes is a saturated solution of ammonium borate $(NH_4)HB_2O_4$. Two sets of plates are immersed in this, care being taken to insulate the conductors as they pass out of the liquid surface; by the application of (say) 100 volts (direct current) for several days to the two sets in parallel against a platinum kathode, the plates are "formed." The two sets are then separated and form an electrolytic condenser which can be used with alternating voltages as high as 90 volts, and with this voltage it shows a power factor of about 0·05. With ten aluminium plates, each having an area of about 160 sq. cm., Schultze thus obtained an effective capacity

[4] A. Campbell and D. W. Dye, *N.P.L. Report*, 1913–14.

[5] C. J. Zimmermann, *L'Éclair. élec.*, 1903, p. 388; and G. Schultze, *Elektrotech. u. Maschinenbau*, 1909.

of about 60 microfarads. The condenser was tested with frequencies up to 800 ∿ per sec., and carried alternating current of the order of 1·5 amperes for a week without apparent alteration. The capacity depends on the forming voltage, being inversely proportional to it (for rising values).

IV. Measurement of Capacity

§ (37) Introductory.—In most of the more modern methods of measuring capacity alternating current is employed, and many of these methods are similar to those used for the measurement of inductance. A full description of suitable sources of alternating current and the measuring or detecting instruments employed will be found in the article on "Inductance, Measurement of." That article also includes descriptions of the resistances and inductances required for the measurements.

Classification of Methods.—The various methods of measuring capacity are best classified according to the chief electrical property in terms of which the unknown capacity is determined. The measurement may be in terms of:

(A) Resistance (including current and voltage);
(B) Another capacity;
(C) Mutual inductance; and
(D) Self-inductance.

The following descriptions of a number of typical methods are given in the order of this classification, some special cases being discussed separately.

(A) *Comparison with Resistance*

§ (38) Impedance Methods.—If a pure sinoidal voltage V applied to the terminals of a condenser gives a current I, then (§ (9)) the capacitance K of the condenser in microfarads is given by

$$K=\frac{10^6 I}{\omega V}, \quad . \quad . \quad . \quad (103)$$

where the pulsatance $\omega=2\pi n$, n being the frequency.

The voltage and current may be measured by ammeter and voltmeter. From Table I. (§ (9)) it will be seen that the ammeter should be a low reading one (*e.g.* of thermal type). The voltmeter should be electrostatic, to ensure that the current taken by it shall be very small. When the capacitance to be measured is small, it is necessary to take account of the capacitance of the voltmeter. This capacitance depends on the deflection [1]; for example, in an Ayrton-Mather voltmeter tested by Campbell the capacitance ranged from 8·1 to 11·9 $\mu\mu$F, corresponding to readings of 500 and 4000 volts respectively. In a Kelvin multicellular voltmeter (to 120 volts) the capacitance at 40 volts is about 70 $\mu\mu$F; in a portable Ayrton-Mather instrument of the same range it rises from about 30 $\mu\mu$F. at zero reading to about 55 $\mu\mu$F. at a reading of 84 volts. In no case should long twisted flexible leads to the voltmeter be used, as they may add more capacitance than the voltmeter itself. The capacitance of the voltmeter must be subtracted from the K obtained by equation (103). If the applied voltage is constant, the correction, if any, may be found by observing the change in the current when the voltmeter is applied to the condenser.

Instead of using an ammeter, a resistance R may be put in series with the condenser, and V_0, the potential difference across this, observed on the voltmeter.

Then $$K=\frac{10^6 V_0}{V\omega R} \text{ (in } \mu\text{F)}. \quad . \quad . \quad (104)$$

Here V and V_0 should if possible be nearly equal. The following examples indicate the order of magnitude of R required:

With $V_1=V_0$,
for $n=50$ ∿ per sec., $R \doteqdot 3180/K$;
for $n=800$ ∿ per sec., $R \doteqdot 200/K$.

If the wave-form of the applied voltage is not a pure sine curve, the current wave-form will be more impure, for the effect of the condenser is to accentuate the harmonics. If the voltage wave-form is known, and the components are V_1, V_3, V_5 . . ., then instead of equation (103) we have

$$K=\frac{10^6 I}{\omega V}\sqrt{\frac{V_1^2+V_3^2+V_5^2+\ldots}{V_1^2+9V_3^2+25V_5^2+\ldots}}, \quad (105)$$

and the same correcting multiplier applies to equation (104).

§ (39) Arnò's Electrometer Method.—In Arnò's null method [2] a sine wave current is sent through the unknown condenser K in series with an adjustable resistance R (*Fig.* 28). The point A is connected to the needle of an electrometer, and B and C to the alternate pairs of quadrants. When a balance is obtained by adjusting R,

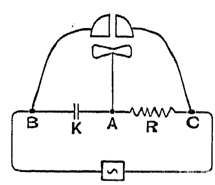


Fig. 28.—Arnò's Electrometer Method.

$$K=\frac{10^6}{\omega R} \quad (\mu\text{F}). \quad . \quad . \quad (106)$$

§ (40) Commutator Methods with Direct Deflection.—Maxwell [3] suggested various

[1] Lord Kelvin, *Roy. Soc. Proc.*, 1892; A. Campbell, *El. Rev.*, 1895, xxxvi.

[2] R. Arnò, *Nuovo Cim.*, 1895, iv. (1), 252.

[3] Maxwell, *Electricity and Magnetism*, 2nd ed. vol. ii. §§ 775 and 776.

methods of determining capacity in terms of resistance by the help of a commutator regularly repeating the process of charge and discharge of a condenser, and various experimenters have developed methods of this kind. In Fleming and Clinton's[1] method a rotating commutator periodically charges the condenser from a battery (of constant voltage) and discharges it through a galvanometer. The commutator is of the secohmmeter type; it is solidly built, and is driven by a directly coupled electric motor, the number of charges per second being determined by a counter geared to the spindle and a stop-watch or chronograph. The commutator consists of three toothed metal discs mounted on the same spindle, but all insulated from one another, the arrangement of the teeth being shown in *Fig.* 29, which gives a view of a portion of the rim. The two discs a and b carry the working contact teeth, while the teeth of the middle disc (shown shaded) are only for the purpose of steadying a contact brush as the other teeth pass under it. This brush (not shown) is of brass gauze and is fixed centrally so as to make contact alternately with a and b as the spindle revolves. Two other similar brushes (c and d) make uninterrupted contact with the sides of a and b.

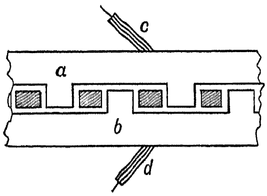


FIG. 29.—Portion of Commutator Rim (Fleming and Clinton).

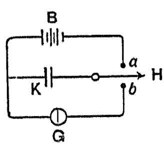


FIG. 30.—Commutator Direct Deflection Method (Fleming and Clinton).

Two systems of working are used:

System (1).—The connections are shown diagrammatically in *Fig.* 30, the commutator H alternately charging the condenser K, through the middle brush, from the battery B and discharging it through the galvanometer G, whose period is long compared with that of the charge and discharge. Accordingly, when the speed is kept steady, the intermittent current (of discharge) will give a steady deflection on the galvanometer. From the ordinary calibration of the galvanometer with steady currents, let this deflection be equivalent to a current I in microamperes.

Then $$K = \frac{I}{nV} \quad (\mu F), \quad . \quad . \quad (107)$$

where V is the voltage of the battery (in volts) and n the frequency of charge.

A moving coil galvanometer only should be used, for reasons to be explained later. The second system is a better one and does not require this restriction.

To ascertain if the condenser has any leakage sufficient to affect the results, the galvanometer is put in series with the battery and a short-circuiting wire put in its place. The galvanometer will now measure the charging current instead of the discharge. If the value of the capacity obtained with the new connections agrees with the former test, there is no error due to leakage.

System (2).—The connections are shown in *Fig.* 30A. Here G and G′ are the equal-acting coils of a special differential galvanometer. Coil G, shunted by S, with a high resistance R in series receives a steady current from the battery B, while the commutator (represented by H), by charging and discharging K, sends an interrupted current in the opposite direction through G′. By adjusting R and S a balance is obtained on the galvanometer, in which case

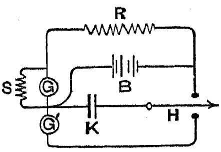


FIG. 30A.—Differential Commutator Method (Fleming and Clinton).

$$K = \frac{(G+S)10^6}{n[R(G+S)+GS]} \quad (\mu F). \quad . \quad (108)$$

As the ordinary type of differential galvanometer with the coils G, G′ wound together does not insulate these coils sufficiently from one another for this method, special galvanometers were constructed. One of these was of moving coil type with the coils G and G′ one above the other, rigidly connected, and on the same suspension, but in separate magnetic fields. Their sensitivities were made equal by adjusting one of the magnetic fields by means of a magnetic shunt of soft iron facing the poles of the magnet.

Leakage can be detected in the same way as in System (1), which is a great advantage in this method. The differential galvanometer method was first used by Klemenčič at Vienna in 1884, and after him by many others. It has recently been used by Rosa and Dorsey[2] in testing guard ring condensers. Their commutator had an extra brush, through which the guard ring was separately charged.

§ (41) MAXWELL'S COMMUTATOR BRIDGE METHOD.—Maxwell[3] described a method in which a reversing commutator periodically

[1] J. A. Fleming and W. C. Clinton, *Phil. Mag.*, 1903, v. 493; and *Phys. Soc. Proc.*, 1903, xviii. 386. See Werner Siemens, *Pogg. Ann.*, 1857, cii. 66.

[2] E. B. Rosa and N. E. Dorsey, *Bureau of Standards Bull.*, 1907, iii. 541.

[3] J. Clerk Maxwell, *Electricity and Magnetism*, 2nd Edition, vol. ii. §§ 776 and 777.

charges and discharges a condenser in one arm of a Wheatstone's bridge, in which the other arms are resistances. The method, with slight modifications of detail, has been used with great success by many different

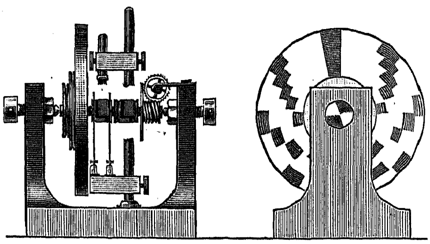


FIG. 31.—Thomson and Searle's Commutator.

experimenters. Instead of a reversing commutator, usually one which merely charges the condenser in the bridge arm and then short-circuits it has been employed. In the earlier experiments the commutator consisted of a vibrator working between two contact pieces, driven by intermittent current from an electrically maintained tuning fork.[1] The vibrating contact maker had certain disadvantages and was replaced by a revolving commutator by Glazebrook [2] and also by Fleming. The revolving type is now in general use. In *Fig.* 31 is illustrated the simple form used by Thomson and Searle [3] in 1890, and in *Fig.* 32 the more elaborate

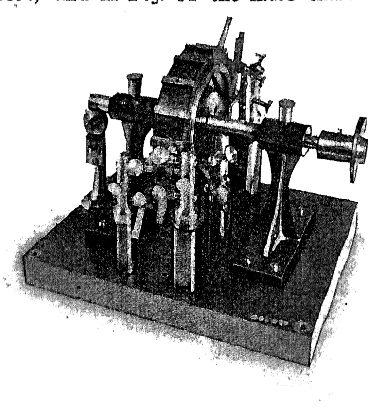


FIG. 32.—Rotating Commutator (Rosa and Dorsey).

type employed by Rosa and Dorsey[4] in 1907. These are both of the three-brush type, similar in principle to that shown in *Fig.* 29, but without the shaded teeth. The middle brush requires careful adjustment; it is advantageous to deaden its vibration by means of india-rubber. For the purpose of determining the speed a small toothed wheel is geared into a screw-thread on the spindle, and closes an electric contact for every fifty or hundred revolutions of the latter. By connection with a chronograph a dot is marked on the recording cylinder exactly at the end of every fifty or hundred revolutions, while at the same time signals every second from a standard clock are also inscribed.

The bridge connections are shown in *Fig.* 33, the middle brush of the commutator being represented by c which is touched alternately by a and b, the first contact introducing K into the bridge arm and the second short-circuiting it.

Let the resistances of the arms be Q, R, S, G, and B, as marked in the figure, and let n be the frequency of charge. By proper choice of R and S and by adjusting Q the deflection of the galvanometer is brought to zero, and then [5] we have

$$K=\frac{S}{nQR}\left\{\frac{1-\dfrac{S^2}{(S+R+B)(S+Q+G)}}{\left(1+\dfrac{SB}{Q(S+R+B)}\right)\left(1+\dfrac{SG}{R(S+Q+G)}\right)}\right\}, \quad (109)$$

where K is in farads if the resistances are in ohms.

This may be written

$$K=\frac{1}{nQ}\cdot\frac{S}{R}\cdot F, \quad . \quad . \quad (110)$$

and by proper choice of the various resistances F may be made a mere correcting factor nearly equal to unity. The battery resistance B can always be kept very small by the use of secondary cells. The ratio R/S should be large, say 1000/10 or 10,000/10, and the galvanometer has usually a moderately high resistance (*e.g.* 500 ohms). When these conditions hold, the correction due to F will in general be less than 1 per cent. It is convenient to tabulate its value for various values of Q and for several values of R and S.

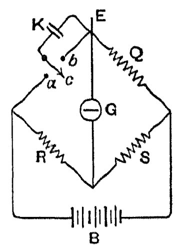


FIG. 33.—Maxwell's Commutator Method.

The resistance in the condenser circuit must be kept as small as possible, as otherwise a correction would have to be introduced due to the condenser not being fully charged in

[1] J. J. Thomson, *Roy. Soc. Phil. Trans.* A, 1883, clxxiv. 707; R. T. Glazebrook, *Phil. Mag.*, 1884, xviii. 98.
[2] R. T. Glazebrook, *Brit. Assoc. Rept., Leeds*, 1890.
[3] J. J. Thomson and G. F. C. Searle, *Roy. Soc. Phil. Trans.* A, 1890, clxxxi. 583.
[4] E. B. Rosa and N. E. Dorsey, *Bureau of Standards Bull.*, 1907, iii. 541.

[5] J. J. Thomson, *Roy. Soc. Phil. Trans.* A, 1883, clxxiv. 707.

each period (see Maxwell, *loc. cit.*). After a test has been made, the condenser is disconnected from its leads and a similar measurement made of the effective capacitance of the leads and commutator. The value obtained is subtracted from that obtained with the condenser, the result giving the capacitance of the condenser alone.

It is best to earth the point E, the outer conductor of the condenser being also connected to that point.

The galvanometer should have a long period of oscillation with good damping. One of moving coil type with a coil having considerable moment of inertia is suitable. A. Campbell found that, when a moving-magnet galvanometer is used, considerable errors may occur unless when at the zero reading the magnet needle is exactly at right angles to the axis of the deflecting coils. The theory of the method assumes that the deflection will be zero when the total electric quantity passing through the galvanometer in each period is zero, that is when the mean current is zero. But if the magnet needle is not perpendicular to the deflecting field, the magnetisation of the needle will not remain constant, but will to some extent rise and fall with the variation of the instantaneous value i of the deflecting current. Thus, instead of the deflecting torque being proportional to i, a certain component of it will be proportional to i^2, and then the zero balance will not be a correct indication that $i_{\text{mean}}=0$. When this source of error is present, reversing the battery gives a change of reading for K, and by taking a mean of the two readings partial correction may be obtained. The effect also may be reduced by shunting the galvanometer with a condenser. In any case it is much better to use a moving coil galvanometer, and even with it care should be taken that the coil is in a position of symmetry for the zero reading, for the square law effect is noticeable in some types of moving coil instruments.

§ (42) MAINTENANCE OF CONSTANT SPEED.—As will be seen from equation (109) the zero balance is for a definite value of the frequency n. If the value of K is constant and the resistances are correct for a given value of n, then if the speed of the commutator is above or below the correct value, the galvanometer will be deflected to the one or the other side of zero. It is therefore necessary to have some means of holding the speed constant. The earlier experimenters regulated it by the help of an electrically driven tuning-fork whose prongs carried screens with overlapping window-slits. A stroboscopic disc on the commutator spindle (as shown in *Fig.* 31) was observed through these slits, and the speed was regulated so as to keep one of the sets of marks on the disc apparently motionless. The regulation was done by the observer applying slight friction with the fingers to the flywheel of the commutator or its driving cord.

About 1902 Campbell introduced a simpler and better method at the National Physical Laboratory. The resistances are first set at appropriate values and the observer controls the speed by simply observing the galvanometer light spot and holding it at zero by friction on the flywheel or other means. The driving motor is set to run normally a little too quick. [The actual speed is determined by a chronograph as already described, the length of run necessary being determined by the accuracy required. For example, if the chronograph can be read to 0·05 second, a run of at least 10 minutes would be required to give accuracy of reading to 1 in 10,000.]

The above method of speed control is not only useful in testing condensers, but affords what is probably the most accurate known system of maintaining a speed constant. At the National Physical Laboratory it has been put to a number of different applications, such as the determination of the frequencies of tuning-forks or of high-frequency sparks, the speed of rotation in the Lorenz method for the ohm, etc. It is only necessary to fix a small commutator on the spindle whose speed is to be held, and to connect it up with a good mica condenser (kept at constant temperature) in a suitable bridge as above described. The accuracy attainable depends on various things, such as the sensitivity of the galvanometer, the inertia of the flywheel, and the constancy of the condenser and the resistance coils used. As the galvanometer must be of relatively long period, small quick changes of speed will not show much on the deflection, and the absolute degree of steadiness must not be estimated too rashly from the steadiness of the deflection.

In similar tests Giebe uses a special automatic speed regulator, which is described in § (6) of the article on "Inductance Measurement."

Effect of Change of Frequency.—If the condenser tested has dielectric loss and capacitance which varies with frequency, then in general a test by the commutator method with frequency n will not give the same result as a test in which sine wave alternating current of frequency n is employed, for in the commutator method the curve of applied voltage is by no means of sine wave form. Campbell[1] attempted to find an empirical relation, but the subject requires further investigation.

§ (43) COMMUTATOR METHOD WITH GUARD-RING CONDENSERS.—When the condenser to be tested has a guard ring the commutator may be duplicated or a second set of brushes added. Rosa and Dorsey ensured that the

[1] A. Campbell, *Roy. Soc. Proc.* A, 1912, lxxxvii. 410.

guard ring should be charged to the same potential as the condenser plate by putting its commutator in a bridge network across the same battery and identical with the main bridge.

(B) *Comparison with another Capacity*

§ (44) INTRODUCTORY.—The older methods of comparing the capacities of two condensers nearly all made use of only a single charge and discharge. By and by, to increase the sensitivity, commutators of the secohmmeter type were introduced so as to obtain a steady condition of periodic charge and discharge (or reversal), still using a direct current source. But since alternating currents have become common in everyday electrical work, the methods have been modified so as to be used with alternating current, and have in this way been rendered on the whole more convenient, more definite, and more sensitive.

§ (45) BALLISTIC GALVANOMETER METHOD.—When a condenser is discharged through a moderate resistance, the greater part of the quantity held by it passes out in a small fraction of a second. To measure the quantity which enters or leaves it at charge or discharge, the most commonly used instrument is a *ballistic galvanometer*,[1] which is simply a galvanometer which has such a slow time of swing that practically the whole charge or discharge quantity has passed through it before the moving part has been appreciably deflected from its zero position. If the angles moved through are small the throw (or kick) of the moving part is very approximately proportional to the quantity that has passed. Thus if q is the quantity (in coulombs) and D the throw (in angle or scale divisions), then

$$q \doteq \mathrm{AD}, \quad . \quad . \quad . \quad (111)$$

where A is the ballistic constant.

If a current of i amperes gives a steady deflection δ, then $i \doteq a\delta$, where a is the steady-current constant of the galvanometer.

Then A and a are connected by the relation

$$Aa \doteq \frac{\mathrm{T}}{2\pi}\left(1+\frac{x}{2}\right), \quad . \quad . \quad . \quad (112)$$

where T is the complete period of the galvanometer (in seconds), and x is the logarithmic decrement of its swings *for one half-period*. [If A has been determined from equation (112), the capacity of a condenser can be found by charging it with a known voltage V and discharging it through the galvanometer. Then $\mathrm{K}=\mathrm{AD}/\mathrm{V}$.]

However, instead of assuming exact proportionality of the scale readings, it is best to calibrate the scale once for all either with known values of i or of q.

To compare the capacities of two condensers each is charged in turn from the same battery and discharged through the galvanometer. Then if the capacities are K_1 and K_2

$$\mathrm{K}_2=\frac{\mathrm{K}_1\mathrm{D}_2}{\mathrm{D}_1}, \quad . \quad . \quad . \quad (113)$$

where D_1 and D_2 are respective throws.

A correction should be applied if necessary from the calibration of the scale.

If the condensers are very different in capacitance, one of the throws will be too small to observe with accuracy, in which case either of the following methods may be used:

(*a*) The condensers are charged with different voltages V_1 and V_2 so that the throws on discharge are nearly equal. The ratio $\mathrm{V}_1/\mathrm{V}_2$ is determined by voltmeter or potentiometer measurement.

Then $$\mathrm{K}_2=\frac{\mathrm{K}_1 . \mathrm{V}_1\mathrm{D}_2}{\mathrm{V}_2\mathrm{D}_1}. \quad . \quad . \quad (114)$$

(*b*) The galvanometer is shunted on the Ayrton-Mather[2] system (which here gives constant damping), and the condensers are charged on the same battery, but the shunt is set in each case so as to make the two discharge throws as nearly equal as possible.

Then $$\mathrm{K}_2=\frac{\mathrm{K}_1 . b_2\mathrm{D}_2}{b_1\mathrm{D}_1},$$

where b_1 and b_2 are the multiplying powers of the shunt in the two cases.

Thus an unknown capacitance can be determined by the use of a standard condenser.

The system of the Ayrton-Mather shunt is shown in *Fig.* 34. A fixed resistance AC is put across the galvanometer terminals. The working terminals are P and Q, and the shunting power is varied by altering the position of B. Thus $(s+r+g)$ is kept constant, which gives constant damping when P and Q are an open circuit.

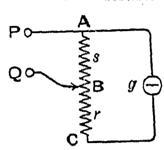


FIG. 34.—Ayrton-Mather Shunt.

For convenience of working the galvanometer should be well damped, and its complete period should, if possible, be about 10 seconds to allow sufficiently deliberate observation of the throws.

The great difficulty in all tests of this kind arises from absorption (see § (8)). When this is present, as it is in all but air condensers, although the greater part of the charge rushes in very quickly when the charging voltage is applied, a small additional amount goes on creeping in, as it were, for a considerable time as long as the charging voltage is maintained. In a similar way for the discharge, after the first rush, in which the free charge quickly comes out, the absorbed charge continues to dribble out for a long time at a gradually

[1] See the article on "Magnetic Measurements."

[2] W. F. Ayrton and T. Mather, *Electrician*, 1893-1894, xxxii. 627.

lessening rate. Thus the value of the capacitance measured by the charge or discharge depends on the amount of time during which either charge or discharge is allowed to continue, and any time elapsing between charge and discharge will allow continued absorption and also affect the result. Thus the discharge is not really quite complete before the galvanometer is appreciably deflected, and so the slow part of the discharge does not have its full effect, and the total quantity indicated depends on the period of the galvanometer used. As an example, *Fig.* 35 shows the effect of varying the time of charge of a standard mica condenser,[1] the variation in apparent capacitance being measured by a galvanometer of 8 seconds' period. It will be noticed that the total variation is of the order of 0·6 per cent.[2] Other materials, such as crown glass or india-rubber, show very much larger absorption effects than this. In all cases, therefore, where capacity is tested by single charge and discharge, it is important that the time conditions should be definitely specified.

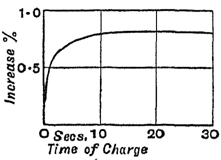


FIG. 35.—Variation of Apparent Capacitance with Time of Charge.

§ (46) THOMSON METHOD OF MIXTURES.—The principle of Thomson's[3] method of comparing two capacities is as follows. If two condensers K and C are charged by different voltages V_1 and $-V_2$ so that their charges are Q and $-Q$ respectively, then, if their corresponding terminals are connected, the charges will annul one another and the condensers will be found to be completely discharged. When this result is obtained

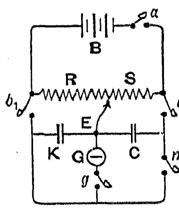


FIG. 36.—Thomson Method of Mixtures.

$$KV_1 = Q = CV_2, \quad \text{or} \quad \frac{K}{C} = \frac{V_2}{V_1}.$$

The connections are shown in *Fig.* 36. At least one of the resistances R and S is adjustable; often they are arranged with a central slider so that the ratio R/S can be varied while (R + S) remains constant. The point E is earthed.

To make the test:

(1) Key *a* is closed, sending a current through R and S, and producing voltages V_1 and V_2 across these resistances, such that $V_1/V_2 = R/S$.

(2) Next, the keys b_1 and b_2 are closed for a definite period and then opened, leaving K and C charged to voltages V_1 and $-V_2$ respectively.

(3) Then the key *m* is closed (for a definite time), which allows the condenser charges to "mix"; and finally the galvanometer key *g* is closed. If a throw is produced on the galvanometer, the ratio R/S is altered and the procedure repeated until by trial the value of R/S is found for which the galvanometer throw is zero. When the galvanometer thus indicates no charge, the condensers have been exactly discharged by the mixing, and then we have

$$K = \frac{CS}{R}. \quad . \quad . \quad . \quad (115)$$

This method has been very extensively used in the testing of electric cables[4] both in the factories and at cable stations. Special keys are generally used to simplify the working of the test. In comparing moderate capacities, Dr. Muirhead recommends a charge of 15 seconds and mixing for 4 seconds. With a long submarine cable of 2000 km. length (about 300 μF capacity), the charge may be for 5 minutes and the mixing for 10 seconds.

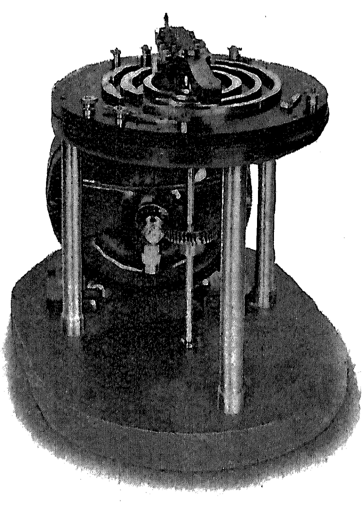


FIG. 36A.—Curtis's Switch for Cyclic Method of Mixtures.

The method has been used by Muirhead, Glazebrook,[5] and others in comparing standard mica condensers against air condensers, and the difficulties due to absorption have been investigated. In order to bring the conditions into a cyclic state Curtis[6] employs a motor-driven switch (shown in *Fig.* 36A) which auto-

[1] Tested at N.P.L. by A. Campbell, 1903.
[2] See A. W. Porter and D. K. Morris, *Roy. Soc. Proc.*, 1895, lvii. 469, and A. Zeleny, *Phys. Rev.*, 1906, xxii. 65.
[3] W. Thomson (Lord Kelvin), *Soc. Tel. Eng. J.*, 1873, i. 397.

[4] See J. Elton Young, *J. Inst. El. Eng.*, 1899, xxviii. 475; also see *Electrician*, 1892, xxviii. 361.
[5] *Electrician*, 1890, xxv. 487, 637.
[6] H. L. Curtis, *Bureau of Standards Bull.*, 1910, vi. 431.

matically performs the operations of the test in a periodic manner until approximately steady conditions are reached.

In all cases when the point E is earthed the battery must be very well insulated.

§ (47) Gott's Method of Comparison.—Another method very largely used in cable work is that of Gott,[1] the connections for which are shown in *Fig.* 37. By closing the key a for a definite time the condensers are charged in series, and the ratio R/S is altered by the slider f until, on closing key b, the galvanometer shows no throw.

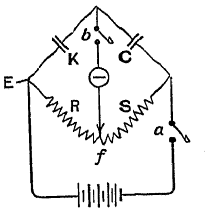


Fig. 37.—Gott's Method of comparing Capacities.

Then $$\frac{\mathrm{K}}{\mathrm{C}}=\frac{\mathrm{S}}{\mathrm{R}}$$

The battery need not be highly insulated, as the point E is earthed. When the condensers are not similar in absorption, difficulties arise just as in the Thomson method. In general the balance on the galvanometer is never perfect, a slight quick kick and a slow drift usually occurring together.

If the galvanometer and battery are interchanged in Gott's method it becomes De Santy's method.

§ (48) Bridge with Alternating Current.—If in one of these last methods (*e.g. Fig.* 37) the battery is replaced by a source of alternating current and the ballistic galvanometer by detector of alternating current, such as a vibration galvanometer or telephone, in most cases it will be found impossible to obtain a balance, and at best only a minimum can be obtained on the detecting instrument. The reason for this is that, unless K and C are both free from absorption and leakage, the impedances of the condenser arms contain both reactance and resistance components. If the leakage and other losses in the condensers are represented by resistances p and q in parallel with them, we have the arrangement shown in *Fig.* 38. The symbolical impedances of the condenser arms are $p/(1+j\omega p\mathrm{K})$ and $q/(1+j\omega q\mathrm{C})$ respectively, and hence, if the bridge balances,

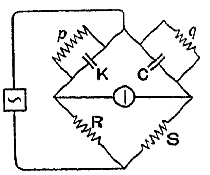


Fig. 38.—Bridge with Leaky Condensers.

$$\frac{p/(1+j\omega/\mathrm{K})}{q/(1+j\omega q\mathrm{C})}=\frac{\mathrm{R}}{\mathrm{S}}$$

or $$\mathrm{S}p(1+j\omega q\mathrm{C})=\mathrm{R}q(1+j\omega p\mathrm{K}),$$

[1] J. Gott, *Soc. Tel. Eng. Proc.*, 1881, x. 278.

which gives the two conditions

$$\left.\begin{aligned}\frac{p}{q}&=\frac{\mathrm{R}}{\mathrm{S}}\\ \text{and}\qquad \frac{\mathrm{K}}{\mathrm{C}}&=\frac{\mathrm{S}}{\mathrm{R}}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (116)$$

Thus it is seen that, for a balance, the parallel resistances must be inversely proportional to the capacitances. In general, in comparing two condensers this condition will not be found to hold for the condensers alone, but a balance can always be got by shunting one or other of them by a resistance of the right value (which can be found by trial). If C (taken as the standard) has negligible losses, then $q=\infty$; and if a balance is got when C is shunted by a resistance Q, we have

$$\mathrm{K}=\frac{\mathrm{CS}}{\mathrm{R}}$$

and $$p=\frac{\mathrm{QR}}{\mathrm{S}}.$$

Thus, by equation (68),

$$\tan\phi=\omega\mathrm{K}p=\omega\mathrm{CQ}, \quad . \quad . \quad (117)$$

where $\cos\phi$=power factor of K.

For example, this method was used by Campbell[2] to determine the capacities and leakage resistances of cellulose condensers.

It is clear that an alternating current method of this kind not only may determine the capacity but also the power loss in the condenser (for given current or voltage). Indeed most of the alternating current methods give information regarding the power factor ($\cos\phi$) as well as the capacitance.

Grover,[3] whose paper on the subject contains very complete information, has discussed the more general case of this method, in which both of the condensers have internal loss, and in the bridge are shunted by external resistances P and Q to obtain a balance. Then, as before,

$$\mathrm{K}=\frac{\mathrm{CS}}{\mathrm{R}},$$

but the power factor of K (or its phase displacement θ) cannot be absolutely determined unless the power factor of the standard condenser C is known. The second condition for balance gives

$$\tan\theta_1-\tan\theta_2=\frac{1}{\omega\mathrm{CQ}}-\frac{1}{\omega\mathrm{KP}}, \quad . \quad (118)$$

where θ_1 and θ_2 are the phase displacements[4] of the condensers K and C. Since the power factor $\cos\phi=\sin\theta$, when θ_1 and θ_2 are small, equation (118) gives $(\theta_1-\theta_2)$, which is approximately the difference of the power factors of the condensers. If the unknown condenser K

[2] A. Campbell, *Roy. Soc. Proc.*, 1906, lxxviii. 196.

[3] F. W. Grover, *Bureau of Standards Bull.*, 1907, iii. 371.

[4] See § (14).

has a very small power factor the shunt required on the standard may be inconveniently high (*e.g.* megohms). It is therefore best, as Grover recommends, to begin with K shunted by 100,000 ohms. The use of high resistances is a weak point in the method, particularly when the capacitances of the condensers are small, for the self capacitances of the resistances may cause error. For this and other reasons the next method is to be preferred.

N.B.—The determination of the power factor is of the utmost importance, for it usually is a criterion of the goodness of a condenser. A condenser with small power factor shows little absorption and its capacity usually changes little with frequency.

§ (49) Wien's Series Resistance Method.—In Wien's [1] method the loss in the unknown condenser is balanced by putting a resistance in series with the standard condenser, which is assumed to have zero power factor. The connections are shown in *Fig.* 39, in which p represents the internal loss in the condenser K under test, and Q is an adjustable resistance in series with C, a standard condenser with negligible internal loss. When the vibration galvanometer (or telephone) shows no current,

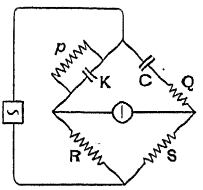


Fig. 39.—Wien's Series Resistance Method.

$$K=\frac{S}{R}\cdot\frac{C}{1+\omega^2C^2Q^2} \quad . \quad . \quad (119)$$

and
$$p=\frac{R}{S}\cdot\frac{1+\omega^2C^2Q^2}{\omega^2C^2Q}; \quad . \quad (120)$$

hence also
$$\omega^2KCpQ=1. \quad . \quad . \quad . \quad (121)$$

If $\cos\phi$ = power factor of condenser K, then

$$\cot\phi=\frac{1}{\omega Kp}=\omega CQ. \quad . \quad . \quad (122)$$

When $\cos\phi$ is small (which is usual in condensers),

$$\cos\phi \doteq \omega CQ \quad . \quad . \quad . \quad (123)$$

and
$$K \doteq \frac{SC}{R}. \quad . \quad . \quad . \quad (124)$$

[Wien used the method not merely to obtain the ratio of K to C, but also to determine both K and C in terms of known resistances and frequency. For this purpose, after making the test giving equations (119), (120), and (121), K is shunted with a known resistance P, and Q is altered to Q_1 to obtain a balance.

Then
$$\frac{\omega^2KCQ_1pP}{(p+P)}=1,$$

which with (121) gives
$$p=\frac{P(Q-Q_1)}{Q_1}.$$

By using this value of p, K and C can be got from equations (119) and (120).]

[1] M. Wien, *Wied. Ann.*, 1891, xliv. 689.

For the comparison the standard C may be an air condenser or a good mica condenser of known power factor. In the latter case the power factor of C will come into the result. It is better then (following Grover) to represent the losses in both the condensers by series resistances ρ_1 and ρ_2, and, in making the test, to use external series resistances P and Q as shown in *Fig.* 40.

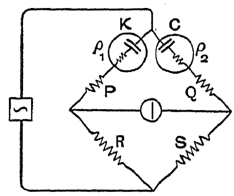


Fig. 40.—Series Resistance Method (Grover).

Then
$$\frac{K}{C}=\frac{S}{R}$$

and
$$\frac{P+\rho_1}{Q+\rho_2}=\frac{R}{S},$$

whence
$$\tan\theta_2-\tan\theta_1=\omega KP-\omega CQ$$
$$=\omega C\left(\frac{SP}{R}-Q\right). \quad (125)$$

As in other bridge methods, it is very desirable to use equal ratio arms (R=S); they can then be interchanged to eliminate want of equality and residual inductances. When very unequal arms are used (*i.e.* when the condensers are not nearly equal), the effects of earth capacities often introduce serious errors, especially with small condensers or at relatively high frequencies. The power factor results are much more affected than those of capacity.

§ (50) Wagner's Earthing Device.—The difficulties due to earth capacities are got over by an ingenious device due to Wagner.[2] As shown in *Fig.* 41, an auxiliary circuit is connected across the alternating source, consisting of a condenser k in series with two resistances r_1 and r_2. The main bridge is first approximately balanced and then, with the point A earthed, r_1 and r_2 are altered until a telephone T shows no current, thus indicating that A and B are at the same potential. The circuit of T is broken, and the main bridge rebalanced more accurately. Then the equality of potentials at A and B is again tried and r_1 and r_2

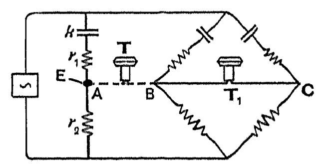


Fig. 41.—Wagner Earthing Device.

[2] K. W. Wagner, *Elektrotech. Zeits.*, 1911, xxxii. 1001: also *Electrician*, 1911, lxviii. 483.

readjusted if necessary. Finally the telephone T is removed, the point A is left earthed, and exact balance on the main bridge is obtained. It will be noticed that since B is at potential zero, when the telephone T_1 shows no current, C is also at zero potential, and so is the telephone, which can therefore be handled without introducing capacity currents to earth. The device ensures that a certain point of the network is virtually at earth potential though not actually connected to earth. It can be used for the same purpose in other methods.

§ (51) Wien's Method.—Wien's series resistance method has been used in a number of important researches. For example, Monasch[1] used it in testing the dielectric losses in glass, ebonite, paraffin, and various kinds of cables at high voltages (up to several thousand volts), using a special air condenser as standard. He showed that the power factors were practically constant over a long range of applied voltage, the losses being exactly proportional to the square of the voltage. Batman[2] used the method with voltages up to 10,000 volts, and employed as detecting instrument a special single-wire (Einthoven) vibration galvanometer in which the wire was of aluminium.

For most purposes the method is one of the best, if good standard subdivided condensers are available.

§ (52) Four-condenser Bridge.—Nernst[3] introduced a bridge in which two condensers were used instead of resistance ratio arms, thus having capacitance in each of the four arms. He used it with radio frequencies up to several million ∿ per sec.

Hertwig[4] used this type of bridge for the measurement of dielectric constants at 10^6 ∿ per sec., arranging it as shown in *Fig.* 42. To avoid as much as possible the inductance of leads, the bridge consists of four similar condensers, two pairs of them having a common plate. The unknown condenser x is put in parallel with K, and is balanced by inserting a sheet of glass between the plates of C. If x shows leakage, this is balanced by the adjustable high resistance Q, which consists of a specially constructed capillary tube containing electrolyte. The source is a quenched spark system, and the detecting instrument a crystal rectifier F and telephone T. Joachim[5] uses a similar bridge with a triode valve source (giving 10^6 ∿ per sec.) and a toothed-wheel interrupter in the telephone circuit.

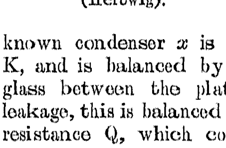


Fig. 42.—Nernst Bridge (Hertwig).

Fleming and Dyke[6] used a four-condenser bridge (*Fig.* 43) in which C, C_1, and C_2 are adjustable air condensers and K is the condenser under test. If p represents the internal loss of K, which is balanced by the series resistance Q, then

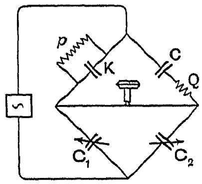


Fig. 43.—Fleming and Dyke's Method.

$$\frac{1}{p}=\omega^2CKQ \quad . \quad . \quad . \quad (126)$$

and

$$\frac{K}{C}+\frac{Q}{p}=\frac{C_1}{C_2}. \quad . \quad . \quad . \quad (127)$$

The condensers were of the order of 0·002 μF.

A high-frequency alternator with a wave-filter furnished a source of pure wave-form, and a telephone was used as detecting instrument.

Hopkinson[7] also employed a four-condenser bridge, but with a quadrant electrometer suitably connected as detecting instrument.

§ (53) Series Inductance Method.—Rosa suggested that instead of balancing the difference of phase for the two condensers by added resistances as in *Fig.* 40, a balance could be obtained by making one or both of the arms R and S inductive, thus avoiding any addition in the condenser arms. The method, as shown in *Fig.* 44, was carried out by Grover,[8] who remarks that it is not so convenient as the series resistance method for condensers over 0·01 μF. If the internal losses of the condensers are ρ_1 and ρ_2, we have

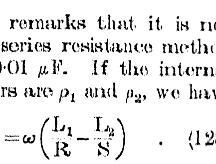


Fig. 44.—Series Inductance Method.

$$\tan(\theta_1-\theta_2)=\omega\left(\frac{L_1}{R}-\frac{L_2}{S}\right) \quad . \quad (128)$$

and

$$\frac{K}{C}=\frac{S}{R}+\omega^2K(L_1\rho_2-L_2\rho_1)$$

$$=\frac{S}{R}\left(1+\frac{\omega L_1}{R}-\frac{\omega L_2}{S}\right). . \quad . \quad (129)$$

Substitution Method.—In both this method and the preceding ones it is sometimes advantageous

[1] B. Monasch, *Ann. d. Physik*, 1907, xxii. 905, and *Electrician*, 1907, lix. 416, 460, 504.
[2] C. A. Batman, *El. World*, 1918, lxxi. 502.
[3] See F. Kohlrausch, *Lehrbuch d. prakt. Physik*, 10th ed., 1905, p. 576; also W. Nernst, *Wied. Ann.*, 1897, lx. 600, and W. Nernst and v. Lerch, *Ann. d. Physik*, 1904, xv. 836.
[4] W. Hertwig, *Ann. d. Physik*, 1913, xlii. 1099.
[5] H. Joachim, *Ann. d. Physik*, 1919, lx. 570.
[6] J. A. Fleming and G. B. Dyke, *J. Inst. El. Eng.*, 1912, xlix. 323.
[7] J. Hopkinson, *Roy. Soc. Proc.*, Oct. 1887.
[8] F. W. Grover, *loc. cit.*

to use a substitution method, testing each of the condensers to be compared against a third.

§ (54) DIFFERENTIAL TRANSFORMER METHODS.—Various experimenters [1] have used a differential transformer (inductor), instead of a bridge, for the comparison of condensers. For further information see the article on "Inductance Measurement," § (94).

§ (55) M'CLELLAND'S IONISATION CURRENT METHOD.—M'Clelland [2] has described a simple method of comparing capacities of any magnitude down to a few micromicrofarads or even less. It is based on the fact that the ionisation current that can be got by the use of a radio-active substance like uranium nitrate is extremely constant, and can be made so small that the time taken to charge a condenser by it can be accurately measured. First one of the condensers is charged up to a given voltage by the small constant current, and then the other is in the same way charged to the same potential difference. The time taken is observed in each case, and from the times the ratio of the capacitances may be deduced. *Fig.* 45 shows the arrangement of the apparatus.

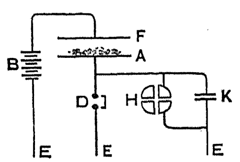


FIG. 45.—M'Clelland's Ionisation Current Method.

A few grams of uranium nitrate are spread on a sheet of paper and placed on a metal plate A, over which is another plate F connected to one pole of a battery B, the other pole of which is earthed. K is one of the condensers, and H is an electrometer across its terminals. The points marked E are earthed. As the voltage across FA is increased, at first the ionisation current increases, but when the voltage is great enough the current comes to a maximum and does not increase further. The voltage required to reach this steady current depends on the distance apart of the plates. It is advisable, however, to be able to reach 200 volts with the battery. To make the test, the earthing switch D, which at first is kept closed, is opened and the time observed which is taken by the electrometer light spot to move a distance of say 100 divisions on the scale. The same process is repeated with the other condenser, and to gain accuracy each may be tested a number of times. If the two condensers have capacitances K_1 and K_2, and if C is the capacitance between F and A, including the electrometer and the leads, then, t_1 and t_2 being the respective observed times,

$$\frac{K_1+C}{K_2+C}=\frac{t_1}{t_2}.$$

To determine C a reading is also taken with the electrometer alone. If t_3 is the time in that case, we have

$$\frac{K_2+C}{C}=\frac{t_2}{t_3},$$

which determines C in terms of K_2 taken as standard. For small condensers an ordinary Kelvin electrometer giving say 60 mm. deflection for 1 volt is suitable; for larger condensers a Dolezalek instrument (giving 5 mm. per millivolt) may be used. The space between the plates A and F should be screened to prevent air currents from blowing away the ionised air. It is better to use uranium than radium or thorium, as it gives off no emanation.

(C) *Measurement in Terms of Mutual Inductance*

§ (56) CAMPBELL'S SIFTER METHOD.—For the determination of a capacity in terms of mutual inductance Campbell's sifter method [3] is the simplest. Two cases arise.

Case (i.) *Condenser free from Loss.*—When the condenser to be tested has zero (or very small) power factor, it is connected with an adjustable mutual inductance m, an alternating source and a vibration galvanometer (or telephone) as in *Fig.* 46. The proper directions to connect the primary and secondary coils of m are found by trial.

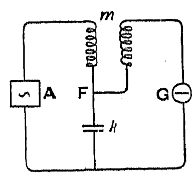


FIG. 46.—Campbell's Sifter Method.

By adjusting m the current in the galvanometer circuit can be made zero, and then

$$k=\frac{1}{\omega^2 m}, \quad . \quad . \quad . \quad (130)$$

where the pulsatance $\omega=2\pi n$, n being the frequency of the alternating source. The capacitance is thus measured in terms of m and the frequency. The frequency may be determined by means of a frequency meter or by measuring the speed of the alternator. When the condenser has power loss, a perfect balance cannot be obtained, only a minimum current being observed on the galvanometer, and the less simple method of Case (ii.) must be used. With good mica condensers, however, Case (i.) is often sufficient.

Case (ii.) *Condenser with Power Loss.*—There are various ways of extending the simple method of Case (i.) so as to obtain an accurate

[1] A. Elsas, *Wied. Ann.*, 1891, xliv. 654.
[2] J. A. M'Clelland, *Roy. Dublin Soc. Proc.*, 1904, p. 168.
[3] A. Campbell, *Phys. Soc. Proc.*, 1908, xxi. 69, and *Phil. Mag.*, 1908, xliv. 155; also *Phys. Soc. Proc.*, 1917, xxix. 350. See also "Inductance, Measurement of," § (84).

balance with an imperfect condenser. *Fig.* 47 shows one of the easiest systems.

In *Fig.* 47 the internal loss in k, the condenser under test, is represented by the series resistance r, m is an adjustable mutual inductance (inductometer), while M_1 and M_2 are mutual inductances whose secondary coils form a closed loop of resistance R and self-inductance L. The coils should be so placed that there is no direct mutual inductance between the primary circuit and the galvanometer circuit other than m. The coils forming M_1 and M_2 should therefore be placed at a good distance from the inductometer m and be turned so as to be conjugate to each other pair and to the inductometer coils.

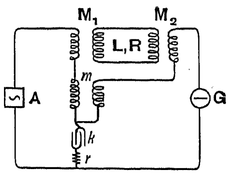


FIG. 47.—Campbell's Method, with Compensation for Condenser Losses.

The current in G can be reduced to zero by adjusting m and M_1 *or* M_2, and then

$$\frac{1}{k\omega^2}=m+\frac{rL}{R} \quad . \quad . \quad . \quad (131)$$

and $$Rr=\left[M_1M_2+\left(m-\frac{1}{k\omega^2}\right)L\right]\omega^2. \quad (132)$$

Hence $$Rr=\left[M_1M_2-\frac{L^2r}{R}\right]\omega^2. \quad . \quad (133)$$

Here M_1M_2 must be greater than L^2r/R, except in the limiting case of a perfect condenser, when $r=0$ and then also $M_1M_2=0$.

From (131) and (132) we obtain

$$r=\frac{RM_1M_2\omega^2}{R^2+L^2\omega^2} \quad . \quad . \quad . \quad (134)$$

and $$\frac{1}{k\omega^2}=m+\frac{LM_1M_2\omega^2}{R^2+L^2\omega^2}. \quad . \quad . \quad (135)$$

When the frequency (and hence ω) is known, these two equations give k and r and $rk\omega$, the power factor of the condenser.

It is best to arrange the auxiliary closed circuit so that L/R is very small, and hence rL/R negligible compared with m. Then by equation (131)

$$k=\frac{1}{\omega^2 m},$$

the same as equation (130) for a perfect condenser. This does not involve a knowledge of r, L, R, M_1, or M_2, and the auxiliary loop circuit now merely introduces a small vector to balance ri_1 without appreciable effect in equation (130).

Power Factor ($rk\omega$).—Also, when L/R is small,

$$r \doteq M_1M_2\omega^2R, \quad . \quad . \quad . \quad (136)$$

and here L need not be known accurately but M_1M_2 must be determined. This can be done directly by adding a known resistance s to the condenser circuit and altering R to R′ to restore the balance.

Then $$M_1M_2\omega^2=\frac{s}{1/R'-1/R}. \quad . \quad . \quad (137)$$

When M_1M_2 is to be determined in this way the loop (LR) may be merely placed over the m inductometer coils if desired.

To illustrate the relative values of the various resistances and inductances required, let us consider the case of measuring the power factor (0·0005) of a good mica condenser of capacitance 0·1 μF at a frequency of 800 ∿ per sec. Since $\omega \doteq 5000$, we have $r=1$ ohm and $m=0{\cdot}4$ henry. For $R=10$ ohms, $M_1M_2 = 4\times10^{-6}$, and so M_1 and M_2 may each be 2 millihenries.

In most cases, however, the Carey Foster method (§ (59)) is the best for the determination of power factor.

§ (57) MEASUREMENT OF FREQUENCY BY CAMPBELL'S METHOD.—Campbell's method affords a very accurate way of measuring frequency, and he has introduced frequency meters working on this principle, in which the inductometer scale is graduated to read the frequency directly or with integral multipliers corresponding to a suitable series of condensers arranged in the instrument. With a long-range inductometer and condensers tested against it by Carey Foster's method, the frequency can be determined with very high accuracy. The value obtained gives the frequency at the instant of balance (which can be set very rapidly), and not the mean frequency over an interval of time such as many other methods give.

For the higher ratio frequencies a telephone forms a convenient detecting instrument for this method. Strong harmonics, if present in the wave-form of the source of current, may somewhat obscure the point of balance, but if the primary self-inductance in the inductometer is kept relatively high, the harmonics cause very little trouble from 200 up to 5000 ∿ per sec.

§ (58) WAVE-FORM SIFTERS.—In *Fig.* 46 it has been seen that if the source gives a current of frequency n, when m (or k) is so adjusted that $mk\omega^2=1$, no current will pass round the circuit G, and this is entirely independent of anything (resistance, inductance, etc.) included in the circuit G. If the current in the first circuit (of A) contains a harmonic of frequency n, then this harmonic will be entirely suppressed in the second circuit. Thus the combination of capacitance and mutual inductance can be used as a wave-form sifter [1] to prevent

[1] A. Campbell, *Phys. Soc. Proc.*, 1912, xxiv. 107.

current of any one desired frequency from passing into a circuit, while allowing currents of all other frequencies not too near n to pass. Other methods in which the balance depends on frequency (*e.g.* that of § (61)) may also be used for this purpose.

§ (59) Carey Foster's Method. (i.) *Theory.*—For the determination of capacitance and condenser power factor by the help of mutual inductance the best system appears to be the method of Carey Foster with Heydweiller's modification.[1] The connections for the most general case are shown in *Fig.* 48, in which K is the condenser to be tested and s represents its internal power loss. R and S are approximately non-inductive resistances having residual self-inductances l and λ respectively. The mutual inductance M may be of fixed value or variable (an inductometer), and P and L are the total resistance and self-inductance of the branch BC, which consists of the secondary coil of the mutual inductance with an added resistance. G is either a vibration galvanometer or a telephone.

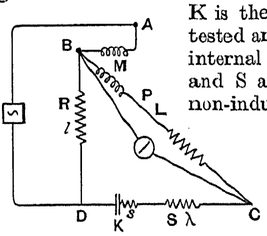


Fig. 48.—Carey Foster Method, Heydweiller's Modification.

In the method as originally described by Carey Foster[2] the branch DC contained only the condenser K, the resistance S being absent, and single make and break (or reversal) of a battery current was used, the condition of balance being that the total quantity passing through the galvanometer should be zero, in which case

$$K=\frac{10^6M}{PR}, \quad . \quad . \quad . \quad (138)$$

where K is in μF when M is in henries and P and R in ohms.

If the condenser is leaky there will be a small steady deflection before the current is reversed, which introduces error.

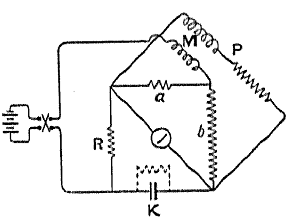


Fig. 48a.—Carey Foster Method, Campbell's Modification.

Campbell (in 1903) got rid of this source of error by the system shown in *Fig.* 48a, in which the resistance a is very small compared to b. By altering the ratio a/b the steady deflection is reduced to zero before the reversal throw is annulled[3] by altering M or P. For the latter balance

$$K=10^6\frac{M}{PR}\left(\frac{a+b}{b}\right)^2. \quad . \quad (138\text{A})$$

For balance at every instant, L should be equal to M.

The original method, however, gives no balance with alternating current. Heydweiller[4] showed that it requires the addition of a resistance S in the condenser branch, which allows two necessary conditions to be satisfied.

When a balance has been obtained (in *Fig.* 48),

$$\frac{10^6M}{K}=PR-\omega^2lL+\omega^2(l+\lambda)M \qquad (139)$$

and

$$M(S+s)=R(L-M)+Pl. \quad . \quad (140)$$

From these two equations K and $(S+s)$ can be determined, and, since S is known, s and hence the power factor ωKs can be found.

Unless ω is large, the terms in l and λ can usually be neglected in (139), which becomes

$$K=\frac{10^6M}{PR}, \quad . \quad . \quad . \quad (141)$$

the same equation as (138).

Sometimes the term in l may be neglected in equation (141), which becomes

$$S+s=\frac{R(L-M)}{M}. \quad . \quad . \quad (142)$$

However, for good condensers in which s is small, to obtain accuracy S should also be made small, and Pl may not be negligible.

(ii.) *Working Conditions.*—In carrying out the method it is best to make R a fixed standard resistance (oil-cooled) of very small residual self-inductance. The final balance may be obtained either (1) by adjusting S and M, or (2) by adjusting S and P.

Case (i.) R *and* P *Fixed,* S *and* M *Variable.*—It will be noticed that P consists of the resistance of the secondary coil of the inductometer (M) in series with an added resistance. It is best to swamp the copper resistance of the secondary coil by making the added resistance very much greater, so that temperature variation may not introduce uncertainty. For the highest accuracy the values of R and P should be measured immediately after a test is made.

The values of R and P should be so chosen as to make PR some integral power of 10. Then K is obtained directly from the observed value of M.

After the reading has been obtained with the condenser in position, the leads should be disconnected from its terminals, and, without disturbing their position, a reading should be taken of their capacitance k. Then the true capacitance of the condenser is $(K-k)$.

Sometimes it is found that no adjustment of S and M will give a balance. Often this is

[1] See also "Inductance, Measurement of," § (85).

[2] G. Carey Foster, *Phil. Mag.*, 1887, xxiii. 121.

[3] Equation (138a) is due to R. Ll. Jones.

[4] A. Heydweiller, *Wied. Ann.*, 1894, liii. 499.

due to L being less than M, in which case a coil must be added in the arm BC to make sure that L is greater than M, otherwise equation (140) cannot usually be satisfied. A difficulty also may arise when M is very small compared with L, for then S may have to be inconveniently large. A lower value of R is then wanted.

A long-range inductometer giving from 0 up to 10 millihenries is suitable for M. In general it is best to earth the point A (*Fig.* 48).

The use of a variable M is extremely convenient in giving a long range of direct-reading measurement, but it has one disadvantage which may sometimes trouble the observer: the adjustments of M and S are not independent and have to be repeated alternately until an *exact* balance is obtained. If it is not perfect, the balance may be a fictitious one corresponding to values of K and S very far from the truth. With a telephone as detector and a current source containing harmonics, in general when the balance is true the harmonics will be silenced almost exactly along with the fundamental (if the condenser is of good quality); if this does not happen, it is often an indication that the approximate balance is fictitious.

Case (ii.) R *and* M *Fixed,* S *and* P *Variable.*—When Pl can be neglected in equation (140), the adjustments of S and P are independent and there is no danger of a fictitious balance. In all other respects, however, the system of Case (i.) is much more convenient.

The modified Carey Foster method has many advantages. With a single inductometer and a few resistance coils to give a series of values of PR, capacities from a fraction of a micro-microfarad up to many microfarads can be directly measured. At the same time condenser power factors can be determined without reference to any standard condenser. When the power factor is very small (*e.g.* 0·0001), L must be known to high accuracy and Pl must not be neglected. The measurement of such a small power factor is difficult by any method, for earth capacities and other small residual effects begin to show themselves, and the experiment often becomes as much a test of the weak points of the apparatus used.

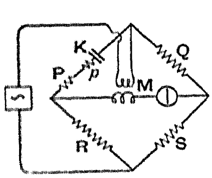


Fig. 49.—Rayleigh's Method.

§ (60) Rayleigh's Method. — Rayleigh [1] has pointed out that the Hughes method can be adapted to the measurement of a capacitance K instead of a self-inductance L. The connections are shown in *Fig.* 49, p representing the power loss in the condenser. The equations for the balance of the bridge, obtained from those of the Hughes method by writing $-1/\omega^2K$ for L, are

$$\frac{M}{K}=QR-S(P+p) \quad . \quad . \quad (143)$$

and
$$\omega^2MK=\frac{S}{(P+p+Q+R+S)}. \quad . \quad (144)$$

Hence
$$\omega^2M^2=\frac{SQR-S^2(P+p)}{P+p+Q+R+S}, \quad . \quad (145)$$

which gives p in terms of M, P, Q, R, and S. By using the value obtained, K can be found from (145).

§ (61) Campbell's Bridge Method. — The arrangement of Campbell's bridge method [2] is shown in *Fig.* 50. The unknown condenser K (with loss represented by p) can be introduced into the arm AB, which contains the secondary coil (P, L) of a variable mutual inductance M. The other arms are non-inductive, Q being adjustable.

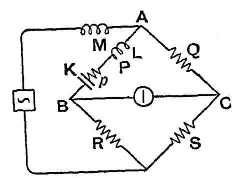


Fig. 50.—Campbell's Bridge Method.

First a balance is obtained without the condenser K, which gives for values M_0 and Q_0,

$$\frac{L-M_0}{M_0}=\frac{R}{S}$$

or
$$\frac{L}{M_0}=\frac{R+S}{S}. \quad . \quad . \quad . \quad (146)$$

Then K is introduced as in the figure, and a balance again obtained with values M_1 and Q_1.

Then
$$\frac{L-1/\omega^2K}{M_1}=\frac{R+S}{S}, \quad . \quad . \quad (147)$$

and hence
$$K=\frac{S}{R+S}\cdot\frac{1}{\omega^2(M_1-M_0)}. \quad . \quad (148)$$

Also
$$SP=RQ_0$$

and
$$S(P+p)=RQ_1,$$

whence
$$p=\frac{R(Q_0-Q_1)}{S}, \quad . \quad . \quad (149)$$

which gives the power factor of the condenser.

This method can also be used for determining frequency or for sifting of wave-form. For this purpose it is better to make it more direct reading by adding self-inductance N in the Q arm so as to make $M_0=0$.

Then
$$M_1K\omega^2=\frac{S}{R+S}, \quad . \quad . \quad (150)$$

and, with K, S, and R fixed, n will be inversely proportional to $\sqrt{M_1}$.

[1] Lord Rayleigh, *Theory of Sound*, 2nd Edition, 1895, vol. i. § 235*q*. See also "Inductance, Measurement of," § (101).

[2] See also "Inductance, Measurement of," § (98).

This method has two advantages over that of § (56): (*a*) the losses in the condenser cause no complication, and (*b*) while in that method very large values of K or M_1 are required for low frequencies, here, by varying the factor $(R+S)/S$, K and M_1 have any desired values. For example, for 100 ∿ per sec. with a value for $(R+S)/S$ of 256, only about 0·01 henry and 1 μF would be required, whereas for the former method with 0·1 henry the value of K would be about 256 μF.

§ (62) Use of Condenser Methods for determining Absolute Value of Ohm.—Any method which determines resistance in terms of mutual inductance can be used to determine the unit of resistance in absolute measure, for the mutual inductance can be referred to a standard calculated from geometrical dimensions alone. (See article on "Electrical Measurements, Systems of.") The methods described above lend themselves to such a determination through the intermediary of a condenser K. If the condenser is first tested at a given frequency by Carey Foster's method, M_1/K is found in terms of resistances. If it is then tested (at the same frequency) by one of the methods of § (56) or (61), M_2K is found in terms of frequency alone, or of frequency and a ratio of resistances, and hence M_1M_2 can be expressed in terms of resistances.

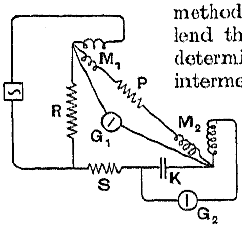


Fig. 51.—Resistance in Terms of Mutual Inductance by Intermediary Condenser (Campbell).

If the first of these methods is used, the two measurements can be carried out simultaneously according to Campbell's system shown in *Fig.* 51. A balance on G_1 is first obtained by adjusting S and M_1 or P with G_2 disconnected. Then G_2 is put in circuit and a balance also obtained on it, and ultimately G_1 and G_2 show a simultaneous balance. Then

$$\frac{M_1}{PR}=K=\frac{1}{\omega^2 M_2},$$

and hence

$$PR=\omega^2 M_1 M_2. \qquad (151)$$

In Rayleigh's method (§ (60)) the intermediary K can be directly eliminated as in equation (145).

§ (63) Determination of Capacitance in Terms of Self-inductance.—The methods for determining K in terms of L are the same as those for L in terms of K, except that usually an adjustable L can be used. A full description of a number of these methods is given in the article on "Inductance Measurement," §§ (106) to (115).

Special Methods

§ (64) Methods for Radio Frequencies.—In general the methods for determining the capacitance or power factor of a condenser at radio frequencies depend upon electrical resonance.

(i.) *Jordan's Method.*—As a typical example may be given Jordan's [1] method of testing the power factors of condensers at high frequencies, as shown in *Fig.* 52. The primary oscillation

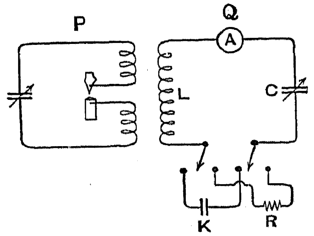


Fig. 52.—Test of Power Factor of Condenser (Jordan).

circuit P consists of a condenser and a self-inductance connected to a Poulsen arc B. To ensure symmetry and thus avoid troublesome earth capacity effects, the inductance consists of two equal coils, and B is connected between them. (The source of current supplying the arc is omitted in the diagram.) A secondary circuit Q is loosely coupled to P, and consists of a self-inductance L, an adjustable condenser C, a high-frequency ammeter A, and a pair of switches which can bring into the circuit either K, the condenser under test, or R, an adjustable non-inductive resistance. First, by setting C, the circuit Q is tuned to resonance, and the reading of the ammeter noted. Then, with R in circuit instead of K, the condenser C is adjusted to give resonance again, and R is altered until the same reading is obtained on the ammeter. The observed value of R gives the effective series resistance of K, and the power factor is ωKR (farads and ohms).

(ii.) *Differential Thermojunction Methods.*—Many null bridge methods are not workable at radio frequencies. The use of differential thermojunctions, however, is the basis of several null methods for high frequencies. For example, Glatzel [2] uses this device in his system of measuring small condensers shown in *Fig.* 53. A circuit A carries current of radio frequency, and two other circuits are loosely coupled to it, containing self-inductances and adjustable condensers K and C.

[1] H. Jordan, *Deutsch. Phys. Gesellsch. Verhandl.*, 1912, p. 451; see also W. Hahnemann and L. Adelmann, *Elekt. Zeits.*, 1907, xxviii. 988, 1010.

[2] B. Glatzel, *Deutsch. Phys. Gesellsch. Verhandl.*, 1907, ix. 151; see also L. Isakoff, *Phys. Zeits.*, 1911, xii. 1224, and A. Hund, *Gen. Elec. Rev.*, 1914, xvii. 981.

A measuring (or detecting) circuit is loosely coupled to the two latter by loops at B and D, with heater thermojunctions at p and q connected in opposite directions to the galvanometer G. By varying K and C a balance is first obtained; then the small unknown condenser c is put in parallel with C, and the capacitance of c determined by the change required in C to bring the galvanometer deflection back to zero.

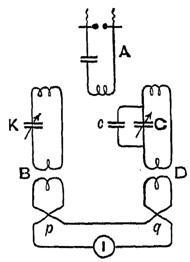


FIG. 53. — Glatzel's Differential Thermojunction Method.

(iii.) *Heterodyne Methods.* — For the measurement of small capacities various observers[1] have used heterodyne methods, in which two separate circuits, containing condensers K and C and inductances, carry currents produced by thermionic valve generators at radio frequencies, which are adjusted to be equal by observing the beats. The unknown small condenser is then put in parallel with C, which is adjusted until the frequencies are again equal. The change in C gives the value of the unknown condenser. In another modification of the method K and C are set to give a difference of frequencies of 1000 ∿ per sec., and the musical note obtained is made to beat against one produced by a third circuit.

§ (65) POWER FACTORS OF CONDENSERS BY WAVE-FORM MEASUREMENTS. — Various observers have deduced the power factors of condensers and the dielectric losses in various materials from determinations of current or voltage wave forms. Two examples may be mentioned.

(i.) Thornton,[2] using a sine wave voltage, took oscillograms of the current and terminal voltage of a number of different condensers (at frequency 39·5 ∿ per sec.); from these he drew hysteresis curves for the various dielectrics used, and deduced the power factor in each case.

(ii.) Nussbaumer[3] determined the quantity and voltage curves for the oscillatory charge (see § (15)) of various condensers, using suitable self-inductances to give frequencies from about 300 up to 2000 ∿ per sec. By means of a Helmholtz[4] pendulum the charging was started and then interrupted after a definitely known short interval (t) of time; the terminal voltage was then read on an electrometer and the charge in the condenser determined by discharging it through a ballistic galvanometer. By repeating the process for a number of values of t, curves of v and q were obtained for one oscillation, and from them hysteresis loops were drawn and the power losses deduced.

§ (66) WATTMETER METHODS.—The power lost in a condenser can be measured by means of a wattmeter, but great care must be taken to avoid errors which easily occur with low power factors. Rosa[5] in 1898 developed a number of methods for this purpose, making use of an electromagnetic wattmeter.

The wattmeter measurements are made much easier by putting in series (or parallel) with the condenser a self-inductance coil of well-stranded wire and having low resistance (R). The combined circuit can thus be arranged to have a power factor not far from unity, and so can be tested with good accuracy. To obtain the power lost in the condenser, the total observed power must be reduced by the amount RI^2, where I is the current in the inductive coil. The series method has the advantage that by means of resonance a high voltage can be applied to the condenser with only a comparatively low voltage on the combined circuit. These methods were used by Rosa and Smith[6] in testing condensers, and they corroborated their results by calorimetric measurements of the actual amounts of heat dissipated in the condensers by the energy losses. Both the series and the parallel inductance systems were used later by Mather[7] in testing the power factors of electric supply cables.

Electrostatic voltmeters have been used successfully for condenser testing by Addenbrooke[8] and others.

§ (67) CAPACITANCE BY MEASUREMENT OF ELECTROLYTIC RESISTANCE.—The electrostatic lines of force in the dielectric of a condenser have the same geometric form as the stream-lines of current produced if the plates of the condenser are immersed in a conducting liquid and a voltage applied to the terminals. Thus the capacitance of a complicated system of conductors (such as a radio antenna) is sometimes determined indirectly by measurement of electrolytic resistance. A properly proportioned model of the conducting system is constructed in metal. This is placed in a large vessel containing a suitable electrolyte of resistivity ρ (in ohm-cm.), and the electrolytic

[1] L. Pungs and G. Preuner, *Phys. Zeits.*, 1919, xx. 543; W. H. Hyslop and A. P. Carmen, *Phys. Rev.*, 1920, xv. 243; G. Leithauser, *Deutsch. Phys. Gesellsch. Verhandl.*, 1920, i. ser. 3, 23.

[2] W. M. Thornton, *Phys. Soc. Proc.*, 1912, xxiv. 301.

[3] H. V. Nussbaumer, *Dissertation, Zürich*, 1907; see also H. Tallqvist, *Wied. Ann.*, 1897, lx. 248, and U. Seiler, *Wied. Ann.*, 1897, lxi. 30.

[4] G. Eichhorn, *Dissertation, Zürich*, 1901, and *Jahrb. drahtl. Telegr.*, 1908, i. 369.

[5] E. B. Rosa, *Bureau of Standards Bull.*, 1905, i. 383; see also C. P. Steinmetz, *Electrician*, July 5, 1901, p. 412.

[6] E. B. Rosa and W. W. Smith, *Phys. Rev.*, 1899, viii. 1, 79, and *Phil. Mag.*, 1899, xlvii. 232.

[7] T. Mather (in discussion on paper by W. M. Mordey), *J. Inst. El. Eng.*, 1901, xxx. 411.

[8] G. L. Addenbrooke, *Phys. Soc. Proc.*, 1915, xxvii. 291, and C. E. Skinner, *Franklin Inst. J.*, 1917, clxxxiii. 667.

resistance R between the two sets of conductors is measured. Then the required capacitance K, in microfarads, is given by

$$K \doteqdot \frac{8 \cdot 84 \times 10^{-9} a \rho}{R}, \quad . \quad . \quad (152)$$

where a is the ratio of the linear dimensions of the actual system to those of the model.

V. Properties of Dielectrics and Behaviour of Condensers

§ (68) By the various methods described in the preceding sections many investigations have been made of the properties of dielectrics and the behaviour of condensers. As regards dielectrics, inductive capacity (dielectric constant) has already been dealt with in §§ (17) to (24), and only power loss (or power factor) requires further consideration. In the case of condensers the most important properties are the variation of capacitance with temperature, voltage, frequency, atmospheric pressure, or lapse of time; also power factor and its variations due to these conditions.

§ (69) Mica Condensers. (i.) *Effect of Frequency.*—For mica condensers used with alternating current the capacitance decreases as the frequency is raised, the rate of decrease becoming very small at the higher audio frequencies. (The lower limit which the capacitance thus tends to approach has been called by Curtis the *geometric capacity.*) The amount of change with frequency depends on the quality of the condenser. In a good condenser of very small power factor the variation is very small indeed from low frequencies of the order of 50 ∿ per sec. upwards, as will be seen from *Fig.* 54, which refers to a standard mica condenser (at 15° C.) tested by Campbell at the National Physical Laboratory. For condensers of poorer quality the change over the same range may amount to 3 or 4 parts in 1000.

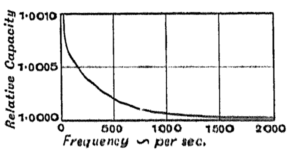


Fig. 54.—Change in Effective Capacity with Frequency (in good Mica Condenser).

(ii.) *Variation of Capacitance with Temperature.*—Mica appears to have a very small positive capacity temperature coefficient,[1] but in most good condensers there is sufficient paraffin wax left along with the mica to give the coefficient a small negative value, which is commonly from −1 to −3 parts in 10 000 per degree C. Condensers made with silvered mica, in which the paraffin does not get between the plates, have a positive temperature coefficient of the order of +1 in 10 000 per degree C.

Curtis [2] has made a very thorough investigation of the behaviour of a number of mica condensers, and for further details the reader is referred to his paper. In *Fig.* 55 are shown the results of some of his tests of a good mica condenser at various temperatures and frequencies. The three upper curves are for single charge of 0·6 sec., and discharge times of 1·0, 0·5, and 0·1 sec. respectively. It will be noticed that these show greater capacities than those for alternating current, and also zero or positive temperature coefficients. The less perfect condensers tested by Curtis show these effects to a much greater degree.

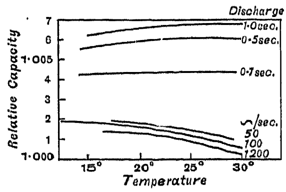


Fig. 55.—Variation of K with Temperature in good Mica Condenser (Curtis).

(iii.) *Alternating Voltage.*—The capacity of a good condenser shows no appreciable change for different alternating voltages of the same frequency. Condensers of silvered mica unfortunately sometimes show slight variation for different voltages; in one case Curtis observed an increase of capacity of 0·1 per cent on raising the voltage from 20 to 100 volts.

(iv.) *Permanence.*—The effect of changes in atmospheric pressure on the capacity of a mica condenser is very slight.

If kept in a room of constant temperature (15° C.) the capacity of a good mica condenser may remain constant to within 1 or 2 parts in 10 000 for years.

(v.) *Power Factors.*—The power factor of a good mica condenser (at 15° C. with 100 ∿ per sec.) is commonly of the order of 0·0003, but in the very best examples values as low as 0·0001 [3] may be found. The temperature coefficient of power factor is positive, and there is decrease with increase of frequency (see Curtis, *loc. cit.*).

The insulation resistance measured with direct current may be as high as 100 000 megohms for a 1 μF condenser.

§ (70) Paraffin Paper Condensers.—Ordinary paraffin paper condensers are as a rule less constant than mica condensers and their power factors are much larger. There are wide differences in quality and behaviour in the types used in commercial work.

(i.) *Change of Capacitance with Frequency.*—The capacitance falls as the frequency is raised, much as with mica condensers, but

[1] W. Cassie, *Roy. Soc. Proc.*, 1889, xlvi 357.

[2] H. L. Curtis, *Bureau of Standards Bull.*, 1911, vi. 431.

[3] This represents a phase displacement of 0·017°.

the variation is much greater. *Fig.* 56 shows the behaviour of a good paraffin paper condenser (of power factor 0·002) at 15° C. The variation with frequency increases as the temperature is raised, and becomes considerable in condensers of large power factor. Very full information on this and the other properties will be found in a paper by Grover,[1] which gives the results of a large series of alternating current tests on fifteen different condensers. Jordan [2] has investigated paper condensers at radio frequencies. In some of them the paper was paraffined, in others it was impregnated with oil.

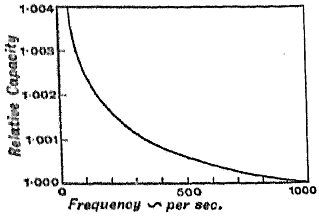


FIG. 56.—Change of Capacity with Frequency in good Paraffined Paper Condenser (Grover).

(ii.) *Variation with Temperature.*—According to Grover, the capacity temperature coefficient of a paraffin paper condenser of small power factor is usually negative and nearly constant, being not more than about −5 parts in 10 000 per degree C. In poorer condensers it is sometimes positive, rapidly increasing at the higher temperatures and lower frequencies (often up to 1 per cent). In other cases it even changes from positive to negative values.

(iii.) *Variation with Voltage.*—Paraffin paper condensers of good make do not appear to vary with change of the applied voltage. For example, a condenser of Mansbridge type tested at the N.P.L. showed constancy of capacitance to within 1 in 1000 for voltages from 10 up to 100 volts (at 100 ∿ per sec.). The power factor was also constant, which is in agreement with the results of Monasch (*loc. cit.* § (51)) and others who have found the power factors of various condensers constant over long ranges of voltage.

(iv.) *Power Factor.*—The power factor depends very much on the quality of the condenser, insufficient drying of the paper during construction causing increased losses. At 15° C. and 100 ∿ per second Grover found values ranging from 0·0017 to 0·017. In general the value rises rapidly at the higher temperatures and lower frequencies. Foiled paper condensers of Mansbridge type, which are now so much used in telephone work, show relatively high power factors. One of these tested in 1912 by Campbell (at 16·5° C. and 100 ∿ per sec.) had a power factor of 0·0084 with a capacity of 2 μF. In this type the resistance of the conducting plates inside the condenser contributes appreciably to the power lost.

According to Fischer,[3] Meirowsky condensers —in which the dielectric is paper impregnated with lacquer—have power factors of about 0·009 at 80 ∿ per second.

§ (71) CONDENSER POWER FACTORS FOR VARIOUS DIELECTRICS.—The power factors of condensers with dielectrics of various other materials have been investigated (for different temperatures and frequencies) by a number of experimenters.[4] The ranges of frequency used in some of these researches are given in Table VII.

TABLE VII

	Frequencies. ∿ per Second.	Temperature ° C.
Hanauer .	128, 256, 512, and 10 000	. .
Monasch .	50 and 86	. .
Fleming and Dyke	920, 2760, and 4600	−18 to 80
	$0{\cdot}5\times10^6$ up to 2×10^6	18
Thornton .	36·5	. .
Wagner. .	500 up to 5000	10 to 50
Austin . .	$0{\cdot}3\times10^6$	. .
Bairsto . .	900 up to 2×10^6	. .

In addition to tests on condensers Monasch also made determinations of the power factors of various cables (for electric supply) with impregnated paper and other dielectrics. Wagner made a very complete investigation of the dielectric constants, power factors, and other properties of various samples of gutta-percha, and of balata and other dielectrics, chiefly in the form of cables, the whole range of telephonic frequency being covered.

The measurements become more difficult at radio frequencies and the results are more uncertain, for in addition to the actual power lost in the solid or liquid dielectric, power losses may often occur due to other causes, such as spraying discharge in the air (particularly near the edges of the conductors). Austin found the power factor to be constant from 4000 up to 20 000 volts in condensers in which spraying is prevented by the nature of their construction; for example, glass plates in oil, oil-immersed jars, etc. But for ordinary Leyden jars in air, between 10 000 and 20 000 volts where brush discharge occurs, the power factors increased roughly in proportion to the voltage. In Fleming and Dyke's experiments very considerable variation in power factor

[1] F. W. Grover, *Bureau of Standards Bull.*, 1911, vii. 495.

[2] H. Jordan, *Deutsch. Phys. Gesellsch. Verhandl.*, 1912, p. 451.

[3] K. Fischer, *Elekt. Zeits.*, 1909, xxx. 601.

[4] J. Hanauer, *Wied. Ann.*, 1889, lxv. 789; B. Monasch, *Ann. d. Physik*, 1907, xxii. 905; J. A. Fleming and G. B. Dyke, *J. Inst. El. Eng.*, 1912, xlix. 323; *Phys. Soc. Proc.*, 1911, xxiii. 117; W. M. Thornton, *Phys. Soc. Proc.*, 1912, xxiv. 301; K. W. Wagner, *Archiv f. Elektrot.*, 1914, iii. 67; L. W. Austin, *Bureau of Standards Bull.*, 1913, ix. 73; G. E. Bairsto, *Roy. Soc. Proc.* A, 1920, xcvi. 363.

was observed in nearly all cases, which appears to have been due to causes of the nature of brush discharge. In work at radio frequencies the dielectric loss is sometimes specified by the logarithmic decrement of the condenser instead of the power factor. If δ is the decrement [1] of the condenser K for a *whole period*, L the resonance self-inductance for pulsatance ω, and R the effective resistance, then $\omega^2 LK = 1$, and

$$\delta = \frac{R}{2nL} = \frac{\pi RK\omega}{\omega^2 LK}$$

$$= \pi RK\omega,$$

or $$\delta = \pi \times (\text{power factor}). \qquad (153)$$

[Sometimes the decrement δ' for *half-period* is used, as in Fleming and Dyke's paper. Then $\delta' = \delta/2 = (\pi/2)RK\omega$.]

In Table VIII. are given a number of values of power factors of condensers according to various observers. As most of the materials are of variable composition, too much stress must not be put upon the absolute values, which must rather be regarded as indicating the order of magnitude to be expected.

TABLE VIII

Condenser or Material.	Authority.	Frequency. ∿ per Second.	Power Factor.
Flint glass jar	Monasch	86	0·0037
Leyden jars	Fleming and Dyke	$1·3 \times 10^6$	0·0005 to 0·0023
,, (in air)	Austin	$0·3 \times 10^6$	0·013 to 0·022
,, (in oil)	,,	$0·3 \times 10^6$	0·003
Glass plates in oil	Campbell	100	0·022
	Austin	$0·3 \times 10^6$	0·005
	Fleming and Dyke	$1·7 \times 10^6$	0·0007 to 0·016
Moscicki glass condenser	Austin	$0·3 \times 10^6$	0·006
	Fleming and Dyke	$0·9 \times 10^6$	0·003 to 0·016
Glass (photographic plates)	Thornton	36·5	0·021
Crown glass (17° C.)	Fleming and Dyke	920	0·018
	Bairsto	$0·3 \times 10^6$	0·022
Sulphur (16° C.)	Fleming and Dyke	920	0·0003
Compressed air (15 atmospheres)	Austin	$0·3 \times 10^6$	0·0016
Micanite	Austin	$0·3 \times 10^6$	0·024
Celluloid (19° C.)	Fleming and Dyke	920	0·026
Paraffin wax	Monasch	50	0·0000
,, paper	Austin	$0·3 \times 10^6$	0·026
Paper (dry), 19° C.	Fleming and Dyke	920	0·007
Paper (air dry), 18° C.	,, ,,	920	0·077
	,, ,,	4600	0·040
Paper core cable (dry), 19° C.	,, ,,	920	0·003
Impregnated paper cable (1)	Monasch	50	0·0039
,, ,, (2)	,,	50	0·009
Vaseline oil	Fleming and Dyke	$2·14 \times 10^6$	0·0024
Ebonite	Thornton	36·5	0·027
,, (in oil)	Fleming and Dyke	$2·0 \times 10^6$	0·016
India-rubber (pure)	Thornton	36·5	0·027
,, ,, (18° C.)	Fleming and Dyke	920	0·005
Vulcanised rubber (17° C.)	,, ,,	920	0·002
	Bairsto	$0·2 \times 10^6$	0·025
	,,	$0·6 \times 10^6$	0·06
Gutta-percha (16° C.)	Fleming and Dyke	920	0·019
Gutta-percha cable (15° C.)	Wagner	800	0·024
Gutta-percha	Bairsto	$0·6 \times 10^6$	0·036
Balata (15° C.)	Wagner	800	0·005

The materials of the india-rubber and gutta-percha class call for special notice.

Gutta-percha.—Rayner [2] discovered that the

[1] See § (15), equation (93).

[2] E. H. Rayner, *J. Inst. El. Eng.*, 1911, xlvi. 412, and 1912, xlix. 3.

temperature coefficient of the power factor of gutta-percha (at low frequencies) is negative above 10° C. Campbell found that at 800 ∿ per sec. it is still negative. The subject was fully investigated by Fleming and Dyke (*loc. cit.*) over an extensive range of temperature for three different temperatures. As appears from their curves, shown in *Fig.* 57,

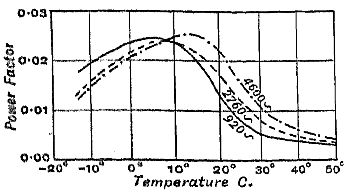


FIG. 57.—Power Factor of Gutta-percha Condenser (Fleming and Dyke).

the power factor has a maximum at a temperature not far from 10° C., the position of the maximum depending somewhat on the frequency. Addenbrooke [1] tested the power factor of gutta-percha at very low frequencies and obtained the values given in Table IX.

TABLE IX

Frequency. ∿ per Second.	Power Factor.
1·5	0·075
3	0·072
6	0·063
12	0·055
46	0·040

Rayner [2] found the power factor of a gutta-percha (cable) condenser at 50 ∿ per sec. to be practically constant from 10 up to 4000 volts.

India-rubber.—As will be seen from *Fig.* 58,

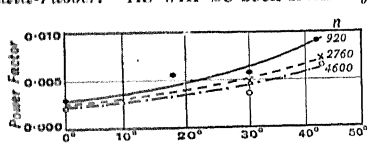


FIG. 58.—Power Factor of Pure Para India-rubber (Fleming and Dyke).

Fleming and Dyke found the temperature coefficient positive for pure Para india-rubber. Wagner's curves indicate a maximum power factor near 30° C.

On the other hand, vulcanised rubber, as shown in *Fig.* 59, has a distinct minimum near 20° C. (Fleming and Dyke).

Balata.—Balata has a much lower power factor than gutta-percha, with a large negative temperature coefficient at ordinary temperatures (Wagner).

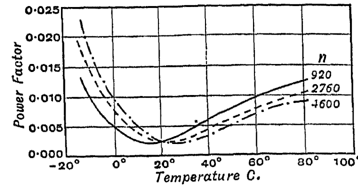


FIG. 59.—Power Factor of Vulcanised India-rubber Condenser (Fleming and Dyke).

VI. USE OF CONDENSERS FOR MEASURING INTERVALS OF TIME

§ (72) An interesting and important application of condensers is their use for the measurement of short intervals of time. If a condenser of K microfarads, with initial charge Q, is allowed to discharge through a resistance R ohms for a time t seconds, then by equation (96)

$$t=10^{-6}\mathrm{KR}\log_\epsilon\left(\frac{\mathrm{Q}}{q}\right), \qquad (154)$$

where q is the quantity left in the condenser at the end of time t.

Or if V and v are the terminal voltages at the beginning and the end of time t, since $\mathrm{K}=\mathrm{Q}/\mathrm{V}=q/v$, we have

$$t=10^{-6}\mathrm{KR}\log_\epsilon\left(\frac{\mathrm{V}}{v}\right). \qquad (155)$$

In 1876 Sabine [3] used a condenser in this way for the measurement of very short intervals of time. In *Fig.* 60 is shown the system as he used it for determining the time of contact of a light hammer H hitting an anvil A. With the key D in the position shown the condenser is charged by depressing key P for a moment. It is then discharged through the ballistic galvanometer G by pressing down key D, the throw measuring the initial charge Q. With key D up, the condenser is again charged, and the anvil hit with the hammer, which is allowed to rebound. The key D is immediately depressed, and the smaller throw now obtained on the galvanometer measures the quantity q left in the condenser. Then t the duration of impact can be obtained by equation (154), Q/q being equal to the ratio of the two throws of the galvanometer.

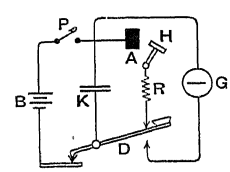


FIG. 60.—Sabine's Method of measuring Duration of Impact.

Radakovič [4] employed the method for the

[1] G. L. Addenbrooke, *Electrician*, 1913, lxx. 673; see also *Phys. Soc. Proc.*, 1915, xxvii. 291.
[2] See *Phys. Soc. Proc.*, 1911, xxiii. 142.
[3] R. Sabine, *Phil. Mag.*, 1876, i. 337.
[4] M. Radakovič, *Wien. Akad. Ber.*, 1900, cix. (II.*a*), 276, 941.

measurement of the speed of bullets. The bullet breaks successively the contacts A and B, which consist of wires (in screens) broken by its passage. The key F is then rocked over to the galvanometer side and the interval between the two breaks of contact determined as already described. Radakovič states that it is desirable to work only with small voltages, and that the best accuracy is got when $Q/q=3\cdot6$.

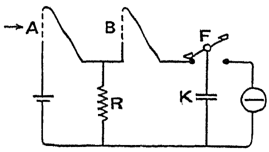


FIG. 61.—Radakovič's System of measuring Speed of Bullets.

Peirce [1] used a similar method to determine the time of contact of a quick tap on a telegraph key, which he found varied from 0·003 to 0·03 second. He worked with time of charge (equation (95)) instead of discharge.

Kennelly and Northrup [2] increased the accuracy of the method by using the method of mixtures (§ (46)) for measuring the charge left in the condenser, setting the resistances so as to give an approximate balance and then reading the small residual throw. They used the method to determine the duration of contact of impacting spheres (chiefly of steel), the time intervals dealt with being of the order of 50 to 300 microseconds.

More recently Klopsteg [3] has used a modification of the method, in which the galvanometer throw for q is made very small by opposing to the voltage of the condenser a nearly equal steady voltage of opposite sign. The system is shown in *Fig.* 62. The two contacts A and B are opened in quick succession at the beginning and end of the short interval to be measured. Then the battery is reversed at D and the key F put down. If the setting of the resistances R and S has been right, the galvanometer throw will be quite small and very exact calibration will not be required. If ΔQ is the small charge indicated by this throw, and V the voltage across (R+S), the interval t will be given by

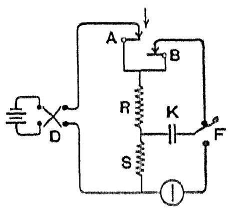


FIG. 62.—Klopsteg's Method of measuring Short Time Intervals.

$$t=10^{-6}KR\log_e\frac{1}{\frac{S}{R}+\frac{\Delta Q}{KV}\left(\frac{R+S}{R}\right)}. \qquad (156)$$

[1] B. O. Peirce, *Am. Acad. Proc.*, 1906, xlii. 95.
[2] A. E. Kennelly and E. F. Northrup, *Franklin Inst. J.*, 1911, xxx. 172.
[3] P. E. Klopsteg, *Phys. Rev.*, 1920, xv. 12.

For example, with a 1 μF condenser, for an interval of 161·5 microseconds R and S were 175 and 70 ohms respectively. Klopsteg investigated the accuracy of the method by the aid of a Helmholtz pendulum, and concluded that in measuring an interval of 250 microseconds the probable error of each observation need not exceed 0·15 per cent. He mentions also a method based on equation (155), in which V and v are measured by means of an electrometer.

Various other experimenters [4] have used similar condenser methods.

§ (73) MEASUREMENT OF RESISTANCE BY LOSS OF CHARGE.—The method of § (72) may be inverted and used to measure an unknown resistance R by observing an interval of time. It is only very high resistances that give t sufficiently long to be observed with accuracy. If R is in megohms and K in μF, equation (155) becomes

$$R\doteq\frac{0{\cdot}4343t}{K\log_{10}(V/v)}. \qquad (157)$$

The high resistance (R) to be measured is put across the terminals of the condenser K in parallel with an electrostatic voltmeter. A charge is given to the condenser so as to make the voltmeter read near the top of its scale, an initial V is observed, and then a number of values of v are read at definite times from the start, as the condenser gradually discharges through R. Both the condenser and the voltmeter must have very high insulation.

The method is sometimes used to find the insulation resistance of a cable, in which case, following equation (154), a ballistic galvanometer may be used, the condenser being the cable itself.

A. C.

APPENDIX

BIBLIOGRAPHY

In addition to those already given, further references, and also bibliographies, will be found in the following original papers and books :

DIELECTRIC CONSTANTS

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Dielectric constants of gases : K. Baedeker, *Zeits. Phys. Chem.*, 1901, xxxvi. 315.
Article by E. Schrödinger in L. Graetz's *Handb. der Elekt. u. des Magnetismus*, i. 157 (J. A. Barth, Leipzig, 1918).

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Article by L. Graetz in Winkelmann's *Handbuch der Physik*, p. 77.
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[4] Edelmann, *Phys. Zeits.*, 1904, v. 461 ; Devaux-Charbonnel, *Comptes rendus*, 1906, cxlii. 1080. For further references see Klopsteg's paper.

1912, xlix. 46; E. Ritter v. Schweidler, *Ann. der Physik*, 1907, xxiv. 711.

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E. B. Rosa and F. W. Grover, *Bureau of Standards Bull.*, 1905, i. 171.

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W. v. Bisicz, *Anwendung u. Zukunft der Kondensatoren in der Wechselstromtechnik* (J. Springer, 1903). This book abounds in references.

CAPACITY MEASUREMENTS: Adaptation of, for the measurement of the capacity of guard-ring condensers. See "Capacity and its Measurement," § (43).
Commutator Methods of. See *ibid.* § (40).
Effect of change of frequency in different methods. See *ibid.* § (42).

CARBON STEELS: Tests on their suitability for permanent magnets. See "Magnetic Measurements and Properties of Materials," § (49).

CARBONS FOR ARCS, THE MANUFACTURE OF

§ (1) THE MATERIALS.—The choice of raw materials for the manufacture of arc lamp carbons is governed by the necessity of producing a finished carbon which is of as high degree of purity as possible, and which is also homogeneous in structure and composition. The purest material which can be used is lamp-black, which is obtained by burning creosote oil under conditions which cause the carbon to be separated. The lamp-black, or soot, thus obtained can be produced with as high a degree of purity as 99·7 per cent or 99·8 per cent carbon, the slight impurities that exist being introduced from the walls of the chambers in which it is collected. It is in the form of an impalpable powder, and of a fairly high specific gravity. Carbons can be made from lamp-black alone with the use of a suitable binder, but it is more general to mix with the lamp-black some other form of carbon. It was formerly general practice to use gas retort carbon as the second raw material. This carbon, which is formed on the inside of the gas retorts in the process of gas manufacture, varies greatly in appearance and purity. Pieces of gas retort carbon may contain less than 0·5 per cent of foreign matter, but the foreign matter may easily run up to 6 per cent or 7 per cent. It is therefore necessary, when gas retort carbon is used, to select only the pure varieties; fortunately, this can be comparatively easily done, as the amount of impurity alters the appearance of the retort carbon, and by carefully selecting only those pieces which have the right appearance indicating high purity, it is possible to obtain gas retort carbon in bulk in which the average amount of impurity will not exceed 0·5 per cent. In recent years, gas retort carbon has been largely displaced as a raw material by petroleum coke or pitch coke, the former being the coke residue left after the complete distillation of crude petroleum, the latter the same residue after the complete distillation of gas-tar. Both these materials, if properly prepared, have a very high purity which should in no case exceed 1 per cent foreign matter, and can easily be kept below 0·5 per cent. These materials, more particularly petroleum coke, as obtained commercially, are liable to have a fairly high content of volatile matter, and in this case it is necessary to resort to the process of calcining, which may easily be performed in any suitable furnace, to drive off all volatile contents.

Lamp-black, of course, does not require any grinding before it is used for the manufacture of carbons. It is sometimes calcined or treated in other ways to increase its density, but when a proper quality of lamp-black has been manufactured in the first instance, such processes are unnecessary. Retort carbon, petroleum coke, or pitch coke require to be ground to very fine powder. Any suitable grinding mills may be used for this purpose, but, on account of the extreme hardness of these materials, they cause great wear on the grinding machinery, and precautions have to be taken to prevent the final powder from being too highly contaminated by impurities from the grinding machines. The crude products may be broken up and ground to fairly coarse powder on steel machines, and the final grinding to a fine powder may also be done with such mills, but in this case it is generally necessary to pass the powders through a magnetic separator to remove any particles of steel which may have been introduced. The finer grinding may alternatively be done on emery or carborundum stone mills, in which case magnetic separation will probably be found to be unnecessary.

The finely ground powder is mixed with lamp-black in the desired proportion, and with a suitable binder such as pitch or tar. The highest grade carbons will contain a very high percentage of lamp-black, say in the neighbourhood of 80 per cent. Lower grade and cheaper quality carbons contain considerably less lamp-black, which may be reduced even as low as 20 per cent. The precise percentage of these two materials used is not only governed by the question of quality, but depends to a certain extent on the conditions under which the carbons are ultimately to be used. Where carbons are to be used with large currents at a high current density, somewhat better results are obtained by decreasing the percentage of lamp-black, which

tends to give the final carbon too china-like a structure, which is liable to crack excessively under the great heat of a heavy current arc.

The choice of a binding material depends largely on the machinery and processes of manufacture. Medium soft pitch, which may be still further softened by the addition of a small percentage of creosote oil to the mixture, requires to be worked at higher temperatures than tar, and somewhat different machinery has to be employed. Tar, when used, is always a refined tar, from which the more volatile constituents have been distilled. It is, of course, necessary that the binding material should be as free from impurities as the main raw materials. The mixture of raw materials and binder is worked up into the form of a very stiff dry paste, so dry that it can be crumbled in the hand into dust, but yet with sufficient binding material to bind it together under high pressure. This mixture is formed under hydraulic or other mechanical pressure into large blocks, often called "cheeses" on account of their resemblance in shape to a large cheese, and these are then pressed in hydraulic presses through a die of the requisite diameter to form a rod of carbon of the desired size. The rods, as they issue from the die, are cut up into lengths suitable for further handling. Where cored carbons are required, the rod, instead of being solid, is in the form of a tube, the hole being formed by a suitable steel needle in the die. The rods are formed at very great pressure, in the neighbourhood of five tons to the square inch.

When the carbons have reached this stage in their manufacture, the final quality has been largely determined, the importance of purity in all the materials has already been emphasised, and there follows, of course, the necessity for preventing contamination during the various processes of grinding and mixing. The fineness of the powder and the thoroughness of the mixing, particularly the thoroughness with which the binding material is distributed throughout the mass, naturally control the final homogeneity of the carbon. The composition of the binder, the temperature and pressure at which the rods are formed, control the percentage of binding material used, and this in its turn largely determines the final porosity of the finished carbon.

§ (2) Baking.—The rods, as they come from the press, are tied up into bundles for calcining. Great care must be taken to keep the rods, which at this stage are rather flexible, as straight as possible throughout the baking process. In the case of the larger carbons, it is usual to perform some of the finishing processes, such as pointing one end and flatting the other end of the carbon, on the green carbon before it is baked, but in the case of the smaller carbons, and often also the larger ones, these processes are carried out after baking. The bundles of rods are packed in suitable fireclay crucibles and surrounded by carbon powder or some other refractory powder, to prevent air getting to the carbons themselves during baking. They are then slowly calcined up to a temperature of about 1200° C.

The baking process must be carried out with great regularity and slowness, the temperature being steadily raised to maximum, and as steadily, though somewhat more quickly, dropped. The process drives off the whole of the volatile material and leaves behind a coke formed from the pitch or tar which has been used as a binder, which cements the whole carbon together, and the baked rod is hard and china-like in character.

Many special types of furnaces have been designed to carry out the baking process, which, however, can be performed in any furnace where there is complete control of the rate of rise of temperature. The differences in design of furnaces only affect the accuracy and economy with which this control can be exercised, and, of course, the economy in the handling of the carbons during baking. The baked carbons, if they are not already cut and pointed, have to pass through this process after baking. Cutting is really a process of breaking the carbons through; pointing and grinding the end flat is done on carborundum or emery stones. A further mechanical process to which the baked carbons must be submitted is that of sorting to eliminate any carbons which have become crooked in baking, and the carbons must also be examined to take out any which are defective in other ways, such as being blistered or cracked or having similar faults.

§ (3) The Core.—At this stage the solid carbons are finished unless they have to be coppered, to which reference will be made later. The cored carbons, namely, those which are in the form of a tube, have to have the core canal filled. The cored carbons which are used in ordinary open type or enclosed type lamps, for the so-called pure carbon arc, have the core canal filled with a mixture of carbon and potassium silicate. The carbon is in the form of a very fine powder, which is mixed with a solution of potassium silicate until it has the consistency of a fairly thick cream. This mixture is then injected into the core canal, which it completely fills, and the carbon is then dried at a temperature sufficiently high to drive off all the moisture in the coring mixture. The core is then in the form of a dry powder held together in its place by the dry silicate of potash. The function of the silicate of potash is not only to act as a binding material, but to lower the resistance of the arc and, as has been ex-

plained elsewhere, to centralise the arc at the end of the carbon. For flame arcs, suitable chemicals are introduced in the form of powder into the coring mixture; calcium fluoride is used when a yellow flame arc is required: cerium fluoride for a white flame arc: other compounds can be used for producing special colours.

§ (4) Conductivity.—The conductivity of the finished carbon is not very high, and varies of course to some extent with its composition. The resistivity is from 0·06 to 0·09 ohm per centimetre cube. Where the diameter of the carbon is small for the current to be carried, or where, as in some types of flame lamps, the length of carbon from the carbon holder to the arc is great, it is necessary to increase the conductivity in order to avoid too large a voltage drop along the carbon. This is effected by copper-plating the carbon electrolytically, sufficient thickness of copper being deposited to give the required conductivity. In long flame carbons another method for lowering the resistance is to introduce metal—generally brass—wire into the body of the carbon: a separate hole parallel to the core canal and about 1 mm. diameter is formed in the carbon when pressing, and the wire is introduced into this after baking, and is generally waved so that it makes contact with the carbon at short intervals throughout its length. Various devices are employed to make direct contact to this wire in the carbon holder of the lamp.

§ (5) Flame Carbons.—Mention has already been made of the flame carbon in which flame-producing material is introduced into the core. The original flame carbon had the flame-producing material distributed throughout the mass of the carbon, the ingredients being introduced into the original mixing before pressing. Experience showed, however, that in open type flame lamps better results could be obtained by concentrating the ingredients in the core. In recent years, however, the development of the enclosed type flame lamp, where the carbons burn in an enclosure from which the air is largely excluded, have necessitated the manufacture of a flame carbon in which the ingredients are introduced into the original mixture, and which is therefore homogeneous throughout its section. Carbons burning under these conditions naturally do not taper to a point but burn with a practically flat end, and it is therefore necessary for the flame-producing material to be evenly distributed instead of concentrated in a central core. Open type lamps have also been recently developed for which the most suitable type of carbon is one consisting of an outer shell of pure carbon with an inner solid carbon containing flame-producing material. To manufacture such carbons, the two parts are prepared separately and the inner rod inserted in the shell, either before or after baking, and suitably cemented to it.

The design of a carbon to be used for a special purpose depends mainly on the lamp and conditions of burning. These determine the diameter and length of carbons required; whether the carbon has to be cored or solid; if cored, the size of the core; and for flame carbons the percentage of flame-producing material which has to be incorporated. These factors, combined, of course, with efficient manufacture, determine the steady and even burning of the carbons. M. S.

Carey Foster Bridge, for the comparison of standards of electrical resistance. See "Electrical Resistance, Standards and Measurement of," § (7) (ii.).

Use of, for the determination of the capacity and power factor of a condenser, in terms of mutual inductance and resistance. See "Capacity and its Measurement," § (59).

Cascade Amplifiers. Arrangements of thermionic valves such that each valve amplifies the output of the preceding one in the series. See "Thermionic Valves," § (14).

Castner Cell used in Electrolysis. See "Electrolysis, Technical Applications of," § (28) (ii.) (*b*).

Cathode: a term used in electrolysis to denote the metallic conductor at which the current leaves the electrolyte. See "Electrolysis and Electrolytic Conduction," § (1).

Cathode Effect on Concrete: the softening of concrete due to stray current electrolysis. See "Stray Current Electrolysis," § (18).

Cathode Fall of Potential, in discharge tube, Mey's values of. See "Electrons and Discharge Tube," § (4).

Cathode Particles, nature of. See "Electrons and the Discharge Tube," § (14).

Cathode Ray Oscillograph. An apparatus for delineating the instantaneous values of the current or voltage in a circuit by the deflection of a fine cathode stream. See "Piezo-electricity"; "Alternating Current Instruments," § (60).

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Conditions for Production and Phenomena exhibited by. § (8).

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— D —

DIELECTRICS

§ (1) MATERIALS USED AS DIELECTRICS.—A dielectric is a type or condition of matter or space in which it is possible to produce and maintain a continuous state of electric stress with little or no supply of energy from outside sources.

Substances are commonly divided into conductors and non-conductors. Non-conductors are often termed dielectrics when their particular function is to separate neighbouring conductors which are at different potentials. Dielectrics may be solid, liquid, or gaseous. Those in common use include such substances as mica, glass, paraffin, porcelain, varnished materials such as papers and cloth, wood, oils, gases at ordinary temperatures, vacuum. The distinction between a dielectric and a conductor is that in the space between two conductors, separated by a dielectric, a state of electric stress can exist without the continuous supply of energy from without the system, whereas in the case of a conductor an appreciable and often very large amount of energy is required to maintain the condition of electric stress. There is no hard and fast line between conductors and dielectrics. Probably all dielectrics allow a very minute current to pass through them, though this may be far beyond the sensitiveness of ordinary measuring apparatus to detect.

Many materials which are efficient dielectrics at ordinary temperatures gradually alter as the temperature is raised, and change imperceptibly to a conducting state. This is common with a number of substances of a mineral nature such as glass, porcelain, oxides of the type used in the Nernst lamp. In fact, at about 2000° C. or less, it may almost be said that insulating materials cease to exist. When conductivity commences in such materials it increases at a relatively very great rate with increase of temperature. It may increase twofold for every few degrees rise.

Many materials when pure, such as water and soluble salts, have the properties of dielectrics, but when mixed become conductors. The work of Faraday on the effect of an electric current passing through solutions of salts gave an insight into the properties of matter in solution and indicated the possibility that a quantity of electricity might not be infinitely divisible, but be found to be of an atomic character. The knowledge derived from observations on electrolysis, osmosis, and vapour pressure of solutions has afforded a more satisfactory basis of an electrical theory of matter in a state of solution than we have possessed of the physical nature of electric stress in materials which are non-conductors. The work of the last thirty years on gases has also given more insight as to the passage of electricity through them than we have of the process of the passage of an electric current through non-conducting solids.

§ (2) DYNAMICS OF ELECTRIC STRESS.—In the article on "Units of Electrical Measurement" definitions have been given of various terms which we shall have to employ. The electrostatic unit of electricity has been defined as the quantity which if concentrated at a point in air—more strictly *in vacuo*—will repel an equal quantity concentrated at a second point 1 cm. away with a force of 1 dyne.[1] The electric intensity at any point of an electric field is the force which would act on unit charged placed at that point without altering the distribution of the field. This quantity we denote by R. The quantity

[1] It may be convenient to recollect that a force of 1 dyne is approximately equal to the weight of 1 milligramme. More strictly, weight of 1 milligramme = ·981 dyne.

$KR/4\pi$ has been defined, for a medium of specific inductive capacity K, as the induction in the field. For air in which K is unity this becomes $(1/4\pi)R$, and we have seen [1] that this quantity measures σ, the surface density of distribution over a conductor at any point of which the intensity of the field is R, so that $R=4\pi\sigma$.

The direction of R is normal to the surface. If the conductor be a plane sheet, then it has been shown that the equation becomes $R=2\pi\sigma$. In these expressions R is measured in electrostatic units, *i.e.* by the fall per centimetre of the electrostatic potential. We have seen that one electrostatic unit of potential is equal to 3×10^{10} electromagnetic units or $3\times 10^{10}\times 10^{-8}$ volts, and this is equal to 300 volts. Hence the electrostatic unit of intensity is equal to 300 volts per centimetre.

Experiment shows that the maximum value possible for R in air is about 100 (which is equivalent to $100\times 3\times 10^{10}\times 10^{-8}$ or 30,000 volts per centimetre). We therefore see that in air and at a distance from other conductors the surface density of charge on a sphere or cylinder cannot exceed $100/4\pi$ or about 8, while in the case of an infinite plane it may theoretically reach double this value. As a plane must in practice have edges, the only way of obtaining values approaching this would be to encase the edges in insulating material of higher electric strength than air.

§ (3) MECHANICAL FORCES ACTING ON CHARGED SURFACES.—The normal mechanical force acting on a charged surface is represented [2] by $F=\frac{1}{2}R\,.\,\sigma$, R being the intensity of the field at the surface. This is equivalent to

$$F=\frac{R^2}{8\pi}=2\pi\sigma^2.$$

Since R has a maximum value of about 100 in air the greatest mechanical force that can be exerted by electrification is about $100^2/8\pi$ or 400 dynes per sq. cm., which is equivalent to the weight of about 0·4 gm. Forces up to something of this order are therefore available for the measurement of voltages.

If a condenser with air as the dielectric has a capacity C, this will be increased by substituting most other dielectrics for the air. If the resulting condenser has a capacity KC, K is termed the *Permittivity*; it is commonly known as the *specific inductive capacity.*

Conductors immersed in a dielectric of permittivity K have the force reduced [3] to 1/K of the value when air is the dielectric, if the electric *charge* is constant. When the electric *voltage* is constant the force is increased K times to $KR^2/8\pi$.

[1] See "Units of Electrical Measurement," § (13); "Electrostatic Field, Properties of," § (4).
[2] See "Electrostatic Field, Properties of," § (5).
[3] "Units of Electrical Measurement," § (11).

In the construction of electrostatic voltmeters and similar apparatus for high voltages there is a direct gain in the forces available in immersing them in paraffin or similar oils (K=2). There is a still greater gain possible in that R can be considerably increased above what is practicable in air, so that KR^2 for good oils may have a working value 20 to 50 times what is practicable in air.

The value of K for air is about 1·0006, for hydrogen 1·0003, while all solid and liquid materials vary from about 2·0 upwards.

The following list, taken from Kaye and Laby's *Physical and Chemical Constants,* which may be consulted for further information, gives the values for a few materials:

SUBSTANCE.	k.
Solids :	
Calcite	7·5-7·7
Ebonite	2·7-2·9
Fluorite	6·8
Glass, Crown	5-7
,, Heavy Crown	7-9
,, Flint	7-10
,, Mirror	6-7
Ice	93·9
Indiarubber	2·1-2·3
Mica	5·7-7
Paper, dry	2-2·5
Paraffin wax	2-2·3
Pitch	1·8
Porcelain	4·4-6·8
Quartz	4·5
Resin	1·8-2·6
Rock Salt	5·6
Selenium	6·1
Shellac	3-3·7
Silica, fused	3·5-3·6
Sulphur	3·6-4·3
Vaseline	2·2
Liquids :	
Alcohol, methyl	35·4/13·4°
,, ethyl	26·8/14·7°
,, amyl	16·0/20°
Aniline	7·30
Benzene	2·29/18°
Carb. bisulphide	2·62
,, tetrachloride	2·25/18°
Ethyl acetate	6
Glycerine, λ=200	39·1/15°
Oil, castor	4·6-4·8
,, olive	3·1-3·2
,, paraffin	4·6-4·8
Petroleum	2·0-2·2
Toluene	2·3
Turpentine	2·2-2·3
Vaseline oil	1·9
Water, λ=∞	81
,, λ=3600 cm.	3·32*
,, λ=1200 cm.	2·79*
	76 cm. Hg.; λ=∞.
Gases :	
Air	1·000586
Hydrogen	1·000264
Helium	1·000074
Nitrogen	1·000581
Carbon dioxide	1·000985

* Beaulard, 1908.

Shellac, oils, rubber, ebonite, have values between about 2 and 3; glass and micas 6 to 9. Many liquids are much higher, water being about 80 at ordinary temperatures, but at very low temperatures the value seems to become constant at 2 to 3 for a variety of

liquids which are widely different at ordinary temperatures.

§ (4) ENERGY STORED IN DIELECTRICS WHEN ELECTRIFIED.—It has been shown [1] that in an electrified dielectric there is stored in it a quantity of energy $KR^2/8\pi$ per unit volume, where R is the electric intensity, *i.e.* volts per cm. $\div 300$.

The fact that the establishment of an electric field requires a definite amount of energy is of great importance in physical phenomena and technical applications.

(i.) *Energy in Space near the Earth's Surface due to Potential Gradient.*—There is usually a vertical electric field [2] in our atmosphere which commonly has a value of about 1 volt per centimetre, and since 1 volt$=1/300$ electrostatic unit, the energy per c.cm. is therefore $1/300^2 \times 8\pi$. In a cubic metre the quantity is about $\frac{4}{9}$ dyne-centimetre. In a cubic kilometre the quantity is $10^9 \times \frac{4}{9}$ dyne-cm.$=4{\cdot}5$ kilogram-metres, or 33 ft.-lbs.

(ii.) *Energy stored in Space in an Overhead Transmission System.*—A very important case of storing of energy in space occurs in the case of high-voltage transmission lines. It is commercially economical to build these up to a length of 200 miles and voltages up to 200,000 where fuel is expensive and water-power is available.

Take as an example a transmission line of 100 kilometres in length, working at 150,000 volts. For simplicity consider a single phase circuit of two wires of 1 cm. radius, 300 cm. apart, supported so high that the capacity to earth may be neglected in comparison with the capacity between the wires. The capacity per cm. of the circuit is $1/(4 \log a/b)$, where a is the distance apart, and b is the radius of the wire, and this is equal to $1/(4 \times 5{\cdot}7)$.

The length of the line is 10^7 cm.

The voltage is $150{,}000 \div 300$ or 500 E.S. units, the peak value of which is $500 \times \sqrt{2}$.

The energy stored in space, when the voltage reaches its highest value, is

$$\tfrac{1}{2}CV^2 = \tfrac{1}{2} \times \frac{10^7}{4 \times 5{\cdot}7} \times (500 \times \sqrt{2})^2 \text{ dyne-cm. (ergs)}$$

$$= 11 \text{ kilowatt-seconds}$$

$$= 1100 \text{ kilogram-metres.}$$

This energy has to be supplied to the line twice every period to keep it charged when no energy is being taken out at the receiving end. A large part of the energy is restored again as the voltage falls; it is therefore of an oscillatory nature, given by an expression of the form ($E \sin \omega t$),[2] and the plant supplying it has to be sufficient to supply the necessary charging current. Suppose the frequency is 25 alternations per second, so that $\omega = 2\pi \times 25$, the effective charging current is $CV\omega$ or $10^7 \times 500 \times 2\pi \times 25/(4 \times 5{\cdot}7)$, and this is $3{\cdot}45 \times 10^{10}$ electrostatic units, which is equal to $(3{\cdot}45 \times 10^{10})/(3 \times 10^{10})$ or $1{\cdot}15$ electromagnetic units. Thus the effective charging current is $11{\cdot}5$ amperes.

The apparent power supplied to the line is $150{,}000 \times 11{\cdot}5$ volt amperes or 1700 kilovolt-amperes. Such a line requires, therefore, plant of the capacity of nearly 2000 kilowatts to be connected to it, even when the load to be supplied is negligible. If a 2000-kilowatt machine were supplying power to such a line it could supply a real load of over 1000 kilowatts as well as the charging current. The kilovolt-amperes would then be

$$\sqrt{1700^2 + 1000^2} = 1950.$$

Actually such a circuit would be capable of transmitting some 50,000 kilowatts.

(iii.) *Energy stored in Insulation of Machinery.*—Similarly in the case of alternating current plant there is an appreciable storing of energy and subsequent emission from the insulating material twice every cycle.

Take, for example, the insulation of a machine working with a mean effective voltage between the copper winding and the iron of 10,000 volts. Suppose the insulation is mainly of mica, 0·33 cm. thick, and that $K=5$. The potential gradient is $10{,}000/0{\cdot}33$ and $R = 10{,}000/0{\cdot}33 \times 300 = 100$.

The energy stored in the insulation will be $(5 \times 100^2)/8\pi$ or 2000 dyne-cm. per c.cm. If the copper circuit is inserted in the iron of the armature in 50 slots, 100 cm. long, there being 2 conductors per slot, and the mean circumference of the insulation is 10 cm., then the volume of the insulation will be $50 \times 100 \times 2 \times 10 \times 0{\cdot}33$ or 33,000 c.c. The total energy stored will be $2000 \times 33{,}000$ dyne-cm. or 0·66 kilogram-metre. This is equal to 4·8 ft.-lbs. If the frequency is 50 cycles per second, the energy is stored up and released 100 times a second, and the 4·8 ft.-lbs. is stored up in 1/200 sec., or at the mean rate of 960 ft.-lbs. per second. There is, therefore, nearly 2 horse-power of oscillating energy in the system irrespective of any loss which may be due to the imperfect qualities of the dielectric. Just as in any mechanical oscillating system energy must in practice be supplied to compensate for losses due to various kinds of friction, so in all dielectrics there must be some electrical loss. This loss appears as heat, which increases the temperature of the dielectric. In some materials such as air, when not over-stressed, the supply of energy required is negligibly small, less than 1/10,000 of the oscillating energy. At the other end there are many materials with large technical applications, such as varnished fibrous in-

[1] See "Electrostatic Field," § (6).

[2] See also "Atmospheric Electricity and Thunderstorms," § (1), Vol. III.

sulations, with values of 5 per cent or more, as mentioned later. The energy absorbed and appearing as heat commonly rises rapidly with increase of temperature and may reach 50 per cent.

It is interesting to note that materials with a moderate dielectric energy absorption have a distinct value when arranged as insulating material in suitable locations in a high-voltage system. Their property of absorbing energy and only giving out an appreciably smaller fraction after the voltage wave has passed, enables them to be of use in absorbing electrical shocks and surges which are inevitable on power systems, such "transients" being commonly caused by short circuits or lightning or by defective plant connected to the system. Materials of this nature act in a manner analogous to air reservoirs and safety-valves on hydraulic pipe systems, which help to prevent excessive pressures which might otherwise be set up by "water hammer" action.

§ (5) Applications of Dielectrics.—Every use of electricity requires conductors at different potentials. The separation of these is effected by the interposition of a dielectric. The qualities of importance in a dielectric depend upon its application.

(i.) *Mechanical Separation of Conductors at Low Voltages.*—In some cases its main function is the mechanical separation of a number of conductors in which the electric stress in the dielectric is relatively low, the necessity for mechanical strength governing largely the thickness chosen. The dry-core telephone cable is an example. In such cables each wire is spirally wrapped with paper ribbon. Two wires which are to be used together are spiralled together, and this process is repeated with pairs, the "lay" or pitch of spiral being modified in the different circuits so as to avoid troubles from inductance.

The voltage is low, not exceeding about 30 volts. Cables may contain 300 or more pairs of wires, and the individual circuits are distinguished by a colour scheme. They are always lead-covered. The dielectric efficiency is dependent on the dryness of the paper, which is dried out, after the cable is laid and jointed, by forcing dried air through it.

Another similar case arises in the submarine telephone and telegraph cable. The insulation is stressed electrically to a very low value. Mechanical strength must be adequate and largely governs the design of the cable, since continuity of service is of first importance. The materials used, gutta-percha and similar substances, have not as good dielectric qualities as mica, paraffin wax, and certain types of mineral oils; but they alone have the necessary mechanical properties such as toughness and flexibility combined with the rare property of being unaffected by salt water, for almost an indefinite time, even under very intense hydrostatic pressures.

For electric circuits such as house-wiring, for lighting and power, india-rubber is largely used as the foundation for the dielectric. The thickness is governed by mechanical considerations. The dielectric is usually protected by braiding which is treated with a wax preparation.

(ii.) *Insulation for Moderate Voltages.*—For machines up to 500 volts a foundation of fibrous materials such as paper and woven fabrics is very largely employed. These in themselves are very poor insulators, and cannot be used alone, as they are not waterproof. If a machine were started up after being cooled below the dew-point, as often happens in practice, it would be full of water in the interstices of the winding. This would result in sufficient electrolytic damage to cause a short circuit. Such fibrous materials must be made waterproof by being suitably impregnated. Shellac, oxidised linseed oil, and varnishes containing bituminous and similar compounds are largely used.

The impregnating is carried out *in vacuo.* After the air is pumped out the hot impregnating compound is run into the vessel and immerses the part of the machine, then air is admitted and forces the material into the interstices.

Synthetic varnishes promise to have a wide application. They are commonly made from phenolic compounds, which condense to more complex substances when treated with aldehydes. The action is brought about by catalysts and heat. In an intermediate stage a form soluble in alcohol may be obtained, which permits of varnished papers and fabrics being made. These may be wrapped in several layers on conductors or laid on one another to form flat insulation of any desired thickness. On being subsequently heated they become hard and waterproof. They resist heat better than materials of a shellac nature, which are liable to soften at high temperatures.

(iii.) *Insulation for High Voltages.*—Though fibrous material is very largely used for low-voltage plant, 500 volts, in which the thickness is often governed more by mechanical than electrical requirements, it is also largely used when the thickness is governed by the electric stress that has to be dealt with. Not only is this the case in such plant as transformers (2000 to 130,000 volts) but also in transmission cables in which the insulation consists of paper saturated with a material such as highly refined mineral oil.

For the electrical circuit of high-voltage moving machinery, such as dynamos driven by steam turbines, it is essential that the insulation should occupy as little space as possible. The high speeds necessary, which are directly fixed by the frequency of supply, put a very definite limit to the diameter of

the rotating part. From a magnetic point of view space occupied by other materials than copper or iron is wasted, and a larger machine is required than otherwise would be the case. The insulation must therefore be as thin as possible and also withstand temperatures of the order of 150° C. Mica possesses the desired properties in the highest degree. Its main disadvantage is that it has to be used in small pieces. In covering the copper conductor of a high-voltage machine a large number of flakes of mica, each of which is one or two thousandths of an inch thick, are cemented together with a varnish to form a large sheet, which can be wound round the copper to the desired thickness. Heat and pressure are used to solidify the wrapping. Sometimes, for convenience of construction, the mica is attached to paper to make the wrapping process easier, the paper being wound on to the bar with the mica. In such cases the paper adds little to the electric strength of the insulation, as it is much inferior to an equal thickness of mica. The best course to adopt depends on general design and method of manufacture.

Mica insulation of this class is far from homogeneous. This is, electrically speaking, a disadvantage, but its use is essential for modern high-power generating plant. Without it designs would have to be seriously modified. There is strong economic inducement to transmit large amounts of power at the highest practicable voltage, which is largely governed by the limitations of the transmission cable and its insulation. The direct generation of high voltages demands an increased amount of insulating material in the plant as compared with the production of the same amount of power at lower voltages. This is of special importance in prime generators, such as turbo-alternators, as it leaves less room for copper and iron, just where it is most valuable. It is therefore not economical to generate power directly at very high voltages; but to wind the machines for any convenient economical voltage, and to transform up to the transmission voltage by static transformers

Generating voltages in such cases depend on local conditions. About 10,000 volts have been frequently used, where there is a local demand which can be fed at this voltage. Distant supplies can be carried out simultaneously by stepping up to 70,000 or 120,000, according to circumstances. These pressures are used for overhead transmission. For underground work 6600 and 11,000 are most common, 22,000 and 33,000 systems are increasing, and 66,000 is being installed.

One of the most valuable commercial dielectrics is mineral oil. As mentioned above, it is used very extensively for impregnating the paper used in power cables. It is also used in very large quantities for insulating transformers and for the immersion of the operating contacts of high-voltage switches. For these purposes it is commercially obtainable with an electric strength of 20,000 to 40,000 volts as the breakdown pressure between half-inch spheres 0·15 inch apart, while by special purification the value may be raised to 80,000.

Fibrous insulating materials may often be used in oil immersed transformers without impregnating materials such as varnishes, the oil performing the function of the varnishes which are necessary if fibrous material is used exposed to the air.

§ (6) Characteristics of Dielectrics.—All solids and liquids fall short of perfection as dielectrics.

(i.) *Leakage Resistance.*—A perfect dielectric would withstand electric stress indefinitely, so that two conductors immersed in it and charged to a definite difference of potential would maintain this difference for an indefinite time. In practice the difference of potential will diminish more or less quickly. In the case of very good insulators the rate may be very slow, condensers may be made so as to lose only a small fraction of their charge in a day when in a dry atmosphere if insulated by amber, quartz, or soapstone. In the case of other dielectrics such as treated fabrics the leakage may be immensely greater.

(ii.) *Dielectric Hysteresis.*—Gases such as air are practically perfect dielectrics if not stressed too highly.

In addition to very high resistance to leakage, it is for very many purposes of the first importance that the energy stored up in a dielectric, when stressed electrically, should be recovered as far as possible when the electric stress is reduced to zero. Just as a definite amount of energy is absorbed in iron when the magnetic field in which it is situated is altered and brought back to its initial value, so a definite amount of energy is required when the electric stress in a dielectric is altered in a similar manner.

In the case of magnetism[1] the energy absorbed per cycle is represented by $\int BdH$. In dielectrics the energy is $\int IdV$ when I is the polarisation and V the voltage. The form of the hysteresis curve naturally differs from that of iron, which is complicated by the great range of permeability. The electrical hysteresis figure is composed of two nearly straight lines meeting at the ends and slightly bowed from one another in the middle. This dielectric hysteresis becomes important when the power available is very small, when the frequency of alteration of the electric stress is high, and when the distance of transmission is long. These conditions hold particularly for

[1] See "Magnetic Hysteresis."

telephone and telegraph transmission, especially when cables are used. The fact that every periodic alteration of the electric stress absorbs a certain amount of energy, requires this energy to be provided by the generating plant (a telephone transmitter, for example) over and above that required by the ohmic resistance of the conductors. The extra energy required may be simulated by connecting the conductors together by an impedance of suitable value, the loss in which is equivalent to the hysteresis loss in the dielectric.

It is to be noted that the energy absorbed in losses of this nature is approximately a constant value per cycle, and therefore the power required to supply this energy absorption is proportional to the frequency. The importance at telephonic frequency (800 cycles per second) with a small amount of power available may therefore be very great, if dielectrics of unsuitable character or condition are employed. At radio frequencies, 10^6 per second, the effect may be so pronounced as to very rapidly raise the temperature, and even set on fire insulating materials which are quite suitable for the same voltages at frequencies, such as 50 per second, in common use for power transmission.

The accurate measurement of the properties of dielectrics is therefore a matter of great importance, especially as the actual ohmic resistance is no guide to the value of dielectric hysteresis. The power absorbed for a given alternating voltage is often many thousand times that which would be the case under constant steady stress of the equivalent value.

§ (7) Dielectrics surrounding Long Cylindrical Conductors. (i.) *Homogeneous Dielectrics; Cables.*—Economy in the transmission of large amounts of power requires high voltages to be used to save copper. The saving of copper is important as a matter of prime cost and standing charges, and also on account of the impracticability of transport and laying of cables of very large section. Cables have to be wound for transport on drums which must be large compared with the diameter of the cable. A limit is set by the size of drum that can be handled and by the cost of making the joints which have to be made after laying. The joints increase in number if heavy cable, which can only be handled in short lengths, is used.

The commercial importance of employing high voltages for power transmission is therefore great, and for underground work the limit is fixed by the efficiency of the dielectric insulating the conductors in the cable.

For the great majority of cases where high-voltage transmission is used three-phase currents are employed, three insulated conductors being covered by a cylindrical lead sheath. Paper wound spirally in strips about 1 inch wide and thoroughly impregnated with an insulating oil is used as the insulation. The number of layers of paper used depends upon the voltage. The successive layers are wound so as to "break joint," and the narrowness of the strip permits of the cable being wound on and off the drum without damage, the layers sliding over each other slightly when the cable is bent.

When the insulation surrounding a cylindrical conductor is thick the electric stress at the inner surface is higher than at the outer, if the insulation is electrically homogeneous. To take a plain concentric cable as an example, the electric intensity at P due to a difference of potential V, between the inner and outer conductor is inversely proportional to the distance, r, of P from the centre (*Fig.* 1). Hence k being a constant we have

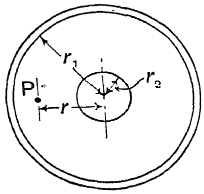


Fig. 1.

$$\frac{dV}{dr}=k\frac{1}{r}, \quad dV=k\frac{dr}{r},$$

$$V=k(\log r_1-\log r_2),$$

$$k=\frac{V}{(\log r_1-\log r_2)},$$

$$\frac{dV}{dr}=\frac{1}{r}\frac{V}{\log r_1-\log r_2}=\frac{1}{r}\frac{V}{\log (r_1/r_2)},$$

(see left side of *Fig.* 2). Suppose $r_1=2r_2$, then dV/dr near the inner surface of the dielectric $=V/r_2 \log 2=V/r_2\times 1{\cdot}44$, and near the outside surface it is $V/2r_2 \log 2=V/r_2\times 0{\cdot}72$.

The mean gradient is V/r_2, so that the maximum stress is 44 per cent above the mean.

For a given outside diameter of the dielectric the electric intensity diminishes with thickness of the dielectric until the radius of the inner conductor reaches a definite fraction of the outer diameter of the insulation. For

$$\frac{dV}{dr}=\frac{V}{r_2}\frac{1}{\log (r_1/r_2)}.$$

This expression has a minimum value when $r_1/r_2=e=2{\cdot}718$. In this case we find at once $dV/dr=V/r_1\times 2{\cdot}718$.

The values of the gradient for $r_1/r_2=2{\cdot}5$ and $r_1/r_2=2{\cdot}9$ are, however, less than 1 per cent in excess of the minimum value. The question whether it is economical to give so much space to insulation at the expense of the size and carrying capacity of the inner conductor is a technical one which will considerably modify the design of the cable.

If the cross-section of the inner cable is increased three times, for instance, so that its radius becomes $r_1\sqrt{3}/2{\cdot}718$, the potential gradient at its surface is only increased some

30 per cent above the minimum value. It is also to be remembered that increasing the thickness of insulation keeps the inner conductor at a higher temperature, which directly limits the current carrying power. The temperature gradient in the insulation is considerable, and if the material used has an appreciable rate of change of permittivity or other characteristic due either to its physical condition or mode of manufacture, the potential gradient will be seriously modified from that indicated by the theory outlined.

(ii.) *Grading of Cables.*—If the inner layers of insulation of a concentric cable are constructed of a material of a higher permittivity than the outer layers, the potential gradient in the space it occupies will be reduced below

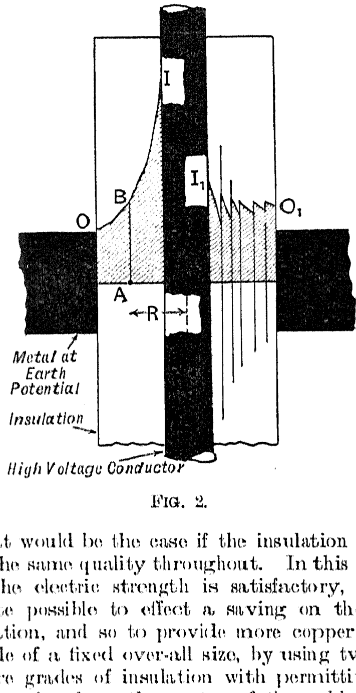


FIG. 2.

what would be the case if the insulation were of the same quality throughout. In this way, if the electric strength is satisfactory, it is quite possible to effect a saving on the insulation, and so to provide more copper in a cable of a fixed over-all size, by using two or more grades of insulation with permittivities decreasing from the centre of the cable outwards, the values of the permittivity being roughly inversely proportional to the radius.[1] The general nature of the result is illustrated in *Fig.* 2.

§ (8) DIELECTRICS SURROUNDING SHORT CYLINDRICAL CONDUCTORS.—The discussion of the case of a long cylindrical dielectric stressed radially shows that the only practicable method of reducing the stress near the inner surface is to increase the value of the permittivity relatively to that of the outer parts.

When the length of a dielectric of cylindrical form is short, not more than a few diameters long, the potential gradient through the dielectric may, however, be equalised by a suitable distribution of conductors embedded in it. Such cases arise when high-voltage conductors have to pass through the metal casings of switches, transformers, etc. When the insulation is thick compared with the diameter of the hole this method becomes of great technical importance.

If a plain cylindrical insulation be used, the potential gradient at any point A across it will be represented by the height AB of the ordinate of the curve IO at A.

The curve has the form AB × R = constant. The potential gradient at the inner surface will be greater than that at the outer surface in the ratio of r_1/r_2. As the inner surface will naturally be the hotter, the material will be considerably weaker, electrically speaking, if made of varnished paper, such as is often used.

It is, therefore, very desirable to equalise the gradient. This can be done when the axial length of the conductor is not very long, by embedding a series of cylindrical metal conductors in the material, of a size so graduated that the capacity between two neighbouring ones is constant, assuming that they are separated by equal radial thicknesses of insulation. As the capacities in series are equal the potential gradient between successive layers will be practically equal. There will be a negligible difference in the exact shape of the curve depending on the radius.

The potential gradient curve takes the form of the saw-toothed line, I_1, O_1, the area of the two shaded portions being equal. It is desirable to connect the inner conductor to the innermost metal foil, and the outer one likewise to the outermost foil. This avoids any danger of brush discharge and surface leakage which is otherwise liable to occur and to damage the material when contact between metal and insulation is imperfect.

As insulation of all kinds depends for its efficiency on being kept dry, it is especially desirable to expose as little as possible of the "end grain" of tubular insulators made of materials of the nature of paper. When they are made of porcelain and moulded materials, insulators are commonly made somewhat taper in shape. This should not be done in the case of paper tubes, which should be of full diameter to the ends. "Condenser" insulators of this type are being used for plant carrying very large amounts of power at high voltages. They have the great advantage over porcelain in not being brittle and they can be made by plant commonly available in electrical works.

To achieve the equality of the successive condenser elements they must be made of equal area and, therefore, the axial lengths must be inversely proportional to their radius—

[1] A. Russell, *The Theory of Electric Cables and Networks.* Constable.

that is, their lengths must follow the ordinates of the curve OI of potential gradient for the homogeneous dielectric.

The lengths need not be symmetrical vertically. If one end is immersed in oil, as will often be the case, it may be more economical to allow most of the tapering to be carried out at the upper end. In practice many more cylinders more closely spaced are used than are shown in the diagram.

The above is only a first approximation to the design of insulators of this type. There is an additional effect due to the part of the central conductor beyond metal cylinders. It has an appreciable capacity effect with the outside areas of the longer cylinders.

§ (9) Limitations of the Application of Dielectrics by the Type of the Electric Stress.—An important technical application of dielectrics is their use for improving the power factor of alternating circuits in which the current would naturally lag behind the voltage, as occurs when induction motors are connected to the circuit. The line current can be reduced by bringing it into phase with the voltage, or approximately so, by connecting to the motor end of the circuit a condenser of suitable size. Such condensers are commonly made of paper, the Mansbridge type being very largely used. Each condenser may be only capable of withstanding a few hundred volts, so that a number have to be put in series to deal with several thousand volts. If they have the same value the voltage will divide itself evenly between them.

Such an arrangement is perfectly suitable for alternating voltages, but is quite impossible for continuous voltages of similar values. If a steady continuous voltage is applied to a number of equal condensers in series, the first effect will be that they will divide the voltage equally between them. The leakage across each condenser will then come into play, and as their resistances will inevitably vary, the voltage across the most leaky one of the chain will gradually fall, increasing the average voltage on the rest. The same effect will go on as regards the remainder, leaving the condenser with the highest resistance to take a very large part of the whole voltage, under which it will fail. Assuming this one to be practically short circuited by such failure the rest will suddenly have an increase in voltage applied to them. The same cycle is gone through, the condensers failing one after another, leaving the most leaky one till the last, other things being equal. Under alternating stress there is no time for this leak to affect the voltage distribution, the capacity current being very much greater than the leakage current.

Condensers for low voltages can only be used in series for high continuous voltages, if subsidiary arrangements are made to prevent the operation of the differential leakage above described. One way of doing this is to short circuit each condenser by a conductance which is fairly large compared with its own conductance. In many cases a megohm or more across each condenser would prevent the voltage distribution becoming seriously uneven if the resistance of the most leaky condenser were several megohms. If such a method is inadmissible it is necessary to design the condenser units so that each will withstand the whole voltage. The Mosciki condensers are made especially suitable for such purposes. They are of tubular form, glass being used as the dielectric, and they can be made to withstand high voltages.

High-voltage D.C. is used in the Thury system of power transmission. A certain design of high-voltage cable may be suitable for alternating current, but unsuitable for high continuous voltages, such as is used in the Thury system of power transmission. The body of the insulation may be considered as equivalent to a number of condensers in series separated by infinitely thin metal laminae. The voltage distribution will be definite under alternating stress, but under continuous potentials it may be quite different, as it will depend to a very important extent on the relative ohmic resistance of the various layers. If this varies with temperature, for instance, the potential gradient will be different when the inner conductor is hot from what it is when it is cold.

Suggestions have been made for controlling the potential gradient across cable insulation, by dividing it into concentric cylinders with thin metal foil between them. The various metal foils are connected to different points of a potential divider which may be a high resistance connected across the whole voltage. The function of the resistance is identical with that of a leak across each of a number of condensers in series previously described.

§ (10) Methods of Measurement of the Properties of Dielectrics.—The investigation of a dielectric requires electric stress to be applied to it by conductors charged to a suitable potential difference. For many purposes dielectrics are used in flat sheets, such as mica and paraffined or varnished paper. These may be conveniently made up into condensers by interleaving them with sheets of tinfoil, alternate sheets being connected together into two groups forming the conductors by which the electric stress is applied. In other cases, as in many details of high-voltage plant, such as the terminals of transformers, machines, and switches, cylindrical forms are used. In such cases internal and external metal coverings can be applied.

For making measurements two principal

methods are commonly used. They are (i.) Bridge methods and (ii.) Wattmeter methods.

§ (11) BRIDGE METHODS FOR TESTING DIELECTRICS.—A condenser constructed of the insulating material to be tested forms one part of a bridge.

(i.) *Direct Current Tests.*—If it is tested by using a battery as a source of power as in *Fig.* 3, T being a detector such as a high resistance galvanometer, the value of the apparent resistance will be given in the usual way, Res. of D = (B/A)C. For a majority of dielectrics made from organic materials, R will increase with the time of application of the electric stress, at first rapidly and later more slowly. It is customary to take the value after one minute's electrification as a criterion; but without a knowledge of the form of the time-resistance curve such an arbitrary figure has not much value.

In practice, measurements of this nature are not usually carried out by bridge methods, as it is impracticable to make a galvanometer, T, sufficiently sensitive. Direct deflection

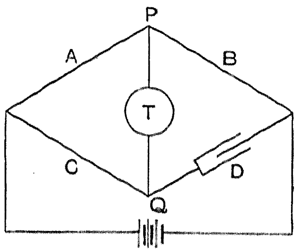


FIG. 3.

methods are employed. Such tests at steady potentials give little or no idea of the performance of an insulator under alternating potentials.

(ii.) *Low Frequency Alternating Current Tests.*—The arrangement shown in *Fig.* 3 cannot be balanced for alternating currents. If A and B are similar resistances or impedances, and D is the condenser under test, C must be given characteristics similar to those of D to obtain a balance, T being a detector sensitive to alternating currents, such as a telephone or vibration galvanometer.

If a variable air or other condenser with practically perfect insulation be substituted for the resistance C (*Fig.* 4), a close approximation to a balance may be obtained if the dielectric of D is of good quality; but in order to obtain a complete balance the power loss in C must be made equal to that in D, assuming A and B are equal. The currents in C and D will then be equal and in phase, so that the potentials at the ends of T are the same at every instant. A and B must be similar. They may be resistances or capacities or inductances. Their function is to act as a potential divider.

The power loss in C may be made equal to that in D by two methods:

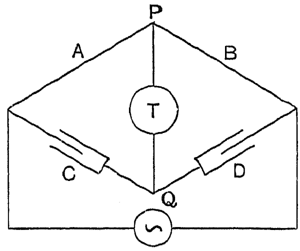


FIG. 4.

(1) By the addition of a resistance in series with C, R_s (*Fig.* 5).

(2) By a resistance in parallel with it, R_p (*Fig.* 6).

The relation of these is such that the power absorbed by the charging current passing

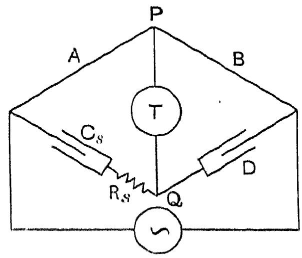


FIG. 5.

through R_s, I^2R_s is equal to the V^2/R_p loss, where V is the voltage on C, either being equal to the power expended in D. In practice R_s will often be small, a few ohms, while R_p will be several thousand ohms. The value of

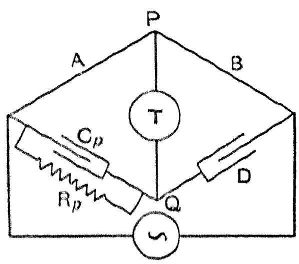


FIG. 6.

capacities C_s and C_p will be slightly different in the two cases.

If A and B are resistances, C and D being condensers with small power loss, it is to be noted that though the voltage across PQ may be zero at every instant, the current in A and

B is very different in phase from the current in C and D. The two currents are approximately in quadrature.

(iii.) *Tests at Audio Frequency.*—Bridge methods of this type are particularly suitable for audio frequencies, a telephone being used as a detector. The impedance of the telephone should be of the same order as that of the bridge arms. These methods are also specially suitable for low voltages and for materials which have very small losses. Condensers made of paraffined paper generally have an energy absorption of 2 per cent or less of the apparent power (voltage × current) passing through them. Mica condensers are often considerably under 1 per cent. The ratio of power absorbed to apparent power is termed the power factor.

In such cases the value of R_s, the resistance in series with a perfect condenser, which is required to produce a balance is quite small and the corresponding value of R_p very large.

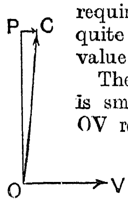


Fig. 7.

The vector, when the power factor is small, is shown in *Fig.* 7 where OV represents the voltage and OC the current vector. The direction of OC differs from OP, the normal to OV by 2° or 3°. The power is PC, parallel to OV, and the apparent power is OC. The power factor is PC/OC.

In such cases OP and OC are practically equal, which is equivalent to considering C_s and C_p equal. Let their values be called K.

The relation of R_s and R_p for equal power losses may be obtained as follows: Let V be the voltage on C or P and I the current passing. Then the power expended in $R_p = V^2/R_p$. The power expended in $R_s = I^2R_s$ and $I = KV\omega$, where $\omega = 2\pi n$; so the power in $R_s = K^2V^2\omega^2R_s$. Eliminating V we have $K^2\omega^2R_sR_p = 1$. The power factor is $V^2/R_pV \, . \, KV\omega = 1/R_pK\omega$, and is also equal to

$$\frac{I^2R_s}{V \, . \, KV\omega} = \frac{(K \, . \, V\omega)^2R_s}{KV^2\omega} = R_sK\omega.$$

To take a numerical example. Suppose a condenser of 0·01 microfarad is being tested against an air or other condenser of negligible power absorption, the frequency being about 800 so that $\omega = 2\pi n = 5000$, and suppose the series resistance R_s required to effect a balance is 400 ohms, then the power factor is

$$R_s \, . \, K \, . \, W = 400 \times 0{\cdot}01 \times 10^{-6} \times 5000 = 0{\cdot}02,$$

i.e. 2 per cent. The same result would be obtained if a resistance of a megohm in parallel with the air condenser were found to be required, for in this case the power factor

$$1/R_pK \, . \, \omega \, . \, \frac{1}{10^6 \times 0{\cdot}01 \times 10^{-6} \times 5000} = 0{\cdot}02.$$

In many cases it is preferable to use the series resistance rather than the parallel resistance arrangement. Smaller values are required, and the error due to the self-capacity of high resistances when used in this manner is avoided. This error is liable to be serious, as it may be an appreciable fraction of the value of the other capacities of the circuit.

A very important precaution which is necessary in all work of this nature is to guard against errors due to distributed capacity between the various parts of the circuit and the earth. This can be avoided by "virtually" earthing certain points of the system by such methods as that suggested by K. W. Wagner.

§ (12) Wattmeter Methods of Testing Dielectrics.—For measurements at high voltages 0·5 to 100 kilovolts or more, Bridge methods become unsuitable and dangerous, especially when working near the voltage at which the insulation breaks down. In addition the power factor changes very rapidly under such conditions, and it is impossible to obtain accurate measurements requiring a double adjustment of condensers and resistances.

A better method is to measure the power directly by means of a wattmeter. For considerable lengths of high-voltage cables it is possible to use a dynamometer wattmeter,[1] having a current coil of a large number of turns designed to carry currents of the order of 0·1 to 1·0 ampere.

A more universal instrument is the electrostatic wattmeter, which can measure the power over a very wide range of currents by simply varying the resistance through which the current passes. By immersing the instrument in a suitable oil, high voltages up to 60 kilovolts have been applied directly to the instrument itself.

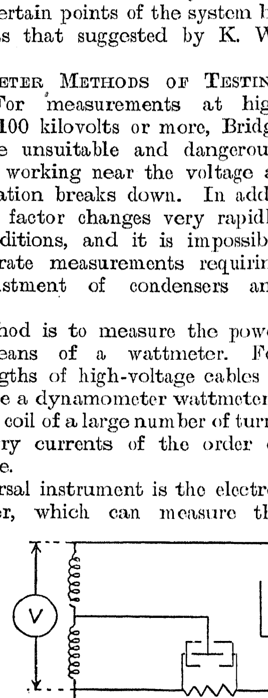


Fig. 8.

One of the most suitable methods of using an electrostatic wattmeter is to supply the needle with a potential equal to half of that applied to the dielectric (*Fig.* 8).[1]

Many types of insulating materials such as fabrics treated with varnishes and gums show great increase of energy absorption under alternating stress as the temperature rises. Wattmeter tests will show that starting from a certain atmospheric temperature the power absorbed rises to commence with, as the temperature rises. If the cooling conditions are suitable, the condition may become stable

[1] See "Alternating Instruments, Wattmeter," § 20.

and the insulation may be able to withstand the electric stress for several hours or days. An increase of atmospheric temperature may, however, so increase the dielectric losses that the temperature stability is no longer possible, more heat being evolved than the cooling conditions can cope with. The temperature will then rise continually, the increase of dielectric power absorbed increasing rapidly all the time, so that failure finally occurs. With the assistance of the indications of a wattmeter the temperature may be so finely adjusted that the stable condition will change to the unstable by increasing the temperature one or two degrees. In such cases the upward tendency of the amount of power absorbed may give an indication of inevitable failure an hour before it actually occurs. Stability or instability of temperature condition may be brought about by starting or stopping an electric fan circulating the air in the neighbourhood of the specimen. If the fan be stopped and the temperature commence to rise at a continually increasing rate, it may be checked and, if not gone too far, the stable condition may be re-established by starting the fan again. The insulating materials, such as varnished cloths, which are largely used for electrical plant, have a much larger power factor than paraffined paper or mica. The power factor may be from 5 per cent to 20 per cent or more. At ordinary temperatures, 15°, the power absorbed is approximately proportional to the frequency, at ordinary commercial frequencies, such as 50 ∿. This proves that the loss is of the nature of a hysteresis effect, the polarisation lagging behind the electric field, as in iron when the magnetic induction lags behind the magnetic field. At higher temperatures, when the loss is very much greater, the loss is only slightly affected by frequency, the energy due to the very great diminution of ohmic resistance of the material becoming a relatively, if not predominantly, important factor.

Information on this point can be obtained by measuring the resistance to continuous potentials while the material is subjected to alternating potentials of several thousand volts. The arrangement is shown in *Fig.* 9. A battery B and a D.C. ammeter A are placed in series with the alternating supply. The battery B may be about 50 to 100 volts, and A is preferably a micro-ammeter, which will only respond to continuous currents. It has to carry a superposed alternating current many times its rated value as a D.C. ammeter. The needle may vibrate under the oscillating torque; but the instrument can be made to give no material deflection under the influence of the alternating current alone. The current supplied by the battery B has to traverse the high-voltage winding of the transformer supplying the power, but its resistance will be negligible compared with that of the insulation under test. Before applying the alternating voltage the ammeter A will give a negligible reading if the insulation is cold. When alternating voltage is applied a measurable continuous current may result, from which and from a value of the voltage of the battery an estimate of the resistance of the insulation to direct currents can be made. This resistance will be found to account for only a small part of the power indicated by the wattmeter to commence with, but as the material warms up a great increase in the ammeter reading occurs, and when the material is nearly at the point of failure the resistance computed from the direct-current readings will, in certain materials, account for practically all the power indicated by the wattmeter.[1]

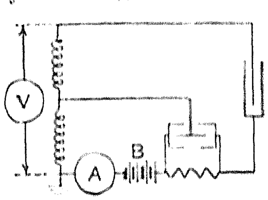


Fig. 9.

§ (13) Measurements at Radio Frequencies.[2]—Accurate measurements at radio frequencies, 10^4 to 10^7, are very difficult, partly because the ear loses its sensitiveness at about 10^4 and direct telephonic methods are no longer available, and also because the distributed and residual capacity and inductance errors increase greatly. It is impossible to make inductances free from resistance or distributed capacity; resistances likewise have inductance and capacity. Condensers are easier to make "pure" than either resistances or inductances. Radiation may also play a serious part in such measurements.

Probably the most suitable method is to measure the energy absorbed thermally. The dielectric may be subjected to a known voltage as measured by an electrostatic voltmeter and its increase in temperature measured thermometrically. With a knowledge of the necessary thermal quantities the dielectric losses may be estimated.

§ (14) Routine Tests on Insulation of Electrical Plant. A.C. Tests.—It is important to subject the insulation of electrical plant to an adequate, and yet not excessive, electric stress before it is put into service. The object of such a test is to prove that construction has been carried out properly and that there are no serious accidental weaknesses in the insulating material. The design of the apparatus ought to provide adequate insulating material, and except in special circumstances, such as the proving of a new

[1] Rayner, *J.I.E.E.* xlix. 47.
[2] See "Radio-frequency Measurements," § (19).

design, the electric stress test should be regarded as a construction—rather than a design—test.

A suitable ratio between the test voltage and working voltage depends upon various factors. In the case of low-voltage plant, 100 to 500 volts, the thickness of the insulation is largely governed by mechanical considerations, and the thickness required ought generally to withstand 2000 volts. A failure under this voltage indicates mechanical weakness such as cracks, imperfect impregnation by insulating varnish, etc. A factor of 10 or 20 may seem large; but a lower test voltage may not reveal important defects. Alternating voltage at 25 to 100 cycles per second is always used, largely on account of its simplicity of production. For plant designed for higher voltages a relatively lower test voltage suffices, and values from 2 to 3 times the working voltages are commonly used. It is generally recognised as reasonable that individual component parts, such as insulators alone, should be capable of withstanding somewhat more than the finished apparatus of which it forms a part, and that a test on an assembly of finished plant such as dynamos, switchboards, transformers all together, should not be subjected to the full pressure applied to each individual apparatus.[1]

The time of application of a test voltage is commonly 1 minute. Sometimes more than 1 minute is required, but there is a distinct danger that, if the time of application is unnecessarily prolonged, the insulation may be appreciably weakened, though the voltage applied may be materially less than what would produce failure, except after a very long time of application. The types of insulation of electric plant are so diverse, and are affected in different ways by high electric stress, the effect of which is also largely dependent on design, that it is practically impossible to say definitely what time of application of a given voltage will prove harmful. It is therefore wise to reduce the time to the shortest which may be considered reasonable. One of the factors to be kept in mind is the possible production of "Corona," *i.e.* ionised air surrounding conductors at high voltages. This may occur under the test voltage while not at the working voltage. The ionised air has highly oxidising properties and organic insulating materials should not be subjected to its action unnecessarily.

Many individual pieces of insulation of electrical plant cannot be tested to destruction in air. The discharge will pass round over the surface before going through. In such cases it is usual to immerse the whole in mineral oil, by which means much smaller pieces can be tested than when in air.

[1] For further details of such tests see the specifications of the British Engineering Standards Association relating to Electrical Machinery.

§ (15) D.C. Tests. Low Voltage up to 1000 Volts.—The measurement of the resistance of the insulation of plant is a very important test. It is made for the purpose of proving the continuity of the insulation and of determining whether it is in a fit state to be operated at the working voltage. If the insulation is damp, its resistance will be a small fraction of what it will be if properly dried. It is frequently impossible to prevent the temperature of electrical plant being below the dewpoint, especially during erection and before being put into service. In such cases the insulation may be thoroughly soaked for days together. Such a condition will generally be indicated by low resistance, and the plant should be dried out before being put into service. If this is not done, permanent damage might be done to the insulation at the working voltage, even though the leakage current were insufficient to produce obvious failure.

For tests of this nature 500 or 1000 volts are commonly used, special insulation resistance measuring instruments being employed giving a direct indication of the resistance. They are provided with a hand-driven magneto generator, and read from about 10,000 ohms to 20 megohms, and specially sensitive types are made to read up to 1000 megohms using generators giving up to 2500 volts.

§ (16) D.C. Tests. High Voltage.—The thermionic valve affords a simple and valuable method of obtaining D.C. voltages, and is used for this purpose in testing electrical plant. Power cables are tested in the makers' works in short lengths, usually by alternating current. After being laid, it is not always a simple matter to provide the necessary portable A.C. testing plant, especially if it is a question of testing long lengths, requiring a large charging current.

The ohmic resistance of the insulation of such cables is high, and only a small amount of power is required to provide the leakage current. Quite a small high-voltage transformer may be connected to such a cable through one or more thermionic rectifying valves. The cable will be gradually charged up by the passage through the valve of the current from alternate half-waves of the A.C. supply, until the value of the D.C. potential applied reaches approximately the peak value of the A.C. voltage wave. Such a method of testing provides a convenient means for proving the absence of serious damage or defective joints in cables after being laid.

E. H. R.

Dielectrics, applications and uses of. See "Dielectrics," § (5).

DIRECT-CURRENT INDICATING INSTRUMENTS

§ (1) INTRODUCTORY.—The Indicating Instruments at present in use for the measurement of Direct Current are almost entirely of the Moving-coil or Moving-iron types. The earlier forms of instrument, such as the Siemens' Dynamometer and Ayrton and Perry Spring Ammeter, are now only of historical interest.

Both types of instruments, in order to measure the current, make use of the magnetic field, produced by the current to be measured when flowing in a coil of suitable form.

In a voltmeter this coil is one of high resistance; the current circulating in it is a small one, and the instrument measures the voltage applied to its terminals; many turns of fine wire are employed.

In an ammeter used to measure currents of moderate amount the whole current may pass through the coil; its resistance is low, being wound with comparatively few turns of coarse wire.

For larger currents the whole current does not pass through the coil but through a shunt of known resistance made of a resistance alloy having a very small temperature coefficient, so that the heating due to the current does not produce an appreciable change in the resistance. The voltage drop between the terminals of the shunt is measured on a voltmeter, or rather a millivoltmeter, an instrument of comparatively high resistance which, since the voltage drop is proportional to the current in the shunt, can be graduated in terms of this current.

In all types of indicating instruments the moving part has a pointer attached which moves over a graduated scale, and hence indicates the applied voltage or the current.

(i.) *Moving-iron Instruments.*—In the moving-iron type the coil in which the current to be measured flows is fixed, and to actuate the pointer use is made of the attraction between this coil and a piece of soft iron of suitable form and suitably mounted. The motion of the pointer is controlled, in some cases by gravity acting on the moving iron, in others by a spring. The equilibrium position of the pointer in any case is that in which the electromagnetic attraction due to the current just balances the control force;[1] the current is thus measured. Many of the requirements described in the sections which follow dealing with moving-coil instruments apply equally to the other type.

(ii.) *Moving-coil Instruments.*—For the highest grade instruments, both for ordinary switchboard use and for precision measurements, the moving-coil instrument is employed.

The current to be measured traverses a coil which is suitably suspended in the field due to a permanent magnet. When a current flows in this coil it experiences a couple tending to set it with its plane at right angles to the lines of magnetic force due to this magnet; this couple is resisted by a spring control, and the pointer attached to the coil takes up a position in which the effect of the current just balances that of the spring; thus the deflection becomes a measure of the current.

The principle of a coil moving in a permanent magnetic field was used by Kelvin in his siphon recorders and by D'Arsonval in a galvanometer.[2] Weston developed the system, and in 1888 produced the first moving-coil ammeters and voltmeters.

This system is now used in the construction of instruments by makers in all parts of the world, and, while there has been considerable general improvement in detail and some

[1] See "Switchgear," §§ (25)-(27).
[2] See "Galvanometers," § (7) (ii.).

difference in design among the various makers, the essential principles of the original Weston instruments are retained. With the exception of the long-scale instruments, described later, a maximum angle of deflection of 90° is generally used, although instruments with an angle of 120° are now being introduced.

The every-day type of instrument consists of a U-shaped magnet (some makers use a horse-shoe pattern), to which are fixed pole-pieces of soft iron, the opposite faces of which are shaped cylindrically. A round core of soft iron is supported in the centre of the pole-pieces, and the coil moves in the annular gap between them. By this means the lines of magnetic force are rendered nearly uniform over the whole surface of the pole-pieces, and consequently the deflection of the coil

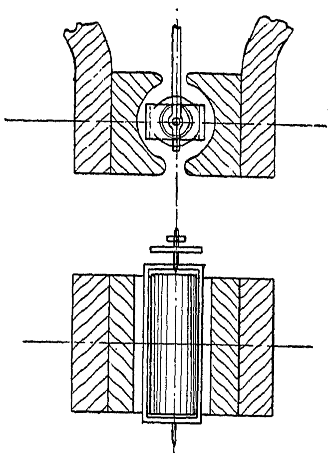


FIG. 1.

is nearly proportional to the current flowing through it. The control is effected by means of springs made of non-magnetic material, which serve also to lead in the current to the moving coil. In some cases one spring only is used, the second connection being made by means of a thin flexible strip connection which exerts little control; but, in general, two springs are employed, wound in opposite directions, thus compensating to a large extent for changes in their mechanical properties or due to variations of temperature.

Steel pivots are fixed to the moving coil, and these work in jewelled bearings fixed usually to the pole-pieces.

A typical moving-coil system is shown in *Fig.* 1, in which the upper figure gives a horizontal and the lower a vertical section through the coil and magnet poles.

§ (2) MAGNETS.—Since the accuracy of a moving-coil instrument depends on the constancy of the permanent magnet employed, the steel must be carefully selected and aged. Most makers employ systems of artificial ageing, and observations, extending over a large number of years, show that a high degree of constancy is obtained. Since it is desirable that the working forces should be large, the magnet should be as long and large as possible and the length of air-gap small. The ratio of the length of the magnet to its cross-section and the section of the air-gap to its length should be as great as possible. Edgcumbe [1] states that the product of this ratio usually varies between 150 and 500, and that it should in no case be less than 100; Fitch and Huber [2] give figures ranging from 190 to 505 for eight types of American-made instruments. The flux density depends on the size and magnetic quality of the steel and the size and shape of the air-gap. A. Campbell [3] gives values for (*a*) an Evershed Ammeter as 700, and (*b*) for a Weston Voltmeter of 870, and considered the latter to be a very high value. Edgcumbe (*loc. cit.*) states that the value in practice ranges from 500 to 2500, and Fitch and Huber (*loc. cit.*) give values ranging from 213 to 2790. The improvement shown in the more recent figures is due to improvement in the steel and the design, particularly in the direction of reduction of the air-gap.

§ (3) MOVING SYSTEM.—The moving coil is almost invariably of copper wire wound in one or two layers on a light copper or aluminium former. The shape of the coil is sometimes round, but more usually rectangular. The dimensions of the system, both former and winding, will, of course, depend on the torque required and the size of the air-gap.

In view of the comparatively large pressure per unit area on the small area of the bearings and the inertia of a heavy moving system, it is desirable that the weight should be reduced as much as possible consistently with mechanical strength and suitable torque. Modern practice has tended to the use of light-moving systems, the requisite torque being obtained largely by decrease of the air-gap. This has the further advantage that the damping, due to the eddy currents set up in the former, is supplemented by the viscous resistance of the air in the gap. But it requires that the workmanship should be exceedingly good, since the slightest contact between coil and magnet pole, or the smallest amount of foreign matter between them, makes the instrument unusable. In practice the advantages of this instrument are, however, so very great that it is satisfactory to note that in the Weston and some other makes, the

[1] *Industrial Electrical Measuring Instruments*, p. 145.

[2] *Bul. Bur. Stds.* vii. No. 3, 420.

[3] *Phil. Mag.*, 1899, xlvii. 1.

moving system is very light and robust, the air-gap very small, and the general behaviour of the instrument most satisfactory.

The actual weight of some moving elements given by Fitch and Huber (*loc. cit.*) ranges for voltmeters from 1·64 grammes to 3·5 grammes, and for millivoltmeters (ammeters) 2·17 grammes to 4·66 grammes. These, however, are all switchboard type instruments: the weight of the moving system of a precision type, such as the Weston or Elliott Standard instruments, would be much less than this, and in some cases probably nearer one gramme.

The size of the windings of moving-coil instruments may vary according to whether the instrument is to be used as an ammeter or a voltmeter. In the case of a millivoltmeter intended to be used with a shunt for the measurement of current, since the current passing through the moving coil is dependent on the pressure drop of the shunt, which must be small, the turns have to be low in resistance, while, in the case of a voltmeter where the current is controlled by a series resistance, the use of a larger number of turns of fine wire gives additional torque and at the same time keeps the total resistance of the instrument high, and reduces the power consumption. Some makers take advantage of this and use different types of coils for ammeters and voltmeters, but others find it possible to use a standard type of coil which will satisfy the requirements of both types of use.

§ (4) Ratio of Torque to Weight.—Janus [1] considered the ratio of turning moment (torque) to the weight of the moving system to be an essential consideration in moving-coil instruments. He took the case of a steel pivot ground to an angle of 60° working in a sapphire bearing, and states that for a deflection of 90° the ratio Torque/Weight should be approximately 0·17.

Fitch and Huber (*loc. cit.*) measured the torque exerted by the moving system and obtained values for the ratio ranging from 0·08 to 0·55. These refer to switchboard instruments, and it is probable that in the case of precision instruments, where the cost allows of better workmanship, the actual values would be considerably higher.

Edgcumbe states that the torque expressed in cm.-grammes for a deflection of 90° should be not less than 0·05 of the weight of the moving system expressed in grammes.

§ (5) Damping.—Efficient damping is one of the essential requirements of a good instrument. The term "damping" is sometimes used in connection with the number of swings which a moving system will pass through before it comes to rest at a definite point. In the use of an instrument, however, it is essential that the pointer should follow small variations rapidly, and since this condition will depend partly on the number of oscillations, and partly on the natural period of the moving system, it is more general to express the quality of the damping in terms of the time required to settle to a definite reading. The time will naturally vary with the size of the instrument, since in a large instrument having the same torque as a small one the free period will be longer, owing to the increased inertia of the long pointer and heavier moving system. This, of course, could be compensated for by increase of the working forces and the control, but the working forces are generally standardised so as to obtain interchangeability, and, in consequence, it is usual to allow a longer damping time for a larger instrument. The damping in moving-coil instruments is effected mainly by means of eddy currents in the metal former, on which the coil is wound, but it is assisted to a certain extent by the fluid friction of the air and also, in the case of ammeters, where the coil is connected across a shunt of low resistance, by the current generated in the coil when moving in the magnetic field. For this reason the damping time for millivoltmeters will apply only when the instrument is connected to the shunt with which it is to be used.

The specification No. 49, 1918, of the British Engineering Standards Association requires (clause 27), that:

"The motion of the pointer of every indicator shall be damped sufficiently to cause it to settle to a definite indication within a reasonable time"; and in an appendix gives a definition of a "reasonable time," based on a large number of tests made at the National Physical Laboratory, as follows:

With a scale not exceeding 4½″ in length, 2½ seconds.
" " 7 " 3 "
" " 12 " 5 "

Fitch and Huber (*loc. cit.*), giving results for a number of typical American moving-coil ammeters and voltmeters, find that in eight types of voltmeters the time varies from 0·8 to 4·6 seconds, and for a similar number of ammeters, from 1·1 to 5·8 seconds. (The actual length of pointer is not given for these cases, but it is probably from 6″ to 8″.) Edgcumbe (*loc. cit.*) gives values for moving-coil instruments of 0·6 sec. for a portable instrument, 2 secs. for an 8″ switchboard instrument, and 6 secs. for a large sector pattern instrument. It should, however, be noted that the figures given do not apply to the best modern types of precision instruments, where it is usual to find that the pointer goes straight to its reading, or, at most, only swings once through it, the time required to take up the final position being considerably less than one second. The same applies in a lesser degree to the better types of switchboard instruments,

[1] *Elektrotechnische Zeitschrift*, xxvi. 561.

in which in general the final position will be taken up in about one second, and the pointer will not swing more than once through its final position.

§ (6) Control Springs.—Control springs are a most important part of a moving-coil system. It is of course necessary to use a non-magnetic material, and hence phosphor bronze is generally used on account of its high elasticity. The series of alloys generally called phosphor bronze have a comparatively high resistivity, and while this does not affect a voltmeter in which there is a large swamping resistance, it may affect the sensitiveness of the low-resistance millivoltmeter considerably, since the resistance of the spring will be comparable with that of the coil, with the consequent loss of working force. For this reason alloys other than phosphor bronze have been, and are still, used in some cases, while in others a low-resistance bronze is used. Edgcumbe (*loc. cit.*), discussing the question, states that "the resistivity of phosphor bronze may be taken as about 10 times the resistance of copper, and that of a high conductivity bronze as about twice that of copper."

Edgcumbe discusses the dimensions of control springs, and states that the stress (of flexure) for ordinary phosphor bronze must not exceed 600 kgs. per sq. cm. if any tendency to permanent set is to be avoided. This limits the thickness of the spring in proportion to the length, and he states that the length should be at least 1500 times the thickness for a deflection of 90°. Increase of torque, therefore, must be obtained by increasing the width of the spring. Janus [1] describes a series of tests made with springs of various types and shows that with phosphor bronze the stress due to flexure may be taken up to 530 kgs. per sq. cm. without permanent set; but he considers that a limit of 280 kgs. per sq. cm. is advisable for instrument work. For a low-resistance bronze having a resistivity of 2·5 microhm-cm., Janus gives the following figures as a basis of calculation for a spring:

Turning moment (90°) = 0·4 dyne-cm.
Temperature coefficient of resistance = 0·35 per cent for 1° C.
Modulus of elasticity = 1,260,000.
Maximum strain of flexure = 200 kgs. per sq. cm.
Resistance of spring = 0·67 ohm.
Breadth = 0·18 cm.
Thickness = 0·0058 cm.
Length (in 8 turns) = 28·5 cm.

Thus the ratio thickness : length is in this case nearly 5000.

For hard-rolled copper Janus considers that the stress due to flexure should not exceed 180 or 200 kgs. per sq. cm.

Existing data regarding materials for springs are by no means complete, and an investigation to determine the elasticity, resistivity, and other constants of the alloys suitable for use as control springs appears to be urgently required.

[1] *E.T.Z.* xxvi. 562.

§ (7) Resistance and Energy Losses.—For moving-coil voltmeters the actual loss of energy involved in the measurement of the pressure is very small. The over-all resistance of a voltmeter is generally of the order of from 50 to 100 ohms for 1 volt, and the actual loss of energy on a 500-volt circuit need not be more than 5 watts.

The resistance of the moving coil itself will vary with different makes of instrument from 3 ohms to 50 ohms, a resistance of about 10 ohms being used by a maker who is employing a standard type of coil for both ammeters and voltmeters. The standard-type instruments made by such firms as Weston and Elliott Bros. have a resistance of approximately 100 ohms per volt, and, in most cases, such instruments will work with a pressure drop of 0·03 volt, or even less, for full-scale deflection, and are thus equally satisfactory for use with shunts as ammeters. The resistance of switchboard-type voltmeters of British manufacture is of the same order, and the figures given by Fitch and Huber (*loc. cit.*) for eight types of American voltmeters, range from 50 ohms to 120 ohms per volt.

In the case of the millivoltmeters used as ammeters, the standard practice in Britain is as defined in the Specification No. 49, 1918, of the British Engineering Standards Association, as follows:

"First Grade and Second Grade Instruments (suitable for Switchboard Industrial use) 0·075 volt; the resistance of the connecting leads specified is 0·025 ohm, and for sub-standard instruments either 0·0005 or 0·001 volt per division of the scale of the indicator, the connecting leads in this case having a resistance of 0·05 ohm."

This apparently allows a large range of pressure drop for sub-standard instruments, but since the use of a 150-division scale for such instruments is almost universal, it amounts in practice to a pressure drop of either 0·075 volt or 0·15 volt for a sub-standard instrument. The resistance of the moving system of the millivoltmeter varies largely with various makes, the values ranging generally from 1 ohm to 5 ohms for switchboard instruments, and from 1 ohm to 10 ohms for sub-standard instruments. The millivoltmeters examined by Fitch and Huber, when they are all reduced to a basis of 0·075 volt drop, had resistances varying from 1·4 to 5·9 ohms. Occasionally instruments have a resistance which is lower than 1 ohm, but since the contact resistance of the connections to the shunt has to be included in the circuit, it is desirable that the resistance of the instrument should be as high as possible.

§ (8) Pointer.—The pointer of a moving-coil instrument is almost invariably made of fine aluminium tube with a flat or knife-edge end, depending on the type of instrument. For precision-type instruments, the pointer is of the knife-edge pattern, the upper edge being painted black or red. A mirror, coming generally to the edge of the scale, serves to eliminate errors due to parallax. For switchboard-type instruments, the tip of the pointer is generally of palm-leaf shape, although some makers use slightly different shapes. Some typical shapes in general use are shown in *Fig.* 2.

To meet the case where a precise reading is required, and, in addition, visibility at a distance, Messrs. Elliott Bros. introduced the type No. 5 (*Fig.* 2). The thin portion of the pointer here moves over the scale fitted with the usual mirror, while the flat circle enables

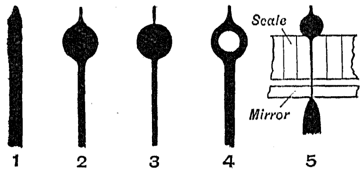


Fig. 2.

an approximate reading to be made from a distance.

The movement of the pointer is restricted by small stops, supported on springs, usually fixed to the scale or magnet. The stops allow the pointer to deflect over the full length of the scale, but serve to prevent damage should an excess current be passed through the instrument. These stops are generally insulated with glass tubing or rubber; of these glass or a similar material is to be preferred, since the rubber perishes and may become sticky with age. The British Engineering Standards Association Specification No. 49, clause 25, requires that the pointer shall be insulated from the electric circuit of an instrument unless the scale plate and stops are so insulated, and that it should be so shaped as to lend itself to ease and accuracy of reading. It specifies further that:

"If an instrument is not provided with means for avoiding errors of reading due to parallax, the clearance between the portion of the pointer which traverses the scale and the scale itself shall be not more than 1/100th part of the length of the scale for all instruments having scale lengths of 150 mm. and upwards, and not more than 1·50 mm. for scale lengths less than 150 mm.

"In the case of First Grade and Second Grade instruments the pointer shall be of such a length that it extends over more than one-half, but not over more than two-thirds, of the length of the shortest scale mark."

The requirements regarding distance of pointer from the scale have not always been adhered to, but the limits specified can be worked to in instruments of good manufacture with a consequent improvement in accuracy and ease of reading.

§ (9) Pivots and Bearings.—The use of two steel pivots, each working in a jewelled bearing is almost universal, although in one or two cases makers use a single bearing of phosphor bronze, or a jewel, the other end of the coil being suspended on fine-strip ligaments on which the coil is hung. Edgcumbe (*loc. cit.*) states that while there is some divergence of opinion as to the best form of pivot, an angle of about 60° is commonly employed. Laboratory (precision) instruments will have a fine point, while with industrial instruments a more obtuse angle is preferred. He also states that sapphires should always be employed for the bearings.

§ (10) Scales.—Examination of a large number of instruments shows that there is great diversity of opinion as to the best type of division and length of it. Not infrequently the dividing lines are far too long, with the result that at a comparatively short distance away the spaces between the lines cannot be distinguished, and it is most difficult to read the instrument. Kelvin drew attention to this malpractice many years ago, and showed that the length of a dividing line should be equal to or not more than 1·5 times the width of the space between two lines. The British Engineering Standards Association Specification deals fully with the question and specifies types of scale suitable for most types of instrument, and particularly for the moving-coil type, where the scale divisions are uniform. The Specification states, clause 22:

"(*a*) Value of Divisions.—The value of each scale division shall be either 1, 2, or 5 of the units measured; or any decimal multiple or sub-multiple of these numbers.

"(*b*) Width of Divisions.—For all Sub-standard instruments and portable type First Grade instruments the angle subtended by a scale division shall be not less than 0·5°. For all First Grade and Second Grade instruments other than those of the

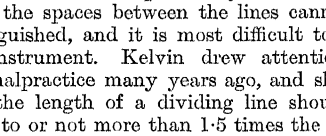


Fig. 3.

portable type the angle subtended by a scale division shall be not less than 1·0°.

"(*c*) Construction of Scale.—The scale shall be divided throughout its effective range and consistently with the requirements of section (*b*) of this clause, with scale marks arranged in accordance with one of the two systems illustrated in *Figs.* 3 and 4 (see also Appendix I.).

"*Fig.* 3 shows a scale formed of the necessary

number of groups of marks, each group consisting of a long mark followed by four normal marks; with a long mark at the end. Scales of this type shall only be used for indicators in which the scale divisions represent 1 or 2 of the units measured, or decimal multiples or sub-multiples of these numbers.

"*Fig.* 4 shows a scale in which a short mark subdivides each of the scale divisions of *Fig.* 3;

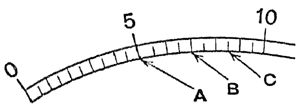


FIG. 4.

thus forming a series of groups of ten divisions followed by a long mark at the end. Scales of this type shall only be used for indicators in which the scale divisions represent five of the units measured or a decimal multiple or sub-multiple of 5.

"(*d*) Figuring.—The figuring of the scale, except in the early part of a non-uniform scale, shall be by steps of 1, 2, or 5, or a decimal multiple or sub-multiple of any of these.

"The figures shall be of such shape as to minimise the risk of different figures being confused with one another; and so spaced as to render individual groups clearly distinguishable from adjacent groups.

"In the case of scales subtending an angle of not more than 100° the figures shall be disposed symmetrically and similarly throughout the scale marks to which they refer.

"Scales of the type shown in *Fig.* 4 shall be figured at every long mark or at every alternate long mark.

"Normal and short-scale marks shall in no case be figured.

"Scales of the type shown in *Fig.* 3 should usually be figured at every alternate long mark, but may be figured at every long mark if the divisions represent 2 units (or a decimal multiple or sub-multiple of 2), and the figures are of such a size that the requirements of the preceding paragraph of this clause are complied with. They may also be figured at every long mark when the value of the divisions is 1 (or a decimal multiple or sub-multiple of 1) if in the absence of such figuring there would be less than one group for every 30 degrees of scale.

"Appendix I.

"Scales.—The following is recommended as the ratios of the lengths of the marks on scales:

Normal marks	. . . .	1·0
Long ,,	. . . .	1·7 to 1·8
Short ,,	. . . .	0·6 to 0·7

"It is also recommended that the length of the normal mark should be not less than 0·8 nor more than 1·5 times the distance between the scale marks defining the widest scale division over the useful range."

The thickness of the dividing lines is not specified; generally, on scales of precision instruments they will be as fine as possible, while with switchboard instruments a bolder line is more usual, and most frequently the lines at the main divisions are much thicker than the others.

§ (11) FIGURING.—Considerable attention has been given to the question of the shape of the figures used to denote the main divisions on a scale, particularly with a view of avoiding confusion between two nearly similar figures, such as 3 and 8. Trotter[1] describes the various systems in use and shows a set of

12345678 0

FIG. 5.

numerals designed to give maximum legibility. This is illustrated in *Fig.* 5.

§ (12) MOVING-COIL INSTRUMENTS WITH NON-UNIFORM OR SET-UP SCALES.—The greater number of moving-coil instruments in use have evenly divided scales graduated from 0 to the maximum value, but in some cases this arrangement is varied to suit the requirements of use. The non-uniform scale instrument has been devised to meet special conditions, such as those arising from the ordinary use of a battery where the normal (10-hour) rate would probably be about one-fifth of the maximum discharge rate, with the result that while the instrument is required to read up to the maximum, under the more normal working load conditions the load is too small to permit of readings being taken to the required accuracy. The ammeters for this special purpose have the zero mark at, or near, the centre of the scale, and the moving system is so arranged that the deflection from a given current at, or near, the zero mark is about three times the amount for the same current value at the upper part of the scale. This is generally effected by shaping the magnet poles so that the coil swings partially out of the field. Consequently, the working forces are decreased and the increase in the size of the lower-scale division is obtained largely by the sacrifice of the controlling forces.

A more common instrument is the voltmeter with a portion of the scale suppressed, so that the visible scale represents only a portion of the total movement of the control springs. In this case the springs are set back so that the pointer is forced back against the stops, and does not move until the pressure across the terminals of the instrument exceeds the value of the suppressed portion of the scale. The reading portion of the scale will be evenly divided, but since the increase of scale length can only be obtained either by increasing the working forces or decrease of control with consequent loss of torque, the extent of the suppression is generally limited. Edgcumbe

[1] *Journal I.E.E.* liv. 273.

(*loc. cit.*) considers that the suppressed portion should be not greater than twice the value of the visible part of the scale, and the Specification of the British Engineering Standards Association requires that

"the suppressed portion of the scale shall be not more than seven-tenths of the maximum scale value."

A usual case is that of a voltmeter for use on a 220-volt circuit, where the scale will be graduated from 180 volts to 240 volts. Voltmeters of this type have the disadvantages that the bending of the spring is much larger than for the ordinary type of instrument, and that there is no zero or other fiducial mark to which the pointer can be set to take up small changes in the spring. It is therefore usual to check them at frequent intervals and to calibrate by means of an adjustable rim or screw.

§ (13) Zero Adjustment.—The provision of an adjustment whereby small changes, due as a rule to the springs, can be adjusted for is most convenient to the user, and the provision of such a device is called for by the British Engineering Standards Association Specification in the following terms:

"A suitable device, accessible from the outside of the case, shall be provided for adjusting the pointer to zero or other fiducial mark, without risk of damage to any of the working parts of the indicator. This clause shall only apply to spring-controlled sub-standard and First Grade instruments.

"On circuits carrying pressures exceeding 250 volts, this device shall be such that it can be employed with safety to the operator when the circuit is alive.

"Any instrument not complying with these conditions shall bear a warning to the effect that the adjustment shall not be made unless the instrument is entirely disconnected from the circuit."

The device usually takes the form of a screw connected to the arm which carries the spring. The screw head is accessible from the outside of the case, a slight turn being sufficient to reset the pointer.

§ (14) Calibration.—The calibration of moving-coil instruments is generally carried out by adjusting the resistance which is in series with the moving coil; other means of adjustment are the variation of the length of the control springs and the more modern practice of using a small piece of iron fixed to one of the magnet poles, which can be moved so as to shunt a portion of the magnetic circuit.

§ (15) Accuracy Characteristics.—Moving-coil instruments are capable of adjustment to a high degree of accuracy. In the highest grades of precision instrument the error is frequently not greater than $\pm 0{\cdot}1$-scale division (150-line scale), while with the more robust switchboard pattern the error with a new instrument does not generally exceed ± 1 per cent of the full-scale reading. The British Engineering Standards Association Specification No. 49, which was drafted by a Committee comprising makers, users, and testing authorities, requires for sub-standard (precision) instruments that the maximum error from full-scale value to the lower limit of the effective range expressed as *a percentage of the maximum scale value* shall not exceed for

	+ or −.
Voltmeters	0·2 per cent
Ammeters (self-contained with shunt permanently connected)	0·5 „
Ammeter indicators (for connection to separate shunts)	0·3 „

The possible sources of error in these high-grade instruments are probably—

(1) Error in calibration;
(2) Inaccuracy of scale drawing;

and, in general, the limits set out meet the case very well, although it is unusual in practice to find a new high-grade ammeter with an error as large as 0·5 per cent of the maximum scale value. For the first grade and second grade (switchboard or portable) instruments a different basis of permissible error is required, since a wider limit can reasonably be allowed for errors of calibration. If, however, for this case the errors over all parts of the scale were expressed as a percentage of the full-scale reading, it would result in limits far too wide for the lower readings on the scale, since with an ammeter having a permissible limit of error of ± 2 per cent the permissible error at half-load would be 4 per cent, and at quarter-load 8 per cent. Therefore, for these instruments the limit of error for the lower half of the scale is only one-half of that allowed for the upper portion, the exact specification being as follows:

Limits of Error in Ammeters and Voltmeters.

Instrument.	From Full-scale Value to Middle Point, expressed as a percentage of the Indication.		From the Middle Point to the Lower Limit of the Effective Range, expressed as a percentage of half the Maximum scale Value.	
	First Grade.	Second Grade.	First Grade.	Second Grade.
	+ or −	+ or −	+ or −	+ or −
Voltmeter, self-contained	1·0	2·0	1·0	2·0
Ammeter, self-contained	2·0	4·0	2·0	4·0
Ammeter, indicator of, permanent magnet, moving-coil type	1·0	2·0	1·0	2·0
Ammeter, indicator of, other types	2·0	4·0	2·0	4·0

In an appendix the Specification indicates the use and class of accuracy for various types of instrument,

and although this table deals with instruments other than those dealt with in this article, it is convenient to reproduce it here to provide comparison of various types.

The following table serves to indicate the class of accuracy which may reasonably be expected with indicating ammeters and voltmeters of different types:

Type.	Class of Accuracy of which capable.	Conditions.
Permanent magnet, moving coil, single range	Sub-standard first grade or second grade	On direct current.
Permanent magnet, moving coil, portable, multi-range, self-contained	First grade, or second grade	On direct current.
Moving-iron . .	First grade or second grade	On alternating current at or about a given frequency.
Moving-iron . .	Second grade	On direct current.
Dynamometer . .	Sub-standard first grade or second grade	On direct current or alternating current at ordinary frequency.
Hot-wire . . .	Second grade	On direct current or alternating current at ordinary frequency.
Induction . . .	Second grade	On alternating current at a given frequency and temperature.
Electrostatic—		
Low tension .	First grade	On direct or alternating current at ordinary frequency.
High tension. .	Second grade	On direct or alternating current at ordinary frequency.

§ (16) TEMPERATURE COEFFICIENT. (i.) *Voltmeters.*—The temperature coefficient of a moving-coil voltmeter is usually so small that it can be neglected for instruments of a moderately high range, say above 30 volts. The actual effects produced by a change of temperature of 1° C., according to Brooks,[1] are—

(*a*) The strength of the spring is reduced by 0·04 per cent.

(*b*) The magnetic flux density in the air-gap is reduced by about 0·02 per cent (this figure represents the average of six instruments of three makes, the individual values ranging from 0·01 per cent to 0·03 per cent).

(*c*) The increase of resistance of the moving coil of 0·4 per cent for 1° C.

To these may be added—

(*d*) The temperature coefficient of the series resistance.

Thus, the effect of temperature on the spring and the magnets produces effects which tend to correct each other, while change in the resistance of the coil, which in itself is only a small portion of the total circuit, can be balanced by the change in the series resistance, which is generally constructed of copper-nickel alloy having a temperature coefficient of about $-0·02$ per cent. Thus, although the limits of variation allowed in the British Engineering Standards Association Specification are 0·1 per cent for a change of 1° C. for sub-standard voltmeters and 0·2 per cent for first grade instruments, the actual variations found in practice for moving-coil voltmeters are very much lower. In a well designed and constructed voltmeter of the sub-standard (precision) type it is difficult to detect any change in the reading for a change of temperature of 20° C., and for first grade or (switchboard) type instruments Fitch and Huber (*loc. cit.*) give values for American instruments ranging from $-0·01$ per cent to $+0·02$ per cent, values which confirm results obtained with a large number of British instruments.

(ii.) *Ammeters.*—In the case of the millivoltmeters used with shunts as ammeters, the temperature coefficient is generally large, and various ingenious devices have been used to eliminate it. The pressure drop required for full-scale deflection—that is, across a circuit comprising the coil and spring only—is of the order of 0·03 volt. A series resistance of material having a small temperature coefficient is used to bring the pressure drop up to the standard values of 0·075 volt or 0·15 volt. So for a switchboard-type instrument the actual change of resistance of the circuit for a variation of 1° C. would be

$$\frac{0·03 \times 0·4}{0·075} = 0·17 \text{ per cent,}$$

while in the case of a sub-standard instrument, using the high-pressure drop, the value would be of the order of 0·08 per cent. These values may be taken as being a fair average of the results of examination of a large number of British-made instruments, which are not provided with additional means for compensation, although with the larger types of switchboard instrument, where larger forces are required to actuate the movement, the temperature coefficient may be as high as 0·3 per cent for 1° C. Fitch and Huber give five values, ranging from 0·08 per cent to 0·32 per cent, for American-made ammeters, and the British Engineering Standards Association

[1] *Trans. I.E.E.* xxxix. 495.

Specification allows variations of 0·1 per cent for 1° C. for sub-standard ammeters, 0·2 per cent for first grade and 0·4 per cent for second grade instruments.

(iii.) *Methods of Compensating for Temperature Coefficient.*—The methods in common use for compensating for changes due to temperature are—

(*a*) The patent springs used by the Edison & Swan Electric Co., the strength of which decreases with increase of temperature to an extent sufficient, or nearly so, to compensate for the change of resistance of the coil.

These have been used almost entirely on switchboard-pattern ammeters, and while they are apparently satisfactory for that purpose, there is no definite information as to their suitability for the higher degree of accuracy required for sub-standard instruments.

(*b*) The method of shunting the magnet with a piece of Guillaume steel, a material the permeability of which has a large magnetic temperature coefficient, was used in some French instruments about 1902. Edgcumbe (*loc. cit.*) states that "the change of permeability with temperature is not large, so that, without shunting the working flux to an excessive amount, sufficient compensation cannot well be obtained." Tests of an actual instrument, however, over a temperature range of from 10° C. to 30° C. showed that the degree of compensation was extremely good, and the same device has been used on ampere-hour meters with very satisfactory results.

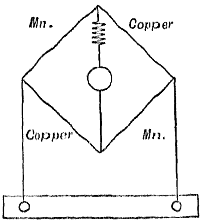


FIG. 6.

(*c*) Campbell [1] perfected a device which allowed of complete compensation for temperature errors in ammeters. The method consists of the arrangement as shown in *Fig.* 6, the millivoltmeter being connected to one pair of opposite corners of a Wheatstone bridge network and the shunt to the other corners. For practical temperature compensation it is usually convenient to make the opposite bridge arms of equal resistance, one pair having a high temperature coefficient and the other negligible. If the total resistance of the millivoltmeter and its series resistance is a, that of the two copper arms na, and if the resistance of these arms is b times that of the manganin arms, then for complete compensation

$$\frac{b^2-1}{b}=\frac{2n(1+a)}{a}.$$

Campbell gives the following practical example: "Let the resistance be in proportion to the numbers shown on *Fig.* 7, those marked 1 being of manganin and the other three of copper, then

$$a=b=3,$$
$$n=1,$$

and it will be seen that these values satisfy the equation for the condition of complete compensation."

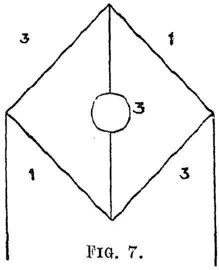


FIG. 7.

(*d*) Another method consists, as will be seen from *Fig.* 8, in shunting the moving coil M, and a portion a of the series resistance $R(=a+b)$ which has zero temperature coefficient by another copper resistance S.

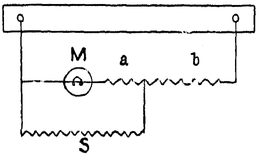


FIG. 8.

The following method of arriving at the conditions for exact compensation is due to R. S. Spillsbury.

Let $r=M+a$,

then the total resistance of instrument and shunt S is

$$\frac{bS+bM+ba+Sr}{S+M+a},$$

and the current through the instrument is proportional to

$$\frac{S}{bS+(a+M)(b+S)}.$$

The differentiation of this with regard to temperature gives

$$\frac{(dS/dT)\{bS+(a+M)(b+S)\}-S\{b(dS/dT)+(a+M)(dS/dT)+(b+S)(dM/dT)\}}{\{bS+(a+M)(b+S)\}^2}.$$

For the temperature coefficient of the instrument to be zero the numerator must vanish. That is, since $dM/dT=M/S(dS/dT)$,

$$ab-MS=0.$$

Thus, if $a=1:b=2$ and $M=1$, the equation is satisfied and exact compensation secured if $S=2$, and even for values of S considerably different from 2 a very close degree of compensation is still obtained.

It should, however, be noted that both of these resistance methods of compensation require that the resistances should be at

[1] *Journal I.E.E.* xxxv. 197.

exactly the same temperature as the moving coil, and therefore should be inside the instrument and as near to the moving coil as possible. If this requirement is not observed, differences of temperature between the compensating resistances and the moving coil may result in errors larger than if the ammeter were not provided with compensation. Further, the types of compensator described are not suitable for use with instruments which are used both for current and pressure measurements, unless provided with special switches, as with the Elliott instrument, for cutting out the compensator when the instrument is used as a voltmeter. Otherwise, the compensator would introduce an error due to temperature when the instrument was used as a voltmeter.

§ (17) LONG-SCALE INSTRUMENTS.—In the instruments previously described the pointer moves over an angle of approximately 90°, but in two cases, by a special arrangement of magnet poles, a scale extending over about 300° has been used. Davis [1] described a special arrangement of magnet poles and coil, designed to give a very long scale, the actual deflection of the pointer being over an arc of from 210° to 270°. The magnet poles were arranged (see *Fig.* 9) to give a long and uniform air-gap, and the rectangular coil, pivoted on one side, was placed with one limb through the centre of the core and the other moving through the air-gap, as shown in *Fig.* 10. Very few of these instruments were made, but later

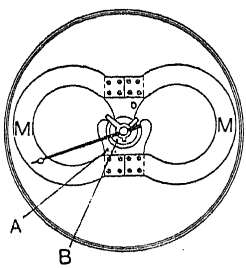


FIG. 9.

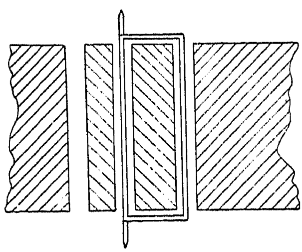


FIG. 10.

Record introduced an improved form. The magnet system and the coil of this instrument is shown in *Fig.* 11, from which it will be seen that extension plates are fitted to both poles of the magnet; the plate from one pole is placed between the two plates fixed to the other pole; the spaces between the inner and outer plates forming the air-gap in which the coil moves. It is claimed for this arrangement that since the two air-gaps are in parallel and not in series, as is the case with the ordinary instrument, more effective use is

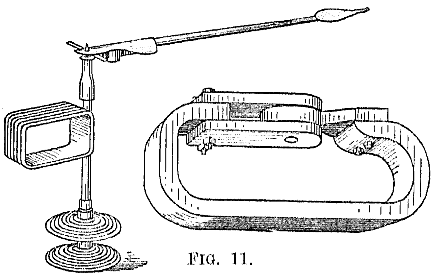


FIG. 11.

made of the magnet. One link of the moving coil passes through the centre of the cores and the other rotates through the air-gap shown in *Fig.* 12.

As the coil embraces a flat plate, the length of the vertical limbs is small, and, in conse-

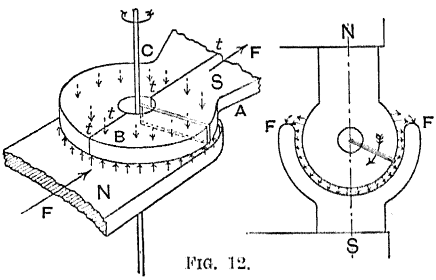


FIG. 12.

quence, the magnetically idle portion of the coil is reduced to a minimum.

The data furnished for an ammeter of this type is as follows:

Weight of movement . . .	2·85 grammes
Torque for deflection of 270° .	0·5 gramme-cm.
B in air-gap, estimated . .	1600
Maximum error due to external magnetic field of 10 gausses .	0·35 per cent

Actual tests gave the following results:

Resistance of instrument and connecting leads . . .	1·42 ohms
Moving coil only . . .	0·39 ohm
Pressure drop for full-scale deflection.	0·075 volt
Temperature coefficient . .	0·25 per cent for 1° C.
Time for pointer to settle to a given reading	2·5 seconds.

The instrument was maintained at full-scale deflection for 18 hours, when it was found that the change of reading was only 0·1 per cent of the full-scale reading.

[1] *Proc. Physical Society*, 1897, p. 425.

The ratio Torque/Weight based on the figures given above has nearly the value 0·17 mentioned by Janus.

§ (18) Other Requirements.—The British Engineering Standards Association Specification specifies a number of other requirements for moving-coil instruments.

(i.) *Insulation.*—" The insulation resistance between all the electrical circuits of an instrument coupled together and the containing-case, or other metal not intended to be insulated from the case when the instrument is in use, shall be not less than 5 megohms."

Where suitable insulating materials are used, there is no difficulty in complying with these requirements.

(ii.) *Polarity.*—Practice, as regards polarity of terminals, has varied considerably, in most instruments the left-hand, or, in the case of terminals fixed one above the other, the bottom terminal has been the positive (*i.e.* connected to the positive pole of a battery or supply pressure), but in many cases the reverse has been the practice. The specification now requires that the left-hand or the bottom terminal, as seen from the front of the instrument, shall be the positive terminal.

§ (19) Variations due to External Magnetic Fields.—Although moving-coil instruments, owing to the large magnetic flux, are affected less than most types of instrument by external magnetic fields, large variations are produced unless due precaution is taken. With an unshielded instrument of the precision-type variations of $\pm 0{\cdot}1$ per cent can be obtained, due to the effect of the earth's field. The effect of an external field, however, only affects the reading of the instrument to an appreciable extent when the direction of the external field is approximately parallel to that of the magnetic flux in the instrument. Precision instruments which are unshielded, therefore, are generally used with the axis of the magnet pointing north and south.

In practice, the effect of the external field due to the current in neighbouring conductors and of placing instruments in close proximity is much greater than that due to the earth's field. Thus, if the instrument is placed at a distance of 1 foot from a straight conductor carrying 1000 amperes, the resultant field would be of the order of 10 gausses as compared with the 0·2 gauss of the earth. The great majority of instruments are therefore fitted with a magnetic shield, this being in most cases formed by making the case of soft iron, and in a few by retaining the non-magnetic case and fitting a shield of soft iron. Even with this protection, however, the effect is not entirely eliminated. In the case of precision instruments used to measure large currents it is usual to provide leads sufficiently long to enable the instrument to be placed at a safe distance, which can always be determined by disconnecting the instrument from the shunt and passing a small current through it from a battery. The effect of the stray field due to the main current circuit is then ascertained by noting the reading of the instrument when turned through a right angle or by switching the main current on and off. For switchboard instruments, however, where these may have to be placed near conductors carrying large currents, it is usual to provide heavy iron cases. The British Engineering Standards Specification provides that

" The variation in the maximum indication of an ammeter or voltmeter indicator, which is intended to be permanently fixed in any position, shall not exceed 3 per cent of that indication, when the indicator is exposed to a magnetic field of 10 C.G.S.[1] units in the direction which produces the maximum error in the instrument readings. For alternating-current instruments the disturbing field shall be alternating and in phase with that in the coil of the indicator.

" Portable First Grade instruments, not purporting to comply with the above requirements, and all Sub-Standard instruments shall bear a statement of the precautions necessary to eliminate or avoid any error due to external fields."

Fitch and Huber[2] obtained values of from 1 per cent to 3 per cent for voltmeters, and from 1 per cent to 4 per cent for ammeters; and Farmer,[3] for the same external field, finds values of from 0·75 per cent to 1·75 per cent for shielded instruments, and 3·5 per cent to 5·5 per cent for unshielded.

§ (20) Shunts. (i.) *Construction.* — The shunts used with moving-coil millivoltmeters are usually made of strips of copper-nickel alloy soldered into brass or sometimes copper lugs. Wires are sometimes used,[4] particularly by German makers, but this form of construction is apparently too expensive, although it possesses many advantages. Tubes also have been used which, when erected in a vertical position, tend to increase the cooling. The thickness of the sheets varies somewhat with different makers, but in a large shunt with many sheets it will be of the order of 1 millimetre. Up to, say, 500 amperes there is little difficulty in making a satisfactory shunt, but the measurement of larger currents presents difficulties (which are not always satisfactorily overcome) in the design of the ends. The effect of stream-line distortion is largely dealt with in the article " The Potentiometer System of Measurement." The points which affect the design of standard resistances apply also to shunts. In the latter, however,

[1] 800 ampere turns will produce a field of approximately 10 C.G.S. units at or near the centre of a plane circular coil 100 cm. diameter.

[2] *Bul. Bureau Standards*, vii. No. 3, 420.

[3] *Book Electrical Measurements in Practice*, p. 54.

[4] See article on " The Potentiometer System of Measurement."

the required accuracy is not so high, but differences sufficiently large to affect the accuracy of a switchboard ammeter can be obtained by slight variation of the method of connecting the main current leads. In the case where a heavy current shunt is

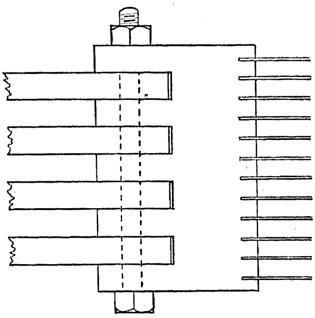


FIG. 13.

designed expressly to fit a certain type of laminated bus-bar, as in *Fig.* 13, it is probable that, if the correct method of connection is strictly observed, there will be no appreciable error due to distortion of stream-lines, but in other cases, and particularly where the lug is in the form of a wide flat plate to which connection may be made on either or both sides, errors of as much as 2 per cent have been found, due to the manner of connecting. The reason for this difference is usually due to the fact that the lugs are not sufficiently long to enable the stream-lines of the current to become uniform. In the case of portable instruments, where weight is a consideration, the case can be met by suitable slotting of the lugs. In the case of large shunts, however, which are intended for permanent connection in bus-bars, it is not easy to understand the reason for the irregularity. Shortening the length of the shunt lugs will reduce the cost of the shunt, but since this has to be made up by an equivalent length of bus-bar, the saving in this respect is not altogether valid.

(ii.) *Shunts for Large Currents.*—The design of shunts for large currents, say for 5000 or 10,000 amperes, presents special difficulties. In the more usual type, as shown in *Fig.* 13, in addition to the shape and size of the ends, it is difficult to ensure that each and every strip is properly soldered into the lug. Field [1] described a multiple-unit shunt which appears to be particularly suitable for measurements of heavy currents. The shunt is built up of a number of units, each bolted to a bus-bar, as shown in *Fig.* 14. Any effect of the resistance of the bolted joints is very largely corrected for by the equalising leads shown in the figure, the resistance of which must be in the same proportion as that of the individual units of the shunt. Field shows that with this arrangement the distribution of current in the individual shunt units may be widely different without appreciable effect on the instrument reading, and gives the result of some tests made by Moore, where four shunts, each for 150 amperes and of a resistance 0·000505 ohm, were connected in parallel to bus-bars.

Each potential terminal was connected to a star-point by a 10-ohm equaliser, and as will be seen

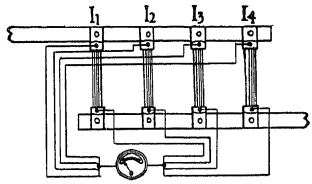


FIG. 14.

from the table below, even when one or more of the shunts is entirely disconnected from the bus-bar, the instrument reading is only slightly affected. In practice the shunts would be bolted tight to the bus-bars, and if this is carried out, as in *Fig.* 14, the current distribution will be approximately equal. Similar results were obtained with equalising leads of much lower resistance, more nearly 0·1 ohm.

Total Current.	P.D. on Shunts, Millivolts.				Mean Shunt P.D.	P.D. between Star-points.	Current calculated from Star-point P.D.
	(1.)	(2.)	(3.)	(4.)			
Amps.							Amps.
A 502·5	64·87	65·67	61·25	61·78	63·39	63·4	502
B 502·5	81·2	80·9	86·1	5·7	63·47	63·4	502
C 502·5	10·8	94·8	86·4	63·5	63·9	63·4	502
D 502·5	79·3	0·0	87·87	86·8	63·51	63·4	502
E 502·5	79·1	0·0	87·75	86·8	63·41	63·4	502

A. All the shunts were clamped tightly on to the copper bus-bars.
B. Shunt No. 4 was just laid on the bars.
C. All the shunts were just laid on the bars.
D. Shunt No. 2 was isolated from one bar and tight on the other.
E. Shunt No. 2 was isolated from both bars.

Another arrangement designed to meet the case of multiple bus-bars is shown in *Fig.* 15.

(iii.) *Testing Sets.*—For testing sets and for

[1] *Journal I.E.E.* lviii. 660.

instruments for laboratory use a number of volt ranges are obtained by means of a series resistance divided at points appropriate to the range required and controlled by a switch. For various current ranges an instrument is usually provided with a number of separate

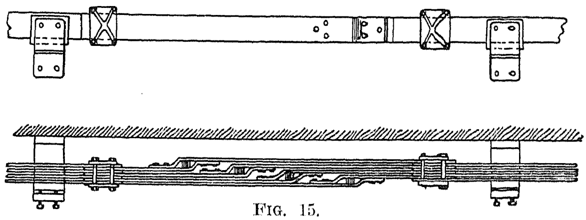


FIG. 15.

shunts which are connected according to the range required. A much more convenient method, however, is that based on the Ayrton-Mather Universal Shunt, in which the millivoltmeter is connected directly across the ends of a shunt and the current led in at the desired point by means of a switch. See *Fig.* 16.

With a small shunt-box of this type eight ranges from 0·15 ampere to 30 amperes are provided. Change from one range to another is effected by movement of the switch, and it will be noted that the contact resistance of the switch does not affect the accuracy of the reading.

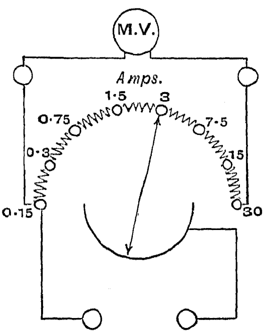


FIG. 16.

In designing a shunt of this kind it is necessary first to determine the total resistance R required for the lowest range I; the value of the intermediate ranges R_1, etc., will then be in the inverse proportion to the current I, *i.e.*

$$\frac{R_1}{R}=\frac{I}{I_1}.$$

Thus, taking a millivoltmeter requiring a pressure drop of 0·1 volt and having a resistance of 10 ohms, the value of R for a range of 0·1 ampere will be 1·01 ohm. For a range of 10 amperes the resistance will be 1·01/10, and so on for the other ranges.

(iv.) *Heating of Shunts.*—The heating of a shunt made up of a number of thin sheets is most irregular, the hottest portion being at the top edge of the central strips. The middle of the shunt will be hotter than the ends, where considerable cooling will take place by conduction along the connecting leads. At full-load variations of temperature of the order of 25° C. are found with large shunts.

(v.) *Thermo E.M.F.*—The copper-nickel alloy generally used as a resister has a thermo E.M.F. of the order of 45 micro-volts for 1° C., and thus a difference of temperature of 17° C. between the two ends will affect the pressure drop across a shunt by 1 per cent. Much larger differences of temperature than this are found in practice, actual measurements showing a difference of temperature between the ends of approximately 30° C., due to Peltier effect only. This may be still greater if the shunt is erected in a vertical position—that is, with one lug above the other—the lower end being connected to the negative of the supply. In this case the increase of temperature at the positive pole of the shunt will be accentuated by the convection currents; thus it is unsafe to connect a shunt in a vertical position unless the positive pole is at the bottom.

The usual method adopted by instrument makers to eliminate thermo E.M.F. is to include in the circuit of the connecting leads strips of the same alloy as the shunt metal. One end of these strips is connected to the potential screws on the shunt, and if of sufficient length to ensure that the other ends are at nearly the same temperature, provide quite satisfactory compensation.

An extension of this method is that generally specified by the Admiralty, in which small lugs of the same material as the shunt are soldered into the end blocks, as shown in *Fig.* 17, the connecting wires being soldered to the curled ends of the strips.

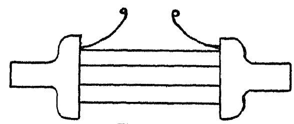


FIG. 17.

The British Engineering Standards Association requirements for shunts are as follows:

"No part of a shunt shall rise in temperature more than 80° C. above the temperature of the surrounding air after carrying the current corresponding to the maximum scale value of the indicator for two hours.

"The main terminals of the shunt shall be so constructed that slight variations in the method of connecting it to the circuit shall not alter the indication of the instrument by more than 0·25 per cent.

"The construction of the shunt and connecting leads shall be such that no thermo-electric force is produced sufficient to alter the indication of the instrument by more than 0·25 per cent."

An appendix gives the following recommendations:

"*Erecting Shunts.*—A shunt dissipates the heat generated when current is passing more by conduction than by radiation. It is therefore necessary to ensure that the ends of the shunts, and any bars to

which the shunts are connected, are of such section and area as to permit of good contact being made, and thus to prevent any rise in temperature due to insufficient section or bad contact.

"When a shunt is built up of a number of sheets, the maximum cooling is obtained when it is erected in a horizontal plane with the sheets in a vertical plane, so as to permit the maximum amount of ventilation between the sheets.

"If by reason of the design of the switchboard it is necessary to erect the shunt with its terminals one above the other, the lower end of the shunt should be connected to the positive pole.

"When the shunt ends consist of flat connecting plates machined either on one or both surfaces, it is essential to ensure that the whole of the surfaces are in contact with the bars to which connection is to be made in order that the lines of flow of current shall not be unduly distorted."

§ (21) Recording (Graphic or Chart) Instruments. — The instruments used for obtaining a graphic record of volts or amperes on a direct-current supply are almost invariably of the moving-coil type, but owing to the comparatively large amount of power required to move a pen over a moving chart with which it is in contact the energy losses are larger and the accuracy lower than in the case of indicating instruments. An ammeter of this type requires a pressure drop of the order of 0·2 volt to operate the pen; such a pressure drop allows of little, if any, swamping resistance, and in consequence the temperature coefficient will be about 0·4 per cent for 1° C. For a recording voltmeter the resistance will be of the order of 20 ohms per volt. An additional source of error is that due to variation of the amount of ink in the pen: this affects the balance of the moving system and may modify the reading considerably.

The Specification No. 90 of the British Engineering Standards Association has the following clauses, which apply particularly to moving-coil recording instruments.

Case or other Protection.—"The recorder shall be contained in a suitable case, of sufficient strength to afford adequate protection against injury when reasonably used, and to exclude dust from the working parts; the accessory apparatus shall be suitably protected when necessary to ensure permanence of the accuracy of the indications. The design shall be such, that complete safety to the operator is ensured when the case is open for the purpose of changing the chart or filling the pen."

Clock or Driving Mechanism.—"Suitable means, easily accessible, shall be provided whereby the timing of the clock or other driving mechanism can be regulated so as not to have an error exceeding 5 minutes in 24 hours."

Chart Ruling.—"The scales on charts employed in conjunction with recording ammeters, voltmeters, and wattmeters shall be divided into 40, 50, 60, or 75 divisions.

"The value of each scale division shall be either 1, 2, or 5 of the units measured, or any decimal multiple or sub-multiple of these.

"A piece of the chart shall be permanently attached to the instrument in such a position as to permit of a ready comparison between the ruling on it and the ruling on a new roll of paper."

Pointer and Pen.—"The pointer and pen shall be insulated from the electrical circuit of the recorder."

Pen to Paper Friction.—"Suitable means shall be provided, when necessary, whereby the pressure of the pen on the paper may be adjusted to prevent variations of pen to paper friction from introducing inaccuracy beyond the limits laid down."

Damping.—"The moving system of a recording instrument shall be provided with means for damping its movement suitably for the conditions in which the instrument is used. Such means shall be independent of the friction between the pen and the paper."

External Shunts.—"The drop in pressure across the terminals of a shunt to which a permanent magnet moving-coil recorder is connected shall not exceed 0·2 volt with the maximum indication on the recorder."

Limits of Error.—"The error in the reading of a recorder, when tested over the effective range at the standard temperature, or at the temperature marked on it, and at ordinary frequency, or at the frequency marked on it, and, with the exception of a wattmeter, when placed in any position in a magnetic field not greatly exceeding that of the earth, and with the pen approximately half full of ink, shall not exceed the limits given in the following table. In the case of voltmeters and wattmeters the pressure shall have been applied for 30 minutes before the test is made."

Description.	Limits of Error expressed as a percentage of the Maximum-scale value.
	+ or −
Voltmeter with free zero	2·5
Voltmeter with partially suppressed zero . .	1·5
Ammeter . . .	3·0
Wattmeter . . .	3·5

Variations in the Indications due to Changes in the Amount of Ink.

"The variation in the indication of a recorder caused by varying the quantity of ink from pen full to pen empty shall not exceed ±3 per cent of the maximum-scale value."

In most of the recorders in general use the pen moves across the paper in the arc of a circle and the paper is ruled accordingly. An exception to this, however, is the "Murday" instrument made by Messrs. Evershed & Vignoles, in which case the pen is pivoted and moves in a straight line across the paper.

Where a record is required to a high degree of accuracy, the "Thread" recorder, described in the article[1] on "Thermocouples," is used. This consists of a suspended galvanometer, the pointer of which swings freely over the paper. A blackened thread is interposed between the

[1] See "Thermocouples," § (16), Vol. I.

pointer and the paper, and a mechanism operated by clockwork forces the pointer down at regular intervals on to the thread, which makes a dot on the paper corresponding to the position of the pointer. In this way the pen-to-paper friction is totally eliminated. An instrument of this kind is sufficiently sensitive to allow of its use in conjunction with a potentiometer with the consequent increase in accuracy. S. W. M.

Direct Current Machines, advantages of. See "Dynamo Electric Machinery," § (11).
Design of. See *ibid.* § (4).

Directional Wireless: the determination of the direction from which signals come in wireless telegraphy. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (11).

Disc Discharge, The: an arrangement for producing sparks of a definite audible frequency, for transmission in wireless telegraphy. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (2).

Discharge through Gases, general theory of. See "Electrons and Discharge Tube," § (7).

Displacement Moment: the torque (per unit current) tending to displace the moving system of a galvanometer. See "Vibration Galvanometers," § (21).

Distortion in Telephone Transmitters,
Electrical: incorrect translation of sounds by a transmitter, due to electrical causes. See "Telephony," § (15).
Mechanical: incorrect translation of sounds by a transmitter, due to mechanical causes. See *ibid.* § (15).

Distribution Networks for Electric Power. See "Switchgear," § (40).

Distributor, High-tension: the component of a magneto, which transmits the sparking voltage from the secondary winding to each spark gap in turn. See "Magneto, The High-tension," § (11) (v.).

Dolezalek Alternator, for audio-frequency work. See "Inductance, The Measurement of," § (6).

Double Current Working. In telegraphy, a system in which signals are transmitted by the reversal of a current. See "Telegraph, The Electric," § (7).

Double-plate Sounder: a telegraphic receiver which emits a characteristic sound for each unit of the code employed. See "Telegraph, The Electric," § (6).

Doublet, The Hertzian: two equal stationary charges of opposite sign, varying in magnitude harmonically with time. Radiation of energy by. See "Wireless Telegraphy," § (4).

Drysdale Galvanometer. See "Vibration Galvanometers," § (16).

Drysdale Permeameter, for testing magnetic qualities in bulk. See "Magnetic Measurements and Properties of Materials," § (38).

Drysdale Universal Standardising Bridge: a bridge for the comparison of standards of electrical resistance. See "Electrical Resistance Standards and Measurement of," § (10).

Drysdale Wattmeter: a precision dynamometer wattmeter of the null type. See "Alternating Current Instruments," § (10).

Du Bois Magnetic Balance. See "Magnetic Measurements and Properties of Materials," § (27).

Du Bois Optical Method, for high magnetisation tests. See "Magnetic Measurements and Properties of Materials," § (44).

Du Bois-Rubens Galvanometer. See "Galvanometers," § (6).

Duddell Alternator, for frequencies of 100 to 2000 ∿ per sec. See "Inductance, The Measurement of," § (7).

Duddell Galvanometer. See "Vibration Galvanometers," § (8).

Duddell Inductor: a special form of mutual inductance used as a standard of magnetic flux. See "Magnetic Measurements and Properties of Materials," § (3).

Duddell and Marchant. Point by point method of delineating A.C. wave forms. See "A.C. Wave Forms," § (1).

Duddell-Mather Wattmeter: a precision dynamometer wattmeter of the null type. See "Alternating Current Instruments," § (11).

Duddell Thermogalvanometer, use of, for the measurement of small currents at radio frequencies. See "Radio-frequency Measurements," § (18).

Dufour Oscillograph: a form of cathode ray oscillograph. See "Alternating Current Instruments," § (62).

Dynamo Electric Machine: a generator the magnetic field of which is produced by an electromagnet. See "Dynamo Electric Machinery," § (3).
Design of. See *ibid.* § (4).
Fundamental Principles of. See *ibid.* § (3).

DYNAMO ELECTRIC MACHINERY

§ (1) Electromagnetic Transformations of Energy.—The purpose of dynamo machinery is to convert mechanical energy into electrical energy, or *vice versa*, the machines used, in the first case, being known as "generators," and, in the second case, as "motors."

The transformation of energy can be effected in two ways, the first, and one which has very little practical application, being by electrostatic action, while the second, and much

more general method, is by electromagnetic forces, or forces which depend on the interaction of electricity and magnetism. A conductor carrying a current in a magnetic field has a force [1] acting on it when at rest, while if moved across the lines of magnetic force it has an E.M.F. set up in it, and this occurs even if there is initially no current in the conductor. These both depend on the strength of the field, which is measured by the number of lines of induction traversing a unit area placed at right angles to their direction, and on the rate of motion of the conductor. The electrostatic effect referred to is employed in such a machine as the Wimshurst influence machine, but the power available is so small that the electromagnetic effect has been adopted to form the basis of all dynamo machines for practical purposes.

As an example of what can be achieved by this electromagnetic action, with copper, iron, and some insulating material such as cotton, a copper rod of about the same diameter and length as a lead pencil, carrying as much current as it can without getting too hot, when in a position at right angles to the lines of induction of a magnetic field, such as can be obtained by the use of iron, will experience a force of from $1\frac{1}{2}$ to 2 lbs., in a direction at right angles both to its own length and to the direction of the field. A motor armature, say, 7 in. long and 7 in. in diameter could carry about sixty such conductors, each insulated with cotton and connected in proper sequence round its periphery. Such an armature, if run at a speed of about 1500 r.p.m., would be capable of exerting from 7 to 8 horse-power.

Electromagnetic action takes place in accordance with laws first discovered by Faraday, and which may be stated thus : [2]

I. A conductor carrying unit current placed in a magnetic field of unit induction or unit flux density (one line per square centimetre), in a direction at right angles to that of the field, experiences a force of 1 dyne per centimetre of its length, in a direction at right angles both to the conductor and to the field.

II. A conductor moving with a velocity of 1 cm. per second, at right angles to the lines of force of a magnetic field of unit induction or unit flux (one line per square centimetre), has, induced in it, unit electromotive force per centimetre of its length.

In order to calculate either the force on the conductor, in the first case, or the electromotive force, in the second case, it is necessary to know the strength of the magnetic field, which is given by the number of lines of induction per square centimetre. This strength of the magnetic field is defined as its magnetic induction or flux density, and since, in most cases, the field is obtained with iron which is magnetised by the action of a current of electricity, it is necessary to determine the relation between the magnetic flux and the magnetising force produced by this current.

If, as is usual, the current is flowing in a wire which is bent into the form of a solenoid, round a mass of soft iron, the magnetising force (H) in C.G.S. units is given by the expression

$$H = 4\pi n i,$$

where i is the current and n is the number of turns of wire per unit length of the solenoid.

In this formula the current is measured in absolute units, which are ten times the value of the practical unit of current, the ampere. Thus, if I be the current in amperes,

$$H = \frac{4\pi}{10} \cdot nI,$$

where the product nI is the number of ampere turns per unit length of the solenoid, and the magnetising force is given in C.G.S. units.

The relation between (B) the magnetic flux density and (H) the magnetising force in C.G.S.

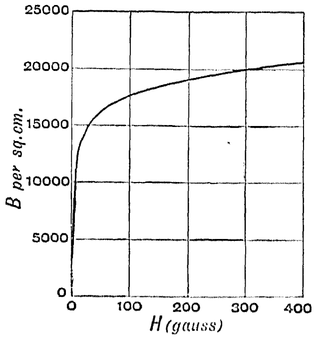


FIG. 1.

units is expressed in the form of a curve which is determined for the material in question by direct experiment [3] (see *Fig.* 1).

The ratio of B to H is defined as the magnetic permeability and is generally denoted by the symbol μ ; its value clearly depends on H.

The two laws previously given may be stated in somewhat different form, which, although easily recognised as equivalent, for certain purposes is more convenient, thus :

(I.*a*) A closed circuit carrying a current, when placed in a field of magnetic flux, is acted on by a force which tends to set the circuit in such a position as to link with itself the maximum amount of flux.

The force in a given direction, measured in dynes, is equal to the product of the current and the increase of the amount of interlinked

[1] See "Electromagnetic Theory," § (9).
[2] See *ibid.* § (12).

[3] For the method of doing this see "Magnetic Measurements," § (18) and following.

flux which would result from a unit displacement of the circuit in the direction specified. In the case of angular displacement the resultant action will be a couple.

(II.a) If there is any variation in the magnetic flux which is linked with a closed circuit, an E.M.F. is induced round the circuit which is equal to the rate of decrease of the interlinked flux.

It should be noted that, provided that the magnetic flux density is measured in absolute units (number of lines of induction per square centimetre), the electromotive force is also given in absolute units. In practice the E.M.F. is measured in volts; and 1 volt $= 10^8$ absolute C.G.S. units.

According to Law I. the mechanical force on a conductor is at right angles to the flux and to the current; the positive directions of the three are related as shown in *Fig.* 2, in which Oz drawn upwards represents the direction of the flux, and Oy horizontal and towards the observer represents the direction of the force. Ox drawn horizontal and towards the right represents the direction of the current. If the current or the on flux be reversed, the direction of the mechanical force is also reversed.

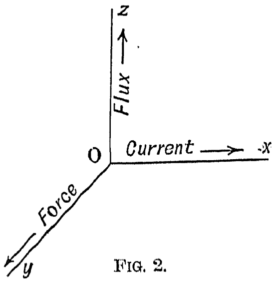


Fig. 2.

§ (2) The Magnetic Field due to a Conductor.—The following method of illustrating the action referred to is also useful.

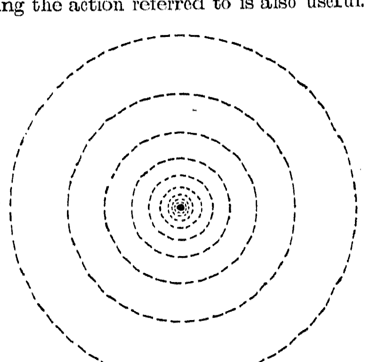


Fig. 3.

In a straight conductor, in air and carrying current, lines of force will surround the conductor symmetrically in the form of concentric circles, as shown in *Fig.* 3.

If another conductor is arranged close to and parallel to the first conductor (*Fig.* 4), and current is also passed through this second conductor, then, if the currents in the two are in the same direction, there will be a differential action in the intervening space, so that the magnetic field will be weak, whereas the total

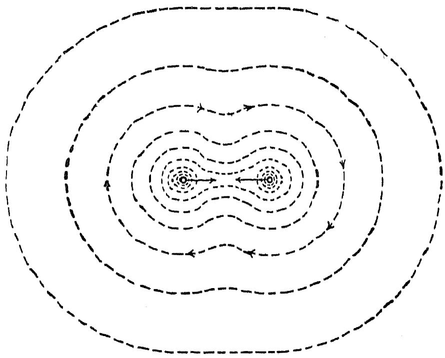


Fig. 4.

field surrounding the two conductors will be increased. If the magnetic lines are assumed to be in tension, then they will tend to draw the conductors together. Similarly, if the conductors carry currents in opposite directions (*Fig.* 5), then the field strength between them

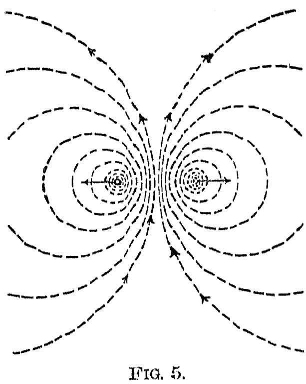


Fig. 5.

will be increased, and the total field surrounding them will be reduced, and the conductors will, therefore, tend to move away from one another. If a single conductor be placed in a magnetic field at right angles to the direction of the magnetic lines, the effect of its own field will be to cause a distortion of the main field (*Fig.* 6), increasing the number of lines on one side of the conductor, and decreasing it on the other, tending to produce motion towards the weaker part of the field. If two conductors are placed close to one another and carry equal currents in opposite

directions, then the distortion of the magnetic field produced by one will be eliminated almost completely by the distortion produced by the other, and the whole magnetic

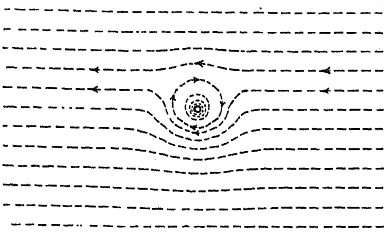


Fig. 6.

field will remain in its original position, but the conductors will tend to move, one in one direction and the second in the opposite (*Fig.* 7).

The case of a single conductor, situated in a magnetic field, experiencing a force and at the same time distorting the field, corresponds to the ordinary direct current generator or alternating current generator, and the case of two conductors, placed near one another and carrying equal and opposite currents, corresponds to the compensated direct current machine or the induction motor.

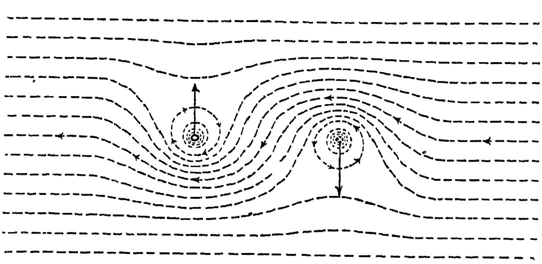


Fig. 7.

§ (3) Fundamental Principles.—A dynamo or motor consists, in its simplest form, of a conducting circuit, lying in one plane, which can rotate in a field of magnetic flux about an axis, also in the same plane, at right angles to the direction of the flux. In *Fig.* 8 the axis is represented by the line AB, and the flux is in the direction CD.

The ends of the conductor are attached to either (*a*) a split tube of copper which rotates with the coil, for a direct current machine, or (*b*) two concentric metal rings, for an alternator.[1] Conducting brushes rub on the tube or rings and convey the current to or from the circuit. In the direct current machine we may suppose the split tube to be so arranged that when the plane of the coil is at right angles to the flux the brushes pass from one half of the tube to the other. In an actual machine, for reasons which are specified later, the brushes do not occupy exactly this position.

If we imagine the conductor as in rotation, in such a manner that the upper part of the circuit is approaching the observer, an electromotive force will be produced in the direction indicated and a current will flow round the circuit. When the coil has made half a turn the direction of the flux will be reversed, and the current, therefore, will be reversed in the coil, so that the end which was positive will become negative.

Owing to the action of the split metal tube—the commutator—in the direct current machine, the connections between the brushes and the coil are reversed also, and the current continues to flow in the same direction through the external circuit; we have a direct current generator.

If two concentric rings are used instead of a commutator, the current in the external circuit reverses with that in the coil; the machine is an alternator.

Returning to the commutating machine, it will be seen that the action is reversible, and if, instead of moving the armature coil as before, an external source of current is applied, the coil will revolve so as to set itself in such a position as to embrace the maximum flux. By means of the commutator, which reverses the current in the coil at the right moment, the motion can be made continuous and a direct current motor is obtained.

To make an alternator act as a motor it is necessary that the current be made to reverse

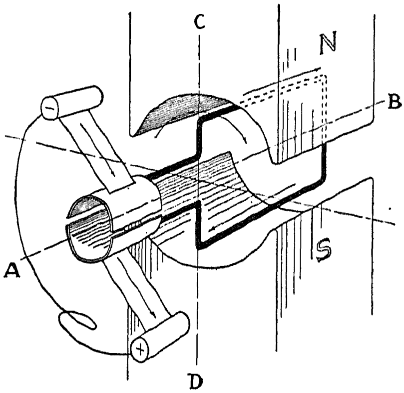


Fig. 8.

its direction at the instant when the revolving coil is at right angles to the direction of the flux. Such a motor must, usually, have means for starting, but if suitably started will run at a speed corresponding to the frequency of

[1] In some alternators the armature is fixed while the field of magnetic flux rotates.

the alternating current and exert considerable power.

The magnetic flux may be obtained by the use of permanent magnets, and under these circumstances the machine is known as a *Magneto Electric Machine* or a *Magneto.*[1] If electromagnets are used the machine is known as a *Dynamo.* The power produced by a magneto is very small in comparison to that obtainable from machines using electromagnets, and in this article it is intended only to deal with Dynamo Machinery.

The coils which carry the current required to produce the magnetic flux are called *Field Coils.* The coils in which the current is induced in a generator, or through which the external current passes in a motor, are built up into a form which is known as the *Armature.*

In most actual dynamos the armature coils like the field coils have an iron core.

§ (4) Designing a Dynamo.—We now proceed to discuss in an elementary way the

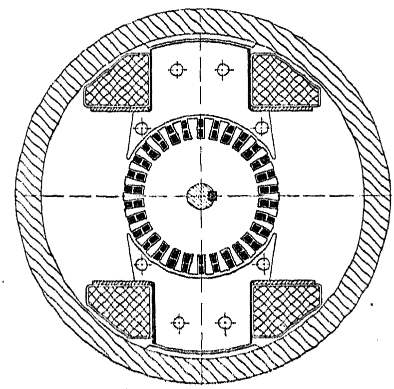


Fig. 9.

design of a small dynamo, and to show the application of some of the principles already discussed to an actual machine; by treating the matter in this way the various factors which affect the design can be most readily understood. The problem is to design a machine to convert electrical energy into mechanical energy, or mechanical into electrical; in the latter case the conditions of supply of electrical power are usually given—this may be 3-phase alternating current with a certain frequency and voltage, or direct current; there may be also some conditions as to the constancy of voltage, etc., and there are also usually conditions qualifying the mechanical power that the motor is required to give. These take the form of specifying the speed, horsepower, torque, speed variation, etc. In the example an armature of 7″ diameter is assumed to have sixty conductors arranged round the periphery, and a simple method of doing this is to arrange the conductors in 30 slots, two in each (*Fig.* 9). It is essential that the conductors should be joined together in such a manner as to form coils which embrace the armature, as shown in *Figs.* 10, 11. Each coil surrounds the armature twice and has its ends connected to adjacent sections of the commutator (*Figs.* 10 and 11). In addition it is necessary to arrange the coils in such a

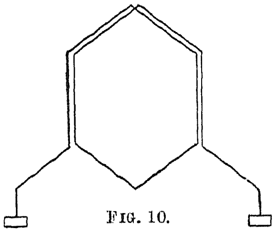


Fig. 10.

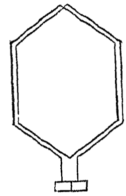


Fig. 11.

manner that they may be built up to form the armature, and for this purpose they should have uniformity of shape so that the coils may lie evenly on the armature.

(i.) *D.C. Machines.*—In a direct current machine this is secured by the arrangement

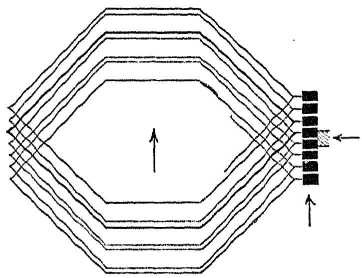


Fig. 12.

shown diagrammatically in *Figs.* 12 and 12A. In *Fig.* 12A each slot is supposed to contain only one conductor bar, and it will be seen that a bar such as 1, 1 in a slot at the top of the armature is connected at one end to the segment 1 of the commutator and at the

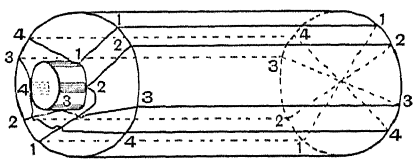


Fig. 12A.

other to the bar 1, 1 diametrically opposite to it. The left-hand end of this bar is connected to segment 2 of the commutator; thus a circuit is formed with its two ends connected to adjacent segments. Segment 2 is connected to 3 through the pair of bars 2, 2, and 2, 2 also diametrically opposite, and so on. The

[1] See article "Magneto, The High-tension."

armature coils thus form a continuous circuit of four complete turns, and from each turn tappings are taken off to two contiguous segments of the commutator. When the brushes are on segments 1 and 3 a current can pass from 1 to 3 either by the path 11111222223 or by the path 14444433333. In each case it circles the armature twice.

In the case of the machine illustrated in *Fig.* 9 in which we have 30 coils in series the connections may be also followed by means of the polar diagram (*Fig.* 13). Here it will be noticed that if the armature is rotated the coils will be connected to the commutator segments in such a manner as to give the combined electromotive force of two circuits, in parallel, each of 30 coils in series. *Fig.* 9 gives a sectional view of a machine such as that of

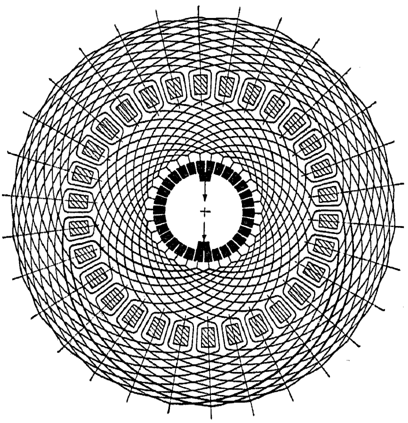


FIG. 13.

the example, designed to run at 1500 r.p.m., and is approximately to scale.

In considering this design it is first necessary to decide what amount of magnetism can be carried by the iron magnetic circuit. For the present it is assumed that such a calculation has been made, that the machine has two magnetic poles, and that it has been found that the amount of magnetism entering the armature on one side and leaving it on the other will be approximately $2{\cdot}3 \times 10^6$ lines of flux, or 2·3 megalines. This amount of flux will give a mean flux density over the surface of the armature of approximately 30,000 lines per square inch (4650 lines per square centimetre). With a rotational speed of 1500 r.p.m. the actual linear speed of the surface of the armature will be 2750 feet per minute, or 550 inches per second, or approximately 1400 centimetres per second. The average rate at which a conductor cuts the magnetic lines of force will be $30{,}000 \times 550$ — *i.e.* 16,500,000 lines per inch of conductor per second, that is to say, the average voltage in a conductor will be ·165 volt per inch, or 1·15 volts for the 7″ conductor assumed. If these conductors are joined up to form coils, and these connected in the manner specified above, there will be two paths in parallel, and each path will have 30 conductors in series, therefore the total voltage generated will be 34·5 volts.

If looked at from another aspect and it is desired to calculate the voltage which would occur due to lines of force entering and leaving a coil, we can assume that when a coil consisting of two conductors is in a plane at right angles to the axis between poles, that such a coil encloses 2·3 megalines. Half a revolution later, which will be accomplished in one 50th part of a second, the coil has turned completely round, and now has lines of force passing through it in exactly the opposite direction. The change of flux which has occurred is from $+2{\cdot}3$ megalines to $-2{\cdot}3$ megalines—that is, a total change of 4·6 megalines—and this has occurred in one 50th part of a second.

The average rate, therefore, is $4{\cdot}6 \times 50$, or 230,000,000 lines per second, which will give rise to an average voltage of 2·3 volts; and as there are 15 coils in each circuit of the armature, the total voltage will be 34·5 as previously determined.

We may write the expression just obtained $34{\cdot}5 \times 10^8 = 4 \times 2{\cdot}3 \times 10^6 \times 25 \times 15$. On the left-hand side we have the volts multiplied by 10^8; on the right $2{\cdot}3 \times 10^6$ is the change of flux per pole—the machine has two poles—25 is the frequency, and 15 the number of circuits. Thus we have

$$\text{Volts} = 4 \times \text{Flux per pole} \times \text{Frequency} \times \text{Number of circuits} \times 10^{-8}.$$

A little consideration shows that this formula is of general application.

Thus let the length of the armature be l centimetres and its radius a centimetres, let it make n revs. per second, let the flux be Φ, and the number of circuits N. The number of conductors will be 2N.

The surface of the armature through which the flux enters is πla cm.², and the average flux density is $\Phi/\pi la$. The speed of a conductor across the flux is $2\pi an$ cm./sec.

Thus the E.M.F. per cm. is $2\pi an\Phi/\pi la$ C.G.S. units, and the E.M.F. per conductor of length l is given by multiplying this by l. Hence

$$\text{Volts per conductor} = \frac{\Phi \times 2\pi anl}{\pi la} \times 10^{-8} = 2n\Phi \times 10^{-8},$$

and

$$\text{Total volts generated} = 2n\Phi \times 2\text{N} \times 10^{-8}$$
$$= 4 \times \Phi \times n \times \text{N} \times 10^{-8},$$

or

$$4 \times \text{Flux per pole} \times \text{Frequency} \times \text{Number of circuits} \times 10^{-8}.$$

(ii.) *Alternators.*—The method of calculation given above is applicable for direct current machines, as the voltage at any instant is the same as the instantaneous voltages of all the coils which are connected in series; that is to say, that if there are N coils, it will be N times the average voltage of each coil—in the present instance $15 \times 2{\cdot}3$, which is 34·5 volts. Machines,

however, are frequently made as alternating current generators or motors, and in such machines the coils, instead of being connected to a commutator which periodically reverses the direction of current, are connected to slip rings in the case of revolving armature machines, or simply brought to terminals in the case of fixed armature machines. In the present case we will assume that the machine has the armature conductors connected up so as to form a 3-phase alternating current machine. There are 60 conductors. They will be connected up in three symmetrical groups, each containing 20 conductors or ten coils, and spaced 120° apart. The form of the voltage supplied by an alternating current machine is usually designed to be very closely sinusoidal. Each conductor as it revolves and cuts the magnetic flux will have induced in it a voltage which will reproduce the shape of the magnetic field under each pole, for the reason that the armature is revolving uniformly. Thus if the curve giving the flux in terms of the time be sinusoidal, the curve connecting the voltage and the time will also be sinusoidal.

The voltage, therefore, of each conductor will have a form similar to that shown on the attached curve (*Fig.* 14). If now two conductors situated in adjacent slots are connected in series, each will have induced in it this same voltage form, but the resultant of the two voltages will be slightly different, in fact it can be obtained by adding together two such curves which have been displaced from one another by a distance corresponding to the pitch of a slot, which

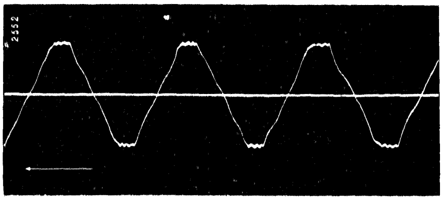


FIG. 14.

in the present instance is 12°. Similarly if three, four, or five conductors, situated in adjacent slots are connected in series, the resultant voltage of the five will be different from that of each individual one. The flux form shown on *Fig.* 14, due to a single slot, may be assumed to consist of a fundamental sine wave with superposed harmonics of various frequencies, and the process just described has the effect of cancelling the higher frequencies, leaving only the fundamental. In many machines not only are coils in adjacent slots connected in series, but these coils frequently have fractional pitch—*i.e.* they do not span the full distance from one pole to another. This fractional pitch winding tends to still further suppress any harmonics in the fundamental voltage wave, and a further refinement resorted to sometimes is the use of a number of slots not integral with the number of poles.

By all these various means the modern alternating current machine usually produces a wave of electromotive force which approximates exceedingly closely to the sine wave (*Fig.* 15). It will be noticed, therefore, in following through this description of the production of electromotive force in an alternator, the exact form of the flux distribution is not of importance as the whole process tends to eliminate any variations from a sinusoidal form, therefore machines built in this way will give a sine wave of

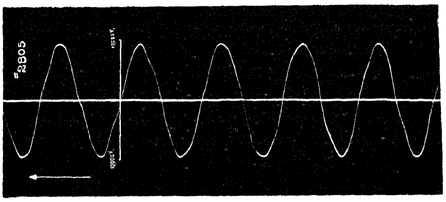


FIG. 15.

electromotive force even when the flux distribution is considerably distorted due to load.

The description given above is applicable perhaps more particularly to large turbo alternators, for in small machines with a small pole pitch, these methods cannot always be adopted, but in such cases it is usual to make the air-gap in the centre of the pole rather smaller than at the sides, and so give a flux distribution which is nearly sinusoidal, and has not many harmonics needing elimination.

If the form of the flux distribution under a pole need not have any particular shape, it will be simplest to assume that it is sinusoidal for the purposes of calculation. If, then, in the example already quoted, the flux distribution follows a sinusoidal form, the maximum value will be higher than the mean value previously assumed of 30,000 lines per sq. inch, in the ratio of $\pi/2$, for this is the ratio of the maximum to the mean value of a quantity which varies sinusoidally; hence the maximum density will be approximately 47,000 lines per sq. inch, and as the mechanical speed is the same as before, *i.e.* 550 inches per second, the maximum voltage per inch of conductor will be $47{,}000 \times 550 \times 10^{-8}$. This is equal to ·26 volt per inch, or 1·8 volts for a 7″ conductor, or 3·6 volts for a coil composed of two such conductors. This is the maximum value voltage, and as it follows a sinusoidal curve and has a frequency of 25 per second (corresponding to 1500 per minute) and two poles, this instantaneous value will be $3{\cdot}6 \sin 2\pi nt$, where n is 25. The effective value of an alternating current is measured by that of the direct current which produces the same heating effect, and this will be the square root of the mean square, or in the case of a current of sinusoidal wave form ·707 of the maximum value, so that the effective value of the voltage per coil will be $3{\cdot}6 \times {\cdot}707$, which is equal to 2·55 volts; and as there are ten

coils in each phase, the total voltage will be 25·5 volts. There is, however, a small factor to be taken into account which slightly reduces this value, and, where the coils, as in the present instance, are assumed to have a full pitch and are distributed in five adjacent slots, reduces the actual voltage by about 4 per cent, making the total effective voltage per phase 24·5 volts. When three phases are connected together in the form of a letter Y, it can be shown that the total voltage is $\sqrt{3}$ or 1·73 times the voltage per phase, so that in the particular case we are considering the voltage between terminals will be 42·5 volts. The same value for the voltage would be obtained by considering the change in flux in any coil, as in the direct current case, treating the flux as varying in a sinusoidal manner.

From the above considerations we have seen that the formula for obtaining the voltage for a direct current machine may be expressed as follows:

$$\text{Volts} = 4 \times \text{Flux per pole} \times \text{Frequency} \times \text{Number of coils in series} \times 10^{-8} \times \text{F}.$$

For an alternating current machine the factors are exactly the same, except that the constant, instead of being 4, is 4·44. This follows from the fact that we have to multiply the mean value of the flux by $\pi/2$ to obtain the maximum, and again by $1/\sqrt{2}$ to obtain the effective value. The product of these two factors is 1·11, thus the 4 of the original formula becomes 4·44.

The quantity F in this formula is the factor which allows for fractional pitch windings and other factors which tend to make the windings less efficient for producing voltage than it otherwise would be. It should not be much less than unity, and for most of the calculations below may be omitted.

(iii.) *The Magnetic Circuit of a Dynamo.*—This, unlike the simple anchor rings used in magnetic testing, is a composite one. In the simpler cases it may be considered as a series of portions formed by materials of different magnetic properties (such as air, soft iron, cast iron, etc.) composing a closed circuit. The component parts have different cross-sections and mean lengths. To a first approximation the total magnetic flux Φ produced by the ampere turns applied to one or more portions may be assumed to traverse the complete circuit without leakage. In that case we have

$$\text{Total flux } \Phi = \frac{\text{Magnetomotive force}}{\text{Total reluctance}}$$

$$= \frac{0{\cdot}4 \times \pi \times \text{ampere turns}}{\text{Sum of series reluctances}}$$

$$= \frac{0{\cdot}4 \times \pi \text{N}i}{\dfrac{l_1}{s_1} + \dfrac{l_2}{\mu_2 s_2} + \dfrac{l_3}{\mu_3 s_3} + \ldots},$$

where l_1, l_2, l_3 are the mean lengths (cm.),
s_1, s_2, s_3 cross-sections (sq. cm.),
$1, \mu_2, \mu_3$ permeabilities

of the air-gap ($\mu = 1$) and the various other parts of the circuit (field magneto cores, plate, armature core, etc.). It should be noted that the above equation

$$\text{Magnetic flux} = \frac{\text{M.M.F.}}{\text{Reluctance}}$$

for the magnetic circuit corresponds to Ohm's law for the electric circuit,

$$\text{Current} = \frac{\text{E.M.F.}}{\text{Resistance}}.$$

§ (5) Armature Reaction.—So far no account has been taken of the effect which may be produced when the armature bars are carrying current. Carrying current does not directly affect the voltage which would be induced in a conductor, but it has an indirect effect in that currents which flow in the armature bars produce a magnetising effect which may considerably distort the original distribution of magnetic flux in the machine. It is usual to calculate the total magnetomotive force of the armature conductors, and combine this with the original magnetomotive force of the field coils or exciting windings, in order to determine what the resultant magnetomotive force on the magnetic circuit will be, and so determine the amount of magnetisation that will occur under loaded conditions. In general the magnetising forces of the armature tend to neutralise the magnetising force of the field winding, so that in order to maintain the same amount of flux in a machine the field winding has to be considerably increased as the load on the machine increases. At any instant the various conductors in an armature will usually be carrying different currents, and they will occupy various positions relatively to the centre lines of the poles. The magnetomotive force due to each coil of the armature acts at right angles to the plane of the coil, and the number of ampere turns to which it is proportional is found by multiplying together the current in the coil and the number of turns. The resultant magnetomotive force may be regarded as due to the armature turns in a coil placed at right angles to the direction of this magnetomotive force; the number of ampere turns in such a coil is known usually as the armature reaction, and is determined in this way, as its principal purpose is for combination with the ampere turns on the field spools, in order to arrive at the resultant magnetising force of the machine.

(i.) *D.C. Machine.*—Consider now a D.C. machine. We require to find the direction and

magnitude of the magnetomotive force due to the armature current. In *Fig.* 16 let the brushes be so set that the current in a circuit AOB rotating in a clockwise direction is reversed each time the circuit is perpendicular to NS, the direction of the flux. The direction of the current in the circuit will be such that the lines of magnetic force due to it run as shown. The circuit in the position shown moves as though its right-hand face possessed south magnetic polarity and its left-hand face north polarity. The armature reaction is from right to left in a direction at right angles to the plane of the coil. The amount of the reaction in any direction making an angle θ with this plane is equal to

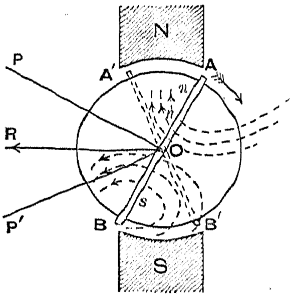


FIG. 16.

$$\text{Ampere turns} \times \sin\theta.$$

As the circuit passes the commutating position the right-hand face becomes the left hand and *vice versa*, but their polarities are also reversed through the change of current. The armature reaction is still from left to right, and of an amount given by the same expression. Now consider the complete armature. We shall have a second circuit A′B′ placed as shown, symmetrically with AB, but on the opposite side of NS. Let OR be perpendicular to NS and let θ be the angle between the plane of either coil and OR. The current is the same in each coil, and the resultant magnetomotive force of the two is clearly along OR, and we obtain

$$\text{Armature reaction} = 2 \text{ ampere turns per coil} \times \sin\theta.$$

This is for one pair of coils; the total number of such pairs with which we have to deal will be half the total number of circuits; these are arranged at various values of the angle θ. The resultant magnetomotive force due to the whole armature is therefore given by the expression 2 ampere turns per circuit $\times \frac{1}{2}$ number of circuits $\times$ average value of $\sin\theta$ between 0° and 90°. This average value, when the circuits are numerous, is $2/\pi$; while the ampere turns per circuit multiplied by the number of circuits is the total number of ampere turns. Now the armature reaction is measured per pole of the machine. Thus the result just found requires dividing by the number of poles of the machine and total ampere turns thus becomes ampere turns per pole.

We have finally for the D.C. machine the expression

$$\text{Armature reaction per pole} = \frac{2}{\pi} \times \text{Armature turns per pole.}$$

The direction in which the armature reaction acts will depend upon the position of the brushes. If, as above, the brushes are set so that commutation takes place in conductors situated midway between poles, then the direction of the armature reaction will be at right angles to that of the magnetising force of the field spools. Armature reaction acting in this direction does not demagnetise the field spools, but has a cross-magnetising effect tending to increase the density of flux at one side of the pole, and decrease it at the other. This distortion of the flux distribution is usually prejudicial in direct-current-machines, in that it causes bad commutation, as will be explained later. In small machines the usual remedy is to make the strength of the field spools high compared to the value of the armature reaction, and so keep the distortion to a value which is found by practice to be permissible.

In the actual machine which we have been considering it will be assumed that each of the armature bars carries 100 amperes (corresponding to 200 amperes total current from the machine as there are two circuits in parallel). There are 60 conductors in the machine, equal to 30 circuits, or 15 circuits per pole as it is a two-pole machine. Therefore the armature reaction expressed in ampere turns per pole will be $100 \times 15 \times 2/\pi = 960$ ampere turns. This armature reaction will be substantially constant in value and direction; as, due to the action of commutation under the brushes, the current is reversed in any coil during the time it passes under the brushes, and coils which occupy similar position and space will always carry the same current.

(ii.) *A.C. Machine.*—In a two or three phase alternating-current machine the currents are carried in two or three sets of windings, and are continually varying in value. In order to determine the value of the armature reaction it is necessary to consider the values of currents at some particular instant, and the position of the coils in which these currents flow. We shall assume that this machine is, as before, wound as a 3-phase machine, having three sets of circuits placed 120° apart, each set of circuits having ten turns or five turns per pole. Suppose that the instant be chosen when the current in one of the circuits is the maximum, and further that the direction in which the armature reaction acts is at right angles to this circuit, so that it has its full magnetising effect. The plane of the

circuit will be parallel to and the direction of the armature reaction perpendicular to the flux. It will further be assumed that each conductor carries 100 amperes (as measured by an ammeter so that 100 amperes is the effective value of the current). The armature reaction depends on the maximum current, and for this circuit containing five turns it will be $5 \times 100 \times 1{\cdot}414$.[1] The other two circuits will be displaced 120° from the first circuit, and they are carrying currents displaced 120° from the current in the first circuit. The value at the instant considered will be the maximum value multiplied by sin 120°, and due to the inclination of the coil, the component of its magnetising force in a direction at right angles to the first coil considered will be further reduced in proportion of sin 120° : 1. Thus the armature reaction due to this coil will be found by multiplying the maximum value by $\sin^2 120°$ or $\frac{3}{4}$. The total value of the armature reaction will therefore be $1{\cdot}414 \times 5 \times 100 \times (1 + \frac{3}{4} + \frac{3}{4})$, which is equal to $2{\cdot}12 \times 5 \times 100$, or 1060 ampere turns.

In this calculation no account has been taken of the small displacement which occurs in consequence of the conductors which form one coil occupying different slots, and as was shown in calculating the voltage, a correction factor amounting to about 4 per cent must be used; this will slightly reduce the armature reaction, and make the value in this particular instance taken 1020 ampere turns.

This armature reaction has been calculated for a comparatively simple position of the phases, but if any other instant were chosen, and the value of the currents corresponding to that instant and the displacement of the coils be taken into account, it would be shown that the armature reaction is substantially the same; in fact the armature reaction of a 3-phase machine remains practically constant in direction and in value relatively to the poles. A calculation for a 2-phase machine could be carried through in the same manner, and would give somewhat similar results.

The value per pole of the armature reaction for a D.C. machine can therefore be expressed as

$$\frac{2}{\pi} \times N_p \times i,$$

where N_p is the number of circuits per pole, and i the current in each armature conductor.

For the 3-phase[2] case this value becomes

$$2{\cdot}12 \times \frac{N_p}{3} \times i,$$

where i is the effective value of the current in each conductor.

The factor 2·12 is the product of 1·5 expressing the effect of the three circuits and 1·414 the ratio of the maximum to the effective current.

The calculation of the voltage from the flux of a machine and the calculation of the armature reaction have been gone into at some length, but these calculations are exceedingly important, and it will be shown presently that the flux and armature reaction of a machine are the two chief factors involved in a design.

§ (6) Output of the Machine.—The electrical output of a machine is the product of voltage multiplied by current, and in the particular example chosen, the direct-current output will be 34·5 volts × 200 amperes = 6900 watts. In the 3-phase case the output is three times the product of volts by amperes in each phase, that is, $24{\cdot}5 \times 100 \times 3 = 7300$ watts. It will be noticed, looking at these figures, that the same size of machine with the same amount of flux and the same value of current in each armature conductor, gives a slightly bigger output as a 3-phase machine than it does as a direct-current machine, but it will also be noticed that the calculated armature reaction for the 3-phase machine is also slightly larger than for the direct-current machine, and that these two figures have a direct proportionality—that is to say, the output is proportional to the armature reaction. The reason why in the two machines the outputs and armature reactions are not identical, is that, due to the disposition of the windings, the conductors are used a little more effectively in a 3-phase machine in this particular case than they are in a direct-current machine.

It is now proposed to show the relationship that the flux and armature reaction bear to the electrical and mechanical output of a motor or generator.

The electrical output of a machine in watts is the product of volts and amperes. From the previous calculations it has been shown that volts are proportional to the flux, frequency, and number of turns; and also that armature reaction is proportional to turns and current, consequently of course current will vary as armature reaction and inversely as number of turns. If, therefore, volts and amperes in the expression for output are expressed in terms of flux and armature reaction, and the equation simplified, it will be found that the output in watts can be reduced to the following expression:

$$\text{Watts} = 2\pi \times \text{Revs. per second} \times \text{Flux} \times \text{Total armature reaction} \times 10^{-8}.$$

Thus in a bipolar D.C. machine we have, omitting the factor F, and putting Armature reaction $= \Psi$, and Revs. per sec. = R,

$$\text{Volts} = 4\Phi \times n \times N \times 10^{-8},$$

$$\text{Amperes} = \frac{\Psi}{2} \times \frac{2}{N} \times \frac{\pi}{2},$$

$$n = \tfrac{1}{2}\,R \,.\, 2.$$

[1] The factor 1·414 measures the ratio of the maximum to the effective value of the current.

[2] See also § (6).

Hence

Watts = volt amperes

$= 2\pi R\Phi \times \Psi \times 10^{-8}$

$= 2\pi$ Revs. per sec. × Flux × Total armature reaction $\times 10^{-8}$.

The formula for an A.C. machine can be obtained similarly.

In the above expression the total armature reaction and the flux are used, being the armature reaction per pole multiplied by the number of poles in the machine and the flux issuing from either pole. Obviously, in this expression, 2π × revolutions per second is the angular velocity, and consequently since the Watts are equal to Torque × Angular velocity, the Torque will be represented by

$$\text{Flux} \times \text{Total armature reaction} \times 10^{-8},$$

which is equal to $(\Phi \,.\, \Psi)/10$ dyne cm.

Another method of deriving the output of a dynamo is as follows.

Each element, *e.g.* a square centimetre of surface of the armature, has a certain number of conductors carrying current; the force, in dynes, acting on the conductors in this element will be the product of the total current in these conductors and the magnetic flux density through the area. From this we can find the torque on the armature and hence the output. Let the radius of the armature in centimetres be a, its length l, the number of circuits each of one turn N, the number of poles 2, the total flux Φ, the current in amperes i, and the armature reaction Ψ.

(i.) *Direct Current.*—The number of conductors concerned is 2N which are connected so as to make N circuits or N/2 circuits per pole. The flux issuing from one pole is Φ, and the surface through which this flux passes is πal. Hence

$$\text{Mean flux density} = \frac{\Phi}{\pi a l}.$$

Current × Number of conductors per cm. of circumference

$$= \frac{2N}{2\pi a} \times \frac{i}{10} = \frac{Ni}{10\pi a}.$$

Hence

$$\text{Mean force per sq. cm.} = \frac{N\Phi i}{10\pi^2 a^2 l},$$

$$\text{Torque per sq. cm.} = \frac{N\Phi i}{10\pi^2 a l},$$

$$\text{Total torque} = \frac{N\Phi i}{5\pi} \text{ dyne cm.}$$

for there are $2\pi al$ sq. cm. on the armature.

But Ψ = Total armature reaction

$$= \frac{4}{\pi}\frac{Ni}{2} = \frac{2Ni}{\pi},$$

for there are two poles and N/2 circuits per pole.

Thus, $\quad \text{Total torque} = \dfrac{\Phi \,.\, \Psi}{10}$ dyne cm.

(ii.) *Alternating Current.*—We assume the positions of maximum current and maximum flux density to coincide and consider a current in a conductor when in a position making an angle θ with this maximum position. The armature has as many phases as there are circuits.

$$\text{The mean flux density} = \frac{\Phi}{\pi a l}.$$

Since the wave form is sinusoidal

$$\text{Maximum flux density} = \frac{\Phi}{\pi a l} \times \frac{\pi}{2}.$$

$$\text{Flux density when in position } \theta = \frac{\Phi}{2al}\sin\theta.$$

If the effective current be i amperes, its maximum value in C.G.S. units is $1{\cdot}414i/10$. Thus Current in position $\theta = (1{\cdot}414/10)i\sin\theta$. Number of conductors in an element $\delta\theta$ of the surface $= (2N/2\pi)d\theta$. Thus the force per unit length of this elementary strip

$$= \frac{1{\cdot}414 N\Phi i}{20\pi a l}\sin^2\theta\, d\theta.$$

Torque on armature of length l

$$= \frac{1{\cdot}414 N\Phi i}{20\pi}\int_0^{2\pi}\sin^2\theta\, d\theta,$$

$$= \frac{1{\cdot}414 N\Phi i}{20}.$$

Now consider one complete circuit made up of the conductors on two opposite elements $d\theta$ of the armature. The current is $1{\cdot}414i\sin\theta$ and the number of such circuits $2Nd\theta/2\pi$. Thus the ampere turns are $(1{\cdot}414Ni\sin\theta)/\pi$. The magnetomotive force due to this circuit acts in a direction perpendicular to it; to obtain the equivalent ampere turns which will produce the same effect in a direction perpendicular to the flux we must multiply the expression by $\sin\theta$, and to find the total armature reaction must sum this for all values of θ between θ and π. We thus find

$$\Psi = \frac{1{\cdot}414Ni}{\pi}\int_0^{\pi}\sin^2\theta\, d\theta,$$

$$= \frac{1{\cdot}414Ni}{2}.$$

Thus $1{\cdot}414Ni = 2\Psi$, and hence as for the D.C. case,

$$\text{Total torque} = \frac{\Phi\Psi}{10}\text{ dyne cm.}$$

If instead of supposing the conductors uniformly distributed over the armature we had supposed them concentrated into three sets of circuits each containing N/3 turns at angles of 120°, we should replace the $\dfrac{N}{\pi}\displaystyle\int_0^{2\pi}\sin^2\theta\, d\theta$ by $\dfrac{2N}{3}(1+\frac{1}{4}+\frac{1}{4})$ and the $\dfrac{N}{\pi}\displaystyle\int_0^{\pi}\sin^2\theta\, d\theta$ by one half this amount and arrive at the same result for the output. We should also

arrive at the result already obtained for the armature reaction.

Now

1 watt $=10^7$ ergs per sec.
$=10^7$ dyne cm. per sec.

Thus

1 dyne cm. $=10^{-7}$ watt sec.
$=10^{-7}$ joules.

And the Torque=
Flux × Total armature reaction $\times 10^{-8}$.

The output in watts is, we have seen, equal to 2π Revs. per sec. × Torque. Thus

Output $=2\pi\times$ Revs. per sec. × Flux
× Total armature reaction $\times 10^{-8}$ watts.

The flux Φ is measured by the number of lines of force issuing from one pole of the field. It is sometimes more convenient to work in terms of the total flux which, since there are two poles, is in this case equal to 2Φ, and then we have

Output $=2\pi\times$ Revs. per sec. $\times\frac{1}{2}$ Total flux
× Total armature reaction $\times 10^{-8}$ watts.

For practical engineering purposes it is sometimes preferable to have the torque expressed in inch-pounds rather than in dyne-centimetres; and if the total flux is expressed in megalines, and the total armature reaction in ampere turns then, changing the units of force and length, the expression for torque in inch-pounds then becomes

Torque in inch-pounds = Total flux × Total
armature reaction × ·0445.

§ (7) Electro-magnetic Induction.—A still further aspect of the conversion of mechanical into electrical energy can be obtained by considering the stored energy in an inductive circuit; by an inductive circuit is meant an electric circuit in a field of magnetic induction. Usually such a circuit consists of a number of turns surrounding an iron core.

When such a circuit carries a current it possesses energy, and the energy depends on the current and the total number of lines of force linked with the circuit. Let the number of turns of the circuit be N, the flux through each turn Φ, and the current i. The current itself will produce flux through the circuit, but we will consider first the case when Φ arises entirely from causes external to the circuit.

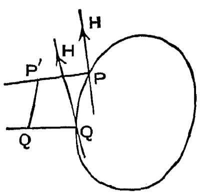


Fig. 17.

Consider a case when the lines of induction are perpendicular to the circuit. Take an element PQ of the circuit (*Fig.* 17) and imagine it displaced to P′Q′ in a direction perpendicular both to itself and to the flux; its ends must be supposed to rest on conducting rails PP′, QQ′. Let there be H lines of induction per cm. of the conductor in the neighbourhood of PQ, and let the current be i.

The force on PQ is $i\,.\,\mathrm{H}\times\mathrm{PQ}$, and the work done in the displacement is $i\,.\,\mathrm{H}\times\mathrm{PQ}\times\mathrm{PP'}$.

But $\mathrm{H}\times\mathrm{PQ}\times\mathrm{PP'}$ = increase of flux through the circuit.

Thus increase of energy $=i\times$ increase of total flux.

From this it follows that the total energy is equal to $i\times$ total flux.

Thus energy[1] $=i\times$ Total flux $=i\times\mathrm{N}\Phi$
= Ampere turns × Flux per turn.

Turn now to the case when the flux arises entirely from the current and denote by L the ratio of the total flux through the circuit to the current producing it.

Then $$\mathrm{L}=\frac{\text{Total flux}}{\text{Current}}=\frac{\mathrm{N}\Phi}{i}.$$

Hence $$\mathrm{L}i=\mathrm{N}\Phi.$$

The quantity L is known as the coefficient of self-induction of the circuit. It is measured in Henrys;[2] it depends on the shape and dimensions of the circuit and of the material with which it is surrounded. A circuit having a self-induction of 1 Henry is such that a change of current of 1 ampere per second produces an E.M.F. of 1 volt.

If the current is made to vary in any way, the flux through the circuit will alter and an electromotive force will be set up which, by Faraday's second law, is equal to the rate of decrease of the total flux, that is to $-d(\mathrm{N}\Phi)/dt$. Thus from the value of $\mathrm{N}\Phi$ already found we have

$$\text{E.M.F. of self-induction}=-\frac{d}{dt}(\mathrm{L}i).$$

To maintain the growth of the current an E.M.F. equal and opposite to this must be applied.

Work will be done and the energy of the circuit increased at a rate measured by

$$i\frac{d}{dt}(\mathrm{L}i),$$

and the product in the case in which L is constant becomes

$$\mathrm{L}i\frac{di}{dt}.$$

Thus the total amount of energy required to raise the current from zero to i is

$$\int_0^{C}\mathrm{L}i\frac{di}{dt}dt,\ \text{or}\ \tfrac{1}{2}\mathrm{L}i^2.$$

[1] If i is measured in C.G.S. units and the flux in lines per square centimetre, the energy will be in ergs. If the current be given in amperes and the flux in megalines, the energy will be in joules $\times 10^{-2}$.

[2] See "Electromagnetic Theory," §§ (4), (5).

This expression, if we substitute for Li its value $N\Phi$, becomes

$$\tfrac{1}{2}N\Phi i, \text{ or } \tfrac{1}{2} \text{ flux} \times \text{Ampere turns.}$$

In the cases we are concerned with we are dealing generally with coils carrying current moving in fields which, in the main, are not due to the current in the coil, and the energy stored in the coil is measured as above in the first case by the product of the flux per turn multiplied by the ampere turns.

In a dynamo, and for simplicity we will assume a D.C. case, although it applies equally well to any dynamo, a coil situated at right angles to the magnetic axis, so that it embraces the total flux of the machine, and carrying a current, will have stored magnetic energy in it. For instance, in the example we have considered already such a coil, consisting of one turn and carrying 100 amperes, embraces 2·3 megalines, and will have stored energy of $2{\cdot}3 \times 10^6 \times 100 \times 10^{-1}$ C.G.S. units or 2·3 watt seconds. Half a revolution later this coil will embrace flux in exactly the opposite direction, *i.e.* the direction of energy will be reversed, and the change in energy will be twice the above value, *i.e.* 4·6 watt seconds; the average rate of change, which, of course, represents power, will be this amount divided by the time during which the reversal occurs, which is 1/50th of a second. Therefore, the rate at which energy is being expended in order to cause this alternating magnetic energy is 230 watts in each coil; and as there are a total of 30 coils, the total of the machine will be $30 \times 230 = 6900$ watts, which was the output of the D.C. machine previously arrived at.

In considering the output of a D.C. machine, the direction in which the armature reaction operates is fixed by the brushes, and the magnetisation due to the armature is generally, for practical purposes, at right angles to the magnetic axis. In an alternating-current machine, however, the armature reaction does not act in this manner; in fact, only when running on a purely energy load does this occur. In a D.C. machine, due to the action of the commutator, the current delivered to the external circuit is a steady one, and though this machine may be, and frequently is, connected to a highly inductive circuit, the current is practically steady, and, therefore, no self-inductive voltage occurs. The voltage of the machine is the whole voltage of the circuit.

In an alternator the current is not commutated, but is applied to the line exactly in the form in which it is generated in the machine, *i.e.* as an alternating current; such an alternating current when put through an inductive circuit gives rise to a voltage induced in this circuit, and external to the alternator. It will thus be seen that the voltage generated in the alternator is only a portion of the whole voltage which is induced in the complete circuit and from which it differs in phase. If the matter is looked at in this way, and it is assumed that the current in the circuit is in phase with the total voltage in the circuit, this current will obviously be considerably out of phase with the voltage generated by the alternator alone. This is the usual case with alternating-current circuits. The current differs in phase very considerably from the voltage generated by the machine. In the more usual alternating-current circuits the effect of capacity can be neglected, and the effect of self-induction only considered. This causes the current to lag in time behind the voltage. In an alternator the effect of current lagging behind the voltage is to shift the direction of the armature reaction and cause it to oppose very largely the magnetising effect of the field spools, and thus to demagnetise the whole magnetic circuit. When considering the torque of an alternator it is necessary to take this factor into account, as it is only the energy component of the total armature reaction which gives rise to torque and requires the corresponding mechanical energy to rotate the machine.

If the value of the angle of lag be ϕ the output is given by the expression

$$\text{Volts} \times \text{Amperes} \times \cos \phi.$$

The above considerations show that in the design of a dynamo the two important factors are the amount of flux, which depends on the amount of iron in a machine, and the amount of armature reaction, which depends on the amount of copper; in designing a machine it may be made with a large amount of copper and a small amount of iron, or a large amount of iron and a small amount of copper; it is the skill and experience of the designer which determines the correct proportions of these two components. It is hardly feasible to discuss fully how a design should be arrived at in any particular machine as to the proportion of these two components, but an attempt will be made to show how each of them influences the design, so as to indicate how a proper relationship can be obtained.

§ (8) THE MAGNETIC CIRCUIT. DETAILS OF DESIGN.—In considering the magnetic circuit the most important feature is that iron becomes saturated at a flux density of about 100,000 lines per square inch, 15,000 to 16,000 lines per square centimetre (see *Fig.* 1). Up to this point the magnetic flux is roughly proportional to the magnetising force, the value of the permeability given by the ratio of B/H is nearly constant. Above this point the amount of flux is increased only slowly by increasing the magnetising force. The other feature is that where reversal of magnetism takes place, as it does in the

armature of dynamos, a loss is associated with this reversal.[1] This loss varies with different grades of iron, and it can be reduced by alloying a small amount of silicon, which increases the specific resistance and also to a small extent helps the permeability. In dynamo machinery also there is a loss due to eddy currents set up in the iron. This is a conductor, and currents are induced in it as it rotates in the field of the machine. These currents are greatly reduced by laminating the iron and separating the laminae by an insulating material. But metallic contact between the laminae cannot be entirely prevented; eddy currents and loss of energy are a necessary consequence resulting in the heating of the iron. In actual practice, and with ordinary frequencies, the losses that occur can be dealt with without excessive rise of temperature, and probably the chief limit in the magnetic circuit of modern machinery is the saturation limit. In a preliminary design of a machine it is necessary to make a sketch of the probable shape of the magnetic circuit. The section of iron can be reduced in the centre of the poles with the advantage that the length of turn of the magnetising coils can be made as small as possible. The section must be increased in the neighbourhood of the air gap so as to reduce the density of flux where it enters the armature, and where the iron is cut away in the form of slots to accommodate the windings.

A knowledge of the voltage, speed, and output of the machine and of the magnetic area and length of its armature and other parts enables us to calculate the total flux required in the armature to give the desired effect. The flux in the field coils and yoke must be greater in order to allow for leakage. By dividing the total flux in each part by the magnetic area of that part we get the corresponding flux density. The magnetisation curves for the various materials enable us to calculate the ampere turns per unit length to produce this flux density, and multiplying these by the length of the iron core in each case we find the total ampere turns.[2]

Allowance must be made for flux which leaks across the intervening space between poles and does not actually enter the armature, and a magnetic circuit has to be designed so as to keep this leakage as small as possible. Such magnetic leakage occurs when the machine is running on open circuit, but the amount is very much increased when currents are flowing in the armature windings which tend to demagnetise the field winding, and which consequently call for larger magnetising force on the field spools to produce the same amount of flux in the armature.

[1] See "Magnetic Measurements," § (1) (xii.); also "Magnetic Hysteresis." [2] See § (4) (iii.).

The next step in the design of a machine after a preliminary sketch of the magnetic circuit, and determination of the amount of flux which such a circuit would carry, is to calculate the number of turns on the armature for the voltage required. Such a calculation will probably involve a modification in the amount of flux in the magnetic circuit in order that a convenient and symmetrical form of winding may be used. For instance, in a three-phase machine the slots must obviously be some multiple of three, and it is also necessary that an integral number of turns should be used in each slot. Having made the necessary modifications, the armature reaction can be calculated from the number of turns used and the amount of current which the machine is to carry. This armature reaction will enable us to determine the amount of leakage which will occur, and also to determine how much of the magnetic circuit will have to be sacrificed in the form of slots to accommodate the winding. These considerations may further modify the design of the magnetic circuit. Having done this it will be necessary to consider the winding itself, whether space can be found for the conductors to carry the currents allowed for, with a reasonable current density in the copper, whether losses in the copper will cause too high a temperature, whether room can be provided for the necessary insulation, and whether in the case of large alternating-current machines excessive eddy currents are likely to occur in the large copper conductors. The value of the armature reaction arrived at will also determine the number of ampere turns which must be put on the field windings, as these ampere turns must not only supply the magnetism, but must be able to overcome the neutralising effect of the armature reaction, and the necessary space must be available on the magnetic structure for field spools of sufficient size. These considerations may lead to modification in the winding, that is to say, possibly a smaller number of turns may be necessary, with a corresponding smaller armature reaction, and consequently a larger flux must be used.

Another important feature in the design is that a machine must conform with patterns, dies, and other parts which may exist in a manufacturing organisation, and also the machine must be capable of being modified for various voltages.

From all these considerations it will be realised that the design of a dynamo is a very complex matter, and very many considerations influence the final design which is chosen, but the two broad features stand out which influence the output obtained from a machine; first, the amount of magnetic flux which can be carried in the iron circuit,

and secondly, the amount of current which can be carried in the copper circuit. If a quality of iron were available which would allow, say, 10 per cent more flux to be carried, then the output could be increased by this amount, but it also could probably be increased still further, as the leakage flux would now be a smaller percentage, and consequently the armature reaction could be increased since the consequent increase in leakage could now be permitted. By this means it is probable that, provided the copper heating is not a limit on a machine, a 10 per cent increase in the permeability in the steel would allow about 20 per cent more output to be obtained from a given size of machine. Researches are being made in alloyed steels, and it is possible that some result of this type may be eventually obtained.

With regard to the other limits, it is improbable that a better conducting material at a reasonable price will be obtained than copper, but continual improvements are being made in means for getting rid of the heat generated in the copper conductors, and also great improvements are taking place in the methods of insulating coils, and economising the space occupied by the insulated conductors. In the preliminary design of dynamos, it is convenient to express armature reaction in the form of ampere turns per unit of circumference, and flux in the form of mean flux density per square inch of circumference of the armature. Permissible values may be assigned to these two quantities, depending on the type and size of machine to be designed, and from them a first approximation can be quickly obtained to the general dimensions.

§ (9) Induction Machines.—Most of the considerations given above apply to machines where the initial magnetisation is supplied by means of direct current. A rather different form of machine is an induction machine, of which the chief example is the induction motor. The machine consists of two parts, the stator, or fixed part, and the rotor which, in a motor, rotates in the field due to the stator. In this case the excitation necessary to maintain the flux in the machine is supplied by alternating current, and, as already mentioned, when alternating current and flux are used, alternating electromotive forces are introduced into the circuit. It is necessary, therefore, in an induction motor, in order that the power factor may be reasonably high, that the magnetising current should be kept small. The same considerations apply in induction motors as in all other dynamo machines, leading to the result that the output is the product of the active armature reaction and active flux. In this type of machine it is not necessary to use a component of the magnetising current to neutralise the rotor or working armature reaction, as this, as we shall see, is automatically done by means of a current differing in phase by a right angle from the magnetising current which is induced by the reaction of the stator windings. It is also very desirable that the amount of leakage flux should be kept a minimum, as obviously if a large leakage flux occurs, the actual working flux in the armature will be reduced, and the output of the machine will fall in consequence.

Another feature in an induction motor which influences its ultimate output is that a slip must occur in order to induce the necessary currents in the rotor bars. Slip is the difference between the speed of rotation of the rotor and the speed of rotation of the magnetic field on the stator, as will be described later. If the rotor current is large, requiring a large slip, the frequency of the rotor currents increase, and a phase displacement of the rotor currents relatively to the working flux will take place which reduces the effective torque of the motor.

Most induction motors are made as 3-phase machines, but it is a little simpler to understand the theory of a 2-phase motor, and therefore this will be discussed. In a 2-phase motor the stator has two sets of coils arranged at right angles to one another and connected respectively to the two phases of the supply circuit. The rotor is assumed to have a large number of phases. If the stator is connected to the supply circuit, currents will flow and flux will be produced. This flux will be just sufficient to induce a voltage in the stator equal to the supply voltage, and the current will be just enough to produce this amount of flux. Current and flux are always in phase with one another, *i.e.* attain their maxima at the same instant. Voltage is, however, greatest when the flux change is greatest, and as sinusoidal values of current, flux, and voltage are assumed voltage will be directly out of phase with current. Voltage will also be induced in the rotor windings due to the alternating flux produced by the stator currents.

Referring to *Fig.* 18, assume for the moment that, due to magnetising currents flowing in the stator winding AA, flux oscillates in the direction BB. This flux induces voltage in the coil AA and also in the rotor coils *aa*. If the rotor coils are connected up so that current can flow through them due to the voltage induced by the flux along BB, and if such currents are in phase with the voltage so induced, then the currents will be at a maximum in the region *aa*, since the coils in this position embrace the largest amount of flux. The magnetising force causing flux to oscillate along axis BB will be the resultant of the currents in AA and *aa*, and this resultant

will not be equal to the original magnetising force of AA until an additional current flows in AA which is equal and opposite to the currents in *aa*. All this is just what happens in an ordinary static transformer, and the

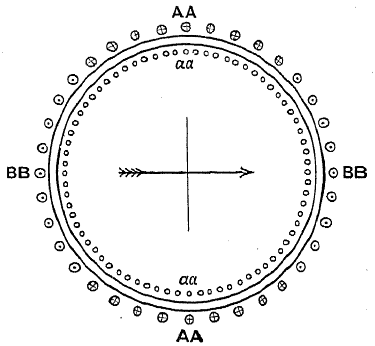


FIG. 18.

net result is that there are two components of current in AA, one an impressed magnetising current in phase with the magnetism along BB and at right angles to the voltage across AA, and the other equal and opposite to the current in *aa*. These two latter currents are at a maximum value when the flux is at a minimum and are in phase with the voltage across AA. Further, they are carried by conductors placed close to one another and they therefore neutralise one another, and as there is no flux when they are at a maximum they do not exert any force on one another tending to cause rotation of the rotor. So far, the effect of the currents in the phase BB has been ignored, and the conditions discussed are those that would occur in a single-phase motor or static transformer. However, the windings BB have the same relationship to the phase B of the supply as windings AA to phase A. Whilst, therefore, the flux along BB is a minimum and the currents in *aa* and their counterpart in AA are a maximum, the flux due to phase B will have a maximum value in the direction AA. The two conductors *aa* and AA carrying equal currents in opposite directions are in a magnetic field at right angles to the direction BB, parallel, that is, to the line joining the two conductors A and *a*. Thus the case illustrated in *Fig.* 7 arises. A strong circumferential force acts on the conductors at *aa* tending to cause rotation in the rotor.

There is another and rather simpler way of regarding an induction motor, which is based on the fact[1] that the currents in the two sets of coils AA and BB give rise to a rotating magnetomotive force which produces a continuously revolving flux. The rotating flux cuts the rotor bars and gives rise to voltages and currents in them. The voltage will be greatest where the rate of cutting is greatest, *i.e.* where the flux is densest, and if the currents in the rotor bars are in phase with the voltages induced these currents will be greatest where the flux is greatest, and consequently mechanical forces will occur as in *Fig.* 6. When the motor is running, its rotational speed will be slightly slower than the rotational speed of the flux, and only the difference in speed will cause cutting of the flux by the rotor bars. The rotor windings have a low resistance, and the slip or speed of cutting will be just sufficient to produce the voltage necessary to force the current required in the rotor windings through the resistance of the windings. The rotor windings consist of bars or wires carried in slots, and are therefore inherently highly inductive, but when the motor is running the frequency of the rotor currents is so low that inductive voltage is very small and the currents in the windings are practically in phase with the voltage induced in the bars, and the phase relations discussed above hold. When starting the rotor is at a standstill, the frequency is the same as in the stator, and the currents tend to become displaced and no longer occur in the bars situated in the densest portion of the flux; further, the low resistance rotor winding tends to prevent flux penetrating into the rotor. The conditions at starting are not, therefore, favourable for high starting torque, but in small motors sufficient initial torque can usually be obtained at the expense of a large initial rush of current. In larger motors resistance is inserted in the rotor windings and cut out after the motor has attained speed.

§ (10) COMMUTATION. — Direct - current machines are usually made with a closed armature winding from which connections are made at regular intervals to the commutator bars. Diagrams, *Figs.* 10 and 11, show portions of two forms of such winding, the first being suitable for high - voltage machines, and the second for low - voltage machines. In each case, the span between the two sides of the coil is practically one pole-pitch on the armature. *Fig.* 12 shows a portion of a winding connected to the commutator, and with a brush in position. This brush is arranged so that the coils make contact with it when they are in the neutral position between poles, that is to say, when there is no voltage induced in them; the coil being connected to the commutator bars which are in contact with the brush, and short-circuited by the brush. Commutation consists in

[1] The magnetic fields due to the two currents may be represented by expressions H sin nt and H cos nt. These are at right angles, and their resultant is H inclined at an angle nt to the direction of the second field. The direction, therefore, rotates, the period being $2\pi/n$.

reversing the current in the coils during the time that they are short-circuited by the brush.

Carbon or graphite brushes are used, as these ensure lubrication of the commutator, and also have a considerable amount of resistance. The resistance between the brush and the commutator segments is practically proportional to the area of segment in contact with the brush. If reversal were considered to take place very slowly, it could be seen that

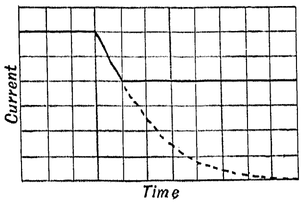


FIG. 19.

the current flowing from a brush into the winding through the commutator bars will divide up amongst the coils with which the brush is in contact in proportion to the contact resistance between the bars and the brush, and consequently the amount of current will gradually change as the coil moves past the brush.

In actual practice commutation has to be effected in a very short interval of time (usually ·001 to ·002 second), and the reversal of a large current in a coil which is embedded in slots in the iron of the armature in a very short interval of time would give rise to a considerable voltage, which tends, as shown earlier, to prevent reversal of the current. It is therefore necessary to use force to cause this reversal of the current in the time allowed, and this is done by causing the coil, during the period of reversal, to rotate in a magnetic field, which will give a voltage equal and opposite to the self-inductive voltage of the coil. Immediately the coil is short-circuited by the brush, the current will tend to change to an ultimate value equal to the voltage induced in the coil divided by the resistance of the coil, and such a curve of current charge is shown on *Fig.* 19.

The initial part of this curve is to a large extent independent of the resistance of the coil and brush, and depends upon the rotational voltage and the self-induction of the coil, and in order that the current may be reversed properly, the rotational voltage must be made proportional to the current which the machine is carrying. The usual means for doing this

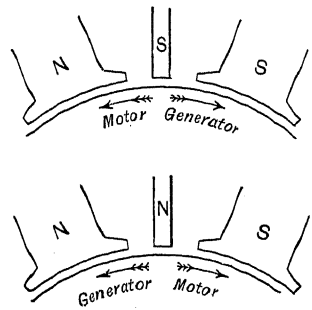


FIG. 20.

is to provide the machine with small auxiliary poles called Commutating Poles, situated midway between the main poles. *Fig.* 20 shows such an arrangement of poles.

These poles are provided with coils which are traversed by the main armature current, so that the commutating flux produced is proportional to the current which the machine is carrying. *Fig.* 21 shows the distribution of flux and armature reaction for the commutating pole machine at no load and when carrying full load. This diagram shows clearly the distortional effect produced by the commutating poles which enables commutation to be effected. *Fig.* 22 shows a similar curve for a machine which has a com-

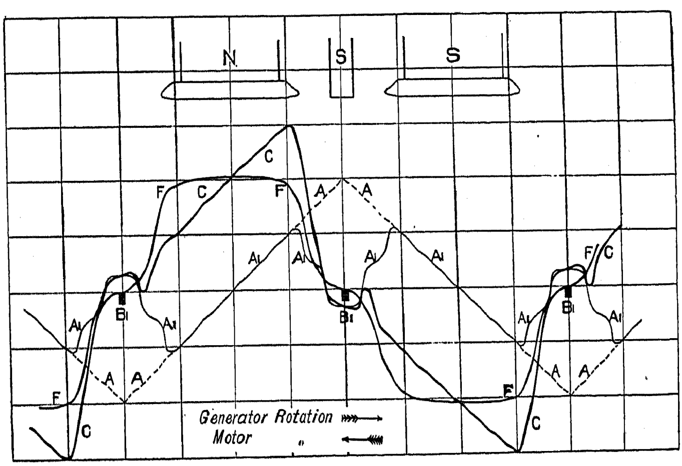


FIG. 21.

pensating winding on the main poles in addition to the commutating poles.

In these figures the letters F and A refer to the flux and armature reaction, while C is the current.

Suppose that the machine is carrying 100 amperes, then the current in the coil would have to change from plus 100 amperes to minus 100 amperes during the commutation period, or during the time that it is short-circuited by the brush. If this has not happened, the coil will be carrying more or less current than should be, and the current will have to change suddenly when it breaks contact with the brush. This sudden change in current at the end of the commutating period gives rise to sparking, and if bad sparking takes place the commutator rapidly deteriorates and the machine becomes inoperative. The important object to be obtained in the proper designing of a machine is that commutation shall take place without appreciable sparking. The means for accomplishing this result have been discussed, but in practice they are not always attainable, and it is therefore undesirable to build machines in which the current per coil exceeds a certain safe value. If a large current machine has to be built, it is necessary to build it with many poles, and a corresponding number of parallel circuits, so that the current in each individual coil is kept to a safe value. In addition to limiting the amount of current in the coil, it is desirable to make the self-induction as low as possible.

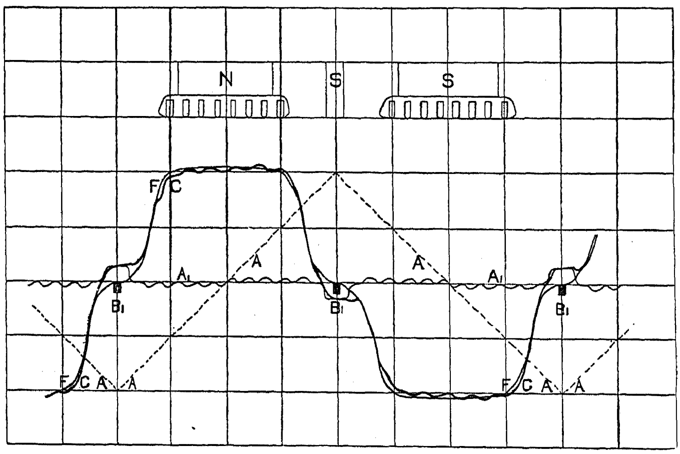


FIG. 22.

In the diagram, *Fig.* 12, is shown a means frequently used in D.C. machines, in which it will be noticed that there are two coils per slot, and that the pitch of the end windings is an integral number plus a half.

The induction of a coil undergoing commutation is not the self-induction of the coil by itself, but its induction relatively to the next coil which is still short-circuited by the brushes. In the arrangement shown on *Fig.* 12 one side of the coil is in the same slot with the coil next to it, and therefore its self-induction is very low, and mutual induction relatively to the next coil high; on the other side the coils occupy adjacent slots, and therefore the self-induction is represented by the flux that can pass through the tooth between them. The energy available for causing sparking and burning of the commutator and brushes is represented by the product of the surplus or deficit current that the coil carries at the moment it leaves the brushes, and the flux associated with this current that can pass through the tooth or teeth between coils. Expressed in other words, it is proportional to the square of the current in a coil multiplied by the coefficient of self-induction. As it is not possible to neutralise the flux of self-induction exactly by means of commutating poles under all conditions of operation, it is desirable to keep the self-inductive energy low. This can be done by means described above and other similar devices, and also by keeping the core length short and by keeping the current in each coil as low as possible. An actual oscillograph of the current in a coil of a comparatively large generator when running heavily overloaded is shown on *Fig.* 23. If

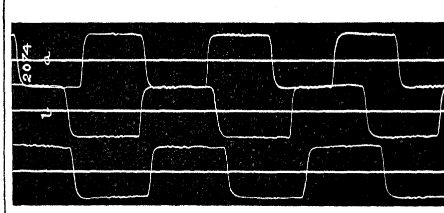


FIG. 23.

commutation is taking place correctly, the current flowing from brushes to commutator will be evenly distributed over the surface of the brush, and consequently the potential difference between brush and commutator will be uniform. Therefore a convenient

means of testing proper commutation is to measure the difference of potential from front to back of the brushes; if this is low, commutation is correct.

§ (11) Direct-current Generators and Motors.—The great advantage of direct-current machines is that:

(i.) They can be made self-exciting.

(ii.) They can be used in conjunction with storage batteries.

(iii.) The speed can be readily varied.

So far as generators are concerned, the tendency at present is to confine these to small machines, though in certain cases large machines are used, especially in connection with electrolytic plants; but, in consequence of the possibility of varying the speed, there is a big field for both large and small direct-current motors.

The magnet frame of a large machine is shown on *Fig.* 24.

The number of poles on a direct-current machine depends on the size, speed, and the voltage. Generally, the larger the output, the greater the number of poles; the higher the speed, the fewer the poles, and the higher the voltage, the fewer the poles. The factors which determine the design and permissible flux density at the air-gap and in other parts of the magnet are: armature reaction; the leakage between the poles, and more particularly between main and commutating poles; the permissible size of armature conductors and slots, and the current in the conductors both from the point of view of heating and the possibility of satisfactory commutation.

Some consideration has been given to these features in the early part of this article. The actual design of the machine cannot be made subject to any definite rules, as the final choice depends on factors, many of which are purely of a manufacturing nature, but in practice it has been found that a number of poles which correspond to a frequency of 20 to 30 cycles give a satisfactory design of machine. The length of the core varies between p and $\frac{1}{2}p$, where p is the pole pitch at the circumference of the armature.

In most D.C. machines the mechanical stresses due to centrifugal force do not form a limit in the design, as limits due to commutating difficulties occur earlier. The armature punchings on small machines are made in a complete circle, and for large machines are made in segments, the segments overlapping one another by a half. For a

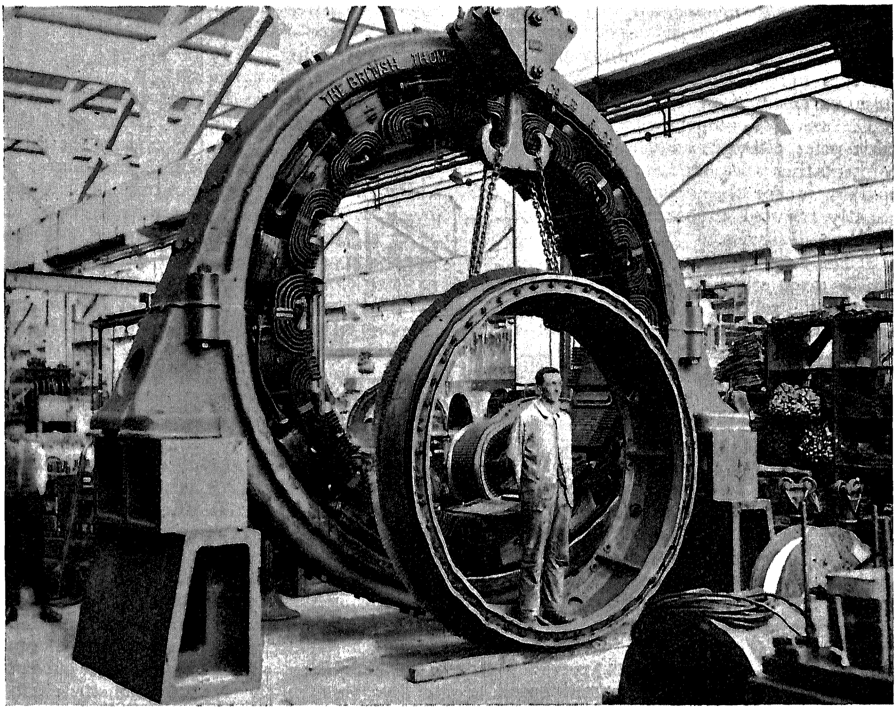


Fig. 24.

given diameter of armature the amount of material will be reduced somewhat by increasing the number of poles, though this gain is most marked when changing from two to four or six poles, but if too great a number of poles are used, there may be excessive magnetic leakage between the poles. The commutator consists of copper bars in width about $\frac{1}{4}$ in. to $\frac{3}{8}$ in. insulated from one another by plates of mica about ·03 to ·04 inch thick. It is undesirable to allow more than 25 to 50 volts per inch of circumference of the commutator, as in case of a very severe or sudden overload excessive sparking may momentarily occur at the brushes, and the copper vapour and carbon particles may form a conducting path round the commutator, thus a flash over may occur from one brush to another.

The use of a compensating winding on the main poles, either in addition to or instead of commutating poles, tends to prevent excessive sparking under these conditions, and is frequently resorted to on machines subject to very severe service.

It has been mentioned that D.C. machines can be made self-exciting; they may also be separately excited. When self-excited there are two ways of arranging the exciting circuit. These are known as shunt and series excitation respectively. In the first case the exciting windings are connected across the terminals of the machine, and in the other case the exciting windings are arranged in series between the armature and the external circuit. In some cases a combination of both these methods of excitation is used. *Fig.* 25 shows the two arrangements.

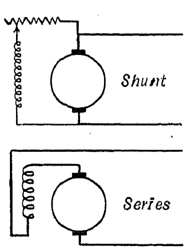


FIG. 25.

(i.) *Shunt Dynamo.*—With the shunt method a dynamo will tend to keep constant voltage with constant speed of rotation, and this is usually the desired condition for a generator, as the line is most conveniently carried at a constant voltage. Actually, the voltage of a shunt machine drops slightly with increase of load, due to the demagnetising effect of the armature in reducing the flux, and also to ohmic drop in the windings of the machine. This drop in the armature voltage of course decreases the excitation current, and the effect is therefore accumulative. In order that a shunt machine may maintain its voltage reasonably steady, it is necessary that the magnetic circuit should be saturated to a certain extent; in other words, the flux is not directly proportional to the exciting current.

In the curve, *Fig.* 26, are shown two so-called saturation curves from a typical machine, *i.e.* curves which show the relationship between the generator voltage and exciting current. These two curves correspond to normal speed and half speed of the machine. On the same diagram are shown also three other curves—the straight lines A, B, C—which correspond to different values of resistance of the exciting circuit. The slope of the lines gives in each case the value of the ratio voltage/exciting current, *i.e.* the resistance.

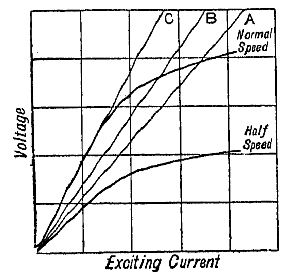


FIG. 26.

If the machine is operating at normal speed, it will be seen that definite voltages occur at the intersections of A and B with the upper curve, but when the resistance of the exciting circuit is increased to correspond to C, no definite voltage is obtained, and the machine becomes unstable. The angle of intersection of these lines is a measure of the stability of the machine. There is nearly always a small amount of residual magnetism in the iron circuit, which gives initial tendency for the machine to excite itself. The connections between the brushes and exciting winding must be such that this initial tendency becomes accumulative and the machine excites itself. It is also necessary that the winding should have a sufficiently low resistance, otherwise self-excitation will not occur. In the curve, which corresponds to the low speed of the machine, it will be noticed that the voltage produced by the machine is not enough to cause sufficient excitation current to flow through the resistance of the exciting windings. At this speed, therefore, the machine will not self-excite. The shunt current of a dynamo may be as low as $\frac{1}{2}$ per cent of the main current on large machines, and 5 to 10 per cent on small machines. In any case it is a small proportion of the total current of the machine, and it is therefore an easy matter to arrange variable resistance in the exciting circuits, whereby the voltage of the machine can be controlled.

(ii.) *Series Dynamo.*—The series method of excitation is unsuitable for generators, except under a few special conditions of operation, but in many cases a few series turns are added to a shunt machine to correct for the inherent drop in voltage that occurs with increased load.

(iii.) *Shunt Motor.*—For motors both forms of excitation are used. The shunt method of

excitation gives a tendency to constant speed for a constant voltage on the supply circuit and varying load, and the series method gives a variable speed—low speeds with heavy loads, and high speed with light loads.

When a machine is running as a motor coupled to a source of supply, the voltage generated in the armature will be approximately equal to the voltage on the line. When this voltage is constant, the excitation of a shunt motor will be constant, and independent of the load; therefore flux will be constant, and as voltage depends on flux and speed, the speed will be constant also independently of the load in the armature.

(iv.) *Series Motor.*—In the series motor, however, if the load increases, the current in the armature increases, so also does the excitation and flux; therefore the speed will decrease, as a lower speed will be sufficient to generate the necessary voltage with increased flux. The torque depends on the flux and armature current, and consequently a series motor is capable of exerting a very large torque at low speeds. This makes it particularly suitable for such duties as operating trains, etc. A series motor must never be entirely without load, otherwise it may run away and attain a dangerous speed. On the other hand, a shunt motor may be operated entirely without load, and will run at a definite speed. All motors, whether series or shunt type, must have a resistance inserted in series with the armature during starting period until the armature can attain sufficient speed to generate the voltage equal to the supply voltage. This resistance is cut out after the motor has attained its speed.

The mechanical construction of an armature is illustrated in *Fig.* 27. This shows the punchings carried on a cast-iron hub or spider, and the means adopted for firmly securing them in place. Ventilating ducts are left at intervals in the punchings. A complete armature is shown on *Fig.* 28.

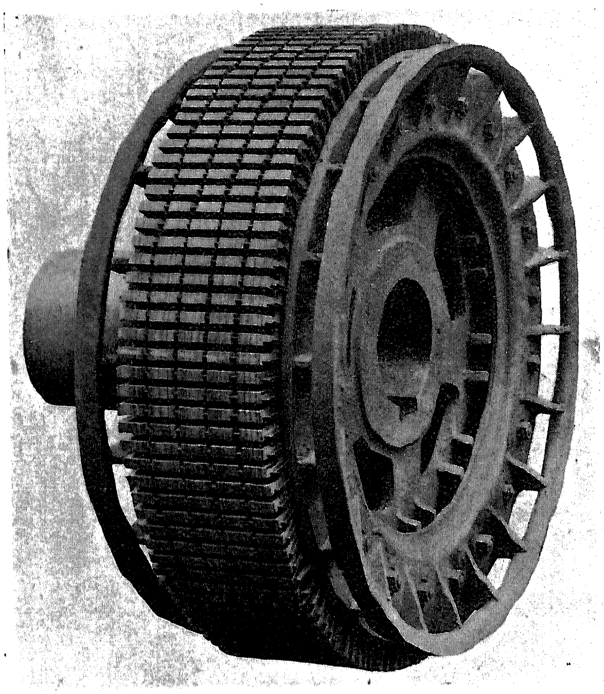


Fig. 27.

Typical views of complete machines are figured in *Figs.* 29 and 30.

§ (12) Alternators.—In the alternator the commutator is eliminated, the ends of the windings being brought direct to the terminals of the machine. The elimination of the commutator also allows the machines to be made much more readily in large sizes. At the present day the alternator is the most usual form of generator which is used. It is coupled to steam turbines, water wheels, steam engines, and gas or oil engines; the total capacity of the machines installed being in the order given.

The steam turbine has a very high rotational speed (necessary owing to the high velocity

which steam attains when issuing from a nozzle), and alternators have been built which can be directly coupled to steam turbines. In this country the usual frequency is 50 cycles, which gives rotational speeds of 3000 r.p.m. for 2-pole machines or 1500 for 4-pole machines, and most of the alternators now built run at either of these speeds.

Alternators are made in almost all cases with the armature stationary, and the field revolving ; the two chief reasons for this being that with a revolving field it is only necessary to carry through the slip-rings the small amount of energy required for exciting the field, and further that it is much easier to put high voltage armature coils on the stator than on the rotor, especially if the rotor runs at a very high speed.

The largest machines which are being built at present have capacities of 10,000-15,000 kw. at 3000 r.p.m., and 30,000-40,000 kw. at 1500 r.p.m. Such machines are usually designed to operate with a power factor of 80 per cent. The usual voltage for such machines is between 6000 and 12,000 volts. It has not been found desirable to build machines on a much higher voltage than 15,000, and if higher voltages are required for transmission purposes, this is easily obtained by means of step-up transformers.

The principal difficulties in connection with the designing of alternators for coupling to steam turbines are mechanical. In order to get the output mentioned above, it is necessary to go to a very high peripheral speed, which would attain in some cases 25,000 feet per minute. The rotor is usually built in the form of a cylinder with slots, which carry the exciting windings. The end portions of the winding are retained in place by stout steel rings. The exciting winding consists of coils formed from a flat copper strip, both the individual turns and the complete coil being insulated with mica.

Great care has to be used in designing these machines and in choosing the right material to deal with the very high centrifugal forces associated with the high speed of rotation. It is necessary to give special attention to the proper balancing of rotors, otherwise serious vibration will occur, as machines very often have to pass through what is called the "critical speeds." This is the speed at which the rotor would naturally vibrate, and depends on its weight and the elasticity on the shaft. Means have to be provided on all machines which have to pass through this critical speed to prevent the vibration becoming dangerous at this point.

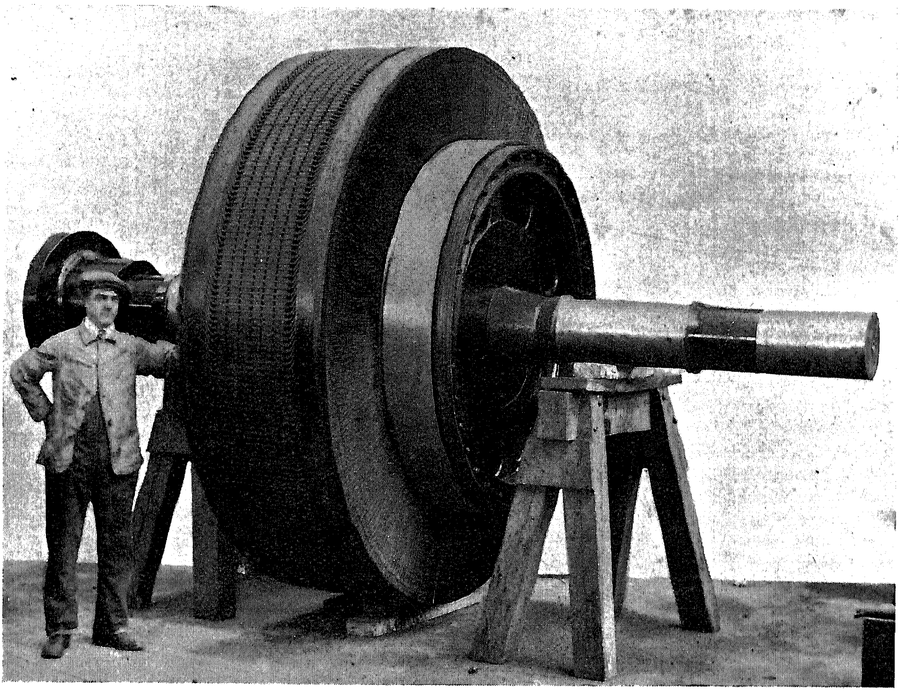


FIG. 28.

The heat generated by the losses in the machine is carried off by means of air which is forced through the machine by means of fans on the end of the rotor, ducts being provided both in the rotor and stator to ensure that the air comes in contact with all parts.

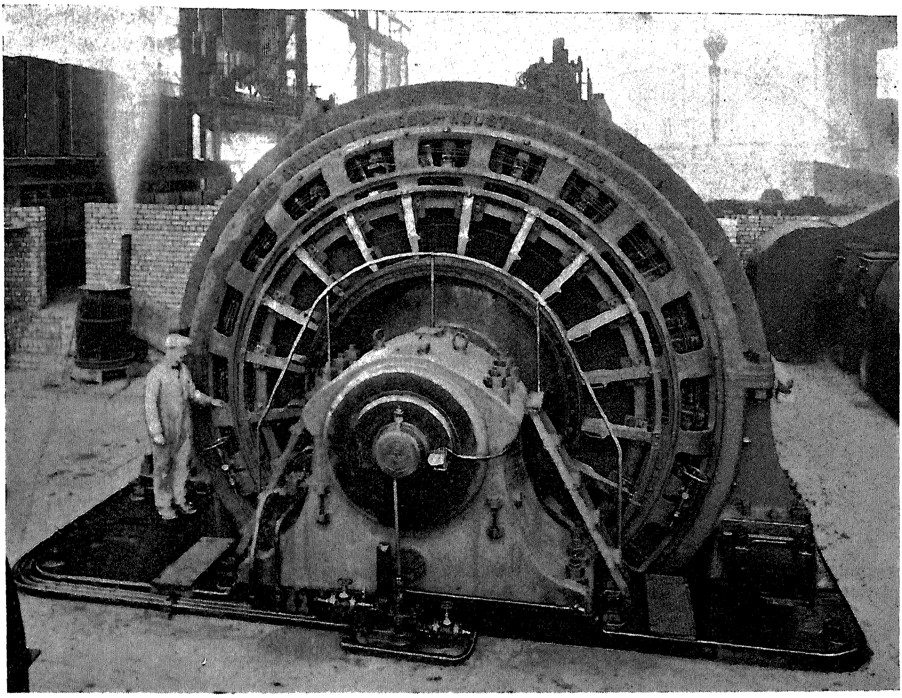


FIG. 29.

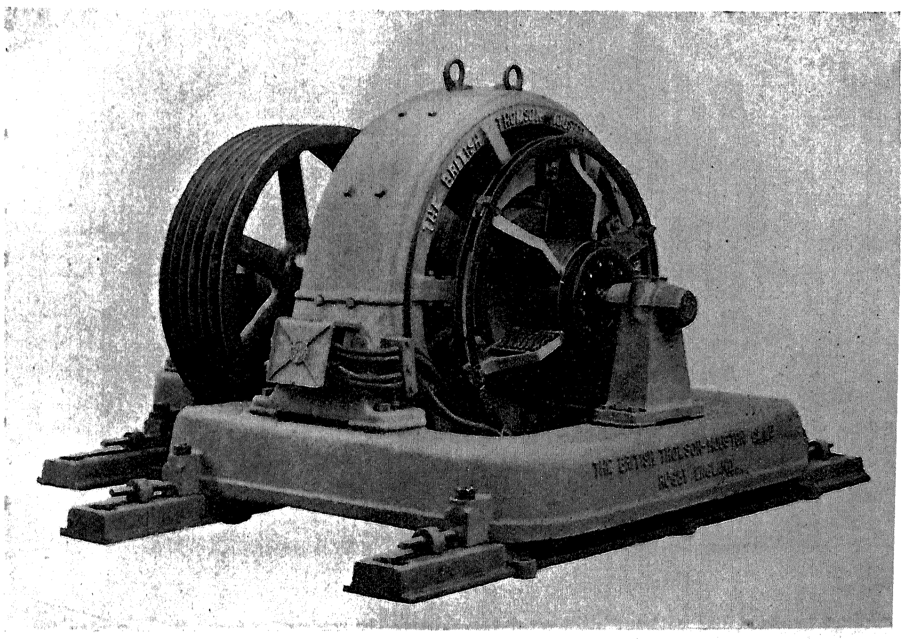


FIG. 30.

Sectional views of typical turbo-alternators are shown on *Figs.* 31 and 32.

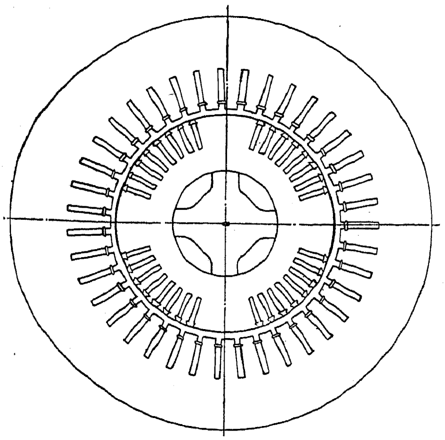


FIG. 31.

Fig. 33 shows a stator frame without the windings, and *Fig.* 34 a similar machine completely wound. A 2-pole rotor with the exciting ring is put in place on *Fig.* 36. The excitation is usually supplied by means of a small direct-coupled direct-current generator, a view of which is shown on *Fig.* 37. Another view of

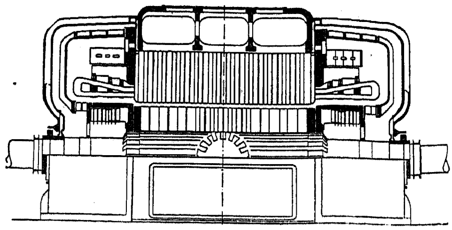


FIG. 32.

a high-speed alternator coupled to a steam turbine is shown on *Fig.* 38.

Alternators which have to be coupled to steam engines rotate at a much lower speed, and therefore have to have more poles. A typical machine is shown on *Fig.* 39, the rotor for such a machine being shown on *Fig.* 40. A partially wound stator is shown on *Fig.* 41.

Machines for coupling to waterwheels are intermediate in speed from those which are

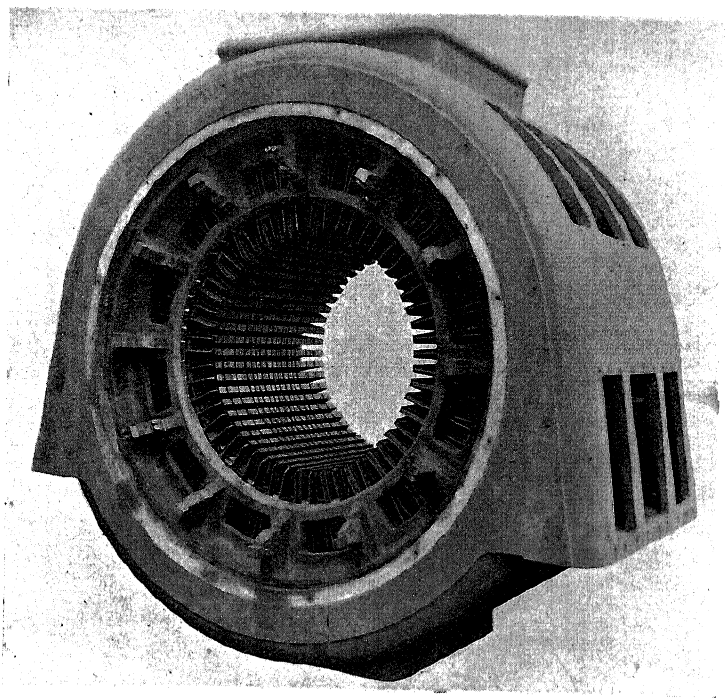


FIG. 33.

windings in place, but without steel retaining rings, is shown on *Fig.* 35, and the retaining driven by steam-engines and steam-turbines. Such machines are built with definite poles,

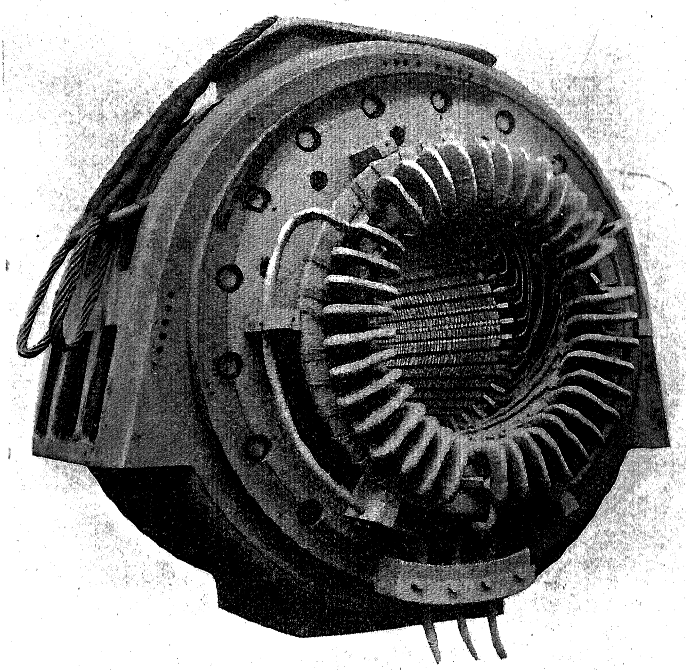


FIG. 34.

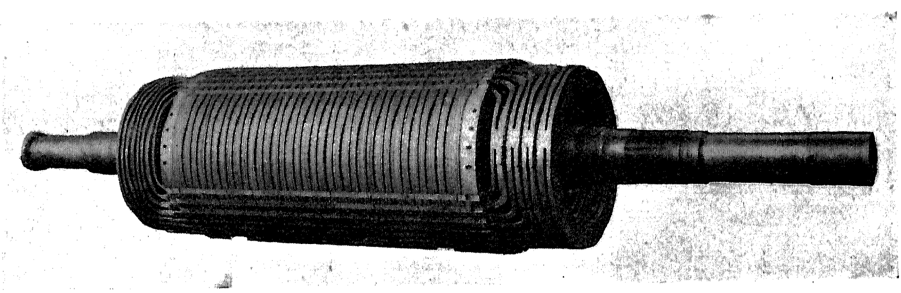


FIG. 35.

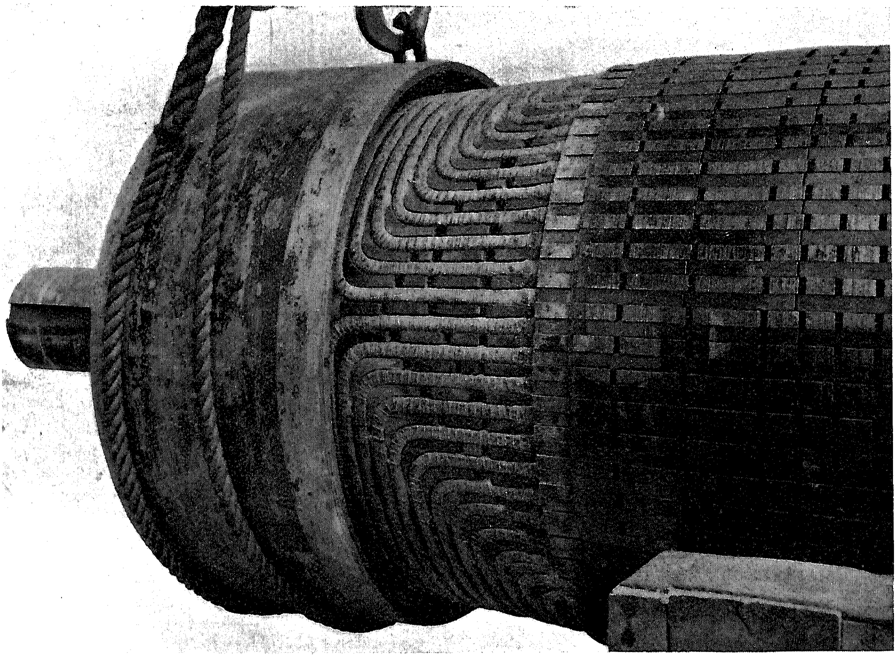


FIG. 36.

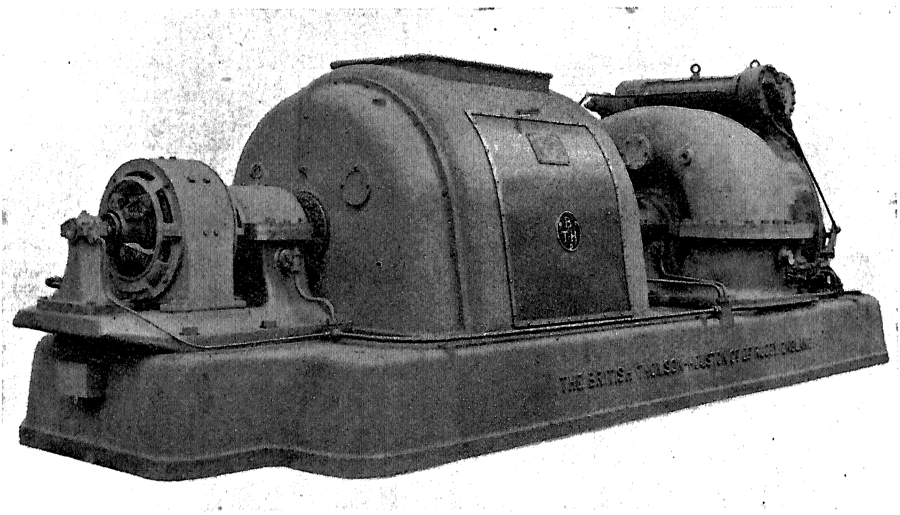


FIG. 37.

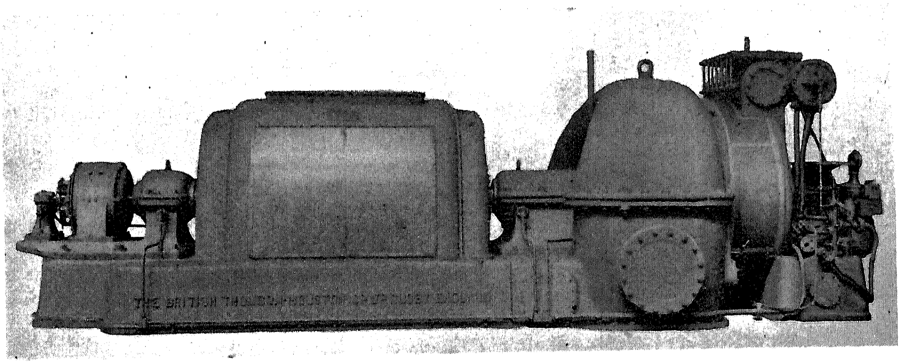


Fig. 38.

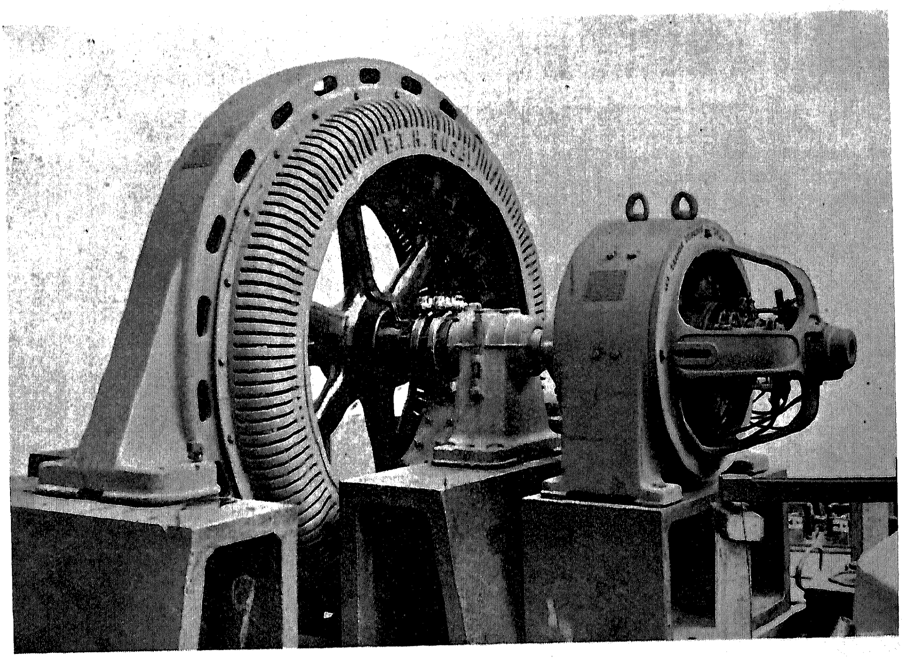


Fig. 39.

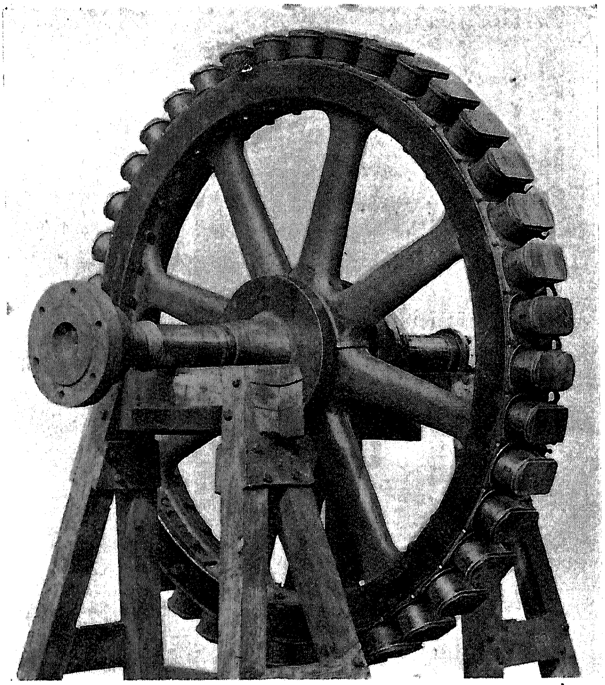


FIG. 40.

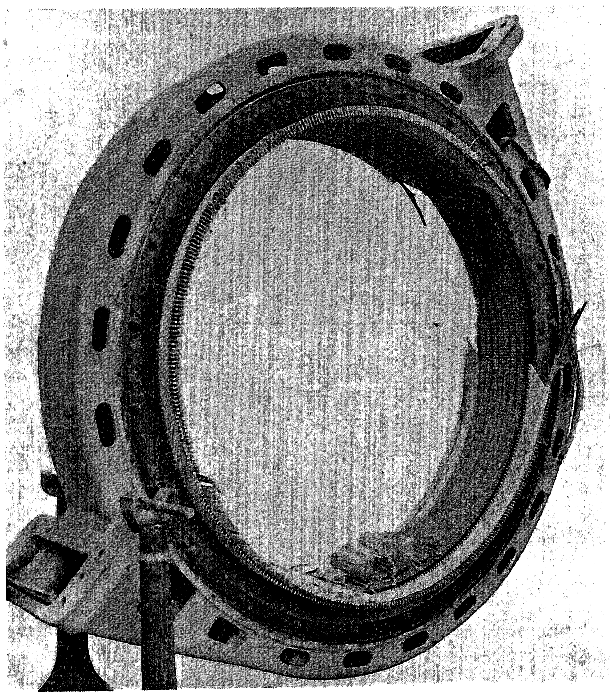


FIG. 41.

which have to be very strongly constructed as the waterwheel is liable to run away if the load is suddenly removed, and construction has to be sufficiently robust so that the machine

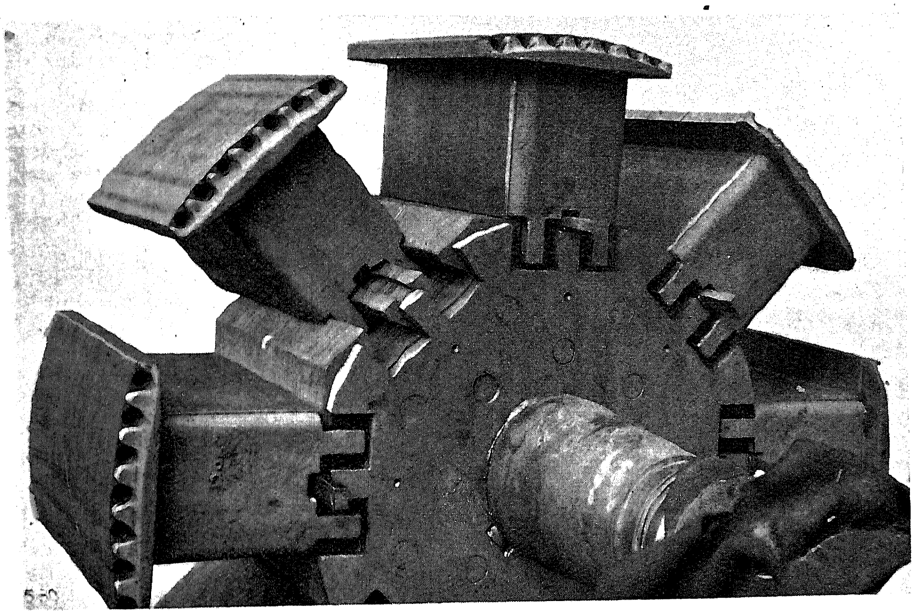


FIG. 42.

will hold together even at speeds twice the usual speed. The construction of rotor used on such machines is shown on *Figs.* 42 and 43.

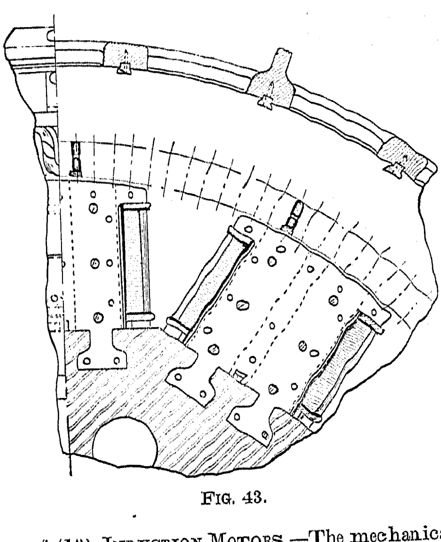


FIG. 43.

§ (13) INDUCTION MOTORS.—The mechanical construction of a typical induction motor is shown on the attached sectional drawing (*Fig.* 44).

The stator consists of a ring of laminations slotted on the inside and carrying the primary windings. The windings are arranged in groups corresponding to the number of phases and number of poles. The form of windings

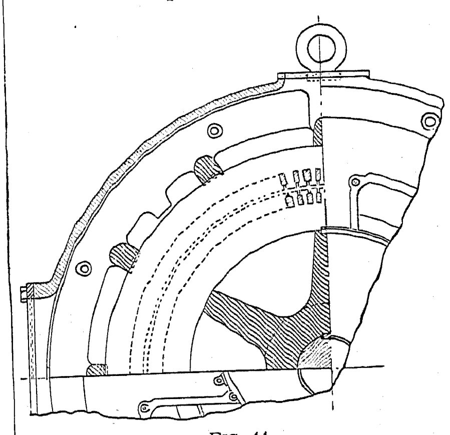


FIG. 44.

depends on the size of the motor and voltage of supply. For small machines, and some high-voltage machines, the conductors consist of round wires formed into coils, and for larger machines the conductors consist of rectangular

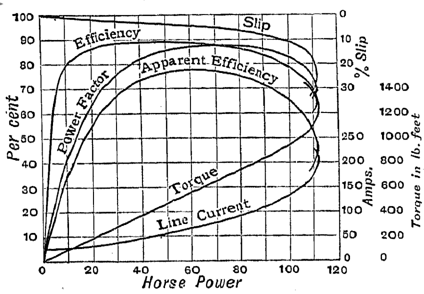


FIG. 45.

bars. The rotor consists also of a ring of punchings of a lesser radial depth than the stator, as higher flux densities can be allowed in the rotor. The rotor winding is usually arranged for low voltage, and consists either of a set of bars short-circuited at their ends by rings, or of rectangular bars connected up to form groups corresponding to the number of poles and phases.

The speed at which the magnetic field of an induction motor revolves, expressed in revolutions per minute, is obtained by multiplying the frequency in periods per second by 120, and dividing by the number of poles; for example: an 8-pole, 50-cycle motor will have

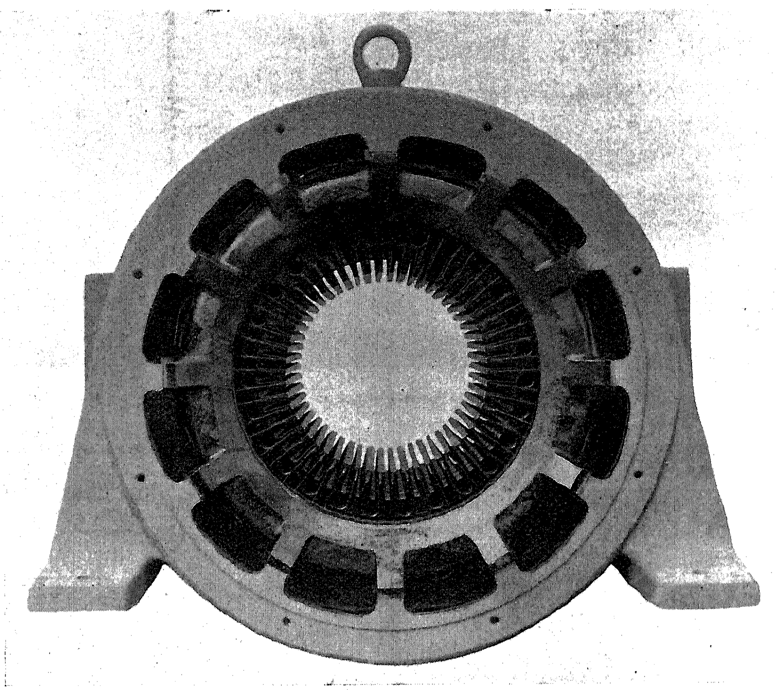


FIG. 46.

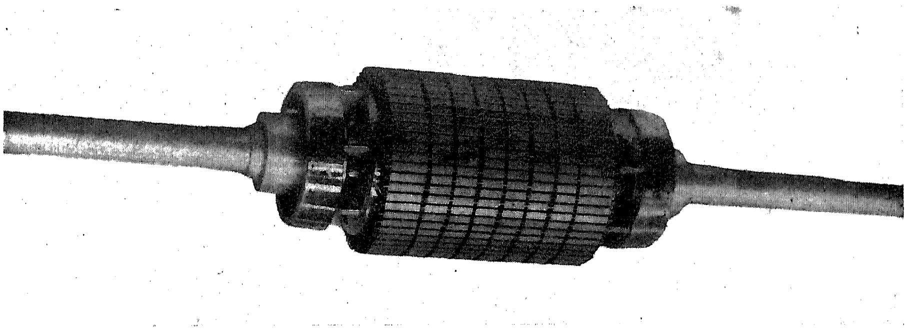


FIG. 47.

a speed of 750 revolutions per minute. The rotor will revolve at approximately this speed when the motor is unloaded, and will have a slower speed when fully loaded. The difference of speed or slip amounts to 2 or 3 per cent in large motors, and about double this amount in small motors.

A curve sheet showing the performance of an induction motor is shown on *Fig.* 45.

An unwound stator is shown on *Fig.* 46, and an unwound rotor in *Fig.* 47. The stator consists of a cast-iron frame with feet. The laminations are carried in this frame and firmly held in place by heavy cast-iron end flanges. The completely wound stator is shown in *Fig.* 48. The coils in this machine are all of the same shape and nest into one another at the ends, forming a basket-like construction. The terminals of the three phase are seen on the near side of the stator frame.

A completely wound rotor is shown on *Fig.* 49.

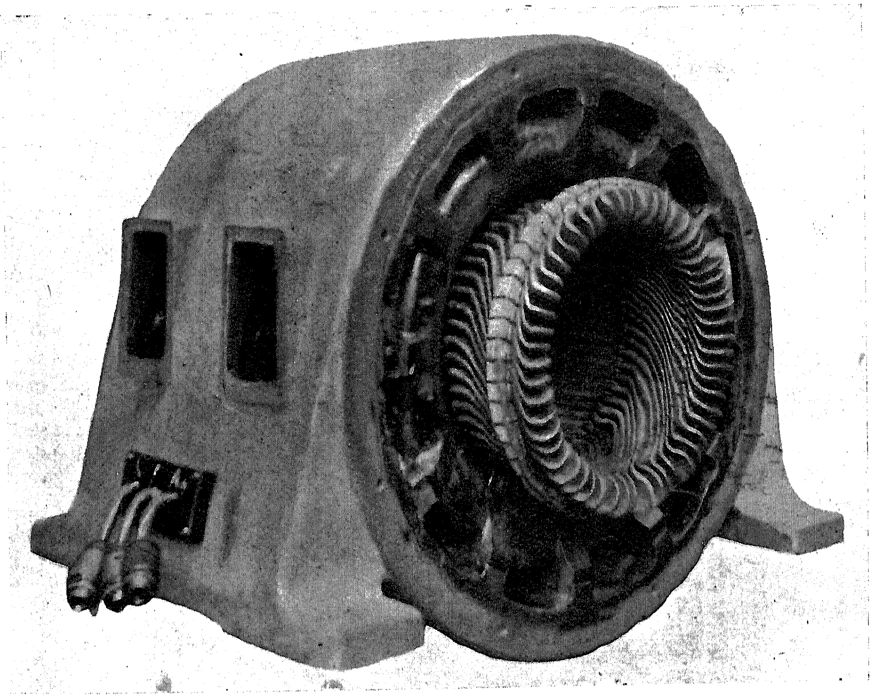


FIG. 48.

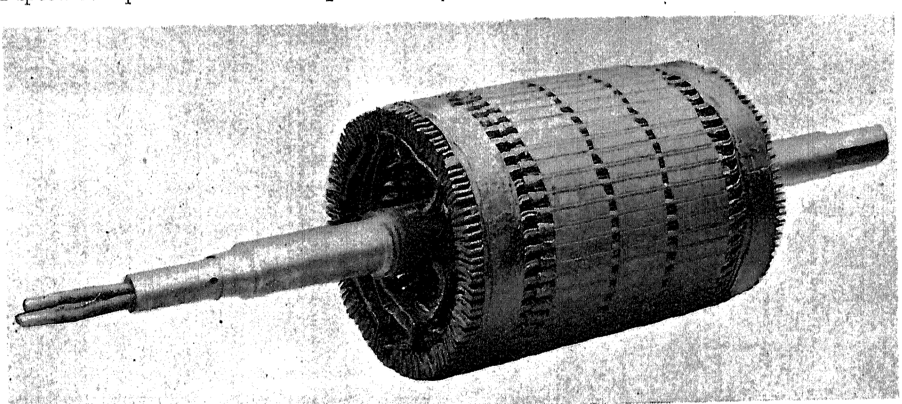


FIG. 49.

A short-circuited or " Squirrel Cage " type of rotor is shown on *Fig.* 50. In this particular rotor the bars are electrically welded to the end short-circuiting ring, so that the whole structure is practically a jointless piece of copper. It has been mentioned already that a squirrel cage induction motor will give a reasonable starting torque at the expense of a large starting current. A motor of normal size and construction will take about five to six times normal current when connected to the mains, provided the supply voltage remains normal. This large current will cause heavy losses in the windings—six times normal current will cause thirty-six times normal losses. Provided this initial rush of current is not detrimental to the voltage regulation of the supply system, the limit of ability to withstand such a current lies in the motor itself, and usually in the rotor winding, as the current density is usually higher in the rotor bars than in the stator winding. Joints in the winding are a source of trouble, as obviously soldered joints are not always suitable if the winding is liable to be overheated, and bolted joints are likely to become loose. The welded rotor illustrated is

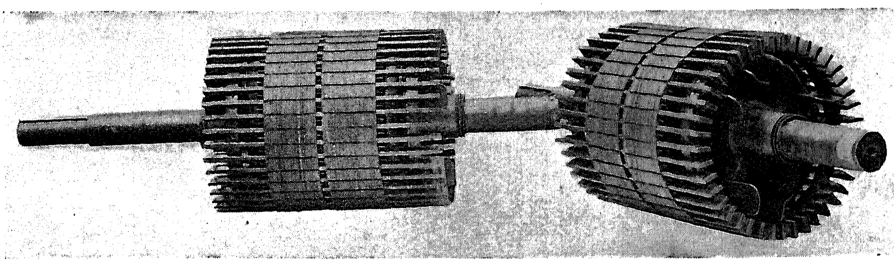


FIG. 50.

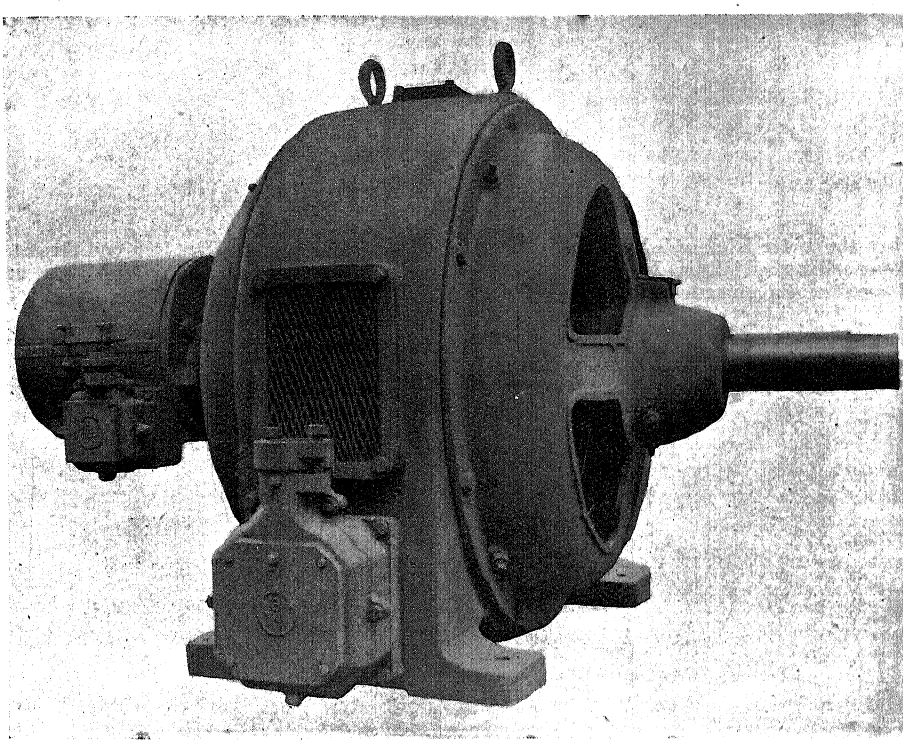


FIG. 51.

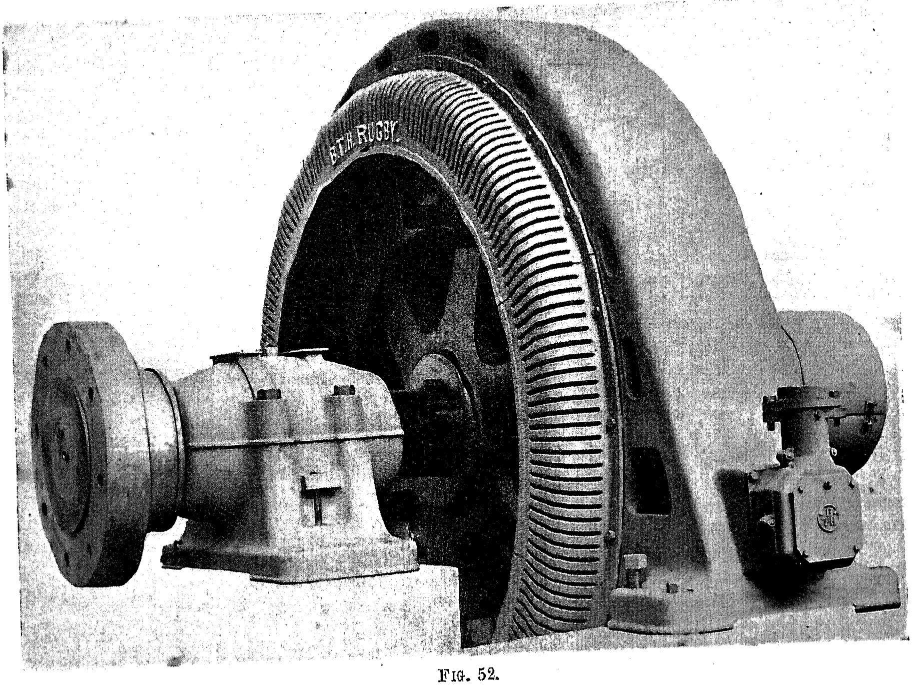


FIG. 52.

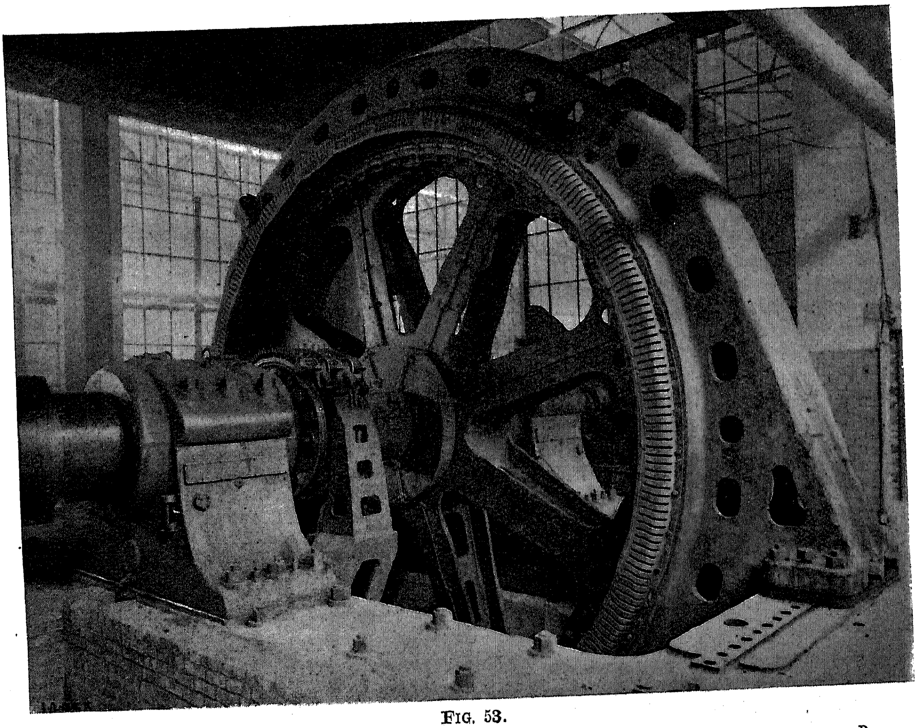


FIG. 53.

most suitable for withstanding the momentary overheating which may occur at starting. It can be shown that the amount of energy dissipated in a rotor winding is equal to the amount of mechanical work done in accelerating a machine up to full speed, but if the motor has to start against a load torque, the amount of loss in the rotor will be more.

Where a motor is only required to start against a comparatively small load, the voltage at the terminals is sometimes reduced at starting and raised again when the motor has attained full speed. A convenient way to do this is to use a small transformer and a throw-over switch, or the windings can be connected $\curlywedge$ when starting, and changed to Δ for running. This arrangement reduces the starting voltage to 58 per cent, and the initial rush of current to one-third of what it would be if started at full voltage.

Where it is necessary to keep the starting current low, and where large starting torques are required, a phase-wound rotor is used. Such a motor is illustrated in *Fig.* 49. The ends of the phase are connected to slip-rings on which carbon or graphite brushes make contact, and resistance is introduced during the starting.

The rings are sometimes short circuited when the motor has attained full speed in order to avoid the contact losses in the brushes and rings. Illustrations of typical induction motors are given in the figures on pp. 208, 209: *Fig.* 51 shows a small motor used for general industrial purposes; *Fig.* 52 is a large motor with separate bearings; and *Fig.* 53 is a motor for driving a continuous steel-rolling mill. This motor is rated at 5000 H.P. at a synchronous speed of 92 revolutions per minute and a maximum output of 12,000 H.P. at about 90 r.p.m. The corresponding torques are $3 \cdot 45 \times 10^6$ inch-lbs. and $8 \cdot 45 \times 10^6$ inch-lbs.

F. H. C.

Dynamometer, Repelled Disc, use of, for the measurement of small radio-frequency currents. See "Radio-frequency Measurements," § (18).

Dynatron: a special type of thermionic valve having "negative" characteristics.
Use of, as Oscillation Generator. See "Thermionic Valve, its Use in Radio-measurements," § 5 (iv.).

— E —

"*e*," the charge on an electron. Electrical and Scintillation Method of Determination of, using α particles. See "Electrons and the Discharge Tube," § (22).
Electrolytic Determination of. See *ibid.* § (23).
Millikan's Determination of. See *ibid.* § (21).

Earth, Currents of Electricity stronger at times of magnetic disturbance and ascribable to induction in the conducting layer of the surface of the earth by the varying magnetic field of the external current sheet. See "Magnetism, Theories of Terrestrial and Solar," § (25).

Earth, General Magnetic Field of the. See "Magnetism, Theories of Terrestrial and Solar," § (1).

Earth Capacities: capacities between the plates of a condenser and the earth. Effects of, on capacity measurement. See "Capacity and its Measurement," § (33).

Earth's Magnetic Field, Effect on Arc of. See "Arc Lamps," § (5).
Measurement of. See "Magnetism, Terrestrial Observational Methods."

Earth-plates: plates buried in the earth, used in "earth-return" telegraph circuits. See "Telegraph, The Electric," § (14).

Earth Resistance, as a factor in the corrosion of underground structures. See "Stray Current Electrolysis," § (15).

Earth-wires, for telegraph poles. See "Telegraph, The Electric," § (14).

Earthing: connection of some point on a circuit to earth in order to fix the potential of the system. See "Switchgear," § (17).

Ebonite Condensers. See "Capacity and its Measurement," § (30).

Eddy Current Disc. See "Meters for D.C. Electricity," § (2) (ii.).

Eddy Current Losses, in static transformers: power losses due to eddy currents in the windings. See "Transformers, Static," § (4).

Eddy Currents: currents induced in masses of metal by changes of magnetic flux.
In Transformer Cores. See "Electromagnet," § (6).

Effective Value. The effective value of any quantity is the square root of the mean square of that quantity. If the quantity vary harmonically and be given by an equation such as $i = \mathrm{I} \sin \omega t$, then the effective value $\bar{\mathrm{I}}$ is given by the equation

$$\bar{\mathrm{I}} = \frac{\mathrm{I}}{\sqrt{2}} = 0 \cdot 7071 \times \mathrm{I}.$$

For $\quad i^2 = \mathrm{I}^2 \sin^2 \omega t = \dfrac{\mathrm{I}^2}{2}(1 - \cos 2\omega t),$

and the mean value of $\cos 2\omega t$ is zero. Hence

$$\bar{\mathrm{I}} = \sqrt{(\text{mean value of } i^2)} = \frac{\mathrm{I}}{\sqrt{2}}.$$

Efficiency, Current: a term used in electrolysis to denote the ratio of the actual yield obtained by electrolytic decomposition, to the theoretical yield calculated from Faraday's laws. See "Electrolysis, Technical Applications of," § (2).

Efficiency, Energy: a term used in electrolysis to signify the ratio of the theoretical amount of energy required to decompose unit mass of any substance, to the amount actually needed for the decomposition. See "Electrolysis, Technical Applications of," § (3).

Efficiency, Telephone, Measure of. See "Telephony," § (9).

Efficiency of Thermionic Valves: the ratio of the power given out at the anode, to the total power taken from the steady source. See "Thermionic Valves," § (8).

Efficiency, Voltage: a term used in electrolysis to signify the ratio of the reversible voltage required theoretically for the decomposition of a substance, to the actual cell voltage necessary. See "Electrolysis, Technical Applications of," § (3).

Einthoven Galvanometer. See "Galvanometer," § (7); "Vibration Galvanometer," § (7).

Critical Damping of. See "Galvanometer," § (10) (vi.).

Electric Charge. See "Units of Electrical Measurement," §§ (2), (3).

Electric Discharge through Gases, at low pressures. See "Electrons and Discharge Tube," § (1).

Electric Displacement: the value of the quantity $KR/4\pi$ where R is the electric intensity and K the inductive capacity at any point of an electrical field. See "Units of Electrical Measurement," § (13).

Electric Field: the portion of space in the neighbourhood of a charged body or current throughout which the electric forces to which it gives rise to are sensible.

Electric Force or Intensity. The force in dynes experienced by a unit charge of positive electricity concentrated at any point in an electric field measures the electric force or intensity of the field at that point. The assumption is made that the presence of the charge does not disturb the field. See "Units of Electrical Measurement," § (2).

Electric Oscillations, damping of, due to radiation and resistance. See "Wireless Telegraphy," § (7).

Electric Potential at a Point. The work necessary to bring a unit of positive electricity from beyond the boundary of the field to the point, without disturbing the existing electrical distribution, measures the potential at the point. See "Potential."

Electric Stress as a limitation to the use of dielectrics. See "Dielectrics," § (9).

Dynamics of. See *ibid.* § (2).

Electric Waves, detectors of, for use in wireless telegraphy. See "Wireless Telegraphy," § (18).

Production of, by the acceleration of a moving charge. See *ibid.* § (1).

Production of, on engineering scale. See *ibid.* § (11).

Electrical Extraction and Refining of Metals. See "Electrolysis, Technical Applications of," VI. §§ (15)-(23).

Electrical Intensity, definition of. See "Electrostatic Field, Properties of," § (7); "Units of Electrical Measurement," § (12).

Distribution of, in discharge tube. See "Electrons and Discharge Tube," § (3).

ELECTRICAL MEASUREMENTS, SYSTEMS OF

§ (1) Fundamental Units.—All electrical measurements are founded on one of two systems, and each of these, when complete, requires four fundamental units. The fundamental units in each of the two systems are those of length, mass, time, and a property of the electromagnetic medium. In the first, or electromagnetic system, the property of the medium utilised is magnetic permeability, and in the second, or electrostatic system, the dielectric constant is used. Frequently, for convenience, these properties are assumed to be of zero dimensions, but really the dimensions are not known.[1]

In mechanics there are only three fundamental units, those of length, mass, and time, and these three would probably serve for a system of electrical measurements if the identity of electrical and mechanical phenomena could be established. This, however, is not possible without knowing the dimensions of "magnetic permeability" and "dielectric constant." The electromagnetic and electrostatic systems of measurement, in common with all others in which length, mass, and time are fundamental units, are called "absolute" systems, and because the units of length, mass, and time generally chosen are the centimetre, the gram, and the second respectively, the systems are also referred to as "Centimetre Gram Second" systems, or more briefly "C.G.S." systems.

§ (2) Definitions.—In the electromagnetic system, unit electric current is defined as that of which the unit length at a unit distance from the unit magnetic pole exerts on the latter a unit of force, the unit magnetic pole being defined as that which exerts unit force on a similar pole at unit distance; the medium

[1] See "Units of Electrical Measurement," § (3).

is supposed to be a vacuum. In the electrostatic system unit current is that which conveys the unit quantity of electricity in unit time, unit quantity being defined as that which exerts unit force on an equal quantity at unit distance. Again the medium is supposed to be a vacuum.

Starting from these two definitions Weber first framed two distinct systems of electrical measurements. Subsequently, Professor W. Thomson (later Lord Kelvin) defined a unit of work on Weber's system and thus allowed all physical measurements to be connected together. Of the two systems of electrical measurements, the electromagnetic one is more convenient than the other for general purposes and is the one used in electrical engineering.

The value of such systems for quantitative measurements cannot be over-estimated. If, instead, units had been arbitrarily chosen, the relations between them would still exist, and by the use of coefficients it would be possible to pass from one kind of measurement to another, but the number and complexity of these coefficients would make such a choice inexcusable.

§ (3) Electrical Units. — The electrical phenomena susceptible of direct measurement are Quantity, Current, Resistance, and Electromotive Force. With respect to Quantity, it has been found by experiment that two equal and similar quantities of electricity concentrated in two points repel one another with a force (F) directly proportional to the quantity (Q), inversely proportional to the square of the distance (d) between the points, and inversely proportional to a quantity called the dielectric constant (K) which varies with the medium. The equation representing the facts is

$$F=\frac{Q^2}{Kd^2}. \quad . \quad . \quad . \quad . \quad (1)$$

(i.) *Electrostatic.*—In the electrostatic system K is usually taken as of unit value in a vacuum, and in such a case the unit of quantity is that which exerts a force of one dyne upon an equal quantity, the two being collected at points one centimetre apart in a vacuum.

With regard to Current, it has been experimentally proved (first by Faraday) that the quantity (Q) of electricity conveyed by any given current (I) is simply proportional to the strength of the current and to the time (t) during which it flows. Hence we have the equation

$$Q=It. \quad . \quad . \quad . \quad . \quad (2)$$

The mechanical effects observed in connection with electricity give the connection between electrical and mechanical units. Whenever a current flows through any circuit it performs work, or produces heat or chemical action equivalent to work, and it has been experimentally proved that the work (W) done is directly proportional to the square of the current (I), to the time (t) during which it acts, and to a quantity known as the resistance (R) of the circuit, which can be proved to be constant so long as the physical conditions are unaltered. This is expressed by the equation

$$W=I^2Rt. \quad . \quad . \quad . \quad (3)$$

It follows that the unit current flowing for a unit of time will perform a unit of work (1 erg) in a circuit of unit resistance.

By experiment, Ohm's law connecting current (I), electromotive force (E), and resistance (R) has been verified, the equation being

$$I=\frac{E}{R}. \quad . \quad . \quad . \quad . \quad (4)$$

From this it follows that the unit electromotive force is that which produces the unit current in a circuit of unit resistance.

If the dielectric constant K of air be assumed to be equal to unity, equations (1) to (4) suffice to measure all electric phenomena by reference to length, mass, and time. Otherwise expressed, it is possible to determine Quantity, Current, Resistance, and Electromotive Force, by reference to Mechanical Units only. Equation (1) determines Quantity in terms of a Force and a distance, and since Force is measured in terms of a mass, length, and time, Electric Quantity can be expressed in a Length-Mass-Time system. Equation (2) determines Current in terms of Quantity and Time. Equation (3) gives Resistance in terms of Current, Time, and Work, and since the last is measured in terms of a Force and a Length it follows that Electric Resistance can also be measured by reference to mechanical units. Lastly, (4) gives Electromotive Force in terms of Current and Resistance. It is clear, therefore, that the system is a coherent one enabling measurements of Quantity, Current, Resistance, and Electromotive Force to be measured in terms of Length, Mass, and Time.

The units based on these four equations are called electrostatic units and are little used, except in electrostatics. The system is founded on a statical phenomenon, the force between two electric charges, and this is not the most convenient scheme for dealing with the many dynamical applications of electricity.

(ii.) *Electromagnetic.*—Now the force exerted by a current on the pole of a magnet in its vicinity is a purely mechanical effect. This force (F) has been experimentally proved to be proportional to the strength of the current (I) and to the strength of the magnetic pole

(m). Further, if the current be at all points at the same distance from the magnetic pole, that is, if the conductor conveying the current is bent in a circle of radius r round the pole, the force is proportional to the length (L) of the conductor and inversely proportional to the square of the distance (r). This relation is expressed by the equation

$$F=\frac{ILm}{r^2}. \qquad (5)$$

From which it follows that in this the Electromagnetic System the unit current is that of which the unit length everywhere at a unit distance from a unit magnetic pole exerts on the latter the unit force (1 dyne).

In a manner similar to that for the electrostatic unit of quantity the magnetic pole strength (m) is defined as that which exerts unit force upon an equal magnetic pole, the two being at points 1 cm. apart *in vacuo* which is assumed to be of unit magnetic permeability. If the medium be of magnetic permeability μ, we have the general equation

$$F=\frac{m^2}{\mu d^2}. \qquad (6)$$

If μ is supposed to have no dimensions and the equations (2), (3), (4), (5), and (6) are adopted as fundamental, they give a distinct system of units called the Electromagnetic System. The Electromagnetic System and the Electrostatic System are both absolute inasmuch as all measurements in them can ultimately be referred to the fundamental units of Length, Mass, and Time. As previously stated, the electromagnetic units are found much the more convenient when dealing with electromagnetic phenomena, and it is this system which is most generally used.

§ (4) Practical Electromagnetic Units.—The units of resistance, of electromotive force, and others in the electromagnetic system are either too large or too small to be convenient in practice. Definite multiples or sub-multiples of these units are therefore used. Some of these practical units and their relation to the C.G.S. units are given below:

	Name of Practical Unit.	Value of Practical Unit.
Resistance	Ohm	10^9 C.G.S. units
Current	Ampere	10^{-1} ,,
Electromotive Force	Volt	10^8 ,,
Quantity	Coulomb	10^{-1} ,,
Capacity	Farad	10^{-9} ,,
Inductance	Henry	10^9 ,,

These practical units are consistent with the four fundamental units, 10^9 centimetres, 10^{-11} gram, one second, and unit magnetic permeability of a vacuum. In this system the Ohm, Ampere, and Volt are of convenient size, but such units as the Farad are inconvenient; the microfarad is generally used instead of the Farad as the unit of capacity. The unit of dielectric constant in this system is 10^{18} C.G.S. unit and is never used; the dielectric constant is always given in electrostatic units.

§ (5) Dimensions of the Electrical Units.—Every quantity which is measured is given a name consisting of two parts or factors. The first is a number and the second is the name of an agreed unit of measurement. Thus we have 14 grams, 50 centimetres, 20 seconds, etc. The gram, centimetre, and second are the names of the units or standards of mass, length, and time, and the numbers 14, 50, and 20 denote the number of such units in the quantity measured. The units of length, mass, and time are usually designated by [L], [M], and [T] respectively, and are independent and fundamental. The electrical units we have seen can be expressed in terms of these, but as they are derived units it is necessary to know at what power each of the fundamental measurements enters into a derived measure. Thus velocity is measured in units of length divided by units of time and the powers of the fundamental measures of length and time in the derived unit of velocity are 1 and -1 respectively. Put otherwise, the dimensions of velocity are $[L/T]$ or $[L^1T^{-1}]$. Acceleration is the rate of change of velocity and the dimensions of the unit of acceleration are $[LT^{-1}T^{-1}]$ or $[LT^{-2}]$.

The product of a mass into an acceleration gives the number of units of Force required to produce the given acceleration in the given mass. The dimensions of the latter are therefore $[MLT^{-2}]$. From equations (1) to (6) the dimensions of the electrical units in the electromagnetic system may readily be obtained. Thus from equation (6) $[MLT^{-2}]=[m^2L^{-2}\mu^{-1}]$. Hence, the dimensions of m, the unit Magnetic Pole, are $[L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}]$. From equation (5) we have $[MLT^{-2}]=[ILL^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}L^{-2}]$. Hence the dimensions of I, the unit of Current, are $[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}]$. Similarly, from equations (3), (4), and (5) it is easy to find that the dimensions of the unit of Resistance are $[LT^{-1}\mu]$; the dimensions of the unit of Electromotive Force are $[L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}\mu^{\frac{1}{2}}]$; and the dimensions of the unit of Quantity are $[L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$. They all involve μ. Similarly, dimensions of the units in the electrostatic system may be written down. These dimensions are indeed given in Table I. and it will be observed that all include K.

TABLE I

DIMENSIONS OF ELECTRICAL UNITS IN THE ELECTROSTATIC AND ELECTROMAGNETIC SYSTEMS

L = Length. M = Mass. T = Time.
μ = Magnetic Permeability. K = Dielectric Constant.

Unit.	Symbol.	Dimensional Formulae.	
		Electrostatic.	Electromagnetic.
Quantity	Q	$\left[\frac{L^{\frac{3}{2}} M^{\frac{1}{2}} K^{\frac{1}{2}}}{T}\right]$	$\left[\frac{L^{\frac{1}{2}} M^{\frac{1}{2}}}{\mu^{\frac{1}{2}}}\right]$
Dielectric Constant	K	$[K]$	$\left[\frac{T^2}{L^2 \mu}\right]$
Current	I	$\left[\frac{L^{\frac{3}{2}} M^{\frac{1}{2}} K^{\frac{1}{2}}}{T^2}\right]$	$\left[\frac{L^{\frac{1}{2}} M^{\frac{1}{2}}}{T\mu^{\frac{1}{2}}}\right]$
Resistance	R	$\left[\frac{T}{LK}\right]$	$\left[\frac{L\mu}{T}\right]$
Electromotive Force	E	$\left[\frac{L^{\frac{1}{2}} M^{\frac{1}{2}}}{TK^{\frac{1}{2}}}\right]$	$\left[\frac{L^{\frac{3}{2}} M^{\frac{1}{2}} \mu^{\frac{1}{2}}}{T^2}\right]$
Magnetic Pole Strength	m	$\left[\frac{L^{\frac{1}{2}} M^{\frac{1}{2}}}{K^{\frac{1}{2}}}\right]$	$\left[\frac{L^{\frac{3}{2}} M^{\frac{1}{2}} \mu^{\frac{1}{2}}}{T}\right]$
Magnetic Permeability	μ	$\left[\frac{T^2}{L^2 K}\right]$	$[\mu]$
Capacity	C	$[KL]$	$\left[\frac{T^2}{L\mu}\right]$
Self Inductance / Mutual Inductance	L / M	$\left[\frac{T^2}{LK}\right]$	$[L\mu]$

§ (6) RATIO OF UNITS.—According to the definition of the unit magnetic pole the magnetic permeability μ of vacuous space is unity, and according to the definition of the unit charge the dielectric constant K of vacuous space is also unity. Now, although we have every right to make these assumptions separately, they cannot be made simultaneously since μ and K are really related. Hence unless μ and K are always inserted in formulae and equations the two systems of units are distinct the one from the other. It is clear from Table I. that the unit of quantity in the electromagnetic system is $1/\sqrt{K\mu}$ times greater than the unit of quantity in the electrostatic system. Further, since the dimensions of electric quantity must be the same however measured, the dimensional expressions for electric quantity in the two systems must be equal. Hence

$$\left[\frac{1}{\sqrt{K\mu}}\right] = \left[\frac{L}{T}\right].$$

The same relation may be obtained by equating the dimensional expressions in the two systems for any electric quantity. The equations do not enable us to find the dimensions of K or of μ, but it follows that $1/\sqrt{K\mu}$ has the dimensions of a velocity. This velocity is usually denoted by v; as already shown,[1] it is the ratio of the Electromagnetic Unit of Quantity to the Electrostatic Unit of Quantity. Its value is 3×10^{10} cm. per sec.

Maxwell proved theoretically that $1/\sqrt{K\mu}$ is the velocity of an electromagnetic wave in air, and when such waves were produced by experiment and their velocities measured, the prediction was verified. Further, it has been experimentally proved that $1/\sqrt{K\mu}$ is also the velocity of light, and that light is an electromagnetic phenomenon.

§ (7) VALUES ADOPTED FOR THE UNITS.—The Absolute or C.G.S. system of units, while being ideal inasmuch as they are independent of the physical properties of any material, was, almost from the time it was suggested until the last few years, deemed by many to be impracticable, owing to the necessarily limited accuracy of absolute measurements.

At the first meeting in 1862 of the Electrical Standards Committee of the British Association the question was very carefully considered, and it was recognised as desirable to refer results to concrete standards, made to represent as nearly as possible the C.G.S. units or multiples of them. The unit of measurement and the standard of reference thus became distinct things, and the general policy for the past sixty years has been to have reference standards, but to adjust them from time to time so as to more nearly approach the C.G.S. units or multiples of the units. In recent years, however, absolute measurements of electrical resistance and current have been made with an accuracy in excess of all commercial requirements, and in the opinion of many scientific workers it is no longer necessary or desirable to state values in terms of such concrete standards as are in use. The precision required in practice being less than that it is possible to obtain in absolute measurements, it is satisfactory in every way to give values in terms of the C.G.S. units or multiples of the units.

§ (8) ABSOLUTE MEASUREMENTS.—The three fundamental electric units are the Ohm, Ampere, and Volt, and since these are related by Ohm's Law the accurate measurement of any two of them fixes the third.

I. ABSOLUTE MEASUREMENT OF RESISTANCE.

§ (9) THE OHM (10^9 C.G.S. UNITS).—Various methods for the absolute measurement of resistances have been used, and experiments based on these have been carried out with

[1] See "'v,' the Ratio of the Units."

very great care. It is difficult to compare accurately the results obtained owing to the variety of concrete standards in terms of which the results are expressed. In some cases mercury columns of known dimensions and temperature were used, but in the majority of cases coils of wire were employed as reference standards.

In the electromagnetic system, taking μ as unity, resistance has the dimensions of a velocity, and any method devised for the absolute measurement of a resistance must therefore involve measurements of length and time, or measurements of other quantities having length and time in their dimensional formulae.

Referring to Table I., when $\mu=1$, Inductance has the dimension [L], and it is largely by the use of inductances combined with measurements of time that all absolute measurements of resistance have been made.

§ (10) KIRCHOFF'S METHOD.—Two coils a and b (Fig. 1), between which there is a mutual inductance M, are connected to a battery B, a

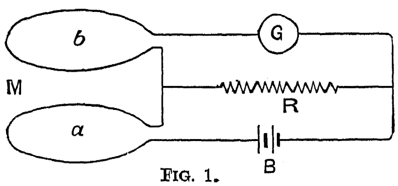


FIG. 1.

galvanometer G, and a resistance R, the value of which is desired. With the circuit closed the *steady* deflection of G is first measured. The mutual inductance M is then reduced to zero by a change in position of one of the coils and the galvanometer kick observed together with the logarithmic decrement.

Kirchoff's first measurement, which was made in 1849, has only historical value. Rowland (*American Journal*, 1878, xv.) improved the method by reversing the current through the coil a instead of removing the coil b; in this case the quantity of electricity passing through the ballistic galvanometer is twice that given above. Glazebrook, Sargant, and Dodds also reversed the current through the coil a and further modified the method by using the same galvanometer for measurement of the steady and induced currents. A shunt was employed for the measurement of the steady current.

In the last case let M be the coefficient of mutual inductance between the coils, T the period of the galvanometer needle, α the deflection due to the primary current, β the throw due to the induced current, and h the factor by which the deflection α must be multiplied in order to correct for the shunting of the galvanometer. Then, if i be the current and G the galvanometer constant,

$$\frac{Mi}{R}=\frac{GT}{\pi}\sin\tfrac{1}{2}\beta, \quad \text{and } i=hG\tan\alpha.$$

Hence the resistance of the secondary circuit in absolute measure is given by

$$R=\frac{Mh\pi}{T}\cdot\frac{\tan\alpha}{\sin\frac{1}{2}\beta}.$$

M/T is of the dimensions of a velocity and $h\pi\tan\alpha/\sin\frac{1}{2}\beta$ is merely a ratio.

The original difficulty lay in the calculation of M with the necessary precision, but, as Glazebrook pointed out in 1890, it should be possible to determine M within 1 or 2 parts in 10,000. Lord Rayleigh showed (*Phil. Mag.*, 1882) that the attainable precision largely depends on the ratio of the diameters of the coils to their distance apart. If the mutual inductance were of the Campbell form (*Roy. Soc. Proc.* A, 1907, lxxix.), the measurements of the diameter and distance apart of the coils could be made with sufficient accuracy to enable M to be calculated within 1 or 2 parts in 100,000. The method is subject to whatever uncertainty attaches to the use of a ballistic galvanometer (*Roy. Soc. Phil. Trans.*, 1882, p. 669), and this probably is where improvement is most desirable. For educational purposes the method is simple.

(i.) *Experiments.*—The induction coils used in Rowland's experiments were wound on brass bobbins. Three coils were used, the mean radii being respectively 13·710 cm., 13·690 cm., and 13·720 cm. These were used two at a time, the bobbins having carefully ground ends so that they could be fitted end to end with their axes in line. The constant of the ballistic galvanometer was determined first by calculation from its dimensions and afterwards by comparison with that of a large double coil of an electrodynamometer constructed on the Helmholtz-Gaugain plan. After a comparison the instruments were interchanged in position so as to eliminate any difference between the values of H at the two places. The probable error in determining this constant appears to have been about 1 part in 10,000. A tangent galvanometer with a circle 50 cm. in diameter was used for measuring the direct current and its constant was compared with that of a single circle of wire about 83 cm. in diameter.

In Glazebrook's experiments (*Roy. Soc. Phil. Trans.*, 1883) no separate measurements for eliminating variations in H were necessary owing to the use of one galvanometer. The coils used by Glazebrook had a mean radius of about 26 cm. and all dimensions were carefully measured. The positions of the mean planes were estimated from the axial breadth of a coil, and any doubt as to the

exact positions was removed by reversal of the bobbins relative to the gauge pieces between them. Each experiment included 8 observations of induced current and 2 of steady current deflection, and these observations were made for each of the four positions in which the coils could be placed by reversing them without changing the distance between their centres.

Further measurements were made by Rowland (*La Lumière Électrique*, 1887, xxvi.) in 1884.

(ii.) *Results.*—In 1878 Rowland made comparisons with certain resistance coils the values of which had been given in B.A. units and concluded that 1 B.A. unit $=0{\cdot}9911 \times 10^9$ C.G.S. units.

Glazebrook made comparisons with standard coils known in B.A. units and gave as the mean of all his results (B.A. Report, 1890) 1 B.A. unit $=0{\cdot}98665 \times 10^9$ C.G.S. units.

In 1884 Rowland found 1 B.A. unit to be equal to $(0{\cdot}98627 \pm 40)\ 10^9$ C.G.S. units.

§ (11) Roiti and Himstedt's Method.—Himstedt (*Wied. Ann.*, 1886, xxviii.) modified Kirchoff's method so that it was not the first deflection produced by the induced current that was measured. The primary and galvanometer circuits (Fig. 2) were separate and were made and

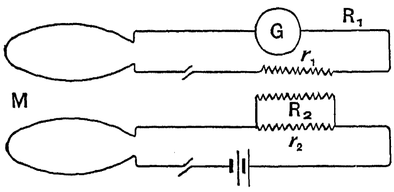


Fig. 2.

broken in rapid succession by circuit breakers. The frequencies of make and break were the same, but the times were so adjusted that the current passing through the galvanometer was either the induced current at make or that at break. A stationary deflection α_1 of the galvanometer was thus obtained, the relation being very approximately

$$G \tan \alpha_1 = \frac{niM}{R_1 + r_1},$$

n being the frequency of the interruptions, i the primary current, M the mutual inductance between the circuits, and $R_1 + r_1$ the resistance of the secondary circuit.

The resistances r_2 and R_2, which are equal respectively to r_1 and R_1, are now removed and r_1 and R_1 substituted. The value of the current is undisturbed and the deflection α_2 of the galvanometer is such that

$$G \tan \alpha_2 = \frac{r_1 i}{R_1 + r_1}.$$

Combining this with the previous equation we have

$$r_1 = nM\frac{\tan \alpha_2}{\tan \alpha_1}.$$

The principal difficulty is that connected with the galvanometer when used to measure an interrupted current.

(i.) *Experiments and Results.*—Roiti in 1884 used an interrupted current method and found 10^9 C.G.S. units of resistance to be equal to that of a column of mercury 105·896 cm. long and 1 sq. mm. in cross-section, all at 0° C.

Himstedt (1886) found that a column of mercury 106·08 cm. long and 1 sq. mm. cross-section at 0° C. had a resistance of 10^9 C.G.S. units.

§ (12) Weber's Method of Transient Currents.—A large coil of effective area A (the sum of the areas enclosed by the turns) is mounted with its plane in the magnetic meridian and is capable of being rotated about a vertical axis. Included in the circuit of the coil is a ballistic galvanometer of known constant. To measure the total resistance of the circuit the coil is rapidly turned about its vertical axis through half a revolution. If H is the horizontal magnetic intensity in the position of the coil, the total change of induction is 2AH and the total quantity of electricity which flows through the circuit is 2AH/R. If N is the number of turns on the galvanometer, a the radius, T the period of the needle, and θ the deflection (corrected for damping) produced by the flow of the quantity of electricity 2AH/R, then

$$R = \frac{A}{aT}\frac{4\pi^2 N}{\sin \frac{1}{2}\theta} \cdot \frac{H'}{H},$$

where H is the horizontal magnetic intensity in the position of the galvanometer.

If $H' = H$, the formula is correspondingly simplified. The part A/aT is of the dimensions of a velocity, the remaining portion being merely a ratio.

There is a difficulty in ensuring that the axis of rotation is vertical, and Lord Rayleigh pointed out that it was a decided advantage to carry out measurements near to the magnetic equator. Glazebrook (*Electrician*, 1890) showed that the effect of the time of duration of the induced current was important. He showed that if the effect of the induced current was prolonged for one second, the period of the galvanometer needle being twenty-three seconds, the deflections are reduced by 0·1 per cent.

For absolute measurements of resistance the method is not recommended, but an adaptation is of interest to students and teachers for measuring the horizontal and vertical intensities of the earth's magnetic field.

(i.) *Experiments.*—Measurements were made by Weber himself, and later by Weber and

F. Zöllner. These latter used in their experiments coils as large as one metre in diameter. The method of recoil was used. Turning the coil first through 180° from the initial position, the first deflection (positive, say) was observed. The needle then swings through its zero position to the negative side and back again to zero. At the instant the needle arrives at zero the second time the coil is turned back to its original position, thus reversing the motion of the needle, which swings to a maximum deflection on the negative side, then back again to the positive side, and as it passes again through zero the coil is again turned through 180°, and so on.

In the experiments made by Weber and Zöllner galvanometer magnets 20 cm. and 10 cm. long were used and the results obtained in the two cases differed by 2 per cent. It is obvious that the needles were far too long. However, the measurements were largely intended as a test of the apparatus.

Later with the same coils Wiedemann (Wiedemann's *Elektricität*, Band 4, p. 913) made a careful determination at Leipzig. The needle was small and carried beneath it an arrangement on which weights could be placed to alter the moment of inertia of the suspended system. Instead of the method of recoil being used, the method of multiplication was employed. (Maxwell, *Elec. and Mag. Art.* § 747 and § 751.) An arrangement of stops was provided enabling the coil to be turned quickly through 180° and back again. The coil was turned suddenly a number of times in succession through this angle, always when the needle had returned to its zero position, so that the deflections were multiplied as far as the limits of the scale would allow. To express the unit of resistance in terms of the resistance of a mercury column, observations were made with and without ten Siemens mercury units in circuit with the coils.

Mascart, De Nerville and Benoit (*Ann. de Chemie et de Phys.*, 1885) have also employed the method, using two coils about 15 cm. in diameter and three coils about 30 cm. in diameter. Both the smaller and larger coils were mounted to admit of being turned through exactly 180°, and at the centre of the larger coil was suspended a small magnetometer needle. By turning the coil round a vertical axis through 90° from its position when arranged for inductive use, it could be used as a galvanometer coil and its constant compared with that of the galvanometer used in the experiments. Thus H/H′ was accurately determined.

(ii.) *Results.*—Wiedemann found that 106·162 cm. mercury 1 sq. mm. in cross-section at a temperature of 0° C. had a resistance of 10^9 C.G.S. units.

Mascart, De Nerville and Benoit found 10^9 C.G.S. units $=1{\cdot}0142$ B.A. unit, or 106·37 cm. mercury of cross-section, etc., as above had a resistance of 10^9 C.G.S. units.

§ (13) Weber's Method of Damping.—A magnet of considerable magnetic moment is oscillated at the centre of a coil, first when the circuit of the coil is open, and again when the circuit is closed. The period of vibration and the logarithmic decrement of the oscillations are observed. In the case when the circuit of the coil is closed the motion of the magnet sets up currents in the coil which react on the magnet and damp its motion. The resistance R of the coil circuit is deduced from the effect which the currents induced in it by the motion of the magnet have in resisting that motion. R is given by the expression (see Gray's *Absolute Measurements*, vol. ii. part ii.)

$$\mathrm{R}=\frac{\mathrm{M}^2\mathrm{G}^2}{2\mathrm{I}(k'-k)}+\frac{\mathrm{L}}{2}\left(3k'-k+\frac{n^2-k^2-a^2}{k'-k}\right),$$

where M is the moment of the magnet, I the moment of inertia of the suspended apparatus, G the principal galvanometer constant of the coil, and L the self inductance of the coil. k and k' are equal respectively to $2\lambda/\mathrm{T}$ and $2\lambda'/\mathrm{T}'$, where λ and λ' are the logarithmic decrements in the two cases and T and T′ the respective periods of the magnet. $a=2\pi/\mathrm{T}'$ and $n=\mathrm{MH}/\mathrm{I}$.

The measurement of the magnetic moment M and of the horizontal magnetic intensity H are of first importance, and at the present time (1920) neither magnetic moment nor magnetic intensity has been measured in C.G.S. units with the accuracy that resistance has been measured by other methods. It will further be observed that R is proportional to the square of M so that a very accurate determination of M is necessary. Again the moment of inertia I is not easy to determine for small magnets, and if a large magnet is employed there is uncertainty as to the distribution of its magnetism. Corrections are required for the arc of oscillation and for temporary induced magnetisation due to the action of the current. Further, the value of R is directly proportional to the difference between the logarithmic decrements.

In the opinion of Lord Rayleigh, Rowland, and Glazebrook, the formula is sufficient to show that the method cannot compete with most of the other methods used for absolute measurements.

(i.) *Experiments.*—The method was used by W. Weber (*Pogg. Ann.* Bd. 82, S. 337), first in 1851, but Weber was interested more in the development of the method than in the accuracy of the measurements.

With modifications the method has been employed by H. F. Weber, Dorn (*Wied. Ann.*,

1882, xvii., and 1889, xxxvi.), Wild (*Mém. de l'Ac. des Sc. St-Pétersbourg*, 1884, tome 32, Nro. 2), and Kohlrausch (*Phil. Mag.* xlvii.). The most important modification is that used by Kohlrausch, which practically amounts to a combination of Weber's Method of Transient Currents with the Method of Damping. Weber had previously used this method. The constant of the galvanometer with which the earth-inductor was connected was eliminated by determining the logarithmic decrement of the motion of the galvanometer needle, first when the circuit of the galvanometer was closed, and again when it was open. Calling these decrements γ and γ_0, and putting a, β for the arcs of vibration in the method of recoil (Maxwell's *Electricity and Magnetism*, § 750), and T the period of the needle when the circuit was open, Kohlrausch's formula is, to a first approximation,

$$R=\frac{32a^4H^2T(\gamma-\gamma_0)}{I}\cdot\frac{a\beta}{(a^2+\beta^2)^2},$$

where R is the resistance of the circuit composed of the inductor and galvanometer, I is the moment of inertia of the magnet and the suspended system, and a is the radius of the galvanometer. A defect pointed out by Lord Rayleigh and by Rowland (*American Journ. of Science*, 1878) is the great number of quantities difficult to observe, which enter the equation as squares and fourth powers. In Kohlrausch's experiments the diameter of the wire occupied 2 per cent of the radius of the coil, making it uncertain to what point the radius should be measured. Three other quantities than the radius, viz. H, γ, and I, are very difficult to determine, and H enters as the square. Rowland sums up a criticism of this method with the remark, "The method is defective because, although absolute resistance has the dimensions of Space/Time, yet in this method the fourth power of space and the square of time enter, besides other quantities which are difficult to determine."

(ii.) *Results*.—Dorn used a Siemens resistance unit verified by Strecker, and found that 106·243 cm. of mercury of 1 sq. mm. cross-section at 0° C. has a resistance equal to 10^9 C.G.S. units.

Wild employed coils by Siemens and Halske as reference units and found that 106·027 cm. of mercury of 1 sq. mm. cross-section at 0° C. has a resistance equal to 10^9 C.G.S. units.

Kohlrausch used a mercury resistance unit. The mean result was that 10^9 C.G.S. units of resistance was equal to that of 106·34 cm. of mercury of 1 sq. mm. cross-section, all at 0° C.

§ (14) METHOD OF ROTATING COIL (British Association Method). —This it appears was originally suggested by Weber, but it was also independently put forward in 1863 by Lord Kelvin (then Sir William Thomson) to the Electrical Standards Committee of the British Association. The method is very fully described in the Reports of the British Association covering the period 1862–1867, and by Lord Rayleigh and Sir Arthur Schuster in *Roy. Soc. Proc.*, 1881.

The method consists in spinning with uniform velocity about a vertical axis, a circular coil at the centre of which is suspended a small magnetic needle. The circuit of the coil is complete, and although the current in the coil alternates as it is rotated in the earth's magnetic field, the reversal of the coil relatively to the needle causes all the impulses to have a unidirectional effect on the needle. Apart from self induction, the current is of maximum value when the plane of the coil is in the magnetic meridian.

Let H = horizontal intensity of earth's magnetic field,
A = effective area enclosed by all the turns of the coil,
θ = angle of inclination of coil with the magnetic meridian,
ω = angular velocity which must be uniform.

Then we may put $\theta=\omega t$.

The rate of cutting of the magnetic lines is $AH\omega\cos\omega t$. If L is the self inductance of the coil, R its resistance, and i the current in it, then

$$L\frac{di}{dt}+Ri=AH\omega\cos\omega t,$$

from which we have

$$i=\frac{AH\omega}{R^2+L^2\omega^2}\{R\cos\omega t+L\omega\sin\omega t\}.$$

If unit current through the coil produces a magnetic intensity G at the centre, and the magnetic intensity due to i is resolved in two directions, (1) at right angles to the meridian and (2) in the meridian, these components are

$$\frac{AHG\omega}{2(R^2+L^2\omega^2)}\{R+(R\cos 2\omega t+L\omega\sin 2\omega t)\}$$

and

$$\frac{AHG\omega}{2(R^2+L^2\omega^2)}\{L\omega+(R\sin 2\omega t-L\omega\cos 2\omega t)\}.$$

It will be observed that the magnetic intensities consist of two parts, one constant and one periodic. If the moment of inertia of the needle at the centre of the coil is chosen sufficiently great the effect of the periodic terms becomes negligible. When such is the case the two components of the magnetic intensity due to the currents in the coil are seen to be $AHG\omega R/2(R^2+L^2\omega^2)$ and $AHGL\omega^2/2(R^2+L^2\omega)$.

The latter magnetic intensity is opposed to that of the earth's field so that the total

magnetic intensity along the magnetic meridian is

$$H-\frac{AHGL\omega^2}{2(R^2+L^2\omega^2)}.$$

Hence, if α is the deflection of the needle,

$$\tan\alpha=\frac{AHG\omega R/2(R^2+L^2\omega^2)}{H-(AHGL\omega^2)/2(R^2+L^2\omega^2)},$$

which reduces to

$$\tan\alpha=\frac{AG\omega R}{2(R^2+L^2\omega^2)-AGL\omega^2},$$

from which R can be determined.

This equation is not quite exact since the effect of the magnetic field due to the magnet at the centre of the coil and also the torsion of the fibre supporting the magnet have been neglected. These corrections are contained in the equation employed by the 1863 British Association Committee, which in the notation hitherto used is

$$R=\frac{AG\omega}{2\tan\alpha(1+T)}\left\{1+\frac{GM}{AH}\sec\phi-\frac{2L}{AG}\left(\frac{2L}{AG}-1\right)\tan^2\alpha\right\}.$$

MHT is the torsion of the fibre per unit of angular rotation.

Maxwell (*Elec. and Mag.* § 763) gives the more complete expression

$$R=\frac{AG\omega}{2\tan\alpha\left(1+T\dfrac{\alpha-\beta}{\sin\alpha}\right)}\left\{1+\frac{GM}{AH}\sec\alpha-\frac{2L}{AG}\left(\frac{2L}{AG}-1\right)\tan^2\alpha-\left(\frac{2L}{AG}\right)^2\left(\frac{2L}{AG}-1\right)^2\tan^4\alpha\right\},$$

where β is the azimuth of the magnet when there is no torsion. Although the equation given in the *B.A. Report* does not include the term involving $\tan^4\alpha$, it is understood that this term was used in reducing the results.

If at first L be assumed to be negligibly small and the correction due to torsion of the fibre be zero, the equation for R may be written

$$R=\frac{AG\omega}{2\tan\alpha}=a\omega\pi^2n^2\cot\alpha,$$

where a denotes the effective radius of the coil, since $A=n\pi a^2$ and $G=2n\pi/a$. From this it is seen that the resistance R is given in terms of the radius of the coil, the angular velocity, the number of turns, and the angular deflection. Only one linear quantity is concerned and on the accurate evaluation of this much depends. Indeed it appears that the accuracy of the method is limited only by errors of measurement of the effective radius of the coil and of the deflection. The speed of rotation can be made very uniform and small slow variations allowed for. With regard to the coil, the radius of a coil of many turns cannot be measured with such accuracy as a single-layered coil. It is probable that the diameter of a many-layered coil cannot be measured with a probable error less than 0·05 mm. amounting in a coil of 500 mm. diameter to 1 part in 10,000.

Lord Rayleigh has pointed out that if the metal ring on which the wire is wound be on a large scale, currents may be developed in it even although it is divided into two parts by insulation, and suggests that the ring should be made of a badly conducting material. A serious error, unless great care is taken, is in a false estimate of L (see complete equation for R). Since the effect of L increases with the square of the speed the effect of self inductance may be eliminated by varying the speed, and this method was used by Lord Rayleigh and Professor Schuster. Lord Rayleigh further suggests the introduction of a second coil in a plane perpendicular to that of a first (*Roy. Soc. Proc.*, 1881, p. 123). By this means the relative correction for self inductance would be reduced to one-quarter, while the deflection would remain unaltered. Rowland (*American Journal*, 1878) studied the method with care, and in addition to the above possible sources of error pointed out that owing to currents being induced in the suspended metallic parts the needle is dragged with the coil. This effect is no doubt minute but cannot be estimated.

(i.) *Experiments.*—In the famous experiment of the British Association Committee the radius of the coil was 15·7 cm. and the coil had 307 turns. The self inductance L, or as it was then called, the electromagnetic capacity, was stated in the *B.A. Report* to be 397,750 metres, but Maxwell in his paper on the "Electro-magnetic Field" gives a corrected value of 430,165.

Lord Rayleigh and Professor Schuster in 1881 again used the B.A. apparatus. Various improvements were introduced and modifications in procedure adopted as checks. In preliminary experiments very consistent results were obtained at constant speeds whether the rotation was in one direction or the other; but when deflections at various speeds were compared, the larger deflections were found to fall very considerably short of proportionality to the speeds. It was proved that this was due to a wrong correction for self inductance.

In Lord Rayleigh and Dr. Schuster's experiments the value of L which best satisfied the observations was 456,748 metres and the result of direct measurement was about 450,000 metres. The value of L was recalculated to be 451,448 metres. When all

the evidence is taken into consideration there can remain little doubt that the reductions of the B.A. Committee are affected by a serious underestimate of L. In 1863 the B.A. Committee employed a governor to regulate the speed, which latter was measured by taking the times at which a bell sounded, the bell receiving a stroke every 100 revolutions. Lord Rayleigh and Dr. Schuster used the stroboscopic method, the vibrating tuning-fork being compared from time to time with a standard fork of Koenig's construction.

In 1882 Lord Rayleigh (*Roy. Soc. Phil. Trans.*, 1882) used an improved rotating coil apparatus, the principal difference being an enlargement of the linear dimensions in the ratio of about 3 to 2, and a general strengthening of all parts. H. Weber (*Der Rotationsinduktor*, 1882) also made measurements in 1882, using an inductor with a horizontal axis of rotation.

(ii.) *Results*.—Until about 1850 all units of resistance were based on the more or less arbitrary size and weight of some conductor in the form of a wire. Weber had made a measurement previous to 1863 in terms of the metre and second, but this differed from the B.A. measurement by about $8\frac{1}{2}$ per cent. Although it was subsequently proved that the measurement of the B.A. Committee was in error by about 1·8 per cent, the advance towards obtaining a coherent system of units was marked and the work has been highly appreciated throughout the scientific world.

Lord Rayleigh and Dr. Schuster in their first experiments found 1 B.A. unit $=0{\cdot}9893 \times 10^9$ C.G.S. units, or 10^9 C.G.S. units $=1{\cdot}0108$ B.A. units.

In 1882, with the improved apparatus Lord Rayleigh concluded that 1 B.A. unit $=0{\cdot}98651 \times 10^9$ C.G.S. units, or 10^9 C.G.S. units $=1{\cdot}01367$ B.A. units.

The result of H. Weber's measurements was 1 B.A. unit $=0{\cdot}9877 \times 10^9$ C.G.S. units.

§ (15) Method of Lorenz.—In this a resistance is compared with a mutual inductance and a time, the latter being the period of rotation of a portion of a conducting circuit in a magnetic field.

The method was proposed and first employed by Lorenz of Copenhagen in 1873 (*Pogg. Ann.*, 1873, cxlix. 251). In the original apparatus (*Fig.* 3) a circular disc of metal was rotated at a uniform rate about an axis through its centre in the magnetic field due to a coaxial coil carrying a current. The disc was touched near its centre and at its circumference by two metal brushes and the circuit completed through a sensitive galvanometer G and a resistance R, the latter being included in the main current circuit.

Let M denote the mutual inductance between the coil and the disc, and n the number of revolutions per second made by the disc. The induced voltage between the centre and edge of the disc is Mni, where i

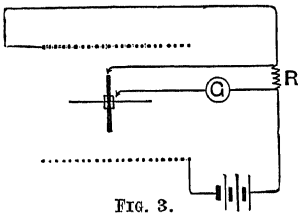


Fig. 3.

is the current through the coil. This induced voltage is balanced against the potential difference Ri at the extremities of the resistance R. Hence when there is no current through the galvanometer

$$Ri = Mni,$$

i.e. $$R = Mn.$$

M is calculated from the dimensions of the coil and disc and n is measured directly.

In no practical case can Mn be very large and therefore R must be small. In the days when experiments were first made by this method there were no low resistance standards and a difficulty presented itself which was likely to lead to serious errors. Because of this Lord Rayleigh in his experiments adopted a method of shunting as shown in *Fig.* 4. The main current from the

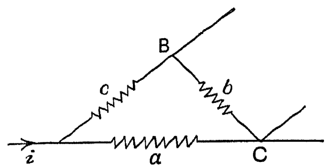


Fig. 4.

battery was divided into two parts, the larger of which passed through a and the smaller part through c and b. If i is the main current, the current through c and b is $ia/(a+b+c)$, and hence the difference of potential between B and C is $iab/(a+b+c)$. It is convenient to think of $ab/(a+b+c)$ as R.

Methods based on this principle have been used by Lorenz (*Wied. Ann.*, 1885, xxv.), Lord Rayleigh and Mrs. Sidgwick (*Roy. Soc. Phil. Trans.*, 1883, clxxiv. 295), Rowland and Kimball (*La Lumière électrique*, 1887, xxvi.), Duncan, Wilkes, and Hutchinson (*Phil. Mag.*, 1889), Jones (*Electrician*, 1890, p. 552), Ayrton and Jones (*B.A. Reports*, 1897), and Smith (*Roy. Soc. Phil. Trans.*, 1913, ccxiv.)

The influence of terrestrial magnetism is not really disturbing if the intensity of the earth's field does not fluctuate rapidly. If the field

is reasonably constant any effect is eliminated by reversing the current. At the brush contacts near the centre of the disc and at the edge there are very appreciable thermoelectric forces which fluctuate rapidly, and can only partially be eliminated by reversing the current unless a very large number of observations are taken. These thermoelectric forces may produce great practical difficulties. The measurement and control of speed is common to most methods, and no serious difficulty presents itself. The main constant error is associated with the linear measurements involved in the calculation of M. This received very careful consideration by Lord Rayleigh (*Phil. Mag.*, Nov. 1882), who investigated the best dimensions of the coil, or coils, and the disc. Smith (*Roy. Soc. Phil. Trans.* A, ccxiv. 37) calculated the rates of variation of the mutual inductance with changes in the radii of coil and disc, and found that in the apparatus experimented with by Ayrton and Jones, errors of 0·01 mm. in the measurements of the radii of coil and disc introduced respective errors of 14 parts and 5 parts in 100,000 in the calculation of M.

(i.) *Experiments.*—In the measurements made by Lord Rayleigh and Mrs. Sidgwick the coils employed were those previously used by Glazebrook in his measurements by the Method of Transient Currents. The two coils were first placed close together with the disc between them so as to give the maximum inductive effect. The axle was mounted vertically in the frame used for the rotating coil determinations, and the arrangements used for driving and measuring the speed were the same as for the rotating coil. The diameter of the disc was about 0·6 of that of the coils. This size was chosen in order that dM/da should not be too large. The edge of the disc was made cylindrical, and contact with the edge was made by a brush of fine copper wires placed tangentially to the edge and amalgamated with mercury. An additional potentiometer circuit was introduced to approximately balance the thermoelectric effects at the brush contacts and the inductive effect of the earth's magnetic field in which the disc rotated. Two series of measurements were made with the coils close together, and a third series with the coils separated to such an extent, with the disc midway between them, that dM/da was very small indeed.

In the measurements made by Jones in 1891 the coil consisted of a single layer of double silk-covered wire wound on a cylinder of brass about 53·5 cm. in diameter, in a screw thread of about 0·6 mm. pitch. Very great care was taken in the machining of the cylinder, and the mean plane of the coil was carefully marked. The disc was 33 cm. in diameter; it was insulated from the axle and was driven by an electric motor coupled direct and was ground true by an emery wheel; its diameter was measured by a Whitworth measuring machine. The stroboscopic method for measuring and controlling the speed was employed, the tuning-fork being bowed and not electrically maintained. The brush finally adopted was a single wire perforated by a channel through which a small stream of mercury flowed. The central brush was fed also with mercury. The resistance directly measured was that of a column of mercury, so that the experiment gave the resistivity of mercury directly. In order to avoid any appreciable reduced voltage due to the earth's magnetic field the axis of rotation of the disc was placed at right angles to the magnetic meridian; practically perfect compensation was obtained by the slight movement of a magnet some distance away from the disc.

In 1897 Ayrton and Jones tested the Lorenz apparatus made for Professor Callendar, who was then at the McGill University (*B.A. Report*, 1897). It was very similar to that previously used by Jones except that the coil was wound on a marble cylinder. Three brushes 120° apart were employed at the circumference of the disc in order to eliminate small errors due to imperfect centring with the coil.

In 1913 F. E. Smith designed and used the apparatus now at the National Physical Laboratory. In his apparatus there are two pairs of coils and two discs. The current is taken off by brushes rubbing on the edges of the discs instead of by one brush at the edge and another on the axis. Thermoelectric effects are thus much reduced. The dimensions of the coils and discs were chosen with full regard to the following considerations.

If A is the radius of coil, a the radius of a coaxial disc, and x the length of the coil, then for small changes dA, da, and dx, in these dimensions the change dM in the mutual inductance can be obtained from the increment formula

$$\frac{dM}{M} = q\frac{dA}{A} + r\frac{da}{a} + s\frac{dx}{x}.$$

The sum $q+r+s$ is always equal to unity, and all three quantities cannot be of the same algebraical sign. For a coil on marble, wound with bare copper wire, the radius and axial length of the coil can be measured with great precision. As the disc must rotate and has brushes in contact, its dimensions when spinning are not likely to be determinable with equal accuracy. Since by properly choosing the diameter and length of coil and size of disc either q, r, or s can be made exceedingly

small, it appears good policy, conditionally that the apparatus is not cumbersome, to make r very small. This condition is fulfilled in the Lorenz Apparatus at the National Physical Laboratory. Changes of 0·1 mm. in A and a produce changes in M of 30·0 and 0·07 cm.

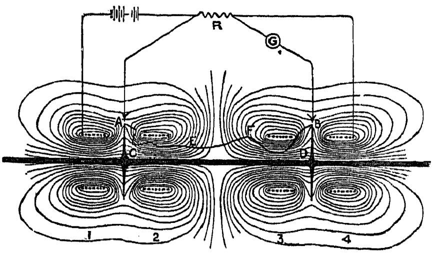


FIG. 5.

respectively, the value of M being about 29,000. Actually a change in the diameter a of the disc of 0·5 mm. is necessary to produce a change in M of 1 part in 100,000.

The Smith Lorenz Apparatus (*Figs.* 5A and 5B) consists of two metallic discs which support ten ends of a single wire, or that between five wires suitably arranged in series, is balanced against the difference of potential between two points on a standard resistance, the current through which is the same as that flowing through the four coils. A diagrammatic sketch of the magnetic field is shown in *Fig.* 5.

The current in the coils 1 and 2 is in the opposite direction to that in the coils 3 and 4, and the resulting magnetic fields are therefore opposite in direction. Each pair of coils and the disc between them is similar in disposition to that of a Campbell standard[1] form of mutual inductance, but the mutual inductance can be changed by varying the distance apart of the two coils. The intensity of the magnetic field at points in the neighbourhood of the edge of a disc is zero, or very nearly so, and it will thus be realised that the measurement of the radii or that of a radial arm is not one of first importance. The discs themselves are used merely as supports of the insulated wires and segments. On the edge of each disc ten segments of stabilite are secured, and to these phosphor-bronze segments of square

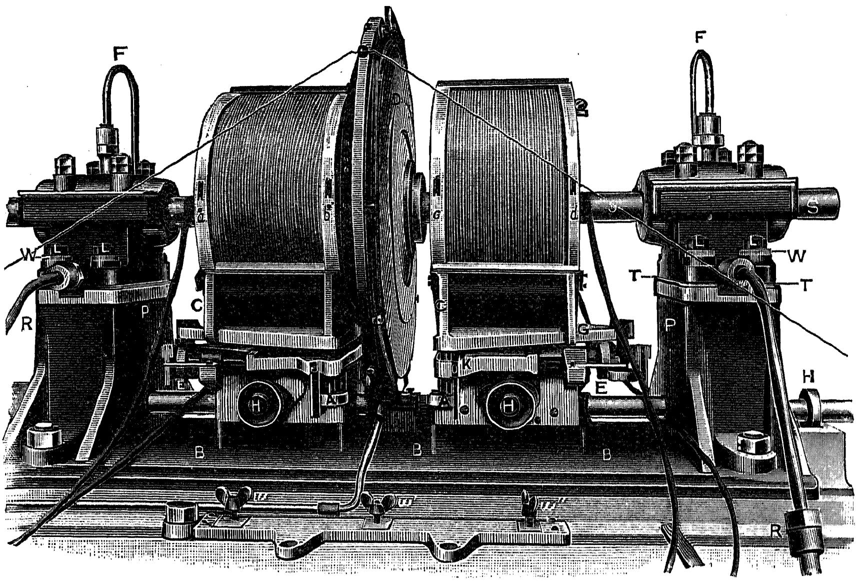


FIG. 5A.

conducting wires insulated from the disc and which rotate in a magnetic field produced by a current in four coils. An electric motor is used as a source of power. Phosphor-bronze wire brushes make contact with segments made of the same alloy attached to the ends of the rotating wires, and the difference of potential between the brush contacts at the section are screwed. The ten phosphor-bronze segments thus form a rim, except that small insulating air gaps separate the segments. Insulated wires pass from the segments on one disc to the segments on the other disc, the wires passing through channels milled in the discs and through central holes drilled in

[1] See "Inductance, Measurement of," § (54).

the shaft. A difference of potential is produced between the ends of a rotating conductor, and its value depends only on the position of the ends of the conductor (in this case the phosphor-bronze segments) and not upon its position, conditionally that the conductor passes through the coils carrying the current. Thus the potential difference at the extremities of a rotating conductor ACDB is not altered if its shape is changed to AEFB (see *Fig.* 5).

The brushes consist of phosphor-bronze wire 0·12 mm. diameter, and to obtain as small thermo-electric effects at the contacts as possible the wire was drawn from other wire of square section similar to that employed for the segments on the disc. The fine wires forming a brush are stretched by two spiral springs and resemble violin bows. The wires meet and leave the segments tangentially. The overall length of a brush is 20 cm., and contact is made with one or two segments over a length varying from 5 to 6 cm. Petrol is fed by means of a wick to the edge of a disc and acts as a cleanser and a lubricant. The rapid variations in the thermo-electric voltage rarely exceeded 0·1 microvolt over intervals as long as 20 minutes.

There are two principal ways of using the apparatus. In the first, the ten brushes (5 on each disc) are included in a circuit so as to be in series. When each brush is in contact with a single segment the differences of potential due to five rotating conductors are added together, the remaining five conductors being ineffective. When each brush connects two neighbouring segments the ten rotating conductors are arranged in five sets of two in parallel, and the total potential difference is the same as before. By having a comparatively large arc of contact between each brush and a segment (or segments), and twice as many segments as brushes, the circuit made through the brush contacts is never broken.

In the second method the brushes are divided into two sets of five in parallel, and the total potential difference is the same as that of a single rotating conductor.

The discs are about 53 cm. in diameter and are of phosphor-bronze. They are mounted on a shaft of copper-aluminium alloy. The bearings, the pedestals, and all metallic parts were tested for magnetic quality to ensure freedom of magnetic material. One disc, two coils, and a part of the supporting framework are shown in *Fig.* 5A.

The coils are wound with bare copper wire on hollow marble cylinders, having double-threaded screw grooves cut on the surface. The two wires on any one cylinder form two adjacent helices which may be connected in series or in parallel. In the general use of the instrument they are connected in series, but they may at any time be disconnected from one another and an insulation test made. Carrara marble was chosen as the material of the cylinders, and No. 24 hard-drawn copper wire for the coils. The mean diameter of a coil is about 35·9 cm. and the axial length is about 16 cm. The dimensions were very accurately determined.

Each cylinder (see *Fig.* 5A) is mounted on a strong metal support, and its position with regard to a disc may be altered by screw adjustments. Engraved platinum plugs are fixed in the cylinders, and by means of observations on these the distance between the mid-planes of two coils is determined.

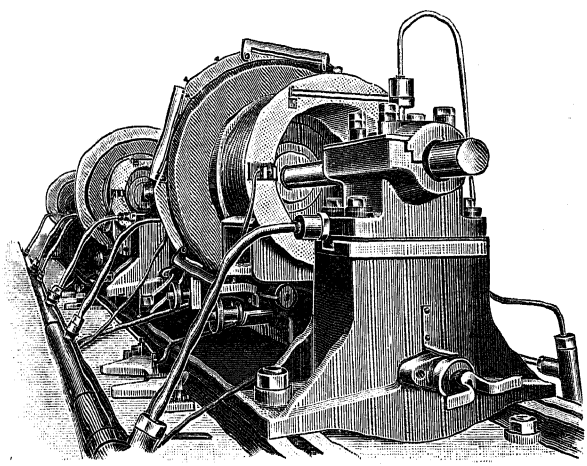


FIG. 5B.

The electric motor used for driving is situated at a considerable distance from any one of the coils, and its influence on the result was calculated and also experimentally proved to be negligibly small. A commutator is fixed to the axle of the motor, and this serves to charge and discharge a condenser placed in one arm of a Wheatstone bridge; by keeping the bridge permanently balanced, the speed of the apparatus is maintained constant and a directly driven chronograph enables the speed to be calculated. A fly-wheel of phosphor-bronze is added to reduce the effect of small sudden disturbances. The magnitude of the observed sudden fluctuations was of the order of 5 parts in 100,000.

The calculation of the mutual inductances between a coil and disc was made by the formula given by Viriamu Jones,[1] viz.

$$M = \Theta(A+a)ck\left\{\frac{K-E}{k^2}+\frac{1-c^2}{c^2}(K-\pi)\right\},$$

where Θ is the angular length of the helix, A its radius, a the radius of the disc or contact circle, and d the axial length of the helix.

$$c^2 = 4Aa/(A+a)^2,$$
$$k^2 = 4Aa/\{(A+a)^2+d^2\}.$$

K, E, and π are complete elliptic integrals.

The changes of mutual inductance, (1) when the axis of a coil is not normal to a disc and (2) when the axes of coil and disc are parallel but coincident, are difficult to calculate but easy to measure electrically, and by such measurements the axes of coils and discs were made to coincide.

(ii.) *Results.*—Lord Rayleigh found as a result of his measurements that

1 B.A. unit $=0{\cdot}98677\times10^9$ C.G.S. units,
or 10^9 C.G.S. units $=1{\cdot}01341$ B.A. unit.

In 1884 Rowland, Kimball, and Duncan found ($0{\cdot}98642\pm18$) B.A. unit equal to 1 ohm, and Duncan, Wilkes, and Hutchinson in 1889 found the ohm equal to 0·9863 B.A. unit.

The final results of five sets of measurements made by Jones in 1891 gave the resistivity of mercury at 0° C. as 94067 C.G.S. units. According to this result a column of mercury 106·307 cm. long and 1 sq. mm. cross-section all at 0° C. has a resistance of 10^9 C.G.S. units.

In 1897 Ayrton and Jones made comparisons with coils of wire, and the general result was that the resistance at 0° of 106 cm. of mercury 1 sq. mm. in area is equal to $1{\cdot}00026\times10^9$ C.G.S. units.

Smith gave the results of 56 measurements made under various conditions, during a period slightly over three months. The mutual inductance, the speed of rotation, the temperature of the coils, and the resistance were all varied. The maximum difference of any result from the mean was 5 parts in 100,000, and the average difference from the mean was ± 1·5 parts in 100,000. The probable observational error was 3 parts in a million. The probable error of the absolute measurements is believed to be not more than 2 parts in 100,000. The result may be stated in the following way:

The ohm, 10^9 cm./sec. is represented by the resistance at 0° C. of a column of mercury $14{\cdot}4446\pm0{\cdot}0006$ grams in mass, of a constant cross-sectional area (the same as for the international ohm),[2] and having a length of $106{\cdot}245\pm0{\cdot}004$ cm.

[1] See "Inductance, Calculation of," § (3).

§ (16) Campbell's Alternating Current Method (*Roy. Soc. Proc.*, 1908, lxxxi., and *Roy. Soc. Proc.*, 1912, lxxxvii.).—This is a direct comparison of a resistance with a combination of mutual inductance and the period of an alternating current in which two-phase alternating currents are used.

In *Fig.* 6, M is a mutual inductometer the value of which must be known with great

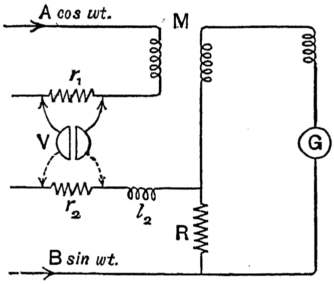


Fig. 6.

accuracy, and R is a resistance the value of which is desired. If A cos ωt and B sin ωt be the instantaneous values of currents in quadrature obtained from a two-phase alternator or phase-splitting device, then when the galvanometer shows no deflection the electromotive force introduced into its circuit is zero at every instant, and hence

$$\frac{d}{dt}(\text{MA}\cos\omega t)+\text{RB}\sin\omega t=0,$$

so that $R=(A/B)\omega M$.

The galvanometer G is a vibration galvanometer tuned to a frequency n where $p=2\pi n$. R is not perfectly non-inductive. Let its self inductance be l causing circuit A to be out of quadrature with B by a small angle x. Then when $A=B$ there will be a balance when

$$\omega M\cos(\omega t+x)-R\cos\omega t-l\omega\sin\omega t=0,$$

and since l is small compared with M

$$\omega M\cos\omega t-\omega Mx\sin\omega t-R\cos\omega t+l\omega\sin\omega t=0.$$

Thus $\qquad R=\omega M$ and $x=\div l/M$.

The inductometer used by Campbell was calibrated against a Campbell standard, and the errors of calibration being only a few parts in a million the error of the standard only is involved. This (*Proc. Roy. Soc.*, 1907, lxxix.) can be made as small as that of any other standard, and when the primary is of bare wire on marble, as was the case in Campbell's experiments, the value is determinable within 1 part in 100,000. The compari-

[2] See § (38), the area of the column assuming the mercury to be of normal density, 13·596 grams per c.c., will be 1 sq. millimetre.

sons were made with alternating current, but there seems no reason to suspect the material on which the coils were wound. The frequency determination could be improved, but the close agreement between the 3 results obtained (greatest difference $=3$ parts in 100,000) indicates that the error due to this cause was also small. Errors may arise due to the wave forms being not exact sine curves, and Campbell has examined this case. When there is a departure from the sine form the instantaneous values of the currents may be written

$$\Sigma A_s \cos(s\omega t+\phi s) \text{ and } \Sigma B_s \sin(s\omega t+\phi s)$$

respectively, where $s=1.\ 3.\ 5.\ .\ .\ .$

In this case the ratio of the effective values will be

$$(A_1^2+A_3^2+A_5^2+\ .\ .\ .)^{\frac{1}{2}}/(B_1^2+B_3^2+B_5^2+\ .\ .\ .)^{\frac{1}{2}},$$

$$i.e.\ A_1(1+a_3^2+a_5^2+\ .\ .\ .)^{\frac{1}{2}}/B_1(1+b_3^2+b_5^2+\ .\ .\ .)^{\frac{1}{2}}.$$

Campbell showed that in his experiments $(a_3+a_5+\ .\ .\ .)$ and $(b_3+b_5+\ .\ .\ .)$ were neither of them of an order greater than 1/100; hence the last written expression will not differ from A/B by more than 1 part in 20,000. The error, if any, can be eliminated by repeating the experiment with the coils of the alternator interchanged.

Campbell thought the probable error of his measurements to be about 1 part in 10,000.

(i.) *Experiments.*—Campbell only has made measurements by this method. His standard of mutual inductance was of the Campbell type but was not suitable for direct comparison with resistance coils. A secondary standard was therefore constructed in the form of a mutual inductometer of range from 11,000 microhenries to 0·01 microhenry, and for permanence the bobbins were made of white marble.

When making measurements the ratio of A to B was found by observing the effective values of the two currents by the respective potential differences produced in the nearly equal resistances r_1 and r_2, these voltages being read alternately on a very sensitive electrostatic voltmeter V. The alternator was run at as steady a speed as possible, the galvanometer tuned, and at every half-minute a balance was obtained by slight adjustment of M and of the phase of the B circuit. The adjustment of phase was made by a small variable self inductance l_2 (*Fig.* 6). A second observer simultaneously switched the voltmeter alternately from r_1 to r_2. As A/B differed from unity by only about 0·5 per cent and the scale of the voltmeter enabled readings to be taken with an error less than 1 part in 50,000 the ratio could be determined with great accuracy. A set of readings extended over about 15 minutes and the average frequency over this interval was taken, the maximum variation being from 2 to 3 parts in 1000. The vibration galvanometer was of the moving coil type and was amply sensitive for the purpose.

In making measurements the connections of the leads to the voltmeters, and also those to the variable inductance l_2, were systematically interchanged in order to eliminate as far as possible the slight inequality in capacity and leakage of the leads and any effect of stray field from the coil l_2. In some of the experiments the connections to the alternator coils were altered so as to interchange the positions of the coils relative to R and M.

(ii.) *Results.*—Campbell found

$$\text{One Ohm} = 10^9 \text{ C.G.S. units}$$
$$= 1{\cdot}0002_5 \text{ International Ohms.}$$

§ (17) Foster's Method.—In 1880 Professor G. Carey Foster made some measurements by a method suggested by him, the principle of which is essentially the same as that of the B.A. Rotating Coil. In the B.A. method the rotating coil forms part of the galvanometer; in Foster's method the current is measured by an independent galvanometer, and the conductor, whose resistance is given by the experiments, is entirely distinct from the rotating coil. So far as this method possesses any particular advantages they arise from the circumstance last mentioned, inasmuch as the resistance to be measured may be a coil of wire of any material, or it may be a conductor of any shape immersed in a bath of oil to keep it at a constant temperature.

The nature and arrangement of the essential parts of the apparatus are shown in *Fig.* 7. In this R is the wire of which the resistance is desired, B a battery, and G a tangent galvanometer. C is a rotating coil, and the ends of the wire of the coil are connected through a commutator K upon the axle, with the extremities of R, a sensitive galvanometer g being inserted in the circuit.

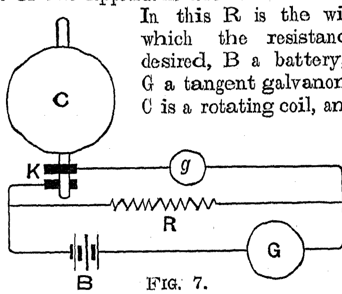


Fig. 7.

The circuit is only completed through K for a portion of the time. The commutator was of ivory with two pieces of platinum on opposite sides. Contact with the external

circuit was made through two platinum-faced gun-metal wheels each about 15 cm. diameter, which revolved in contact with the ivory cylinder. This arrangement was adopted in order to avoid the heating, and consequent thermo-electric action, which would probably have resulted from the use of rubbing contacts.

The platinum contact pieces had an angular breadth of about 20 degrees so that the coil was in connection with the rest of the circuit during about $\frac{1}{9}$th of each revolution, and the middle of the period of contact was made to coincide with the instant of maximum induced E.M.F. in the coil. The extreme variation of induced E.M.F. during contact was 1·83 per cent.

The results may be expressed in terms of the experimental data as follows:

Let A be the total area included by all the turns of the rotating coil, H the horizontal magnetic intensity, ω the angular velocity of the coil, and $2a$ the arc of contact made by the commutator. Then E the effective E.M.F. produced by the coil is given by the equation

$$E = HA\omega \frac{\sin a}{a}.$$

If G is the galvanometer constant (*i.e.* the intensity of the magnetic field in the centre of the galvanometer system produced by unit current in the galvanometer coils) and θ the deflection of the galvanometer, then

$$I = \frac{H}{G} \tan \theta$$

and

$$R = \frac{E}{I} = \frac{HA\omega G(\sin a/a)}{H \tan \theta},$$

or

$$R = \frac{AG}{T} \frac{2(\sin a/a)}{\tan \theta},$$

where T is the period of one revolution of the coil. The expression assumes that the experiment is made in a region of uniform magnetic intensity; failing this, a correction is necessary.

The coil used was somewhat similar to that of the B.A. Committee, but stouter in construction. Foster made only preliminary experiments and attached no great importance to his results.

The main advantage of the method lies in the elimination of self inductance, but against this there is a possibility of disturbance from mutual inductance between different parts of the circuit. The thermo-electric effects at the points of contact might also be troublesome. The method is probably inapplicable for precise measurements owing to uncertainty of the arc of contact, but the simple theory should render it useful to teachers. If bow-wire brushes (see Lorenz method) were employed, the thermo-electric effects would be small.

§ (18) ROSA'S METHOD.—Rosa (*Bureau of Standards Bull.*, 1909, v. No. 4) has made preliminary measurements with an apparatus in which two revolving coils at right angles to one another rotate in a strong magnetic field produced by two stationary coils, set somewhat farther apart, relatively, than the coils of a Helmholtz-Gaugain galvanometer, the whole constituting a kind of two-phase alternator without iron (*Fig.* 8). The wave

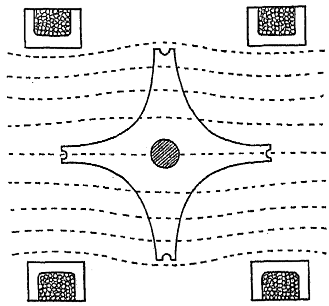


FIG. 8.

form of the induced electromotive force is not a sine; instead of varying at a maximum rate as it passes through zero, the electromotive force varies at a minimum rate. A revolving coil moves parallel or nearly parallel to the magnetic lines for an appreciable distance at the region of minimum electromotive force, thus giving a very small induced voltage for a considerable angle. This permits the electromotive force to be commuted without sensible loss. In making an absolute

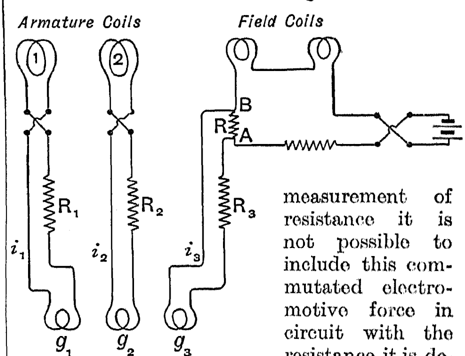


FIG. 9.

measurement of resistance it is not possible to include this commutated electromotive force in circuit with the resistance it is desired to measure; instead, a differential galvanometer with 3 windings stranded together is used. One of these, g_3 (*Fig.* 9), carries a constant current from the terminals AB of the resistance R, through which flows the current passing through the field coils. The other windings, g_1 and g_2, carry the respective currents from the armature coils 1 and 2. The field coils are so disposed as to make the

electromotive force at 45° of either side of the maximum equal to one-half the maximum, and under these circumstances the sum of the two currents through g_1 and g_2 is constant within about 3 per cent, and the fluctuation has a frequency four times that of either of the main induced currents. The effect on the differential galvanometer needle is the same as though the two currents i_1 and i_2 were combined in a single winding. Each circuit has its two-part commutator, and each has the same resistance as the circuit carrying the constant current. These resistances do not, however, have to be known. When the galvanometer is balanced, the sum of the average induced voltage generated by the two coils is equal to the voltage at the extremities of the resistance R which is the resistance to be determined absolutely.

If M_1 and M_2 be the respective mutual inductances of the revolving coils with the fixed coils, when each is in the position of maximum inductance, n the number of revolutions per second, and I the field current, the average value of the induced currents will be

$$i_1=\frac{4nM_1I}{R}$$

and

$$i_2=\frac{4nM_2I}{R},$$

if the effect of self inductance is negligible. The third current i_3 is RI/R_3, where R_3 is the total resistance of the circuit, including R.

Hence, when the constant current i_3 is balanced against the pulsating currents,

$$R=4n(M_1+M_2).$$

With appropriate interchanges any want of balance in the galvanometer windings may be eliminated.

No results have yet been published and some experimental difficulties cannot therefore be anticipated. It should be possible to measure directly by this method resistances as great as 10 ohms, and in consequence the thermo-electric effects at the commutators should be nearly or wholly negligible. The resistances R_1, R_2, and R_3 might also be as great as 1000 ohms, and in such case the variation of contact resistance at the brushes might be negligible. There will be temperature changes of resistance which could be appreciably reduced by making the circuits largely of manganin. There should be no trouble in keeping the speed constant. To determine M, Rosa suggests a comparison with a standard of mutual inductance such as that of Campbell's, or a mutual of two coaxial coils in the same plane.

§ (19) Grüneisen and Giebe's Method.—In this a self inductance L consisting of a single-layered coil on marble has its value in C.G.S. measure calculated from its dimensions, and its value in international units determined by measuring it in terms of a resistance and a frequency. The dimensions are inductance [L], resistance [LT^{-1}], and frequency [T^{-1}]. For the measurement of L in international units its value is first determined in terms of a capacity and two resistances, the capacity is subsequently measured in terms of a frequency and three resistances.

Grüneisen and Giebe (*Ann. d. Physik,* Sept. 1920, lxiii.), working at the Physikalisch Technischen Reichsanstalt, employed three coils A, B, and C, each of which was 35·5 cm. in diameter. The coils are of bare copper wire 0·5 mm. in diameter and each consists of one layer wound on marble. The pitch of coil A is 1 mm. and B and C are of $\frac{3}{4}$ mm. pitch. Coil A has 162 turns and a self inductance of about 0·01 henry; B and C each have 447 turns and a self inductance of about 0·05 henry; each of the latter can be subdivided by applying leads to $\frac{1}{3}$ and $\frac{2}{3}$ of their lengths, and by appropriate connections 13 different self inductances can be built up, all the coils of which are, however, of approximately the same diameter.

To calculate the self inductance of a coil it is necessary to know the radius R, the pitch p, the diameter of the wire d, and the number of turns N. The self inductance L can then be calculated by the Lorenz equation as modified by Rosa (*Bureau of Standards Bull.*, 1911, viii.):

$$L=\frac{8\pi}{3}RN^2Q\left(\frac{R}{Np}\right)-4\pi RN\left\{A\left(\frac{d}{p}\right)+B(N)\right\}.$$

Here Q is a function of R/Np, A a function of d/p, and B a function of N. Q contains elliptic integrals of the first and second order, which can be obtained from Legendre's tables.

The effects of errors in the measurement of R, p, and d can be obtained from the equation

$$\frac{\Delta L}{L}=\alpha\frac{\Delta R}{R}+\beta\frac{\Delta p}{p}+\gamma\frac{\Delta d}{d},$$

the coefficients α, β, and γ depending somewhat on the shape of the coil. For coil B they are

	α.	β.	γ.
Coil B . .	+1·672	−0·670	−0·002
$\frac{1}{3}$ B . .	+1·456	−0·4526	−0·0032.

If the error in the calculated value of L has to be within 1 part in 100,000, then R must be determined within about 6 parts in a million, p within 15 parts in a million, and d within about 3 parts in 1000.

The cross-section of the wire used was determined partly from its weight, length, and density, and partly from its electrical resistance, length, and resistivity. The pitch was measured directly by means of microscopes, and numerous measurements of diameters were made by special contact methods. Irregularities in diameters were allowed for when computing L. A measurement of the

mean diameter was also made by measuring the length of wire and number of turns.

To measure the self inductance in international electrical units the following procedure was adopted. A coil of self inductance L and resistance r_3 was first evaluated in terms of a capacity C and two resistances r_1

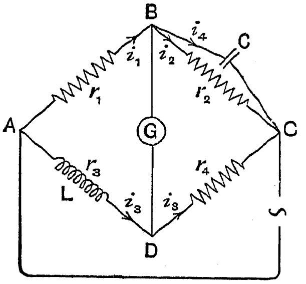


FIG. 9A.

and r_4 by a modified Maxwell bridge method (*Fig.* 9A).

Neglecting small distributed capacities, we have, when no current passes through the galvanometer,

$$L\frac{di_3}{dt}+r_3i_3=v_A-v_D=v_A-v_B$$
$$=r_1i_1.$$

Now $$v_B-v_C=r_2i_2=r_4i_3$$

and $$i_4=C\frac{d}{dt}(v_B-v_C)=Cr_4\frac{di_3}{dt}.$$

After substitution we have

$$Li_4=Cr_4(r_1i_1-r_3i_3)=Cr_1r_4i_1-Cr_2r_3i_2$$
$$=Cr_1r_4(i_1-i_2),$$

and since $i_4=i_1-i_2$ we have

$$L=Cr_1r_4.$$

If K_1, K_2, etc., represent the small capacities in the various parts of the bridge and C be increased by a small adjustable rotary condenser C_a, then the complete expression is

$$L=r_1r_4(C+C_a)-\frac{r_1r_4}{r_2}(r_1K_1+r_4K_4-r_2K_2).$$

To eliminate K_1, K_2, etc., the coil of self inductance L was replaced by a bifilar wire of equal resistance and of small calculable self inductance l. C was removed and the bridge balanced by adjustment of the rotary condenser. Then

$$l=r_1r_4C_1-\frac{r_1r_4}{r_2}(r_1k_1+r_4k_4-r_2k_2).$$

Combining this with the previous equation we have

$$L-l=r_1r_4(C+\Delta c),$$

where Δc is the change made in the capacity of the rotary condenser.

r_1, r_2, and r_4 were special resistances of small capacity and small self inductance. Alternating current was used, the frequencies employed being 332, 500, and 720 per second. G is a tuned vibration galvanometer and was capable of detecting 10^{-8} ampere at a frequency of 300 per second. All leads to the bridge arms were arranged in a bifilar fashion as shown in *Fig.* 9B.

The capacity C consisted of a number of air condensers built up of circular discs of an

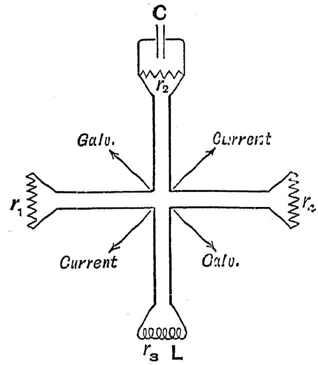


FIG. 9B.

aluminium alloy. The value of C was determined by the Maxwell interrupter method, the relation being

$$C=\frac{1}{nr},$$

where n is the number of discharges per second and r a resistance in international ohms. r is not, however, a single resistance.

From the two equations

$$L=Cr_1r_4 \text{ [farads ohms}^2\text{]}$$

and $$C=1/nr \text{ [seconds/ohms]}$$

we have

$$L=\frac{r_1r_4}{nr} \text{ int. henries [int. ohms seconds].}$$

The values obtained on various dates for the self inductance of a coil of approximately 0·05 henry are as follows :

Date.	$L=0{\cdot}05\times$	Differences from Mean. Parts in 1,000,000.
20.4.1914	$1+66\cdot10^{-6}$	− 7
28.4. ,,	1+82 ,,	+ 9
29.4. ,,	1+72 ,,	− 1
1.5. ,,	1+87 ,,	+14
2.5. ,,	1+91 ,,	+18
4.5. ,,	1+67 ,,	− 6
5.5. ,,	1+78 ,,	+ 5
7.5. ,,	1+69 ,,	− 4
8.5. ,,	1+73 ,,	± 0
12.5. ,,	1+67 ,,	− 6
13.5. ,,	1+65 ,,	− 8
14.5. ,,	1+66 ,,	− 7
23.5. ,,	1+70 ,,	− 3
25.5. ,,	1+74 ,,	+ 1

The ratio of the value of the self inductance L in 10^9 centimetres to its value in international measure is the ratio of the ohm (10^9 C.G.S. units) to the international ohm.

Grüneisen and Giebe give the following summary of their results:

Coil.	L in 10^9 cm.	L in Int. Henry.	$\frac{\text{L abs.}}{\text{L int.}}$	Mean Ratio for each Combination.
Section A	0·010 138 20_1	0·010 133 06	1·000 50_7	1·000 50_7
B	0·050 029 66	0·050 004 0	1·000 51_3	1·000 51_3
B	28 681 96_3	28 667 24_9	51_3	1·000 51_3
B	28 684 07_6	28 669 39_0	51_2	
B	10 261 05_5	10 255 89_4	50_3	
B	10 261 09_0	10 255 80_0	51_6	1·000 50_5
B	10 262 03_0	10 256 95_3	49_5	
C	0·050 009 65	0·049 983 1	1·000 53_1	1·000 53_1
Section C	28 672 28_4	28 656 77_7	54_1	1·000 52_0
C	28 672 63_3	28 658 32_7	49_0	
C	10 257 73_2	10 251 98_6	56_1	
C	10 258 06_9	10 252 88_0	50_8	1·000 52_1
C	10 257 60_7	10 252 50_3	49_8	

The mean ratio is $1{\cdot}00051_6$.

After comparing the mean effective diameters of the coils and sections of same by an electrical method, Grüneisen and Giebe conclude that

1 international ohm

$$=1{\cdot}00051 \text{ ohm } (10^9 \text{ C.G.S. units}),$$

and estimate the probable error to be ± 3 parts in 100,000. The corresponding length of the mercury column is 106·246 cm.

This value is almost identical with that found by Smith, but there is reason to believe that the units obtained from the mercury standards of resistance of the National Physical Laboratory and the Reichsanstalt differ by 3 parts in 100,000. If this is so the difference between the results of the two absolute measurements may be 4 parts in 100,000.

§ (20) Summary of Results.—A summary of the results of the principal measurements by all methods are given in the table on following page.

§ (21) The B.A. Unit.—It will be seen from the table that nearly half of the results are given in terms of the B.A. unit, and it may be useful to state clearly what is meant by that unit. Originally the B.A. unit was supposed to be equal to the ohm (10^9 C.G.S. units) within about 1 part in 1000, and the original B.A. unit was the result of absolute measurements made by Maxwell, Jenkin, and Balfour in 1863. In 1864 Matthiessen and Hockin constructed a number of coils of various materials to represent the B.A. unit. The resistance of these coils did not keep absolutely constant, and in after years the B.A. unit was taken as the mean of the values of six of these coils. The B.A. unit of one period is not therefore necessarily the same as that of another period. Every precaution was taken to ensure constancy, but with wire standards of resistance great difficulty is experienced. In 1908 (*British Association Report of Elec. Stands. Committee*, 1908) it proved possible to trace the changes in these coils with a fair measure of success, and the corrections to results of absolute measurements due to changes in the coils can in a number of cases be calculated. This has been done by Smith (*Roy. Soc. Phil. Trans.*, 1913, ccxiv. 32), in the case of the coils used by Lord Rayleigh, Glazebrook, and Jones.

§ (22) Final Results.—The corrected results of the absolute measurements expressed in terms of centimetres of mercury are:

Observer.	Ohm in Centimetres of Mercury.
1882. Rayleigh . . .	106·26 cm.
1882. Glazebrook . . .	106·25 ,,
1883. Rayleigh and Sidgwick .	106·24 ,,
1891. V. Jones . . .	106·31 ,,

Unfortunately it is not possible to deal in the same way with the standards of other investigators as the particulars available are insufficient for the purpose. It is, however, of particular interest to note that these corrected values are in very good agreement with the results of recent measurements made by Smith, by A. Campbell, and by Grüneisen and Giebe. These are tabulated below, together with the values obtained by Wiedemann in 1885 and by Dorn in 1889.

Observer.	Ohm in Centimetres of Mercury.
1882. Rayleigh . . .	106·26 cm.
1882. Glazebrook . . .	106·25 ,,
1883. Rayleigh and Sidgwick .	106·24 ,,
1885. Wiedemann . . .	$106{\cdot}16_2$,,
1889. Dorn	$106{\cdot}24_3$,,
1891. V. Jones . . .	106·31 ,,
1912. Campbell . . .	$106{\cdot}27_3$,,
1913. Smith	$106{\cdot}24_5$,,
1920. Grüneisen and Giebe .	$106{\cdot}24_6$,,

The mean of all these values is $106{\cdot}24_8$, and it appears that within 1 part in 10,000 the ohm (10^9 C.G.S. units) can be represented by the resistance at 0° C. of a column of mercury of the same cross-section as the international ohm and having a length of

$106{\cdot}24_5$ centimetres.

RESULTS OF ABSOLUTE MEASUREMENTS OF RESISTANCE

Date.	Observer.	Results given by Author.		
		B.A. Unit in Ohms.	Siemens Unit (100 cm. of Mercury) in Ohms.	Ohm in cm. of Mercury.
	1. KIRCHOFF'S METHOD			
1878	Rowland	0·9911	..	..
1883	Glazebrook	0·98665	..	..
1884	Rowland	0·98627 ± 40	..	106·34
	2. ROITI AND HIMSTEDT'S METHOD			
1884	Roiti	..	..	105·896
1886	Himstedt	..	..	106·08
	3. WEBER'S METHOD OF TRANSIENT CURRENTS			
1885	Wiedemann	..	..	106·162
1885	Mascart, de Nerville, and Benoit	0·9860	..	106·37
	4. WEBER'S METHOD OF DAMPING			
1884	Wild	..	0·94315	106·027
1888	F. Kohlrausch	..	..	106·34
1889	Dorn	..	..	106·243
	5. METHOD OF ROTATING COIL			
1863	Maxwell, Jenkin, and Balfour (For B.A. Committee)	1·000	..	..
1881	Rayleigh and Schuster	0·9893	..	..
1882	Rayleigh	0·98651	..	..
1882	H. Weber	0·9877	..	..
	6. METHOD OF LORENZ			
1873	Lorenz	..	0·9337	..
1883	Rayleigh and Sidgwick	0·98677	..	..
1884	Rowland, Kimball, and Duncan	0·98642 ± 18	..	106·29
1889	Duncan, Wilkes, and Hutchinson	0·9863	..	..
1891	V. Jones	..	..	106·307
1913	Smith	..	..	106·245
	7. CAMPBELL'S METHOD			
1912	A. Campbell	..	..	106·273
	8. GRÜNEISEN AND GIEBE'S METHOD			
1920	Grüneisen and Giebe	..	..	106·246

II. ABSOLUTE MEASUREMENT OF CURRENT

§ (23) DIMENSIONS. — The dimensions of current in the electromagnetic system are $[L^{\frac{1}{2}} M^{\frac{1}{2}}/T\mu^{\frac{1}{2}}]$. If, as is customary, μ be assumed to have no dimensions, the dimensions of current are $[L^{\frac{1}{2}}M^{\frac{1}{2}}/T]$. The square of these dimensions is $[LM/T^2]$, which is that of a force. Hence to measure a current it may suffice to measure a force only. If instead other quantities are measured these must, together, include the dimensions of a force. Thus the product of the dimensions of magnetic intensity (commonly called magnetic force) $[M^{\frac{1}{2}}/L^{\frac{1}{2}}T\mu^{\frac{1}{2}}]$ and magnetic pole $[L^{\frac{3}{2}}M^{\frac{1}{2}}\mu^{\frac{1}{2}}/T]$ is of the dimensions of a force $[ML/T^2]$, and this suggests that current may be measured in C.G.S. units in two ways: (i.) By galvanometer methods in which the intensity of the magnetic field produced by an electric current is compared with the intensity of the earth's horizontal field (independently determined) by the action of the field on a magnet; and (ii.) by a current weigher in which the mutual action between a current and a magnet is measured by comparing it with the force exerted by gravity on a known mass.

It is also possible to measure current by measuring the force due to one portion of a circuit on another portion of the same circuit. Such measurements include (iii.) those in which the mutual action between the currents in two or more coils takes the form of a torque, which may be measured by the torsion of a wire or with a bifilar suspension; and (iv.) current weighers in which the mutual action between the coils may be balanced by

the force exerted by gravity on a known mass.

It is clear that if in a measurement of current a force is measured with accuracy no other measurement of length, of mass, or of time need be made absolutely, but accurate ratios may be necessary. In such a case, since measurements of force are in general based on measurements of the earth's gravitational intensity any error associated with a determination of "g" is necessarily introduced in half-measure in the determination of current.

§ (24) Galvanometer Methods.—In these the intensity of the magnetic field produced by an electric current is compared, by means of a galvanometer system of coils, with the horizontal intensity of the earth's magnetic field.

The general expression for the couple acting on a magnet (*Fig.* 10) with its centre on the axis of a coil and deflected through an angle θ from

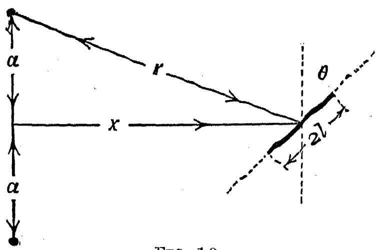


Fig. 10.

the magnetic meridian, the plane of the coil being also in the magnetic meridian, is as follows:

$$\text{Couple} = 2\pi \mathrm{N} i \mathrm{M} \frac{a^2}{r^3} \cos\theta$$

$$\left\{1 + \frac{3}{2^2}\frac{l}{r^4}(a^2 - 4x^2)(1 - 5\sin^2\theta) + \frac{3^2}{2^2}\frac{5}{4_2}\frac{l^4}{r^8}\right.$$

$$\left.(a^4 - 12a^2x^2 + 8x^4)(1 - 14\sin^2\theta + 21\sin^4\theta + \ldots)\right\},$$

where N = number of turns in coil,
i = current in coil,
M = moment of magnet,
$2l$ = length of magnet,
θ = angle of deflection of magnet,
x = distance between centre of coil and centre of magnet,
a = effective radius of coil,
r = distance of effective edge of coil from centre of magnet.

For the use of the method this couple is balanced against that due to the earth's magnetism, which is given by $\mathrm{MH}\sin\theta$.

The value of l is definite only in the case of ideal magnetisation of the magnet. In practice the terms l, l^2, l^4, etc., depend on the distribution of magnetism. When l is very small, as it is in good modern instruments, and when also $x=0$, as it is in the tangent and sine galvanometers, the couple is very approximately equal to

$$2\pi \mathrm{N} i \mathrm{M} \cos\theta / a.$$

So far the assumption has been made that the N turns of wire are coincident, but in general the coil cannot be regarded as a simple circular conductor, and corrections have to be made for its cross-section. If the coil is of rectangular cross-section, $2b$ being its axial length and $2d$ its radial depth, and if n is the number of turns crossing unit of area of cross-section, the intensity of the field produced at a point on the axis is

$$2\pi n i \left\{(x+b)\log\frac{a+d+\sqrt{(x+b)^2+(a+d)^2}}{a-d+\sqrt{(x+b)^2+(a-d)^2}}\right.$$

$$\left. -(x-b)\log\frac{a+d+\sqrt{(x-b)^2+(a+d)^2}}{a-d+\sqrt{(x-b)^2+(a-d)^2}}\right\},$$

which, when $x=0$, reduces to

$$\frac{\mathrm{N}\pi i}{d}\log\frac{a+d+\sqrt{(a+d)^2+b^2}}{a-d+\sqrt{(a-d)^2+b^2}},$$

where N is the total number of turns in the coil (see Gray, *Absolute Measurements*, vol. ii. part i.).

§ (25) Tangent Galvanometer.—In the ideal case of a tangent or sine galvanometer not only is $x=0$ but the centre of the magnet is coincident with the centre of the coil. When this is not the case but nearly so, and the co-ordinates of the centre of the magnet relative to the centre of the coil are δx, δy, and δz, δx being measured parallel to the axis of the coil, the correcting factor is

$$1 + \tfrac{3}{2}\frac{\delta y^2 + \delta z^2 - 2\delta x^2}{a^2},$$

a being the radius of the coil. Thus, if $a=20$ cm., $\delta y=0{\cdot}1$ cm., $\delta z=0{\cdot}1$ cm., and $\delta x=0$, the correcting factor is $1{\cdot}00007$.

It is apparent that l the length of the magnet cannot be made equal to zero, and in accurate measurements the terms involving l^2, etc., may not be negligibly small, but it will be observed that in the equation for the couple the second correction term involving l may be made to vanish if $5\sin^2\theta=1$. When $\theta=26°\ 34'$ this relation is satisfied, and some observers have arranged for θ to have this value in their measurements.

The chief difficulty with galvanometer methods is associated with H the horizontal intensity, for not only does H vary, but its value is as difficult to determine as that of current. Indeed in recent years galvanometer apparatus has been constructed to determine H when the value of the current has been found by some independent method. Owing to variation in the intensity of the field

near the centre of a coil, the magnet used must be very small or the distribution of its magnetism accurately known; if a small magnet is used it is of great importance to eliminate the torsional control due to the fibre. In all methods the effective radius of the coil must be measured with great accuracy, and there is a distinct gain in using a coil of one layer.

(i.) *Measurements.*—Measurements of current by means of the tangent galvanometer have been made by W. Weber (1840), Bunsen (1843), and by Joule (1851). More precise measurements have been made by F. Kohlrausch (*Pogg. Ann.*, 1873, cxlix.) and later by F. and W. Kohlrausch (*Wied. Ann.*, 1886, xxvii. 1). The most recent experiments are those by van Dijk and Kunst (*Roy. Acad. Proc.*, Amsterdam, 1904) and by Haga and Boerema (*Konink. Akad. Wetensch. Amsterdam Proc.*, 1910, p. 587).

(ii.) *Results.*—These are sometimes expressed in terms of the weight of silver deposited by 1 coulomb, and at other times the electromotive force of a standard cell is given. This latter involves knowledge of the absolute unit of resistance.

Date.	Observer.	Electro-technical Equivalent of Silver.	E.M.F. at 20° C. of Weston Normal Cell.
		mgm.	
1873	F. Kohlrausch	1·1363	..
1886	F. and W. Kohlrausch	1·11833	..
1904	Van Dijk and Kunst	1·1180	..
1910	Haga and Boerema	..	1·01825 *

* The international ohm was used as the standard of resistance.

§ (26) Sine Galvanometer.—In this instrument the coil, or coils, can be moved round a vertical axis, and when the magnet is deflected the coil is turned until its plane contains the axis of the magnet. The difficulties and probable errors are of the same order of magnitude as those for the tangent galvanometer.

(i.) *Measurements.*—In 1872 Latimer Clark (*Roy. Soc. Phil. Trans.*, 1874, Part I.) measured the E.M.F. of a Clark cell by means of a sine galvanometer, and in 1886 Thomas Gray (*Phil. Mag.*, 1886) designed a special sine galvanometer and determined the electrochemical equivalent of silver. In Gray's instrument the coil was about 10 cm. in diameter and 100 cm. long. Such a coil, if uniformly wound with n turns per cm. length, produces a field at its centre of intensity equal to $4\pi nil/(a^2+l^2)^{\frac{1}{2}}$, where l = half the length of the coil and a the radius. As l was great compared with a the exact determination of the radius was not of very great importance. Great care was taken to obtain a uniform winding.

(ii.) *Results.*—In 1872 Latimer Clark found the E.M.F. of the Clark cell to be 1·4562 volts at 15°·5 C. The B.A. unit was the standard of resistance. In 1886 T. Gray found the electro-chemical equivalent of silver to be 1·118 mgm. per coulomb.

§ (27) Helmholtz-Gaugain Galvanometer.—In order to eliminate the correction involving l (see general equation for couple on the magnet) Gaugain constructed a galvanometer in which $4x^2$ was equal to a^2, *i.e.* the magnet was suspended not at the centre of the coil but at an axial distance from it equal to half the radius. It was, however, of great importance to place the magnet in the correct position.

Helmholtz improved Gaugain's system by placing a second coil, similar to the first, on the other side of the magnet. If for one coil (a^2-4x^2) is slightly positive, then for the other coil it is equally negative, and the total correction term involving l becomes negligibly small. The necessity for accurately locating the axial position of the magnet is thus avoided. When $a^2=4x^2$ the expression for the couple reduces to

$$4\pi NiM\frac{a^2}{r^3}\cos\theta\left\{1-\frac{3}{2}\cdot\frac{45}{64}\frac{l^4a^4}{r^8}(1-14\sin^2\theta+21\sin^4\theta)\right\}.$$

As a further simplification the correction term involving l^4 can be made negligibly small by choosing a small magnetic needle, or it can be made to vanish by making $14\sin^2\theta$ equal to $1+21\sin^4\theta$. This is the case when $\theta=16°\ 34'$, and also when $\theta=49°\ 55'$.

The principal difficulty in using the instrument is in the measurement of H and of the dimensions of the coils. The latter should be single layer coils on marble or other non-magnetic material.

Measurements.—Gehrcke and Wogan (*Verhandlungen der Deutschen Phys. Gesellschaft*, Jan. 1911) used a Helmholtz-Gaugain galvanometer and found 1 C.G.S. unit of current equal to 10·002 international amperes.

Watson (*Roy. Soc. Phil. Trans.*, 1902, cxcviii.) used the method to find H, the current being known. More recently Schuster (*Terr. Mag.*, 1914, xix.) has proposed to measure H with such a galvanometer. This [1] instrument has been constructed at the National Physical Laboratory and a similar instrument has been set up by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington.

§ (28) Electromagnetic Current Weigher.—In this the mutual action between a current and a magnet is measured by comparing it with the force exerted by gravity on a known mass.

This method fails as one of precision because of the lack of knowledge of the distribution of magnetism in a magnet. A very small magnet cannot be used or the force to be measured would be too small.

[1] See "Magnetism, Terrestrial, Electromagnetic Methods of Measuring."

Measurements were made by this method by A. Becquerel in 1837, and by Lenz and Jacobi in 1839. In recent years the method has been revived by W. Hibbert for teaching purposes. In Hibbert's balance the beam is made of a long thin cylindrical magnet and a coil is placed under one end of it in such a position that one pole of the magnet lies approximately on the axis of the coil. The force due to the current is measured by a rider weight on the beam. The pole strength of the magnet is determined in C.G.S. units by the use of two similar magnets and directly measuring the forces between combinations of any two of them. The forces and distances between the poles being known, three equations are obtained which enable the pole strength of any one of the magnets to be calculated.

§ (29) Electrodynamometer Methods.—In these the mutual action between the currents in two or more coils takes the form of a torque which may be measured by the torsion of a wire or by a bifilar suspension.

§ (30) Gray Electrodynamometer.—Gray (*Absolute Measurements*, vol. ii. part i. p. 274) shows that the torque between two concentric coils can be written

$$\text{Torque} = \pi^2 n_1 n_2 i^2 a_1^2 a_2^2 \sin\phi(K_1 k_1 Z_1 + K_3 k_3 Z_3 + K_5 k_5 Z_5 + \ldots),$$

where

$$K_1 = \frac{4\rho_1}{a_1 D}, \qquad k_1 = 2\rho_2 a_2,$$

$$K_3 = \frac{-\rho_1 a_1}{D^5}, \qquad k_3 = \rho_2 a_2^3(4\rho_2^2 - 3),$$

$$K_5 = \frac{-\rho_1 a_1^3}{4D^9}(4\rho_1^2 - 3), \qquad k_5 = 2\rho_2 a_2^5(2\rho_2^4 - 5\rho_2^2 + \tfrac{15}{8}),$$

etc., and Z_1, Z_3, Z_5 are the zonal surface harmonics corresponding to the respective terms, the angle between the axes of the coils being the argument. ρ_1 and ρ_2 are the respective ratios of length to diameter, n_1 and n_2 the number of turns per centimetre, a_1 and a_2 the radii of the fixed and movable coils respectively, and D the half diagonal of the fixed coil (see *Fig.* 11). i is the current, supposed to be the same in the two coils.

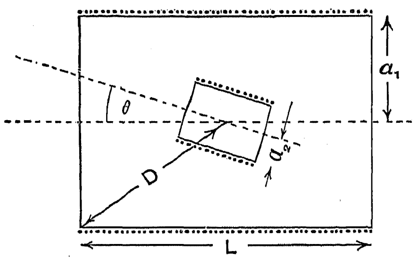


Fig. 11.

Gray further shows that if the dimensions of the coils are so chosen that the length and the radius of each are in the proportion $\sqrt{3/1}$, the terms involving K_3 and K_5 vanish. The next terms in the series involving K_7, K_9, etc., may be made negligibly small if the moving coil is small compared with the fixed coil. Thus the term involving K_7 amounts to about 1/4700 if the ratio of the radii of the coils is 2/3 and 1/27000 if the ratio is 1/2. The expression for the couple then reduces to

$$\frac{2\pi^2 N_1 N_2 i^2 a_2^2}{D} \sin\phi.$$

N_1 and N_2 are respectively the total number of turns on the fixed and movable coils. An electrodynamometer constructed so that the ratio of length to radius of each coil is $\sqrt{3}$ is known as the *Gray* type of instrument.

Electrodynamometers have an advantage over galvanometer systems inasmuch as no magnet is necessary and there is therefore no uncertainty as to distribution of magnetism. A further advantage is that no knowledge of the intensity of the earth's magnetic field is required in order to reduce the observations. The disadvantages are inferior sensitiveness and the difficulty of accurately measuring force by a torsional or bifilar suspension. In all measurements of current, coils are used, but, as Lord Rayleigh has shown, it is not necessary in all cases to measure the dimensions of the coils; a ratio of radii sometimes suffices. In the Gray electrodynamometer, however, the lengths and diameters must be measured. Rosa (*Bureau of Standards Bull.*, 1906, ii. No. 1) has shown that the spiral winding of the coils is practically equivalent to a current sheet, and further shows that large errors may be produced by irregular winding of the coils. The correction due to this effect in Guthe's experiments (*Bureau of Standards Bull.*, 1906, ii. No. 1) was particularly large. Regularity of pitch may, however, be obtained if the bobbins are first screw cut on a precision lathe.

When the couple between the coils is balanced against the torsional moment of a wire, the latter moment is determined by vibrating a cylinder of calculated moment of inertia and taking the period of a complete vibration. The probable error of such a determination is not negligible, as has been shown by Guthe, who made experiments with four cylinders and obtained results differing by as much as 1 part in 3000.

(i.) *Measurements.*—Latimer Clark (*Roy. Soc. Proc.*, May 30, 1872), in determining the electromotive force of his standard cell, used a bifilar electrodynamometer in which the Helmholtz-Gaugain arrangement of two coils was adopted both for the fixed and suspended systems. The suspension consisted of two

wires attached to a silk thread passing over a small pulley; in this way the tension was maintained reasonably uniform and the suspension wires allowed of the passage of the current. The suspended coil could be moved vertically and horizontally in two directions by means of screw adjustments.

A similar instrument was employed at the McGill University, Montreal, by R. O. King (*Roy. Soc. Phil. Trans.*, 1902), and later by A. N. Shaw (*Roy. Soc. Phil. Trans.*, 1914). The experiments with this instrument merit special attention as the modifications introduced enable measurements of very high accuracy to be made. The principal sources of error in previous instruments of this type were (1) the uncertainty of insulation of the coils; (2) the difficulty of determining the mean radii of the coils which were wound with silk-covered wire; (3) the want of rigidity of the suspension arrangement for equalising the tensions of the suspending wires; and (4) the imperfect elasticity of the control, which depended too much on torsion. To overcome these difficulties Shaw wound the coils with a double winding to give a perfect check on the insulation. The large coils were wound with a carefully measured length of hard rolled copper tape, which gave a very high order of accuracy in the determination of the mean radius. The coils were also made reversible and interchangeable to eliminate possible errors of symmetry, especially in the measurement of the distance between their planes. The dimensions of the suspended coils were determined by a method based on that due to Bosscha, and great accuracy was obtained in this way. A good deal of time was spent in selecting a suitable suspension, and Callendar, who was associated with the measurements, estimated the error introduced by it to be very small.

A Gray electrodynamometer in which the coils were of single layers of wire was constructed by Patterson and Guthe (*Phys. Rev.*, 1898, vii.) for the determination of the electro-chemical equivalent of silver and was also used by Carhart and Guthe (*Phys. Rev.*, 1899, ix.) for the measurement of the E.M.F. of the Clark cell. In these experiments the couple was balanced by the torsional moment of a single wire the mechanical properties of which had been previously determined. Guthe (*Bureau of Standards Bull.*, 1906, ii. No. 1) used single layer coils wound on bobbins of plaster of Paris, the larger coil being about 50 cm. in diameter and the smaller coils 7·5 cm. and 9·9 cm. respectively. Two movable coils were used in order to increase the accuracy of the result. A great deal of time was spent in the attempt to find a suspension which would show the smallest elastic after-effect and not change its elastic properties in course of time. The attempt was only partially successful.

(ii.) *Results.*—In 1872 Latimer Clark found the electromotive force of the Clark cell at 15°·5 C. to be 1·4573 B.A. volts. The standard of resistance was the B.A. ohm.

Patterson and Guthe in 1898 measured the electro-chemical equivalent of silver to be 1·1192 mgm. per coulomb, and in the following year Carhart and Guthe obtained the value 1·4333 volts for the electromotive force of the Clark cell at 15° C.

Guthe (1906) arrived at the values 1·43296 volts for the Clark cell at 15° C. and 1·01853 volts for the Weston normal cell at 20° C. He concluded that the electro-chemical equivalent of silver is 1·11773 mgm. per coulomb.

King (1914) found the electromotive force of the Weston normal cell at 20° C. to be 1·01831 volts.

§ (31) McCollum Type of Electrodynamometer.—B. McCollum (*Bureau of Standards Bull.*, Nov. 1910) has described a type of electrodynamometer which appears to be free from most of the difficulties associated with the Guthe type with torsional or bifilar control.

The instrument (*Fig.* 12) consists essentially of a relatively large fixed coil C_1 with horizontal axis, and a smaller suspended coil C_2 with its axis coincident with that of C_1 when the system is at rest. Rigidly attached to C_2 is a cylinder K of non-magnetic homogeneous material placed with its axis vertical and in line with the suspending wire W. When a current is passed through the two coils appropriately connected in series the effect is to hold the movable coil with its axis coincident with that of the fixed coil, and if the movable coil be given an angular displacement and released, it will oscillate as a torsion pendulum. If the restoring couple is proportional to the angular displacement, the period is independent of the amplitude and the system is subject to the laws of a damped oscillation. The time T of vibration of the moving system is given by the equation [1]

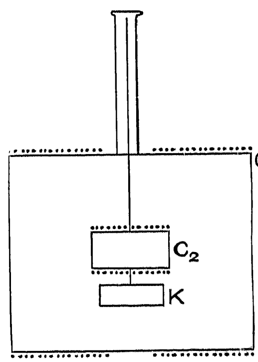


Fig. 12.

$$T_1=\frac{2\pi}{\sqrt{\frac{c}{K}-\frac{b^2}{4K^2}}},$$

[1] See "Galvanometers," § (10).

where c is the restoring couple per unit angular displacement, b is the counter torque due to the damping forces at unit angular velocity, and K is the moment of inertia of the moving system.

When the coils are so proportioned that the torque due to the current i is proportional to the deflection

$$c = Ci^2 + a,$$

where C is a constant of the coil system, a a constant depending on the suspending wire, and R the total restoring couple. The period T_1 may therefore be written

$$T_1 = \frac{2\pi}{\sqrt{\frac{Ci^2+a}{K} - \frac{b^2}{4K^2}}}.$$

With no current through the coils the time of vibration T_2 is

$$T_2 = \frac{2\pi}{\sqrt{\frac{a}{K} - \frac{b^2}{4K^2}}}.$$

From the last two equations it follows that

$$i = \frac{2\pi}{T_1}\sqrt{\frac{K}{C}\left(1 - \frac{T_1^2}{T_2^2}\right)}.$$

McCollum shows that the torque is proportional to the deflection when the coils are so wound that the ratio of length to diameter of the fixed coil is $\sqrt{3/2}$ and that of the movable coil is either 2·062, 0·92, or 0·38. For deflections of 8° from the central position the time of vibration is then independent of amplitude within about 1 part in 100,000. The constant of the instrument can be either calculated or determined by measuring the mutual inductance in the coaxial position. In the Gray type of electrodynamometer the torque is proportional to the sine of the angle of deflection, and the period would not be isochronous except for values of θ not exceeding about one-half degree.

Unfortunately the foregoing method has not yet been subjected to experimental tests, so that the limits of accuracy cannot be stated with precision. It is clear, however, that the torsion of the suspending wires is eliminated. The value of K must be accurately known and the determination may present difficulties; also much depends on the correct measurement of the times of vibration. McCollum describes the method he would adopt for measuring T_1 and T_2 which involves observations over a considerable period of time (20 minutes), but has the advantage that if the current varies its mean value is accurately measured.

The writer has made somewhat similar measurements with a current balance and is of opinion that the McCollum type of electrodynamometer can be used for the accurate measurement of steady currents lasting less than a minute. In such a case the period should be taken photographically on a rapidly running film on which an accurate time trace is simultaneously photographed, the trace enabling time intervals to be measured within 0·00001 second. The writer further suggests that in order to remove the uncertainty attending the determination of K the suspended coil should be mounted centrally on the beam of a balance and appropriate measurements made to determine the distance between the centre of support and the centre of mass. K can then be calculated.

§ (32) Current Weighers.—In these the mutual action between currents in coils is balanced by a force exerted by gravity on a known mass.

The acceleration of gravity at any place can be determined with great accuracy, and therefore force can be measured accurately by means of a balance. For this reason instruments in which the mutual action between two circuits, or two portions of the same circuit, is measured by means of a gravity balance have been much favoured.

In most current weighers two or more coils are placed in series with their planes parallel and their axes coincident. If M is the mutual inductance between the coils and x the distance between their centres, the mutual attraction[1] between them is $i^2(dM/dx)$, where i is the current. dM/dx is calculated from the dimensions of the coils and the force $i^2(dM/dx)$ is balanced by the force exerted by gravity on a mass on the balance pan. Thus i can be determined. This type of current weigher in which the axes of the coils are coincident will be considered first.

In a primitive form the first current weigher appears to have been constructed by Cazin (*Ann. de Chim.* i.) in 1863. This consisted of two rectangular coils with their planes parallel, one hanging from the beam of a balance directly above the other. In 1864 Joule made a current balance having three circular flat coils, one being suspended from the beam of a balance so that its mean plane, which was horizontal, was midway between those of the other two fixed coils. The principal constant of the instrument was determined by comparison with a standard tangent galvanometer. Joule employed it in his electrical determination of the mechanical equivalent of heat.

In 1882 Mascart (*Journ. de Physique*, March 1882) suspended a long solenoid from one end of a balance beam, the lower end of the solenoid being in the mean plane of a large flat coaxial coil of much larger radius. If the solenoid is uniformly wound it is equivalent to a simple magnet, with poles at the terminal

[1] See "Electromagnetic Theory," § (10).

faces. The mutual action then depends upon the difference between the mutual inductances of the fixed coil and the lowest and uppermost windings of the movable coil.

§ (33) The Rayleigh Balance.—In 1882 Lord Rayleigh (*B.A. Report*, 1882) pointed out that in a measurement of current depending on the mutual attraction between two coils the absolute dimensions of the system are of no importance. If M is the mutual inductance and i the current, the force between the coils is $i^2(d\mathrm{M}/dx)$, where x is the axial distance between the coils. In this expression i^2 is already of the dimensions of a force and $d\mathrm{M}/dx$, though a function of the radii of the coils and their distance apart, is itself a pure number. If A and a be the mean radii of the coils the value of $d\mathrm{M}/dx$ depends only on the ratios a/A and x/A. If we write $(d\mathrm{M}/dx)=f(\mathrm{A}ax)$ and consider the variation of f as a function of the three linear quantities, the coefficients in the equation

$$\frac{df}{f}=q\frac{d\mathrm{A}}{\mathrm{A}}+r\frac{da}{a}+s\frac{dx}{x}$$

are subject to the relation $q+r+s=0$.

Lord Rayleigh pointed out that if the coils are placed at such a distance apart that the attraction is a maximum, $s=0$, and the calculation is independent of small errors in the value of x. In these circumstances $q+r=0$, so that proportional errors in A and a affect the result in the same degree and in opposite directions. This is a result of great importance, as the attraction becomes practically a function of the ratio A/a only. In this way all that is necessary for the absolute determination of currents can be obtained without measurements of length, or of moments of inertia, or even of absolute angles of deflection. To measure the ratio of the radii the method of Bosscha (*Pogg. Ann.*, 1854) can be used. This consists in placing the two coils concentric and with their planes in the magnetic meridian, the coils being connected in parallel and in series with suitable resistances. When a current is passed through this compound circuit the magnetic fields at the centre will be opposed; equality is obtained by adjusting one of the resistances until the torque exerted upon a small magnetic needle suspended at the centre of the coils is zero. For a very short magnetic needle the ratio of the galvanometer constants is then equal to the ratio of the two resistances. In determining this latter ratio the difficulty due to the heating of the coils by the measuring current is overcome by arranging the coils and resistances in such a way that, after balancing the magnetic fields, the removal of a single link suffices to place the resistances, of which the ratio is desired, into the adjacent arms of a Wheatstone bridge.

All current weighers have not been constructed on the Rayleigh system, but there is so much in common that the probable errors of all can be considered together.

In any current balance the coils must have a sufficient number of turns and the wire be of such cross-section as to permit of the passage of such a current as will produce a force large enough to be measured with precision. As the coils are warmed by the current the dimensions change slightly, and by whatever method the dimensions, or ratio of dimensions, are determined, the same value current should be used as in the weighing experiments. In the case of coils with many layers the heating may be very troublesome and will vary with the position of the coils; some uncertainty may therefore be introduced in the Bosscha method, because in this the coils must be vertical while in the balance they are horizontal. With a view to avoiding this difficulty Heydweiller in 1891 placed the coils with their common axis horizontal, the moving coil being carried directly below the centre of the balance beam in the position usually occupied by the pointer. The heating of the coils also produces convection currents of air inside the balance case, which in precision measurements may be very serious in their effects; duplicating the system on the other side of the balance arm is a partial remedy. Any attempt to increase the force by increasing the number of turns is limited not only by the heat developed in the coil but also by the increase of voltage required to overcome the resistance of the coils. When this voltage is moderately great it gives rise to serious electrostatic attractions due to differences of potential between various parts of the balance. Again, since the coils are of copper and the current must usually be maintained very steady, a ballast resistance of small temperature coefficient has to be employed. In the cases of current balances where single layer coils have been used, it is necessary to know the radii and lengths of all coils with very great accuracy, and in the Rayleigh balance corrections must be made for the cross-sections of the coils. In all current weighers care must be taken that no magnetic material is used which may invalidate the results, and the current in all leads to and from the coils must have a negligible or calculable effect.

(i.) *Measurements.*—In 1884 Lord Rayleigh (*Roy. Soc. Phil. Trans.*, 1884, part ii.) made measurements of the electro-chemical equivalent of silver using a current weigher to determine the mean value of the current. The coil system was a single one, *i.e.* on one side only of the balance, two equal fixed coils about 25 cm. in radius and one suspended coil about 10 cm. in radius being employed. The number of turns on each fixed coil was

225, and there were 242 turns on the suspended coil. The current used usually was about 0·3 ampere, and on reversal through the suspended coil the change of force was about one gram. The weighings were recorded to milligrams only, but the accuracy of weighing was considerably greater than one part in 1000. The current was led into the suspended coil by means of fine flexible copper wires, bent so as to place themselves naturally in the required positions before soldering them; these wires would carry a current of an ampere without getting unduly hot. Lord Rayleigh concluded that, apart from errors in the constant of the instrument, the determination of the mean value of a current of half an hour's duration should easily be correct within 1 part in 10,000.

Janet, Laporte, and Jouast (*Bull. de la Soc. Internat. des Électriciens*, 1908) used a current balance approximating to the Rayleigh type in experiments made at the Laboratoire Central d'Électricité. The system was a duplex one, two fixed coils and one suspended coil being arranged on each side of the balance. The distance between the fixed coils was greater than that which would give the maximum force upon the suspended coil, and hence the force varied less rapidly for small variations in the axial position of the latter. The exact placing of the suspended coil therefore was of less importance than in the Rayleigh system, but the distance between the fixed coils had to be determined with greater accuracy; an error of 0·01 mm. in this distance produced an error of 5 parts in 100,000 in the measurement of the current. The radii of the coils were determined from the length of wire wound on them, the mean radius of a fixed coil being about 36 cm. and that of a suspended coil being about 19 cm. Each of the fixed coils consisted of 414 turns of wire, and each of the suspended coils had 192 turns.

In their first calculation of the constant of the instrument Janet, Laporte, and Jouast assumed that the axial breadth of a coil was given by the total breadth of the channel in which it was wound, but that the radial depth was given by the distance from the axis of the wire in the bottom layer to the axis of the wire in the top layer. Later (*Comptes Rendus*, Oct. 1911, p. 718) Janet, Laporte, and Jouast recalculated the value of the constant of their coils, using the correct sectional dimensions. In their current measurements the total resistance of the circuit was 94 ohms, the current employed about 0·3 ampere, and the balancing mass about 4·5 grams.

Guillet (*Bull. de la Soc. Internat. des Électriciens*, 1908) also used a duplex current weigher with multi-layered coils, but the fixed coils were placed very close together, and the moving coils were but slightly smaller than the fixed coils. There were 960 turns on each of the fixed coils and 452 turns on each of the moving coils. The constant of the instrument was not determined from its dimensions, but the mutual inductance between the fixed and moving coils for various axial positions of the latter was determined by direct comparison with an absolute standard of mutual inductance of the Lippmann type. An equation enabling dM/dx to be calculated was thus obtained. This method has the advantage that the standard of mutual inductance may consist of coils of very few turns wound so as to enable their dimensions to be accurately determined, whereas the coils in the current balance may be wound irregularly conditionally that the dimensions keep reasonably constant. A disadvantage in practice appears to be the difficulty of determining dM/dx with sufficient accuracy owing to the rapid variation of M with x, and the practical impossibility of measuring x with the requisite accuracy.

Pellat and Potier (*Journ. de Physique*, 1890) in 1890 used an electrodynamometer with a very long fixed coil and a short moving coil, and in 1908 Pellat (*Bull. de la Soc. Internat. des Électriciens*, 1908) used an instrument similar in principle. The fixed coil consisted of a long multi-layered solenoid placed with its axis horizontal. Inside this a small single-layered coil is supported on a knife edge with its axis vertical and its centre coincident with that of the fixed coil; it is free to oscillate about an axis at right angles to the axis of the two coils. In addition to the small coil the knife edge supports the beam of a balance, and the torque between the two coils is measured by means of weights in the ordinary manner.

Ayrton, Mather, and Smith (*Roy. Soc. Phil. Trans.* A, 1908, ccvii.) made measurements with an elaborate current weigher designed to measure current in absolute measure with a probable error of a few parts in 100,000. The instrument was constructed and erected at the National Physical Laboratory.

The system is a duplex one, and both fixed and suspended coils are of single layers of bare wire wound in spiral grooves cut on marble bobbins. Each cylinder has double-threaded screw grooves cut on the surface, and into these grooves separate coils are wound with the wire under tension. The coils form adjacent helices, which in the use of the instrument are connected in series and act as one coil; they can, however, readily be disconnected and an insulation test made between them. This applies to each of the six coils forming the current weigher; there are, therefore, 12 helices in all, and these are

connected in series by means of small concentric cables running to a plug board and commutators outside the balance case. The mean diameter of the fixed coils is about 33 cm. and that of the suspended coils is about 20 cm., the respective axial lengths being 12·7 cm. and 13 cm. The number of turns on each fixed coil is 90 and on each suspended coil 92. The normal current is about 1 ampere, and the change of balancing mass on reversal of this current is about 16 grams.

The fixed cylinders are supported on phosphor bronze brackets, the tops of which are similar to the tables of small milling machines, and the motion of the fixed coils is controlled by screws of large diameter enabling their positions to be read within about 0·0025 mm. An adjustment in an axial direction can also be made. The fixed and suspended coils are set coaxial and coplanar by electrical methods which are exceedingly sensitive; the scheme is to arrange the circuits so that a suspended coil is subject to no restoring force when it is coplanar and coaxial: the rate of change of force for a small displacement is then a maximum. In this electrical method of setting the coils in their correct positions one-half of the system, *i.e.* a system on one side of the balance, is arranged to exercise no force, whatever be the relative position of its coils; the current flows through these coils, however, and compensates approximately for the disturbance due to convection currents of air and also for any effect due to the earth's magnetic field. Changes in position of the fixed coils on the other side of the balance are then made until they are coaxial and coplanar with the suspended coil. It is concluded that the vertical position can be fixed within less than 0·01 mm., and that the coils can be set concentric so that the greatest error due to faulty radial setting is 1 part in 5 millions.

The advantages of the duplex system were investigated experimentally. The results show that marked compensation for the effect of convection currents is obtained. In addition, the duplex system has the advantage that two independent determinations of current can be made by using the sets separately; also the setting of the coils in their correct positions is much facilitated.

Current was led into and out of the suspended coils by means of 80 silver wires, each 0·025 mm. in diameter and 10 cm. long. The sensitiveness of the balance was reduced 6 per cent by these wires, but there was no disturbing effects due to convection currents of air.

The force between the fixed and suspended coils was calculated by an elliptic integral formula due to J. Viriamu Jones (*B.A. Report*, 1889), which is rigorously exact for a helix and a current sheet and a very close approximation for two helices of fine pitch. With a duplex system the fixed coils of one system exert a force on the suspended coil of the other system, but by properly arranging the circuits these secondary forces can be added to or subtracted from the principal force. This was done and the secondary effects eliminated. Dr. G. F. C. Searle has calculated the value of this secondary effect with great accuracy.

Electrostatic effects of appreciable magnitude were measured and found to be due principally to the electrostatic forces between the suspended coils and metal guard discs underneath them. The effects were easily eliminated, and in any case would have produced no error, as in the observations they occurred twice with opposite signs.

The physical balance was specially constructed, the principal novelties being an arrangement for removing the beam from the central agate plane while leaving the suspended cylinders and scale pans swinging and a method of hanging of the cylinders from planes separate from those supporting the scale pans. Any motion of the scale pans had no effect, therefore, on the cylinders.

Considerable difficulty was experienced in obtaining all metal parts sufficiently free from magnetic material, and very exhaustive tests were made to ensure that the error due to magnetic material should be negligible. After completion a current was sent through one suspended coil only, and the rest point of the balance observed. The same test was made with the current reversed but no change in the rest point could be detected.

Ayrton, Mather, and Smith conclude that the determination of current by means of their current balance is subject to errors of the following magnitude:

(1) Due to uncertainty in the dimensions of the coils: possible error about $\pm 0{\cdot}001$ per cent.

(2) Due to uncertainty in the value of "g": possible error about $\pm 0{\cdot}0015$ per cent.

(3) Other possible errors less then $\pm 0{\cdot}001$ per cent.

The total probable error is therefore about ± 2 parts in 100,000.

Rosa, Dorsey, and Miller (*Bureau of Standards Bull.*, 1912, viii. No. 2) have made measurements at the Bureau of Standards, Washington, with a beautifully constructed and carefully designed current balance of the Rayleigh type. It has a single system of coils of large proportions, and to allow for independent sets of measurements to be made four suspended coils and six fixed coils were used. The suspended coils varied in radius from 10 to 12·5 cm. and the fixed coils from 20 to 25 cm. All the coils are on forms

of cast brass and are wound bifilarly with wire insulated with enamel, the channels of the forms being lined with paper soaked in paraffin wax. The fixed coils are cooled by a water-jacket, and the moving coil is surrounded by a cylindrical copper jacket, double walled on the sides, completely closed at the bottom and covered by a lid having a central aperture through which passes the tube from which the moving coil is suspended. The space between the two cylindrical walls is filled with circulating water, which carries away the heat generated by the current in the moving coil and enables a steady state to be quickly brought about. The coils are not contained in the balance case proper, but in a lower chamber, and no disturbing effects on the beam of the balance were experienced.

The ratio of the radii of the coils was measured in several ways. In one of these, called the potentiometer method, the ratio measured was that of the currents through the coils when the same field intensity was produced at the centres. In another measurement, called the shunt method, which is applicable only to coils having very nearly the same galvanometer constant, the two coils are connected in series, and the coil with the larger galvanometer constant, together with an added resistance, is shunted so as to obtain a zero field at the centre of the coils. One of the coils was so constructed that when its two windings were in series it had nearly the same galvanometer constant as another coil with its two windings in parallel. A third method, being a combination of the potentiometer and shunt methods, was also experimented with and satisfactory results obtained. The probable error of the measurements was about 2 parts in a million.

In computing the force between the coils it was shown that ample accuracy is secured by assuming that a fixed coil is equivalent to the eight turns obtained by quartering it and replacing each quarter by its two equivalent turns, and that the suspended coil is likewise equivalent to eight turns. The effect of errors in the sectional dimensions was most carefully considered, and it is concluded that the uncertainty due to errors in measuring the cross-section of the coils may amount to about 5 parts in a million.

When making current measurements, the windings of the suspended coil were directly connected to the water-jacket by means of a wire running from the commutator. Thus the water-jacket and all of the surrounding framework were brought to the potential of the moving coil; electrostatic effects were thus eliminated. The authors conclude that the possible uncertainty of the mean result obtained by them is not greater than 2 parts in 100,000.

(ii.) *Results with Current Weighers.*—The results are expressed either as the mass of silver deposited by one coulomb, or as the electromotive force of a standard cell (Clark or Weston normal cell), the value of the resistance used being assumed as known in C.G.S. units, or known in terms of some other standard. The standard employed is stated in columns 4 and 5 of the table.

Date.	Observer.	Electro-chemical Equivalent of Silver.	E.M.F. of Clark Cell.	E.M.F. of Weston Normal Cell.
		mgm.		
1882	Mascart	1·1156	..	..
1884	Lord Rayleigh	1·11794	1·435 at 15° C. (Ohm = 10^9 C.G.S.)	..
1890	Pellat and Potier	1·1192	..	..
1908	Ayrton, Mather, and Smith	1·11827	..	1·01818 at 20° C. (Int. Ohm)
1908	Janet, Laporte, and Jouast	1·11821	..	1·01836 at 20° C. (Int. Ohm)
1908	Guillet	..	..	1·01812 at 20° C. (Int. Ohm)
1908	Pellat	..	..	1·01831 at 20° C. (Int. Ohm)
1911	Rosa, Dorsey, and Miller	1·11804	..	1·01822 at 20° C. (Int. Ohm)

The differences between the results given in column 3 are no doubt largely due to the different forms of voltameters employed and to the use of impure solutions of silver nitrate.

The differences between the results given in column 5 may be due in part to a real difference in the cells measured.

§ (34) Smith's Pendulum Balance.—In 1912, F. E. Smith made some experiments with the current balance at the National Physical Laboratory with a view to measuring the mean value of a slightly varying current. The balance was used as a pendulum system, and in addition to the mutual inductance of the system of coils, the moment of inertia

of the swinging mass and the periods of swing have to be known.

In a current balance of suitable form let m be the total mass of the swinging system, K its moment of inertia about the central knife edge, and h_1 the distance of the centre of gravity below the central knife edge. Then, if b is the counter torque at unit angular velocity due to the damping forces, and T_1 the period, we have

$$T_1=\frac{2\pi}{\sqrt{\frac{mgh_1}{K}-\frac{b^2}{4K^2}}}.$$

Let M be the mutual inductance between the coils, and suppose the latter to be such that for relatively large displacements dx of a moving coil, the force tending to restore the coil to the central position is $i^2(dM/dx)$, where i, the current, circulates through all the coils in the same direction. Under these conditions the new period T_2 is given by

$$T_2=\frac{2\pi}{\sqrt{\frac{mgh_1+i^2l\frac{dM}{dx}}{K}-\frac{b^2}{4K^2}}},$$

where l is the length of an arm of the balance.

From the equations for T_1 and T_2 we have

$$i^2=\frac{4\pi^2}{T_2^2}\cdot\frac{K}{l\frac{dM}{dx}}\left(1-\frac{T_2^2}{T_1^2}\right).$$

In this equation dM/dx is calculated from the dimensions of the coils, T_1 and T_2 are observed, and l is easily measured with great accuracy. It remains to find K the moment of inertia. This would be exceedingly easy if the beam did not bend with varying load and so alter the position of the centre of gravity. However, as this does happen it is better to find K indirectly by finding the change of position of the centre of gravity when there is a redistribution of mass without any change of K.

Let there be two small masses m_2 and m_3, the one being vertically above the central knife edge and the other directly below it. Let m_2 be much smaller than m_3, and let both be provided with such markings that the distance between them can be readily measured. Let m_2 be centrally situated at a distance l_2 above the knife edge and m_2 at a corresponding distance l_3 below it. l_2 should be equal, or nearly equal, to l_3. Let the remainder of the swinging system have a mass m_4, so that

$$m=m_2+m_3+m_4.$$

If the centre of mass of m_4 be at a distance l_4 below the central knife edge, then

$$m_4l_4+m_3l_3-m_2l_2=mh_1.$$

Let m_2 and m_3 be now interchanged in position, thus altering the period of the balance (without current) to T_3. Then

$$T_3=\frac{2\pi}{\sqrt{\frac{mgh_2}{K}-\frac{b^2}{4K^2}}}$$

and $$m_4l_4+m_2l_3-m_3l_2=mh_2.$$

Hence $$(m_3-m_2)(l_3+l_2)=m(h_1-h_2).$$

Eliminating $b^2/4K^2$ from the equations for T_1 and T_3 we have

$$K=\frac{T_1^2}{4\pi^2}\cdot\frac{mg(h_1-h_2)}{\left(1-\frac{T_1^2}{T_3^2}\right)}.$$

Hence $$K=\frac{T_1^2}{4\pi^2}\cdot\frac{g(m_3-m_2)(l_3+l_2)}{\left(1-\frac{T_1^2}{T_3^2}\right)}.$$

(m_3-m_2) can be determined with great precision, and if m_3 and m_2 are supported from pivots, the distance between the effective centres of mass can also be accurately measured.

Preliminary measurements of current have been made with satisfactory results, and it has been planned to use the current balance at the National Physical Laboratory both as a "weigher" for steady currents and as a pendulum balance for slightly varying currents.

§ (35) Summary of Results.—The table on following page summarises the results of absolute measurements of current made since 1872.

§ (36) Electro-chemical Equivalent of Silver.—In the section[1] dealing with the International ampere many sources of error in the silver voltameter are explained. In many of the very early voltameter experiments there is little doubt that impure silver nitrate solutions were used, and these led to more silver being deposited than would have resulted from pure solutions. Filter paper as a septum between anode and cathode was quite common, and the reducing effect of this resulted in abnormal deposits being obtained from otherwise pure solutions. The presence of acid causes a reduction in the weight of silver, but on the whole there is little doubt that previous to 1910 most of the values given for the electro-chemical equivalent of silver are too large because of the presence of impurities in the electrolyte. With very pure silver nitrate solutions, and using no septum between the anode and cathode, the true value for the electro-chemical equivalent of silver is obtained. A close review of recent determinations leaves little doubt that within 1 part in 10,000 the electro-chemical equivalent of silver is

1·1181 milligrams per coulomb.

[1] See § (40).

RESULTS OF ABSOLUTE MEASUREMENTS OF CURRENT

Date.	Observer.	Results given by Author. Electro-chemical Equivalent of Silver in mgm.	E.M.F. of Clark Cell.	E.M.F. of Weston Normal Cell at 20°·0 C.
	1. METHOD OF TANGENT GALVANOMETER			
1873	F. Kohlrausch	1·1363	..	..
1886	F. and W. Kohlrausch	1·11833	..	..
1904	Van Dijk and Kunst	1·1180	..	..
1910	Haga and Boerema	1·11802	..	1·01826 *
	2. METHOD OF SINE GALVANOMETER			
1872	Latimer Clark	..	1·4562 † B.A. Volts at 15°·5 C.	..
1886	T. Gray	1·118	..	..
	3. ELECTRODYNAMOMETER METHODS			
1872	Latimer Clark	..	1·4573 † B.A. Volts at 15°·5 C.	..
1898	Patterson and Guthe	1·1192	..	..
1899	Carhart and Guthe	..	1·4333 * at 15°·0 C.	..
1906	Guthe	1·11773	1·43296 * at 15°·0 C.	1·01853 *
1913	Shaw	..	..	1·01827 *
	4. CURRENT WEIGHER METHODS			
1882	Mascart	1·1156	..	..
1884	Rayleigh	1·11794	1·435 at 15°·0 C.	..
1890	Pellat and Potier	1·1192	..	..
1908	Ayrton, Mather, and Smith	1·11827	..	1·01818 *
1908	Janet, Laporte, and Jouast	1·11821	..	1·01836 *
1908	Guillet	..	..	1·01812 *
1908	Pellat	..	..	1·01831 *
1910	Smith	1·11815	..	1·01818 *
1911	Rosa, Dorsey, and Miller	1·11804	..	1·01822 *
1914	Shaw	..	..	1·01831 *

* The E.M.F. is expressed in terms of the ampere (10^{-1} C.G.S.) and the international ohm.

† The E.M.F. is expressed in terms of the ampere (10^{-1} C.G.S.) and the B.A. unit.

Various forms of voltameters were used and the numbers given for the electro-chemical equivalent do not enable a good comparison to be made (see § (40) on "International Ampere").

§ (37) E.M.F. OF WESTON CELL.—In recent years the electromotive force of the Weston normal cell at 20° C. in terms of the ampere (10^{-1} C.G.S. unit) and the international ohm has been determined by eight observers, the results being as follows:[1]

Year.	Observers.	E.M.F. of Weston Normal Cell at 20° C.
1908.	Ayrton, Mather, and Smith	1·01818
1908.	Janet, Laporte, and Jouast	1·01836
1908.	Guillet	1·01812
1908.	Pellat	1·01831
1910.	Smith	1·01818
1910.	Haga and Boerema . .	1·01826
1911.	Rosa, Dorsey, and Miller .	1·01822
1913.	Shaw	1·01827
	Mean .	1·01824

The most probable value of the international ohm is $1{\cdot}0005_2$ ohms (10^9 C.G.S. units), so that the most probable value of the Weston normal cell in C.G.S. measure is $1{\cdot}0182_4 \times 1{\cdot}0005_2$ or

1·0188 volts (10^8 C.G.S. units) at 20°·0 C.

[1] For a further discussion of the latest values see § (41).

III. INTERNATIONAL ELECTRIC STANDARDS

§ (38) INTRODUCTION.—It is generally agreed that the fundamental electric units should be defined in the C.G.S. system, and the usual definitions of the ohm, ampere, and volt are universally accepted. Very few countries have, however, made measurements of the ohm and ampere in absolute measure, and it is only in recent years that such measurements have been made with comparatively great accuracy. While, therefore, the world possesses ideal definitions for the electric units, material standards do not necessarily closely represent these units.

In the section on the absolute measurement of resistance it will be seen that the first attempt of the British Association Committee in 1863 to measure a resistance in C.G.S.

units was in error by about 1½ per cent. The resistance actually measured was that of a coil of wire, and this resistance was compared with those of other coils which afterwards served as reference standards. In particular, one coil of German silver wire known as "June 4" was compared with the resistance of the rotating coil, and its value was given by the observers as 107,620,116 metres per second. This coil afterwards served as a "standard." When subsequently other coils were compared with this and other "standards" the observed values, instead of being stated in metres per second, were stated in B.A. units. While therefore the magnitude of a standard was determined on the electromagnetic system of measurement a new unit was at the same time adopted which probably was not identical with, but for all practical purposes was sufficiently equal to, the C.G.S. unit.

The ampere cannot be represented in a concrete or material form in the sense in which the ohm can, but having once measured within certain limits the electro-chemical equivalent of silver it is the silver voltameter which afterwards forms the basis of reference, and therefore acts as a "standard."

Now it has always been possible to compare electric quantities with greater accuracy than to determine their values in absolute measure. If, therefore, for either purely scientific or commercial purposes the value of an electric quantity must be known in terms of some definite unit with a greater precision than is possible if that unit is one in the C.G.S. system, the obvious course is to measure the value of some concrete material in C.G.S. units and to assume the measured value to be correct within the desired limits. In practice this has been done, and the "unit" and "standard" have thus become distinct. Fortunately, with progress of time new standards have been set up and endeavours made to reduce the difference between the assumed and real values. To prevent confusion, qualifying names have been given to the standards, such as "legal ohm," "Siemens' unit," "international ohm," etc.

The history of the efforts made to bring about international uniformity in electric standards is a very long one, and for full information reference should be made to the Reports of the Electrical Standards Committee of the British Association. The last Conference to consider the subject met in London in 1908, and delegates were present from Australia, Canada, India, the Crown Colonies, and twenty-two foreign countries. The conference was presided over by Lord Rayleigh. As a result of its deliberation the Conference passed the following resolutions:

I. The Conference agrees that, as heretofore, *the magnitudes of the fundamental electric units shall be determined on the electromagnetic system of measurement with reference to the centimetre as the unit of length, the gramme as the unit of mass, and the second as the unit of time.*

These fundamental units are (1) the ohm, the unit of electric resistance which has the value of 1,000,000,000 in terms of the centimetre and second; (2) the ampere, the unit of electric current which has the value of one-tenth (0·1) in terms of the centimetre, gramme, and the second; (3) the volt, the unit of electromotive force which has the value 100,000,000 in terms of the centimetre, the gramme, and the second; (4) the watt, the unit of power which has the value 10,000,000 in terms of the centimetre, the gramme, and the second.

II. *As a system of units representing the above, and sufficiently near to them to be adopted for the purpose of electrical measurements and as a basis for legislation, the Conference recommends the adoption of the international ohm, the international ampere, and the international volt defined according to the following definitions:*

III. The ohm is the first primary unit.

IV. The international ohm is defined as the resistance of a specified column of mercury.

V. The international ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of a constant cross-sectional area and of a length of 106·300 centimetres.

To determine the resistance of a column of mercury in terms of the international ohm, the procedure to be followed shall be that set out in Specification I. attached to these resolutions.

VI. The ampere is the second primary unit.

VII. The international ampere is the unvarying electric current which, when passed through a solution of nitrate of silver in water, in accordance with Specification II. attached to these resolutions, deposits silver at the rate of 0·00111800 of a gramme per second.

VIII. The international volt is the electric pressure which, when steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere.

IX. The international watt is the energy expended per second by an unvarying electric current of one international ampere under an electric pressure of one international volt.

It will be seen that a distinction is drawn between the fundamental C.G.S. units and the international units which are regarded as representing them with a sufficient degree of accuracy. One of the important questions discussed at this Conference was: Shall the ohm and the ampere, or the ohm and the volt be defined for practical purposes by material standards? The international ohm as defined

by the resistance of a specific column of mercury under standard conditions is acceptable to all. The international ampere cannot be represented in a material form in the same sense in which the ohm can, but the international volt might be specified in terms of the electromotive force of a standard cell. The accuracy and ease of reproduction of the voltameter was thought to be not well established, and it was urged that the standard cell had many advantages. The difficulties of both are dealt with in sections which follow, but the Conference resolved that the ampere should be the second primary unit.

§ (39) THE INTERNATIONAL OHM.—More than sixty years ago Dr. W. Siemens (*Poggendorff's Annalen*, cx. 1) proposed to adopt as a unit of resistance that of a column of mercury at 0° C. a metre long and one square millimetre in cross-section, and in a letter to the British Association Committee on Electric Standards (1862) he described the method by which he had constructed such standards. The measurements involved were those of length and mass, but a knowledge of the density of mercury was required in order to determine the cross-section. In 1892 Dr. von Helmholtz pointed out that any difficulty due to uncertainty of the density of mercury could be avoided by defining the mass of the mercury column of a given length which has unit resistance. The length recommended by the B.A. Electrical Standards Committee in 1892 was 106·3 cm. and 14·4521 grammes was chosen as the mass. The London Conference in 1908 modified this definition; the correction to be applied for the ends was given, and the length was stated to six significant figures, viz. 106·300 cm. The more essential parts of the 1908 specification are as follows:

"*The international ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice,* 14·4521 *grammes in mass, of a constant cross-sectional area and of a length of* 106·300 *centimetres.* The resistance which has to be added to the resistance of the tube to allow for the effect of the end vessels is to be calculated by the formula

$$A=\frac{0\cdot80}{1063\pi}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\text{ ohm},$$

where r_1 and r_2 are the radii in millimetres of the end sections of the bore of the tube.

"For the purposes of the electrical measurements the end vessels are to carry connections for the current and potential terminals. These end vessels are to be spherical in shape and of a diameter of approximately 4 centimetres.

"The mean of the calculated resistances of at least 5 tubes shall be taken to determine the value of the international unit of resistance."

If L_0 be the length, and s_0 the uniform cross-section of a column of mercury at 0° C. its resistance R_0 is given by the equation

$$R_0=\rho_0 L_0/s_0,$$

where ρ_0 is the resistivity of mercury.

By definition of the international ohm (London Conference, 1908), when $L_0=106\cdot300$ cm. and the mass of mercury is 14·4521 grammes, the resistance R_0 is unity; hence the resistance of any uniform column of mercury is given by

$$R_0=\frac{14\cdot4521}{(106\cdot300)^2}\cdot\frac{L_0^2}{M_0^2},$$

where M_0 is the mass of the column. To realise the international ohm, therefore, any length of column can be taken, and it is sufficient to measure the length and mass provided the column is of uniform cross-section and the end corrections are known.

The simplicity of the above formula is disturbed by the inevitable departure of the mercury column from the truly cylindrical form. In a well-chosen glass tube the column is conical, preferably uniformly so, but in general it may be regarded as a series of truncated cones of various lengths. As a result a correction, termed the conical correction, or calibre factor, has to be applied.

The true resistance R_0' of a column of variable cross-section is given by the expression

$$R_0'=\rho_0\int_0^{L_0}\frac{dl}{s}.$$

If Δ_0 is the density of mercury at 0° C. the mass M_0 is given by

$$M_0=\Delta_0\int_0^{L_0}s\,dl,$$

and we may write

$$R_0'=\rho_0\frac{\Delta_0}{M_0}\int_0^{L_0}s\,dl\int_0^{L_0}\frac{dl}{s}.$$

The resistance R_0 calculated on the assumption that the cross-section is uniform is $\rho_0 L_0/s_0$, or since $M_0=\Delta_0 L_0 s_0$ we have $R_0=\rho_0\Delta_0 L_0^2/M_0$, and therefore

$$\frac{R_0'}{R_0}=\frac{1}{L_0^2}\int_0^{L_0}s\,dl\int_0^{L_0}\frac{dl}{s}.$$

The ratio R_0'/R_0 is slightly greater than unity and is called the calibration factor.

(i.) *Calibration.*—It is clearly impossible to determine the cross-section of a tube from point to point, and therefore mean cross-sections of finite lengths must be taken. The methods adopted for the calibration of tubes intended for mercury standards of resistance vary considerably. If the tubes vary in cross-section by more than about 4 per cent a very complete and often laborious calibration is necessary, but simpler calibrations suffice for more uniform tubes.

In a simple calibration the length of a short mercury thread approximately l cm. long is measured when placed in n sections of the tube, the length of the tube being nl. The lengths are corrected for the menisci, and the cross-sections are proportional approximately to the reciprocals of the lengths. In a complete calibration $(n-1)$ mercury threads are employed; these have lengths L/n, $2L/n$, $3L/n$ . . . $(n-1)L/n$ respectively, so that when n is large the work is very long and tedious. Details of a complete calibration are given by Benoît (*Construction des étalons prototypes de resistance électrique*, Paris, Gauthier-Villars, 1885). A very simple yet accurate method of calibrating tubes has been used by Smith, who divides the tube first into two, three, or more equal lengths and introduces approximately equal lengths of mercury to occupy these sections. These mercury threads are weighed and the relative mean cross-sections are proportional to the masses. The number of such separate lengths into which the tube is divided is governed by the condition that none of these lengths shall vary in cross-section by more than 1 per cent. In a well-chosen tube the number of such separate sections is rarely more than four. Each section is then divided into centimetre lengths, and the length of a mercury thread approximately 1 cm. long is measured in each division; the menisci are ignored. Percentage differences from the mean length are then tabulated, and the calibre factor obtained from tables previously prepared. By this method a tube can be calibrated and its calibre factor calculated within about four hours.

In recent years tubes have been chosen with very great care. In some cases the uniformity of section is so good that the calibre factors are as small as 1·00002. Values of the factor for mercury standards recently constructed in Great Britain, Japan, and America are given in the following table:

Great Britain.		Japan.		United States.	
No. of Tube.	Calibre Factor.	No. of Tube.	Calibre Factor.	No. of Tube.	Calibre Factor.
2	$1{\cdot}00010_1$	1	1·000021	1	1·000089
6	$1{\cdot}00007_7$	3	1·000051	2	1·000066
9	$1{\cdot}00011_6$	4	1·000052	3	1·000047
11J	$1{\cdot}00018_1$	5	1·000079	4	1·000096
27	$1{\cdot}00007_3$	6	1·000093		
S	$1{\cdot}00002_6$				
G	$1{\cdot}00012_8$				
1	$1{\cdot}00002_9$				
130	$1{\cdot}00007_3$				
137	$1{\cdot}00000_8$				
5	$1{\cdot}00007_1$				
11V	$1{\cdot}00007_0$				
12	$1{\cdot}00010_0$				
13	$1{\cdot}00013_3$				

An uncertainty arises from lack of knowledge of the condition of the axis of the tube. The specification assumes the axis to be straight, but since the cross-section varies it is probable that the axis also departs from the straight line. Thus if the axis of a tube is of an undulatory character, such that the curvature is everywhere equal to 1/40 cm. and points of inflexion occur on the assumed axis at equal distances of 0·7 cm., the resistance of a mercury column filling the tube will be greater by 0·0036 per cent than that calculated on the assumption of an absolutely straight axis. The true axis, if drawn on paper, could not be distinguished by the unaided eye from a straight line.

(ii.) *End Correction.*—A correction which does not admit of absolutely definite solution is that required to take account of the resistance offered by that part of the mercury in the terminal cups. The problem is practically identical with that of the correction necessary in calculations of pitch for the open ends of organ pipes (see Rayleigh's *Theory of Sound*, § 307). Lord Rayleigh calculated that the maximum correction would be equivalent to adding to the actual length of the tube 0·82 of the mean diameter of the ends; this supposes that the diameter of the mercury column suddenly becomes infinite. It has been found by experiment for terminal bulbs a few cm. in diameter that the correction is equivalent to adding to the length of the tube 0·795 of the mean diameter of the ends. For specified terminal vessels the London Conference chose as the correction 0·80 of the mean terminal diameters.

(iii.) *Measurement of Length.*—The length of a tube is comparatively easy to measure conditionally that the ends are either ground or polished. In recent years the end planes of such tubes have been polished plane within a wave-length of light, and this enables good measurements to be made by direct sighting on the ends, or by contact methods.

(iv.) *Measurement of Mass.*—Three methods have been used to measure the mass of mercury filling the tube. In the first of these a mass of mercury in the form of a continuous column is introduced into the tube so as to approximately occupy the whole length chosen for standard purposes. Its length and the menisci are measured, and these, together with the mass of the column and the calibration data, enable the mass required to fill the tube to be calculated.

The second method consists in filling the tube with mercury, closing one end by a plane surface and removing the excess mercury at the other end. The plane surface is clamped to the tube by means of a steel screw with an intermediate ball-and-socket joint. After exhaustion of the tube sufficient mercury is

admitted to completely fill it, and after closing the lower end it is removed to an ice bath, suitable precautions being taken to prevent the introduction of moisture. The excess of mercury at the upper end is removed by the motion of a plane glass plate in a gimbal mounting. This method was perfected at the Reichsanstalt and has also been used in the United States and Japan.

The third method devised by Smith (*Researches of the National Physical Laboratory,* 1909, v.) consists in rendering the meniscus correction negligible and at the same time reducing the influence of errors in the measurement of thread length by means of capillary extensions coupled to the ends of the tube. Measurements can be rapidly made with the tube immersed in ice, but it is desirable that the tubes have thick walls in order that good connections can be made.

No single institution has made measurements on the same tube by all of these methods, but the Bureau of Standards have made measurements on one tube by the Reichsanstalt method and by that proposed at the National Physical Laboratory by Smith. The means of six measurements by each method agreed within 7 parts in a million, the probable error by the Reichsanstalt method being 2 parts in a million, and that by Smith's method being about 4 parts in a million.

(v.) *End Vessels.*—To compare the resistance of the mercury column with wire coils the spherical terminal vessels must be so fitted to the tube that there is no fear of contamination of the mercury. In addition the latter should be at 0° C. The glass tube and vessels containing mercury should therefore be surrounded by melting ice, and the terminal vessels and junctions should be such that there is no danger of leakage of water into the tube. Also it must be possible to measure the insulation resistance between the mercury column and the mixture of ice and water surrounding it. These requirements are satisfied when the terminal vessels and connections are of the form shown in *Fig.* 13. This form of terminal vessel is used at the National Physical Laboratory and modifications of it are in use in other countries.

A is one end of the standard tube and B is one end of the auxiliary tube, which serves, when partly filled with mercury, as a current lead. C is a second auxiliary tube with a platinum wire sealed in its lower end: C serves as a potential lead. The ends of the three tubes are ground slightly conical and are a good fit into the ground necks of the terminal vessel. The shaded pieces represent rubber bungs which are first slid over the glass tubes, and after these latter have been adjusted in position the bungs are pressed into the enlarged portions of the necks of the vessel. The external surfaces of the bungs

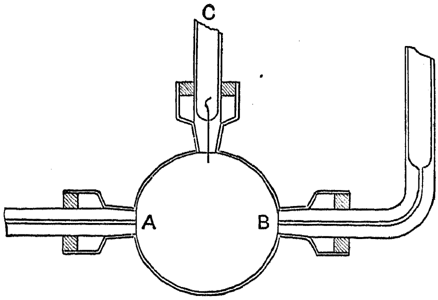


Fig. 13.

may, if necessary, be coated with paraffin wax.

(vi.) *Erection.*—With such end vessels there is no fear of contamination of the mercury by the ingredients of the rubber, and there is no danger of water leaking into the tube. The filling of the tube with mercury, which must be carefully purified, may be made under atmospheric pressure, or the tube may be first exhausted of air; the pressure should be equal to that holding when the mass was determined of the mercury filling the tube. As a change of temperature of mercury of 0°·01 C. is accompanied by a change of resistance of 8 parts in a million, the cross-sections of the current and potential leads must be sufficiently small to prevent the flow of heat through them raising the mean temperature of the mercury column by more than 0°·001 C. A limiting value is likewise placed on the testing current unless a correction for its heating effect is made. In the latter case two different currents, preferably in the ratio of 1 to 2, are used; as the increase of resistance is proportional to the square of the current used, it follows that for a current of negligibly small intensity the resistance is that of the smaller value measured less one-third of the difference between the two measured values.

(vii.) *Measurement of Resistance.*—The comparison of wire resistance coils with mercury standards of resistance presents some difficulties which do not arise in general resistance comparisons. The difficulties are due to the comparatively great resistance of the current leads and the considerable thermoelectric effects in the circuit.

Lord Rayleigh, Glazebrook, and Smith made measurements by the Carey Foster bridge, using special terminal vessels which allowed of high conductivity current leads being employed. The principal difficulty was due to the heat conducted through the current leads, and because of this a correction had to

be made. At the Reichsanstalt the Kohlrausch differential galvanometer method has been used, and modifications of the Kelvin double bridge have been employed at the National Physical Laboratory and the Bureau of Standards. A simple modification of the Wheatstone bridge has also been used at the National Physical Laboratory and at Tokyo. As the modifications of the Kelvin double bridge are not in general use for other resistance comparisons, one form is described here.

The leads of the mercury standard are indicated by L_1, L_2, L_3, and L_4, and in the bridge

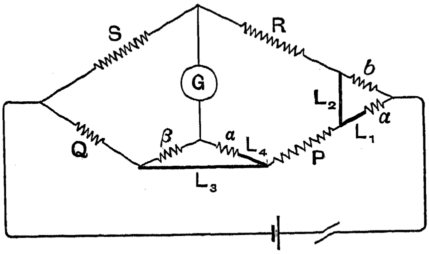


FIG. 14.

shown in *Fig.* 14 L_2 is shunted by $(b+a+L_1)$ and L_3 is shunted by $(\beta+\alpha+L_4)$.

When the connections are as shown in the figure the condition for balance is

$$\frac{P+\dfrac{(\alpha+L_4)L_3}{(\alpha+\beta+L_3+L_4)}+\dfrac{(a+L_1)L_2}{(a+b+L_1+L_2)}}{Q+\dfrac{\beta L_3}{\alpha+\beta+L_3+L_4}}=\frac{R+\dfrac{bL_2}{a+b+L_1+L_2}}{S}, \quad (1)$$

and the expression for P is

$$P=\frac{QR}{S}+\frac{\beta L_3}{\alpha+\beta+L_3+L_4}\left(\frac{R+\dfrac{bL_2}{a+b+L_1+L_2}}{S}-\frac{\alpha+L_4}{\beta}\right)+\frac{bL_2}{a+b+L_1+L_2}\left(\frac{Q}{S}-\frac{a+L_1}{b}\right). \quad . \quad . \quad . \quad (2)$$

In practice the second and third terms on the right-hand side of this equation are made negligibly small, and then

$$P=QR/S. \quad . \quad . \quad . \quad (3)$$

To do this the resistance coils are so chosen that approximately $R/S=\alpha/\beta$ and $Q/S=a/b$. In the general case the bridge is adjusted before commencing comparisons of the mercury standard with wire coils. In this adjustment α is disconnected from L_4, and by adjustment of Q and a a balance is obtained whether L_2 is connected to $R{\cdot}b$ or not. The following two equations then hold good:

$$\frac{P+L_3+a+L_1}{Q}=\frac{R+b}{S} \quad . \quad . \quad (4)$$

and

$$\frac{P+L_3+(a+L_1)\left(\dfrac{L_2}{a+b+L_1+L_2}\right)}{Q}=\frac{R+b\dfrac{L_2}{a+b+L_1+L_2}}{S}. \quad (5)$$

From these we have

$$\frac{a+L_1}{b}=\frac{Q}{S}. \quad . \quad . \quad . \quad (6)$$

Afterwards, by adjustments of Q and a and of α or β, a balance is obtained which holds good whether L_3 is connected to $Q{\cdot}\beta$ or not. In these circumstances

$$\frac{P+\alpha+L_4+\dfrac{(a+L_1)L_2}{a+b+L_1+L_2}}{Q+\beta}=\frac{R+\dfrac{bL_2}{a+b+L_1+L_2}}{S}, \quad (7)$$

and the relation (1) also holds good. From (1) and (7) we have

$$\frac{\alpha+L_4}{\beta}=\frac{R+\dfrac{bL_2}{a+b+L_1+L_2}}{S}, \quad . \quad (8)$$

and from (1), (6), and (8) it follows that

$$P=QR/S. \quad . \quad . \quad . \quad (9)$$

One of the coils Q, R, S is shunted in order that this equation may hold good.

The value of the combination having been obtained, a 1-ohm wire coil is substituted for P and the bridge rebalanced by altering the shunt on either Q, R, or S. From the change in the shunt the value of the wire coil is calculated in terms of the mercury standard.

It is convenient to choose as nominal values R=5 ohms, S=500 ohms, $\alpha=9{\cdot}85$ ohms, $\beta=1000$ ohms, and $b=500$ ohms. Q and a are equal variable resistances and are changed together. In such a case, if the resistance of each of the leads of the mercury standard be not greater than 0·15 ohm, and if the ratio R/S is equal to $(\alpha+L_4)/\beta$ within 4 parts in 1000, and the ratio Q/S is equal to a/b within 1 part in 10,000, the greatest error introduced by neglecting the preliminary adjustments cannot exceed 4 parts in a million.

Other modifications of the Kelvin double bridge for the comparison of mercury standards with wire coils are described by Smith (*Roy. Soc. Phil. Trans.*, 1904, p. 85) and by Wolff (*Bureau of Standards Bull.*, 1915, No. 256). A simple modification of the Wheatstone bridge is also described by Smith (*British Association Report*, 1906).

(viii.) *Measurements.*—Mercury standards of

resistance were first set up by W. Siemens in 1860, who was followed by Matthiessen and Hockin in 1863 (*B.A. Report*, 1863). These two authorities differed considerably, and Lord Rayleigh in 1881 (*Phil. Trans.*, 1883, Part I.) prepared four tubes for the purpose of determining the resistivity of mercury in absolute measure. The maximum difference between the mean value and those obtained from individual tubes was about 8 parts in 100,000. Carefully prepared standards were set up by Benoît in 1884 (*Construction des étalons prototypes de résistance électrique*, Paris, Gauthier-Villars, 1885), and a determination of the change of resistance with temperature was made by Guillaume (*Bureau International des Poids et Mesures*, 1892). In 1888 Glazebrook (*Phil. Trans.* A, 1888) erected six tubes and found that a column of mercury 1 metre long and 1 sq. mm. in cross-section had a resistance at 0° C. equal to 0·95352 B.A. unit. The largest calibre factor was 1·00079 and the smallest 1·00002. In more recent years most of the National Standardising Laboratories have set up mercury standards of resistance and the results obtained are in remarkable agreement. The principal references are Jaeger and Kahle (*Wiss. Abhandlungen Phys. Techn. Reichsanstalt*, 1900, iii.), Smith (*Roy. Soc. Phil. Trans.* A, 1904), Wolff, Shoemaker, and Briggs (*Bureau of Standards Bull.*, 1915), Obata (*Report of Electrotechnical Laboratory*, Tokyo, 1914), and *Report of the Bureau of Weights and Measures*, St. Petersburg, 1911.

The reproducibility of the international ohm appears to be very satisfactory, and mercury standards set up during the past forty years form our most satisfactory links with past standardising work. This is well illustrated in the B.A. Electrical Standards Committee Report for 1906, in which it is shown that if the mean value of six standard wire coils is taken as remaining constant from 1881 to 1908 two of the coils (of platinum wire) decreased in resistance by 63 parts in 100,000. These actual coils were, however, compared by Lord Rayleigh in 1881, by Glazebrook in 1888, and by Smith in 1908 with mercury standards newly constructed in these respective years, and the resistance values of the two platinum coils as given by these observers differ by only 3 parts in 100,000. The obvious conclusion is that the platinum coils remained reasonably constant in resistance and that the mercury standard of resistance is reproducible with a high degree of accuracy. The latter conclusion is also borne out by recent comparisons of mercury standards between Great Britain, the United States, Japan, Russia, and Germany. Taking the mean value as correct, the differences between it and the mean values of the standards of the various countries are as follows:

	Mean Mercury Unit minus that of the respective Countries.
Great Britain	0 parts in a million
United States	−10 „ „
Japan	− 4 „ „
Russia	− 5 „ „
Germany	+18 „ „

Measurements of the change of resistance of mercury with change of temperature have been made by Guillaume (*Bureau International des Poids et Mesures*, 1892), by Kreichgauer and Jaeger (*Wiedemann's Annalen*, 1892, xlvii.), and by Smith (*Roy. Soc. Phil. Trans.* A, 1904). According to these measurements the resistances of a constant volume of mercury at 0° C., 10° C., and 20° C. are in the following proportions:

Observers.	Resistance at		
	0° C.	10° C.	20° C.
Guillaume	1·000000	1·008987	1·018167
Kreichgauer and Jaeger	1·000000	1·008953	1·018158
Smith	1·000000	1·008984	1·018175

(ix.) *Relation between the International Ohm and the Ohm* (10^9 *C.G.S. Units*).—Absolute measurements of resistance show that the most probable value of the international ohm is

$$\mathbf{1{\cdot}0005_2}\ \text{ohm}\ (10^9\ \text{C.G.S. units}).$$

§ (40) The International Ampere.—By resolution of the London Conference on Units and Standards (1908):

"*The international ampere is the unvarying electric current which, when passed through a solution of nitrate of silver in water, in accordance with Specification II., deposits silver at the rate of* 0·00111800 *of a gramme per second.*"

Specification II. reads:

"The electrolyte shall consist of a solution of from 15 to 20 parts by weight of silver nitrate in 100 parts of distilled water. The solution must only be used once, and only for so long that not more than 30 per cent of the silver in the solution is deposited.

"The anode shall be of silver, and the cathode of platinum. The current density at the anode shall not exceed 1/5 ampere per square centimetre and at the cathode 1/50 ampere per square centimetre.

"Not less than 100 cubic centimetres of electrolyte shall be used in the voltameter.

"Care must be taken that no particles which may become mechanically detached from the anode shall reach the cathode.

"Before weighing, any traces of the solution adhering to the cathode must be removed, and the cathode dried."

This specification was not regarded as more than a provisional one. It was hoped that further experiments would be carried out and a more complete and more satisfactory specification drawn up. The conditions imposed by the specification were, however, believed to be sufficiently stringent to ensure reproducibility. Immediately previous to the Conference, researches at the National Physical Laboratory, the Reichsanstalt, and by Professor Kohlrausch indicated that the electrolyte does not alter appreciably because of electrolysis, but the researches of most previous investigators indicated that such a change does result. The specification makes no reference to temperature or pressure. While some observers have obtained results varying with temperature and with pressure, researches at the National Physical Laboratory indicated that the temperature and pressure coefficients are either zero or negligibly small. The silver nitrate, silver, and water are not specified as "pure," but it is to be concluded that all materials should be as pure as it is possible to obtain them.

A very large number of papers dealing with the silver voltameter have been published since Mascart (*Journ. de Physique*, 1882, Series II.) made the first absolute measurement of the electro-chemical equivalent of silver. In 1884 Carhart (*Am. J. Sci.*, 1884, p. 374) used the silver voltameter to measure the E.M.F. of a Daniell cell, and in the same year Lord Rayleigh and Mrs. Sidgwick (*Roy. Soc. Phil. Trans.* A, 1884) made a systematic study of the instrument. They employed large platinum bowls for the cathodes and silver sheet enclosed in filter-paper for the anodes. This type became known as the "Rayleigh form" of voltameter.

In 1886 F. and W. Kohlrausch (*Wied. Ann.*, 1886, xxvii.) published some results obtained with a form of voltameter in which a small platinum cup formed the cathode, a silver rod the anode, and underneath the latter a small glass dish was supported. This form is known as the "Kohlrausch form." They also used a siphon-form voltameter.

In 1899 Richards, Collins, and Heimrod (*Amer. Acad. Proc.*, 1899 and 1902) suspended a porous cup of porcelain under the anode, thus abolishing the use of any organic material in the electrolyte and at the same time providing a more efficient septum. This has been known as the Richards, or porous cup form.

The Rayleigh, Kohlrausch, and Richards voltameters together with the syphon form were in use up to the time of the London Conference in 1908. At times the forms were slightly modified, as by the use of silk as a septum in the Kohlrausch type.

(i.) *Principal Measurements up to* 1908.—In 1884 Lord Rayleigh and Mrs. Sidgwick, wishing to make the deposit adhere better to the platinum bowl, added to the silver nitrate solution a very small proportion of acetate of silver. They found the deposit to be much more adherent and of much closer texture, but the mass of silver deposited was greater than when silver nitrate alone was used. In addition chlorate of silver was employed, but in this case the deposits substantially agreed with those obtained from nitrate solutions alone. For a current of 0·25 ampere Lord Rayleigh thought that a 4 per cent solution was strong enough, but in general he urged the use of solutions containing 15 or 30 per cent of silver nitrate. A deposit made at 50° C. was found to be heavier than one at 4° C. by about 0·04 per cent.

In 1892 Schuster and Crossley (*Roy. Soc. Proc.* l. 344) discovered that the mass of silver deposited was related to the pressure and also to the size of the anode. The deposits *in vacuo* were about 0·04 per cent heavier than at atmospheric pressure, a result which they attributed to the effect of oxygen.

In 1892 Kahle (*Brit. Assoc. Report*, 1892) found that the addition of silver oxide caused an increase in the mass of the deposit. He made a solution of silver nitrate saturated with silver oxide and found an increase in the deposit of 0·05 per cent. He also confirmed Schuster and Crossley's discovery that the deposit *in vacuo* is about 0·04 per cent greater than at atmospheric pressure.

In 1892 Glazebrook and Skinner (*Roy. Soc. Phil. Trans.* A, 1892) followed the procedure of Lord Rayleigh. Two voltameters were included in the circuit and the general difference in the mass of the deposit was about 0·05 per cent.

Novak (*Proc. Roy. Bohem. Ac. Sci.*, Prague, 1892) appears to have been the first to suggest that a cause of disturbance in the silver voltameter was the formation of a complex silver ion at the anode, and in 1895 Rodger and Watson (*Roy. Soc. Phil. Trans.* A, 1895, p. 631) independently put forward the same idea. The continued use of the same solution was found to increase the deposit, but new solutions gave the same weight of deposit. They further proved that the change was not due to impurities in the silver of the anode. The following are, in chronological order, the results obtained with an old solution :

0·09983	0·09995
0·09987	0·10002
0·09990	0·10005
0·09999	0·10006
0·09995	0·10002
0·09993	

Behn (*Wied. Ann.*, 1894, li. 105) studied the striations which others had observed in silver deposits. He changed the concentration of the electrolyte and the density of the electric current. The striations were more distinct and farther apart in the stronger solutions and also more distinct as the current density at the cathode was reduced. With increasing temperature the striations were more marked, but with purer solutions the tendency was for striations to disappear. By having a horizontal cathode at the top and the anode underneath he succeeded in producing deposits without striae, and concluded that when there are no convection currents in the electrolyte near the cathode, striae cannot occur.

Myers (*Wied. Ann.*, 1895, lv. 288) verified the effect of pressure previously observed by Schuster and Crossley, and by Kahle, and reported a decrease in deposit in an atmosphere of carbon dioxide.

In 1898 Kahle (*Zeits. Instrumentenk.* xviii. 229-267) made a very large number of measurements using both platinum and silver bowls as cathodes, and silver nitrate solutions treated in various manners and from many sources, as electrolytes. He found the deposit of silver per coulomb to be greater on a silver surface than on one of platinum; that it increased with the continued use of a solution, and that the nature of the deposit also varied with the solution employed. His results for the electromotive force of the Clark cell at 15°·0 C. are

1·4327 volts from new electrolyte,
1·4341 volts from used electrolyte.

In 1899 Kahle (*Ann. d. Physik*, 1899, lxvii. 1) used several large voltameters of the Kohlrausch type and found many of the deposits to be striated. In some cases he found acid was liberated, which he concluded accompanied the formation of oxidation products at the anode.

Merrill (*Phys. Rev.*, 1900, x. 167) studied the influence of temperature, pressure, used solutions, and size of anodes. He found no definite effect due to temperature and no marked effect due to pressure alone. The results of three experiments on the effect of pressure were:

Experiment No.	1.	2.	3.
Pressure in Atmospheres	103	95	103
Deposit in grams at atmospheric pressure	1·1371	1·23745	1·08205
Deposit in grams under pressure	1·1370	1·23740	1·08190
Difference in grams	0·0001	0·00005	0·00015

Used solutions were found to give heavier deposits than fresh solutions, and this was thought to be due to some reduction of the valence of the silver in the electrolyte. The relative sizes of anodes, except possibly in extreme cases, was concluded to have no appreciable influence.

Another point, tested by several experiments, was whether the filter-paper wrapped about the anode in the Rayleigh form of voltameter has any influence on the deposit. It was found that the deposits were either the same in amount or their difference was within 0·2 milligram.

In 1899 Richards, Collins, and Heimrod (*Am. Acad. Sci. Proc.*, 1899, xxxv. 123) published a paper on the electro-chemical equivalents of copper and silver. The most noteworthy change was the introduction of a porous cup of unglazed porcelain between the anode and cathode. They concluded that the Rayleigh voltameter always gave erroneous results because a complex silver ion was formed at the anode and passing through the filter-paper deposited silver on the cathode. The porous pot was supposed to serve as a more perfect separator between anode and cathode. Under certain circumstances silver was found to be deposited on the cathode from an anode solution without an electric current. Heavier deposits were obtained at 60° C. and 0° C. than at 20° C., the difference between the masses deposited at 60° C. and 20° C. being about 6 parts in 10,000.

In 1900 Leduc (*Journ. de Physique*, 1902, I. 561) concluded that the anode current density should be small, and that with high current densities the formation of acid occurs. The presence of hydroxide of silver in solution was believed to be free from objection. A decrease in the mass of the deposit was found to result from an increase of temperature.

Guthe (*Bureau of Standards Bull.* Reprint 1, 1904, and Reprint 16, 1905) compared the Rayleigh form with that due to Richards, using the usual electrolytes and also others saturated with silver oxide. He also made special experiments with large silver anodes. The use of filter-paper was avoided in the preparation of the solutions, and only in the Rayleigh type was filter-paper employed. He found the Rayleigh form gave a heavier deposit than the Richards form by 0·48 per cent, and concluded that the cause was due to the heavy anode liquid. He fully corroborated Richards' results.

Van Dijk and Kunst (*Ann. d. Physik*, 1904, xiv. 569) determined the electro-chemical equivalent of silver by a tangent galvanometer method. The Rayleigh form of voltameter was used and twenty-four deposits were made, the resulting values of the electro-chemical

equivalent being in satisfactory agreement. Later, van Dijk (*Arch. Néerland. des Sci.* Series II. ix. 442) made a careful comparison of the Rayleigh, Richards, and syphon forms. He found the Richards type to give a deposit lighter by 0·023 per cent than the deposit in a Rayleigh voltameter and 0·015 per cent lighter than the deposit in a syphon form voltameter. In addition he observed a difference due to the size of the cathodes, the smaller cathode invariably giving the lighter deposit for the same form of voltameter.

Duschak and Hulett (*Am. Electrochem. Soc. Trans.*, 1907, p. 257) made some very careful comparative experiments using the Richards voltameter. They concluded that a deposit from a water solution of pure silver nitrate free from air and *in vacuo* is distinctly lighter than one formed under ordinary conditions; the observed difference was about 7 parts in 100,000. Further, they believed that with proper care the Richards form was trustworthy within 1 part in 100,000.

In 1907 Smith, Mather, and Lowry (*Roy. Soc. Phil. Trans.*, 1908, ccvii.) made an extended investigation, using various forms of voltameter and various electrotypes. They concluded that samples of silver nitrate could be prepared to give in various types of voltameter, values of the electro-chemical equivalent not varying by more than 3 parts in 100,000. A method of purifying silver nitrate was described. High values for the electrochemical equivalent were obtained when the solution contained oxide, carbonate, chloride, or hyponitrite. The presence of acid caused low values. The impurities which increase the mass of the deposit were found to be, in general, substances which are insoluble in water, but soluble in silver nitrate solutions; they are therefore precipitated during electrolysis from the impoverished solution at the cathode. They concluded that there might be slight changes in the electrolyte due to its interaction with filter-paper, but did not believe the mass of the deposit to be seriously affected during one electrolysis in the size of voltameter they had used. At higher temperatures it was thought the effects might be important. The most important conclusions were: (1) The Rayleigh, the Richards, and the syphon form voltameters give identical results when the electrolyte is pure. (2) The deposit is independent of the pressure. (3) The deposit is independent of the temperature. (4) The complex ion effects observed by Richards and Guthe could not be repeated, and they believed that no complex ions were formed. (5) Striae indicates an impure solution. (6) In general the size of anode has no effect. (7) The deposits are independent of the nature of the cathode, *i.e.* whether of platinum or of silver. In most of the experiments anodes of electrolytic silver were used for the first time. They found it difficult to free porous pots from acid without baking, and a theory explaining the formation of striae was given. The electro-chemical equivalent was found to be 1·11827 mgm. per coulomb.

In 1908 Jaeger and von Steinwehr (*Zeits. Instrumentenk.*, 1908, p. 327) gave a detailed account of experiments carried out at the Reichsanstalt. The Richards and Kohlrausch forms of voltameter were found to be in agreement.

Also in 1908 Janet, Laporte, and de la Gorce (*Bull. Soc. Int. Élect.*, 1908, viii. 523), working at the Laboratoire Central d'Électricité, found 1·11821 mgm. as the electrochemical equivalent of silver using the Rayleigh form of voltameter.

It was largely on the evidence obtained from the national laboratories of Great Britain, America, France, and Germany, that the London Conference of 1908 based its provisional specification. It was, however, obvious that the very appreciable differences between the results of many observers demanded further experiments.

In 1910 an important step was taken. By invitation of the United States Bureau of Standards an International Technical Committee consisting of E. B. Rosa (Bureau of Standards, United States), F. Laporte (Laboratoire Central d'Électricité, Paris, France), F. E. Smith (National Physical Laboratory, Great Britain), and W. Jaeger (Physikalisch-Technische Reichsanstalt, Germany) made experiments with standard cells and silver voltameters at Washington in order to discover the causes of certain discrepancies. Previous to this meeting much work had been conducted at the Bureau of Standards and the results communicated to other laboratories. One of the main discoveries was the effect of filter-paper, which in certain cases was proved to produce very important results. This influenced the work elsewhere and must be regarded as an important discovery. There were, however, decidedly different opinions respecting the various forms of voltameters, etc., as is shown in the Reports which were submitted by the delegates before the commencement of the experiments (*Report to the Int. Committee on Electrical Units and Standards*, Washington, 1912, and Supplement to same). The general conclusions arrived at, together with the conclusions of the special Technical Committee, are given under appropriate headings in the following sections. These also contain the results of the investigations carried out by Rosa, Vinal, and McDaniel at Washington (*Supplement to Report to Int. Committee*, 1912, and *Bureau of Standards Bull.*, 1912, ix. 154-207, 208-282,

495-551; 1913, x. 475-536, and Reprint No. 285, 1916), and by others. The work at Washington covered a much wider field than had ever before been anticipated, and the number of silver deposits made total more than the combined totals of all previous investigations.

(a) *Effect of Filter-paper.*—Kahle (*Zeits. Instrumentenk.*, 1902, xxii. 155) was the first to point out that filter-paper may have an effect on the electrolyte, but it was not until the work of Rosa, Vinal, and McDaniel that any definite evidence of its effect was obtained. These latter urged that the effect of filter-paper was highly detrimental for general voltameter work, although in certain cases with large voltameters the effect might be small. This was subsequently confirmed by the National Physical Laboratory, who admitted that their work suffered slightly because of filter-paper. In the experiments carried out by Smith, Mather, and Lowry, all silver nitrate solutions were filtered before using in any form of voltameter. Thus, although silver nitrate recrystallised by them was probably pure, the filtering of the electrolyte contaminated the solution sufficiently to increase the masses of the deposits in all forms by about 1 part in 10,000. The small quantity of filter-paper used in the large Rayleigh form appeared not to increase the contamination sufficiently to affect the result appreciably. The Reichsanstalt also agreed with regard to the general effect of filter-paper, and stated that their experiments showed no effect on the deposits when small quantities of filter-paper were present, but appreciable effects when large quantities were used. Rosa, Vinal, and McDaniel repeated the difference of 1 part in 10,000 under the conditions holding for the National Physical Laboratory experiments, but by increasing the contamination with large quantities of filter-paper differences of 1 part in 1000 were obtained. The results of experiments carried out by the International delegates confirmed, in general, the effect of filter-paper, although, owing to many of the electrolytes being impure from sources of contamination other than filter-paper, the results were far from consistent. In all 10 experiments were made with the Rayleigh form of voltameter, and the mean error of these was about 2 parts in 100,000; 35 experiments were made with voltameters of about the same size not containing filter-paper, and the mean deposits were less than with the Rayleigh form by 11 parts in 100,000; the mean error was about the same.

In later experiments Rosa, Vinal, and McDaniel showed that the constituents of the filter-paper which are active in the voltameter are soluble in water since aqueous extracts of filter-paper produce even more pronounced effects than the filter-paper itself. These active substances are not foreign impurities in the paper, since repeated extraction with water does not diminish the activity of the filter-paper in the voltameter. A preliminary washing of the filter-paper with dilute alkali and water reduces the effect, but does not eliminate it.

As first pointed out by Smith, Mather, and Lowry, impure solutions invariably give striated deposits, and the contamination of the electrolyte with "filter-paper extracts" causes the deposits to be markedly striated when the contamination is excessive. Other substances can, however, also produce these striations. Filter-paper effects are further dealt with in the section dealing with impurities in the electrolyte.

The International Technical Committee agreed not to recommend the Rayleigh form as a standard type.

(b) *Effect of Silk.*—Raw silk used as a septum instead of filter-paper gives at first, according to the Bureau of Standards, an effect somewhat similar to that of filter-paper, due to its decomposition into an aldehyde; if used repeatedly, acid is produced and this acts in the opposite direction, decreasing the weight. Sometimes, therefore, the effect of silk may be to increase and sometimes to decrease the weight of the deposit, according to how long it has been used. Pure raw silk thoroughly washed produced the effect only slightly. Measurements made by the International Technical Committee were not conclusive on this point: the mean of 14 deposits with silk as a septum agreed within 2 parts in 100,000 with the mean deposit in voltameters without septa of any kind, and from previous measurements made at the Reichsanstalt the conclusion was drawn that the effect is negligible.

(c) *Impurities in the Electrolyte.*—In the chemical part of their work Smith and Lowry showed that if the electrolyte contains oxide, carbonate, chloride, or hyponitrite, the mass deposited is heavier than the mass deposited from a pure solution of nitrate. Subsequently, Lowry (*Roy. Soc. Proc.* A, 1914, pp. 53-71) determined the solubility of many silver salts in solutions of silver nitrate of different concentrations and at different temperatures. Smith and Lowry found that solutions saturated with silver oxide gave deposits 0·025 per cent heavier than those from pure solutions; they found the chloride to be freely soluble in concentrated silver nitrate solutions, especially when hot, but on dilution the major portion is precipitated and then there is little effect on the deposit. Sulphide also is very soluble in silver nitrate solutions, but the mass of the silver deposit is not affected to any appreciable extent. The presence of

silver nitrite appeared to have no appreciable influence, a result afterwards confirmed by Rosa, Vinal, and McDaniel, although Richards and Heimrod found an effect of the order of 50 parts in 100,000. It is probable that these latter observers were using impure solutions. On the other hand Smith and Lowry found a solution saturated with hyponitrite of silver to give a striated deposit and to be 0·046 per cent heavier than from a pure solution, while Rosa, Vinal, and McDaniel found no striations in such a deposit. The latter observers proved, however, that electrolytes containing nitrates of hydrazine and hydroxylamine gave deposits which are distinctly striated. It appears highly probable that these impurities were present with the hyponitrite in Smith and Lowry's experiment. The fact that these two reduction products of nitric acid are definite chemical compounds which possess the peculiar property of producing striated deposits caused a careful examination to be made of solutions contaminated with filter-paper in order if possible to detect the presence of these compounds. No trace was discovered. It was finally concluded that a strongly reducing character was an essential property of any impurity which produces striations, and a large number of strong reducing agents, mostly aldehydes and phenols, were separately added to the electrolyte and their effect noted. The deposits were striated in all cases, the more marked effects in general being produced by the stronger reducing agents of the group. The similarity in the effects of filter-paper and the reducing agents already referred to furnished evidence that the activity of the cellulose is due to its action as a reducing agent. This was confirmed by experiments in which permanent colloidal solutions of metallic silver were prepared from filter-paper and silver nitrate, and the metallic silver coagulated and identified by its properties. It is concluded, therefore, that paper, cotton, linen, etc., act chemically upon silver nitrate solutions with the production of colloidal metallic silver. Upon electrolysis these colloidal particles are deposited partly by the current and partly by gravity upon the cathode, thus increasing the weight of the silver deposit above the true electrochemical equivalent of silver.

(*d*) *Effect of Free Acid.*—Leduc, and Smith and Lowry made experiments with free acid in their voltameters, and in general concluded that abnormally low values could only be explained by the presence of free acid. No systematic quantitative work on the subject was however attempted until the experiments of Rosa, Vinal, and McDaniel. The International Technical Committee at Washington tested their solutions for acidity, but made few special experiments; in general the attempt was to have neutral solutions. The results of the special experiments are:

Parts of Added Acid.	Proportional Decrease in Deposit.
5×10^{-6}	$-\ 5\times 10^{-5}$
10 „	− 8 „
100 „	− 12 „
100 „	− 6 „

To test the acidity of silver nitrate solutions the silver is first precipitated with neutral sodium chloride or potassium chloride and the filtrate titrated with one-thousandth normal H_2SO_4 or NaHO, using as indicator iodeosine in ether water solution or an alcoholic solution of methyl red. It is of great importance to test the effect of the filter and all glass vessels upon the neutrality of "conductivity" water. Especially must the experimenter guard against the use of newly bought vessels unless of high-resistance glass. Acidities and alkalinities within less than 1 part in a million can be readily measured.

Rosa, Vinal, and McDaniel (*Bureau of Standards Bull.*, 1914, x. 482) made experiments with the Richards and Smith form of voltameters, and found that for acidities (y) below 10 parts in 1,000,000 equivalents of HNO_3, the correction x (in parts per million) to be applied is given by the formula

$$x=-y/4{\cdot}5.$$

For acidities ranging from 10 to 100 parts in 1,000,000 the following formula represents the results

$$y=-4{\cdot}5x+0{\cdot}02x^2.$$

Dr. von Steinwehr (*Zeits. Instrumentenk.*, 1913, xxxiii. 321) denies the effect of acid, but Obata (*Math. Phys. Soc. Tokyo Proc.*, May 1916), using the Smith form voltameter, confirmed the Bureau of Standards work. He obtained the following relation:

$$y=-4{\cdot}2x+0{\cdot}02x^2,$$

where x and y represent the same quantities as before.

There are two possible ways by which acid may lower the mass of the deposit. The one is by direct action on the deposit and the other by the deposition of hydrogen ions in place of silver ions. Experiments have been made in which slightly acid in water solutions have remained in contact with silver deposits for many hours. At the Bureau of Standards no definite conclusion could be drawn from these experiments, but Obata (*Report No. 11 of the Tokyo Electrotechnical Laboratory*, 1917) states with certainty that acidified water dissolves a much larger amount of silver than natural water. It should be noted, however, that such

experiments present great difficulties, and the change in mass of deposit which is recorded does not in any instance amount to 1 milligram. The second possibility is also largely discredited by experiments made by Rosa and Vinal, who made experiments with acid solutions and deposited only 1 milligram of silver. Preliminary deposition of hydrogen was not caused by the acid. It appears, however, to be established that the presence of acid causes a diminution in the mass of silver, and this is of importance in choosing a type of voltameter for precision work.

(*e*) *The Porous Cup as a Septum.*—Richards originally used a porous cup to prevent the heavy liquid formed at the anode from reaching the cathode. Smith and Mather proved that no complex silver ions were contained in this liquid, and concluded that it had no detrimental effect; this was afterwards verified by Rosa, Vinal, and McDaniel. The porous cup continued to be employed, however, as a convenient means of separating the anode and cathode liquids. In 1910 Rosa and Vinal (*Supplement to Report to Int. Committee on Units*, Washington, 1912) were of opinion that some form of septum was necessary, but Smith, Laporte, and Jaeger were of the opposite opinion.

That porous cups are very efficient separators between anode and cathode chambers is shown by the fact that contaminated liquid may be placed inside the porous cups without altering the appearance or weight of the deposit on the cathode. Smith, Mather, and Lowry found difficulty in removing completely the acid from the pores of a cup, and later Smith produced pronounced stenolysis in such septa. This latter phenomenon is an electro-deposition of silver in the capillaries of the cups which results in the formation of acid and a complex silver salt having the formula Ag_7NO_{11}. If platinum is used as an anode in a silver voltameter (*Roy. Soc. Phil. Trans.*, 1908, ccvii. 588) crystals of this substance separate at the anode and the electrolyte becomes strongly acid, resulting in a diminution in the mass of the silver deposit.

In 1911 Smith made a study of the electrostenolytic action of porous cups, and with very dilute silver nitrate solutions succeeded in making the effects visible. With all capillaries the phenomenon of "endosmose" must take place to some extent during electrolysis, and hence some electrolyte must flow through the pores of the cup. In the usual voltameter experiments Smith calculated that the silver ions travel through the pores of a cup at a velocity of about 4 mm. in a minute, and in the course of an experiment about 2 grams of silver ions pass through the pores. Following Helmholtz, it is assumed that even before electrolysis a double layer charge is formed at the surface of contact of pot and electrolyte. When a current passes, a fall of potential is produced in the electrolyte and the positive part of the double layer moves towards the cathode leaving the walls of the capillaries negatively charged. The positive silver ions in the solution may now give up their charge to neutralise the negative charge on the walls, and hence silver may be deposited in the capillaries. Smith did not suggest that the whole of the capillaries are thus covered with an exceedingly thin layer of silver, but suggested that the deposit would be in patches owing to the capillaries being irregular in section and the velocity of the ions through them being variable. As the conductivity of silver is more than a million times that of the electrolyte, the silver patches will take an appreciable part in the conduction of the current. They will not, however, necessarily increase in size because of this; they will only do so if the anode ends of the patches are covered with an insoluble substance such as Ag_7NO_{11}. Even if the action takes place to a slight extent the deposition of silver in the capillaries will result in an excess of hydrogen ions in the electrolyte, *i.e.* the latter will become slightly acid.

The possibility of detecting a minute quantity of silver in the capillaries of a porous cup was attempted by chemical means and some positive evidence was obtained. However, Smith concluded that the passage of a strong current through a weak solution of silver nitrate contained in the capillaries should produce complex actions where the current leaves the silver patches, and this might lead to their detection. In such a case the silver would grow towards the anode of the voltameter and should become visible. Twenty separate experiments were made and in 6 of these the insides of 6 different cups were lined with silver within ten minutes from the start; in the remaining 14 experiments no silver was detected. In 5 of the 6 cases a small quantity of Ag_7NO_{11} was formed on the outside of the cups. Some of the cups were broken, and microphotographs showed a dark line of silver deposited on the inner wall. In another experiment with a new cup no silver was observed, but crystals of Ag_7NO_{11} were discovered on the outside. In order to prevent any slime from the anode reaching the cup the anode was, in some of the experiments, enclosed in well-washed silk. In other experiments cracks were produced in glass tubes and similar effects produced. Experiments of this latter nature had previously been made with success by Coehn (*Zeitschr. f. Elektrochemie*, 1898, pp. 501-503).

Smith further found that filtering a solution of silver nitrate through a porous cup rendered the formation of visible silver far more diffi-

cult. The process of filtering leads probably to the action of stenolysis and deposits some silver in the capillaries; in such a case the filtrate should be slightly acid. When tested this was found to be so. The final conclusion was that the behaviour of the Richards form with pure electrolyte depended on the condition of the cup. If the latter were clean but quite new the resulting silver deposit would probably be too low; if much used and kept continually soaked in electrolyte the resulting deposits would be much nearer normal. These conclusions appear to be largely confirmed. With freshly prepared porous cups the deposits are in general smaller than with cups much used and permanently stored in electrolyte.

A very complete investigation of the Smith and Richards forms of voltameter has been made by Obata (*Electrotechn. Research Lab. Tokyo*, Reprints No. 71 (1918) and No. 76 (1919)), who measured the acidity of the electrolyte before and after each run. He used very fine-grained Pukal cups which required from twenty to thirty hours to filter half their volume of water and an alundum cup which filtered the same volume in a few seconds. With the latter cup stenolysis effects would be small. In 7 out of 7 measurements the mean observed increase in acidity in the Smith form was 1 part in 10 millions; Obata concludes that with this form the acidity does not change. In 4 experiments with a small porous cup (in which stenolysis should be most marked) the mean increase was 77 parts in 10 millions. Before comparing the masses of the deposits Obata corrected them for the acidity of the electrolyte by assuming that the mean acidity during the experiment is twice that calculated from the initial and mean acidities. His general conclusion is *not* that the formation of acid in the Richards form must be produced by and subsequently give rise to some disturbance but that the anode liquid in the Smith form contains a small quantity of some complex silver ion. As the acidity remains constant this assumption appears to be untenable.

Smith concludes that deposits in the Richards form will in general be lighter than in the syphon and non-septum forms. The difference should be greatest when the cups are new and least when they have been much used and kept immersed in neutral silver nitrate solutions. The procedure adopted at the Bureau of Standards for getting the cups in a condition of equilibrium (*Bureau of Standards Bull.*, 1910, x. 513) probably gives very satisfactory results, and the difference between the Richards form and non-septum forms will be in such cases very small; experiments indicate differences from 2 to 3 parts in 100,000.

(*f*) *Complex Silver Ion in Anode Solution.*—From the time of Richards' experiments in 1899 until the work of Smith, Mather, and Lowry in 1907, it was the generally accepted view that certain anomalous substances were formed at the anode which interacted with the silver at the cathode and by deposition of silver increased the mass of the deposit. Smith, Mather, and Lowry first conclusively proved that the effects found by Richards could not be due to the formation of a complex silver ion. The results obtained by the International Technical Committee in 1910 confirmed this conclusion. With a non-septum (Smith) form of voltameter in which anode liquid may pass to the cathode the mean deposit in 11 experiments was 1 part in 100,000 less than the mean deposit in 4 voltameters of the Richards form containing the same amount of electrolyte, and 2 parts in 100,000 greater than the mean deposit in 9 voltameters of the latter form containing a smaller volume of electrolyte. The Committee agreed that there was no evidence to support the theory advanced by Richards and supported by Guthe.

Buckner and Hulett (*Am. Electrochem. Soc. Trans.*, 1912, xxii. 367-383) made experiments with four Richards voltameters in series. In most cases the anode liquid was lower in level than the cathode liquid, and the former was stated not to flow through the porous cups except in special cases. In these cases deposits heavier by 0·019 per cent were obtained, and they concluded the anode liquid had a detrimental effect. Richards and Anderegg (*Am. Chem. Soc. Journ.*, 1915, xxxvii. 675-693) also do not entirely accept the view of an inert anode liquid, but believe that the anode liquid may really augment the weight of the silver deposit. Rosa, Vinal, and McDaniel have failed to confirm the conclusions of Richards, and there is now a fairly general consensus of opinion that with the usual sized anodes no complex silver ions are formed.

(*g*) *Purity of the Deposit.*—The question of inclusion of electrolyte in the silver deposited in a voltameter has received much attention and the results are contradictory. Lord Rayleigh found a loss in weight of 0·01 per cent on heating the deposits to incipient redness. Richards and Heimrod found a loss of 0·018 per cent. Van Dijk heated the platinum bowls with their silver deposits in an electric furnace and found no appreciable loss in weight. Smith, Mather, and Lowry, using very large bowls, confirmed the results found by van Dijk. Jaeger and von Steinwehr also found the loss to be insignificant. Duschak and Hulett made elaborate measurements and concluded that the total inclusions averaged about 0·011 per cent, and were distributed

through the crystals. Later, Laird and Hulett by a new method found the included silver nitrate to be not greater than 0·005 per cent. In 1916 Vinal and Bovard (*Bureau of Standards Bull.* Reprint 271) heated a number of silver deposits to temperatures slightly above 600° and found the loss in weight to indicate inclusions of foreign matter in the deposits of about 0·004 per cent. It appears that with large cathode surfaces the included matter must be very small, and Smith in more recent experiments has again failed to detect any loss on heating the deposits when large bowls are used and the deposit is not too heavy.

(*h*) *Temperature Coefficient of Silver Voltameter.*—As already stated, Lord Rayleigh obtained a higher deposit at 50° C. than at 15° C., and a higher deposit at 15° C. than at 4° C. The temperature coefficient was about +0·001 per cent per 1° C. Leduc found a negative coefficient. Richards, Collins, and Heimrod found a positive coefficient between 20° C. and 60° C. Merrill, using fused silver nitrate, found no temperature effect. Smith, Mather, and Lowry used the syphon forms only in two experiments at 15° C. and 90° C., and found the coefficient, if any, was less than 1 part in a million. They concluded that the temperature coefficient is zero. This was also the opinion of F. Kohlrausch and of the Reichsanstalt. The International Committee made no experiments on the effect of temperature, but in 1910 Rosa, Vinal, and McDaniel made two experiments with four voltameters and found in the first experiment a positive coefficient of 3 parts in a million and in the second experiment one of 1 part in a million. They concluded that the coefficient is probably zero. For a pure electrolyte there is no doubt that this is so, but when the electrolyte contains reducing impurities these will be more active at the higher temperature and therefore a positive temperature coefficient will appear, as in the experiments of Lord Rayleigh and Richards and Heimrod.

(*i*) *Preparation of Pure Silver Nitrate.*—The first serious attempt to prepare pure silver nitrate for work with the silver voltameter was made by Smith, Mather, and Lowry in 1907. Their method was one of repeated crystallisation, and there is little doubt that the salt obtained was very pure. Laporte also prepared the salt by recrystallisation, and some of this was used by him at Washington. In 1908 Smith studied the effects of small quantities of acid in the electrolyte, and finding that a relatively small quantity of acid had an appreciable effect he recrystallised silver nitrate from distinctly acid solutions and subsequently fused the salt in an electric furnace. From 500 to 1000 grams of the slightly acid salt were fused in a platinum bowl screened by a clock glass, and in order not to decompose the nitrate the heating current was switched off when about half of the salt was melted. Time was then allowed for the whole mass to become fluid when the nitrate was removed and allowed to cool. The salt thus prepared was beautifully white throughout, except, at times, on the surface; before using for voltameter work the surface salt was therefore removed by washing with distilled water.

When the International Technical Committee met at Washington in 1908 this fused salt was the only nitrate which satisfactorily complied with all the tests made for detecting reducing impurities and acid and alkali, and the method adopted by Smith forms the basis of the present methods of purification. Previous to 1910, while hundreds of experiments had been made with the silver voltameter and various causes of error detected, very few of the electrolytes could have been quite pure except by chance. This naturally led to a number of false conclusions being drawn. Subsequently, at Washington, Dr. McDaniel prepared some fused silver nitrate by Smith's method, and this was found to answer satisfactorily to all the tests made. The electrolytes containing silver nitrate prepared by Smith and McDaniel were the only ones which gave practically the same deposits in large and in small voltameters. In further experiments made at the Bureau of Standards (*Bureau of Standards Bull.*, 1913, ix. 549) it was found practicable to fuse the salt at considerably higher temperatures than the melting-point, but there appears to be no advantage in this except to save a little time; in fact, Rosa, Vinal, and McDaniel say that their best results have been obtained by removing the salt as soon as melted.

The tests for acidity and alkalinity have already been described. A sensitive test for reducing agents and colloidal silver has been suggested by Dr. McDaniel and is most convenient. The test is made with a solution of potassium permanganate and is as follows: 10 c.c. of a 66·6 per cent solution of the $AgNO_3$ to be tested is placed in a glass stoppered cylinder of 25 c.c. capacity and acidified by the addition of 1 c.c. of concentrated HNO_3 (free from reducing substances). After thorough mixing of the acid and solution, 0·5 c.c. of 0·001 normal permanganate solution is added and the mixture again shaken. The addition is continued in portions of 0·5 c.c. until the colour persists undiminished in intensity for five minutes or longer. From the amount of permanganate added the quantity of reducing substances present is calculated.

(*j*) *Explanation of Striations.*—In 1894 Behn (*Wied. Ann.*, 1894, v. 105) published a work on striated deposits in the silver voltameter,

and gave photographs of deposits illustrating the nature of these striations. He found that the direction of the striae was determined by the convection currents circulating in the liquid, and when the cathode was horizontal and at the top of the electrolyte no striae appeared. He further investigated the effect of varying the concentration of the electrolyte and the density of the electric current. With increase in concentration the striae were more distinct but farther apart, and the same effect could be produced by reducing the current density.

Smith, Mather, and Lowry in 1907 found that in pure electrolytes striae did not appear and regarded the formation of striations as evidence of an impure electrolyte. They traced out the general direction of the convection currents near the cathode and showed why the upper portion of a deposit may have a matt surface and the lower portion a striated surface. They explained striations by assuming instability in the thin layer of liquid near the cathode which rapidly loses much of its silver. When this film is very thin it is supposed to be unstable and break up into cylindrical columns of liquid which ascend near the cathode surface. Hence in contact with the cathode surface there are columns of liquid of low concentration, and in between these the electrolyte is of approximately normal concentration. The latter electrolyte has the higher conductivity, and since in addition there is an electromotive force acting from the columns of low concentration towards the main body of the electrolyte, the current passes into the cathode through the liquid between the columns. If these assumptions are correct, an increase in the current should result in the cathode film becoming thicker and more stable, and when it is sufficiently stable to remain as a film a striated deposit should not be formed. This was tested by experiment and found to be so. In special experiments conducted in glass vessels the breaking up of a layer into cylindrical columns was observed.

Rosa, Vinal, and McDaniel confirmed the conclusion that the direction of the striae is determined by the convection currents of the electrolyte, but believe the production of striae is due in the first place to the deposition of colloidal silver on the silver crystals. From a pure solution the silver is deposited in a crystalline form and these grow with perfect faces and angles in a pure electrolyte. With impure electrolytes the crystalline character disappears, the deposits being masses which under the microscope look like molten metal. It is supposed that in a pure electrolyte the force of crystallisation constrains the silver atoms to be deposited in a regular order, in spite of the tendency of the convection currents in the liquid to distort the crystals by building them upwards. In an impure solution, however, the regular crystalline growth is interfered with by the deposit upon the initial crystals of particles of colloidal silver. It is found, however, that if the relative velocity of the liquid be great, as it may be with forced rotation of the cathode, a striated deposit may be obtained from pure electrolytes.

(*k*) *Forms of Voltameters.*—In 1910 the International Technical Committee recommended that the Rayleigh form be not adopted as a standard owing to the effect of the filter-

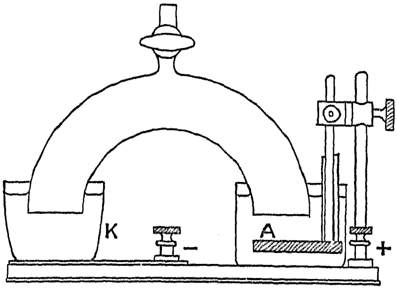


FIG. 15.

paper. Four other forms have, however, been frequently used, and the respective merits of these have been much discussed.

The first form (*Fig.* 15) is the Syphon form. This is for comparison purposes only, but is regarded by Smith as the form by which all others should be judged. If there are any impurities of a reducing nature in the electrolyte these will cause the deposit to be too heavy, but no complications due to actions at the anode can have the slightest effect. The resistance is too high and variable to maintain a steady current through it with ease, and it is used therefore for comparison purposes only. As modifications of this type, Smith, Mather, and Lowry put a porous cup over the anode or cathode end of the syphon. The same procedure was afterwards adopted by Rosa, Vinal, and McDaniel.

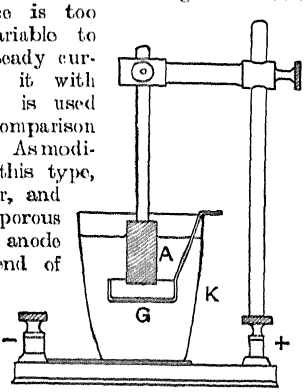


FIG. 16.

The Kohlrausch form (*Fig.* 16) consists of a silver rod for the anode and a platinum bowl for the cathode, the only partial separa-

tion between anode and cathode being a glass dish placed under the anode to catch any particles falling from it. In practice this dish is not found sufficient to confine the anode slime which is often carried to the top of the electrolyte by bubbles of gas. In the experi-

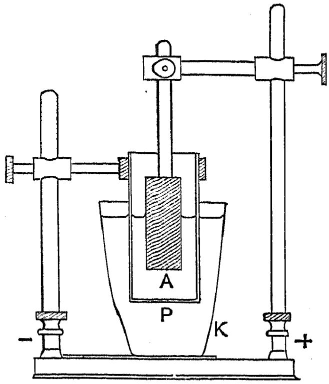


FIG. 17.

ments made by the International Technical Committee Dr. Jaeger used the form on two occasions only; on fourteen other occasions, owing to trouble with floating scum, he surrounded the anode with silk. In a modified form, introduced by Rosa, the difficulty is partly surmounted by supporting a ring of glass in the surface of the liquid.

In the Richards type (*Fig.* 17) a porous cup of unglazed porcelain serves as a means of separating the anode and cathode. The cups used are of the finest porcelain, have thin

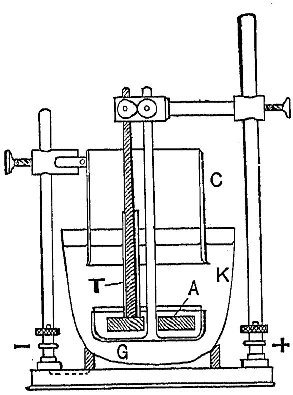


FIG. 18.

walls, and are of very fine grain. Pukal cups made by the Königlich Porzellan Manufaktur of Berlin have been used by the majority of experimenters.

The Smith voltameter (*Fig.* 18) is designed to avoid the use of a septum of any kind. It differs from the Kohlrausch form inasmuch as there is no trouble with anode slime even if the latter floats with tiny bubbles to the surface of the electrolyte, and no difficulty is experienced in removing the slime from the voltameter at the conclusion of an experiment. The anode is in the form of a silver disc coated with electrolytic silver and afterwards baked in an electric oven. This disc is contained in a shallow glass basin with a ground edge, the basin being supported by a glass rod passing through a hole in the centre of the disc. A glass cylinder, the lower end of which is ground, fits over the basin and is used before and after the electrolysis to separate the electrolyte into two parts. A silver rod supports the silver disc (anode) and a glass tube fits over its lower end to prevent undue electrolysis of the rod. During electrolysis the glass cylinder is raised, but its lower end is always immersed in the electrolyte.

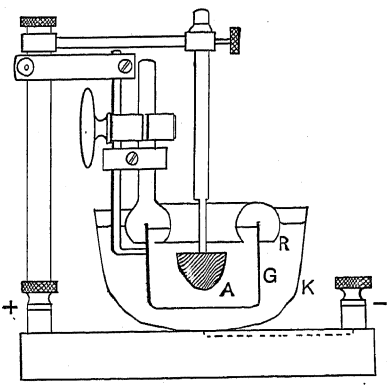


FIG. 19.

A modified syphon voltameter (*Fig.* 19) has been used by Rosa, Vinal, and McDaniel. In this a small glass dish supported by a glass rod is submerged in the electrolyte. From the top edge of this dish there project four short glass rods that support an annular syphon whose lower edge is just below the surface of the electrolyte. The chief difficulty is in the escape of anode slime when the voltameter is dismantled.

With impure electrolytes all of these forms will give deposits which are abnormal. Examples of this are numerous in experiments made previous to 1910.

In 1907 Smith, Mather, and Lowry stated that all forms of voltameter experimented with by them gave identical values within a few parts in 100,000. In 1909 Rosa, Vinal, and McDaniel (*Supplement to Report of Int. Committee*, 1912, p. 7) found that the Kohlrausch form without silk or other septum gave an abnormally heavy deposit. They

expressed the opinion that some form of septum was necessary. The syphon form was found to give a larger deposit than the Richards and Kohlrausch forms, but they did not believe it to be a reliable criterion by which to judge other forms. Pure silver nitrate had not then been used by them. With the electrolytes employed they found larger deposits in large voltameters than in small ones, but with purer electrolytes the difference became smaller.

The results obtained with initially pure electrolytes by the International Technical Committee (1910) are as follows:

No. of Measurements.	Form of Voltameter.	Relative Masses of the Deposits.	E.M.F. of Weston Normal Cell at 20° C.
11	Smith's form, 370 c.c.	$1{\cdot}00000_0$	$1{\cdot}01829_5$
9	Richards' form, small, 100 c.c.	$0{\cdot}99997_7$	$1{\cdot}01827_2$
4	Richards' form, large, 300 c.c.	$1{\cdot}00000_9$	$1{\cdot}01830_4$

A special sub-committee made a comparison between the syphon form and the Smith and Richards forms, and found as the result of one experiment the relative deposits to be

Syphon.	Smith's Form.	Richards' Form.
1·00000	0·99988	0·99990

In 1910 and 1911 Smith and Vinal working at the National Physical Laboratory and using pure electrolyte obtained the following relative values for the silver deposits:

No. of Measurements.	Form of Voltameter.	Relative Masses of the Deposits.	E.M.F. of Weston Normal Cell at 20° C.
11	Smith's form, 370 c.c.	$0{\cdot}99996_8$	$1{\cdot}01829_4$
8	Smith's form, 120 c.c.	$0{\cdot}99999_1$	$1{\cdot}01831_7$
13	Richards' form, small, 100 c.c.	$0{\cdot}99988_3$	$1{\cdot}01820_9$
2	Richards' form, large, 300 c.c.	$0{\cdot}99994_7$	$1{\cdot}01827_3$
2	Syphon form	$1{\cdot}00000_0$	$1{\cdot}01832_6$
1	Syphon. Porous pot over anode end	$0{\cdot}99999_7$	$1{\cdot}01832_3$
1	Syphon. Porous pot over cathode end	$0{\cdot}99987_7$	$1{\cdot}01820_3$

In January 1911 three further experiments showed that the syphon and Smith forms agreed within 1 part in 100,000, but the Richards form gave deposits which were lighter by about 7 parts in 100,000. In further experiments made by Smith in 1911 a difference of only 4 parts in 100,000 was measured, and the same difference was independently found by Vinal at Washington. Since April 1910 a large number of most carefully planned experiments have been carried out by Rosa, Vinal, and McDaniel, who particularly studied the Richards and Smith forms, using pure electrolytes. The results are given in the following table:

No. of Measurements.	Form of Voltameter.	E.M.F. of Normal Cell at 20° C.	Weighted Mean.
32	Smith, large size	$1{\cdot}01826_3$	
21	Smith, medium size	$1{\cdot}01827_3$	$1{\cdot}01827_4$
2	Smith, small size	$1{\cdot}01830_4$	
47	Richards, large size	$1{\cdot}01826_9$	
22	Richards, medium size	$1{\cdot}01824_4$	$1{\cdot}01826_7$
87	Richards, small size	$1{\cdot}01826_0$	
6	Syphon	$1{\cdot}01832_4$	$1{\cdot}01832_4$
7	Modified Syphon	$1{\cdot}01834_6$	$1{\cdot}01834_6$
9	Kohlrausch	$1{\cdot}01829_7$	$1{\cdot}01829_7$

This is the largest number of observations so far made with pure electrolyte, and the results indicate that in the experiments at Washington the porous cups were carefully prepared and pure electrolytes used. Rosa, Vinal, and McDaniel state in their Summary (*Bureau of Stands. Bull.* Reprint 285, 1916) that with electrolyte of highest purity the syphon voltameter probably agrees with other forms. This agrees with the conclusions of Smith and Mather. The present position is therefore that the deposits obtained with all forms of voltameters agree conditionally that the electrolyte is pure and does not become contaminated during electrolysis.

§ (41) Specification of the Silver Voltameter.—The International Conference on Units and Standards (London, 1908) did not specify the silver voltameter in detail, but recommendations for a complete specification were expected from the International Technical Committee. Unfortunately the work of this Committee was not completed at Washington, but in 1910 Rosa and Smith agreed on a specification, and work since then has shown that if the specification is complied with accurate results are obtained.

The specification is as follows:

Specification

1. The electrolyte shall consist of a solution of silver nitrate in distilled water, having from 10 to 20 grams of silver nitrate in 100 c.c. of the solution.

2. The electrolyte must be free from organic or other reducing substances, as shown (*a*) by a suitable chemical test, (*b*) by giving a crystalline deposit free from striations, and (*c*) by giving the same weight of deposit in a large and in a small voltameter.

3. The silver nitrate is purified by crystallisation from slightly acid solutions and fusion, and if the chemical test for purity is omitted it should be purified until further crystallisation does not change the weight of the deposit.

4. The voltameter should contain not less than 75 c.c. in the cathode chamber, and the deposit should not continue long enough to reduce the mean concentration of the electrolyte in the cathode chamber below 5 per cent. If no septum is used, no greater weight of silver should be deposited in a single experiment than is contained in the electrolyte at the start.

5. The electrolyte when ready for use must be neutral or very *slightly* acid, as tested by iodeosine. As one part in a million of alkali may increase the deposit appreciably, it may be better to have a slight acidity (say one part in a million) than to take the risk of slight alkalinity in attempting to make it strictly neutral. The electrolyte must be neutral or slightly acid at the end of the experiment, no alkalinity, and only a trace of acid, if any, being present. Any septum or other substance which contaminates the electrolyte or produces appreciable alkalinity or more than a trace of acid must be avoided.

6. The cathode should be a crucible or bowl, preferably of platinum (although gold may be used) of from 125 c.c. to 400 c.c. capacity. The surface should preferably be smooth and bright, and the deposited silver should be removed by electrolysis or by acid, without scratching or marring the surface of the platinum by any instrument.

7. The anode should be of pure silver and is preferably coated with electrolytic silver. This is conveniently done when a previous deposit is being removed from the cathode bowl, using a relatively small current. The anode should have as large an active area as the size and type of voltameter permit.

8. The current during a deposit should be maintained constant, and is preferably not more than one ampere, and the time not less than one hour.

9. If the surface of the platinum is perfectly clean and the electrolyte pure the silver will be adherent and there will be little if any loose silver. After thorough washing the cathode bowls are dried at about 150° C., preferably in an electric oven, and after cooling are weighed. In the weighing a similar platinum dish adjusted to the same weight is advantageously used as a tare.

10. The electromotive force of the standard cell employed is calculated from the weight of silver deposited, the resistance, and the time, using 1·11800 mg. per second as the electro-chemical equivalent of silver.

Rosa, Vinal, and McDaniel add an additional paragraph specifying:

11. If a septum between the anode and cathode is used, it must not contaminate the electrolyte with organic or reducing impurities; it must not produce acid or alkali in the electrolyte, and it must be of sufficiently fine grain to hold back the anode slime without introducing any high resistance into the voltameter.

§ (42) Results.—The results obtained from time to time have usually been expressed in terms of the electromotive force of the Weston normal cell at 20° C. Since 1908, the time of the meeting of the London Conference, the values given in the annexed table have been obtained:

Date.	Observer.	Form of Voltameters.	No. of Deposits.	E.M.F. of Weston Normal Cell at 20° C.
1908–10	Rosa, Vinal, and McDaniel	Richards	86	$1·01828_1$
1909–10	Smith	Smith and Richards	17	1·01827
1910	International Technical Committee	Smith	8	$1·01828_7$
		Richards	14	$1·01828_6$
1910	Smith and Vinal	Smith	19	$1·01830_4$
		Richards	15	$1·01821_8$
1910	Von Steinwehr	Kohlrausch	40	$1·01829_0$
1910–12	Rosa, Vinal, and McDaniel	Smith	55	$1·01826_7$
		Richards	156	1·01826
1912–13	Haga and Boerema	Smith	32	1·01826
1914	Foehringer	Smith	4	$1·01829_5$
1916	Obata	Smith	40	$1·01826_9$
1918	Obata	Richards	61	$1·01826_6$

By a Resolution of that Conference:

The international volt is the electric pressure which when steadily applied to a conductor whose resistance is one international ohm will produce a current of one international ampere.

The results obtained since 1908 show that to a high degree of accuracy the electromotive force of the Weston normal cell at 20° C. is

1·0183 international volts at 20°·0 C.

The mean of all the values given in the last table is 1·01827, and the greatest difference

from this mean is 5 parts in 100,000. The measurements were made in England, the United States, Holland, Japan, Germany, and Russia. In 1910 the value 1·0183 international volts at 20° C. was recommended for universal adoption by Lord Rayleigh's Committee on Electrical Units and Standards, and this value is now adopted in all civilised countries.

The agreement is remarkably good, especially since there must be slight differences between the cells employed. The international ampere may therefore be regarded as one which may be satisfactorily specified.

In terms of the volt (10^8 C.G.S. units) the most probable value of the electromotive force of the Weston normal cell at 20° C. is 1·0188. This follows from recent absolute measurements of current, which show that the electromotive force of the Weston normal cell at 20° C. in terms of the ampere (10^{-1} C.G.S. unit) and the international ohm is $1{\cdot}0182_4$. The most probable value of the international ohm is $1{\cdot}0005_2$ ohms (10^9 C.G.S. units), so that in C.G.S. measure the electromotive force of the Weston normal cell at 20° C. is $1{\cdot}0182_4 \times 1{\cdot}0005_2$ or

1·0188 volts (10^8 C.G.S. units).

§ (43) THE INTERNATIONAL VOLT AND STANDARD CELLS.—As a working method for the realisation of the international volt the Conference recommended the use of a Weston normal cell whose electromotive force has been determined in terms of the international ohm and the international ampere. A complete specification of the Weston cell was not, however, approved by the Conference, but the duty of preparing such a specification was assigned to a scientific committee nominated by Lord Rayleigh. An International Technical Committee which met at Washington in 1910 determined the electromotive force of the Weston normal cell to be 1·0183 international volts at 20° C. A specification for the cell, which it is believed will meet all needs, is given at the end of this section.

§ (44) TYPES OF STANDARD CELLS.—A standard cell of any type should comply with the following conditions:

(*a*) It must be made up of chemicals which can be purified and reproduced with great exactness.

(*b*) No chemical or electro-chemical reactions must take place inside the cell except when a current passes through it.

(*c*) When the cell passes through a cycle of temperature it must give the same electromotive force at any stated temperature whatever its past thermal history has been, subject to certain limitations of temperature.

(*d*) When current passes through the cell its electromotive force may vary slightly, but it must recover completely within a reasonable time.

No cell is known which satisfies these conditions absolutely, but the Clark and Weston systems do so very nearly.

Cells which have from time to time been used as standards are given in the following table:

Name.	Positive Element.	Negative Element.	Electrolyte.	Depolariser.
Daniell	Copper	Zinc	Zinc Sulphate	Copper Sulphate
Fleming type of Daniell	”	”	”	”
Clark	Mercury	Zinc	”	Mercurous Sulphate
Gouy	”	”	”	Mercurous Oxide
De la Rue	Silver	”	Zinc Chloride	Silver Chloride
Helmholtz	Mercury	”	”	Mercurous Chloride
Hibbert	”	”	”	”
Weston	”	Cadmium	Cadmium Sulphate	Mercurous Sulphate

The Daniell cell as commonly used is an imperfect standard, for when a current passes through the cell the concentrations of the electrolytes are permanently changed. Also copper sulphate is an easily soluble salt, and it diffuses in appreciable quantities into that part of the cell containing the zinc. If a porous partition is added to keep the two liquids apart, it serves as a separator for a short time only. The other cells tabulated are great improvements with regard to the depolariser, and in the case of the Clark, Gouy, Helmholtz, and Weston cells, diffusion of the depolarising liquid to the negative element serves only to further amalgamate the latter and so produces a negligible change in the electromotive force.

In practice the cells containing mercurous sulphate as a depolariser are more satisfactory than the Gouy cell with mercurous oxide as depolariser; they are also better than the Helmholtz and Hibbert cells; this is due to the mercurous salt in the latter three cells being so insoluble as to incompletely protect the cells from polarisation. At the present time the Weston and Clark cells are the only ones used as standards, and these only will be described.

§ (45) Clark and Weston Cells.—The Clark cell is a voltaic combination which has a saturated solution of zinc sulphate as its electrolyte, mercury as the positive element, and zinc or zinc amalgam as the negative element. To ensure saturation of the solution at all working temperatures crystals of zinc sulphate are added, and to prevent polarisation of the cell the positive element is covered with mercurous sulphate. Latimer Clark first introduced this cell in 1872,[1] and investigations of it were first conducted by Lord Rayleigh,[2] and by Glazebrook and Skinner;[3] these investigators proved that the principal difficulty lay with the mercurous sulphate.

The Weston cell was patented by Dr. Edward Weston in 1892. As originally made it contained an electrolyte which was an unsaturated solution at most working temperatures, but since 1908 the cell known as the Weston normal cell, the electrolyte of which is saturated at all working temperatures, has been most used. The Weston normal cell is a voltaic combination which has a saturated solution of cadmium sulphate as its electrolyte, mercury as the positive element, cadmium amalgam as the negative element, and mercurous sulphate as the depolariser. Crystals of cadmium sulphate are added to ensure saturation of the solution.

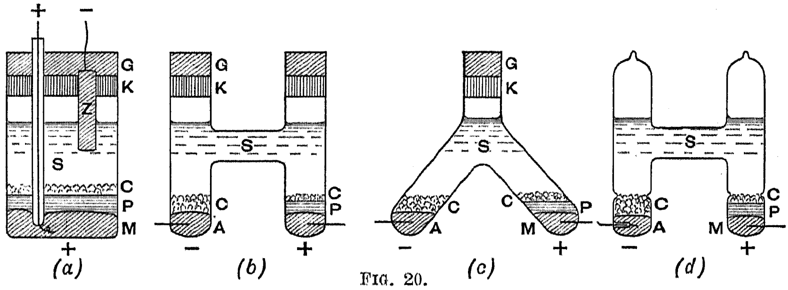


Fig. 20.

Researches on standard Clark and Weston cells have revealed many causes of variations, but the chief difficulty is with the mercurous sulphate. Minor variations have been traced to the negative element and the electrolyte, especially when the latter contains a minute quantity of acid. The type of cell, the preparation of the chemicals, and the causes of instability are considered in the following sections:

(i.) *Form of Cell.*—The original shape of Clark cell was tubular (*Fig.* 20 (*a*)), mercury being contained in the base of the tube, and a zinc rod supported by a cork occupied the upper part. Lord Rayleigh pointed out that this form was objectionable because the concentration of the mixed solutions of zinc and mercurous sulphates might be greater over the mercury surface than over the surface of the zinc rod; moreover, zinc amalgam could not be used in this form of cell. He devised therefore the H form (*Fig.* 20 (*b*)), in one leg of which there is zinc amalgam and in the other pure mercury; electrical contacts with the amalgam and with the mercury are made by platinum wires sealed into the glass. In the original H vessel evaporation was prevented by closing the upper ends of the tubes with corks and marine glue, but Lord Rayleigh pointed out that the cell could if necessary be hermetically sealed. Whether the cell be tubular or of the H form there is a tendency to aggregation of the zinc sulphate crystals by alternate melting and crystallisation as the temperature rises and falls. It is desirable, therefore, that only sufficient crystals be introduced to ensure saturation.

In 1904 F. E. Smith[4] showed that Kahle's conclusions[5] that the E.M.F. of the H form of cell was 0·0004 volt less than that of the tube form was erroneous. No such difference exists because of any difference of form.

Wright[6] introduced the inverted Y form of cell (*Fig.* 20 (*c*)). This is a variation of the H form, and in practice is a little more difficult to fill.

In 1886 Lord Rayleigh observed that in many H cells breakages occur in the amalgam limb, and he concluded that some alloying takes place between the zinc amalgam and the platinum wire making contact with it, resulting in an expansion of the wire which may crack the glass near the point where the platinum is sealed into it. To delay this action he introduced a little marine glue into the cell so as to protect part of the platinum wire from the amalgam. Hulett substituted a glass bead for the marine glue, and Smith sealed a fine glass tube about 4 mm. long over the platinum wire inside the cell (*Fig.* 20 (*d*)). This glass tube forms an extension of the H vessel and may be cracked by the alloyed platinum wire, and in time no doubt the cracking of the H vessel by the same cause

[1] *Roy. Soc. Proc.*, 1872, xx. 144; and *Roy. Soc. Phil. Trans.*, 1874, clxiv. 1.
[2] *Roy. Soc. Phil. Trans.*, 1885.
[3] *Ibid.*, 1892, clxxxiii.
[4] *Brit. Assoc. Report*, 1914.
[5] *Ibid.*, 1892.
[6] *Phil. Mag.*, 1883, xvi.

will take place. In practice, however, Clark cells so protected have been kept at the National Physical Laboratory for twelve years and not one has cracked. Recently M'Kelvy and Shoemaker [1] have employed platinum wire previously subjected for a short time to the action of hot 10 per cent zinc amalgam and have used a lead sealing-in glass. The first hermetically sealed cells appear to have been made by Callendar and Barnes.[2]

The universal practice now is to employ H vessels for standard cells, but there is no variation in electromotive force due to the *type* of cell, conditionally that the electrolytes in contact with the positive and negative elements are saturated.

(ii.) *The Positive Element, Mercury.*—All observers have used distilled mercury. Mercury which has been much used in a laboratory should be submitted to a preliminary purifying electrolytic process. The latter consists in making the mercury the anode in an electrolytic cell containing a mixture of dilute sulphuric and nitric acids as the electrolyte; the current density at the anode is conveniently about 0·002 ampere. Impurities of cadmium, zinc, etc., are thus removed. After being so treated the mercury should be washed, dried, and distilled *in vacuo.*

An alternative method of distillation which removes all readily oxidisable metals such as zinc, cadmium, bismuth, tin, copper, lead, etc., has been developed by Hulett.[3] An ordinary distilling flask with suction attachment is arranged so that a little air bubbles through the mercury in the still and passes over with the vapours. The air pressure is adjusted to be about 25 mm. and the temperature about 200° C. Any metallic vapour will completely oxidise under these conditions if the dissociation pressure of its oxide is less than the partial pressure of the oxygen maintained in the still. This is particularly true of all the common base metals. However, mercury vapour does not appear to oxidise at all under the conditions stated.

(iii.) *The Negative Element of the Clark Cell. Zinc or Zinc Amalgam.*—Hockin and Taylor [4] first showed that the presence of mercury on the surface of zinc makes practically no difference to its electromotive behaviour. They made cells with zinc as the negative element and a zinc amalgam as the positive element, and found that even when the mercury concentration reaches 98 per cent the electromotive property is the same, within a fraction of a millivolt, as that of pure zinc. Rayleigh [5] confirmed this result.

[1] *Bureau of Standards Bull.*, 1920, xvi.
[2] *Roy. Soc. Proc.*, 1897, lxii.
[3] *Phys. Rev.*, 1911, xxxiii.
[4] *Jour. of Society of Telegraph Engineers*, 1879.
[5] *Roy. Soc. Phil. Trans.*, 1886.

In 1892 Glazebrook and Skinner traced some of the irregularities of the tube form of Clark cell to the indifferent amalgamation of the zinc rods. They found that when the zinc was in the clear zinc sulphate solution amalgamation did not always take place, but when contact was made with the mercurous sulphate amalgamation invariably resulted.

In 1909 Cohen and Tombrock [6] made an elaborate study of the electromotive properties of zinc amalgams. They used H vessels to contain the amalgams and used a dilute solution of zinc sulphate as electrolyte. All measurements were carried out at 0·5° C. As metallic zinc does not show a sharply defined potential towards its salts except when it has been deposited electrolytically, a 10 per cent zinc amalgam was used as a standard of reference. The results showed that after a few hours the electromotive difference between amalgams containing from 2·57 per cent to 10 per cent of zinc was less than 0·0001 volt, but with amalgams containing 60 per cent of zinc differences of 0·0009 volt were observed.

J. L. Crenshaw [7] studied the electromotive behaviour of liquid zinc amalgams at 25° C. and of two solid amalgams containing 2·5 and 5 per cent of zinc respectively. After two months the electromotive effects of the latter were identical within one microvolt.

In the general use of the Clark cell, amalgams containing 10 per cent of zinc are used. The amalgams are preferably made by the electrodeposition of zinc on a mercury cathode using acid zinc sulphate solution as an electrolyte. In practice no investigator has traced abnormalities in the electromotive force of a Clark cell to zinc amalgams prepared by electro-deposition.

(iv.) *The Negative Element of the Weston Cell. Cadmium Amalgam.*—Dearlove [8] first showed that the electromotive properties of cadmium amalgams depend on the percentage content of cadmium.

W. Jaeger [9] made measurements at 20° C. with amalgams varying in content from 1 to 20 per cent of cadmium, and concluded that at 20° C. all amalgams containing from 5 to 14·3 per cent of cadmium could be used in the anode limb of a Weston cell with the same resulting E.M.F. In 1902 H. C. Bijl [10] examined the electromotive properties of a series of cadmium amalgams at 20°, 25°, 50°, and 75° C., and from the data he obtained the conclusion may be drawn that an amalgam containing 12½ per cent of cadmium cannot

[6] *Konink. Akad. Wetensch. Amsterdam, Proc.*, Aug. 1909, xii.
[7] *Jour. of Phys. Chem.*, 1910, xiv.
[8] *Electrician*, 1893, xxxi.
[9] *Wied. Ann.*, 1898, lxv.
[10] *Zeitschr. f. physik. Chemie*, 1902, xli.

be usefully employed below 14° C. In 1908 E. Cohen and H. R. Kruyt[1] found differences of 0·04, 0·23, 0·13, 0·16, 0·20, and 0·20 millivolt between amalgams containing 10 and 12½ per cent of cadmium when the electrolyte between the amalgams was a solution of cadmium sulphate and the temperature was maintained at 0° C. Cohen suggested the general use of an 8 per cent amalgam for Weston normal cells.

F. E. Smith[2] made experiments with amalgams containing from 1 to 25 per cent of cadmium at temperatures from 0° to 60° C. Some of the amalgams experimented with were chilled and others were allowed to cool in a normal fashion.

Later S. W. J. Smith[3] pointed out the limitations of the Weston cell as a standard of electromotive force owing to small variations in the cadmium amalgams.

The results given by F. E. Smith show that an all-solid amalgam gives a higher E.M.F. when solidified by chilling; they show also that the amalgam in a Weston cell must be partially liquid if the E.M.F. is to remain constant. When, by increasing the temperature, a solid amalgam is converted into a mixture of liquid and solid phases there is an abrupt change in the electromotive properties of the amalgam. When stable, however, the E.M.F. of such an amalgam towards a solution of cadmium sulphate does not depend on the relative amounts of the two phases: hence, stable amalgams containing different percentages of cadmium, but possessing the two phases, have the same E.M.F. towards a cadmium sulphate solution. With amalgams all liquid or all solid the E.M.F. varies with the cadmium content.

Experiments on chilled and slowly cooled amalgams indicate that cadmium amalgams of such composition that, if homogeneous, they would be completely solid below certain temperatures, may not, in a Weston normal cell, have that E.M.F. towards a cadmium sulphate solution corresponding to such a solid. The general result is a lowering of the E.M.F. of the cell, which is due to a lack of homogeneity of the amalgam, the surface in contact with the cadmium sulphate solution being of smaller cadmium concentration than parts of the interior. Diffusion tends to restore uniformity, and in consequence such a cell is unstable for a very considerable time, the E.M.F. rising. On the other hand, in a cadmium amalgam of such composition that, if homogeneous, it would be all liquid above certain temperatures, the upper part may be a two-phase system and the lower part an all-liquid system of less cadmium concentration than the average. Diffusion quickly equalises the concentration and the E.M.F. falls to a normal value.

Cadmium, as purchased, often contains a trace of zinc which is very injurious in a Weston normal cell, and to avoid this it is essential to prepare the cadmium amalgam by electro-deposition of cadmium on mercury, the electrolyte being a slightly acid (H_2SO_4) solution of cadmium sulphate.

The fall of voltage from anode to cathode should not exceed 0·3 volt, and the approximate weight of cadmium deposited should be calculated from measurements of the current and the time. From the weight of mercury used as cathode and the final weight of the amalgam the percentage of cadmium in the latter can be calculated. Mercury should be added to reduce the cadmium content to 10 per cent. *Fig.* 21 shows the range of temperature over which amalgams of various cadmium content may be safely used.

Temperature ° C.	Percentage of Cadmium in the Amalgam									
	2.	4.	6.	8.	10.	12.	14.	16.	18.	20.
0	−3·65	+0·34	+0·35	+0·36	+0·36	+0·40	+2·49	+5·60	+12·95	+16·13
5	5·80	0·35	0·36	0·37	0·37	0·41	1·23	4·31	11·63	14·76
10	7·98	0·29	0·30	0·30	0·30	0·34	0·61	2·92	10·23	13·33
15	10·24	−0·79	0·17	0·18	0·18	0·22	0·63	2·44	8·70	11·88
20	12·50	2·91	−0·01	0·00	0·00	0·04	0·13	0·70	7·16	10·35
25	14·83	5·07	0·24	−0·23	−0·23	−0·18	−0·15	0·00	5·55	8·77
30	17·17	7·29	1·40	0·49	0·49	0·44	0·42	−0·40	3·72	7·10
35	19·59	9·54	3·62	0·80	0·79	0·76	0·74	0·72	1·97	5·39
40	22·02	11·84	5·83	1·14	1·12	1·12	1·11	1·10	0·24	3·60
45	24·46	14·15	8·04	2·96	1·46	1·45	1·45	1·44	−1·30	1·69

The above table shows the differences in millivolts between Weston normal cells containing amalgams of different cadmium content and a Weston normal cell (always at 20° C.) containing an amalgam with 10 per cent of cadmium.

(v.) *The Electrolyte of the Clark Cell. Zinc Sulphate.*—At ordinary temperatures the salt occurs in the form $ZnSO_4 \cdot 7H_2O$ and is apt to contain sulphates of lead and iron and also free sulphuric acid. The latter has the greatest

[1] *Zeitschr. f. physik. Chemie*, 1909, xlv.
[2] *Phil. Mag.*, 1910, xix.
[3] *Phys. Soc. Proc.*, 1910, xxii.

effect upon the electromotive force of a Clark cell and promotes the formation of gas over the zinc amalgam. Lord Rayleigh added a little carbonate of zinc to neutralise the free

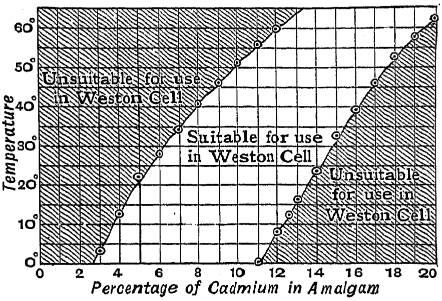


FIG. 21.

acid and found it a good plan to let the solution stand for a time, as a small quantity of iron salt was usually deposited even when so-called *pure* zinc sulphate was used. Examination of some cells of which the E.M.F. was low led him to believe that in these the zinc sulphate was supersaturated, but it appears probable that the low E.M.F. was due to the formation of the hexahydrate $ZnSO_4 \cdot 6H_2O$. The salt $ZnSO_4 \cdot 7H_2O$ when heated to 39° C. suffers the transition

$$ZnSO_4 \cdot 7H_2O \rightleftarrows ZnSO_4 \cdot 6H_2O + H_2O.$$

Below 39° C. a Clark cell containing $ZnSO_4 \cdot 7H_2O$ has the greater E.M.F. and is stable; above 39° C. the cell is unstable and one containing $ZnSO_4 \cdot 6H_2O$ has the greater E.M.F. These changes in the E.M.F. of a Clark cell in the neighbourhood of 39° C. were first discovered by Callendar and Barnes,[1] and were further investigated by Jaeger.[2] Callendar and Barnes found that the E.M.F. of cells with the heptahydrate could be measured at a temperature of about 42° C. but thereafter the E.M.F. became uncertain. A cell prepared with the hexahydrate can be measured down to 0° C. owing to the extreme slowness with which the hexahydrate changes to the heptahydrate form. Because of this, Clark cells should never be raised to temperatures above 39° C., and if the salt is recrystallised when purifying, the temperature of the solution should be less than 39° C.

Zinc sulphate, as purchased, often contains less than the theoretical amount of water of crystallisation owing to efflorescence. Callendar and Barnes[3] found the solubility of $ZnSO_4 \cdot 7H_2O$ to be 41·1 per cent of the solution at 39° C. and 29·4 per cent at 0° C. To purify the salt it is best to dissolve it in hot water and to add an excess of pure zinc oxide and hydrogen peroxide to oxidise any ferrous iron present. After keeping near 100° C. for several hours the solution should be filtered, acidified slightly, and evaporated on a water bath (electrically heated) until zinc sulphate commences to crystallise out. The mixture is then cooled with ice water, when much more of the zinc sulphate separates. The crystals which separate are washed, and dissolved in sufficient warm water at 35° C. to form a saturated solution. The solution is then made acid with H_2SO_4 to about 1 part in 10,000 and cooled to near 0° C. The crystals which now separate are drained in a funnel, using a suitable platinum or glass cone, and washed twice with a small quantity of cooled water.

(vi.) *The Electrolyte of the Weston Normal Cell. Cadmium Sulphate.*—At temperatures up to 74° C. a saturated solution yields crystals having the composition $CdSO_4 \cdot \frac{8}{3}H_2O$; above this temperature $CdSO_4 \cdot H_2O$ separates instead. The salt is very soluble and has a minimum solubility at a temperature of about 1° C. Over the ordinary range of temperature the increase of solubility with temperature is slight; thus at 1° C. about 75·5 parts by weight of $CdSO_4$ are dissolved in 100 parts of water and 76·8 parts at 25° C.

Kohnstamm and Cohen[4] concluded that a saturated solution of cadmium sulphate in water underwent a change in constitution at a temperature of about 16° C. and this they believed led to erratic changes in the E.M.F. of a Weston cell. This was disproved by H. v. Steinwehr[5] who made very careful solubility measurements and showed that such a change does not take place.

Hulett[6] states that 100 c.c. of water dissolves the following amounts of cadmium sulphate at the indicated temperatures.

Temp.	$CdSO_4$.	$CdSO_4 \cdot 8/3H_2O$.
	grams.	grams.
0° C.	75·5	112·5
15° C.	76·1	113·4
25° C.	76·8	114·7
40° C.	78·5	117·9

Cadmium sulphate does not appear to be isomorphous with any known salt and it can therefore be obtained in a very pure state by crystallisation. As purchased it often contains iron and some free acid. The latter may be eliminated by raising the temperature of the salt to about 700° C.; after such treatment the salt is dissolved in water, the solution filtered, and after making acid with H_2SO_4 to about 1 part in 10,000 the cadmium

[1] *Roy. Soc. Proc.*, 1897, lxii.
[2] *Wied. Ann.*, 1897, lxiii. 354.
[3] *Roy. Soc. Proc.*, 1897, lxii.
[4] *Wied. Ann.*, 1896, lix.
[5] *Ann. der Physik*, 1902, ix.
[6] *Phys. Rev.*, 1911, xxxii.

sulphate is crystallised out, very pure salt being thus obtained. Another method of purifying is to dissolve the purchased salt in hot water and render basic by adding an excess of cadmium oxide and some hydrogen peroxide. After heating the mixture at 100° C. for about four hours the solution is filtered, acidified slightly, and evaporated until the volume of liquid is reduced to rather less than one-quarter. The salt which separates is drained in a funnel having a glass or platinum cone and washed twice with cold water; it is then dissolved at room temperature in a slight excess of water containing about 1 part in 10,000 of H_2SO_4. The solution is filtered and set aside in crystallising dishes for the separation of cadmium sulphate crystals. After draining, the latter are washed twice with distilled water.

(vii.) *The Depolariser for Clark and for Weston Cells. Mercurous Sulphate.*—More than half of the researches on standard cells have really been investigations of the depolariser. In his first work on the Clark cell [1] Lord Rayleigh found irregularities due to the mixture of mercurous and zinc sulphates. He set up H type cells charged with pure mercury and depolariser in *both* limbs and filled up as usual with saturated zinc sulphate solution. There should, if the depolarisers were the same, have been no electromotive force, but he found one such cell to have an E.M.F. of 0·0059 volt and it remained tolerably constant for several days. To another such cell zinc carbonate was added and this greatly reduced the E.M.F. Believing the mercurous sulphate to be initially acid, cells were therefore set up with zinc carbonate added to neutralise the free acid and this reduced the irregularities. On neutralisation he observed that the paste turned yellow.

Glazebrook and Skinner [2] found that mercurous sulphate could not easily be purchased in a pure state and considered the depolariser to be the principal source of error in standard cells. The commercial preparations they met with were either grey or very white; the grey powder contained mercury in excess which was not harmful, but the white salt was found frequently to contain mercuric sulphate. When water is added to the latter it turns yellow in colour due to the formation of basic mercuric sulphate $3HgO{\cdot}SO_3$. This was believed to be not very harmful in small quantities, but at its formation a soluble acid mercuric sulphate $HgSO_4{\cdot}2SO_3$ is also formed. They recommended that the whole of the $HgSO_4{\cdot}2SO_3$ should be removed by washing. Further they recommended that mercurous sulphate contaminated with mercuric sulphate should be treated with mercury so as to convert the latter into mercurous salt.

[1] *Roy. Soc. Phil. Trans.*, 1885.
[2] *Ibid.*, 1892, clxxxix.

In 1901 Jaeger and St. Lindeck [3] confirmed the observations of Rayleigh and Glazebrook. They showed that cells containing mercurous sulphate from different sources may differ by more than 200 microvolts.

In 1904 Carhart and Hulett [4] advocated a standard method of preparing mercurous sulphate and expressed themselves strongly in favour of an electrolytic method of preparation. About the same time F. A. Wolff [5] also urged the adoption of a standard method of preparation and gave details of an electrolytic method used by him. F. E. Smith [6] made mercurous sulphate by three methods and found the products identical. Since that time many investigations have been made, and it is convenient to classify the methods and afterwards consider the electromotive properties of solutions of the salt.

(viii.) *Methods of Manufacture.* (*a*) *Electrolytic Method.*—Carhart, Hulett, and Wolff appear to have been the first to make mercurous sulphate by an electrolytic process and to use the product in a standard cell. The method has been largely employed at the Bureau of Standards, Washington. In a flat-bottomed glass vessel pure mercury is placed and is covered with dilute sulphuric acid to a depth of about 10 cm., the acid being prepared by adding one part by volume of concentrated H_2SO_4 to from 6 to 20 parts of distilled water. The solution is electrolysed by making the mercury the anode and a sheet of platinum hung in the acid the cathode; a current density of about 0·5 ampere per 100 sq. cm. surface of anode may be employed. A stirrer consisting of a glass rod bent at right angles at the bottom must be used to keep the surface of the mercury exposed and to stir the solution and the salt formed. A little over 4 grams an hour can be prepared in this way with a current of half an ampere. The preparation should be conducted in a dark room and the salt removed from the mercury by means of a separating funnel.

(*b*) *By Chemical Precipitation.*—This method has been much used at the National Physical Laboratory. Protonitrate of mercury is first made by the addition of nitric acid to mercury and the mercurous nitrate thus formed is added to hot dilute sulphuric acid; mercurous sulphate is precipitated. Fuller details of the method are given in the Specification at the end of this section.

(*c*) Other methods have been used but they are troublesome and are principally of interest

[3] *Zeitschr. f. Instrumentenk.*, 1901, xxi.
[4] *Trans. Am. Electrochem. Soc.*, 1904, v.
[5] *Ibid.*
[6] *British Association Report*, 1904.

because the products have been shown to have the same electromotive properties as the salts obtained by the methods already described. In one of these processes a purchased sample of mercurous sulphate is heated with concentrated H_2SO_4 to a temperature of about 150° C. and the hot clear acid is very carefully poured into dilute sulphuric acid (1 to 6), when precipitation of pure mercurous sulphate results. In another method fuming sulphuric acid is added to pure mercury and stirred well until the action between the two is practically at an end. Mercurous sulphate is thus formed in the cold and appears in the crystalline form after a few minutes.

Smith[1] first showed that the products obtained by these four methods had uniform electromotive properties, and other observers have confirmed this. Laporte[2] has employed an electrolytic method using alternating current of a frequency of about 40 cycles per second, and the product obtained has been proved to be quite satisfactory.

A special technical committee which met at Washington[3] set up standard cells with samples of mercurous sulphate made by chemical precipitation, by direct current electrolysis and by alternating current electrolysis. Cells set up with the mercurous sulphate prepared by the alternating current method were at first greater in E.M.F. by about 1 part in 10,000, but cells set up by the two other methods agreed within 1 part in 100,000.

(ix.) *Hydrolysis of Mercurous Sulphate.*—When pure mercurous sulphate is washed with water a yellow basic salt is produced the composition of which has been determined by Gouy[4] to be $(HgHO)_2 \cdot Hg_2SO_4$. This yellow product was formed and must have been present in most of the depolarisers of standard cells made previous to 1904, for until that date it was the invariable practice to wash the mercurous sulphate with water before using it as a depolariser.[5] Swinburne in 1891 favoured the washing of mercurous sulphate with zinc sulphate (for Clark cells), and Hulett[6] insists on the exclusion of the hydrolysed salt both in the preparation of Hg_2SO_4 and during the washing process. For Weston normal cells, saturated solutions of cadmium sulphate are recommended by Hulett for washing the salt free from acid, etc.

Measurements made by Hulett and by F. E. Smith, by totally different methods, show that 6 grams of Hg_2SO_4 are completely hydrolysed by about 3600 grams of water. Gouy has shown that the following equations represent the chemical actions:

$$3Hg_2SO_4 + 2H_2O = (HgHO)_2Hg_2SO_4 + Hg_2SO_4 + H_2SO_4,$$

or

$$3Hg_2SO_4 + 2H_2O = (HgHO)_2Hg_2SO_4 + 2HgHSO_4,$$

from which it follows that 6 grams of Hg_2SO_4 produce 3·75 grams of the hydrolysed salt. Further the addition of 0·395 gram of H_2SO_4 to this quantity of hydrolysed salt should result in the formation of Hg_2SO_4 and water. The presence of 0·395 gram of H_2SO_4 in 3600 c.c. of water (*i.e.* about 1 part in 9000 or 0·00224 normal H_2SO_4) should, therefore, prevent hydrolysis. For reasons which will be given later it is advisable for the final washing of mercurous sulphate to be made with 0·1 normal H_2SO_4, and if it is desired to preserve the salt for a considerable time it should be kept in a stoppered bottle in contact with acid of this strength.

The hydrolysis of mercurous sulphate in contact with cadmium sulphate solution has been studied by Smith, who added pure mercurous sulphate to solutions containing from ·06 x to 1·00 x parts of cadmium sulphate where x is the quantity of the salt in a saturated solution. With solutions containing up to 0·20 x parts of cadmium sulphate the mercurous sulphate was visibly hydrolysed, and with saturated solutions there was, after ten years, visible evidence of hydrolysis at the surface. Smith concludes, therefore, that even with saturated cadmium sulphate solutions mercurous sulphate is slightly hydrolysed, but the action is a slow one. In practice hydrolysis of the salt must be avoided. The reason for this will be apparent after consideration of the action of acids on solutions of cadmium and mercurous sulphates.

(x.) *Solubility of* Hg_2SO_4 *in Acid Solutions.*—The solubility of Hg_2SO_4 in water solutions of H_2SO_4 has been determined by Hulett and by Smith. Starting with water the solubility is about 0·47 gram of mercury in a litre. With the addition of acid the solubility at first diminishes to a minimum of 0·30 gram per litre, the normality of the acid being then about 0·04. Here there is a sharp change, and with a further increase in concentration of the acid the solubility increases to a maximum of about 0·44 for twice normal sulphuric acid. Hulett[7] says that hydrolysis of Hg_2SO_4 is effective unless the concentration of acid is as great as twice normal, but Smith

[1] *Roy. Soc. Phil. Trans.*, 1908, ccvii.

[2] *Report to Int. Committee on Electrical Units and Standards*, Washington, 1912.

[3] *Ibid.*

[4] *Compt. Rend.*, 1900, cxxx.

[5] See "Specification of Clark Cell," *Report of British Association*, 1894.

[6] *Amer. Electrochem. Soc. Trans.*, 1904, vi.

[7] *Amer. Electrochem. Soc. Trans.*, 1904, vi.

regards hydrolysis to be impossible if the normality exceeds 0·0023. *Fig.* 22 shows the change of solubility (grams of mercury per litre) with varying concentration of H_2SO_4. The most interesting portion of the curve is that corresponding to a minimum

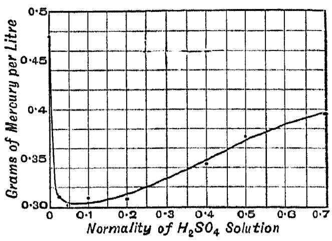


FIG. 22.

solubility of Hg_2SO_4 with a concentration of H_2SO_4 equal to about 0·05 normal.

(xi.) *Solubility of Mercurous Sulphate in Solutions of Zinc Sulphate.*—A saturated solution of zinc and mercurous sulphates contains about 0·84 gram of the latter. No measurements of the solubility of mercurous sulphate in unsaturated solutions of zinc sulphate appear to have been made.

(xii.) *Solubility of* Hg_2SO_4 *in Cadmium Sulphate Solutions.*—Smith has measured the solubility of mercurous sulphate in solutions containing various amounts of cadmium sulphate. The results obtained are:

Grams of $CdSO_4$ per Litre of Solution.	Approximate Weight of Mercury per Litre.	Grams of $CdSO_4$ per Litre of Solution.	Approximate Weight of Mercury per Litre.
	grams.		grams.
701*	1·1	7·0	0·25
561	1·00	1·8	0·27
350	0·92	0·9	0·30
140	0·61	0·45	0·31
35	0·33	0·00	0·45

* Saturated.

It will be observed that the minimum solubility corresponds to a normality of the cadmium sulphate solution of about 0·033. The change in solubility of mercurous sulphate with change of concentration of H_2SO_4 is, therefore, very similar to that occurring with change of concentration of cadmium sulphate, and since with the stronger solution the weight of mercurous sulphate in solution is greater than with the weaker solutions a complex salt is probably formed.

(xiii.) *Effect of Acid on the E.M.F. of the Clark Cell.*—Free acid in the Clark cell reduces the effective life of a cell owing to the formation of gas near the zinc amalgam. Hulett[1] gives the following values for Clark cells made with saturated solutions of zinc sulphate in x normal sulphuric acid:

E.M.F. (Volts).	x.	E.M.F. (Volts).	x.
1·42043	0·000 N	1·41810	1·012 N
1·42015	0·202 N	1·41595	2·024 N

(xiv.) *Effect of Acid on the E.M.F. of Weston Normal Cells.*—Hulett[2] rotated various cathode systems, with and without acid, for several days, and measured the change of E.M.F. produced by the acid, together with the weight of mercurous sulphate dissolved in a litre of the solution. The results are as follows:

Concentration of Sulphuric Acid.	Cathode Systems rotated for	E.M.F. of Cell at 25° C.		Weight of Hg_2SO_4 per Litre.
		Before rotation.	After rotation.	
Molar.	Days.			Gram.
1·012	8	1·01671	1·01678	0·90
0·506	5	1·01745	1·01747	0·93
0·2024	6	1·01802	1·01808	1·00
0·1012	10	1·01821	1·01823	1·04
0·0506	7	1·01834	1·01834	1·08
0·0202	13	1·01835	1·01847	1·13
0·0000	21	1·01836	1·01905	1·34
0·0000	21	1·01836	1·01940	1·47
0·0000	20	1·01836	1·02015	1·51

It is well known that acid decreases the E.M.F. of standard cells, and this may be seen at once from the tabulated results. On plotting the electromotive forces as a function of the acid concentration, curve A, *Fig.* 23,

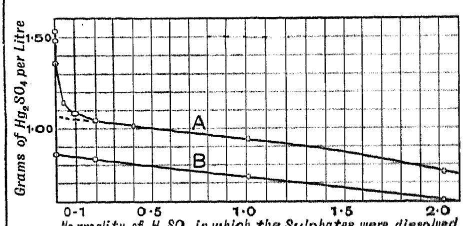


FIG. 23.

results. Curve B relates to corresponding observations with Clark cells, and it will be observed that the abnormal values with low acid concentration which occur in the Weston normal cell do not occur in the Clark cell. Indeed as a "standard" the Clark cell is undoubtedly better than the Weston. The high values obtained with non-acid cathode systems did not remain constant when the rotation was stopped, but decreased on standing.

[1] *Phys. Rev.*, 1908, xxvii.

[2] *Amer. Electrochem. Soc. Trans.*, 1908, xiv.

Smith has made a very extensive study of the effect of acid on the E.M.F. of the Weston normal cell. A large number of cells were set up with electrolytes consisting of cadmium sulphate dissolved in acids of varying strength, and these were kept under observation for ten years. *Fig.* 24 shows the relation between

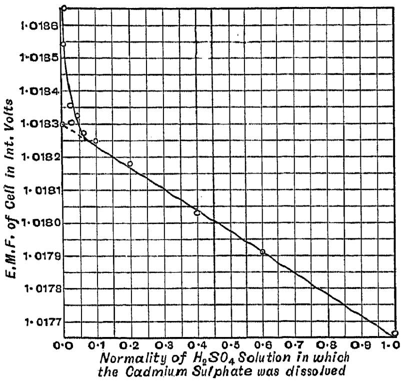


FIG. 24.

the E.M.F. of these cells and the normality of the sulphuric acid in which the cadmium sulphate was dissolved. When the solutions were made up with acid stronger than 0·4 normal, gas was produced after three years over the cadmium amalgam, but by gently warming the cell the gas could be displaced. The curve shows the values of the cells after the latter had been made up for two weeks. For cells containing from 0·10 normal sulphuric acid to 4·0 normal acid Smith found the following relation holds—

$$y = -(0{\cdot}00060x + 0{\cdot}00005x^2) \text{ volts},$$

where y is the change in E.M.F. and x is the normality of the sulphuric acid in which the cadmium sulphate is dissolved.

If the acid is in the negative limb (amalgam side) only the change in E.M.F. within the same limits of acidity is

$$y = 0{\cdot}01090x - 0{\cdot}00125x^2 \text{ volts},$$

and if the acid is in the positive limb only the change in E.M.F. is

$$y = -0{\cdot}01150x + 0{\cdot}00120x^2 \text{ volts}.$$

Thus, if there is 0·1 normal sulphuric acid in the positive limb only, the E.M.F. will be lowered because of it by 0·00114 volt. When, therefore, cells are made with acid solutions care should be taken to ensure that the acid is uniformly distributed.

Cells containing less than 0·05 N sulphuric acid are often very abnormal in behaviour. Cells containing no acid at all are at first usually quite normal, and often remain normal for many years if kept at a constant temperature. In many cases, however, the E.M.F. falls with time, and hysteresis effects can be obtained when the cells are subjected to a temperature cycle. Cells made up with cadmium sulphate dissolved in 0·1 normal sulphuric acid do not show such anomalous behaviour.

(xv.) *Interaction between Mercury and the Sulphates of Cadmium and Mercury in Solution.*—Hulett [1] found that when the cathode system (consisting of mercury, mercurous sulphate, and cadmium sulphate solutions) of a cell was rotated for a few hours a very decided change in the electromotive force was produced. In all cases the voltage rose, and in some cases a rise of two millivolts was experienced. After various experiments the conclusion was arrived at that the mercurous sulphate was hydrolysed by the cadmium sulphate solution, the action being a slow one. By direct analysis the amount of mercury in solution was also found to be increased by agitation of the cathode system for several days.

Wolff and Waters [2] rotated cathode systems of mercurous and cadmium sulphate solutions and solids together and found the changes in the E.M.F. of cells not to exceed 1 part in 10,000 even when the rotation was continued for four months. They concluded that all samples of mercurous sulphate do not behave abnormally.

Smith made a large number of experiments with rotated cathode systems, and concludes that no hydrolysis of Hg_2SO_4 is possible in the presence of 0·0023 normal sulphuric acid, but that the presence of this quantity of acid does not prevent an interaction between the two sulphates in solution which is detrimental to the Weston cell. At times the result of such interaction is to increase the electromotive force by 3 millivolts, an increase which is probably due to other mercury compounds being produced in solution the mercury ion concentrations of which are different from that of mercurous sulphate. In the general body of the solution (excluding that in contact with mercury) the mercury ion concentration appears to be greater than that corresponding to mercurous sulphate only, and as a result the electromotive force of the cell may be increased by agitation. This solution of high mercury ion concentration is not, however, in contact with mercury, and suffers a reduction on being brought into such contact. At the surface layer of the mercury, where solid mercurous sulphate is in contact with mercury, there appear to be exceedingly minute differences of potential causing parasitic currents which lower the mercury ion concentration in the surface layer of liquid, and as a result the E.M.F. slowly falls below the normal value.

To produce such parasitic currents in the

[1] *Amer. Electrochem. Soc. Trans.*, 1908, xiv.
[2] *Bureau of Standards Bull.*, 1907, iv.

surface layers Smith made cells containing pastes which were mixtures of hydrolysed and unhydrolysed mercurous sulphates. It was anticipated that the solutions of these salts in contact with the mercury would be similar and independent of the proportion of hydrolysed and unhydrolysed salts, since both were in excess. This was very nearly the case when the cells were first set up, the maximum difference between the E.M.F.'s of any two cells being 3 parts in 10,000. The second anticipation was that, since a cell containing only hydrolysed salt has an E.M.F. about 0·02 volt lower than a cell with unhydrolysed salt, and further since both of these salts were in contact with mercury in the cells under consideration, there must have been a slightly higher mercury ion concentration in the solution near particles of the unhydrolysed salt than in that near particles of the hydrolysed salt. At the mercury surface such a difference in the mercury ion concentration must result in a difference of potential between parts of the conducting surface. Such a difference cannot be maintained, and the tendency must be to *lower* the mercury ion concentration in the liquid near the surface of the mercury; also near the surface the proportion of unhydrolysed salt must gradually decrease. If this anticipation is correct the speed of the action should be related to the proportion of hydrolysed salt present in the original mixture. *Fig.* 25 shows the results obtained with six

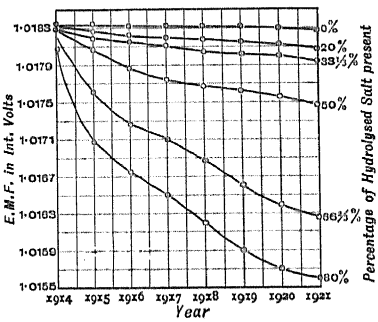


FIG. 25.

cells over a period of seven years. The results are in accordance with the explanation given. It follows also that finely divided mercury distributed throughout the paste should result in the elimination of abnormally high E.M.F.'s, but will not prevent a fall of E.M.F.

From these experiments it is concluded that hydrolysis and interaction between cadmium and mercurous sulphates must be avoided, and that the best way of doing this is by using an electrolyte of cadmium sulphate dissolved in 0·1 normal sulphuric acid.

(xvi.) *Influence of Size of Crystals.*—H. von Steinwehr [1] pointed out that the size of the mercurous sulphate crystals might influence the E.M.F. of a standard cell, and concluded from his experiments that the variations in E.M.F. were due almost entirely to variations in the size of the crystals. Smith [2] further investigated this matter and took microphotographs of crystals prepared by several methods; the conclusion is that no large crystals of mercurous sulphate which are sufficiently soluble to act as an efficient depolariser can result in the E.M.F. of a cell being appreciably less than when crystals from 0·003 to 0·030 mm. long are used. It is possible that in von Steinwehr's experiments the very small crystals more rapidly interacted with the cadium sulphate solution and produced a complex salt, and thus by their size indirectly affected the E.M.F.

Wolff and Waters [3] examined many samples of mercurous sulphate under the microscope and found very few particles possessed a length less than 0·001 mm., the average dimension being considerably greater. They concluded that, in general, any effect due to the size of the crystals may be disregarded.

(xvii.) *Change of E.M.F. with Temperature of the Weston Normal Cell.*—Wolff [4] made a very complete investigation on 200 cells for a cycle of temperature from 0° to 40° C. Some of the cells were abnormal in their behaviour, and the results with these were rejected. The conclusion is that the relation between E.M.F. and temperature is best represented by the formula

$$E_t = E_{20} - 0{\cdot}00004075(t-20) - 0{\cdot}000000944(t-20)^2 + 0{\cdot}0000000098(t-20)^3.$$

This formula was recommended for general use by the International Conference on Electrical Units and Standards (London, 1908). Other formulae have been obtained by Jaeger and Lindeck [5] and by Smith,[6] but the number of cells examined by these investigators was comparatively small, and the formulae are not, therefore, so reliable. The following table gives the electromotive force of a Weston normal cell from 0° C. to 40° C. when its E.M.F. is 1·01830 international volts at 20° C.:

Temperature ° C.	E.M.F. in International Volts.	Temperature ° C.	E.M.F. in International Volts.
0	1·01866	25	1·01807
5	1·01866	30	1·01781
10	1·01860	35	1·01761
15	1·01848	40	1·01718
20	1·01830		

[1] *Zeitschr. f. Instrumentenk.*, 1905, xxv.
[2] *Roy. Soc. Phil. Trans.* A, 1908.
[3] *Bureau of Standards Bull.*, 1907, iv.
[4] *Ibid.*, 1908, v.
[5] *Ann. d. Phys.*, 1901, v.
[6] *Roy. Soc. Phil. Trans.*, 1908.

(xviii.) *Temperature Coefficient of each Limb.*—Smith[1] showed that although the temperature coefficient of the complete Weston cell is small, the temperature coefficients of the separate limbs are comparatively large, and in consequence it is important that in use the cell be at a uniform temperature. At 20° C. the temperature coefficient of the positive limb is + 0·00031 volt for a rise of temperature of 1° C., and that for the negative limb is − 0·00035 volt for a rise of temperature of 1° C.

(xix.) *Polarisation Effects.*—In 1884 Lord Rayleigh made some experiments on the polarisation of Clark cells and showed that the effect of short circuiting for a few minutes passed rapidly away. Smith[2] made tests on four Weston normal cells short circuited for one minute, five minutes, five hours, and five days respectively. The cell short circuited for one minute recovered within one ten-thousandth of a volt one minute afterwards; but forty minutes were occupied in its recovery within 1 part in 100,000. The cell which was short circuited for five minutes was nearly 0·001 volt low one minute afterwards; at the end of two minutes it was 0·0005 volt low, and after five minutes it had recovered within 0·0002 volt. The recuperation of the cell short circuited for five hours was very slow; after one minute its E.M.F. was about 0·1 volt; at the end of four minutes it was 0·9 volt, and five hours afterwards it had recovered within 0·0004 volt. It became nearly normal three weeks afterwards. The cell which was short circuited for five days had an E.M.F. less than 0·05 volt five minutes after breaking the circuit, but within twenty-four hours its E.M.F. was normal within 2 parts in 10,000. It appears clear, therefore, that accidental short circuits of standard cells do not, in general, result in permanent injury to them.

§ (46) Change of E.M.F. with Temperature of the Clark Cell.—Lord Rayleigh (1886) gave the equation

$$E_t = 1{\cdot}435\{1 - 0{\cdot}00082(t - 15)\}$$

to connect temperature and E.M.F. In 1892 Glazebrook and Skinner made observations with cells mostly in the neighbourhood of 0° to 15° C., and gave 0·00076 as the average value of the coefficient between these temperatures. Their experiments did not enable them to decide whether the relation between E.M.F. and temperature is linear or not.

§ (47) Stability of the Weston Normal Cell.—The stability of the Weston cell has been studied by many investigators, but more particularly by Hulett, Smith, Wolff, Carhart, Shaw, and Reilley. When made with neutral cadmium sulphate solution Hulett and Smith believe the cell to be slightly unstable, the electromotive force falling with time; they recommend the inclusion of acid to reduce the instability. There are, however, numerous instances of cells made up with neutral cadmium sulphate solutions remaining stable for many years, and it has been definitely established that new cells set up by observers in England, Canada, America, France, Japan, Holland, Russia, Germany, and other countries have, when first set up, electromotive forces which are the same within 20 or 30 microvolts.[3] The following table gives the results of some comparisons made in Washington in 1910 of Weston cells made in England, America, France, and Germany:

Place of Manufacture.	No. of Cells tested.	Mean Variation between Cells.	Difference of Groups from Mean of all.
		Microvolts.	Microvolts.
National Physical Laboratory . .	34	± 9	− 3
Bureau of Standards	40	± 6	− 8
Laboratoire Central d'Électricité . .	15	±25	+10
Physikalisch Technischen Reichsanstalt	15	±10	+ 1

All the cells were comparatively new ones.

In 1908 Smith[4] gave details of 10 cells which, although prepared in an apparently normal manner, fell slowly but steadily in electromotive force. In five years the fall in E.M.F. of one of these cells was more than a millivolt. Other cells of the same age kept constant in E.M.F. within 30 microvolts. Hulett[5] found a large number of cells to continually fall in E.M.F., and in one instance measured a change of over 5 millivolts in less than two years. Wolff and Waters[6] and Carhart[7] observed but very small changes in E.M.F., and Shaw and Reilley[8] observed changes not greater than 140 microvolts in ten years.

Smith made special cells containing 0·1 gram only of mercurous sulphate, and found the fall of E.M.F. to be more pronounced. From the results so far published it appears that while many cells may remain constant within 100 microvolts for many years, the chemical system of a Weston cell is not a stable one, and that the E.M.F. is subject to fluctuations owing to an interaction between

[1] *Phys. Soc. Proc.*, 1910, xxii.
[2] *Roy. Soc. Phil. Trans.*, 1908, ccvii.

[3] See Smith, *Roy. Soc. Phil. Trans.*, 1908, ccvii.; Shaw and Reilley, *Roy. Soc. Canada Trans.*, 1919; *Report of International Committee, Washington*, 1912; *Travaux de Laboratoire Central d'Électricité*, 1908; *Report of Electrotechnical Laboratory, Tokyo*, 1913; *Konink. Akad. van Wetensch. Amsterdam, Proc.*, 1910, xiii.
[4] *Roy. Soc. Phil. Trans.*, 1908, ccvii.
[5] *Phys. Rev.*, 1908, xxvii.
[6] *Bureau of Standards Bull.*, 1907, iv.
[7] *Phys. Rev.*, 1908, xxvi.
[8] *Roy. Soc. Canada, Trans.*, 1919.

neutral solutions of mercurous and cadmium sulphates. With old cells and others comparatively new but abnormal, most investigators have observed that the E.M.F. increases slightly after agitation but falls again with time. The same effect is produced by increasing the temperature of the cell for a short time and then cooling to the original temperature. Both actions result in the surface layer of liquid, of low mercury ion concentration, being displaced by other liquid of higher mercury ion concentration with a consequent rise in E.M.F. The E.M.F. is not stable, however, but very slowly falls. A number of such cells were noted by Wolff [1] when measuring the mean temperature coefficient of a large number of Weston cells.

It follows that if the liquid of high mercury ion concentration is not stable when in intimate contact with mercury and mercurous sulphate, the distribution of mercury in fine globular form throughout the paste is advantageous. Data obtained by numerous experimenters appear to support this view. Such distribution of mercury throughout the paste does not, however, prevent a slow fall in electromotive force. To prevent the latter, free sulphuric acid must be present, the saturated cadmium sulphate solution being made up in 0·1 normal sulphuric acid.

§ (48) Specification for Clark and Weston Standard Cells.—The International Conference on Electrical Units and Standards, London, 1908, agreed on a brief but incomplete specification for the Weston normal cell, and assigned the duties of specifying more particularly the conditions under which the cell was to be set up to another Committee. No specification giving details of procedure has yet been issued. The following specifications are based on the work of numerous experimenters: that relating to the Weston cell contains the instructions of the London Conference together with notes on procedure, etc., based on information obtained since the Conference. The instructions are indicated by a numeral, and the added notes by a numeral and a letter.

§ (49) Specification for the Clark Cell.—(No specification was prepared by the London Conference, 1908.)

1. The Clark cell is a voltaic combination which has a saturated solution of zinc sulphate as its electrolyte, mercury covered with mercurous sulphate and zinc sulphate crystals as the positive element, and zinc amalgam covered with zinc crystals as the negative elements.

2. To prepare pure zinc sulphate, dissolve the purchased salt in hot water to form a saturated solution and add pure zinc oxide in excess and about 1 per cent of hydrogen peroxide. Heat over a water bath for about four hours, filter, acidify slightly with H_2SO_4, and again heat over the water bath until the salt begins to be deposited. Remove the mixture and cool with ice water or very cold liquid to ensure much of the zinc sulphate separating. Filter, re-dissolve the crystals in warm water (not exceeding 35° C.) to make a saturated solution, make slightly acid with H_2SO_4 (acidity of solution should not exceed 1 in 10,000), and again cool with ice water to separate the zinc sulphate. The latter crystals are filtered off, rinsed twice with cold water, and are then ready to use. Acid solutions must not be used.

3. Mercury.—This is prepared in the same manner as for the Weston cell.

4. Zinc Amalgam.—This is prepared in the same manner as for the Weston cell, except that slightly acid zinc sulphate solution is used as the electrolyte and a zinc rod as the anode. The final amalgam should contain 10 per cent of zinc.

5. The depolariser is a paste consisting of mercurous sulphate, fine globules of mercury, zinc sulphate crystals, and saturated solution of zinc sulphate. The mercurous sulphate is prepared in the same way as for the Weston cell, but to remove all traces of acid, saturated neutral zinc sulphate solution should be used as the washing liquid.

6. For setting up the cell the H form is the most suitable, but the platinum wire in the limb containing the zinc amalgam should be covered with a thin sheath of glass at least 4 mm. long (see *Fig.* 20 (*d*)). The general process of filling the cell is the same as for the Weston cell. Acid must be avoided.

7. The electromotive force of the Clark cell is

1·4326 international volts at 15·0° C.

and

1·4333 volts (10^8 C.G.S. units) at 15·0° C.

At any other temperature t between the limits 10° and 25° C. the electromotive force may be obtained from the equation

$$E_t = E_{15} - 0{\cdot}00119(t-15) - 0{\cdot}000007(t-15)^2.$$

§ (50) Specification for Weston Normal Cell.—1. The Weston normal cell is a voltaic cell which has a saturated aqueous solution of cadmium sulphate ($CdSO_4 \cdot 8/3H_2O$) as its electrolyte. The electrolyte must be neutral to Congo red.

1*a*. Cadmium sulphate crystals should be dissolved in hot water and the solution rendered basic by the addition of cadmium oxide and about 1 per cent of its volume of hydrogen peroxide. The mixture should be heated over a water bath for about four hours, after which the solution should be filtered,

[1] *Amer. Electrochem. Soc. Trans.*, 1908.

acidified slightly with H_2SO_4 and evaporated until its volume is reduced to about one quarter. The salt which has separated should be drained in a funnel, using a glass or platinum cone, and rapidly rinsed twice with cold water; it should then be dissolved at room temperature in a slight excess of water containing about 1 part in 10,000 of H_2SO_4, and after filtering, be set aside in crystallising dishes for the separation of cadmium sulphate crystals. The latter should be rinsed with cold distilled water. To make a saturated solution the crystals are to be powdered and agitated with the solvent for a long time.

For neutral solutions of cadmium sulphate the salt is dissolved in distilled water. For acid solutions the salt is finely powdered and dissolved in 0·1 normal sulphuric acid. No solution should be used until saturated.

2. The positive electrode of the cell is mercury.

2*a*. The simplest way to purify commercial mercury is to treat it electrolytically and afterwards distil it. Place the mercury in a large crystallising dish, and in the centre place a small dish or crucible of glass or porcelain also containing a little mercury. Over the mercury surfaces pour a sufficient quantity of a mixture of dilute sulphuric and nitric acids (90 of water, 5 of H_2SO_4, and 5 of HNO_3) to cover the top of the smaller dish at a depth of about 2 cm. Platinum wires sealed into glass tubes should make contact with the two lots of mercury, and a current of about half an ampere should be passed for two hours per 1000 grams of mercury, from the mercury in the large dish through the electrolyte to the mercury in the small dish. The mercury in the large dish should then be removed, washed and dried, and distilled *in vacuo*.

3. The negative electrode of the cell is cadmium amalgam, consisting of 12·5 parts by weight of cadmium in 100 parts of amalgam.

3*a*. The amalgam specified by the London Conference should not be used. An amalgam consisting of 10 parts by weight of cadmium in 100 parts of amalgam should be substituted.

To prepare the amalgam, deposit cadmium on mercury by electrolysis. Place a small dish containing a weighed quantity of mercury in a large crystallising dish, and add sufficient dilute slightly acid cadmium sulphate solution to cover the top of the dish at a depth of about 2 cm. A platinum wire sealed in a glass tube should make contact with the mercury. A rod of commercial cadmium enclosed in a sheath of filter-paper has its lower end immersed in the solution. A current of about half an ampere should be passed from the cadmium to the mercury sufficiently long to ensure that more than one-tenth of the cathode is cadmium. The cathode mixture should be removed, melted over dilute H_2SO_4 (1 of acid to 20 of water), washed, and weighed. Sufficient mercury should then be added to make the cadmium content 10 per cent, the mixture being again heated and stirred to render homogeneous.

4. The depolariser, which is placed in contact with the positive electrode, is a paste made by mixing mercurous sulphate with powdered crystals of cadmium sulphate and a saturated aqueous solution of cadmium sulphate. The different methods of preparing the mercurous sulphate are described in the Notes. One of the methods there specified must be carried out.

4*a*. The Notes referred to in 4 described chemical and electrolytic methods of preparation. The simplest method is that of chemical precipitation.

Protonitrate of mercury is first formed by adding about 30 c.c. of concentrated nitric acid to 200 grams of mercury; when the action is over or nearly over, the resulting salt and solution is added to about 400 c.c. of warm dilute nitric acid (1 of acid to 40 of water). This acid solution of mercurous nitrate is filtered, and then run as a very fine stream into two litres of hot dilute sulphuric acid (1 of acid to 3 of water), the liquid being well stirred during the mixing. A small quantity of mercury is added to the dilute H_2SO_4 before running in the mercurous nitrate. The dilute acid should be prepared immediately previous to the precipitation, as its temperature is then suited for the precipitation of well-defined crystals of mercurous sulphate. The salt thus formed is washed two or three times by decantation with dilute H_2SO_4 (0·1 normal) and filtered. After several washings of the filtrate with 0·1 normal sulphuric acid it is washed several times with saturated cadmium sulphate solution. The latter is to be neutral if the electrolyte of the cell is to be neutral, but if the electrolyte is to be an acid one the cadmium sulphate solution is likewise acid. In the case of an acid electrolyte, the mercurous sulphate, after washing, is placed in a clean stoppered bottle together with some of the acid cadmium sulphate solution, some powdered cadmium sulphate crystals, and about its own weight of mercury. These are agitated in order to produce a uniform distribution of acid.

If the electrolyte is to be neutral it is desirable to have a large number of fine globules of mercury distributed throughout the paste. This result may be achieved by the violent agitation of the mercurous sulphate with mercury, powdered crystals of cadmium sulphate, and sufficient saturated cadmium sulphate solution to form a thick paste. After such agitation the paste is ready for making up a cell.

5. For setting up the cell, the H form is the most suitable. The leads passing through the glass to the electrodes must be of platinum wire, which must not be allowed to come into contact with the electrolyte. The amalgam is placed in one limb, the mercury in the other.

The depolariser is placed above the mercury and a layer of cadmium sulphate crystals is introduced into each limb. The entire cell is filled with a saturated solution of cadmium sulphate and then hermetically sealed.

5*a*. The platinum wires inside the cell are to be amalgamated by passing an electric current from a platinum anode through an acid solution of mercurous nitrate to each of the wires as cathode. Afterwards, the vessel should be washed out twice with dilute nitric acid and several times with distilled water; it should be dried in an oven. The cadmium amalgam is fused, and sufficient of it to cover one of the amalgamated wires completely is introduced into one of the limbs of the H vessel. Into the other limb of the vessel sufficient mercury is introduced to cover the amalgamated wire; then the paste, finely divided crystals of cadmium sulphate, and saturated cadmium sulphate solution are added in the order named by means of small pipettes with wide orifices. To the cadmium amalgam, lime, cadmium sulphate crystals and electrolyte are added. Whenever, in filling the cell, the materials accidentally come in contact with the part of the glass which is to be heated in sealing, it is first cleaned by wiping with filter paper slightly moistened with distilled water and then with dry paper.

When cells with acid electrolyte are set up it is desirable, after making a saturated solution of cadmium sulphate in 0·1 normal sulphuric acid, to add to this solution all the cadmium sulphate crystals it is intended to use in the solid form. The crystals should be finely powdered. The whole of the solution and some of the cadmium sulphate crystals (about $\frac{1}{5}$ of the volume of mercurous sulphate) should be added to the washed mercurous sulphate, and the whole agitated to ensure a uniform distribution of acid. After standing, the liquid should be poured off and filtered, leaving the mixed solid sulphates in the form of a thick paste ready to insert in the cell. The filtrate is added to the residue of cadmium sulphate crystals, and these are introduced in the cell in the manner already described.

§ (51) E.M.F. of Weston Normal Cell.—The electromotive force of the Weston normal cell is

1·0183 international volts at 20·0° C.

and

1·0188 volts (10^8 C.G.S. units) at 20·0° C.

At any other temperature t between the limits 0° and 40° C. its electromotive force may be obtained from the equation

$$E_t = E_{20} - 0{\cdot}0000406(t-20) - 0{\cdot}00000095(t-20)^2 + 0{\cdot}00000001(t-20)^3.$$

Acid cells containing electrolyte made by dissolving $CdSO_4{\cdot}8/3H_2O$ in 0·1 normal sulphuric acid have a lower electromotive force by 62 microvolts.

Weston normal cells with neutral cadmium sulphate solutions have been set up in most countries. The electromotive force in international volts has been determined in England, the United States, Holland, Japan, Germany, and Russia. The mean value is 1·01827 international volts at 20° C. In 1910 the value 1·0183 international volts at 20° C. was recommended for universal adoption by Lord Rayleigh's Committee on Electrical Units and Standards. As previously stated, Weston cells set up with neutral electrolyte often remain constant for many years, but cells containing electrolyte made by dissolving cadmium sulphate in 0·1 normal sulphuric acid are much more constant. Clark cells if set up with care are even more reliable, but suffer the disadvantage of a large temperature coefficient.

Bronson and Shaw[1] found the ratio of the E.M.F. of the Weston normal cell at 25° C. to that of the Clark cell at 25° C. to be 0·71696.

F. E. S.

Electricity Meters. See "Watt-hour and other Meters for Direct Current. I. Ampere Hour Meters. II. Watt-hour Meters"; "Alternating Current Instruments," Parts IV. and V.

Electro-chemical Electricity Meters. See "Watt-hour and other Meters for Direct Current. I. Ampere Hour Meters," § (2) (ii.).

Electro-chemical Equivalent: the ratio of the mass of any substance deposited in an electrolytic cell to the quantity of electricity which has passed.

Electro-chemical Equivalent of Silver, Determination of, and Results. See "Electrical Measurements," § (36).

Electrode: the means by which an electric current finds its way into or out of a conductor, usually a plate of metal immersed in a liquid, or supported in a tube containing gas at low pressure.

Electro-deposition of Metals. See "Electrolysis, Technical Applications of," § (7).

Electrodes and shape of samples used in measurement of Insulation Resistance materials. See "Measurement of Insulation Resistance," § (7) (ii.).

[1] *Electrician*, 1911, lxvi.

ELECTRODYNAMOMETERS: instruments depending for their action on the mechanical force existing between two coils carrying current. See "Alternating Current Instruments," §§ (2)-(13). For application to absolute current measurement see "Electrical Measurements," § (29) *et seq.*

Use of, as Detecting Instrument in Alternating Current Measurements. See "Inductance, The Measurement of," § (25).

ELECTROLYSIS: the decomposition of a conducting medium—usually a liquid—by the passage of an electric current. See "Electrolysis and Electrolytic Conduction," § (1) *et seq.*

Earliest Systematic Study of, made by Faraday. See *ibid.* § (1).

Experimental Results. These are found to be in accord, in certain particular cases, with the relation

$$\left(\frac{\lambda}{\lambda_\infty}\right)^2\left(\frac{N}{1-(\lambda/\lambda_\infty)}\right)=K,$$

where N is the number of gramme molecules per unit vol., λ the conductivity per equivalent per c.c., and K a dissociation constant; but there are many electrolytes for which the relation between λ and N is not that predicted by this formula. See *ibid.* § (9).

Faraday's Electrolytic Constant: the quantity F, the same for all ions, in the expression of Faraday's laws of electrolysis in the form $w=(W/nF)q$, where w is the weight in grammes of the ion deposited upon the electrode during the passage of q coulombs, W is the chemical atomic weight of the element from which the ion is derived (if it is elementary, or the group weight if it is complex), and n its chemical valency. The value of F is found by experiments upon an electrolyte for which the experimental data can be fixed with the greatest precision; within 1 part in 1000 F = 96,500 coulombs. See *ibid.* § (2).

Faraday's Laws of. The weight of the deposit upon either electrode during electrolysis is directly proportional to the quantity of electricity passed through the electrolytic cell, and also to a quantity known as the "electro-chemical equivalent" of the substance deposited. See *ibid.* § (2).

ELECTROLYSIS AND ELECTROLYTIC CONDUCTION

§ (1) INTRODUCTORY.—When an electric current passes through a metallic conductor there is no obvious change in the composition of the conductor either where the current enters or leaves, or indeed in any part of the conductor through which the current flows. It is otherwise when the conductor is an "electrolyte," for example a solution of a salt in water. In this case the conduction of the current is accompanied by the convection of material substances, parts of the electrolyte, which are frequently deposited upon the metallic "electrodes" by which the current enters and leaves the solution.

The deposit upon one electrode is different from that on the other, so that two currents of matter, flowing in opposite directions, accompany the passage of electricity through the solution.

The earliest systematic study of electrolysis was made by Faraday. It was through his work that the terms "anode" and "cathode," "anion" and "cation," came into use. The anode is the metallic conductor at which the current enters the electrolyte, and the cathode is that at which it leaves. The anion is the constituent of the electrolyte which migrates towards the anode, and the cation that which migrates towards the cathode.

The electrolytes with which we shall be concerned almost exclusively in this article are those in which the solvent is water. Perfectly pure water is a very feeble conductor of electricity. According to Kohlrausch, its conductivity at ordinary temperatures is about $0 \cdot 04 \times 10^{-6}$ ohm.$^{-1}$ cm.$^{-1}$ (cf. Washburn and Weiland, *Am. Chem. Soc. J.*, 1918, xl. 106, 131). Special precautions are, however, required in order to obtain water of this purity. The conductivity of ordinary carefully distilled water is usually thirty or forty times as great, say, from 1 to 2×10^{-6} ohm.$^{-1}$ cm.$^{-1}$.

Small quantities of dissolved substances produce large increases of conductivity. For example, the conductivity of a solution containing 1·5 parts by weight of KCl dissolved in 10,000 parts of water is about 250×10^{-6} ohm.$^{-1}$ cm.$^{-1}$. In this case, as in many others, the added substance has no appreciable conductivity before it is dissolved.

§ (2) FARADAY'S LAWS.—Faraday showed that the weight of the deposit upon either electrode during electrolysis is directly proportional to the quantity of electricity passed through the electrolytic cell, and also to a quantity known as the "electro-chemical equivalent" of the substance deposited.

The laws which he established can be expressed by the equation

$$w=\frac{W}{nF}q,$$

where w is the weight in grammes of anion deposited upon the anode, or of cation deposited upon the cathode, during the passage of q coulombs. In this equation the electro-chemical equivalent, or the deposit

per coulomb, is written in the form W/nF, in order to show how it depends upon the chemical properties of the substance deposited. W is the chemical atomic weight of the element from which the ion is derived (if it is elementary, or the group weight if it is complex), and n is its chemical valency; while F is a quantity, the same for all ions, known as the Faraday electrolytic constant.

Assuming the truth of Faraday's laws, the value of the electrolytic constant can be found by experiments upon any electrolyte; but preferably, of course, upon one for which the experimental data can be fixed with the greatest precision. If for silver we take $w/q=0{\cdot}0011183$ gramme per coulomb and $W/n=107{\cdot}88$ we get for F the value 96,470. For most practical purposes F can be taken as 96,500 coulombs.

§ (3) Theoretical Interpretation.—Faraday's laws are the result of direct experiment, and refer only to what happens at the electrodes; but they suggest a theory as to what happens within the solution. Thus suppose we take the case of a dissolved salt, say MA, which when electrolysed deposits the metal M at the cathode and the acid radicle A at the anode. Let us suppose also, for simplicity, that the chemical valencies of M and A are the same and each equal to n, and that their atomic weights are m and a respectively.

The experimental fact is that one gramme molecule ($m+a$ grammes) of the salt is electrolysed when nF coulombs of electricity are passed through the solution.

We may represent this fact by stating that m grammes of the metal carry $+nF$ coulombs of electricity with them from the solution to the cathode, during deposition, and that a grammes of the acid ion carry $-nF$ coulombs with them from the solution to the anode.

We may extend this result, hypothetically, to the interior of the solution and say that every m grammes of the metal taking part in the electrolytic conduction carry the charge $+nF$ coulombs, while every a grammes of the acid radicle taking part carry the charge $-nF$ coulombs.

In order to account for such charges on the ions we may suppose that originally every molecule of the substance MA is electrically neutral, but that the forces which bind together the basic radicle M and the acid radicle A are, partly at least, of electrostatic origin; so that when the components of the molecule separate they carry with them equal and opposite charges.

§ (4) Ionisation.—Granting the existence of such oppositely charged ions in solution, we see at once why there should be two currents of matter, in opposite directions, when an electromotive force is applied; but it still remains to account for their presence.

The "ionisation," as it is called, must be either a spontaneous effect accompanying solution or an effect produced by the electric field which causes the current to flow. If it were the latter, electrolytic conduction would not, as it does, obey Ohm's law. For the strength of the current must obviously depend upon the number of molecules split up at any moment and also upon the rates at which the separated portions move. The latter clearly depend upon the applied electromotive force, and if, by hypothesis, the former does also, then the current could not, in general, be proportional to the first power of the electromotive force, *i.e.* Ohm's law could not apply.

We are compelled, therefore, to turn to the other hypothesis, namely, that the ionisation is spontaneous and results from solution.

We have here to discriminate between various possibilities. Thus the ionisation may be complete or it may be partial. All the molecules or only a fraction of them may be ionised. In the latter case the ionisation may be permanent or it may be intermittent. It may be always the same molecules that are split up or merely the same fraction of the total number.

Later we shall see how far it is possible to decide between these different possibilities.

§ (5) Calculation of Conductivity.—Assuming the existence of ionisation, we can find a quantitative expression for the conductivity of a solution. For this purpose it is unnecessary to know the actual weights of the charged particles. It is sufficient (keeping to the case already considered) to assume only that the masses m and a grammes carry the charges $\pm nF$ coulombs. For convenience, such masses with their associated charges are called "gramme ions."

Suppose the solution to contain p grammes of a substance whose molecular weight is q dissolved in a volume r c.c. of the solution. The number of gramme molecules per unit of volume is then $N=p/qr$. Suppose that of this number the fraction αN is ionised and that each gramme molecule splits into two gramme ions of valency n carrying equal and opposite charges $\pm nF$ respectively. When a potential gradient is applied these ions will begin to move. The manner in which they move and their average velocities while under the influence of the electric field can be regarded from different points of view which lead to the same result, namely that the velocities are proportional to the potential gradient. Hence, if u_0 and v_0 denote the velocities of the cation and anion (in cms. per sec.) under unit potential gradient (1 volt per cm.), the velocities under any potential gradient dE/dl will be u_0dE/dl and v_0dE/dl respectively.

Across any small area S, within the solution at right angles to the direction in which the current is flowing, at which the potential gradient is dE/dl, the current (in amperes) will be

$$\alpha N(u_0 + v_0)\frac{dE}{dl}SnF.$$

This result is obtained by regarding the current as the sum of (1) a flow per sec., across S in one direction, of $+\alpha Nu_0(dE/dl)SnF$ coulombs, and (2) a similar flow, in the opposite direction, of $-\alpha Nv_0(dE/dl)SnF$ coulombs.

But, if Ohm's law holds, another expression for the current will be $\kappa SdE/dl$, in which κ is the conductivity of the solution, *i.e.* the reciprocal of its resistivity.

Hence, by comparison,

$$\kappa = \alpha N(u_0 + v_0)nF.$$

The physical significance of this formula is obvious, for it is merely an expression of the fact that, if we grant the ionisation hypothesis, the conductivity of a solution must be directly proportional to the concentrations of the ions, to the sum of the velocities with which they move, and to the charges which they carry.

In deducing the formula we have, for simplicity, assumed the oppositely charged ions to be of equal valency n; but it can easily be shown that the same expression holds generally.

[If the ions are of unequal valencies n_1 and n_2, the formula of the substance will be M_xA_y, in which $n_1x = n_2y = n$. For example, in ferric sulphate $Fe_2(SO_4)_3$, the value of n for the conductivity equation is 6.]

§ (6) Equivalent Conductivity.—By transposition from the conductivity equation we get

$$\kappa/nN = \alpha(u_0 + v_0)F.$$

As we have seen, n is the aggregate valency of each part of the molecule. Each gramme molecule may be said to contain n equivalents. In this sense nN represents the number of equivalents in unit volume of the solution and κ/nN is the conductivity per equivalent per c.c. It is called the "equivalent conductivity" of the solution, and is usually denoted by the symbol λ. It is a quantity of great interest in connection with the ionic theory, since its magnitude, according to that theory, depends only upon the values of the ionic velocities and of α, the "coefficient of ionisation."

§ (7) Variation of Conductivity with Concentration.—The relation between the conductivity and the concentration has been carefully examined for many electrolytes, much of this work having been carried out by Kohlrausch (see *e.g.* Kohlrausch and Holborn, *Leitvermögen der Elektrolyte*, Teubner, Leipzig, 1898). Some general results may be indicated. Solutions of inorganic acids are, as a rule, the best conductors. After these come solutions of the alkalis and finally solutions of salts. In every case the conductivity increases with the concentration at first; but the rate of increase decreases afterwards and, in the case of very soluble substances, the conductivity passes through a maximum value—usually when the solution strength is somewhere between 4 and 6 gramme equivalents per litre.

The relation between the equivalent conductivity and the concentration has also been the subject of careful study, particularly in the case of dilute solutions. For these, in every case, the equivalent conductivity increases with dilution, the rate of increase becoming very rapid in very weak solutions. If, however, the value of λ be plotted against the cube root of the concentration it is found that, in most cases, a nearly linear relationship obtains when the concentration is small. In such cases the limiting value of λ when the concentration is infinitely small would appear to be given very approximately by the equation

$$\lambda = \lambda_\infty - aN^{\frac{1}{3}},$$

where λ_∞ is the "equivalent conductivity at infinite dilution," λ being the equivalent conductivity when the concentration is N and a a constant peculiar to each electrolyte but often not varying much from one electrolyte to another. Naturally, in these very dilute solutions, the question of the purity of the solvent becomes of great importance.

§ (8) The Variation of the Equivalent Conductivity.—The fact that the equivalent conductivity increases with dilution means that $d\lambda/dN$ is negative, and hence, according to the expression for λ already given, that $\{\alpha d(u_0+v_0)/dN + (u_0+v_0)d\alpha/dN\}$ is negative. If so, one at least of the differential coefficients $d(u_0+v_0)/dN$ and $d\alpha/dN$ must be negative, *i.e.* either the ionic velocities or the coefficient of ionisation must increase as the solution becomes more dilute. The latter alternative is the more probable cause of the variation; the position with respect to the former is less certain. It is known, for example, that the temperature coefficient of the resistance of very dilute solutions is almost identical with that of the viscosity of water. This is what would be expected if the velocities of the ions determined the conductivity and were inversely proportional to the viscosity of the solvent. Under such circumstances we might expect that in dilute solutions we should have $d(u_0+v_0)/dN = 0$. On the other hand, evidence is not wanting in support of the view that the ions are "hydrated," *i.e.* that they move through the solution, not

alone, but accompanied by molecules of the solvent. It may be that the number of these attached molecules increases with dilution. In that case the velocity of the ion would diminish at the same time, *i.e.* du_0/dN or dv_0/dN, or both, would be positive. It is sometimes assumed that u_0 and v_0 are proportional to the viscosity of the solution; $d(u_0+v_0)/dN$ would then be negative.

If it be assumed that $d(u_0+v_0)/dN$ is very small in the case of dilute solutions, the negative value of $d\lambda/dN$ must, if the expression for λ is correct, be due to the negative value of $d\alpha/dN$. It is not difficult to find reasons why $d\alpha/dN$ should be negative. Of these the most direct is that based upon the "law of mass action." We assume a tendency of the molecules to break up either spontaneously through some inherent instability or in consequence of collisions with other molecules. Of the molecules taking part in such collisions with a molecule of the solute, those of the solvent would be by far the most numerous, so that the number of effective collisions per second would be practically proportional to the number of solute molecules present in each unit of volume. Hence, whether the dissociation be spontaneous or due to collisions, we should expect the amount taking place per second to be proportional to the concentration of the undissociated molecules. On the other hand, the number of recombinations per second, between separated ions, would be proportional to the product of the concentrations of the separated parts. In the steady state, if α is the coefficient of ionisation, the concentrations of the undissociated and dissociated portions will be $N(1-\alpha)$ and αN respectively. The rate of decomposition, being proportional to $N(1-\alpha)$, can be written $k_1N(1-\alpha)$. Similarly the rate of recombination, being proportional to $(\alpha N)^2$, can be written $k_2(\alpha N)^2$. These rates must be equal. Hence

$$(\alpha N)^2 = k_1N(1-\alpha)/k_2 = KN(1-\alpha),$$

where K is a "dissociation constant," depending upon the nature of the solute, and probably also upon the properties (*e.g.* the dielectric constant) of the solvent.

From this result, which may be written $\alpha^2N/(1-\alpha)=K$, and which is known as "Ostwald's dilution law," it follows that α must increase as N diminishes, *i.e.* that $d\alpha/dN$ must be negative. It remains to examine whether the observed relation between λ and N corresponds with the theoretical relation between N and α. From the expression for λ already deduced, it follows that

$$\lambda/\lambda_\infty = \alpha(u+v)/\alpha_\infty(u_\infty+v_\infty),$$

in which α_∞, the value of α at infinite dilution (N=0), must in accordance with the dilution law be equal to unity. Hence, if we assume

$$u+v=u_\infty+v_\infty,$$

we get $$\lambda/\lambda_\infty=\alpha,$$

and hence

$$(\lambda/\lambda_\infty)^2N/(1-\lambda/\lambda_\infty)=K.$$

[If we assumed

$$(u+v)/(u_\infty+v_\infty)=\eta_\infty/\eta,$$

where η_∞ and η are the coefficients of viscosity of the solvent and solution, we should get $\eta\lambda/\eta_\infty\lambda_\infty=\alpha$ with a corresponding change in the equation connecting λ and N.]

§ (9) Experimental Results.—The relation thus deduced has been tested very thoroughly. It is found to give results in accord with experiments in certain particular cases. For instance, for solutions of various organic acids and bases. These are characterised by relatively low values of α at moderate dilutions and form the class known as "weak electrolytes." But there are many electrolytes for which the relation between λ and N is not that predicted by the dilution formula. These include the solutions of most of the inorganic acids, bases, and salts. They are characterised as a class by the fact that the calculated values of α are relatively high and are known as "strong electrolytes." For these, as a rule, the value of $(\lambda/\lambda_\infty)^2N/(1-\lambda/\lambda_\infty)$, instead of being constant, is much larger when N is large than when N is small.

§ (10) The "Abnormality of Strong Electrolytes."—Many attempts have been made to find the cause of this discrepancy, and various formulae have been devised with the object of finding one which shall fit the facts more closely than the simple formula already given. As a rule, it is assumed that the equation $\alpha=\lambda/\lambda_\infty$ is probably not seriously in error. In that case the object is to explain why $(\alpha N)^2/(1-\alpha)N$, instead of remaining constant, increases as N increases. The expression $(\alpha N)^m/(1-\alpha)N$ would obviously increase less rapidly than $(\alpha N)^2/(1-)\alpha N$, as N increased, if m were less than 2. It was found (van't Hoff) that, taking $m=3/2$, this expression is approximately constant over a considerable range for many strong electrolytes.

Reverting to the original argument (§ (8)), even if the rate of ionisation depends upon the concentration of the un-ionised molecules, there is no certainty that the factor k_1 will not increase as N increases. In the absence of exact knowledge of the mechanism of ionisation it may be possible, for example, that the ionised parts of the molecules already dissociated influence the conditions of ionisation of the others. In order to allow for this possibility, an equation

$$\alpha^2N/(1-\alpha)=K+k(\alpha N),$$

where k is an additional constant, has been derived (Larmor, Partington). An empirical modification of this is

$$\alpha^2N/(1-\alpha)=K+k(\alpha N)^m,$$

which has been found (Kraus and Bray) to be applicable to solutions of strong electrolytes in many solvents other than water. An analogous equation

$$\alpha^2N/(1-\alpha)=K+k\{(1-\alpha)N\}^m$$

has also been suggested (Szyszkowski), on the ground of evidence which supports the view that the un-ionised molecules themselves exert an influence upon the ionisation.

It should be remarked that the dielectric constant of the medium, whether modified or not by the presence of the solute, may be expected to influence the degree of ionisation. For if the forces which bind the components of the molecule together are in part of electrical origin, a molecule will stand a greater chance of ionisation, under any disruptive influence, in a medium of which the dielectric constant is large than in one of which the constant is small (J. J. Thomson, Nernst). Much successful work in connection with this view has been done by Walden.

With respect to the causes of disruption it has been suggested (Perrin) that the origin of the instability of undissociated molecules is the absorption of external radiation rather than impacts with other molecules.

Returning to the equation

$$\alpha^2N/(1-\alpha)=K+k(\alpha N)^m,$$

it has been pointed out (Arrhenius) that for weak electrolytes in water k may be small enough to be negligible. For strong electrolytes, on the other hand, K may, except for extremely high dilutions, be negligible. Under such circumstances, taking $m=\frac{1}{2}$, we obtain van't Hoff's equation which agrees with experience. On the other hand, very dilute solutions of strong electrolytes would obey Ostwald's law if $k(\alpha N)^m$ were then negligible compared with K. The experimental evidence seems to agree with this view (Arrhenius, Weiland).

§ (11) Inter-ionic Forces.—Many workers have attempted to find, in the electric forces between charged ions, an explanation of the deviations of strong electrolytes from the simple dilution law. In connection with this, it is to be remembered (Arrhenius) that some weak electrolytes obey the law even when their ionic concentrations are greater than those of strong electrolytes which do not. Nevertheless a great deal of interesting work has been done from this point of view, *e.g.* by Jahn, Nernst, Sutherland, Milner, Ghosh, and others. The most recent contributions are those of Milner (*Phil. Mag.*, 1918) and Ghosh (*Chem. Soc. J.*, 1918). Each adopts the view previously taken by others (*e.g.* Sutherland), that strong electrolytes are completely ionised at all concentrations. Milner ascribes the decrease in equivalent conductivity with increasing concentration mainly to a reduction in the average mobilities of the ions in consequence of inter-ionic forces which increase the frequency of their effective association. Ghosh's work has attracted attention, partly because of the method by which he arrives at the conclusion that a certain fraction only of the ionised molecules is unrestrictedly free to move under the influence of a potential gradient, and partly because of the evidence that his theory fits the facts better than any other that has been proposed. Information with respect to these, and other contributions to the problem presented by the "abnormalities of strong electrolytes," will be found in a recent discussion of the present position of the theory of ionisation (*Faraday Soc. Trans.*, 1919, xv. pt. 1), which contains references to nearly all the important papers upon this subject.

§ (12) Average and Individual Ionic Velocities.—The conductivity equation already given (§§ (5), (6)) can be written

$$\lambda/F=\alpha(u_0+v_0)=\alpha u_0+\alpha v_0.$$

This equation might have been deduced directly in the form

$$\lambda/F=u+v,$$

where u and v are the effective velocities of the ions, *i.e.* the average velocities in cms. per sec. per volt per cm. at which the oppositely charged components of the dissolved molecules move when conduction is taking place. For, upon the hypothesis of intermittent ionisation, all the molecules take part in the conduction process, although not all at the same time, and the effective velocities which determine the current or the conductivity are the average velocities of all the molecules concerned, *i.e.* of all the molecules dissolved.

The quantities αu_0 and αv_0 of the previous argument are thus equivalent to the average velocities of the components of the N gramme molecules dissolved in each unit of volume of the solution, and could have been replaced by u and v as above.

Whichever mode of presentation be adopted, the conductivity is represented as the sum of two terms, one depending upon the cation and the other upon the anion.

If we assume that, at infinite dilution, the solute is completely ionised, every ion will be effective in current conduction at any instant. The effective velocities of the ions will then be their actual velocities. Hence ($\alpha=1$), we should have

$$\lambda_0/F=\lambda_0\times1\cdot036\times10^{-5}=u_0+v_0.$$

As an example we may take the case of the chlorides KCl, NaCl, and LiCl. For these, in round numbers, the values of λ_∞ in ohm.$^{-1}$ cm.$^{-1}$ per gramme equivalent per c.cm. are 130, 110, and 101. Whence the values of u_0+v_0 in cm. per sec. for 1 volt per cm. are approximately $1{\cdot}34\times10^{-3}$, $1{\cdot}14\times10^{-3}$, and $1{\cdot}05\times10^{-3}$. The velocities of the ions are thus of the order 1/1000th cm. per sec. under unit potential gradient; but, since (u_0+v_0) is not the same for the different salts, the velocity of an ion must depend upon its chemical constitution. Hence, although conductivity data may serve to give the sum of the ionic velocities, they are not sufficient by themselves to give the separate velocities of the different ions.

§ (13) Ionic Velocity-ratio by "Migration" Methods.—The additional experimental information necessary before single velocities could be determined was first obtained by a method due to Hittorf, who showed, in effect, how the ratio of the ionic velocities could be found.

If, when a current is passed through an electrolyte MX, the products of electrolysis M and X escape or are deposited at the electrodes, it is obvious that the total quantity of MX contained in the solution must decrease continuously. As a general rule it is found that the changes in concentration are confined for some time (diffusion being minimised as much as possible by the arrangement of the apparatus) to the regions surrounding the electrodes, *i.e.* the weakening of the solution spreads from the electrodes outwards into the solution.

What happens in such experiments can be regarded in the following way. Suppose that the current is flowing uniformly between two parallel electrodes A and C and that the electrolyte contains αN ionised molecules per unit volume. Let u be the velocity of the cations and v the velocity of the anions under the gradient causing the flow. Then across any section in the interior of the electrolyte at right angles to the current there will pass per unit area, in the unit of time, αNu cations in the direction from A towards C and αNv anions in the direction from C towards A.

The current passing per unit area will be $\alpha N(u+v)nF$, and in the unit of time, by Faraday's laws, if we consider unit area of each electrode, $\alpha N(u+v)$ cations must pass over from the electrolyte to the electrode C, and in the same way $\alpha N(u+v)$ anions must pass over into the neutral state at the electrode A.

It is obvious that into any space, bounded by two perpendicular sections of the current stream, within the interior of the solution, the number of ions of one kind which enter through one section in the unit of time will be identical with the number leaving through the other. Consequently, within such portions of the solution there will be no changes in concentration during the passage of the current.

Consider, however, an imaginary section in the immediate neighbourhood of either electrode, *e.g.* the cathode.

Across this inwards, in the unit of time, there will pass αNu cations which are in motion owing to the potential gradient. But the total number of cations leaving by the cathode in the unit of time is greater than this, namely $\alpha N(u+v)$.

Thus αNv cations must move out of solution at the cathode, along with the αNu cations brought up by the current. These must come from the cathode region. The αNv anions left unpartnered by them are carried away by the current to take the place of other anions, since, as already seen, there is a drift towards the anode, across every unit area, of αNv anions per unit of time.

In the unit of time, therefore, there must ionise, in the immediate neighbourhood of the cathode, a sufficient number of molecules to give at least αNv positively charged cations and αNv negatively charged anions. The former go out with the αNu cations brought up by the current. The latter are carried away under the influence of the potential gradient. It is thus the molecules which split up in the immediate neighbourhood of the cathode that are "lost" by the cathode region.

As a result, when $\alpha N(u+v)$ cations leave at the cathode, αNv molecules leave the portion of the solution near the cathode, and, in an exactly similar way, when $\alpha N(u+v)$ anions leave at the anode, αNu molecules leave the portion of the solution near the anode.

Hence, αNv molecules leave the cathode space and αNu molecules leave the anode space when, by electrolysis, $\alpha N(u+v)$ molecules of dissolved substance leave the solution. Thus, in this case, the ratio "cathode loss"/"anode loss" would, if measured, give the ratio v/u of the ionic velocities.

It frequently happens that, owing to chemical action, measurement of the "anode loss" is not practicable. The ratio of the ionic velocities cannot then be determined in the way just described. For example, when electrodes of some particular metal are used, and the electrolyte is a salt of that metal, the metal itself is deposited upon the cathode, but the acid radicle is not deposited at the anode. Instead, the metal of the electrode enters the solution. In this case the current is carried over from the anode to the solution by metallic ions only, just as it is from the solution to the cathode, and therefore, by Faraday's laws, the amount of metal entering at the anode must be exactly equal to that leaving at the cathode. The total quantity of salt in solution therefore remains constant.

There is now no "anode loss"; but, instead, an "anode gain" equal in amount to the "cathode loss."

Fortunately, however, the ratio of the ionic velocities can always be found without reference to what occurs at the anode. It is sufficient to compare the cathode loss of salt with the total deposit at the cathode. Thus, returning to the case considered above, the ratio of the weight of metal deposited at the cathode to the weight of metal contained in the salt molecules carried away from the cathode space is represented by the expression $\alpha N(u+v)/\alpha Nv=(u+v)/v$. Hence, if each of them is measured, the ratio $v/(u+v)$ called by Hittorf the "migration constant" for the anion or, alternatively, its "transport number," can be determined. The weight of the cathode deposit can be measured either directly or by measurement of the total quantity of electricity passed through the solution (if the electro-chemical equivalent of the metal be known) or by means of a silver voltameter in series with the cell in which the electrolysis occurs. The cathode loss is measured by comparing the salt content of the solution round the cathode at the end of the experiment with the known salt content at the beginning. In the latter measurement allowance has, in general, to be made for the change in volume accompanying dilution.

It is impossible here to enter into any detail concerning the results of measurements made in this way.

As an example of their application, however, we may take the case of the three chlorides already cited. For these in very dilute solutions the values of $v/(u+v)$, again in round numbers, are 0·5, 0·6, and 0·65 respectively. With the values of (u_0+v_0) already mentioned these give for the value of v_0, the velocity of the chlorine ion, $0{\cdot}67\times10^{-3}$, $0{\cdot}68\times10^{-3}$, and $0{\cdot}68\times10^{-3}$ cm. per sec. respectively, and for the velocities of K, Na, and Li, the values of $0{\cdot}67\times10^{-3}$, $0{\cdot}46\times10^{-3}$, and $0{\cdot}37\times10^{-3}$ respectively.

The measurements upon less dilute solutions treated in a similar way give corresponding results. Thus for solutions containing one-tenth of a gramme molecule per litre, the values of $\alpha(u_0+v_0)$ are $1{\cdot}159\times10^{-3}$, $0{\cdot}958\times10^{-3}$, and $0{\cdot}859\times10^{-3}$ respectively. The corresponding values of $v/(u+v)$ are 0·508, 0·617, and 0·69; whence the values of αv_0 work out at $0{\cdot}588\times10^{-3}$, $0{\cdot}592\times10^{-3}$, and $0{\cdot}593\times10^{-3}$ respectively.

In other cases, particularly of relatively concentrated solutions, the results obtained are not so easily interpreted as those just given. These are none the less of great value, as showing that the ions are not always of the simplest possible type. To cite one striking example from Hittorf's work, concentrated solutions of CdI_2 give values for $v/(u+v)$ which are greater than unity. This suggests that the iodine ions carry with them molecules of the undissociated salt, or perhaps that the cadmium ions carry with them molecules of the solvent. In any case, such results yield one of the many reasons for supposing that complex ions exist. The frequent variation of the value of the migration constant with the concentration of the solution is another indication of the same thing.

§ (14) Direct Determination of Ionic Velocities.—The method of finding values for the effective velocities of the ions by combination of conductivity and migration data is not the only one available, as was first pointed out by Sir Oliver Lodge (*B.A. Report*, 1886). He devised and made use of another method, subsequently extended by Whetham, Masson, Steele, and others. We can only refer to it very briefly. It is a direct method depending in its final forms upon the determination of the velocities of moving boundaries between different electrolytes in series. For instance, to take an example from Masson's work, the electrolytes in series were solutions of copper sulphate, sodium chloride, and potassium chromate, and the current was passed in the direction from the blue sulphate solution through the colourless chloride solution to the yellow chromate solution. Under these circumstances the sodium ions move in one direction, followed by the copper ions, while the chlorine ions move in the opposite direction, followed by the chromate ions. It is essential, if the sharpness of the boundaries between the solutions is to be preserved during the passage of the current, that the following ion should be slower than the ion which precedes it. This condition is satisfied in the present case, because sodium ions move faster than copper ions, and chlorine ions move faster than chromate ions. In the experiment the boundary between the blue solution and the colourless solution moves, let us say, from left to right, while that between the yellow solution and the colourless solution moves from right to left. The velocities of movement give the average velocities of the sodium and chlorine ions under the same potential gradient. This gradient can be determined from a knowledge of the strength of the current passing, of the diameter of the tube containing the salt solution, and of the conductivity of the latter. Hence it is possible to determine αu_0 and αv_0. The value of u/v, and hence of $v/(u+v)$, can of course be determined by direct comparison of the rates of movement of the two boundaries if the containing tube be of uniform bore. Steele extended the applicability of this method by showing that difference of refractive index could be used instead of colour as a means of fixing the boundaries between

the main electrolyte and the indicating solutions.

Results have been obtained in this way in excellent accord with those obtained by the method first described (cf. *e.g.* Denison and Steele, *Roy. Soc. Proc.*, 1905). In the case of the more concentrated solutions, however, there is often a considerable divergence between the value of $v/(u+v)$ found by Hittorf's method and that found by the method just described. It has been suggested (by Nernst and others) that a possible cause of this divergence is the hydration, to different degrees, of ions of different kinds (for references to some of the literature, see a paper by the present writer, *Phys. Soc. Proc.*, 1916, xxviii. 327 *et seq.*). The problem has been examined from this point of view by Washburn (*l.c.*). From his experiments with the three chlorides already mentioned he deduced, for example, that the degree of hydration increases from K to Li. This result accords with what might be expected on chemical grounds. The probability that hydration is the cause of the observed decrease in ionic velocity in the order from K to Li is increased by the fact that the atomic volumes are in the inverse order, and would lead us to anticipate that but for hydration the order of the ionic velocities would be Li > Na > K. In agreement with this, the rates of diffusion of the metals in a medium, mercury, in which hydration cannot arise are in the order Li > Na > K. Washburn shows also how, from his point of view, the different values of $v/(u+v)$, obtained by the two methods, can be reconciled.

The effect of hydration in experiments of this kind is a differential one, depending upon the relative degrees of hydration of the cation and anion. Attempts have been made (*e.g.* by Riesenfeld and Reinhold, *l.c.*) to find the degree of hydration in each case by assuming that the velocity is inversely proportional to the radius of the (spherical) ionic envelope under a given moving force. From such considerations it would appear, for example, that the H ion is unhydrated, while the K ion carries with it 22 molecules of water and the OH ion 11.

§ (15) Speculations with respect to the Actual Carriers.—Reference should perhaps be made to a method of deducing the radius of the ion, used by different investigators, which is of interest not only in connection with the previous paragraph but also in connection with the expression for the conductivity of an electrolyte derived in § (5). In finding that expression it was unnecessary to go beyond the gramme ion, *i.e.* beyond the fact that the total weight of the ions of a particular element which suffice to carry 96,500 coulombs is represented by its chemical equivalent in grammes. It was sufficient, for example, to know that 1 gramme of hydrogen ions carries approximately $0{\cdot}96\times10^5$ coulombs or $0{\cdot}96\times10^4$ electromagnetic units. We can, however, suppose that the actual carriers are charged atoms of hydrogen, and then, if the number of atoms contained in a gramme of hydrogen be known, we can deduce both the atomic mass and the atomic charge. The agreement between the various experimental estimates of the number of molecules in a gramme of hydrogen is not yet as perfect as it might be; but we may assume, provisionally, that one gramme contains about 6×10^{23} atoms. In that case the atomic charge will be $1{\cdot}6\times10^{-20}$ electromagnetic units, and the force on an ion carrying this charge in a field in which the potential gradient is 10^8 electromagnetic units (one volt) per cm. will be $1{\cdot}6\times10^{-12}$ dynes. Algebraically the mechanical force on the ion will be $f=edE/dl$, where e is the atomic charge and dE/dl is the potential gradient. The inference with respect to the behaviour of the ion under the action of this force depends upon the assumption made as to the conditions of its motion. If we assume that the motion can be likened to that of an isolated sphere moving through a viscous medium, subject to a resistance proportional to its velocity, the equation of motion will be

$$f-kv=mdv/dt,$$

where k is a constant and m is the mass of the ion. If we assume that Stokes' law applies, the value of k will be $6\pi\eta r$, where η is the coefficient of viscosity of the medium and r is the radius of the ion, assumed spherical. The solution of the equation of motion is

$$v=\frac{f}{k}\left(1-\epsilon^{-\frac{k}{m}t}\right),$$

from which it follows that the time which elapses before the velocity becomes constant is the time taken by $\epsilon^{-\frac{k}{m}t}$ to become vanishingly small. The steady velocity $(dv/dt=0)$ will be $\bar{v}=f/k$, whence

$$6\pi\eta r\bar{v}=edE/dl.$$

If we take the value of $\bar{v}$ to be $3{\cdot}2\times10^{-3}$ cm. per sec., the estimated value of u_0 for hydrogen in very dilute solution, at 18° C., and the value of η to be 0·011, the coefficient of viscosity of water at the same temperature, we get

$$\begin{aligned} r&=(1{\cdot}6\times10^{-12})/(6\pi\times{\cdot}011\times3{\cdot}2\times10^{-3})\\ &=2{\cdot}4\times10^{-9}\text{ cm.}\end{aligned}$$

Under these circumstances the value of k/m, which is equal to $6\pi\eta r/m$, would be 3×10^{14}, and hence the time taken by the ion to acquire its steady velocity within 0·1 per cent would be given by

$$1000=\epsilon^{t\times3\times10^{14}},$$

i.e. by $\quad t=2{\cdot}3\times10^{-14}$ sec. approx.

For any other ion of valency n and ionic velocity v, relative to hydrogen, the radius calculated in the above way would be

$$r = 2\cdot4 \times 10^{-9} \times n/v \text{ cm.}$$

This analysis of the phenomena, based upon the applicability of Stokes' law, is not the only one that has been put forward. Another view, for example, is that the motions of the ions are controlled by collisions with other particles and that their velocities accelerate continuously between these collisions. Under such conditions the effective velocity would be a function of the mass of the ion instead of depending only upon its size, as it would if the view above described were correct. It is obvious, however, from the experimental data, that there is no simple relation between the velocity of an ion and its mass.

It may be noted in connection with the above estimate of the time required by the ion to acquire its steady velocity that there is no difficulty, on that view, of accounting for the fact that alternating currents of high frequency give the same value as continuous currents for the resistivity of an electrolyte.

§ (16) Polarisation.—We have been concerned, so far, mainly with what takes place in the solution itself when a current is passed through an electrolyte. Thus we considered first how the current is carried through the interior of the electrolyte, and second how the phenomena which occur near the electrodes can be interpreted. We come now to the consideration of what happens at the actual surfaces of separation between the electrolyte and the electrodes.

When an electromotive force is applied between the terminals of a cell in which the electrodes are composed of some metal M, and in which the electrolyte is some salt of that metal MA, a continuous current passes, however small the applied E.M.F. may be; but with this restriction: from the moment at which the E.M.F. is applied changes in the concentration of the electrolyte around the electrodes begin to occur in the way and for the reasons already described. And if the E.M.F. is kept applied for a sufficient time these concentration changes will become considerable. Concurrently, an appreciable counter E.M.F. will arise between the cell terminals and will cause the current to diminish. Only when the concentration of the solution is kept uniform, *e.g.* by stirring, will the current remain steady while the E.M.F. is applied.

This, however, is a particular case of electrolysis. We may consider first the more general case, in which the electrodes are composed of some other metal not chemically acted upon either by the salt MA (or the acid HA) or by the products of its decomposition. Thus, suppose we take the case of an electrolytic cell in which the electrodes are of platinum and the electrolyte is dilute sulphuric acid. We shall find that the phenomena to be observed are intimately connected with the magnitude of the applied E.M.F.

Suppose the cell, connected in series with a ballistic galvanometer, to form a branch circuit, between the ends of which, a and b, an E.M.F. of any desired magnitude can be applied. So long as this E.M.F. is small, what happens is as follows: on the completion of the circuit, there is an immediate throw of the galvanometer needle; but, after the oscillation has died down, the needle comes to rest very nearly in its zero position. There is thus an initial rush of electricity through the cell, which converts it into a kind of secondary "battery" having an electromotive force equal to that applied between a and b.

If the experiment be repeated with a greater E.M.F. the initial throw is increased. The final, steady deflection becomes greater at the same time. Eventually however, with increase in the applied E.M.F. beyond a fairly definite value (which depends in general upon the nature and concentration of the electrolyte and upon the electrodes), the final deflection of the galvanometer becomes rapidly larger. The counter E.M.F. no longer keeps pace with the applied E.M.F., and at the same time visible decomposition of the electrolyte ensues.

In experiments of this kind the electrolytic cell is said to have been "polarised." The passage of electricity through it has produced a "back electromotive force of polarisation" between the electrodes.

It will be seen that the existence of this phenomenon complicates the measurement of the ohmic resistance of electrolytes. For instance, in the case just considered, when a small E.M.F. is applied, the ratio of this E.M.F. to the final current is so large as to make the apparent resistance of the electrolyte extremely high. Yet it is obvious from the initial flow that, up to a point, electricity passes through it readily enough.

The phenomenon has been studied by many observers, particularly by Kohlrausch, in connection with the measurement of resistance.

It may be assumed as a first approximation that the back E.M.F. is proportional to the total quantity of electricity which has passed through the cell. In that case the relation between the applied E.M.F. E and the current i, at any instant during the brief interval of charging, would be given by

$$E = Ri + P\int_0^\tau i\,dt,$$

where R is the true resistance of the electrolyte, τ is the time elapsed since the E.M.F. was applied, and P is a "polarisation constant" for the given cell.

It is easy to picture an electrical arrangement which would give a charging equation of this type. For, suppose we take two condensers of capacities c_1 and c_2, connected in series by a wire of resistance R, and apply a potential difference E between the unconnected plates, the equation of charging will be

$$E = Ri + \left(\frac{1}{c_1} + \frac{1}{c_2}\right)\int_0^T idt,$$

identical with the previous equation if we replace $(1/c_1 + 1/c_2)$ by P.

Hence, by analogy we may liken the process which occurs, when electricity passes through an electrolytic cell, to the charging of a pair of condensers in series. We may suppose that when q units of electricity pass round the cell circuit, $+q$ units flow into the anode A and $-q$ units into the cathode C, and that, simultaneously, anions carrying $-q$ units come up to the surface separating the electrolyte from A, while cations carrying $+q$ units come up to the surface separating the electrolyte from C. We may suppose further that these charges remain apart in condenser-like layers at the electrode surfaces, the surface densities of the charges rising until the sum of the potential differences across the two layers is equal to the applied E.M.F. In order to make the analogy complete as far as it goes, we must, however, suppose that the electrolyte-electrode condensers are not quite perfect, that their charges leak away to some extent, so that a continuous supply of electricity is necessary if these charges are to be maintained. At first the required continuous supply, represented by the steady deflection of the galvanometer, is relatively very small; but it increases with the applied E.M.F., *i.e.* with the surface densities of the charges, and becomes relatively very large when the applied E.M.F. reaches the value required to produce visible decomposition. This final stage might be likened to the discharge which occurs in ordinary condensers, *e.g.* by sparking, when the potential gradient exceeds that which the dielectric can support.

The modifications which must be made in this view of polarisation phenomena will appear later. Our immediate object is to show how the resistance of an electrolyte can be measured in spite of their existence. For this, the above preliminary sketch of the phenomena will suffice.

§ (17) The Measurement of Electrolytic Resistance.—The methods of measurement can be divided broadly into two classes: in one the effects of polarisation are avoided, in the other they are reduced to a minimum.

Comparatively few methods belong to the first class. Theoretically, the best example is that in which the circuit is wholly electrolytic and the electromotive force is supplied by electromagnetic induction (cf. Guthrie and Boys, *Phil. Mag.*, 1880); but very few measurements have been carried out in this way. Another method which may be placed in this class is that in which auxiliary or potential electrodes are used, particularly when the potential drop is measured by an electrostatic voltmeter of negligible capacity. This method has been used occasionally and measurements with it have been described, *e.g.* by Bouty (1884) and Sheldon (1888). A particular form of it, devised by the writer (1899), in which mercury electrodes (which have certain advantages) are used, is described in Watson's *Text-Book of Practical Physics* (Longmans). Further applications of the method in which both continuous and alternating currents were used have been described by Smith and Moss (*Phys. Soc. Proc.*, 1913, xxv. 133). The same method has recently been used by Newbery with the object of testing the accuracy of Kohlrausch's measurements (*Chem. Soc. J.*, 1918).

An early example of the second class was that due to Horsford in which the electrolyte was contained in a vessel of uniform cross-section between parallel electrodes, each perpendicular to its length. This vessel was connected in series with a battery and galvanometer. A measurement of the current was taken with the electrodes at a distance d_1 apart. This distance was then reduced to d_2, and at the same time known resistances were inserted to maintain the current at its original value. On the assumption that the polarisation effects remained unchanged, the added resistance was equal to that of a column of the electrolyte of length $d_1 - d_2$. This method is only reasonably accurate when the polarisation is relatively small, *e.g.* when the electrolyte is a solution of a salt of the metal of which the electrodes are made. It was improved by converting it into a bridge method with similar vessels in opposite arms, the distance d_1 between the electrodes in the one being greater than the distance d_2 between those in the other. The ratio arms being equal, the resistance required to be inserted in series with the shorter column in order to obtain a balance is equal to that of a column of length $d_1 - d_2$. (Cf., *e.g.*, Kohlrausch and Holborn, *Leitvermögen*, p. 71; Stroud and Henderson, *Phil. Mag.*, 1897, xiii. 19.)

The most widely used method of the second class is that, developed by Kohlrausch, in which the effects of polarisation are overcome by the use of alternating currents. If we assume that the polarisation arises in accordance with the equation $E = Ri + P\int idt$, the effect of alternation is easily represented. In the simplest case the applied E.M.F., instead of remaining constant, alternates in accordance with the equation $E = E_0 \sin pt$, the number of

alternations per second being given by $n=p/2\pi$. Under this condition the current through the electrolyte settles down to a value given by

$$i=E_0 \sin(pt+\theta)/R\sqrt{1+P^2/p^2R^2},$$

where $\tan\theta=P/pR$; while the corresponding values of e, the back E.M.F., are given by

$$e=-PE_0\cos(pt+\theta)/pR\sqrt{1+P^2/p^2R^2}.$$

The quantities i and e are thus always in quadrature while the phase difference between E and i depends upon the value of P/pR. If this can be made negligibly small, the phase difference between E and i and, at the same time, the existence of e can be ignored. The value of R can then be obtained by a bridge method if a suitable indicating instrument be employed.

For a given value of P/R the effects of polarisation are dependent upon the frequency of alternation n. Alternation makes the effects of polarisation non-cumulative: rapidity of alternation reduces the maximum amount. It can be seen that even if the variations of E are not simply harmonic, but only such as to ensure that the quantities of electricity passing in opposite directions in successive half-periods are nearly the same, the effects of polarisation may be overcome.

For a given value of p the effects of polarisation are dependent upon the value of P/R. To a first approximation we can assume from an earlier paragraph that P is inversely proportional to the effective area of either electrode, assuming for simplicity that the areas are equal. Hence it is relatively easy to make polarisation negligible when the electrodes are large (or when their effective areas are increased, *e.g.* by platinisation) and when the resistance of the electrolyte between them is great. Conversely, it is practically impossible to eliminate polarisation when P/R is large, and a cell for which this is true should not be used if an accurate value of R is desired. The question whether sufficient precautions have been taken can be answered by means of the expression for i already given. From this the effect of polarisation upon the resistance can be expressed by

$$R'=R(1+\tfrac{1}{2}P^2/p^2R^2),$$

where R′ is the apparent resistance. In order to attain an accuracy of 0·1 per cent, we must therefore have $\frac{1}{2}P^2/p^2R^2<1/1000$. Now if we make the approximate assumption $P=2/cA$, where A is the area of each electrode and c is the "electrode capacity" per unit area, we get $\frac{1}{2}P^2/p^2R^2=\frac{1}{2}4/c^2A^24\pi^2n^2R^2$. Hence we must have $(nAR)^2>1000/2\pi^2c^2$. If the electrodes are of mercury, copper, or platinum, c is of the order 10 microfarads per sq. cm. (more rather than less). Hence the condition for 0·1 per cent accuracy is approximately $nAR>7\times10^5$, where A is expressed in sq. cms. and R in ohms. When platinised electrodes are used the value of c appears to be increased from 25 to 50 times, and the value of nAR, necessary for 0·1 per cent accuracy, is correspondingly reduced.

In many of his measurements Kohlrausch used a small induction coil (with solid core and no condenser) as source of E.M.F. and a telephone as indicator. The conditions for a sharply defined minimum sound in the telephone were carefully examined, but cannot be discussed here. It is, however, important to observe that, in some of his standard measurements, he not only used alternating currents to avoid polarisation, but also carried out the measurements in such a way that, as in Horsford's method, any uneliminated effect appeared merely as a difference between two measurements in the final result.

In recent years the number of appliances suitable for the production and the measurement of alternating currents has greatly increased. For instance, generators capable of supplying an E.M.F. closely following a sine law and of adjustable frequency are now available, as are detectors, such as the vibration galvanometer, which can be tuned to give maximum response to a current of definite period. In consequence, new measurements have been undertaken, especially in America, in which such instruments are employed. In these measurements a double adjustment is necessary in order to balance the capacity effect in the arm containing the electrolyte, as well as the resistance. The most convenient method of balancing the capacity is by means of a variable inductance in series with the electrolyte. If the polarisation constant P were exactly equivalent to a capacity 1/C, the self induction $L=1/p^2C$ would compensate its effect and both the resistance and the capacity could be measured if L and p were known. As we have seen, however, the analogy is not exact on account of leakage. The effect of this is to alter the phase of e with respect to i so that these are not in quadrature as they would be if $e=P\int_0^\tau i\,dt$ were true. In consequence, the condition of balance is not $L=1/p^2C$ but $L=\cos\phi/p^2C'$, where C′ is greater than C, and ϕ is the amount by which the phase difference between e and i departs from $\pi/2$. Also, when the bridge is balanced, the apparent resistance of the electrolyte arm is not R, the true resistance, but $R(1+\sin\phi/C'pR)$. It will therefore appear to be a function of p if the quantity $\sin\phi/C'pR$ (which can also be written in the form $\tan\phi/C_apR$, where C_a is the apparent capacity $1/Lp^2$) is not negligible in comparison

with unity. A further reference to this method and to the literature will be found in the paper by Smith and Moss already mentioned (*Phys. Soc. Proc.*, 1913). Among later papers are those by Taylor and Curtis (*Phys. Rev.*, 1915, vi.); Taylor and Acree (*Am. Chem. Soc. J.*, 1916, xxxviii.). The effect of polarisation in this method is not, as in the first, inversely proportional to p^2; but diminishes much less rapidly as p increases. In some cases the value of ϕ is practically independent of p, and extrapolation to the case in which p is very great is easily performed by plotting the apparent resistance $R_a = R + \tan\phi/C_a p$ against $1/C_a p$. If ϕ is constant, a straight line cutting the resistance axis at the distance R from the origin will be obtained.

In connection with the measurement of resistance the double-commutator method (see, *e.g.*, Whetham, *Theory of Solution*, p. 201) deserves mention, although it is not essentially different in principle from Kohlrausch's method.

§ (18) Gas Concentration Cells.—Fresh light is thrown upon polarisation by the study of "concentration cells." One such cell, similar in construction to Grove's gas battery, contains two platinised platinum electrodes, each enclosed within a glass vessel open at the bottom. The upper parts of the vessels contain a gas, *e.g.* hydrogen, while the lower parts contain dilute acid. Each electrode lies partly in the gas and partly in the electrolyte. The vessels are arranged side by side in a wider vessel partially filled with the acid solution so that the electrolyte extends continuously from one electrode to the other. This "cell" has an E.M.F. if the pressures of the gas, p_1 and p_2, in the electrode vessels, A and B, are unequal. If p_1 exceeds p_2, the potential of B is higher than that of A. When the cell acts, gas passes through the platinised platinum into the solution at A and out of the solution at B. The masses passing in and out are equal and, for given pressures, the action is reversible, the E.M.F. required to produce the reverse charge being in the limit identical with E, the E.M.F. of the cell. Energy can be taken from the cell, at constant temperature T, by connecting it in opposition with a condenser, charged so that the potential difference between its plates is E, and allowing the capacity of the condenser to rise indefinitely slowly, *e.g.* by allowing the plates to approach against the action of the force by which they are kept apart. Concurrently the pressures p_1 and p_2 may be supposed maintained constant by means of pistons working in cylinders, one attached to each electrode vessel. Let v_1 be the volume of gas which disappears at pressure p_1 from A, and appears at volume v_2 and pressure p_2 in B, when the quantity of electricity dq flows round the circuit. The work supplied from the cell during this change is $Edq + p_2v_2 - p_1v_1$, neglecting the irreversible loss of heat, which can be made vanishingly small. The cell can be restored to its original condition by compressing isothermally the gas which has appeared round B, from v_2 and p_2 to v_1 and p_1, and then adding it to the gas round A. The work done on the gas during this process is $\int_{v_1}^{v_2} pdv$. The net work obtained from the cell during this constant temperature cycle must be zero. Hence

$$Edq + p_2v_2 - p_1v_1 - \int_{v_1}^{v_2} pdv = 0,$$

which may be written

$$Edq = \int_{p_2}^{p_1} vdp.$$

If the gas obeys the law $pv = kT$, this becomes

$$Edq = kT \log \frac{p_1}{p_2},$$

where k is a constant of which the value depends upon dq. If the gramme molecule of the gas gives rise to two gramme ions, each of valency n, then

$$\frac{k}{dq} = \frac{R}{2nF},$$

where R is the gas constant per gramme molecule. Hence

$$E = \frac{RT}{2nF} \cdot \log \frac{p_1}{p_2}.$$

This result may be taken to mean that the potential of the liquid with respect to the electrode increases with p and that E represents the difference between two "contact potential differences." This view may be expressed by the equation

$$E = ({}_S\pi_H)_{p_1} - ({}_S\pi_H)_{p_2} = \frac{RT}{2nF}(\log p_1 - \log p_2),$$

where ${}_S\pi_H$ represents the amount by which the potential of the solution exceeds that of the electrode.

If there is a value p_0 for the pressure of the gas such that $({}_S\pi_H)_{p_0} = 0$; then we could write for any pressure p the equation

$$({}_S\pi_H)_p = \frac{RT}{2nF} \log \frac{p}{p_0}.$$

Throughout it is assumed that the gaseous law, $pv/T = \text{constant}$, is obeyed.

When the gas employed in the cell is not hydrogen, but, *e.g.*, oxygen or chlorine, it gives rise to negative ions in solution and the sign of the E.M.F. is reversed. The lower pressure electrode is now the negative pole of the battery, but the value of E is obtained as

before. For this case, subject to the same simplifying assumptions, we get

$$({}_{S}\pi_{A})_{p}=\frac{RT}{2n'F}\log\frac{p_0'}{p},$$

where A represents the gas yielding the anions and the electrolyte is, *e.g.*, a solution of the acid HA.

These results can be used to improve the account given earlier of the way in which polarisation arises. For brevity we take the simplest example, electrodes of platinised platinum in a solution of HA, giving ions of unit valency. Let a polarising E.M.F. E be applied and imagine that incipient electrolysis occurs, however small E may be. The gases A and H will accumulate in and around the anode and cathode respectively. When their partial pressures have risen to p_H and p_A, the back E.M.F. e will be

$$({}_{S}\pi_{H})_{p_H}-({}_{S}\pi_{A})_{p_A}=\frac{RT}{2F}\log\frac{p_H p_A}{p_0 p_0'}.$$

The current will fall as p_H and p_A increase, and would vanish if they reached the values required to make $e=E$. They can never quite reach these values, however, for the gases will be continuously escaping, *e.g.* by diffusion and convection, and the final value of $E-e$ will be that required to allow the supply to balance the loss.

With each successive increase of E the pressures of electrolysed gas in anode and cathode will rise. A limit will be reached when p_H and p_A are each equal to the atmospheric pressure. The back E.M.F. will not increase further with increase of E and visible decomposition will ensue—the earlier stages preceding this final one in much the same way as evaporation precedes boiling.

We have taken a hypothetical case, but there is much experimental evidence, which cannot be given here, to prove its value as an indication of what occurs in general. It shows that while the simple condenser theory already described may be correct as far as it goes, it omits one essential fact. The potential difference between an electrode and a solution cannot be altered while everything else remains the same. Concomitant changes, either in the electrode or in the electrolyte, must occur at the same time. Hence the first rush of electricity, observed when an E.M.F. is applied to an electrolytic cell, represents not surface effects only but volume effects as well. This is clearly seen in the experiments described by Rothé (*Journ. de Physique*, 1904, p. 685; also *Ann. de Chim. et de Physique*, 1904) in which an oscillograph was used instead of a ballistic galvanometer and the charging current recorded photographically.

When the electrodes are not made of platinised platinum, and even then in many cases, the phenomena antecedent to gas evolution can be much less simple than we have supposed. It would appear that the complications are due in part to physical causes, such as supersaturation, and in part to chemical causes, such as the formation of compounds between the electrodes and the deposited ions. The existence of such effects, which are said to result in "overvoltage," is of considerable technical importance; but is outside the scope of the present article (see, *e.g.*, Allmand, *Applied Electrochemistry* (Arnold); Newbery, *Chem. Soc. J.*, 1916, cix. pt. 2).

§ (19) Liquid Concentration Cells.—Another aspect of polarisation, important in connection with electro-deposition, is seen in the case in which the electrolyte is a salt of the metal of which the electrodes are composed.

Here polarisation occurs because of the changes in the concentration of the electrolyte which arise when the current flows. This case is elucidated by consideration of a second type of concentration cell in which the concentration difference occurs in the electrolyte and not in the electrodes. The E.M.F. can be calculated in this case by a method analogous to that employed before.

The general arrangement of the cell is—metal M: solution of MA: less concentrated solution of MA: metal M. For simplicity, suppose that the solutions, containing N_1 and N_2 gramme molecules per c.c. respectively, are so dilute that each is completely ionised. When this cell is in action, the current flows through the electrolytes from the less concentrated solution towards the stronger, the cathode loss tending to make the latter weaker and the anode gain to make the former stronger. During the passage of dq units of electricity round the circuit, it can be seen, by the aid of § (13), that $\frac{v}{u+v}\cdot\frac{dq}{nF}=dN$ gramme cations and anions leave the solution round the electrode in the stronger solution, and that the same numbers appear in the region round the electrode in the weaker. It is necessary to find a method by which the concentrations of the solutions can be kept at their original values. For this purpose, in each case, we use a piston, operating on the electrolyte, in a cylinder of which the open end communicates with the solvent through a "semi-permeable" membrane. Through this the solvent, but not the solute, can pass. The concentration of the stronger solution is maintained by expelling solvent through one membrane; that of the weaker solution is maintained by allowing solvent to enter through the other. Work is done against the "osmotic pressure" in the former operation and by means of it in the latter. These operations correspond with those giving the

terms $p_1\ v_1$ and $p_2\ v_2$ of the calculation for the gas concentration cell. After the flow dq there is thus less of the stronger solution and more of the weaker. To restore the cell to its original condition the excess of the weaker solution, containing dN gramme molecules, is separated from the rest, and solvent is expelled from it through a semi-permeable membrane until the concentration is that of the stronger solution. To this it is then added, bringing the cell back to its initial state. The work done in this operation corresponds with the term $\int_{v_1}^{v_2} pdv$ of the earlier calculation.

It is known by experiment that the osmotic pressure of a very dilute solution containing dN gramme molecules of a salt in V c.c. is given, very nearly, by $PV = 2dN \cdot RT$. Assuming this law, the terms corresponding with $p_1\ v_1$ and $p_2\ v_2$, as above, balance out, and that corresponding with $\int_{v_1}^{v_2} pdv$ is

$$2dN \cdot RT\int_{dN/N_1}^{dN/N_2}\frac{dV}{V} = 2dN \cdot RT \log\frac{N_1}{N_2}$$

$$= \frac{2v}{u+v}\cdot\frac{dq}{nF}RT\log\frac{N_1}{N_2}.$$

For the same reason as before, this must equal Edq, where E is the E.M.F. of the cell, whence

$$E = \frac{2v}{u+v}\cdot\frac{RT}{nF}\log\frac{N_1}{N_2}.$$

The reason why $2v/(u+v)$ appears in this expression is seen when, as in the preceding case, we consider the factors of which E is composed. In the present case we have

$$E = ({}_S\pi_M)_{N_2} - ({}_S\pi_M)_{N_1} + {}_{N_1}\pi_{N_2},$$

where ${}_{N_1}\pi_{N_2}$ represents the contact potential difference between the solutions of concentrations N_1 and N_2. The latter potential difference owes its existence, in all probability, to the fact that the rates of diffusion of the ions are unequal, or, to put it in another way, to the fact that $2v/(u+v)$ is not, in general, equal to unity.

Without loss of generality in what follows we can, however, assume the electrolyte so chosen that $v = u$; in which case ${}_{N_1}\pi_{N_2}$ will be zero, and we obtain

$$E = \frac{RT}{nF}\log\frac{N_1}{N_2} = ({}_S\pi_M)_{N_2} - ({}_S\pi_M)_{N_1},$$

an equation showing how the potential difference between a metal and a solution of one of its salts depends upon the concentration of the latter.

If, as in the preceding case, there is a value N_0 of N, for which $({}_S\pi_M) = 0$, we can write

$$({}_S\pi_M)_N = \frac{RT}{nF}\log\frac{N_0}{N}.$$

This equation, like that just obtained, is, however, dependent upon the assumption that every osmotic pressure involved is related to N in the way already described. Similar calculations may be made by utilising the fact that the vapour pressure of a solution depends upon its strength (cf. Helmholtz, *Berl. Sitzber.*, 1882). The semi-permeable membrane used in this method is the surface of the solution, through which molecules of the solvent only can pass.

Such results, it will be seen, can be used without difficulty to account for the polarisation which occurs when a current passes, by electrodes of a given metal, through a solution of one of its salts.

A summary of the results of further study of polarisation, along the lines here indicated, will be found in a paper by Krüger (*Phys. Zeits.*, 1910, xi. 719), who gives also (*Zeits. f. Elektrochem.*, 1910, xvi. 533) many references to the literature.

S. W. J. S.

ELECTROLYSIS, TECHNICAL APPLICATIONS OF

I. General Introduction

This article deals with the applications of electrolysis to the extraction, production, and refining of chemical products, omitting those subjects which are of primary interest to the electrical engineer, such as (*a*) primary cells, with the attendant great possibility of the direct conversion of the chemical energy of fuels into electrical energy, (*b*) secondary cells or storage batteries, and (*c*) electrolytic alternating current rectifiers. Electro-chemical analysis is also left out, but a section is included on electroplating and electrotyping.

§ (1) Electrolytic Processes.[1]—Before proceeding further, it will be well to consider broadly the features which distinguish electrolytic from ordinary chemical processes. The most obvious one is that technical electrolytic processes all lead to the production of systems of higher chemical potential than the reacting systems, and that the increased energy content is supplied as electrical energy, not in the form of heat or as the result of a diminution in chemical energy of other added substances. The electric current never plays the part of a *catalyst*. Further, it is well known that electrical energy can be changed to chemical energy in a way which frequently approaches thermodynamic reversibility, and even under technical conditions electrolytic processes are generally characterised by a far higher conversion factor of the energy supplied into chemical energy than is the case when heat

[1] See also "Electrolysis and Electrolytic Conduction."

is used. Except, however, under very exceptional circumstances, the price of electrical energy is considerably above that of heat energy, which largely nullifies this advantage. Another feature of electrolytic processes is partly inherent, and partly arises from the necessity of keeping as low as possible the irreversible losses of the expensive electrical energy. The currents which are found economic in practice are of such magnitude as to furnish comparatively small quantities of chemical product in a given time for the size of plant used. In other words, the time-space yield of the plant is low, and as the auxiliary chemical plant is usually large, owing to the fact that, in electrolytic reactions, we are to a great extent not dealing with matter in concentrated form—*i.e.* in solid or fused state—but in aqueous solution, the first costs of the whole installation for a given annual output are liable to be high. On the other hand, the substitution of low temperature electrolysis for high temperature fuel-fired processes means a great saving in upkeep of plant. Finally, owing to the greater chemical simplicity of the reactions, electrolytic products are usually of a high degree of purity, and the same result follows from the use of electrical energy as against that of fuels. If impure raw materials are used, the impurities must either be removed beforehand or constantly eliminated during the process—otherwise disturbances will arise, electrolytic processes being more sensitive to such influences than chemical processes.

II. Current and Energy Efficiency

We have spoken above of the conversion factor of the electrical energy supplied into the chemical energy of the final product. This is termed the *energy efficiency* of a process, and is composed of two factors, the *current efficiency* and the *voltage efficiency*.

§ (2) Factors affecting Current Efficiency.—In all technical electrolyses the quantity of the desired product finally obtained falls short of the theoretical quantity calculated from Faraday's law. This is not due to a breakdown of the law, but to a variety of other reasons. Losses occur from current leaks and short circuits. Frequently two or more electrode reactions, resulting in different products, only one of which is required, take place simultaneously. Thus, in addition to chlorine, oxygen is sometimes evolved at the anode during brine electrolysis. And when an impure metal is being refined, the impurities, as well as the main constituent, will probably be dissolved at the anode, and in small quantity deposited on the cathode. More important are the losses due to secondary chemical reactions between the primary products formed at the electrodes and other substances present, *e.g.* water, or substances dissolved in the bath, perhaps the electrode material itself. Further, the primary products may be liberated under such conditions that they diffuse away rapidly from the neighbourhood of the electrode, and perhaps recombine with substances produced at the other electrode, or they may be volatile or for some reason difficult to collect. For example, in the Castner process for making sodium by electrolysis of fused caustic soda, some of the metal is lost by vaporisation, some burns away before and whilst being collected, some dissolves in the molten electrolyte and diffuses to the anode, where it combines with the oxygen being liberated there, and some reacts with the water which is simultaneously produced during the electrolysis. Lastly, either the final product or an intermediate product may be itself unstable and decompose before it can be removed or transformed into its final form. Examples are respectively furnished by sodium hydrosulphite and by the rôle played by sodium hypochlorite in the manufacture of sodium chlorate.

The ratio of the yield obtained to the theoretical yield calculated from Faraday's law is the *current efficiency*. In technical practice it nearly always varies between 40 per cent and 100 per cent.

§ (3) Factors affecting Voltage. Energy Efficiency.—The voltage taken by a technical cell is chiefly made up of three factors. Firstly and most important is the theoretical voltage necessary to bring about the change under thermodynamically reversible conditions. This is known as the *reversible decomposition voltage*. In many cases it can be closely measured by gradually increasing the potential difference across the terminals of the cell, and plotting it against the resulting current. The latter will at first increase very slowly ("diffusion" or "residual" current), but will suddenly rise when the applied voltage has reached a certain definite value, sufficient to bring about decomposition at the electrodes. This value approximates to the reversible decomposition voltage. In other cases this magnitude cannot be measured, but is calculable thermodynamically from the free energy relations of the different substances involved. In any case it is the algebraic sum of the reversible potential differences at the two electrode-electrolyte surfaces.

The second factor influencing voltage is the existence of specific irreversible effects at the electrodes. Of these, two in particular should be mentioned, viz. *overvoltage* and *passivity*. The former is met with when one of the electrode products is a gas, such as hydrogen, chlorine, or oxygen. If the potential difference electrode-electrolyte be measured at such an

electrode during the passage of current, it will be found that it differs from the reversible value, sometimes considerably, in the direction indicating increased polarisation. This difference is known as the overvoltage, and is dependent in the first instance on the specific nature of the evolved gas and of the electrode material. As would be expected from such an irreversible effect, it increases with fall of temperature and with increased current density, and often exceeds a volt in magnitude.

Passivity is frequently encountered when a metal is made the anode in an electrolyte. Such an electrode, when brought to a potential which should cause it to go into solution reversibly, does not dissolve, and an increased anodic polarisation is necessary before current will pass. In extreme cases no solution of the metal at all occurs, but another electrode process, *e.g.* gas evolution, takes place. The ferrous metals—iron, nickel, chromium, etc.—exhibit passivity in a marked degree. So also do gold and platinum. In the last case the property is a valuable one, as it enables the metal to be used as anode in brine electrolysis cells. As with overvoltage, the magnitude of the passivity polarisation depends on the specific nature of the electrode, and is affected in the same way by changes in temperature and current density. The nature of the electrolyte also has an effect. Oxidising ions, such as NO_3' and ClO_3', assist, whilst $H^{\cdot}$ ions and halogen ions diminish passivity. The causes of both overvoltage and passivity effects are still in dispute, and cannot be discussed here. Their importance in technical practice is great.

Apart from the above, the voltage of a cell is of course largely determined by the ohmic resistance of the electrolyte. This is reduced as far as practicable by bringing the electrodes near to one another and by raising the temperature, limits being set by increased liability to diffusion and chemical losses, as well as by the possibility of short circuits.

Finally, two minor factors affecting voltage must be mentioned—concentration polarisation in the electrolyte brought about by electrolysis (this is generally neutralised by effective circulation of the bath liquors) and losses in leads and terminals, depending on their resistance and on the efficiency of the contacts made.

The ratio of the reversible decomposition voltage to the cell voltage is the *voltage efficiency* of the process, and the ratio of the theoretical amount of energy required per unit of substance actually produced or recovered to the actual amount expended, *i.e.* the product of the current efficiency and the voltage efficiency, gives the *energy efficiency* of the process. Such energy efficiencies are of course lower than the corresponding current efficiencies. Thus in brine electrolysis (alkali and chlorine, chlorates, hypochlorites), they vary between 20 per cent and 70 per cent.

III. The Technical Electrolysis Bath

§ (4) General.—A technical electrolysis bath with aqueous electrolyte may absorb up to six or seven volts—seldom more. Using D.C. generators, furnishing say 200 volts, it is therefore necessary to couple up a large number of cells in series. The current taken by a single cell will be determined by the nature of the electrolysis, the most convenient dimensions of unit from other points of view, the efficiency of circulation required, the best working temperature, labour charges, etc., etc. Thus a small unit with plenty of surface for cooling is best for low temperature work. If, further, there is, as is usual, a continuous flow of electrolyte throughout the plant, a small unit, involving frequent changes of direction of flow, will mean relatively good circulation of the liquors. Technical units frequently take 1000-4000 amperes, seldom more, and often less. If they are of small capacity, it may be necessary to connect up two or more paralleled series of cells with a single generator. The disadvantage of having too many cells in series is that a mishap to one unit puts a large number out of action until it is rectified.

The bath itself is usually made of cement, concrete, slate, or lead-lined wood. Glazed earthenware and glass are also employed, whilst iron and lead are used on occasion, serving at the same time as an electrode. To avoid voltage losses, the separate cells are carefully insulated on earthenware or porcelain, and are usually mounted so that they can be inspected from underneath. The arrangement of the electrodes varies considerably. In metal production and refining, a large number of anodes are paralleled by means of a common bus-bar, and arranged

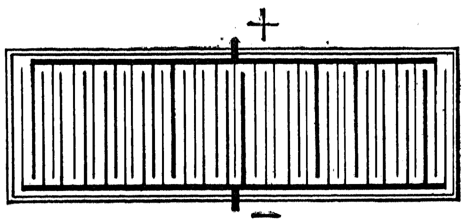


Fig. 1a.—Paralleled Electrodes.

alternately with a similar set of cathodes, thus ensuring a big surface with a corresponding large current, and, at the same time, permitting the use of a bath of convenient dimensions (*Fig.* 1a). On the other hand, in the "series" system of copper refining, the

electrodes are arranged in the bath parallel to one another in such a way that they act as *bipolar* electrodes, one side being the anode and the other the cathode, the two end electrodes being connected to the external source of current (*Fig.* 1B). In such an arrangement the current is of course small, the voltage high. And if the electrolyte resistance is not high compared with the electrode polarisation, shunt current losses will occur, owing to a fraction of the electrolysis not taking place between adjacent electrodes. Electrolytic baths for other purposes are all designed on one of these two principles (generally the former, *i.e.* high amperes, low volts), though the actual details of the arrangement adopted depend largely on the particular process concerned. The Finlay alkali-chlorine cell has a number of alternate anodes and cathodes separated by diaphragms and paralleled. Most other alkali-chlorine cells have one anode or set of anodes, with a cathode on each side or below. Certain hypochlorite and chlorate cells, also water electrolysers, have bipolar electrodes. Details will be discussed later.

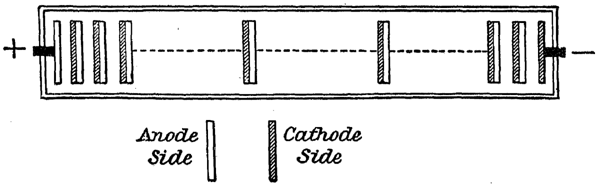


FIG. 1B.—Bipolar Electrodes.

Copper, with its high volume electrical conductivity, is the metal most frequently used for leads and bus-bars. Aluminium, which has a higher weight electrical conductivity than copper, is also employed to a considerable extent. It has the advantage of being less attacked in an atmosphere containing chlorine. Where practicable, bus-bars and electrodes are securely bolted together to avoid contact losses. If, however, the electrodes are being constantly removed and replaced, as in metal refining, they are simply suspended from the bus-bars, care being taken to secure contact between clean and plane surfaces.

§ (5) ELECTRODES.—The important points to be considered in technical electrodes, other than soluble anodes or cathodes on which metal is being deposited, are (*a*) their chemical resistivity, (*b*) their overvoltage for the gas or gases that are liable to be liberated at their surfaces. Iron, platinum, lead, mercury, and graphite are the most important cathode materials. Iron is generally used in alkaline solutions, *e.g.* in many alkali-chlorine cells. It is chemically stable, and has a low hydrogen overvoltage. Smooth platinum is used in the form of bipolar electrodes in various types of electrolytic bleach cells. It too has a low hydrogen overvoltage, though greater than that of iron. Platinised platinum at normal current densities has a practically negligible hydrogen overvoltage, but cannot be used technically, as its surface readily disintegrates. Lead is used in the Schoop water electrolyser, which employs dilute sulphuric acid as electrolyte. It has a very high hydrogen overvoltage (*e.g.* 1·2 volts at a current density of 10 amps/dm.^{2} in $2n.H_2SO_4$ at room temperature). The most commonly used cathode material when working with acid solution is, however, graphite, which is chemically stable, and has a low hydrogen overvoltage. Mercury is used as cathode in a particular class of alkali-chlorine cells. The object here is to produce a sodium or potassium amalgam, without simultaneous evolution of hydrogen, and the fact that hydrogen overvoltage at mercury is as high as, or higher than, that at lead, makes this possible. In practice a high current density is used, and 95 per cent to 99 per cent of the cathodic current in such cells is carried by the alkali metal ions.

The subject of technical anodes is of greater importance, in view of their general liability to attack by anodic chlorine or oxygen. The important anode materials used in chlorine cells are platinum, hard carbon (similar to gas carbon), graphite, and magnetic iron oxide (Fe_3O_4). Platinum is used in chlorate manufacture, in certain mercury cells, and also, as bipolar electrodes, in bleach electrolysers. It shows considerable resistance to the action of chlorine, but is slowly attacked in course of time, particularly at higher temperatures. An alloy containing 10 per cent iridium is far more resistive, and is generally employed. Under technical conditions, the chlorine overvoltage amounts to 0·7 volt in alkali-chlorine cells. Owing to the high price of the metal, and the necessity of utilising its surface to the maximum extent, platinum electrodes are frequently of net or gauze construction. The use of graphite has almost completely replaced the use of hard carbon anodes in brine electrolysis. Both materials are very resistive to chlorine. But varying quantities of oxygen are also invariably produced anodically in brine cells, and the hard carbon is far more readily attacked by this gas than is graphite. When once such anodes have started to burn away they disintegrate more or less rapidly, and foul the electrolyte. Hard carbon anodes which are partly graphitised are in use, and

have of course intermediate properties. Graphite possesses the additional advantages of having a higher electrical conductivity, and of being soft and easily machined. By means of a suitable preliminary chemical treatment, the exact nature of which is unknown, anodes can be made with a continuous life of several years before replacement is necessary. Their chlorine overvoltage value is very low. From the point of view of chemical resistivity, cast magnetite anodes are the most satisfactory, and are now coming more generally into use. They are made by fusing up decopperised iron pyrites (chiefly ferric oxide) in the electric furnace. Impurities are volatilised, and partial decomposition into ferrous oxide occurs. To the melt is added sufficient powdered ferric oxide to make a homogeneous mass of Fe_3O_4 on solidification, and the anodes are then cast in the form of hollow cylinders, closed at the bottom, and about $\frac{1}{4}''$ thick. They are coated internally with copper electrolytically, and contact thus made with a copper ring at the upper end. Their conductivity is not high, but is sufficient. Unfortunately they have a high chlorine overvoltage, greater than that at platinum.

For use with alkaline electrolytes evolving oxygen, iron or nickel can be employed. In acid solution, iron can sometimes be used, but platinum is, from a chemical point of view, more satisfactory. Lead is widely used, and electro-deposited lead and manganese peroxides, as also magnetite, have been utilised with success in certain instances. Except in the case of platinum in alkaline solution, the oxygen overvoltage values are low, and seldom exceed 0·5 volt.

§ (6) Diaphragms.—These are extensively employed in cases where it is necessary to prevent mixing of the anolyte and catholyte. To be satisfactory, they must have a low electrical resistance, and be at the same time mechanically strong and unattacked chemically. A great many substances have been tested or recommended from time to time. For use in alkaline solutions, cement and asbestos are the usual starting materials, the porosity being adjusted by suitable treatment. For use in acid solutions, diaphragms have been devised consisting of siliceous material or artificial corundum (alundum), or mixtures of the two. The porosity can be regulated by the deposition of hydrated silicic acid. Good quality canvas or linen is also used.

IV. Electrolytic Solution and Deposition of Metals

§ (7) Electro-deposition.—Before proceeding to the subject of electro-plating, a few special points in connection with the electro-deposition and anodic solution of metals must be discussed. If a solution of a metallic salt be electrolysed, using a cathode of the same metal, metal deposition should take place as soon as the cathodic polarisation exceeds the value corresponding to the equilibrium potential difference electrode-electrolyte, assuming the metal to be more readily deposited than hydrogen. In certain cases irreversible effects come into play, necessitating a considerably greater polarisation. Examples are the deposition of iron and nickel from solutions of their sulphates, the deposition of copper and zinc from solutions of their alkali double cyanides, etc. More important are the phenomena which may occur if the cathode consists of another metal which can form an alloy with the metal present in the electrolyte. In this case depolarisation will take place—the deposited metal will not exert its full electrolytic solution pressure, and can be plated out, initially at all events, at a lower polarisation than that which corresponds to its normal equilibrium value. An extreme case is the depolarising action of the mercury cathode in mercury alkali-chlorine cells, which, together with the high hydrogen overvoltage at a mercury surface, renders the formation of alkali metal amalgams possible. Other cases of alloying occur very frequently, and probably play a big part in determining whether or not, when one metal is plated out on another, the deposit will adhere well.

The physical state of a deposited metal is of great importance. It is nearly always desirable to have it in coherent form, and the electroplater in addition wants a deposit that can be polished. There are many factors that determine the form of an electro-deposit, and, by changing them, the state of aggregation, hardness, etc., of one and the same metal can undergo very considerable variations. The more concentrated the electrolyte, the more coherent and fine-grained is the deposit. A dilute electrolyte favours the formation of loose crystals. An electrolyte containing the metal in the form of a complex anion, *e.g.* double cyanides, tartrates, etc., will, as a rule, give a denser and smoother deposit than will an electrolyte containing the metal as cation. If the metal is present as cation, a slightly acid electrolyte is desirable to prevent the precipitation of basic salts. The effect of current density is somewhat complex. If it is very low, an adherent coarsely crystalline deposit is the rule. As it is raised, the deposit becomes finer in grain, harder, and brighter. If raised still higher, the concentration of metal ion in the electrolyte in the immediate neighbourhood of the electrode is depleted more rapidly than it can be regenerated by diffusion, stirring, etc., the effect being to give a non-adherent deposit.

It follows that firm deposits are favoured by good circulation of the electrolyte. The effect of temperature is again twofold. If the concentration and current density are of the right magnitude, a fine-grained hard deposit is favoured by a low temperature, a higher temperature giving bigger crystals. If, however, concentration is somewhat low, and current density somewhat high, a more adherent deposit can be got at a high temperature than at a low, owing to the increased rate of diffusion of ions into the layer of electrolyte immediately surrounding the cathode. One last point is the enormous effect often exerted on the nature of a deposit by the addition of a small quantity of some organic substance, often colloidal in nature, to the electrolyte. If the right addition agent is chosen—the action is specific for different metals—a very fine-grained and smooth deposit will result under conditions which would otherwise furnish a coarsely crystalline metal. If too much be added, the deposit becomes brittle, and contains appreciable amounts of organic matter. Examples are the addition of gelatine to a copper bath, of resorcinol to a zinc bath, and of pyrogallol to a lead tank. A discussion of the reasons for this effect would lead us too far—it suffices to say that it is undoubtedly connected with the adsorption of the addition agent on the surface of the depositing particles of the metal.

The physical state of the metal also influences its anodic behaviour. Thus a cast metal will dissolve more readily than metal that has been worked by drawing or hammering, and metal that has been electro-deposited dissolves usually less easily than other forms, though more evenly. Both cast and wrought metals are less homogeneous than electro-deposited metal, and tend, when used as anodes, to leave their less soluble parts undissolved, in the form of "slimes." "Slimes" are of course chiefly produced when anodes are used containing other metals as impurities. If these are less noble than the main constituent, they will enter solution. If more noble, they will remain undissolved. The relations may be somewhat complicated if such less noble foreign metals are dissolved in the main constituent of the anode in the form of a homogeneous alloy. In this case a metal may find its electrolytic solution pressure so decreased, and its "noble" character so enhanced, that it may not dissolve, but remain in the "slimes."

V. Electroplating and Electrotyping

§ (8) General.—Electroplating is the art of covering a metallic surface by electro-deposition with an adherent coating of some other metal, the form of the original surface being fully retained. The second metal may be deposited for decorative purposes, or because of its superior resistance to chemical and atmospheric influences. Electrotyping is the art of reproducing the form of an object by electro-deposition on a cast or negative.

As an electro-deposited metal layer must before all be firmly adherent, all articles to be electroplated are very carefully cleaned. They are first "buffed," *i.e.* all file marks, irregularities, etc., produced during manufacture are removed by suitable polishing machines. Then they are cleansed from grease by dipping into a boiling caustic alkali solution, and the last traces of rust or oxide are removed by pickling in a solution, usually acid, of composition depending on the nature of the article to be treated. From there they go to the plating bath, in some cases after a further mechanical preparation of their surface. When the acid pickling bath is inadmissible (the evolved hydrogen may dissolve in the metal and alter its mechanical properties), the oxide layer is removed by wire brushes or by sand-blasting. Electrolytic methods of cleaning have also been devised and appear to be coming into favour. The articles are made the cathodes in a solution of either acid or alkali, and both grease and scale are very efficiently removed. In some cases the connections are so arranged that the current can be switched over, the articles becoming the anodes for any desired period. Both in electrolytic cleaning and in the pickling bath, the gas evolution removes much of the scale mechanically.

As the objects in the plating vats are being continually removed and replaced, an arrangement of any considerable number of tanks in series would prove very inconvenient. They are therefore connected in parallel as far as possible, and current taken from a low voltage dynamo of high amperage. Each tank should be supplied separately with voltmeter, ammeter, and regulating resistance. When working under standard conditions, the voltmeter can be eliminated, and an ampere-hour meter conveniently used for a single tank or set of tanks. The objects under treatment are suspended by hooks from copper rods, resting on bus-bars passing round the edges of the vat. By means of an eccentric, they can be gently moved to and fro inside the bath. Additional circulation can be provided by "beaters" similarly connected up, or by blowing in air. Small objects are contained in metal baskets. The anodes, which consist when possible of a pure sample of the metal to be plated, are similarly connected up in parallel, and arranged alternately with the cathode articles. Care is taken to utilise the whole cross-section of the bath and to dispose the anodes symmetrically, in order that the cathode articles shall be evenly plated. Auxiliary anodes are placed for this purpose

in special positions when required. If necessary, the anodes are enclosed in linen or parchment bags to retain the slimes. If the electrolyte is to be heated, this is done by leaden steam coils.

The current density used varies, according to the particular process and conditions of electrolysis, between 0·1 and 2 amps/dm.^{2} The thickness of the deposit is very small—usually measured in thousandths of a mm. for cheap articles, and rarely exceeding a few tenths of a mm. At the end of the plating the articles are washed, dried, and finally polished. In some cases, particularly if comparatively thick deposits are being plated, the articles are occasionally removed from the bath during the course of the electrolysis, and the surface polished by suitable means.

§ (9) Zinc Plating.—Zinc plating (wet galvanising) is replacing hot galvanising (dipping the articles in a bath of molten zinc) to an increasing extent, for the purpose of protecting iron and steel surfaces against atmospheric influences. More resistant coatings of easily controllable thickness can be produced and there is less waste of zinc. The electrolyte is nearly always a fairly strong solution of $ZnSO_4$—say 30 grams zinc per litre—with the addition of some other salt such as Na_2SO_4 or $MgSO_4$ to increase the conductivity. A little $FeSO_4$ or alum improves the deposit, probably owing to the presence of small quantities of colloidal hydroxide, and the addition of boric acid has been recommended to regulate the acidity. An average bath will be worked at 50°, with a current density of 2 amps/dm.^{2} and will take about 3·5 volts. The current efficiency is almost 100 per cent.

§ (10) Nickel Plating.—This is of great importance, owing to the mechanical strength, chemical resistivity, and capacity for polish of the metal. The electrolyte consists of the sulphate—10 to 15 grams/litre of the metal—with the addition of a large quantity of ammonium sulphate to increase the conductivity. A little boric or citric acid confers the right acidity, which must be carefully regulated by the addition of sulphuric acid or by hanging bags containing nickel carbonate in the bath. A certain amount of acid is produced at the anodes, which do not dissolve in amounts quite corresponding to Faraday's Law, and some hydrogen is always produced at the cathodes. This must not be allowed to become too great, or the deposit will become brittle and non-adherent. Hence the need for careful regulation of the acidity. This can partly be effected by adjusting the relative numbers of cast (readily soluble) and rolled (less readily soluble) anodes. For ordinary work, the vats are run at 20°-25°. If thick deposits are required, the temperature should be raised to 70°, or special solutions used. The current density will average 0·3-0·5 amp/dm.^{2}, and the voltage 3 volts. A higher voltage is usually applied for the first few moments the articles are in the bath, until they are completely covered with a film of metal. The current efficiency approaches 95 per cent.

The above two metals are plated out from sulphate solutions. In all other important electroplating processes, viz. deposition of silver, gold, copper, and brass, a solution of the corresponding double cyanide with potassium is used, prepared by the addition of potassium cyanide to an aqueous solution of an appropriate salt. A smooth dense deposit is obtained which can readily be polished.

§ (11) Silver Plating.—This is probably the most important branch of the electroplater's art. The solutions used contain 12-15 grams/litre of metallic silver, present as $KAgCy_2$, together with a considerable excess of KCy. Owing to the action of atmospheric CO_2, gradually increasing amounts of K_2CO_3 are formed in them. Unless present in excessive quantities, this does not affect the plating qualities of the bath. When necessary, the liquors can be regenerated by the addition of $BaCy_2$, which precipitates the carbonate as insoluble $BaCO_3$. The anodes are rolled sheets of "fine" silver, and dissolve quantitatively. Their surface should be as nearly as possible equal to the surface of the cathodic articles to be plated. The electrolysis is best carried out at room temperature. Silver deposition is nearly quantitative, a current efficiency of 99 per cent being readily obtained. The most suitable current density is about 0·15 amp/dm.^{2}, at a voltage of one volt. The deposit is of a dull white colour which can be readily polished. The addition of a trace of carbon bisulphide to the bath effects a remarkable change, giving a bright shining deposit of close texture. Its action is doubtless analogous to those of other addition agents already mentioned.

§ (12) Gold Plating.—This is carried out in a bath containing 6-8 grams/litre of gold, with an excess of KCy. Depending on the class of work to be done, and the colour of the deposit required, the baths are worked at room temperature or at, say, 70°. Objects of intricate design are best plated at a high temperature, in consequence of the higher rate of diffusion of the electrolyte into the interstices of the articles. The voltage will of course depend on the temperature—at 20° four volts is about normal. The anodes are of fine gold in the form of very thin sheets, in order to obtain the maximum surface for a given weight. They exhibit passivity to a marked degree, and dissolve very slowly under normal current conditions. It is therefore necessary from time to time to add gold chloride to the electrolyte, as otherwise the deposit loses its usual rich yellow colour.

§ (13) Copper and Brass Plating.—These processes are of importance as being frequently employed in giving a preliminary coating to articles before plating with other metals which would not directly adhere to the original object. They owe their use in this connection to the marked tendency of copper to alloy formation. Thus, iron and steel articles are coated with copper or brass preparatory to silver or nickel plating, as also are Britannia metal and lead-zinc alloys before bringing them into the nickel bath. A copper bath contains about 25 grams/litre of copper as $KCuCy_2$, with 20 per cent excess KCy. The anodes are of pure sheet copper. The normal bath voltage is 3-4 volts, and cathodic current density 0·3 amp/dm.^{2}. There is considerable hydrogen evolution at the cathode surface during plating, the current efficiency rarely exceeding 50 per cent. The case of brass-plating is interesting from an electro-chemical point of view. In acid solution, the single potentials of the two metals lie far apart, and an electrolyte containing the two metals only deposits copper. In a cyanide solution, however, owing to the greater tendency to anionic complex formation of the copper, their electrode potentials can be equalised, or, with a large excess of KCy, the zinc can actually be made more noble than the copper. With the depolarising alloy effect also coming in, it becomes quite possible to deposit alloys corresponding in composition and properties to commercial brasses (*i.e.* with 20-40 per cent zinc). A suitable electrolyte contains about 12 grams/litre each of copper and zinc, present as $KCuCy_2$ and K_2ZnCy_4, with about 10 per cent KCy in excess. Anodes of rolled brass are used, corresponding in composition to the brass which is being plated out. The bath is usually worked cold, taking 4-6 volts. The current density is variable, perhaps about 0·35 amp/dm.^{2} at the cathodes, and still lower at the anodes. There is marked hydrogen evolution at the cathodes, the current efficiency averaging 50 per cent.

§ (14) Electrotyping.—Most electrotypes are of copper, sometimes coated for durability with a thin layer of iron or nickel, and are deposited from an acid copper sulphate bath, containing about 50 and 30 grams/litre of copper and free sulphuric acid respectively. The negative casts of the original are usually prepared from gutta-percha, though other materials, as plaster of Paris or a suitable wax, are sometimes used. Before putting in the bath, they are coated over with a thin layer of graphite which gives conductance, and are furnished with copper leads. Forms of fusible metal can also be used. The anodes are rolled sheets of pure electrolytic copper. The electrolysis is carried out at room temperature or somewhat above it, with a current density of about 3 amps/dm.^{2} and a voltage of one volt. When the necessary thickness of metal has been deposited, the article is taken from the bath, the negative cast detached by gently warming, and the electrotype backed with a low melting fusible alloy to give it the necessary rigidity.

Iron electrotypes are also made, for subsequent use as dies. The most suitable electrolyte is a strong $FeCl_2$ solution, containing in addition NH_4Cl or $CaCl_2$ to increase the conductivity. Great care must be paid to the exact acidity. The electrolyte should be just acid and no more, as otherwise a very brittle metal which easily rusts is produced, owing to the simultaneous formation of hydrogen at the cathode. If, on the other hand, the bath is not kept faintly acid, basic precipitates will form in the electrolyte. The anodes should be of the purest soft iron, and of area greater than that of the cathodes. A high temperature is favourable (70°-90°) and a high current density is employed—5-10 amps/dm.^{2}, or even greater if the circulation of the electrolyte is efficient.

Attention should finally be directed to the use of copper electrotypy for making seamless copper tubes of high tensile strength. The cathode negative is a revolving drum or cylinder, coated with a thin film of oil or graphite. The bath is of acid copper sulphate, similar to the one mentioned above, and is usually run at room temperatures. The anodes are electrolytic copper. In order to ensure a dense, regular, and strong deposit, the cathode is continually burnished by agate brushes, by glass beads kept suspended in the electrolyte, or other means. Ingenious methods have been proposed for making copper wire or copper sheet from the cylinders which result. Cowper-Coles recommends working with rapidly revolving cathodes and high temperatures. The increased friction allows of the use of very high current densities. The same inventor has successfully developed the production of seamless iron tubes by electrotypy.

VI. Electrolytic Extraction and Refining of Metals

§ (15) Copper Extraction.—The electrolytic refining of copper, one of the most important of the electro-chemical industries, is being dealt with in a special article of this Dictionary.[1] The extraction of the metal from its ores by electrolytic means has been the subject of much investigation, but can only recently be said to have become a commercial success. It was laid down early in this article that one of the essential factors for the success of an electrolytic process is an electrolyte of regular composition, the impurities in which can be readily kept within defined limits. And in wet processes of metal extraction these con-

[1] See "Electrolytic Refining of Copper," Vol. V.

ditions are not easily satisfied. The composition of the ore fluctuates. Such fluctuations, even when of small magnitude considered absolutely, are of particular importance with those ores which are too poor for pyro-treatment, and which are generally the ores treated by wet methods. Further, the preliminary roasting to which the ores are usually first submitted is difficult to regulate exactly. These causes lead to variations in the constitution of the electrolyte which make a controlled electrolysis very difficult, and which have led to the abandonment, after long trials, of, for example, several processes of copper extraction which promised well on the laboratory scale. The Marchese process utilised anodes cast from a copper matte containing copper, iron, lead, and sulphur in varying proportions, together with smaller quantities of other substances. The electrolyte contained copper and iron sulphates, with sulphuric acid. When a current is passed, such a sulphide anode sends its metallic constituents into solution, leaving a residue of free sulphur. At the cathode, copper is the chief metal deposited, whilst the iron remains in the electrolyte, and the lead is precipitated as insoluble sulphate. In practice, impurities accumulated in the electrolyte beyond permissible limits, the copper deposit became contaminated, the anodes disintegrated, and the voltage rose on account of the sulphur layer formed on them. Using a purer matte, the results were better, but the process never became commercially practicable. Similar reasons account for the abandonment of the ingenious Siemens-Halske and Höpfner processes. The finely-ground partly-roasted ores were treated respectively with a solution of ferric sulphate and one of $CuCl_2$ containing an excess of common salt. The copper sulphide dissolved, giving in the first case a solution containing $CuSO_4$ and $FeSO_4$, in the second case a solution of Cu_2Cl_2 dissolved in excess of salt. The purified liquors were electrolysed in a diaphragm cell containing copper cathodes and lead or carbon anodes. Metallic copper was deposited on the cathodes whilst at the anodes ferrous sulphate was oxidised to ferric sulphate and Cu_2Cl_2 to $CuCl_2$ respectively, the liquors then being utilised for leaching further quantities of ore. Expressed ionically, the two cell reactions are

$$Cu^{\cdot\cdot} + 2Fe^{\cdot\cdot} \longrightarrow 2Fe^{\cdot\cdot\cdot} + Cu$$

and $$2Cu^{\cdot} \longrightarrow Cu^{\cdot\cdot} + Cu,$$

and, in consequence of the depolarisation without gas evolution which takes place at the anodes, require considerably lower voltages than would be necessary, using pure solutions of say cupric sulphate or chloride. In spite of this advantage, the difficulty of controlling the electrolyte composition, together with diaphragm troubles, led to their abandonment. The only successful commercial processes are those in which a $CuSO_4$ solution, obtained by lixiviating the fully roasted ore with dilute H_2SO_4, followed by careful purification, is electrolysed using insoluble anodes (lead or magnetite) with oxygen evolution. The Laszczynski process may be taken as a type. The filtered electrolyte contained about 3 per cent copper as sulphate with 1 per cent free acid and much iron. The cathodes were thin copper sheets. The anodes were of lead, which soon became covered with PbO_2, and were surrounded by closely fitting thick cloth diaphragms, by which means the oxidation of the ferrous to ferric sulphate was practically eliminated. Using a current density of 0·5-1 amp/dm.^{2} and 2·2-2·5 volts, the copper could be reduced to 1 per cent with a 90-95 per cent current efficiency. The liquors, containing 4 per cent H_2SO_4, were then withdrawn from the cells and used for extraction of further portions of ore. The electrical energy consumption in such processes amounts to about 2200 K.W.H. per ton of copper produced, and their economic situation is liable to be somewhat precarious.

§ (16) Silver Refining.—Electrolytic methods in the metallurgy of silver are practically confined to silver refining. There are several different cases to consider. The crude material may consist chiefly of silver, with a small proportion of gold, and still smaller amounts of other metals, *e.g.* anode slimes from copper refining, or silver concentrates from the Parkes lead process. Or it may contain considerably larger proportions of gold, as in crude bullion. Or it may consist of scrap jewellery, plate, etc., in which case there will be high percentages of copper and of base metals. To these differences correspond well-marked differences in the methods employed. In any case, the crude material is cast and used as anodes in vats containing a mixture of various nitrates with some free HNO_3. Of the different anodic constituents, silver, copper, iron, nickel, zinc, and cadmium will dissolve, together with some of the lead, tin, and bismuth. The remainder of these last substances will enter the slimes as PbO_2 and basic salts of bismuth and tin. The slimes will also contain unattacked gold, platinum, other precious metals, and tellurium. At the cathode the only metal likely to deposit, if appreciable quantities of silver are present, is copper, and even then only under exceptional conditions—high current density, high copper concentration, and poor circulation of electrolyte. The other metals will simply accumulate and must be removed from time to time.

The Moebius process is used for refining crude silver from copper and lead refineries.

It is carried out in tanks of stoneware, divided into several compartments. Each compartment contains a number of anodes, enclosed in bag-diaphragms of canvas. They are arranged in rows alternately with rows of cathodes of "fine" silver foil, electrodes of like sign being paralleled, and the different compartments being connected in series. The anodes contain 95-98 per cent silver, 0·5-3 per cent gold, together with copper, bismuth, lead, and other metals. Interest charges lead to the use of high current densities, in order to shorten the time the silver remains in the bath. This favours copper deposition, and the copper concentration in the bath must therefore be kept below certain limits. An average electrolyte is kept at about 15-20 grams/litre and 30-40 grams/litre of silver and copper respectively. With a current density of 4·4 amps/dm.^{2} the voltage is high, about 1·5 volts. The anodes are in the bath for 24-36 hours. The silver comes down in the form of long crystals which tend to grow over towards the anodes. To avoid short circuits and to circulate the electrolyte, the cathode surfaces are continuously scraped with a wooden scraper, the crystals falling to the bottom of the vessel and being there collected. They are very pure, at least 99·9 per cent silver. The current efficiency is 94-95 per cent. A ton of refined silver requires about 420 K.W.H.

The only important modification of this process, the Balbach-Thum process, is used in the U.S.A. A shallow glazed porcelain trough, lined with graphite (the cathode), is used as the cell, and the anodes (enclosed in canvas bags) are arranged on shelves some distance above the cathode. The electrodes are further apart than in the original Moebius process, as there is no mechanical arrangement for brushing the silver from the cathode. It is merely pressed down and removed from time to time. Working with much the same current density, twice the voltage and energy expenditure are required as in the Moebius process. But there is a more complete separation of silver (the anodes are not removed until completely consumed) and the cell is of simpler construction.

With anodes which are rich in gold, such as are used at the different mints, it becomes more necessary than ever to lower interest charges, and this is effected by the use of higher current densities. Thus, at the San Francisco mint, where anodes with 30 per cent gold, 60 per cent silver, and 10 per cent base metals are employed, a modified Balbach-Thum process is used, with a high current density, and with a higher silver concentration in the electrolyte than when using anodes poor in gold. In some cases a trace of gelatine is added to the electrolyte, which furnishes the silver in coherent form.

When the percentage of copper in the crude material is very high, the process is essentially modified, the anode liquors leaving the cell and having the silver removed from them chemically before passing into the cathode compartments, where copper is deposited. For example, Dietzel employs a cell in which anolyte and catholyte are separated by a horizontally arranged canvas diaphragm. The anodes, in slab form, are below this. The anolyte is drawn off from the bottom of the cell, passes over scrap copper where the silver is deposited, and re-enters the cell by the cathode compartment, which contains the slowly rotating cylindrical cathodes, arranged above the diaphragm. Typical anodes contain 5 per cent gold, 40 per cent silver, 50 per cent copper, and 5 per cent other metals. The base metals which accumulate in the liquors must be kept down by withdrawing a part of the electrolyte at intervals and adding copper and nitric acid. With an anodic current density of 1·5 amps/dm.^{2}, a voltage of 2·5-3 volts is required.

§ (17) Gold Extraction.—Electrolytic methods are used in gold metallurgy both in extraction and in refining processes. As is known, gold is extracted from low-grade ores and from the "tailings" or "slimes" of the amalgamation process by means of a weak solution of potassium or sodium cyanide in presence of air, the metal dissolving in the form of a double salt $KAuCy_2$. From this solution it is generally removed by means of metallic zinc, the gold coming down in the form of a finely divided coloured semi-colloidal precipitate. In some mines, however, this precipitation is carried out electrolytically. The filtered liquors, carrying 3-10 grams/cubic metre of gold are passed slowly, with frequent changes of direction, through long wooden vats, containing paralleled anodes and cathodes arranged alternately. The anodes are large iron sheets, 3-4 mm. thick. As they are slowly attacked by the electrolyte, with formation of Prussian blue, they are enclosed in canvas bags. They last for years. The cathodes consist of strips of sheet lead, suspended from bus-bars of zinced iron. At the low current densities employed—about 0·4 amps/metre^2—the gold adheres to them satisfactorily. When they contain about 10 per cent of gold, which does not happen till they have been in the vats for weeks or months, they are removed, cupelled, and refined. About 85 per cent of the gold is deposited, the liquors then returning to the leaching tanks. The current efficiency is exceedingly low, perhaps 1 per cent, much hydrogen being produced. The baths take 4-5 volts. The disadvantages of the process are the slow rate of work, and the enormous plant required. On the other hand, there is a great saving in

zinc and in cyanide, and the metal is obtained in a more workable and easily purified form.

§ (18) Gold Refining.—The above process uses a cyanide electrolyte, as in electro-gilding. In the refining of gold, as now carried out at several mints and also in the big metal refineries of the U.S.A., the electrolyte is an acid $AuCl_3$ solution, consisting essentially of $HAuCl_4$ and HCl. A gold anode exhibits somewhat complex behaviour in this solution, depending on the current density, temperature, and acid concentration. At room temperature, with a small anodic polarisation, the gold goes quantitatively into solution. If the current density (and anode potential) be raised, the metal becomes passive. If the electrode be still further polarised, chlorine is evolved. The more acid present (the Cl' ions are the active agent) the less easily does the metal become passive, and the higher can the current density be raised before chlorine evolution begins. Rise of temperature, which counteracts passivity, has the same effect. For example, at 70°, with 3 per cent free HCl in the electrolyte, the limiting current density is 30 amps/dm.^{2} The metal dissolves giving a mixture of auric (trivalent) and aurous (monovalent) chlorides, the proportion of the former being the greater. With high current densities, the production of aurous salt becomes very small. If the solution contains much of it, it tends subsequently to deposit metallic gold ($3Au^{\cdot} \longrightarrow Au^{\cdot\cdot\cdot} + 2\ Au$). At the cathode the relations are also complex. With a small polarisation we get no gold deposited, but a reduction of auric to aurous salt. Further polarisation leads to discharge of a mixture of auric and aurous ions, whilst a still further increase results in the elimination of the aurous ion discharge, and in quantitative precipitation of metal from trivalent ions. At higher temperatures, particularly if the electrolyte has been used for some time, there is no noticeable reduction of trivalent to monovalent ions, and only a slight deposition of monovalent gold, nearly all the current being taken by the discharge of auric ions to metal.

In practice, typical gold anodes contain 94 per cent of gold and 5 per cent of silver, the remainder being copper, lead, platinum, etc. They are cast into thin slabs and suspended by gold wires in the electrolyte. This contains about 80-85 grams/litre of gold and 30 per cent free hydrochloric acid, a trace of gelatine being sometimes added to improve the nature of the deposit. It is circulated through a series of tanks by gravity. The cathodes are of thin gold foil, arranged alternately with the anodes, like electrodes being paralleled. The cells are of glazed porcelain, rest on sand baths, and are heated by steam pipes. The electrolysis is best carried out at 70°. This permits of the use of a current density of about 15-17 amps/dm.^{2}, and thus facilitates rapid work and low interest charges. 0·6-0·8 volt is required under these conditions. The gold, which comes down in coherent form, is deposited with a current efficiency exceeding 100 per cent, calculated on the assumption that the sole reaction is auric ion discharge. The excess is clearly due to the discharge of aurous ions.

All the constituents of the anode dissolve. The silver is immediately precipitated as insoluble chloride, and the lead by adding H_2SO_4 to the electrolyte. Copper, platinum, and palladium accumulate in the solution, the gold content of which falls. On this account the electrolyte must be continually regenerated by withdrawing a fraction and replacing it by $AuCl_3$ solution. The slimes consist chiefly of AgCl, $PbSO_4$, and gold. If the amount of AgCl is excessive, it may be necessary to lower the current density in order to avoid chlorine evolution, which normally is hardly perceptible. Wohlwill avoids this difficulty, and at the same time renders the use of still higher current densities possible by superposing on the direct current used for refining an alternating current of rather greater r.m.s. value. The way in which this acts is not clear, but its effect in practice is undoubted. Anodes of high silver content can be treated, larger current densities used, and the proportion of gold in the slimes much reduced. The cathodic current efficiency is unaffected, and the cost negligible.

§ (19) Zinc Extraction.—Owing to the ease with which zinc can be distilled, electrolytic methods are not used for refining the metal. After, however, much experimental work, and many failures, electrolytic extraction processes appear to have acquired a firm footing. We have seen that in wet galvanising a $ZnSO_4$ electrolyte is employed, fairly concentrated, and very slightly acid, and that a high current density is used. Before proceeding to the discussion of the technical extraction processes, we must consider zinc electrodeposition in further detail, as it presents several features peculiar to itself. Zinc, different from gold, silver, and copper, is far more electropositive than hydrogen, should dissolve freely in acid, and conversely should not be deposited cathodically from a solution containing appreciable quantities of $H^{\cdot}$ ions. In practice, pure zinc is unattacked by acids, and the metal can be deposited cathodically under certain conditions with excellent yields. The apparent contradiction is due to the high overvoltage necessary for hydrogen discharge at a zinc surface, higher for any ordinary metal except mercury. If metallic zinc contains traces of such metals as copper, silver, nickel, or arsenic, it will readily dissolve in acid, in consequence of the low hydrogen

overvoltage of these metals. If, further, an electrolyte contains traces of these metals—more electronegative than zinc and hence deposited first—we shall get hydrogen evolution and not zinc deposition at the cathode. We see then the importance of a pure electrolyte in zinc deposition. As further, overvoltage, like other irreversible effects, increases with rise of current density and lowering of temperature, such factors should favourably affect zinc deposition. This is borne out in practice.

Apart from the yield, the nature of the deposit is affected by the conditions of electrolysis. If proceeding favourably, one obtains a coherent, compact greyish-white metal, but often the deposit becomes spongy, voluminous, and dark coloured, a form which is difficult to handle and to melt up, which encloses electrolyte and causes short circuits, and which has a low hydrogen overvoltage. The formation of this sponge is apparently closely connected with the production of basic salt in colloidal form in the electrolyte, and hence anything tending to decrease the acidity by facilitating $H^{\cdot}$ ion discharge will also assist sponge formation. Such factors are the presence of electronegative metals in the electrolyte, a low zinc ion concentration, a low current density, and a high temperature. Taking these points into consideration, the following are the conditions which will favour zinc deposition: (*a*) high zinc concentration, say 60 grams/litre; (*b*) acid concentration not too high, but appreciable, say 0·01-0·1 n.; (*c*) complete absence of impurities less electropositive than zinc; (*d*) current density fairly high, say 1·2·5 amps/dm.^{2}; (*e*) good circulation; (*f*) low temperature.

If the current density be raised still higher, good current efficiencies can be obtained even with very high acid concentrations (150-200 grams/litre H_2SO_4). And if a small quantity of certain colloids be added to the electrolyte, the deposits are hard, bright, and satisfactory from a physical point of view. The presence of the acid increases the conductivity of the electrolyte and keeps the voltage within reasonable limits. Further, under these conditions, the deleterious effect of impurities in the electrolyte is much reduced. Such processes are now coming into use, current efficiencies of the order of 95 per cent being obtained with current densities of 20-50 amps/dm.^{2} in a sulphate electrolyte, the bath voltage being about five volts. Detailed descriptions have not been published, and the accounts apply to the older established processes. These can be divided into two classes, those using a sulphate and those using a chloride electrolyte. In the former case, the roasted ore is leached with dilute H_2SO_4, and the liquors, after purifying and almost neutralising with zinc oxide, circulated through lead-lined wooden vats and electrolysed between aluminium or zinc cathodes and lead anodes, the latter surrounded by closely fitting diaphragm bags. These anodes are not entirely satisfactory, and are best coated beforehand with a layer of PbO_2. Fe_3O_4 and electro-deposited MnO_2 have been tried with promising results. Working at 30°-35°, with a current density of 2-3 amps/dm.^{2} and a voltage of 3·5 volts, a ton of very pure zinc is obtained at an expenditure of about 3000 K.W.H. The extraction factor is considerably higher than when using the ordinary distillation processes.

Zinc chloride processes are also in operation, the electrolyte being prepared by roasting the ore with salt, or by treating the calcined ore with $CaCl_2$ solution in presence of CO_2. After careful purification, the liquors, slightly acidified with HCl, and carrying a small quantity of an addition agent, are electrolysed between graphite anodes and large revolving disc-shaped iron cathodes. Details of the plants now working are not known, but a cathodic current efficiency of 80 per cent is said to be obtained, using a current density of 3 amps/dm.^{2} and a voltage of about five volts. The losses are largely due to chlorine dissolved in the electrolyte. The metal produced is very pure. The energy expenditure amounts to about 5000 K.W.H. per ton.

§ (20) Tin Recovery.—The pyrometallurgy of this metal is very simple, and the only application of electrolysis needing mention is its use in the recovery of the metal from tin scrap, old tin cans, etc., of which it forms on the average 2 per cent by weight. Several electrolytic methods have been proposed, but only one is of importance—the Goldschmidt process—in which the scrap is made anode in a caustic soda solution, the dissolved metal being deposited on an iron cathode. The scrap is first broken up by cutting rolls, cleaned by a bath of hot NaOH, washed, and any solder removed by heating in a furnace. It is then packed loosely into iron cages which are suspended in iron tanks, the latter serving both as bath and as cathode. The electrolyte, which is kept at a strength of 8 per cent NaOH by continual regeneration, is heated to 70° by steam coils, and a current of about 80-100 amps/metre^2 of cathode passed through. The voltage is 1·5-2 volts per tank. The metal is mostly deposited in spongy form. After washing, pressing, and fusing, it is about 99 per cent pure, the remainder being chiefly lead. About 90 per cent of the tin in the scrap is thus recovered. The detinned material is sent to the steel-maker. The cathode current efficiency is perhaps 80 per cent, the deficiency being due to hydrogen discharge at the iron surface, which, as we have seen, has a low overvoltage.

The process is rendered possible by the fact that iron readily becomes passive in caustic soda solution. Otherwise it would dissolve preferentially to tin, being much more electropositive than this metal. In practice, owing to the presence of dissolved chlorides, a certain amount of solution does take place, with resulting fouling of the bath, but the chief process occurring, apart from solution of the tin, is oxygen evolution. The tin itself dissolves first as stannous ($Sn^{\cdot\cdot}$) ions. These are, however, rapidly oxidised by the oxygen to tetravalent tin ions, which are transformed in the electrolyte into sodium stannate, and the current efficiency given above is calculated on this basis.

§ (21) Nickel Extraction and Refining.—Much experimental work has been done on the extraction of nickel by electrolytic methods. The general difficulties encountered are similar to those already dealt with under copper and zinc, and it is only recently that real technical success has been achieved.

With regard to the anodic behaviour of the metal, we have already noted that it exhibits passivity phenomena to a marked degree. For every set of conditions, there is a limiting current density, above which the metal ceases to dissolve, and oxygen evolution begins. Cl' and $H^{\cdot}$ ions tend to activate the metal, and thus higher current densities are permissible in $NiCl_2$ than in $NiSO_4$ solutions, as also in acid than in neutral solutions. The differences shown by different types of anodes must also be recalled, electrolytic nickel dissolving least easily, then the rolled or worked metal, and cast metal the most easily. If the metal has been charged with hydrogen, either by treatment with acid, or by cathodic polarisation, its solution is considerably assisted. If the structure is not homogeneous, and some parts are more active than others, then it will dissolve unevenly, tend to fall to pieces, and will give slimes with high percentages of nickel.

As it is more electropositive than hydrogen, and as the hydrogen overvoltage at a nickel surface is very low, it cannot be quantitatively deposited from solutions containing much free acid. In the case of zinc we saw that by increasing the current density and thereby the hydrogen overvoltage, good current efficiencies could be got even from strongly acid solutions. In the case of nickel this is not possible, as irreversible effects, corresponding to passivity, occur during its deposition, necessitating a cathodic polarisation considerably more than the equilibrium value. This excess polarisation, like that of overvoltage, increases with the current density, and hence the effect of raising the latter is less than with zinc solutions. The same applies to lowering the temperature. A low acid concentration is therefore essential, a lower limit being set by the necessity of preventing basic salts separating out. This readily happens from neutral solutions. It stands to reason that the more electropositive metals, such as copper, must be absent from the electrolyte if a pure deposit is to be got. Iron and cobalt, if present, will be deposited, their electrochemical behaviour being very similar to that of nickel. One last point is that it is difficult to get thick deposits of nickel at room temperatures unless very low current densities are used. Otherwise the metal flakes off. This is of minor importance for the plater, but not so in winning or refining. The cause appears to be the presence of small quantities of iron in the electrolyte, deposited in slight excess at the start, and setting up strains. If the electrolyte and anode be perfectly iron free, or if the temperature be raised to 50°-60°, the difficulty disappears and high current densities can be used.

Copper and iron are the two impurities of most importance technically. If anodes are available fairly free from them, they can be used directly in a nickel bath, and the nickel plated out. At the Balbach refinery (U.S.A.) such a process, employing a nickel sulphate electrolyte, was in use for some years. The chief impurity in the anodes was carbon, which was satisfactorily separated in this way. The reason for abandoning the process was the difficulty of making homogeneous anodes. Those used were brittle, crumbled in the bath, and gave a very high percentage of slimes. In most processes, however, the copper and iron have been removed first by electrolytic and chemical means, and the purified nickel solution subsequently electrolysed, using insoluble anodes. In the Höpfner process the roasted ore was extracted with spent anode liquors (essentially a solution of $CaCl_2$, $NiCl_2$, and $CuCl_2$), and, after removing silver and iron chemically, furnished an electrolyte containing $NiCl_2$ and Cu_2Cl_2. This was electrolysed in a diaphragm cell between copper cathodes and carbon anodes. At the anodes Cu_2Cl_2 was oxidised to $CuCl_2$ the liquors then being returned to the leaching plant. In the cathode compartment copper was deposited. The last traces of this metal were removed chemically, and the resulting liquors electrolysed between sheet nickel cathodes and graphite anodes, giving chlorine and nickel of excellent quality. In the Savelsberg-Wannschaff process a matte containing nickel and iron, with only a trace of copper, is extracted with chlorine and a $CaCl_2$ solution. After purification the solution of nickel and iron chlorides is freed from iron by treatment with fresh ore whilst blowing in air, and the pure nickel solution electrolysed as above.

Using an electrolyte containing initially 100 grams/litre nickel, with a current density of 1 amp/dm.^{2} and four volts, a 93 per cent current efficiency is obtained. The product is very pure. One ton requires some 4000 K.W.H. In the Browne process anodes containing chiefly copper and nickel, with some iron and sulphur, were used in a chloride solution. They dissolved readily, copper being deposited on the cathodes. The liquors, charged with nickel, and containing also iron and some copper, were purified chemically and then electrolysed with graphite anodes as previously described. The current efficiency was about 93 per cent at 3·5 volts. The nickel was of excellent quality.

The Hybinette process successfully treats matte anodes with 65 per cent Ni, 30 per cent Cu, and 5 per cent S, arranged alternately with graphitised iron cathodes. Both anodes and cathodes are contained in canvas diaphragm bags. The electrolyte, a neutral $NiSO_4$ solution with 45 grams/litre nickel and traces of copper, is led into the cathode bags, and from there circulates through the tanks. With a current density of 1 amp/dm.^{2}, 3-4 volts are used, the liquors being heated. A product up to 99·9 per cent pure is obtained, the copper being separately recovered. About two-thirds of the anodes are consumed. The sulphur causes no difficulty.

§ (22) Iron Refining.—We have already discussed (Section V.) the conditions necessary for obtaining satisfactory electro-deposits of iron. It is only necessary to add here that, even before the war, iron was refined electrolytically on a large scale in Germany for use in transformer cores, etc., and that, during the war, this industry, owing to lack of copper, received a great impetus. Its great purity and ductility, when free from hydrogen, render it suitable for many purposes. To get the ductile material directly, electrolysis must be carried out at a high temperature.

§ (23) Lead Refining.—The extraction of lead, as of tin, is very simple and does not offer much scope for electrolytic methods. It is different with refining. In consequence of its high electro-chemical equivalent, the energy expenditure per ton of metal should be small. Further, the silver content of crude lead is an important source of this metal, its recovery by the usual refining processes is incomplete, and there are other valuable constituents, such as bismuth and antimony, which are not recovered at all. As a matter of fact, electrolytic lead refining is now well established, and a serious competitor of the older processes. Two methods are in use, one employing as electrolyte a solution of lead perchlorate, and the Betts process, using a lead fluosilicate solution. We shall deal with the latter only.

Lead is somewhat more electropositive than hydrogen. There is, however, no difficulty in depositing it from acid solution, owing to its high hydrogen overvoltage, which approaches the values given by mercury and zinc. The difficulty lies rather in the choice of a suitable electrolyte. The sulphate and the chloride are too insoluble. The nitrate is chemically attacked by metallic lead, and, further, would tend to dissolve any silver or copper present in the anode. In addition, it gives very crystalline non-coherent deposits, which form excrescent growths, causing short circuits between anode and cathode. Betts found that all these difficulties could be overcome by using lead fluosilicate, with the addition of free H_2SiF_6 to improve the conductivity. A sufficiently strong solution can be obtained, electronegative impurities, when present in normal quantities, do not dissolve at normal working current densities, and a coherent deposit, which shows no treeing effect, results. If, in addition, a trace of gelatine be added to the electrolyte, the deposit is fine and silky in texture, with no tendency to crystalline growths.

Technically, a bath with 75 grams/litre lead and 100 grams/litre free H_2SiF_6 is used, with the addition of not more than 1 gram/litre of glue or gelatine. The liquors are circulated by gravity through a series of tanks of tarred wood, each containing a number of paralleled anodes and cathodes, alternately arranged. Anodes are wedge-shaped, thicker at the top than at the bottom, and their lugs rest on the positive copper bus-bars. The cathodes are thin sheets, cast from electrolytic lead, and are suspended from copper rods. At a current density of 0·8-1 amp/dm.^{2} and a temperature of 30°-35°, the initial voltage is about 0·25 volt, rising to 0·4 volt as the anodes dissolve, owing to the increased resistance of the slimes. A very pure lead is obtained at 90 per cent current efficiency and an expenditure of 100 K.W.H. per ton. The slimes retain the form of the original anodes, of which they form a rather high proportion. Together with 10 per cent lead, they hold practically all the silver, bismuth, antimony, arsenic, tin, and copper, together with adsorbed electrolyte, and their subsequent working up is important. Zinc and iron chiefly accumulate in the electrolyte. In addition, the latter gradually loses its acidity, and the gelatine is continually consumed. Periodical regeneration is therefore necessary. The disappearance of the acid is due to slow chemical action on the lead, the same factor being chiefly responsible for the loss in current efficiency.

The chief advantages of the Betts process are the production of metal free from bismuth and antimony, the recovery of these metals, and the increased silver recovery. The process

can, moreover, be conveniently used on a small scale.

VII. Electrolysis of Brine

§ (24) General.—By passing a current through a solution of common salt, and suitably varying the conditions, a number of highly important products can be made, viz. caustic soda, chlorine, and the hypochlorite, chlorate, and perchlorate of sodium. If KCl be used, the corresponding potassium compounds result. Some of these chemicals are already made exclusively by electrolytic means, and the importance of such methods for the manufacture of the remainder continually grows.

The primary electrode reactions occurring when a brine solution is electrolysed between indifferent electrodes (platinum, graphite) are very simple. At the cathode, $H^{\cdot}$ ions are discharged and hydrogen liberated, the solution becoming alkaline. $Na^{\cdot}$ ions, owing to the very pronounced electropositive nature of the metal, are only deposited if their discharge is depolarised. At the anode Cl' ions are discharged and chlorine gas liberated. If the relations here were strictly reversible, we should get OH' ions discharged and oxygen formed. This, however, does not take place, as anodic oxygen evolution requires a very considerable overvoltage, far greater than that of chlorine. We obtain thus as primary products caustic alkali (with hydrogen as a by-product) and chlorine, and an apparatus which allows these substances to be collected and worked up separately is known as an alkali-chlorine cell. It is well known that, if brought together, they readily react, and that, according to the conditions, a hypochlorite or a chlorate will result. Both these reactions can be carried out inside the cell in which the primary products are formed, whilst, if the electrolysis be continued after the transformation of all the brine into chlorate, perchlorate will finally result.

In order to understand the principles underlying the design and construction of these types of cells, we must first discuss briefly the chemical reactions occurring between chlorine on the one hand and water or alkali on the other, and the different factors by which these reactions and their products can be modified. Chlorine, when passed into water or alkali, reacts according to the equation

$$Cl_2 + OH' \longrightarrow Cl' + HClO, \quad . \quad . \quad \text{(i.)}$$

the reaction proceeding the further the higher the temperature. About 30 per cent of the chlorine is hydrolysed in a saturated solution in water at 0°. The HClO produced is practically undissociated, but the solution reacts acid owing to the presence of $H^{\cdot}$ ions produced in equivalent quantity to the Cl' ions. If caustic alkali be used instead of water, one equivalent being present to a molecule of chlorine, the reaction is almost complete in dilute solution, though in stronger solution both chlorine and free alkali can be detected. If now more alkali be added, it will react with the HClO, and if the new addition amount in all to another equivalent the total equation will practically be

$$Cl_2 + 2OH' \longrightarrow Cl' + ClO' + H_2O \quad . \quad \text{(ii.)}$$

or

$$Cl_2 + 2NaOH \longrightarrow NaCl + NaClO + H_2O \quad \text{(iii.)}$$

the amount of free HClO in such a solution being very small. The hypochlorite solutions in this form are fairly stable. They can, however, slowly decompose, particularly on raising the temperature, in two ways, one of which (equation (x.)) is for us of minor importance. The other reaction, on the contrary, is very important, has been carefully investigated, and proceeds as follows:

$$2HClO + ClO' \longrightarrow ClO_3' + 2Cl' + 2H^{\cdot}. \quad \text{(iv.)}$$

It will be seen that the hypochlorous oxygen disappears, and that chlorate results, also that, as $H^{\cdot}$ ions are produced, which will at once regenerate HClO from the ClO' ions, the concentration of the HClO will remain practically constant throughout, only the ClO' ions disappearing. So that the net result of the reaction will be

$$3ClO' \longrightarrow ClO_3' + 2Cl'. \quad . \quad . \quad \text{(v.)}$$

If the differential reaction velocity equation for (iv.) be written, it will be clear that the concentration of the HClO is of more importance than that of the ClO' ions in determining the rate of reaction, and it is precisely this substance which is present in such low concentration in a solution prepared according to (iii.). If, however, a small extra amount of HClO be present, this means a large *relative* increase in the HClO concentration, and will greatly increase the velocity of the reaction. This can be done by adding a little HCl or a slight excess of chlorine, the HClO coming respectively from the ClO' ions or from the OH' ions of the water. Such a solution, whether at a high or a low temperature, will give chlorate far more quickly than the solution resulting from (iii.).

§ (25) Hypochlorites. (i.) *Theory.*—The above discussion has put us in a position to consider the reactions taking place in hypochlorite electrolysers or electrolytic bleach cells. In such cells, brine is electrolysed without a diaphragm between indifferent electrodes, the object being to obtain a high hypochlorite concentration at as high an

efficiency as possible. Any formation of chlorate is to be regarded as a loss. The primary products of electrolysis, one molecule of chlorine to two molecules of caustic soda, mix with one another, and reactions (i.) and (ii.) take place. The result of the passage of two faradays of electricity through the cell is, in theory,

$$NaCl + H_2O \longrightarrow NaClO + H_2. \quad . \quad \text{(vi.)}$$

At low temperatures, the chemical decomposition of such a hypochlorite solution is very slow, and it should therefore be possible to obtain a saturated solution of the same. At both anode and cathode, however, disturbing electrode reactions play a part. The ClO' ions are readily reduced at the cathode, whatever the material of the latter, to Cl' ions by nascent hydrogen.

$$2ClO' + H_2 \longrightarrow 2Cl' + H_2O. \quad . \quad \text{(vii.)}$$

As the concentration of NaClO rises, so does the rate of reduction, until finally this will tend to equal the rate of production according to (iii.), and the yield will fall to zero. In practice, this is very largely avoided by the addition of some Na_2CrO_4 to the electrolyte, its action being ascribed to the formation of a thin diaphragm of chromium chromate around the cathode, which prevents the diffusion of ClO' ions to the surface of the latter. Other addition agents are also in use technically, with equally satisfactory results.

At the anode, ClO' ion discharge will begin when its concentration in the electrolyte has reached a certain value. It takes place in a somewhat unusual manner, according to the equation

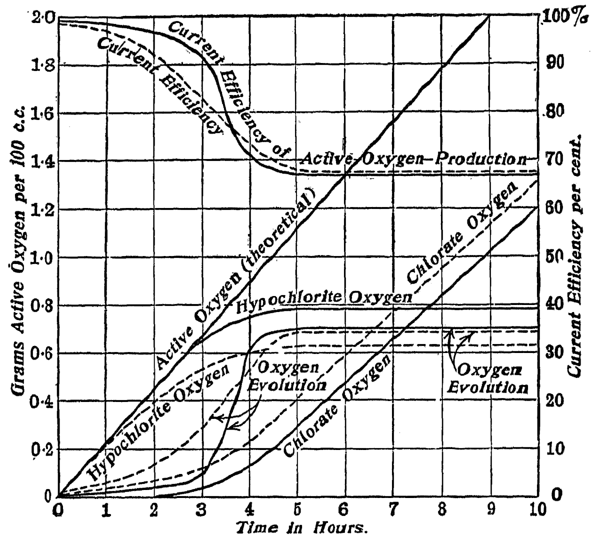


FIG. 2.

$$6ClO' + 3H_2O \longrightarrow 6H^{\cdot} + 2ClO_3' + 4Cl' + 1\tfrac{1}{2}O_2 + 6\ominus \quad \text{(viii.)}$$

the $H^{\cdot}$ ions subsequently neutralising the OH' ions produced in equivalent amount at the cathode. Unlike the cathodic reduction, it is not possible to eliminate this anionic discharge as a source of loss of hypochlorite. So, as the concentration of the latter increases, it will approach a limit at which the rate of its formation according to (iii.) becomes equal to its rate of disappearance in accordance with (viii.), assuming cathodic reduction to be negligible. The six ClO' gram-ions discharging in equation (viii.) require the passage of twelve faradays of electricity for their formation. On discharge, they furnish for the passage of six faradays, six atoms of chlorate oxygen and three atoms of molecular gaseous oxygen. At the stationary state then, we shall have a system of constant hypochlorite concentration, in which sodium chlorate is being formed in increasing concentration at 66·6 per cent current efficiency, the remaining 33·3 per cent of the current giving gaseous oxygen. The net cell equation for this complex of reactions is

$$9H_2O + 2Cl' \longrightarrow 9H_2 + 2ClO_3' + 1\tfrac{1}{2}O_2. \quad \text{(ix.)}$$

The value of the NaClO concentration at the stationary state will depend on several factors. A high NaCl concentration means a smaller anodic polarisation, and therefore a higher hypochlorite concentration before ClO' ion discharge takes place. A high current density causes efficient mixing owing to gas evolution, and prevents a local concentration of ClO' ions around the anode, followed by their premature discharge. Further, the condition of the anode exerts an influence. Cl' ion discharge takes place at a much lower potential at a platinised than at a smooth platinum anode, and this favourably affects the NaClO concentration in the same way as does a high salt content. Lastly, a low temperature is favourable. Otherwise, on cooling down from the temperature of electrolysis, there will be a re-formation of chlorine and alkali in the sense of equation (ii.) read from right to left. The influence of these factors is shown in *Fig.* 2 and the table on following page. The former exhibits the results obtained during a ten-

hours' electrolysis of a brine solution with anodes of smooth platinum (dotted curves) and platinised platinum (full curves). The conditions are otherwise identical.

Concentration of NaCl.	Temperature.	Anodic Current Density in Amps. cm.^{2}	Grams of Hypochlorite Oxygen per 100 c.c.	
			Platinised Platinum Anode.	Polished Platinum Anode.
4·8 n.	13°	0·017	0·61	0·34
..	..	0·17	0·89	0·68
..	50°	0·017	0·31	0·17
..	..	0·17	0·64	0·42
1·7 n.	13°	0·017	0·48	0·28
..	..	0·17	0·65	0·47
..	50°	0·017	0·23	0·15
..	..	0·17	0·40	0·35

The most favourable electro-chemical conditions for the production of a concentrated bleach solution are therefore: (*a*) strong brine, (*b*) low temperature, (*c*) high anodic current density, (*d*) chromate in electrolyte, (*e*) platinised platinum electrodes. In practice these conditions are modified. A strong brine means a high salt consumption, as only a fraction of the dissolved chloride is converted into hypochlorite. Owing to the high current densities and voltages, much heat is produced in the cells, and even artificial cooling cannot always keep the temperature as low as is otherwise desirable. And platinised electrodes are too fragile—the surface deposit readily drops away. In practice, moreover, solutions of moderate bleaching power suffice for most purposes.

(ii.) *Technical.*—A large number of technical electrolysers have been designed, mostly using bipolar electrodes. These allow of a compact design of apparatus and also avoid the use of exposed copper connections. This is important, as traces of copper compounds (also nickel and cobalt) catalytically decompose hypochlorite solutions with oxygen evolution, a reaction which in their absence proceeds at a negligible rate.

$$2NaClO \rightarrow 2NaCl + O_2 \quad . \quad . \quad . \quad (x.)$$

They of course present the drawback of shunt current losses and consequent diminished current efficiencies, particularly serious in the present case owing to the comparatively high voltages employed. We will describe two forms only, which are typical of the remainder, and demonstrate the points brought out in the above discussion.

The Kellner cell is shown in *Figs.* 3 and 4. It consists of a long cement trough, open at the top, and divided by a series of glass walls into a number of chambers arranged terrace-wise. The brine circulates through these chambers by gravity, is cooled, and pumped back again until the required concentration has been reached. The bipolar electrodes are of platinum-iridium network, making connection under the glass separators, and kept in position by glass rods. The anode half lies very close to the bottom of the cell, the cathode half about 5 mm. above the anode. A 220-volt 60-ampere unit will have 36 such compartments. The electrolyte is 15 per cent brine with some addition to avoid cathodic reduction. The temperature of the entering liquors is about 21°. The highest concentration obtainable is about 50 grams/litre of active chlorine, at an expenditure of 9·3 K.W.H. per kilo. of chlorine. Normally the cell is worked to give a solution with 25 grams/litre of active

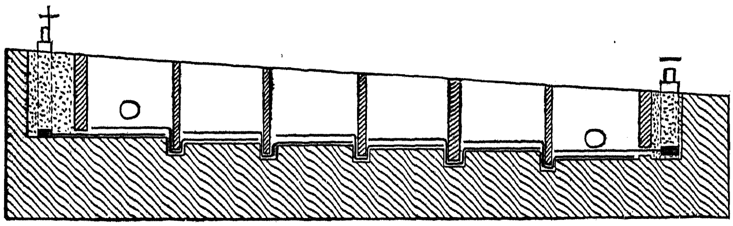


FIG. 3.—Kellner Cell (Side Elevation).

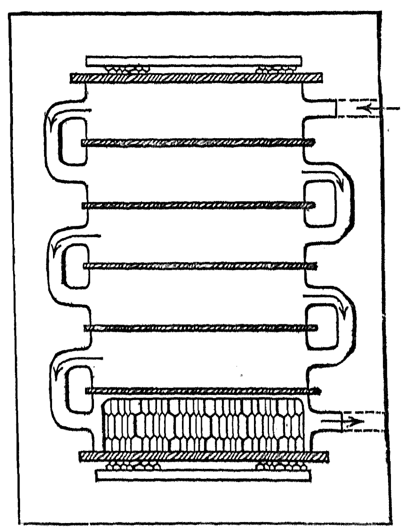


FIG. 4.—Kellner Cell (Plan).

chlorine, at an energy expenditure of 6 K.W.H., and a salt consumption of 4 to 6 kilos. per kilo.

The Haas-Oettel cell (*Figs.* 5 and 6) also uses bipolar electrodes but of carbon, not platinum. It consists of an earthenware trough, open at the top, and divided into compartments by vertical plates of hard carbon or graphite *a a a*, current being led in and out of the end plates. From each side of the trough projects out a series of open earthenware channels *d d d*, corresponding to the cell compartments and separated from one another by earthenware ribs. They serve

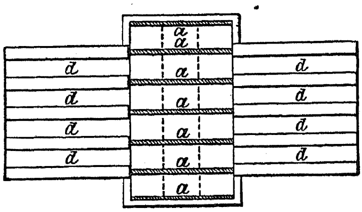


FIG. 5.—Haas and Oettel Cell (Plan).

as outlets for the liquors from the different chambers. The base of the trough contains an entrance for the brine, common to all the compartments. The electrolyser stands in an outer vessel, containing brine up to a level rather lower than that of the discharge channels. This outer tank is provided with cooling coils. When the current is switched

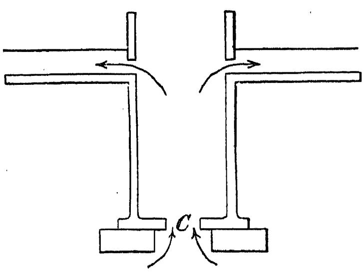


FIG. 6.—Haas and Oettel Cell (End Elevation).

on, the gas evolution discharges the liquors through the side channels into the outer tank, fresh cooled brine entering underneath. The circulation is thus effected and regulated by the electrolysis. With 17 per cent brine, one kilo. of active chlorine at a concentration of 12 grams/litre can be obtained at an expenditure of 14 kilos. of salt and 6·4 K.W.H. In spite of the lower overvoltage at the electrodes the results are seen to be much less favourable than those given by the Kellner electrolyser. This is due to the absence of any "cathode diaphragm" addition agent, and to rather high shunt current losses. The current efficiency in the case quoted is only 46 per cent. The apparatus has, however, the advantage of a far smaller first cost.

Electrolytic bleaching liquors are coming into increasing use for many kinds of fine work. They act rapidly, do not expose the fabric to acid or alkali, can readily be prepared, and their composition can be exactly and reliably regulated.

§ (26) CHLORATES. (i.) *Theory.*—We have studied the possibilities which occur when a neutral brine solution is electrolysed, and the catholyte and anolyte mixed, and have seen that chlorate can result in two ways, viz. chemically,

$$2HClO' + ClO' \longrightarrow ClO_3' + 2Cl' + 2H^{\cdot} \quad \text{(iv.)}$$

and electrochemically,

$$6ClO' + 3H_2O \longrightarrow 2ClO_3' + 4Cl' + 6H^{\cdot} + 1\tfrac{1}{2}O_2 + 6\ominus. \quad \text{(viii.)}$$

These two reactions form the basis of the two classes into which technical chlorate processes can be divided, viz. the *acid* and the *alkali* processes respectively, and before we proceed further, we must discuss the respects in which the electrolysis of a brine solution is modified if, instead of being neutral, it is acid or alkaline.

If the original solution is slightly acid, the concentration of the free HClO produced on electrolysis is, as we have seen, proportionately greatly increased. So is the rate at which reaction (iv.) proceeds. If, in addition, the electrolysis is carried out at high temperatures —70° or over—this formation of chlorate becomes very rapid, quite dwarfing the production by ClO′ ion discharge. The concentration of the latter indeed at the stationary state is very small. If, further, chromate be added to avoid any residual cathodic reduction, and a platinised platinum electrode be used, practically a 100 per cent current efficiency of chlorate oxygen should result, as no oxygen is evolved at any stage. In practice, using smooth platinum anodes, a 90 per cent current efficiency is obtained, the deficiency being due to ClO′ ion discharge.

If, on the other hand, a solution containing free alkali is used, the ClO′ ion concentration, *particularly at the anode*, is increased. Its discharge commences at a lower hypochlorite concentration *in the bulk of the solution* than is the case when using neutral brine. The more alkali added, the smaller the ClO′ ion concentration at the stationary state, until a very low figure is reached. Still more alkali results in the production of oxygen by OH′ ion discharge, and the chlorate current efficiency, never above 66 per cent, falls. *Fig.* 7 shows the dependence of hypochlorite and chlorate production on the initial alkali concentration when a given quantity of electricity is passed through 20 per cent

brine solution at 5°. Other factors which assist anionic discharge at a low anode potential by reducing the overvoltage, will also decrease the chlorate yield, lead to in-

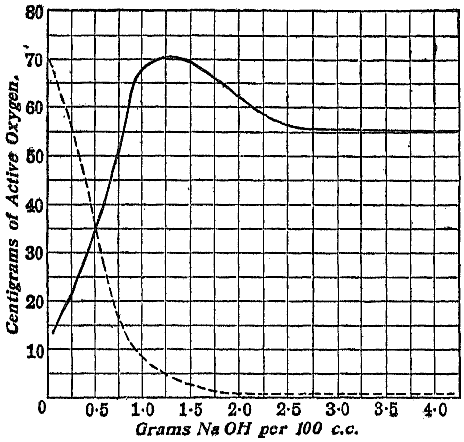


FIG. 7.

Dotted Curve—Hypochlorite: Full Curve—Chlorate.

creased OH′ ion discharge and to higher equilibrium hypochlorite concentrations. Such are the use of platinised instead of polished platinum electrodes, a low current density, and an increased temperature. These points are brought out in the following table and in *Fig.* 8.

Anode.	Anodic Current Density.	Current Evolving Oxygen.	Grams of Chlorate Oxygen.	Grams of Hypochlorite Oxygen.
	amps./cm.²	per cent.		
Platinised	0·067	75	1·049	0·087
Smooth .	0·067	58	1·639	0·0012
Smooth .	0·017	58	1·586	0·0032

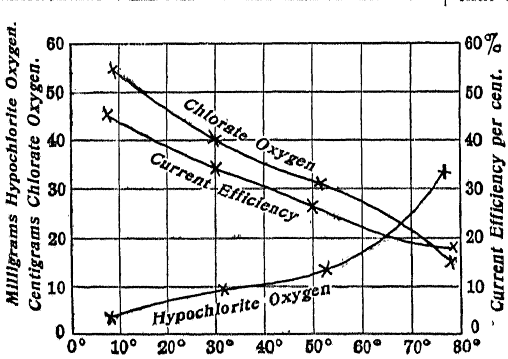


FIG. 8.

The conditions for the successful operation of the alkali process are therefore (*a*) low temperature, (*b*) a polished platinum electrode, (*c*) fairly high current density, (*d*) 1-1·5 per cent NaOH in brine, (*e*) chromate to prevent cathodic hypochlorite reduction. In practice, the alkali processes, with their maximum possible 66 per cent current efficiency, have been almost entirely discontinued, although they were the first introduced, and were used for many years.

(ii.) *Technical.*—The acid processes are usually carried out in long rectangular iron or cement cells, carefully insulated, and arranged in terraces to permit of gravity circulation. They contain a row of anodes between two rows of cathodes, like electrodes being paralleled. Such a bath will take 1000-1500 amperes. The anodes are of smooth platinum-iridium foil, the cathodes of graphite or of iron. The electrolyte contains originally 25-30 per cent KCl or NaCl and about 10 grams/litre $K_2Cr_2O_7$, and is kept at the right acidity by constant small additions of HCl. The anodic current density is 20 amps/dm.² and the temperature 70°-95°. Depending on this last factor, the voltage can vary between 4·5 and 5·5 volts. Chlorate formation proceeds very rapidly at these temperatures. The liquors are circulated until near saturation, drawn off and crystallised. With 90 per cent current efficiency and five volts, one ton of $KClO_3$ will require about 7300 K.W.H., which corresponds to a 25 per cent energy efficiency.

§ (27) PERCHLORATES.—If a brine solution which has already been converted into chlorate be further electrolysed under conditions at which the anode potential is very high, further oxidation to perchlorate takes place, the total anodic reaction being

$$ClO_3{}' + H_2O \longrightarrow ClO_4{}' + 2H^{\cdot} + 2\ominus.$$

At the cathode, $H^{\cdot}$ ions are discharged, so that the cell reaction is

$$ClO_3{}' + H_2O \longrightarrow ClO_4{}' + H_2.$$

The necessary high anode potential can be obtained by using a smooth platinum anode, a high current density, and a low temperature. The last condition also means a low electrolyte conductivity, so, in practice, the bath is frequently run at 30°-50° C., and the current density raised still further. This has the advantage of utilising the expensive platinum to the best advantage. In order to prevent direct OH′ ion discharge, involving oxygen evolution and current loss, the electrolyte is best kept slightly acid. The chlorate concentration is as high as possible in order to reduce the resistance, and consequently the very soluble $NaClO_3$ is used in practice, not $KClO_3$. Initially it

contains 600-700 grams/litre, and current is passed until conversion is almost complete. Iron cathodes are generally used. As the ClO_3' ions are readily reduced at this metal to Cl' ions, chromate is added to the electrolyte. With good circulation and an anodic current density of 45-50 amps/dm.^{2} practically complete conversion can be obtained with an 85-90 per cent current efficiency and an average voltage of six volts. The energy used is 3·2-3·6 K.W.H. per kilo perchlorate. Towards the end of the electrolysis, large quantities of ozonised oxygen are produced. There is a slight loss owing to cathodic reduction, and the final liquors contain some chloride. The electrolyte needs a regulated addition of acid throughout.

§ (28) ALKALI-CHLORINE CELLS. (i.) *Theory.*—In these cells the problem is, not how most efficiently to bring about the combination of the primary anodic and cathodic products of brine electrolysis, but how best to keep them apart, and the influence of such factors as electrode material, concentration, current density, and temperature on their exact nature must first be considered.

Normally we have seen that $H^{\cdot}$ ions, not $Na^{\cdot}$ ions, are discharged cathodically when a brine solution is electrolysed. If, however, a cathode is used which will depolarise the $Na^{\cdot}$ ion discharge by forming a sodium alloy, and which at the same time opposes a considerable overvoltage to $H^{\cdot}$ ion discharge, the reverse can take place. Mercury is such a cathode material, and is in fact used as cathode in a certain class of alkali-chlorine cells, the alkali amalgam being subsequently decomposed with water outside the cell. A high salt concentration obviously assists $Na^{\cdot}$ discharge, and is therefore favourable. So are a high current density and a low temperature, as the hydrogen overvoltage is thereby increased. With the more usual type of cell, on the other hand, we wish to reduce the hydrogen overvoltage, and from this point of view a low current density and high temperature are advantageous.

At the anode we have relations qualitatively similar to those discussed in the preceding section of this article. The liberated chlorine forms HClO with the water, and this furnishes ClO' ions, particularly if any alkali (formed at the cathode) is present. These ClO' ions can be discharged, producing ClO_3' ions and oxygen, and it has been shown that HClO can act anodically in a similar way.

$$6HClO + 3H_2O + 6\oplus \longrightarrow 2ClO_3' + 4Cl' + 12H^{\cdot} + \tfrac{3}{2}O_2.$$

There is in addition the possibility of OH' ion discharge, occurring the less easily the more acid the solution. In practice it is found that, as would be expected, the lower the concentration of the Cl' ions, the smaller the proportion of current concerned in their discharge, and the lower the chlorine current efficiency. On the other hand, the fraction of current concerned with OH' ion discharge increases with decrease in electrolyte concentration, owing to the accompanying rise in the anode potential. Finally, the concentration of hypochlorite and the fraction of current producing chlorate at first increase as the brine solution is made more dilute, pass through maximum values, and then decrease. Figures for the electrolysis of an HCl solution, which shows a similar behaviour, are given below:

Concentration of HCl.	Percentage of Current giving Chlorate Oxygen.	Percentage of Current giving Oxygen Gas.
1·0 *n*	1·04	0·9
0·33	6·54	9·7
0·1	34·62	34·4
0·033	26·50	53·6

The nature of the anode (platinum, magnetite, and graphite are used) is also important in this connection. At platinum or magnetite we have higher overvoltages for Cl' or OH' discharge than at graphite, and hence ClO' ion discharge and chlorate formation are facilitated. On the other hand, at graphite, though there is less chlorate formation, OH' ion discharge is proportionately easier, and takes a larger fraction of the current, which is augmented by the porosity of the material. Electrolysis takes place, not only at the surface of the electrode, but also inside its pores. The electrolyte there soon gets depleted of Cl' ions, and oxygen evolution commences. Not only is there a current loss, but the electrodes are attacked, with CO_2 formation and subsequent disintegration. This circumstance has in the past been very troublesome.

A rise in temperature is found experimentally somewhat to decrease chlorate formation. But on the other hand it facilitates OH' ion discharge and also (with graphite anodes) CO_2 formation. It is advantageous to have it low. The current density is determined more by the nature of the anode than by other considerations. With the expensive metal platinum it must be high on account of interest charges. With magnetite it may be low, which is favourable from the point of view of voltage. With a carbon anode which is at all porous it should be high, as the greater part of the electrolysis will then take place on the surface of the cathode, not in its pores. The better the quality of the electrode, the lower it can be.

The best conditions for getting a pure chlorine gas at high current efficiency are

therefore (*a*) non-porous anode, (*b*) strong brine, (*c*) high current density, (*d*) low temperature.

There are two main reasons which tend to cause the cathodic alkali and the anodic chlorine to mix and interact. Such mingling may be due to convection and the mixing effect of the gas evolution. This can satisfactorily be counteracted by a diaphragm between catholyte and anolyte. More important is the electrical migration of the negative OH' ions from the catholyte towards the anolyte. They will share with the Cl' ions in the transport of negative electricity in a proportion determined by their concentration and ionic mobility. The latter is nearly three times as great as for the Cl' ion, and consequently, in strongly alkaline solutions, the greater part of the negative current will be carried by the OH' ions, which will at once disappear when they reach the anode. The only way of counteracting this is by causing the electrolyte to flow steadily from the anode towards the cathode. Losses in alkali will then be determined by the amount of chlorine dissolved in the anode liquors, which is relatively small, less in a brine solution than in pure water. All modern cells, except mercury cells, are of this type, some with diaphragms, some without. We can therefore classify alkali-chlorine cells as follows:

Mercury cells.

Counter-current cells with diaphragm.

Counter-current cells without diaphragm.

(ii.) *Mercury Cells.* (*a*) *General.*—Owing to the practically complete exclusion of alkali from the anolyte of these cells, their electrochemical theory is very simple. At the mercury cathode $Na^{\cdot}$ or $K^{\cdot}$ ions are discharged, forming a dilute amalgam. In the case of sodium a 0·5 per cent amalgam can on no account be exceeded, and technically it is far lower. In order to prevent $H^{\cdot}$ ion discharge, a high cathodic current density, with high overvoltage, must be employed. This varies in practice between 5-25 amps/dm.^{2}, and is generally over 12 amps/dm.^{2}. At the anode, the potential is determined by the Cl' ion discharge. With platinum, current densities are necessarily high, and so is the overvoltage. With graphite, low current densities and voltages can be employed. The temperature of the electrolyte must not be too high, as the cathodic overvoltage would thereby be destroyed, and also appreciable chemical action would take place between the amalgam and the brine solution. In practice 50° C. is rarely exceeded. The necessary heat is supplied by the ohmic resistance of the electrolyte. The total cell voltage averages about five volts using platinum anodes, and 4·3-4·5 volts with graphite. The electrolyte, originally containing 30 per cent of dissolved salt, has only 20 per cent when it leaves the cells for re-saturation. It should be free from iron and calcium salts, and, if graphite anodes are used, from sulphates, which tend to give anodic oxygen.

The current efficiencies in these cells are of the order of 95 per cent for both alkali and chlorine, and are rather higher with platinum than with graphite anodes. The cathodic deficiency is mainly due to hydrogen evolution. This can happen either by chemical action between the solution and the amalgam, or by direct $H^{\cdot}$ ion discharge. Both processes are favoured by high temperature and by a too slow rate of removal of the amalgam from the cell. The second source of loss is also increased if the brine solution is too weak or the current density too low. The difference between cells using graphite and those using platinum electrodes is due, firstly, to the anodic production of small quantities of acid at the former, and, more important, to small particles of carbon falling on to the mercury surface, and, with their low overvoltage, facilitating $H^{\cdot}$ ion discharge and amalgam decomposition. At the anodes there is a certain amount of oxygen evolution, particularly if they are of graphite, when some CO_2 will also be formed. Lastly, directly affecting both anodic and cathodic processes, we have chemical action between dissolved chlorine and the mercury. A chloride of mercury will be formed, dissolve in the brine, and at once be decomposed, the net result being simply a current loss. Owing to these causes, the anode gases will always contain some hydrogen and oxygen. If graphite is used, the hydrogen content may amount to as much as 2-3 per cent, with the same amount of CO_2. Platinum electrodes give a far purer gas.

(*b*) *Castner Cell.*—The differences between the various mercury cells lie essentially in the means used to remove and decompose the amalgam. In the original mercury cell designed by Castner this was effected by rocking the whole cell, and decomposing the amalgam electrochemically, making it the anode in a caustic soda solution, when the sodium dissolved, and hydrogen was evolved at the iron cathode. Such an amalgam decomposition cell is a *primary* cell, furnishing current, and Castner's idea was that, by coupling it against the brine cell, the impressed voltage required for the latter would be materially reduced. His apparatus took the form of a slate box, divided into three compartments by two vertical partitions reaching nearly to the bottom of the cell (*Fig.* 9). A layer of mercury was put into the box, and by gently rocking the cell by means of an excentric, could be made to flow from one compartment to another underneath the partitions. The

two outer compartments were closed above and contained the graphite anodes, by which the positive current entered. The open middle compartment contained an iron grid cathode by which current left the cell. Brine flowed continually through the outer compartments, and water entered and caustic soda left the middle one. On passing the current, the mercury, charged with sodium, was passed by means of the rocking motion into the middle chamber, where it was denuded and returned to the brine compartments. Owing to the current efficiency in the latter being only 90-92 per cent, it was found necessary to shunt about 8-10 per cent of the current through a resistance placed in parallel with the middle compartment, as otherwise mercury went into solution there and gave a deposit of mercurous oxide. These cells required comparatively little mercury, but their small size and complicated design led finally to their abandonment.

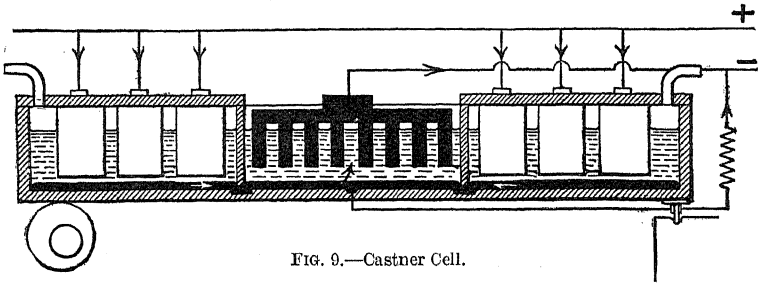


FIG. 9.—Castner Cell.

(c) *Kellner Cells.*—In the various types of Kellner cell the circulation is effected by more simple means, and no attempt is made to reduce the voltage by utilising the energy of the sodium amalgam, the latter being simply short-circuited by iron or graphite in contact with water. The Kellner-Solvay cell (*Fig.* 10) consists of a large cement trough, through which continually flow a stream of brine and a thin layer of mercury, the fresh brine entering just above the mercury. The platinum wire-net anodes are close above the anode surface. Current density and rate of flow are so chosen that the amalgam is charged to the right degree on leaving the cell. It passes by gravity into a separate adjacent trough of iron, where it is decomposed to concentrated alkali by a stream of water, the regenerated mercury returning to the cell by means of a well-wheel. The Castner-Kellner cell is similar in design, but uses graphite anodes, and the mercury is circulated by means of an Archimedean screw. The Kellner-Jaice cell (*Fig.* 11) consists of a long shallow cement trough, divided by partitions as in the original Castner rocking cell. In this case, however, the middle compartment contains the platinum anodes, carried by a series of cement hoods, whilst in the two outer compartments the amalgam is short-circuited by iron grids through which the negative current enters the cell. The circulation is effected by compressed air, applied by means of conical iron vessels dipping into mercury troughs situated at the ends of the cell, and in connection with the mercury cathodes. By alternately compressing and exhausting the air in these vessels the mercury is very efficiently driven from side to side of the cell. All these cells are simple in construction, large units can be built, and high current densities employed.

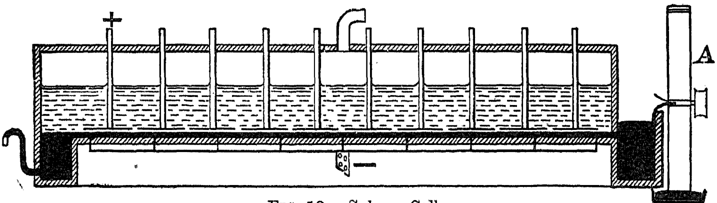


FIG. 10.—Solvay Cell.

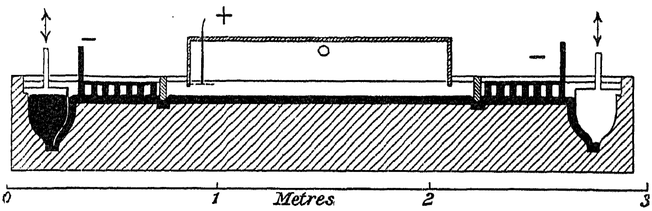


FIG. 11.—Jaice Cell.

The advantages of mercury cells are that

very concentrated and pure caustic liquors—up to 40 per cent NaOH—can be made, current efficiencies are high, and graphite anodes, when used, have a long life. On the other hand, voltages are high, and the first cost of an installation very considerable.

(iii.) *Counter-current Cells with Diaphragm.* (*a*) *General.*—In these cells, the OH′ ions formed at the cathode are prevented from reaching the anode by means of bodily moving the electrolyte towards the cathode, and at the same time mechanical mixing of anolyte and catholyte is eliminated by means of a suitable diaphragm. This permits of shortening the liquid layer between the electrodes, and hence a diminished resistance, which in practice is at least partially neutralised by the extra resistance of the diaphragm itself. More important is the fact that higher current densities can be used without mixing and that cells of compact construction can be more readily designed. The diaphragm can be arranged either horizontally, with the cathode below it, or vertically. With the former arrangement, if any alkali does happen to percolate through, there is less chance of its being lost by mixing with the bulk of the anode liquors. Further, and for the same reason, whilst the anode side of a vertical diaphragm is always in contact with an acid chlorine solution, with a horizontal diaphragm this may not be so. This is important from the point of view of durability as it is more easy to make alkali-resisting than acid-resisting diaphragms. On the other hand, a horizontal diaphragm sacrifices the compact cell construction referred to above, can less easily be changed, and, moreover, has a greater tendency to catch and filter out impurities in the brine, which in time will increase its resistance.

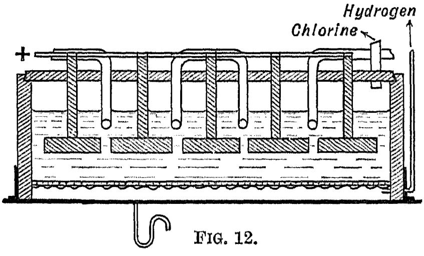


FIG. 12.

(*b*) *Billiter-Siemens Cell.* — Of the various cells with horizontal diaphragms, this is the only one needing mention, and is shown in *Fig.* 12. The active diaphragm material consists of a mixture of $BaSO_4$ and asbestos wool, spread out uniformly on a sheet of woven asbestos cloth. The whole rests on a sheet of heavy iron gauze (the cathode), in electrical connection with and supported about four inches above the bottom of a cast-iron shell, lined with chlorine-resisting brick. Such a diaphragm is of low electrical resistance, and very uniform in its permeability. The top of the cell is of cement, and carries the graphite anodes, the chlorine delivery pipe, and a number of glass U-tubes, the horizontal portions of which are arranged above the anodes, and through which hot brine solution flows in order to raise the temperature of the cell. Hydrogen and alkali are removed from the cathode compartment, the latter by means of a siphon overflow. Using concentrated purified brine, at a diaphragm current density of 4-6 amps/dm.^{2} and a working temperature of 85°, 3-4 N. alkali can be made at a current efficiency of 90-95 per cent and a bath voltage of 3·4-4 volts. The chlorine is pure, and the anodes very slowly attacked. Every few months the diaphragms must be remade and the cell cleaned.

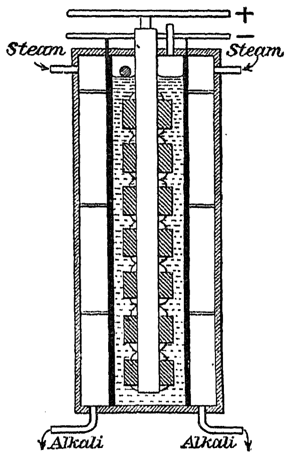


FIG. 13.—Hargreaves-Bird Cell.

(*c*) *Hargreaves-Bird Cell.* — This is the oldest of the cells with vertical diaphragms (*Fig.* 13). It consists of a large iron box, cement-lined, and divided by two cement-asbestos diaphragms into three compartments. The two outer compartments contain the cathodes in the form of an iron network in contact with and supporting the diaphragm. The inner chamber contains the carbon anodes, cemented in through the roof, which also carries the chlorine delivery pipe. Brine is led into the anode compartment, and percolates through the diaphragms to the cathodes. It is there charged with alkali, and is drawn off at the bottom. It is usual to supply steam and also CO_2 to the cathode compartments. The former heats the cell to about 85°, and lowers the voltage. The latter converts the caustic alkali into the less valuable carbonate, but also by destroying most of the OH′ ions, diminishes the losses due to their migration, and decreases the cell polarisation. A unit $10' \times 5' \times 2'$ will take about 2000 amps, and furnish 16 per cent Na_2CO_3 containing excess of salt at a current efficiency of 85-90 per cent and at a voltage of 3·5-4 volts. The diaphragms are not a good feature of the cell

design. They must be fairly dense, or else the flow of liquors would be too rapid. This means a high resistance. Secondly, the hydrostatic pressure increases from top to bottom and so also the rate of flow. If, at the top, the rate is just sufficient to counteract the migration of the OH′ ions, at the bottom it will be too great, and the resulting solution will be unnecessarily weak.

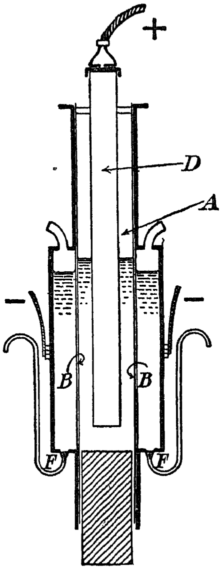


Fig. 14.—Townsend Cell.

(*d*) *Townsend Cell.*—This cell (*Figs.* 14 and 15) marks a distinct improvement on the above, both in design and performance. The central anode chamber A is of cement, and is provided with channels for entry of brine and for exit of chlorine, as also for cleaning. On either side is bolted an iron cathode chamber *b*, of which the side facing the anode compartment consists of an asbestos diaphragm B in close contact with an iron grid C. The anodes D fill up the greater part of A. The feature of the cell is that the cathode compartments are filled with kerosene oil, at a rather lower level than that of the brine in A, which is regulated by E. This has two functions.

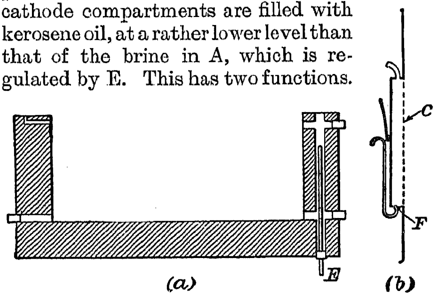


Fig. 15.—Townsend Cell.

It decreases the hydrostatic pressure difference between the two sides of the diaphragm, equalises it all the way down, and allows the diaphragm to be made of low resistance. Further, the alkaline cathodic solution does not remain in the neighbourhood of cathode or diaphragm. Owing to the surface tension effect, it at once forms spherical drops which are carried away by the hydrogen, and are finally collected at F and siphoned off. Losses owing to OH′ migration are avoided, and a high current efficiency, a strong caustic liquor almost free from chlorate, and a pure anode gas, almost free from carbonic acid, are obtained. A 2500-ampere unit measures 8′ × 3′ × 1′ and takes 15 amps/dm.^{2} at the diaphragms. 15 per cent NaOH can be made at 90-95 per cent current efficiency, and with 4-4·5 volts at the above current density. The working temperature is 50°-60°. Both diaphragms and anodes have a long life. The asbestos of the former is specially treated with a mixture of asbestos fibre, Fe_2O_3 and $Fe(HO)_3$, to ensure uniformity.

(*e*) *Nelson Cell.*—This (Fig. 16) will be described as a third typical cell. A 1000-ampere unit ($7\frac{1}{2}' \times 3' \times 1'$) consists of a rectangular steel tank, the outer cathode compartment, supporting a U-shaped perforated steel plate (the cathode), inside of which is spread

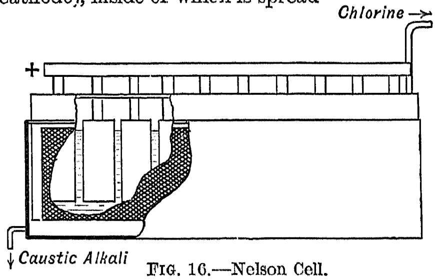


Fig. 16.—Nelson Cell.

the asbestos diaphragm. The cathode plate is closed at the ends by cement mortar blocks, and at the top by slate slabs, which in their turn carry the graphite anodes. The carefully purified brine is led into the anode compartment and percolates through the diaphragm into the cathode chamber. As in the Hargreaves-Bird cell, there is no opposing resistance to its passage. Steam is led into the outer compartment, and the caustic alkali drawn off at the bottom. 10-12 per cent NaOH is produced with a 90 per cent current efficiency and a voltage of 3·7 volts. The chlorine is 99 per cent pure. The anodes are specially treated and last for years, whilst the diaphragm has a life of 6-8 months.

(iv.) *Counter-current Cells without Diaphragm.*—Two cells only need be considered under this head—the Aussig "bell-jar" cell and the Billiter-Leykam "membrane" cell. They are so designed that the hydrogen and chlorine gas streaming away from the cathode and anode compartments respectively do not disturb the bulk of the electrolyte between the electrodes, and the rate at which the electrolyte streams towards the cathode is so chosen as just to overcome the movement of the OH′ ions with the negative current in the opposite direction. Neutralisation of the acid and chlorine from the anode by the OH′ ions will take place within a certain layer

between the electrodes. On the cathode side of this layer the solution will be alkaline, on the anode side it will contain acid and chlorine, and the maintenance of the neutral layer in a constant position is necessary if good current efficiencies and at the same time cathode liquors of a desirable alkali concentration are to be obtained.

(*a*) *Bell-jar Cell.*—The Aussig cell is shown diagrammatically in *Fig.* 17. The shallow rectangular bell jar A of sheet iron is lined with cement and carries a graphite anode which almost fills its cross-section. It further is provided with a pipe for leading off chlorine and is connected by two other tubes with the neighbouring bell jars in order to equalise the pressure. Some 25 such anode chambers are contained in a shallow cement trough. The brine enters each of them from above and is distributed evenly over the cross-section of the bell jar by means of a horizontal tube provided with a number of holes. The outer iron coatings of the bell jars serve as cathodes. All like electrodes in the one trough are connected in parallel. The caustic liquors are drawn off from the top of the trough and the "neutral" layers are formed inside and a little above the bottom level of the jars. Each bell jar takes 25-30 amperes, so the units are small. The current density is about 2 amps/dm.^{2}. 12 per cent NaOH can be made with a 90 per cent current efficiency and a bath voltage of four volts. The anodes last for years, the chlorine being very pure.

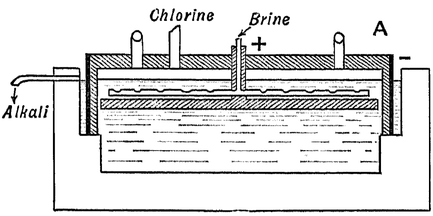


FIG. 17.—Aussig Cell.

(*b*) *Billiter-Leykam Cell.*—In this cell the above process has been modified in several ways. A large unit has only one bell jar instead of many. The distance between the electrodes is reduced by putting the cathodes beneath the anodes, disturbance of the neutral layer by hydrogen evolution being avoided by covering them with asbestos screens. These are diaphragms only in the sense that all the current must pass through them, but much of the electrolyte passes *between* adjacent cathodes and in any case the resistance of the "membrane" is very small. Finally, heat is applied by steam pipes (not shown in the diagram) and the cell is worked at 80°-90°. *Figs.* 18 and 19 explain the construction of this cell. A and B are of cement. The cathodes C are of T-iron, the asbestos tubes being braced above them by strips of non-conducting material. The most convenient current density in the bell jar is 5 amps/dm.^{2}. Making 15 per cent NaOH, a 90 per cent current efficiency at a bath voltage of 3·2 volts is obtained. The anodes have a long life and the asbestos tubes are practically unaffected by the electrolysis.

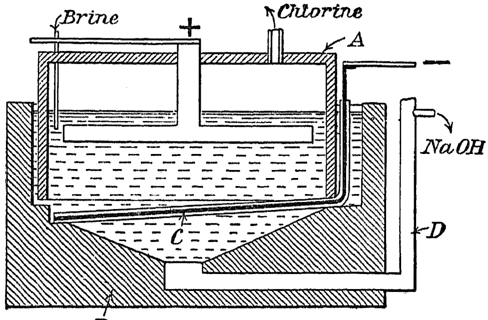


FIG. 18.—Billiter Bell-jar Cell.

It is unnecessary in both the above processes to purify the brine, and this is a very considerable advantage.

Considerations of space have made it necessary to omit various cells which, although efficient enough, have not yet found very extended use. Such are the Edser-Wildermann and Whiting mercury cells and the Allen-Moore and Finlay diaphragm cells. The important but inefficient Griesheim cell has also been omitted. The following table summarises the performances of those cells discussed. As a conventional theoretical voltage for the production of caustic soda and chlorine from brine, 2·3 volts has been taken.

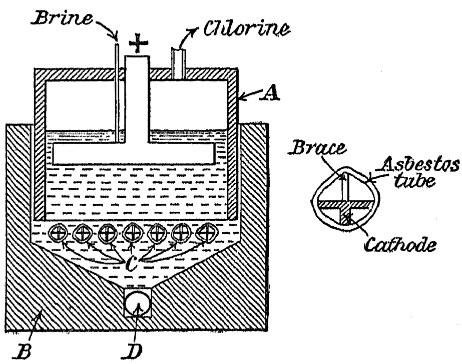


FIG. 19.—Billiter Bell-jar Cell.

Cell.	Normality of Alkali.	Cathodic Current Efficiency.	Voltage.	Energy Efficiency.	K.W.H. per kilo. NaOH.
		per cent.	volts.	per cent.	
Castner	5	92	4·2	50	3·1
Kellner (platinum anodes) .	5-6	97	5·0	45	3·4
Kellner (carbon anodes) . .	5-6	95	4·5	49	3·1
Bell jar	2	85	4·0	49	3·1
Billiter-Leykam	3	92	3·1	68	2·3
Billiter-Siemens	3-4	93	3·7	58	2·6
Hargreaves-Bird . . .	3 (Na_2CO_3)	85	3·7	. .	. .
Townsend	4	94	4·8	45	3·4
Nelson	2·5-3	90	3·7	56	2·7

VIII. Other Technical Electrolytic Processes employing Aqueous Electrolytes

The processes already dealt with are by far the most important technical applications of electrolysis of aqueous solutions, not only in respect of the substances produced, but also because of the amounts of energy utilised in their production. A few minor applications of electrolytic methods will now be discussed.

§ (29) Electrolytic Hydrogen and Oxygen.—Both these gases can be technically prepared more cheaply than by electrolysis, except where exceptionally cheap power is available. As, however, the use of the oxy-hydrogen flame has created a certain demand for the gases in the proportions in which they are liberated by the electrolysis of water, a number of technical electrolytic processes are in use. The electrolyte is usually 10-25 per cent NaOH, though 20 per cent H_2SO_4 is also employed. In the former case the conductivity is somewhat lower, but as iron electrodes can be used, whereas lead electrodes with considerably higher overvoltages must be used with the acid electrolyte, this disadvantage is more than compensated for. The gases produced are pure (97-99 per cent) and can readily be further purified. The current efficiency is nearly 100 per cent. With an alkaline electrolyte the voltages vary between two and three volts; with an acid electrolyte, between three and four volts. The theoretical decomposition voltage of water is 1·23 volts. Hence the energy efficiencies vary between 30 and 60 per cent.

The only well-known electrolyser using H_2SO_4 is that of Schoop (*Fig.* 20). In an outer lead-lined vat, containing the electrolyte, are partly immersed a number of vertical lead tubes, open at the bottom and perforated with a number of holes below and above the surface of the electrolyte, to allow of circulation of the same and to permit of the hydrogen being drawn off from the tops of the tubes. These form the electrodes, and are generally filled with fine lead wire to increase the active surface. Each is surrounded by a hood of clay or glass in order to collect the gases. Like electrodes are connected in parallel. 1000 cubic feet of hydrogen (plus 500 cubic feet of oxygen) require 230 K.W.H. With an alkaline electrolyte this is reduced to about 140 K.W.H.

Other processes employ an alkaline electrolyte. The Garuti electrolyser also uses hooded electrodes, in this case of iron, but in addition, the hoods themselves, insulated from the electrodes, are also of iron. By keeping the voltage below three volts, no decomposition occurs at their surfaces, and consequently no mixing of the gases. They are perforated near the bottom to facilitate circulation of the electrolyte. 1000 cubic feet of hydrogen require 160 K.W.H.

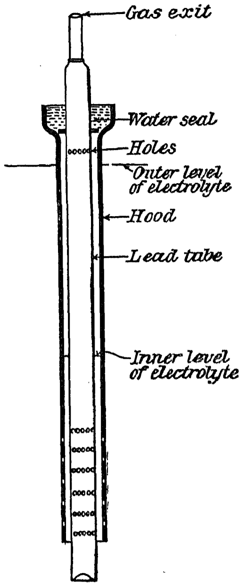
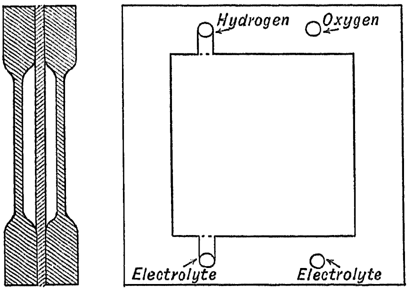


Fig. 20.
Schoop Electrolyser.

In the Schuckert apparatus, iron hoods are also used, and, in addition, unlike electrodes are separated by partitions of insulating material which extend from the top about three-quarters of the way down the cell. The resistance is correspondingly increased above that of the Garuti electrolyser, and 1000 cubic feet of hydrogen require 250 K.W.H.

The most efficient processes are probably those which make use of bipolar electrodes, the electrolysers being built up in filter-press form. In the Schmidt-Oerlikon apparatus (*Fig.* 21), designed for 110 volts, there are 40 bipolar electrodes, each taking the form of an iron plate with thickened edges or rims, so that when they are put together there

is a cavity in the middle of and between each pair of adjacent plates. This cavity is divided into two compartments, anode and



Cathode Compartment

FIG. 21.

cathode, by means of an asbestos diaphragm reinforced at the edges with rubber, which also serves to insulate the plates. By means of channels through the edges of the plates and diaphragms all like compartments are connected together for the purposes of introducing electrolyte and leading off gases. Current is led in and out of the two end plates. The voltage is low, as are also shunt current losses, and 1000 cubic feet of hydrogen require 150 K.W.H.

The electrolyser of the International Oxygen Company is very similar. By corrugating the electrodes, thereby increasing the surface, and by plating the anode sides with nickel, thereby still further reducing the overvoltage, the energy consumption is reduced to 125 K.W.H. per 1000 cubic feet of hydrogen. A unit designed for 120 volts will have 60 bipolar electrodes.

§ (30) PERMANGANATES. — Potassium permanganate is now almost exclusively made by electrolytic oxidation of the manganate. The latter is prepared by fusion of KOH and MnO_2 in presence of air.

$$MnO_2 + 2KOH + \tfrac{1}{2}O_2 \longrightarrow K_2MnO_4 + H_2O.$$

Formerly CO_2 was passed into the dissolved melt.

$$3K_2MnO_4 + 2CO_2 \longrightarrow 2K_2CO_3 + MnO_2 + 2KMnO_4.$$

It will be seen that a third of the manganese was reconverted into MnO_2, and that the alkali was turned into carbonate. In the electrolytic method no MnO_2 is formed, and caustic alkali, not carbonate, is regenerated. The anodic process is

$$MnO_4'' \longrightarrow MnO_4' + \ominus,$$

the cathodic process

$$H_2O \longrightarrow OH' + \tfrac{1}{2}H_2 + \oplus,$$

the total cell reaction therefore being

$$2K_2MnO_4 + 2H_2O \longrightarrow 2KMnO_4 + 2KOH + H_2.$$

In practice the electrolysis is carried out without a diaphragm, using sheet-iron anodes and iron-rod cathodes. The electrolyte contains 80-100 grams/litre of manganate, together with excess of alkali, and is kept at about 60°. The anodic current density is 9 amps/dm.^{2}, that at the cathode ten times as great, as cathodic reduction is thereby decreased. The voltage is 2·8-3 volts. The current efficiency falls off rapidly as conversion proceeds, owing to simultaneous cathodic reduction. With an average current efficiency of 67 per cent only one-third of the manganate is oxidised, about 0·7 K.W.H. being necessary per kilo of product. The power cost is thus negligible, which is probably the reason that the process, as technically described, is so inefficient. An improvement on the above figures has been shown by laboratory experiments to be easily possible.

In addition to the above process, permanganates have been prepared from time to time, particularly during the war, by direct treatment of ferro-manganese anodes in alkaline solution. The manganese goes into solution, whilst the iron remains undissolved. Diaphragm cells are employed to prevent reduction at the iron cathodes. Current efficiencies are poor and voltages high. One kilo of $KMnO_4$ requires about 30 K.W.H.

§ (31) CHROMIC ACID AND ANTHRAQUINONE. —The oxidation of anthracene to anthraquinone, used in alizarin manufacture, is frequently carried out by a solution containing H_2SO_4 and CrO_3, the latter being reduced in the process to $Cr_2(SO_4)_3$. Generally this is worked up and sent to tanneries, but sometimes the CrO_3 is regenerated electrolytically. The best-known process is that of Le Blanc. A cell is used divided into anode and cathode compartments by a special acid-resisting diaphragm. The electrodes are of lead. The spent liquors, containing 100 grams/litre Cr_2O_3 and 350 grams/litre of H_2SO_4, are fed into the cathode compartment, and leave by the anode compartment. At 50°, with 3·5 volts across the cell and an anodic current density of 3 amps/dm.^{2}, an 85 per cent current efficiency can be obtained, 87 per cent of the $Cr_2(SO_4)_3$ in the issuing liquors being oxidised. The process is simple, cheap, and effective. The lead anode soon becomes coated with a lead peroxide layer, and it is the latter which really effects the oxidation, being continuously regenerated electrochemically.

Chromic acid oxidises anthracene slowly compared with the rate at which its reduction product, $Cr_2(SO_{43})$, is reoxidised electrochemically. If, however, a rapid oxidising agent could be found which could also be readily regenerated anodically, the oxidation of anthracene could be carried out by suspending it in a solution of this substance as anolyte,

and passing a current. Such a substance, ceric sulphate, actually exists, and is used technically. It very rapidly and smoothly oxidises anthracene, being reduced to cerous sulphate, and the latter is readily reoxidised. The electrolyser, also acting as the anode, is a lead-lined vessel. The electrolyte is 20 per cent H_2SO_4 with 2 per cent $Ce(SO_4)_2$. The anthracene is suspended in this, and the liquors agitated and kept at 70°-100°. With an anodic current density of 5 amps/dm.^{2}, and about three volts on the bath, very pure anthraquinone is produced with practically the theoretical current efficiency.

§ (32) Persulphates.—Ammonium and potassium persulphates, which are themselves oxidising agents, and are also used to some extent for making H_2O_2, are prepared electrochemically by the oxidation at a high anode potential of the corresponding sulphates. The anode reaction is

$$2SO_4'' \longrightarrow S_2O_8'' + 2\ominus.$$

If OH′ ions are discharged, the yield will fall. From what has been already said, it is clear therefore that the first conditions for good yields are (*a*) smooth platinum anode, (*b*) high current density, (*c*) low temperature, (*d*) solution acid, or at any rate not alkaline. Further, as the S_2O_8'' ions are readily reduced, they must be kept from the cathode. In practice this is best done by the addition of a little K_2CrO_4, as in other processes. The electrolyte is a concentrated solution of $(NH_4)_2SO_4$ or $KHSO_4$. In the former case, H_2SO_4 must be continually added. With an anodic current density of 50 amps/dm.^{2}, a lead cathode, and an electrolyte kept below 15° by cooling coils, $(NH_4)_2S_2O_8$ can be made with a 70 per cent current efficiency and about seven volts across the cell. One kilo of the salt requires 2·4 K.W.H. The yield of $K_2S_2O_8$ is less good.

§ (33) Perborates.—Sodium perborate, $NaBO_34H_2O$, is a bleaching agent which has been coming into extended use during recent years. Usually it is made by the action of Na_2O_2 or H_2O_2 on $NaBO_2$, but electrolytic oxidation methods are also being employed with success. The electrolyte must be alkaline, as perborates are unstable in acid solution. Using a solution containing simply borax and NaOH, practically no perborate is produced. If, however, the NaOH be replaced by Na_2CO_3, and moderately strong solutions used, the salt can be produced with fair electro-chemical efficiencies. In practice a solution with 40 grams/litre borax and 120 grams/litre anhydrous Na_2CO_3 is electrolysed between a platinum gauze anode and a tin or aluminium cathode. The temperature should be kept below 18°, and this can be done by using a hollow cathode, and allowing water to stream through it. No diaphragm is used, but 0·1 per cent chromate is added. With an anodic current density of 100 amps/dm.^{2}, the bath takes about six volts. Either solutions of perborate or the solid salt can be prepared with satisfactory current efficiencies. The formation of perborate is probably a secondary one, sodium percarbonate being produced in the first instance.

§ (34) Iodoform.—This was formerly made by chemical interaction between iodine, alcohol, and an aqueous Na_2CO_3 solution.

$$C_2H_5OH + 8Na_2CO_3 + 5I_2 + 2H_2O \longrightarrow CHI_3 + 9NaHCO_3 + 7NaI.$$

It will be noted that 70 per cent of the expensive iodine goes to iodide and must be worked up again. Electrolytic methods avoid this loss, and are now almost exclusively used. The electrolyte consists of a solution containing alcohol, NaI and Na_2CO_3. The anode is smooth platinum and the cathode lead, surrounded by a parchment or other diaphragm. With an anodic current density of 1-2 amps/dm.^{2} and a voltage of 2-2·5 volts, very pure iodoform can be readily produced with a 90 per cent current efficiency. The successive reactions are—(*a*) electrolytic formation of alkali and hydrogen at the cathode and iodine at the anode; (*b*) partial interaction of these, giving hypoiodous acid (HIO) and sodium hypoiodite (NaIO); (*c*) substitution of the hydrogen of the alcohol by iodine, giving $CI_3 . CH_2 . OH$; (*d*) oxidation of this product by the hypoiodous acid to iodoform and CO_2.

The net result, expressed as an equation, is

$$C_2H_5 . OH + 3NaI + 3H_2O \longrightarrow Na_2CO_3 + CHI_3 + NaOH + 5H_2.$$

Fresh alcohol and NaI must be continually added, and the alkali formed neutralised by passing in a stream of CO_2.

IX. Fused Salt Electrolysis

§ (35) General.—Molten salts are good conductors of electricity, and their conduction is electrolytic. Consequently an electric current will decompose them, as it does when they are dissolved in water. There are, however, several important differences in the phenomena observed. The actual chemical decompositions brought about, owing to the absence of water and of consequent secondary reactions, are of a more simple nature than with aqueous solutions. The high temperatures used influence both current efficiency and voltage. The former tends to be adversely affected. Velocity of diffusion and of chemical reaction, as also vapour pressure, are all enormously increased, and unless anodic and cathodic products are very carefully separated from one another and from the action of the

electrolyte and the air, yields will be low. The higher the temperature the worse the yields, as has been many times shown. If the current density be increased, and therewith the rate of formation of the products at the electrodes, it is found that the current efficiency increases. This is natural, as vaporisation and diffusion losses are unlikely to increase in the same ratio. If the current density be very low and temperature high, however, the current efficiency may fall to zero. Apart from these sources of loss, there is another, peculiar to molten salt electrolysis —the so-called "metal fog" formation. A metal such as zinc or lead, if melted under a layer of one of its fused salts at a high temperature, is seen to give out dark clouds which apparently dissolve in the melted salt until a state of equilibrium has been reached. On cooling, they settle and are reabsorbed by the metal. The cause of this behaviour is to be sought in the surface energy relations between the molten salt and the molten metal. The former brings about peptisation of the latter, and produces from it a disperse phase, analogous to a dilute emulsion. Even very dark-coloured melts contain less than 0·1 per cent of metal, and quite a small quantity of an oxidising agent will destroy the colour. These phenomena will occur during fused salt electrolysis, and are a very important source of loss, as the peptised metal gets carried over to the anode, and is destroyed. It is interesting to note that the addition of certain other salts to the melt lessens or prevents "metal fog" formation, though the mechanism can hardly be the same as in the precipitation of colloids from aqueous solution. The effect of their addition is shown by a marked improvement in the current efficiency.

The voltage in fused salt electrolytic processes is affected in several ways by the high temperatures. The decomposition voltage necessary for a particular cell reaction is lower than that for the same reaction in aqueous solution at room temperature. The conductivities of the electrolytes used are higher. Further, the irreversible effects so common at the electrodes in the electrolysis of aqueous solutions practically disappear. From the point of view of voltage alone, therefore, there is much to be said in favour of high temperatures. One special effect only observed with fused salts must be noted—the "anode effect" remarked when carbon or graphite anodes are used in fused metallic halides at high temperatures. If the current density exceeds a certain value—about 4-5 amps/cm.^{2}. with hard carbon and 7-8 amps/cm.^{2} with graphite—the voltage rises very considerably and quickly, and the anode appears to glow. This is due to the electrode becoming covered with a film of gas, through which the discharge can only pass in the form of an arc. The phenomenon is general, most pronounced with fluorides, least so with iodides. It is sometimes accompanied by loss of weight of the carbon anodes, owing to formation of halides of carbon.

As far as possible fused salt baths are not externally heated, but the temperature is kept up by the current, a solidified layer of electrolyte forming next the walls. They are usually made of iron, lined if necessary with a suitable type of resistive brick. The cathode is of iron, or else a layer of the precipitated metal. With fused halides, graphite is by far the best anode material. Iron has occasional application. In the case of aluminium production hard carbon anodes are used, which burn away in the anodic oxygen.

The most important fused salt electrolytic process, viz. the production of aluminium from a solution of alumina in fused cryolite, is separately dealt with in this Dictionary. Of other processes, we shall discuss only those which are actually in technical operation, viz. the production of sodium, magnesium, and calcium, all exclusively prepared by electrolysis.

§ (36) Sodium.—The obvious raw material for the production of this metal would be common salt, were its melting-point, 800°, not so high. This in practice causes great difficulties which have not been completely overcome. By far the greater proportion of sodium is made by the electrolysis of sodium hydroxide, the melting-point of which is 327° when pure, but about 300° or a little over for the ordinary commercial products.

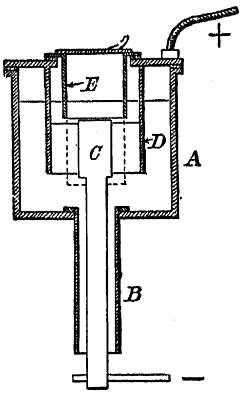


Fig. 22.—Castner Sodium Cell.

The Castner cell (*Fig.* 22) is a cast-iron pot A, embedded in brickwork to prevent loss of heat, and so dimensioned that, when working under normal conditions, it is lined with a solid layer of NaOH. The cathode C consists of an iron rod sealed into an extension B by solid NaOH. The ring-shaped nickel anode D rests on the top of the cell, from which it is suitably insulated. Between anode and cathode is hung an iron curtain E, the bottom part of which is made of gauze. Inside this curtain the sodium, lighter than the NaOH, collects, and is ladled out by perforated metal

spoons. The gauze allows of free passage of electrolyte and current between the electrodes, but prevents globules of sodium from reaching the anode. The units are of small size, take about 500 amperes at a cathodic current density of 200 amps/dm.^{2}, and absorb 4·5 volts. The working temperature is as low as the solidifying point of the alkali permits—about 315°-320° in practice. If it rises, the yield very rapidly falls off and becomes zero at 325°. The current efficiency is about 45 per cent.

The mechanism of the main process is simple. At the cathode, $Na^{\cdot}$ ions are discharged to metal. At the anode, OH' ions are liberated, giving water and oxygen. This water diffuses throughout the mass. Some of it attacks the sodium, which not only forms a "metal fog," but, as has been proved, dissolves in metallic form in the NaOH. The greater part is decomposed by electrolysis, hydrogen being produced at the cathode. The net result, therefore, of the passage of four faradays of electricity through the cell is

$$2NaOH \longrightarrow 2Na + H_2 + O_2,$$

and the maximum possible current efficiency under the cell conditions is 50 per cent. The deficiency observed from this figure in practice is due to volatilisation of the metal and to small quantities burning to form Na_2O_2. If the temperature be raised still further, so as to increase the rate of diffusion of the dissolved sodium, the yield, as has been mentioned, falls to zero. Under these circumstances the dissolved sodium ionises at the anode ($Na + \oplus \longrightarrow Na^{\cdot}$) as rapidly as it is produced at the cathode, no gas being liberated.

It is obvious that if the water liberated at the anode could be kept away from the sodium formed at the cathode, higher yields than 50 per cent would be possible. Unsuccessful attempts have been made to do this by employing cathode diaphragms, and by continuously dehydrating the NaOH by blowing air through. A series of small explosions (accounting for the use of small units) is liable to occur during the electrolysis. When it is remembered that hydrogen is liberated not only at the cathode but also around the anode (chemical action), and that metallic sodium is very reactive towards air or oxygen, this will be understood.

The decomposition voltage of pure fused NaOH is about 2·2 volts at the temperature of electrolysis. Combining this with the above figures, we have an energy efficiency of 22 per cent. One ton of sodium requires about 11,700 kilowatt hours.

§ (37) Magnesium.—This metal is made by the electrolysis of a fused mixture of $MgCl_2$ and KCl. The decomposition voltage of $MgCl_2$ in such mixtures, provided their magnesium content is not too low, and the cathodic current density not too high, is less than that of KCl. Hence, on electrolysis, pure magnesium separates, chlorine being formed at the anode.

Exact descriptions of the method used are not available. An iron pot is employed, lined under working conditions with a solidified layer of the electrolyte. This contains an iron cathode and carbon anode, separated by some sort of an anode screen or diaphragm in order to prevent access of the liberated magnesium (lighter than the electrolyte) to the anode. The best working temperature is about 700°. As magnesium fuses at 633°, it is collected in the molten state. There is some difficulty in making the small molten globules coalesce to larger ones. In practice this is overcome by the addition of some CaF_2 to the electrolyte. The current efficiency is about 75 per cent. With a cathodic current density of 30 amps/dm.^{2} the bath voltage is about six volts. The reversible decomposition voltage of the melt is about 3·2 volts, which gives us an energy efficiency of 40 per cent and an expenditure of 17·7 K.W.H. per kilo of metal.

§ (38) Calcium.—This metal is technically made on a small scale by the electrolysis of the molten chloride at 750°-800°. Metallic calcium melts at 800°, and is therefore deposited in the solid state. In order to minimise the surface of contact between the metal and the electrolyte, and thereby diminish the metal fog formation, the cathode takes the form of an iron rod which just touches the surface of the melt, and is moved upwards by a gearing at such a rate that a continuous rod of solid calcium is drawn up out of the cell. The current density at the cathode is very high, about 100 amps/dm.^{2} This causes the cell voltage also to be very considerable. Nothing is known of the actual material used for the bath, nor of the arrangements employed for preventing interaction between the metal and the anodic chlorine. The anodes are of graphite. Current efficiency and voltage, judging from laboratory scale results, are probably about 80 per cent and 25 volts respectively. With a decomposition voltage of 3·24 volts, we have an energy efficiency of 10 per cent and a consumption of 42 K.W.H. per kilo of metal.

It should finally be mentioned that similar "contact cathode" processes are also in use for the production of both sodium and magnesium, the object being to reduce the contact between the metal and the electrolyte In the former case, liquid metal results; with magnesium, a solid rod is produced.

A. J. A.

Literature References

For a fuller discussion of the subject in its broader and theoretical aspects, see *Applied Electrochemistry*, by A. J. Allmand (Edward Arnold). For fuller

details on actual plant, the following may be consulted:

Section V.—*Electroplating*, by Barclay and Hainsworth (Edward Arnold).

Section VI.—Silver and Gold Refining: *Metall. Chem. Engin.*, 1911, ix. 443; *Am. Electrochem. Soc. Trans.*, 1919, xxxv. 251; Zinc Extraction: *Zeits. Electrochem.*, 1904, x. 688; 1909, xv. 456; Lead Refining: *Electrochem. Ind.*, 1903, i. 407.

Section VII.—Hypochlorites: *Technische Electrochemie*, vol. ii., by Askenasy (Vieweg); Alkali-chlorine Cells: *Technische Electrochemie*, vol. ii., by Askenasy (Vieweg); also *Journ. Soc. Chem. Ind.*, 1919, xxxviii. 285 R.—Billiter-Siemens Cell: *Electrochem. Ind.*, 1907, v. 209, and 1909, vii. 313—Townsend Cell; *Chem. Met. Engin.*, 1919, xxi. 17—Nelson Cell; *Faraday Soc. Trans.*, 1913, ix. 3—Billiter-Leykam Cell.

ELECTROMAGNET, THE

An electromagnet is an arrangement for producing magnetic effects by means of an electric current flowing in a coil of wire wound round a core of iron or other magnetic material. The fact that an iron bar can be magnetised by an electric current was discovered by Arago and independently by Davy in 1820, but the electromagnet was first constructed in a practical form by William Sturgeon in 1825.

The core of an electromagnet is sometimes straight, but is more usually bent into a "horseshoe" so that its ends, or poles, are near each other. A typical form of electromagnet consists of two parallel vertical cores attached at their lower ends to a massive iron yoke. The coils are wound on bobbins which are slipped over the cores, and are so connected that the current flows in opposite directions round them. By means of pole-

pieces resting on the upper core-ends the space between the poles can be reduced to a narrow air-gap, or the space between the core-ends may be partially or completely bridged by an iron armature.

In their various applications electromagnets are used for producing magnetic flux, intense magnetic fields, attraction of small magnetic bodies, tractive force, movement of an armature or core (in "electromagnetic mechanisms"), and induced currents in a secondary circuit. We will consider these effects and the conditions favourable to their production.

§ (1) ELECTROMAGNETS FOR PRODUCING MAGNETIC FLUX.—The most important application in this class is found in the dynamo magnet, used for producing a given flux (or total induction) in an armature placed between the pole-pieces. The relation between the magnetic flux Φ in an electromagnet and the ampere-turns in the exciting coils is expressed by the equation of the magnetic circuit, viz. the magnetomotive force in a magnetic circuit is equal to the product of the flux and the magnetic reluctance of the circuit.[1] The magnetomotive force, or the line-integral of the magnetic force along any closed path, threading once through all the n turns of the exciting coils, the current in which is i amperes, is equal to $4\pi ni/10$, or approximately $1{\cdot}257 \times$ ampere-turns. The reluctance of a magnetic circuit is equal to the sum of the quotients $l/\mu s$ for the various portions of the circuit, where l is the mean length of the lines of induction in any portion, s the cross-section of that portion, and μ its magnetic permeability. The fact that an electromagnet does not usually form a perfect magnetic circuit, *i.e.* that there is magnetic leakage, so that the flux is not strictly the same in all sections of the iron and gap, may be allowed for by the introduction of leakage coefficients. These are numbers expressing the ratio of the mean flux in the various portions of the magnet to the flux in some one portion.

Thus if it be required to find the ampere-turns which will produce a flux of Φ lines in the armature, let l_1 be the mean length of the flux-path in the armature, s_1 its mean section at right angles to the lines of flux. Also let l_2, s_2; l_3, s_3; l_4, s_4; l_5, s_5, represent similar quantities for the air-gaps between the armature and the pole-pieces, for the pole-pieces, the cores, and the yoke respectively, and let q_2, q_3, q_4, q_5 represent the leakage coefficients for these portions of the magnetic circuit. Then the equation of the magnetic circuit is

$$1{\cdot}257\ ni = \Phi\left(\frac{l_1}{\mu_1 s_1} + \frac{q_2 l_2}{s_2} + \frac{q_3 l_3}{\mu_3 s_3} + \frac{q_4 l_4}{\mu_4 s_4} + \frac{q_5 l_5}{\mu_5 s_5}\right).$$

The permeability of each part of the magnetic circuit being known for the flux-density existing therein, and the leakage coefficients for the various portions of the circuit being found by experiment (for methods see S. P. Thompson's *Dynamo-electric Machinery*, 1903, chap. vi.), the ampere-turns required to produce a given flux in the armature can be calculated by the above equation. This is essentially the method suggested by J. and E. Hopkinson, and by G. Kapp, in 1886, for the calculation of the field windings of dynamos. The Hopkinsons also gave a graphical method for finding the ampere-turns, which consists in determining for each portion of the circuit a curve showing the relation between the flux and the ampere-turns required to maintain it. When these curves are obtained for all parts of the circuit they are combined by adding their abscissae, the result being a characteristic curve for the magnetic circuit, the ordinate of which represents the flux in the armature and the abscissa the ampere-turns. These methods can be easily applied to cases in which the magnetic circuit is branched, as in some forms of dynamo magnets.

Methods for determining magnetic permeability are described in Ewing's *Magnetic Induction in Iron and other Metals*, 1900, and in other works referred to below.[2] In *Fig.* 1 are shown the magnetisation curves (from data in Miles Walker's *Specification and Design of Dynamo-electric Machinery*, 1918) for specimens of four kinds of iron used in the construction of electromagnets, viz. I., cast iron; II., cast steel; III., forged ingot iron; IV., silicon steel. In these curves the abscissae represent the magnetising force H, the ordinates the magnetic induction, or flux-density, B, in C.G.S. units. Curves I.-IV. show how the flux-density increases with the magnetising force, the material being initially

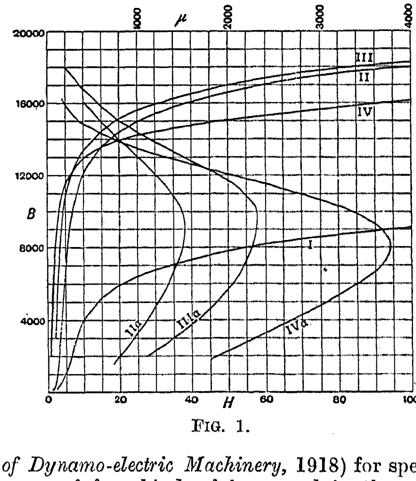


FIG. 1.

[1] See "Electromagnetic Theory," § (6).

[2] See also "Magnetic Measurements," §§ (17) *et seq.*

in the neutral unmagnetised state. The curves II.*a*, III.*a*, IV.*a* represent the permeability B/H (upper horizontal scale) for the three last-named materials at various values of B.

Cast iron is much inferior in its magnetic properties to the low-carbon steels, the maximum permeability of the specimen represented in curve I. being 410 at B = 4000. Cast iron is used for the yokes of large dynamo magnets.

Cast steel, or ingot iron, is used for the yokes and cores of electromagnets. Curve II. represents the properties of an annealed casting containing 0·2 per cent of carbon. Its maximum permeability (curve II.*a*) is 1525 at B = 9000.

Forged ingot iron is a still better (but more costly) material than cast steel for magnet cores. Curve III. is the magnetisation curve, III.*a* the permeability curve for an annealed specimen containing 0·15 per cent of carbon. Its maximum permeability is 2320 at B = 10,000.

The remarkable magnetic properties of certain iron-silicon alloys discovered by Sir Robert Hadfield are illustrated in curves IV. and IV.*a*, the magnetisation and permeability curves for a specimen containing 4·8 per cent of silicon and 0·2 per cent of carbon. The maximum permeability is about 3790 at B = 8000. The improvement in permeability at low flux-densities increases with the proportion of silicon if this proportion is between 1·8 and 4·8 per cent. At inductions over about 14,000, however, the presence of the silicon lowers the permeability. Its high permeability at low inductions, and certain other properties referred to in § (6), make alloyed steel particularly suitable for transformer cores and other magnet cores in which the flux is rapidly varying. The qualities of alloyed steel generally used for these purposes contain 3 to 4 per cent of silicon and less than 0·1 per cent of carbon.

For the laminated armature cores and pole-pieces of dynamos sheet steel (about 0·09 per cent C and 0·01 per cent Si) is preferable to alloyed steel, owing to its higher permeability at high inductions. The magnetisation curve of a good specimen of dynamo sheet steel is very slightly below curve I. (*Fig.* 1).

Considerably higher values of the maximum permeability than those shown in *Fig.* 1 have been found in carefully prepared and annealed iron-silicon alloys; values up to 12,000 are recorded by Gumlich and others. The process of melting or annealing *in vacuo* has been found to effect great increase in the maximum permeability of iron, but the amount of such improvement does not appear to have been as yet well ascertained. The iron-cobalt alloy Fe_2Co was shown by P. Weiss to have remarkably high magnetisation in strong fields, the saturation value of $(B-H)/4\pi$ at ordinary temperatures being 10 per cent higher than that of iron.

§ (2) Electromagnets for producing Intense Magnetic Fields.[1]—In the dynamo magnet no attempt is made to produce very highly concentrated magnetic fields—the average field in the air-gaps between the armature and pole-pieces does not usually exceed about 15,000 gauss (or C.G.S. units)—but for some purposes very intense fields are required, as for instance in the "isthmus" method of Ewing and Low for the examination of the magnetic properties of materials in strong fields, and in magneto-optic experiments. In these cases the pole-pieces must be so shaped as to concentrate the field as much as possible.

The form of pole-pieces giving greatest concentration was calculated by Stefan and by Ewing in 1888 on the assumption that the pole-pieces are uniformly and longitudinally magnetised, the result being that the pole-pieces should take the form of cones of semi-vertical angle 54° 44′. In practice the cones for maximum concentration should, as pointed out by Ewing, have a rather greater angle, owing to the fact that the magnetisation of the pole-pieces is not quite uniform.

As to the other parts of the magnetic circuit these should be so designed as to reduce leakage to a minimum, and therefore should have as few joints and sharp bends as possible. The exciting coils should be so placed as to produce by the direct action of the current the greatest possible field in the neighbourhood of the pole-pieces in order to magnetise them to the highest degree of saturation, and also to increase the contribution of the coil field to the total field in the gap. For it should be remembered that the field due to the magnetisation of given pole-pieces cannot be increased beyond a certain limiting value which depends upon their saturation intensity of magnetisation. The field due directly to the current is, however, proportional to the current, and it therefore becomes an increasing fraction of the total field as the current is increased.

Much attention has been given to the scientific design of electromagnets for producing intense fields by H. du Bois and by P. Weiss. In 1891 du Bois designed a large ring electromagnet, having a core of Swedish iron with coils wound nearly uniformly round its circumference, which gave a field of about 40,000 gauss in a gap 1 mm. wide and 6 mm. in diameter.

With the introduction of highly magnetic cast steel du Bois subsequently designed a less costly and more convenient form of electromagnet known as the half-ring type, which is in use in many laboratories. Further

[1] See "Magnetic Measurements," §§ (39)-(45).

improvements were introduced in a more recent design illustrated in *Fig.* 2 (from the *Zeitschrift für Instrumentenkunde*, Dec. 1911), which shows, in about $\frac{1}{15}$ the actual size, the latest type of du Bois half-ring electromagnet.

Each of the two curved cast-steel cores corresponds to about one-third of a complete

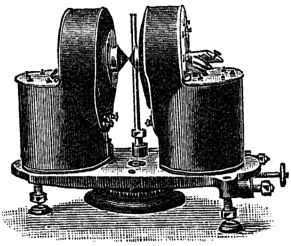


FIG. 2.

toroid; the diameter of its vertical circular section at the polar end is 93 mm., the diameter of the core increasing near the base. Conical borings, 15 cm. long and tapering 1 in 5, pierce the cores to admit a converging beam of light axially to the gap for magneto-optic experiments. For other experimental work the borings can be filled in with iron plugs, one of which is shown in *Fig.* 2. A copper tube, for water cooling, surrounds the polar end of each core.

The coils (wound for 40 amperes at 200 volts) are so arranged as to allow optical or other appliances to be brought close up to the borings. A pair of extra polar coils can be slipped over the pole-pieces so as to increase their saturation and add to the field due directly to the current.

A variety of pole-pieces are supplied with the instrument—plane for uniform fields of

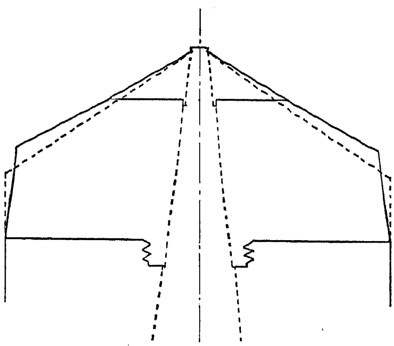


FIG. 3.

considerable extent, conical for more concentrated fields. The form of pole-piece found to give the strongest fields is illustrated in *Fig.* 3. It does not differ much from the 54° 44′ cone (represented by the broken line), but has a rather greater angle, this angle increasing slightly towards the base of the cone. Special pole-pieces are provided for various kinds of magneto-optic and other observations in strong fields.

The total weight of the magnet (large model) is 360 kilogrammes. With the extra polar coils, each of 500 turns, and the maximum current (150,000 ampere-turns in all), the field in a 1×6 mm. gap is 50,000 gauss. When the gap is reduced to $0{\cdot}5 \times 3$ mm. and the pole-pieces are provided with ferro-cobalt tips, the field is 59,000 gauss.

A still larger model of the same type, weighing 1400 kilogrammes and having a pole-base of 20 cm., gives in the $0{\cdot}5 \times 3$ mm. gap (with ferro-cobalt pole-tips) 65,000 gauss.

Without the extra polar coils the fields are about 10 per cent smaller than the above values. Polar windings have, in fact, a greater effect per ampere-turn in saturating the pole-pieces than windings situated on more distant parts of the magnetic circuit.

The du Bois magnet is also made in two smaller sizes, weighing respectively 200 and 50 kilog. and giving maximum fields of 52,000 and 40,000 gauss in a $0{\cdot}5 \times 3$ mm. gap.

Powerful electromagnets have also been recently designed by Weiss, who has adhered to the Ruhmkorff pattern of magnetic circuit having two horizontal coaxial cores supported by a massive yoke, on the ground, already referred to, that coil windings near the air-gap are more effective than those on other parts of the circuit. Weiss recommends, with a view to further improvement in the saturation of the pole-pieces, that the cross-section of the iron should gradually diminish from the distant parts towards the gap, and emphasises the importance of adequate arrangements for cooling the exciting coils. In a large electromagnet constructed on these lines in 1907 the windings of copper strip were immersed in oil cooled by water circulating in a spiral tube. The weight of this magnet was 1300 kilog., it had 3360 turns capable of carrying 60 amperes, and gave a field of 46,000 gauss in a $2 \times 3{\cdot}6$ mm. gap. The diameter of the cores was 15 cm., the length of each (excluding the pole-pieces), 52 cm. In a later and still larger model the windings were composed of copper tubing, which served to carry both the electric current and the cooling water.

With regard to the effect of a change of dimensions of an electromagnet on the field which it produces, this is governed by the principle of similarity as stated by Kelvin, viz. electromagnets of geometrically similar form give equal magnetic fields at corresponding points if their currents are proportional to their linear dimensions. In similar systems, however, the area of the cross-section of the

wire, and therefore its current-carrying capacity, is proportional to the square of the linear dimensions. A small electromagnet would thus require more windings to produce the same field at corresponding points than a large one, otherwise the current would cause excessive heating. It follows also that any further considerable increase of the field, in a gap of given dimensions, produced by increasing the size of an electromagnet of the usual type (the field of which is mainly due to the magnetism of the pole-pieces) could only be attained with an exceedingly great increase in the weight, since the weight increases as the cube, the field at the most only as the logarithm, of the linear dimensions.

The problem of producing very intense magnetic fields has been attacked in a different way by Deslandres and Perot, who increased the field due to the direct action of the current instead of that due to the magnetism of the pole-pieces. With a current of 5000 amperes flowing in a water-cooled spiral of bare silver ribbon a field of 49,900 gauss was attained even without the use of an iron core, the energy consumption in the coil being at the rate of 340 kilowatts. With an iron core a field of 63,700 gauss was reached. Doubtless still higher field intensities can be produced by such methods, the only objection to them being the cost of the heavy expenditure of energy which they involve.

§ (3) Electromagnets for attracting Small Magnetic Bodies.—It is sometimes required to exert by means of an electromagnet a force of attraction on a small magnetic body placed near it. This is the case, for example, in the torsion balance method used by Curie for measuring the magnetic susceptibility of substances at various temperatures. The same action is put to practical use in the surgical electromagnets used for extracting small particles of iron or steel from the eye or other parts of the body, and in the magnetic separators used for separating iron or iron-containing ores from other materials.

The force acting on a magnetic particle placed in a field of intensity H depends on the non-uniformity of the field; if the particle is magnetically saturated, so that its magnetism is independent of the field, the force in any direction x is proportional to dH/dx, if it is unsaturated and the susceptibility is constant the force is proportional to HdH/dx. The force on magnetic particles is directed towards the stronger, on diamagnetic particles towards the weaker, parts of the field.

For the examination of the magnetic properties of paramagnetic or diamagnetic elements du Bois recommends the arrangement indicated in *Fig.* 4. The pole axes of the electromagnet are inclined at 25° to each other, and the rather pointed pole-tips are set 3 mm. apart. At any point in the transverse axis Oy the magnetic force is parallel to Ox, its greatest value occurring at a point P within the smaller angle between the axes. The greatest value of HdH/dy is found to be at a point Q slightly to the left of the point of intersection of the axes. At this latter point Q should the specimen under examination be placed in order to experience the greatest force.

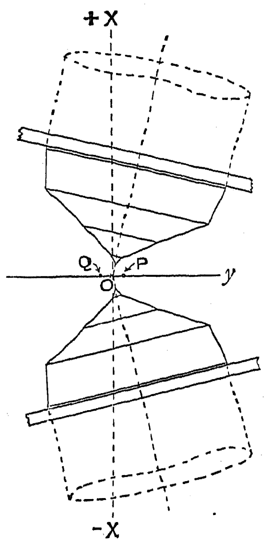


Fig. 4.

A surgical electromagnet designed by du Bois is shown in *Fig.* 5 (from the *Elektrotechnische Zeitschrift*, May 2, 1918). The "single pole" arrangement is more convenient in this case, the core of the magnet being straight but increasing in diameter towards the rear end where it expands into a wide flange. For eye applications the pole-piece takes the form of a truncated cone, which may have an axial boring to contain a small glow-lamp. The best form of pole-piece for attracting saturated or unsaturated particles was investigated by du Bois, who came to the conclusion that a cone of about 40° semi-vertical angle was suitable in both cases. A flexible extension may be attached to the pole, consisting of a bundle of very thin annealed iron wire, tapering towards the end and terminating in a suitable probe. The weight of the eye magnet is about 50 kilog.; for abdominal applications a larger model, weighing 100 kilog., is made, which is provided with a concave pole-face shaped to the surface of the body.

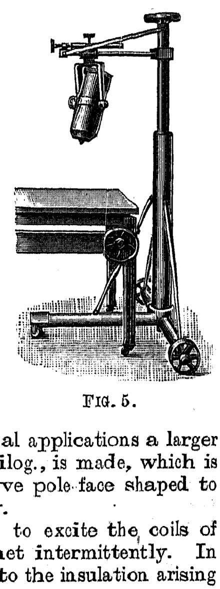


Fig. 5.

It is often desirable to excite the coils of a surgical electromagnet intermittently. In order to avoid damage to the insulation arising

from the high induced E.M.F. at "break," this should be effected by periodically shunting the magnet coil by a non-inductive resistance. The precaution of inserting a shunt should always be observed before opening the circuit of a large electromagnet.

§ (4) Electromagnets for exerting Tractive Force.—The statical tractive force exerted by an electromagnet on an armature in contact with its poles is put to a variety of practical uses, as, for example, in lifting magnets, magnetic clutches, brakes, and chucks.

The physical principle underlying this action is expressed by Maxwell's law[1] of the tractive force of magnets, that the normal traction between two plane, uniformly and normally magnetised pole-faces, separated by an infinitely narrow air-gap, is equal to $B^2A/8\pi$, where B is the flux-density and A is the area of the surface of contact. It should be observed that the above expression represents the total pull associated with all the tubes of induction that cross the area A. If, for example, we imagine a long transversely divided bar of iron to be placed in the uniform field of a very long magnetising solenoid, in which one part of the bar is fixed and the other is freely movable, the pull on the inner end of the movable part is $(B^2 - H^2)A/8\pi$, where H is the magnetic force due to the current in the solenoid. The term $H^2A/8\pi$ represents that portion of the attraction of one part of the coil on the other which is exerted across the area *A* of a plane coinciding with the section of the bar. The expression for the pull on the inner end may be written $(2\pi I^2 + IH)A$, where I is the intensity of magnetisation of the bar. In the present case the force IHA is balanced by an equal and opposite pull on the outer end of the bar, so that the force required to separate the bars reduces to $2\pi I^2A$.

If, however, the movable bar is long enough to extend well beyond the end of the coil, as in some forms of "plunger" electromagnet, the force on the outer end is much reduced, and the pull on the plunger is in this case approximately $(B^2 - H^2)A/8\pi$. This result has been experimentally verified for inductions up to 39,000 C.G.S., at which value the pull was over 700 lbs. wt. per sq. inch. A tension of over 1600 lbs. wt. per sq. inch has been observed at a higher flux-density. These observations were made in the concentrated field of a powerful electromagnet, where H is a considerable fraction of B. In ordinary circumstances H^2 is negligible in comparison with B^2, and the approximate expression for the pull reduces to $B^2A/8\pi$. This expression holds for the tractive force exerted across each pole-face of a horseshoe electromagnet on an armature attached to both its poles, if the induction across the pole-face is uniform and normal.

[1] See "Electrostatic Field," § (5).

Modern lifting magnets are generally of the bell or "iron-clad" type, circular or rectangular in form, the coil being placed round a central core, the casing forming the yoke, and the poles therefore being at the centre and the rim. The object lifted, attached at the central pole and the rim, forms the armature. *Fig.* 6 (kindly supplied by the Witton-Kramer

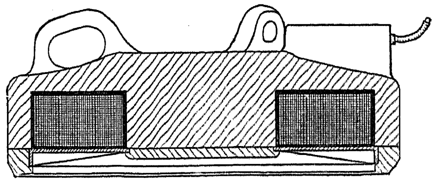


Fig. 6.

Electric Tool and Hoist Works, Birmingham) shows an axial section of a modern lifting magnet, *Fig.* 7 its manner of suspension.

The core and shell of a lifting magnet are of cast steel, the shell being usually provided with ribs which serve the purposes of strengthening the frame, reducing the reluctance of the magnetic circuit, and increasing the area of the cooling surface. The coil, generally of copper wire or ribbon—in some cases of aluminium wire insulated with a coating of oxide—is wound on a bobbin which is set in place over the core and bolted to the frame. The coil and shell are

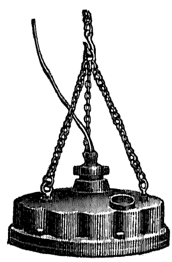


Fig. 7.

thoroughly impregnated with insulating compound *in vacuo* and covered in by a plate of phosphor bronze or manganese steel. The whole magnet is made thoroughly watertight, as it has sometimes to operate under water.

The problem of determining how the core should be shaped, and what ampere-turns should be provided in order that a magnet, constructed from a given total weight of material, should exert the greatest possible lifting force, has been considered by G. Kapp (*Principles of Electrical Engineering*, 1916, i. 195). Among the conclusions arrived at are that the best induction in the core is that corresponding to the "knee" of the magnetisation curve, *i.e.* about 17,000 or 18,000 lines per sq. cm., that in no case should the area of the pole-face be smaller than that of the core section, and that when there is a considerable non-magnetic gap between the

pole-faces and the armature it is advisable that the pole-faces should be of greater area than the section of the core.

As to the effect of a change in the size of a lifting magnet, it can be shown that in similar systems, with currents proportional to the linear dimensions so as to give the same induction, the ratio of the lifting force of a magnet to its weight is inversely proportional to the linear dimensions. Thus small magnets have a higher "load ratio" than large ones. S. P. Thompson refers to a small magnet weighing a grain and a half which could lift 2500 times its own weight.

A modern lifting magnet of the bell type, 60 inches in diameter, and weighing 3 or 4 tons, can support about five times its own weight of material. Lifting magnets are used largely in iron and steel works for loading and unloading pig iron, steel ingots, girders, plates, scrap iron, etc., also iron ore if it contains not less than about 60 per cent of iron.

Similar principles underlie the action of electromagnetic clutches and brakes. In the former the electromagnet may be contained in a pulley which runs loose on the shaft, the armature being a disc keyed to the shaft which attracts the magnet when the latter is excited. The friction between the armature and a friction plate provided on the loose pulley is sufficient to transmit power on a large scale. In magnetic brakes, such as those used on tramways, an electromagnet carried by the car is held just above the rail to which it is attracted upon excitation. The friction between the track-shoes of the magnet and the rail acts as a powerful and easily controllable brake.

Recently, magnetic chucks have come much into use in connection with shaping machines, lathes, and other workshop machines. In

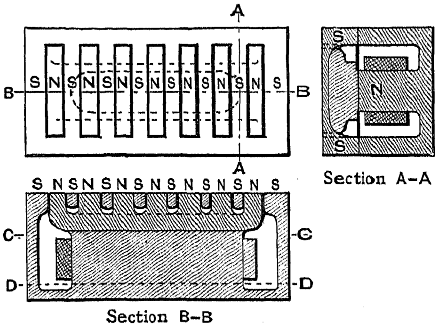


Fig. 8.

these chucks the face plate is composed of a large number of alternately positive and negative poles to which the "work" firmly adheres as an armature when the magnet is excited. *Fig.* 8 (from a paper by O. A. Kenyon in the *Electrical World*, July 5, 1919) shows diagrammatically the form of the magnetic circuit of one type of rectangular chuck. The negative poles are formed by a grid into the interstices of which project the extensions of the positive pole, these being separated from the sides of the openings by non-magnetic material. In this type the single core and coil are within the body of the chuck, and, as in the bell magnet, the casing forms part of the magnetic circuit.

§ (5) Electromagnetic Mechanisms.—There are a great many appliances (such as electric bells, indicators, relays, telephone receivers, and arc-lamp regulators) in which some required movement is produced in the armature or core of an electromagnet by varying the exciting current. In all such cases the movement which the electromagnet tends to produce is such that the magnetic reluctance of the circuit is thereby diminished. Thus in purely electromagnetic systems the air-gap between the armature and the core-ends is reduced when the magnet is excited by a current flowing in either direction, the armature (supposed held by a spring) returning to its original position when the excitation is withdrawn.

The magnitude of the attraction on the armature, represented approximately by $B^2/8\pi$ per unit area of its surface where the normal induction is B, falls off rapidly as the width of the gap increases, owing both to the reduction of flux in the whole magnetic circuit accompanying the increase of reluctance (the magnetising ampere-turns being assumed constant) and to increased magnetic leakage. Thus to obtain any considerable attractive force on the armature this must be placed very close to the ends of the magnet core. Special electrical or mechanical devices have, however, been invented for reducing the variation of the force with width of gap, so as to increase the effective range of action. Among these are the "coil and plunger" arrangement and its modifications, used in some arc-lamp regulators, in which an iron core is drawn into a coil, and in which the range of effective action is much more extended than in those mechanisms where an armature is attracted by an electromagnet having a fixed core.

Numerous attempts have been made to produce reciprocating motion of an electromagnet core by supplying intermittent or alternating current to the coil. Among these devices may be mentioned the electromagnetic hammer of Schüler—a small portable hammering machine suitable for chiselling and riveting —which is driven by an alternating E.M.F. applied to the coil through a contact breaker. The current is "on" for one period, during which the core, or hammer, delivers its blow, and "off" for the next two periods, during

which the hammer is drawn back by a spring. One of the chief difficulties in such arrangements is that of opening the circuit without causing a spark to appear at the contact breaker. This difficulty can be overcome, as pointed out by T. F. Wall, by superposing a suitable constant E.M.F. upon the alternating E.M.F. applied to the inductive coil. By this means the current and the total E.M.F. can be made to become zero simultaneously, thus allowing the circuit to be opened sparklessly.

In telephone receivers, and some relays and electric bells, the magnetic circuit is formed partly of soft iron and partly of permanently magnetised steel, the iron core, round which the coil is wound, being therefore magnetised to a certain degree by the steel magnet. In these "polarised mechanisms" the direction of displacement of the armature is reversed by reversing the exciting current, and the magnitude of the displacement of the armature for a given current (*i.e.* the sensitiveness of the arrangement) may be considerably greater than in those arrangements in which such permanent magnetisation is absent.

Assuming Maxwell's law and neglecting leakage, the pull F on the armature is represented by $B^2A/8\pi$, where B is the induction in the iron core and in the armature due mainly to the field of the permanent magnet. If a small current flows in the coil, causing a proportional change dH in the magnetising force, the change in F (upon which the displacement of the armature depends) is given by

$$\frac{dF}{dH}=\frac{A}{4\pi}B\frac{dB}{dH},$$

i.e. the sensitiveness is proportional to the product of B and dB/dH. The coefficient dB/dH is determined by the form of a small

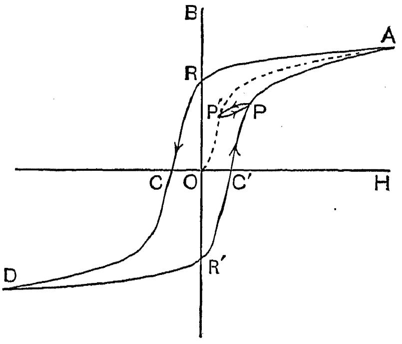


FIG. 9.

cycle (such as PP′ in *Fig.* 9) in the B, H diagram for the iron.

Measurements of dB/dH for small cycles of magnetisation superposed upon various constant values of B have been made by H. Hoffmann, who found that as the induction increases from zero the quantity BdB/dH first increases to a maximum and then diminishes, and that of the materials examined the one that possessed the highest permeability (an iron-silicon alloy) also showed the greatest maximum value of BdB/dH. In this connection reference may be made to some observations by K. W. Wagner, who found that the sensitiveness of a telephone was improved by making the cores and the diaphragm—the diaphragm here takes the place of the armature—of alloyed iron.

§ (6) ELECTROMAGNETS FOR INDUCING CURRENTS IN A SECONDARY CIRCUIT.—In transformers and induction coils an electromagnet of varying strength is employed to generate by induction electric currents in a secondary coil wound round the same core or round a part of the same magnetic circuit. The induced E.M.F. in the secondary coil, supposed to have n turns the flux through each of which is N lines, is according to Faraday's law of electromagnetic induction ndN/dt. In the transformer the flux is caused to change periodically by supplying alternating currents to the primary coil. In the induction coil the variations of flux arise from the electrical oscillations which take place in the circuits after the primary current is interrupted. In both cases electrical energy is supplied to the primary circuit, and it is desired to cause as much as possible of this energy to reappear in the secondary. It is therefore desirable to keep down as much as possible the dissipation of energy, arising from hysteresis and eddy currents, which always occurs in an iron core in which the flux is rapidly varying.

§ (7) HYSTERESIS.—If a piece of steel or other magnetic material is subject to a cyclical magnetising force, the magnetic induction corresponding to any value of the force is found to be greater as the force decreases through that value from higher values than it is as the force increases through the value from lower values. The induction appears to lag behind the force.[1] To this phenomenon Ewing gave the name of Hysteresis.

Fig. 9 shows a typical "hysteresis loop" representing a cycle of magnetic values between the limits of field and flux density represented by the points A and D. If the material be originally unmagnetised, then as the field is increased from zero to the point represented by A the induction increases along a curve from the origin lying between C′A and CA. As the force decreases from H at A to −H at D, the changes in the induction are represented by the curve ACD. If the

[1] See "Magnetic Measurements," § (1) (xii.), also "Magnetic Hysteresis."

force be now increased from $-H$ to H the induction is represented by DC′A, and these curves are repeated as the cycle continues.

OR is the residual magnetism, *i.e.* the flux-density remaining when the magnetising force has been reduced to zero, while OC is the coercive force, *i.e.* the force it is necessary to apply in a direction opposite to the original magnetising force in order to annul the residual force and demagnetise the specimen. It was shown by Warburg and by Ewing that the area of the loop representing any closed cycle of magnetisation, divided by 4π, represents the energy dissipated in the cycle per cubic centimetre of the material. The area of the loop increases with the induction limits between which it extends, being represented fairly well, when the induction alternates between moderate limits $\pm B$, by the expression proposed by Steinmetz, $hB^{1\cdot6}$. At very high inductions the index of B is smaller, at very low values greater, than 1·6.

The coefficient h is called the hysteretic constant. Its value varies with the material; in soft iron it may be 0·002, in cast iron five to eight times as great. It is very small in the iron-silicon alloys. In *Fig.* 10 (represent-

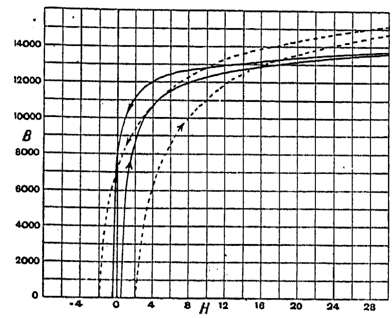


FIG. 10.

ing measurements by H. Hoffmann) the full line curve is part of a hysteresis loop for an annealed specimen of alloyed iron for which the hysteretic constant was 0·001. The broken line curve shows the hysteresis of a specimen of ordinary sheet steel for which $h = 0\cdot0024$. In a very pure specimen of alloyed sheet iron 0·5 mm. thick, containing 4·09 per cent of silicon and 0·07 per cent of carbon, annealed at 800° C. and 20 mm. of mercury pressure, Gumlich found the value 0·0006 for h. The hysteresis loss in "stalloy" sheet in very weak fields (H about 0·01 C.G.S.) was found by A. Campbell to be represented in ergs per cycle per cu. cm. by the expression $6\cdot6\ B^{2\cdot4} \times 10^{-6}$.

The eddy currents induced in the core by the changes of flux not only cause dissipation of energy but also, by their shielding action, prevent the flux changes from penetrating deeply into the core, the effective permeability of which is thus diminished. These effects are much reduced by constructing the core of thin sheets or wires parallel to the flux and insulated from one another. The theory of eddy currents in laminated or wire cores has been given by J. J. Thomson, who showed that the energy thereby dissipated per second per cubic centimetre of iron is proportional, when the quantity $\pi d\sqrt{n\mu/\sigma}$ is small, to $d^2n^2B^2/\sigma$, where d is the thickness of each sheet (or diameter of each wire), n is the frequency, B the maximum induction, and σ the specific resistance of the material. For ordinary low-frequency alternations, therefore, the eddy-current loss is smaller the higher the specific resistance of the iron. The fact that its specific resistance may be five or six times that of ordinary iron is another reason for the superiority of alloyed steel for transformer and other cores in which rapid changes of flux are taking place. In 4·8 per cent silicon steel plates 0·5 mm. thick, at a maximum induction of 10,000 C.G.S. and frequency 50, the combined hysteresis and eddy-current losses are of the order 0·5 watt per pound of the material. In ordinary sheet steel of good quality the losses in similar conditions are two or three times as great.

E. T. J.

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ELECTROMAGNETIC FORCE ON A CIRCUIT. The force is equal to $i(d\phi/dx)$ if ϕ is the magnetic flux through the circuit and i the current maintained constant during the change dx. See "Electromagnetic Theory," § (10).

ELECTROMAGNETIC THEORY

THE experimental laws of electromagnetic action are due to Faraday. It is important to establish the connection between these and the energy of the field in which the action is taking place, and to show how they follow from general mechanical principles. For this purpose an expression is required for the potential energy of the field; this we proceed to investigate.

§ (1) POTENTIAL DUE TO A SMALL MAGNET.—Consider a small magnet NOS (*Fig.* 1), of length l, with its centre at O, and having poles m and $-m$ at N and S respectively. Let P be a point at a distance r from O at which we wish to find the potential. Let OP make an angle θ with the axis SON of the magnet, and let r_1, r_2 be the distances of N and S from P. Describe a circle, with P as centre, PO as radius, cutting PN produced and PS in N′ and S′ respectively.

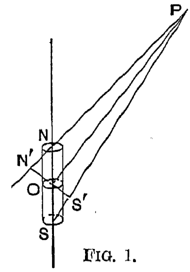


FIG. 1.

Then remembering that NS is small compared with OP, and that the angles at N′ and S′ are right angles, while N′NS and S′SN are each very approximately equal to θ, we have

$$r-r_1=\mathrm{OP}-\mathrm{ON}=\mathrm{NN'}=\frac{l}{2}\cos\theta,$$

$$r_2-r=\mathrm{OS}-\mathrm{OP}=\mathrm{SS'}=\frac{l}{2}\cos\theta.$$

Thus

$$r_1=r-\frac{l}{2}\cos\theta=r\left(1-\frac{l}{2r}\cos\theta\right),$$

$$r_2=r+\frac{l}{2}\cos\theta=r\left(1+\frac{l}{2r}\cos\theta\right).$$

Then if V be the magnetic potential at P due to the small magnet,

$$\begin{aligned}V&=\frac{m}{r_1}-\frac{m}{r_2}=\frac{m}{r}\left(\frac{1}{1-(l/2r)\cos\theta}-\frac{1}{1+(l/2r)\cos\theta}\right)\\&=\frac{ml\cos\theta}{r^2}\left(\frac{1}{1-(l^2/4r^2)\cos^2\theta}\right)\\&=\frac{\mathrm{M}\cos\theta}{r^2}, \qquad (1)\end{aligned}$$

if M be the magnetic moment of the magnet, for we may neglect $l^2\cos^2\theta/4r^2$ in the denominator in comparison with unity.

Now let δS be the area of a section of the small magnet at right angles to its axis, and imagine the poles to be due to magnetism distributed over the ends of the magnet with density σ and $-\sigma$ respectively.

Then $$m=\sigma\delta S$$

and $$V=\sigma l\frac{\delta S\cos\theta}{r^2},$$

and if $\delta\Omega$ be the small solid angle subtended at P by δS we know that

$$\delta\Omega=\frac{\delta S\cos\theta}{r^2}.$$

Hence $$V=\sigma l\delta\Omega. \qquad (2)$$

§ (2) POTENTIAL OF A MAGNETIC SHELL.—Imagine now a thin sheet of magnetic material of any form magnetised normally in such a way that the surface density of magnetisation σ is always inversely proportional to its thick-

ness, so that σl is constant over the sheet. Denote this product by ϕ.

We may consider the sheet as made up of a large number of small magnets such as NS (*Fig.* 2) each of constant moment ϕ per unit of area of the sheet, and if $\delta\Omega$ be the angle which the north end of one of these subtends at P, the potential δV due to this element of the shell is given by

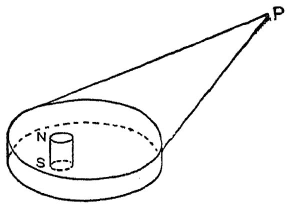


FIG. 2.

$$\delta V = \phi\delta\Omega.$$

But the potential due to the whole shell is the sum of those due to the elements. Thus for the whole shell

$$V = \phi\Omega, \quad . \quad . \quad . \quad . \quad (3)$$

where Ω is the solid angle subtended by the positive side of the shell at P.

Now imagine a magnetic pole m_1 at P, then the mutual potential energy of the shell and the pole is $\phi m_1\Omega$. Lines of magnetic induction issue from the pole, and at all points on a sphere of unit radius with its centre at the pole the induction is m_1. But the induction is measured by the number of lines which cross unit area on the surface of the sphere, thus there must be m_1 lines to each unit of area, and since there are 4π units of area on a sphere of unit radius the number of lines issuing from the pole m_1 is $4\pi m_1$. These lines are uniformly distributed, hence of the $4\pi m_1$, a number equal to $m_1\Omega$ are included within the solid angle Ω and fall within the area of the shell, passing through it in the negative direction from its north to its south side. Hence the mutual potential energy of the pole and the shell depends on the product of the strength of the shell and the number of lines of induction which cut the shell.

The potential energy is positive if the pole is on the north side of the shell; the lines of induction due to the pole cut the shell from north to south; the positive direction through the shell is from south to north, and the number of lines cutting the shell from south to north is equal in magnitude but opposite in sign to those which cut it from north to south. Hence the mutual potential energy of the pole and the shell is equal to minus the product of the strength of the shell and the number of lines of induction which cut the shell from south to north.

§ (3) Potential Energy of a Shell in a Magnetic Field.—Now consider, instead of a simple pole, any system of magnetised matter near the shell. We can treat this as made up of a series of magnetic poles—positive and negative—distributed according to definite laws, and the result just reached is true for each of these poles.

The mutual potential energy is the sum of that due to the individual poles, and the total number of lines of induction cutting the shell is also the sum of those due to the individual poles.

Hence we may write

$$\text{Mutual potential energy of shell and field} = -N\phi, \quad (4)$$

where ϕ is the strength of the shell and N the number of lines of induction due to the system which cut the shell from south to north.

Again let the second system be another shell of strength ϕ', and let N' lines of induction due to the first shell cut it. The mutual potential energy is $-N'\phi'$. Thus for the mutual potential energy of two shells we have the two expressions $-N\phi$ and $-N'\phi'$, and these must be equal. This condition is satisfied if $N = -M\phi'$ and $N' = -M\phi$, where M depends only on the relative position and form of the two shells, for then we have

$$\text{Mutual potential energy} = +M \, . \, \phi \, . \, \phi'. \quad (5)$$

M is known as the coefficient of mutual induction of the two magnetic shells.

§ (4) Equivalence of a Magnetic Shell and a Current.—Again Ampère showed [1] —following Oersted's discovery of the magnetic action of a current—that the magnetic field due to any current of strength i flowing in a small circuit was the same as that due to a small magnet with its axis normal to the plane of the circuit, provided the moment of the magnet is equal to $i\delta S$, where dS is the area of the small circuit.

If we look upon the magnet as a portion of a shell of strength ϕ, bounded by the circuit carrying the current, then its moment is $\phi\delta S$. Hence $\phi = i$.

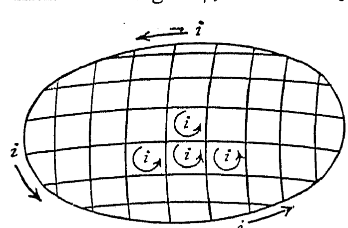


FIG. 3.

Consider now a finite shell as in *Fig.* 3 with its north side uppermost.

Draw a series of lines across the shell,

[1] The real proof of this lies in the fact that it leads to results which are fully verified by experiment.

dividing it into a number of elementary areas, and imagine a current i to circulate as shown round the boundaries of each of these elements. The potential due to each of these current elements is, we have just seen, equal to that of the corresponding shell element if ϕ, the strength of the shell, is equal to i. But a little consideration shows that except at the boundary of the whole shell there are two equal and opposite currents in each of the elementary arcs on its surface. Thus except at the boundary the effects of currents in contiguous elements are mutually destructive. Moreover, the elements of current in the boundary constitute a continuous current i circulating round the boundary of the shell. Thus the magnetic potential due to a shell of strength ϕ and a current of strength i circulating round its boundary are equal provided $i=\phi$.

Hence the mutual potential energy of two circuits carrying currents i, i' may be written Mii', where M depends solely on the geometrical relations of the circuits, and is known as the coefficient of mutual induction between them.

§ (5) Electromagnetic Energy of the Field.—Again, if during a time dt the current i increases by an amount di while i' remains constant, the mutual potential energy increases by $Mi'(di/dt)$, and Mi' is the number of lines of induction due to the second circuit which thread the first.

But lines of induction due to the first circuit also thread that circuit, and these are proportional to the current in that circuit. Denoting them by Li, we see that the rate of increase in energy due to an increase of current in the circuit is $Li(di/dt)$, and the total energy due to the growth of i from zero to its final value is given by $\int Li(di/dt)$ or $\frac{1}{2}Li^2$.

The quantity L is known as the coefficient of self-induction of the circuit.

Thus if we have two circuits of self-inductions L and L′ and mutual induction M carrying currents i, i', the electromagnetic energy of the system is

$$\tfrac{1}{2}Li^2 + Mii' + \tfrac{1}{2}L'i'^2. \qquad (6)$$

We have thus established the identity of the magnetic fields due to a magnetic shell and a current; it should be noted, however, that this identity does not extend to the interior of the shell.

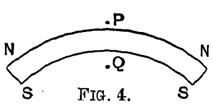


Fig. 4.

§ (6) Work done in circling a Current.—Again let P and Q be two points close together but on opposite sides of the shell shown in section in *Fig.* 4, and let Ω be the solid angle which the positive side of the shell subtends at P. Then the angle subtended by the shell at Q is $4\pi-\Omega$. If ϕ be the strength of the shell the potential at P is $\phi\Omega$, and that at Q is $-\phi(4\pi-\Omega)$.

Hence the work done in taking a unit pole by any path not cutting the shell from Q to P is $\phi\Omega+\phi(4\pi-\Omega)$ or $4\pi\phi$.

If we replace the shell by a current i circulating round the boundary the work done in making a complete circuit round the current is $4\pi i$; if the pole compass the current n times we find that work done in circling n times round a current i

$$=4n\pi i. \qquad (7)$$

In this expression the current is measured in C.G.S. units if it be equal to I amperes.

Then $\qquad I=10i$

and the work is

$$\frac{4\pi}{10}nI=\frac{4 \text{ Ampere turns}}{10}. \text{ Units of work.} \quad (8)$$

§ (8) Relation between the Direction of the Current and the Magnetisation of the Shell.—We can determine this relation thus:

Let the current flow (*Fig.* 5) in the anticlockwise direction round the circuit supposed to lie in a horizontal plane. Then, experiment shows that a north pole when near the plane of the circuit will tend to move upwards when within the area of the circuit, downwards if outside that area; the lines of induction due to the current run downwards outside the circuit, upwards within it; the upper face of the equivalent shell is a north face, while the lower face is south.

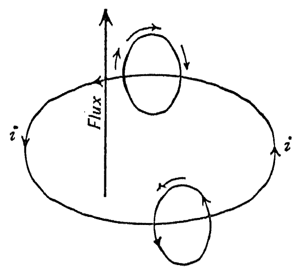


Fig. 5.

Again, as we have seen, the mutual potential energy of a circuit carrying a current i and a pole m is $-m\Omega i$, that of the circuit when in a magnetic field is $-i\Phi$, where Φ is the total flux passing through the circuit in the direction from south to north. Now it is a general rule that any system of bodies free to move under their mutual reactions set themselves so as to make the potential energy of the system a minimum; this will apply to the circuit, which will if free set itself so that $-i\Phi$ is a minimum, *i.e.* so that Φ is a maximum, and the circuit will then embrace the maximum number of lines of induction which will traverse it from its south face to its north.

The direction of the field and the direction of the current will be related as shown in *Fig.* 5.

§ (9) ELECTROMAGNETIC FORCE ON AN ELEMENT OF THE CIRCUIT.—Moreover, if we imagine the conductor to be flexible, it will alter in shape so as to include as many lines of force as possible; if, for example, the field be uniform the conductor would take the form of a circle, for with a given perimeter a circle includes the maximum area.

We can find the force on an element of the wire ds in the following way:

Consider an element PQ (ds), *Fig.* 6, and let it be displaced a distance PP′ ($=dx$) parallel to itself in a direction making an angle α with PQ—we may imagine PP′, QQ′ to be conducting rails on which P and Q slide, thus maintaining the circuit complete—Let the direction of B, the magnetic field at PQ, make an angle θ with the normal to PP′Q′R.

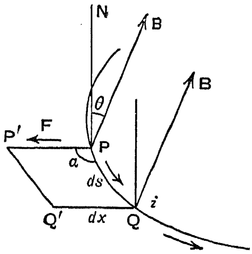


FIG. 6.

The area of PP′Q′Q is $dxds$ sin α, and the total induction through this area is

$$B \cos\theta \sin\alpha dxds.$$

The increase in potential energy is then

$$-Bi\cos\theta\sin\alpha dxds.$$

But if F is the force on ds in the direction PP′ then Fdx is the work done, and this must be equal to the decrease in potential energy.

Hence $Fdx = B\cos\theta\sin\alpha \times idxds$

or $$F = B\cos\theta\sin\alpha \times ids. \quad . \quad . \quad (9)$$

F is clearly a maximum if $\theta = 0$ and $\alpha = \pi/2$, for then $\cos\theta\sin\alpha$ is unity. Thus the resultant force $\overline{F}$ is at right angles both to the element of current and also to the magnetic field, and we then have

$$\overline{F} = Bids, \quad . \quad . \quad . \quad (10)$$

or on each element of the current there is a force per unit of length equal to the product of the current and the magnetic induction.

The relations between the directions of the three quantities involved, the current, the magnetic field, and the force, are as shown in *Fig.* 7.

These relations are perhaps most easily remembered thus: Let the direction of the magnetic field B be vertically upwards, and let the current i be flowing in a horizontal wire at right angles to the field; on one side of the wire the field due to the current is in the same direction as B, on the other side it is opposite to that of B; the effect of the current is to strengthen the field on one side and to weaken it on the other. Then the electromagnetic force is at right angles both to the field and the current and tends to move the wire from the side where the field is strong to that on which it is weak.

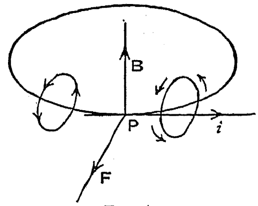


FIG. 7.

§ (10) ELECTROMAGNETIC FORCE ON A CIRCUIT.—We can find an expression for the electromagnetic force on a complete circuit carrying a current thus: We have seen that we can represent the potential energy of the circuit in a magnetic field by the expression $-\Phi i$. Now let some displacement of the circuit denoted by δx take place, and let X be the force resisting this displacement, then the work done is $X\delta x$ and the increase in potential energy is $-\delta(\Phi i)$. If the work is done at the expense of the energy we must have

$$X\delta x - \delta(\Phi i) = 0$$

or $$X = \frac{d(\Phi i)}{dx} = i\frac{d\Phi}{dx}, \quad . \quad . \quad (11)$$

if i is maintained constant.

§ (11) ELECTROMAGNETIC INDUCTION.—So far we have considered the mechanical forces on a conductor in a magnetic field; electromotive forces also are set up if the circuit is moved in the field or the amount of magnetic induction linked with it made to vary in any way.

Let the motion be such that the flux through the circuit is increased by $d\Phi$; the work done by the electromagnetic forces is $i\delta\Phi$, and if this occurs in time δt the energy used in heating the circuit of resistance R is $Ri^2\delta t$; both these supplies of energy come from the battery or other source of the current, and if E be the E.M.F. the whole amount supplied is $Ei\delta t$.

Thus $$Ei\delta t = Ri^2\delta t + i\delta\Phi,$$

and ultimately we find

$$i = \frac{1}{R}\left\{E - \frac{d\Phi}{dt}\right\}. \quad . \quad . \quad (12)$$

Thus whenever the flux through the circuit varies an electromotive force is set up which is measured by the rate of decrease of the flux.

If the current, the induction, and the electro-

magnetic force be related as in *Fig.* 7 the induced E.M.F. will be as shown in the opposite direction to the primary current as in *Fig.* 8.

If, for example, we consider the case when the field is due to a current i' in a second coil

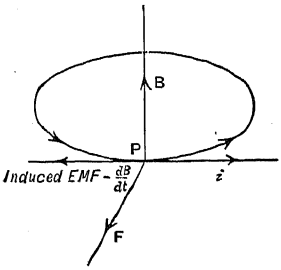


FIG. 8.

we have for the value of Φ the quantity $Li+Mi'$, and if L and M can be treated as constants

$$\frac{d\Phi}{dt}=L\frac{di}{dt}+M\frac{di'}{dt},$$

so that the equation connecting the electromotive force and the current is

$$L\frac{di}{dt}+M\frac{di'}{dt}+Ri=E. \quad . \quad . \quad (13)$$

We must, of course, know how both E and i' vary with the time before we can solve this; if we have a second circuit in which i' is flowing under an E.M.F. E′ we have

$$L'\frac{di'}{dt}+M\frac{di}{dt}+R'i'=E', \quad . \quad (14)$$

and if E and E′ are both known functions of the time we can solve these two equations.

§ (12) FARADAY'S LAWS OF ELECTROMAGNETISM.[1]—The two results we have obtained, giving respectively the electromagnetic force on a conductor and the electromotive force set up in it when the flux linked with it is varied, contain the theory of Faraday's discovery of electromagnetic induction, the laws to which they lead may be put into various forms; they are made use of in the design of electric generators and motors in transformers and in all kinds of electric machinery. We may state the laws thus:

(i.) A closed circuit carrying a current when free to move in a field of magnetic flux sets itself so that the number of lines of magnetic induction through the circuit is the greatest possible.

From this it follows that—

(i.*a*) A conductor carrying a current i placed in a field of magnetic induction B in a direction at right angles to that of the field experiences a force of Bi dynes per unit length in a direction at right angles to both the current and the field.

(ii.) If there is any variation in the magnetic flux linked with a closed circuit an E.M.F. is induced round the circuit which is equal to the rate of decrease of the flux.

And this leads to—

(ii.*a*) A conductor moving in a magnetic field with velocity v at right angles to the lines of magnetic induction of the field has induced in it an E.M.F. equal to $-Bv$ per unit of length.

The various quantities occurring in these statements must, of course, be measured in some consistent system of units,[2] *e.g.* either the C.G.S. system or the ohm-ampere-volt system of practical units.

§ (13) THE MAGNETIC FORCE WITHIN A SOLENOID.—Consider now a circuit like that shown in *Fig.* 9 in which the conductor carrying a current i is wound uniformly round the surface of a ring in such a way as to form a spiral enclosing the whole ring, and let the axis of the ring be a circle of circumference l; let there be n turns of wire on the ring. Then, as we have just seen, the work done in carrying a unit pole once round the axis of the ring, and thus threading all the coils of the solenoid, is given by the expression $4\pi ni$. But it is clear from the symmetry that the magnetic intensity H is the same at all points of the axis of the ring, while the length of the path is l; thus the work is also equal to Hl.

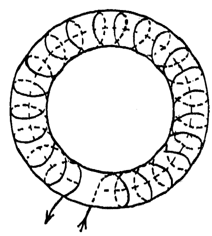


FIG. 9.

Hence $$Hl=4\pi ni$$

and $$H=4\pi\frac{ni}{l}. \quad . \quad . \quad . \quad (15)$$

But n/l is the number of turns per unit length of the axis, and ni/l is the number of current turns per unit of length.

Hence $H=4\pi\times$number of current turns per unit length of the axis of the coil.

In this expression the current is measured in C.G.S. units; if we measure it in amperes, then since 1 ampere$=10^{-1}$ C.G.S. units, we have for the value of H in terms of the Gauss

$$H=\frac{4\pi}{10}\times\text{ampere turns per unit length.} \quad (16)$$

In this form the result is strictly true.

Now let us suppose the ring to be of very large diameter, its cross-section remaining unaltered, so large that we may treat a

[1] See "Dynamo Electric Machinery," § (1); "Transformers," § (7).

[2] See "Units of Electrical Measurement," §§ (2), (21).

limited portion of the axis as though it were straight. The field near the centre of this straight portion will be mainly due to the coils in its immediate neighbourhood and will not be seriously modified if we suppose the more distant parts of the ring removed. If we do this we have a straight coil of wire or solenoid, the length of which is considerable in proportion to the radius of its cross-section. The magnetic intensity within the central portion of such a coil is then still given by the same expression

$$H=\frac{4\pi}{10}\text{ ampere turns per unit length.} \quad (17)$$

Electrometer: an instrument for measuring differences of electric potential. See "Alternating Current Instruments," § (18).

Use of, for the measurement of small currents at radio frequencies. See "Radio-frequency Measurements," § (18) (vi.).

Use of as a Wattmeter. See "Alternating Current Instruments," § (20).

Vibration, an alternating voltage-detecting instrument suitable in cases where the circuits have high impedances. See "Inductance, The Measurement of," §§ (36)-(37).

Electromotive Force. The energy required to convey a unit of positive electricity from one point to another measures the electromotive force between the two.

In electrostatics E.M.F. is measured by the potential difference between the points.

When a steady current i is flowing in a conductor the E.M.F. is measured by the product of its strength and the resistance R of the conductor,

or $$E=Ri.$$

If the current is variable an E.M.F. is produced equal to $-L\,di/dt$, L being the coefficient of self induction of the circuit in which the current i is flowing, while in a neighbouring conductor there is experienced an E.M.F. equal to $-M(di/dt)$, where M is the coefficient of mutual inductance between the two conductors.

The practical unit of E.M.F. is the volt. One volt exists in a conductor when one watt is expended in the passage of one ampere. See "Units of Electrical Measurement," §§ (7), (8), (21); "Electrical Measurement, Systems of," § (37).

Electromotive Force, Thermal, in an electric cell, and its effect upon the E.M.F. of the cell. See "Batteries, Primary," § (14).

Electromotive Force of a Voltaic Cell, Seat of. See "Batteries, Primary," § (13).

Electron: the smallest known charge of negative electricity, equal to $4{\cdot}7\times10^{-10}$ electrostatic units or $1{\cdot}59\times10^{-20}$ electromagnetic units. It has a mass of $9{\cdot}0\times10^{-28}$ grammes and a radius of about $1{\cdot}9\times10^{-13}$ cm. See "Electrons and the Discharge Tube," §§ (25), (26).

Bucherer's experimental verification of Lorentz formula for electromagnetic mass. See *ibid.* § (19).

Charge carried by, and relation with charge on monovalent ion in electrolysis. See *ibid.* § (14).

Determination of charge on. See *ibid.* § (20).

Determination of e/m of an, from various sources of emission. See *ibid.* § (17).

Discharge in high vacua. See "Photoelectricity," § (1).

Electrical origin of mass of. See "Electrons and the Discharge Tube," § (19).

Electromagnetic mass of. See *ibid.* § (18).

Emission from Hot Bodies, latent heat of. See "Thermionics," § (5) (iii.).

Emitted from Hot Bodies, Kinetic Energy of: latent heat of emission and absorption of electrons. See *ibid.* § (5).

Emitted by metals under action of light. See "Photoelectricity," § (1).

Identification with cathode rays. See "Electrons and the Discharge Tube," § (14).

Kaufmann's experiments on variation of mass with velocity of. See *ibid.* § (19).

Method of calculating mass and radius of. See *ibid.* § (26).

Also a Magneton, endowed with specific magnetic as well as electrostatic properties. See "Magnetism, Modern Theories of," § (3) (i.).

Nature of. See "Electrons and the Discharge Tube," § (18).

Production by X-rays, positive rays, and α-rays. See *ibid.* § (15).

Relation to positive charge on nucleus. See *ibid.* § (29).

Sources of production other than in discharge through gases. See *ibid.* § (15).

Total magnetic energy associated with. See *ibid.* § (18).

Ultra-violet light and emission of. See *ibid.* § (15).

ELECTRON THEORY AND SPECTRUM ANALYSIS

In recent years the results of experimental research on the properties of electrons have accumulated with startling rapidity. As knowledge grows, the importance of the part played by the electron in the mechanics of the world becomes ever clearer.

It helps to a right appreciation of the position

as regards the electron if we observe its strong resemblance to the older state of things when first the atomic theory of matter was clearly defined. Just as chemistry has grown and prospered on its recognition of the unit of matter, so electrical science has already begun a new life, and to all seeming a most vigorous one, based on the understanding of nature's unit of electricity. There are many different atoms of matter; nearly a hundred are distinguishable by their different chemical reactions; but the number of different kinds of electrical atoms is very much more limited. We have for some years been clear as to the existence of the electron, nature's unit of negative electricity. More recently the work of Rutherford and of Aston indicates that the nucleus of the hydrogen atom is to be regarded as the positive counterpart.

The first suggestion of the atomic character of electric charge came, it is well known, from observation of the laws of electrolysis. Since the movement of atoms or atom clusters or ions across the electrolytic cell was accompanied by a simultaneous transfer of electricity, in which each ion, of whatever nature, bore always the same charge or at least a simple multiple of it, there was a clear indication that this division of electricity into parcels of constant magnitude implied the existence of some natural unit charge. No progress, however, was made or could be made so long as the charge could only be observed as an attachment to an ion: it was not even clear that it could ever have a separate existence. In the long series of researches which finally led to the isolation of the electron and the determination of its properties, there were certain that marked definite stages in the forward movement. Crookes examined the electric discharge in bulbs exhausted to a high degree by the new air pumps which he had succeeded in making; and he observed the so-called cathode rays, streaming away from the negative electrode. He showed that they possessed the properties to be expected from a stream of particles projected across the bulb, and carrying negative electricity with them; for, on the one hand, they could heat up bodies on which they fell, and on the other, they were deflected in crossing a magnetic field. Crookes spoke of a fourth state of matter and defended his view against the opposing hypothesis, held largely on the Continent, that the stream consisted of electromagnetic waves in some form or other. Hertz showed that the rays could pass through thin sheets of matter such as aluminium leaf, and Lenard took advantage of this to coax them outside the bulb and display their effects in the air outside.

In the later years of the last century came the great experiments of Wiechert, J. J. Thomson, and many other well-known observers, who weighed the electron and measured its charge, and showed that there was only the one electron, though it was to be found everywhere and in every body. Since then, the measurements of these quantities have been repeated many times with increasing skill and understanding. They have reached their present high-water mark perhaps in the experiments of Millikan at Chicago, who gives for the value of the charge in electromagnetic units $e=1{\cdot}591\times10^{-20}$, the mass being $0{\cdot}900\times10^{-27}$ gramme or 1/1850 of the mass of the hydrogen atom.

So we arrive finally at an accurate comparison of these unique and fundamental units of nature with the units which we ourselves have chosen for our convenience, and without, of course, any consideration of the former. We infer from experiments, such as those of Kaufmann and Bucherer, that the energy of the moving electron may be considered to exist wholly in the form of electro-magnetic energy such as is necessarily present when an electrical charge is in motion; and that its mass is in this way perfectly accounted for. But this conclusion sets a limit to the size of the electron and we must assume that its radius, if its form is spherical, is very small compared with the radius of any atom. Also as the velocity of the electron approaches that of light, its mass increases; imperceptibly at first, but at the end very rapidly.

Why, we may well ask, have these measurements of charge and mass never been made before? The electron is everywhere: the transfer of electricity from place to place consists always in the transfer of electrons. The electric current is a hurrying stream of electrons: all our electrical machinery concerns itself with setting them in motion, giving them energy, and again withdrawing it. In the processes of electrolysis the electrons are handed to and fro. Everywhere they fill the stage: why have we not noticed hitherto their qualities, which so far can be expressed so simply?

The answer is that we have never, until recently, been able to make them move fast enough in spaces sufficiently empty of air or other gases. It is only when an electron has a sufficient speed that it can escape absorption in the atoms which it must be continually meeting. Unless an electron has a speed exceeding about a three-hundredth of the velocity of light, that is to say, such a speed as it acquires in falling through a potential of a few volts, it sticks to the next atom it runs up against: even with 10 times that speed it can only move a fraction of a millimetre through air at ordinary pressure before it loses its velocity and, therefore, its power

of going through the atoms. When Crookes first saw the cathode-ray stream in full course, it was because he had reduced the number of gas molecules in his bulb to such an extent that an electron could fly in a straight line from end to end of the bulb without going through more than a hundred atoms or so, and the induction coil had given it quite enough speed to do that without turning out of its course, no matter what sort of atoms they were. Incidentally, since atoms can be traversed in this way, we naturally think of an atom as a very empty affair.

Electrons flying still faster than in the discharge tube are found to constitute a part of the radiation from radioactive substances. Some of the β-rays have velocities nearly equal to that of light, and can pass through millions of atoms before their energy is spent. In open air a β-ray may have a course of metres in length, though it is generally broken, by encounters with traversed atoms, into a path full of corners and irregularities.

It is speed which gives separate existence to the moving electron, and speed which also betrays its presence to us. For, on its way, the electron here and there chips away another electron from an atom which it is crossing and leaves behind it a separation of electricities which may subsequently influence chemical action as in the case of the phosphorescent screen or photographic plate, or provide a current for the ionisation chamber. We do not know exactly how this removal of electrons is effected, nor why some atoms part with electrons more easily than others so that the flying electron loses less energy as it goes through: there is much that is obscure in the whole process. But it gives us a ready means of observation, without which indeed our knowledge of the electron would be far less than it is.

These electrons which are so made manifest by speed form but a minute fraction of the whole number in existence. They are to be found in every body, and in every atom of every body. They form one of the elements of construction of the atom; and it is one of the most immediate aims of present research to find in what way they are built into atomic structure. In every atom there are certain electrons of which one can be removed at the cost of an amount of energy of the order of 10^{-11} ergs. The potential through which an electron must fall so that it acquires this energy is of the order of a few volts. There are other electrons within the atom which are intrinsically far more difficult to remove. On the other hand, some atoms, for example those of a metal in the solid or liquid condition, have each one or more electrons which are little more than hangers on, and are indeed removed with very little trouble. A block of pure metal is full of such loosely bound electrons, so that if an electric potential difference is maintained across the block an electron flow or electric current is produced. The metal "conducts."

At sufficiently high temperatures all bodies become conductors; we must imagine that the violent thermal agitation shakes electrons free from their ties to the atoms even when at low temperature the bonds ordinarily remain unbroken. At high temperature, too, the electrons acquire high velocities as they move to and fro with their proper share of heat energy. At the surface of the hot body the electrons may break away; and hence the "thermionic emission" investigated by O. W. Richardson. So copious is this supply of electrons at the surface of a hot body that if the latter is made negative in potential relative to its surroundings there is a current discharge which may sometimes be measurable in amperes.

There is the most remarkable connection between moving electrons and electromagnetic waves. The one, it seems, can always call up the other; and the action obeys certain precise numerical laws.

Let us take as an example the production of X-rays in a Coolidge bulb. A plentiful supply of electrons is provided at the cathode by heating a fine spiral of tungsten wire to a high temperature. A high potential difference between cathode and target is provided by some appropriate means, and the electrons are hurled at the target, each possessing an amount of energy equal to the product of the electron charge and the applied potential. Where the electrons strike, some of their energy is converted into electromagnetic waves of very high frequency, the so-called X-rays. Let the energy supplied to each electron be measured—not an easy matter with the usual arrangements, but very easily done if, as in certain experiments of Duane and Hunt at Harvard University, the potential is derived from a great storage battery of 40,000 volts. Further, let the X-ray radiation that issues from the target be analysed by the X-ray spectrometer. It is found that the frequencies of the emitted rays may have a wide range of values, but that the upper limit of the frequencies is always proportional to the energy of the electron and, therefore, to the potential imposed on the tube. The ratio remains the same no matter what is the intensity of the electron discharge, and no matter what the nature of the target. This ratio of electron energy to maximum frequency is a number which has turned up in previous cases where the emission of radiation energy has been measured: it is known as Planck's constant and is denoted by "h." Its value is $6{\cdot}55 \times 10^{-27}$. Although the constant has

been met with before, there is probably no instance where the transformation of energy which it governs is so simply displayed or so easily measured as in the case just described.

In certain measurements made by Duane and illustrated in *Fig.* 1, the X-ray spectrometer was set to observe the presence of a

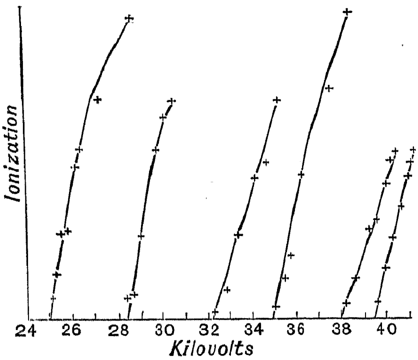


FIG. 1.

Each curve shows the rise of intensity of X-rays of one particular wave-length as the volts on the bulb are increased. The wave-lengths are (left to right) 0·488, 0·424, 0·377, 0·345, 0·318, 0·308, all in Ångström units (10^{-8} cm.). (From Duane and Hunt, *Physical Review*, 1915, p. 166.)

certain frequency as soon as it appeared. The potential on the tube was then increased by degrees. The rays of the given frequency appeared as soon as the energy supplied to the electron was equal to the frequency multiplied by h. As the potential was increased still further these rays increased in intensity, as the *figure* shows. The product of a frequency and the constant "h" is known as the quantum energy of that frequency.

It is to be observed that the production of X-rays is no aggregate of individual efforts by separate electrons: each electron produces its own train of X-rays when it strikes the target. There is no sign of any combined action, as indeed is evident from the fact that the intensity of the cathode-ray stream is without influence on the frequencies of the X-rays produced.

The crucial point is that when the energy of an electron is handed over in whole or in part, the frequency of the X-ray waves that take over the energy is determined by the quantity of energy handed over. This explains why there is a limit to the frequency of the X-rays: it is because there are some electrons, though only a fraction of the whole number, which give up all their energy to the formation of X-rays at the moment of striking, before they have lost energy in collisions. The rest of the rays, all those which have lesser frequencies, will come from electrons that have lost speed in this way, or possibly transfer only part of their energy. The atom of the target is playing the part of a transformer, and does not determine the frequency, so far as these effects are concerned.

This remarkable effect is reciprocal. Just as the swiftly moving electrons excite X-rays, so X-rays when they strike any substance lose their energy, which now appears as the energy of moving electrons. And, again, the same variation is found in the result and the same limit to that variation. Among the electrons so set in motion, if examined as soon as possible after their motion has begun, occurs every variety of energy-content up to a certain critical value which is equal to the frequency of the X-rays multiplied by the same constant h. It is to be observed that it is impossible to measure all the electron velocities as soon as they exist, because some of the motions begin in the body of the substance into which the X-rays have penetrated, and speed is lost on the way out. Again, therefore, there is nothing against the hypothesis that the energy of every electron set going by waves of given frequency is originally the same, and is determined by the standard condition already given.

Not only in the case of X-rays are these effects observed, but also in the case of light. The only difference is that the frequencies of light vibrations are some 10,000 times less than those of X-rays, and the electron energies are correspondingly smaller. When the light waves produce the electrons we have what is known as the photo-electric effect. The production of light by electrons has been much studied recently in experiments to find "resonance-potentials," that is to say, the magnitudes of potentials which must act on electrons so as to give them enough energy to excite certain radiation from atoms on which they fall.

Exactly how this strange transfer of energy from one form to another takes place is not known: the question is full of puzzles. The magnitudes involved are hard to realise; it helps to alter their scale of presentment. Suppose that the target of the X-ray bulb were magnified in size until it was as great as the moon's disc, that is to say, about a hundred million times. The atoms would then be spheres a centimetre or so in diameter. But the electrons would still be invisible to the naked eye. The distance from earth to moon would correspond roughly to the distance that ordinarily separates the bulb from an observer or his apparatus. The enlarged electrons are now shot at the moon with a certain velocity: in every second each square yard or square foot or square inch, it does not matter which, receives an electron. A radiation now starts away from the moon which immediately manifests itself (there is

no other manifestation whatever) by causing electrons to spring out of bodies on which it falls. They leap out from the earth, here one and there one; from each square mile of sea or land, one a second or thereabouts. They may have various speeds; but none exceed, though some may just reach, the velocity of the original electrons that were fired at the moon. That, reduced again to normal size, is the process that goes on in and about the X-ray bulb: which is part of a universal natural process going on wherever radiation, electron or wave, falls on matter, and which is clearly one of the most important and most fundamental operations in the material world.

Keeping these results in mind it is possible to appreciate a very remarkable development of electron theory which has been made in the last few years. Spectrum analysis has long been occupied with the extraordinary complications of the light radiation emitted by the various atoms. As a result it appears that the frequencies of the lines in a spectrum often display curious and exact numerical relations, in the form generally involving differences of frequencies of similar lines or groups of lines. For instance, the famous Balmer equation:

$$\text{Frequency} = \nu = \mathrm{N}\left(\frac{1}{n^2_1} - \frac{1}{n^2_2}\right),$$

where $\mathrm{N} = 3{\cdot}290 \times 10^{15}$,

gives the frequencies of series of lines in the hydrogen spectrum. When n_1 is put equal to 2, and n_2 to 3, 4, 5 in succession, the series of values for ν represent the frequencies of the lines in the visible spectrum. If $n_1 = 3$ and $n_2 = 4, 5, 6 \ldots$, in succession, the frequencies are those of lines in the infra-red (Paschen), and if $n_1 = 1$, $n_2 = 2, 3, 4 \ldots$, the frequencies correspond to those recently shown by Lyman to exist in the ultra-violet.

Now there is nothing in the older conception of the origin of radiation within the atom to give a clue as to why differences of frequencies should come into these empirical though most useful formulae. Our imagination has pictured to us vibrating systems, mechanical or electric, and waves arising therefrom. But no connection between masses or electricities gives in any simple way equations involving the addition or subtraction of frequencies. It is reasonable, therefore, to abandon preconceptions as to the origin of those lines which are found in the light spectrum and to suppose that here also they arise in the same fashion as in the cases considered above. Suppose that the energy of an emission of radiation is derived from the energy of an electron. It may be the only way in which radiation ever does arise, but it is not necessary to suppose so much at present. It is sufficient to carry into the atom the whole process which in X-rays and the photo-electric effect are observed to take place in part outside. Suppose that within the atom there are certain positions or conditions in which electrons *may* be, each postulating a certain energy associated with the electron; and suppose that sometimes an electron slips from one position to another of lower energy, and that the difference in energies is transformed into wave radiation according to the same law as before, *i.e.* energy transferred $= h \times$ frequency. Let the energy in these states be $\mathrm{N}h/1^2$; $\mathrm{N}h/2^2$; $\mathrm{N}h/3^2$; etc., and so on. Then all the series yielded by the Balmer formula are accounted for at the same time.

What may these states be? Why not, as Bohr suggests, so many different orbits in which electrons can move round the central positive nucleus in the atom, the nucleus whose existence has been established by Rutherford? At one time, if the existence of these orbits had been assumed, they would have been connected with the direct emission of radiation; and the frequency of that radiation with the number of revolutions in a second. But now it is assumed these orbits persist without radiation, and that radiation arises when the electron steps from one orbit to another; moreover, the frequency of the issuing radiation is determined by the simple rule: frequency is equal to change of electron energy divided by h. There is no more in this than supposing a process to exist in one place which is already known to exist in another.

It is a very remarkable fact[1] that the number N is equal to $2\pi^2 me^4/h^3$ within small errors of experiment. Spectrum measurements show that N is equal to $3{\cdot}29033 \times 10^{15}$ and $2\pi^2 me^4/h^3$ is equal, taking the most recent determination of m, e and h, to $3{\cdot}289 \times 10^{15}$.

Imagine an electron revolving in a circle about the positive nucleus of the hydrogen atom with kinetic energy $2\pi^2 me^4/n^2h^2 = \mathrm{N}h/n^2$. Its velocity, v, $= 2\pi e^2/hn$; the radius, r, of the circular orbit is found by putting $mv^2/r = e^2/r^2$, and is equal to $n^2h^2/4\pi^2 me^2$. The angular momentum is $mvr = nh/2\pi$. If the electron changes its orbit from $n = n_2$ to $n = n_1$, where n_2 is greater than n_1, its kinetic energy in the new orbit is *greater* than in the old by $\mathrm{N}h(1/n^2_1 - 1/n^2_2)$. But an amount of potential energy has been set free equal to $e^2(1/r_1 - 1/r_2)$, and this is equal to twice the change in kinetic energy, as is easily seen by substituting for the r's their values as found above. Consequently the right amount of energy is available for radiation.

It is possible, therefore, following Bohr, to define the necessary separate states as those of motion in circular orbits in which the

[1] See "Quantum Theory" and "Radiation Theory," Vol. IV.

angular momentum is an integral multiple of $h/2\pi$. The simplicity of these expressions is very attractive. But the matter is far from ending here. During the last few years Bohr and Sommerfeld have led an inquiry into the possibilities of this theory which has furnished very remarkable results. These are due to a slight modification in the original conception. The different circular orbits which Bohr first pictured have become groups of orbits fixed by laws which are somewhat arbitrary, but not without foundation. A group contains a limited number of orbits in which the electrons may move, and each group corresponds to one of the original circular orbits. Some of the orbits in each group are elliptical. It appears that the energy of the electron would be the same in all the orbits of any one group were it not that when an electron moves in an ellipse its velocity is not always the same. Now a fast-moving electron shows a variation in mass when its speed alters, and this does affect slightly the energy of the orbit. Consequently, the electron that steps from an orbit belonging to one group to an orbit belonging to another group may part with an amount of energy which is not always exactly the same. The frequency of the consequent radiation may, therefore, have two or more values differing slightly from each other; the single spectrum line is doubled or trebled. This is what Sommerfeld calls the "fine structure" of the lines.

Now there is far more than mere speculation in this. The formula which Sommerfeld gives as the result of an analysis which is as reasonable as can be expected does more than account for known effects; it has predicted the existence of numerous lines, and even their intensities, and the predictions have been verified by experiment in the most remarkable way.[1]

The interchange of energy between wave and electron has recently been examined from a new point of view with very interesting results. It is well known that every atom can be stimulated under proper conditions to the excitement of X-rays characteristic of the atom. For instance, tin atoms can be made to emit a certain series of "lines" known as the K series, provided that the incident and exciting radiation is, if a wave, of shorter wave-length than 0·421 Ångstrom units; or, if an electron, of the corresponding quantum energy.

The actual measure of the "critical quantum energy" of the wave or the kinetic energy of the electron is

$$6{\cdot}55 \times 10^{-27} \times 3 \times 10^{10} \div 0{\cdot}421 \times 10^{-8} = 4{\cdot}67 \times 10^{-8} \text{ ergs.}$$

When the quantum energy of the exciting radiation exceeds this amount the whole K series is excited: the quantum energies of the principal members of the series are 4·55 and 4·02, the unit being 10^{-8} ergs.

Tin, like other atoms, has other characteristic radiations; in particular, there are certain so-called L series whose quantum energies are about one-eighth of those of the K series; and again, there are M series of still smaller energy, and no doubt others amongst which quanta of visible light are included. It is sufficient for present description to assume that the L series have on the average a critical quantum energy 0·65 and the M series 0·12.

Two questions now arise: (1) If the critical energy 4·67 represents, as is natural to assume, an energy given up by the incident radiation to the atom, what becomes of the balance when the radiation 4·55 or alternatively 4·02 is excited in the atom? (2) If the energy of the exciting radiation exceeds 4·67, what becomes of the excess?

The answer to the first question appears to be supplied by the observation that $4{\cdot}67 - 4{\cdot}55 = 0{\cdot}12$ (M) and $4{\cdot}67 - 4{\cdot}02 = 0{\cdot}65$ (L).

It seems to be the general rule, verified carefully by Duane,[2] that the difference between two critical quanta is equal to a wave quantum.

An answer to the second question is given by de Broglie,[3] whose experimental results agree with the hypothesis that the excess of the incident quantum energy over the critical quantum energy appears subsequently as the energy of a high speed electron. De Broglie examined (with other similar cases) the magnetic spectrum of the electron radiation arising from the incidence of rays characteristic of tungsten upon a tin radiator. The two principal incident radiations had quantum energies 10·62 and 9·40: they were, in fact, the K lines of tungsten. In the magnetic spectrum were found five groups of electrons: one of these had a maximum energy 5·9, which is equal to $10{\cdot}62 - 4{\cdot}67$ within experimental error: the maximum energy of the other was 4·7, which is very nearly $9{\cdot}40 - 4{\cdot}67$.

In this case, then, the difference between the energies of the exciting quantum and of the critical quantum is equal to the maximum energy of a group of electrons which is found in the magnetic spectrum.

Three other groups of electrons were found by de Broglie on his plates. Their maximum energies were 4·4, 3·9, and 3·4. These were to be expected, since the characteristic rays of tin 4·55 and 4·02 were present, having been excited by the tungsten rays of superior quantum energy; and in further sequence to the rule just stated there ought to be electron groups having respectively maximum energies equal to $4{\cdot}55 - 0{\cdot}12 = 4{\cdot}43$,

[1] Sommerfeld, *Atombau*.

[2] *Physical Review*, December 1920.

[3] *Comptes Rendus*, March 29, 1921.

4·55 – 0·65 = 3·90, 4·02 – 0·12 = 3·90, and 4·02 – 0·65 = 3·37.

Parallel results in the case of γ-rays have been obtained by C. D. Ellis under the direction of Rutherford. They are described in the *Proceedings of the Royal Society*, June 1921.

W. H. B.

ELECTRONS AND THE DISCHARGE TUBE

§ (1) ELECTRIC DISCHARGE THROUGH GASES AT LOW PRESSURE.—When an electric discharge is passed through a gas at a fairly low pressure, say below 6 mm. of mercury, some very beautiful and interesting phenomena are to be observed. The effects can be studied by sealing two electrodes, generally of aluminium, in a glass vessel which is gradually exhausted by means of a vacuum pump. If the electrodes are raised to a sufficiently high potential difference, either by means of a small induction coil or a battery of 1000 volts, and the pressure of the gas is gradually reduced, an electric discharge will eventually take place accompanied by luminosity in various portions of the gas. Tubes specially constructed to exhibit these phenomena are known as Geissler tubes, and the discharges themselves are sometimes referred to as Geissler discharges.

The appearances presented by the discharge, and the gas pressure and potential difference necessary to excite them, depend not only on the nature of the gas, but also on the shape and size of the discharge tube, the current passing through it, and to a smaller extent the nature of the electrodes. The simplest form of Geissler tube, and therefore the form most suited for studying the phenomena, consists of a simple cylindrical glass tube fitted with a pair of plane parallel electrodes, the electrodes being sufficiently large to cover the whole cross-section of the tube. A tube of from 2 to 3 cm. in diameter is convenient (though very much wider ones have been used by some observers), and if the tube is to be excited by a 1000-volt battery the distance between the electrodes should not exceed 10 or 12 centimetres. The longer the tube the greater the potential difference necessary to produce the discharge.

With a simple discharge tube of this kind the appearances presented by the discharge may be divided into four main types, which change gradually from one to another as the conditions are varied. The most important factor in determining the appearance of the discharge is the pressure and nature of the gas in the tube, but the exact pressures at which the changes take place depend also on the shape and size of the discharge tube and the current passing through it. An increase in the potential applied to the tube will generally produce an effect on the discharge similar to that which would be produced by a further reduction of the gas pressure. The following description refers to air enclosed in a discharge tube 3 centimetres in diameter, and having plane parallel electrodes 22 centimetres apart. The numerical data are due to Townsend.[1]

As the pressure of the air in the tube is gradually reduced, say to 4 or 5 millimetres of mercury, a discharge passes between the electrodes accompanied by luminosity in the gas. If the potential difference across the tube is not much greater than that required to maintain a current through it, the luminosity is at first confined to the surface of the electrodes (*Fig.* 1),[2] each of which is

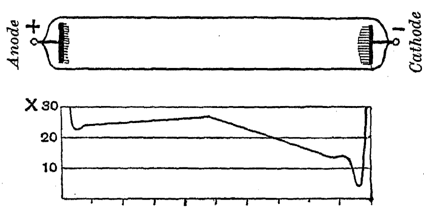


FIG. 1.

seen to be covered by a luminous glow. As the pressure is still further reduced these glows extend outwards into the tube, especially that at the positive electrode, which, at pressures of between 1 and 2 millimetres, will be found to occupy the greater part of the length of the tube. The colour of the two glows is quite distinct. In air the positive column is orange red and the negative blue. The positive column when examined spectroscopically gives the characteristic spectrum of the gas in the tube, and this portion of the discharge is therefore employed in spectroscopic work. The two luminous portions of the gas are separated by a non-luminous portion known as the Faraday dark space.

When it first makes its appearance the positive column is a uniformly luminous column of light. As, however, the pressure is still further reduced to about 0·5 millimetre of mercury it tends to break up into striae, the striation taking place more readily if the applied potential difference is small. The tendency to break into striae increases as the pressure is reduced until, at a pressure of about ·25 millimetre, the column is completely striated for a large range of currents. The luminous striae are button-shaped in appearance with their convex sides facing the cathode. They are spaced at regular intervals, their number and distance apart depending on the

[1] Townsend, *Electricity in Gases*, p. 398.

[2] The lower part of the diagram gives the electric intensity in the tube measured in volts per cm. See § (3).

size of the discharge tube and the current passing through it. The striae are more widely separated in a wide than in a narrow tube.

Somewhere about this stage it will be noticed that the negative glow is beginning to extend outwards into the tube, driving the positive column backwards towards the anode. As soon as the phenomena at the negative end of the tube have become sufficiently well developed it will be seen that the glow consists of two portions separated by a dark interval known as the Crookes dark space. The portion immediately around the surface of the cathode is known as the cathode glow. The other portion, lying between the two dark spaces, is known as the negative glow. The typical appearance of the discharge at this stage is indicated diagrammatically in *Fig.* 2.

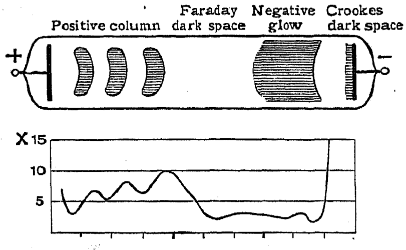


FIG. 2.

If the tube is still further exhausted the Crookes dark space increases rapidly in length at the expense of the positive column, which is soon represented merely by a glow (known as the anode glow) on the surface of the anode. The anode glow generally persists over some considerable range of gaseous pressures, but finally disappears, leaving only the negative glow and the glow on the surface of the cathode (*Fig.* 3). The pressure at which these

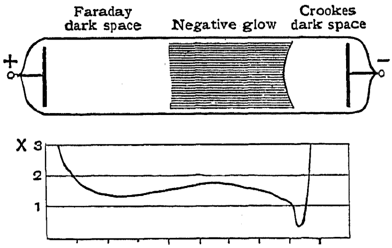


FIG. 3.

changes occur depends mainly on the distance between the electrodes. It is found that the length of tube occupied by the Crookes dark space and negative column is independent of the distance between the electrodes. At a pressure of ·1 millimetre in air the Crookes dark space is about 1·5 centimetres wide and the negative glow about 10 or 12 centimetres long. Thus if the distance between the electrodes is less than from 11·5 to 13·5 centimetres the negative glow will reach the anode and there will be no positive column. If the pressure is still further reduced, the Crookes dark space continues to extend, driving the negative glow before it. The cathode glow also disappears, the whole of the gas eventually becoming non-luminous. At this stage, which is employed in X-ray tubes, the discharge is entirely dark, save for the vivid fluorescence excited on the walls of the discharge tube itself. If the exhaustion is pushed to extreme limits the tube ceases to conduct.

§ (2) POTENTIAL NECESSARY TO PRODUCE THE DISCHARGE.—The potential necessary to initiate a discharge is not a very definite quantity, owing, possibly, to electrification on the inner walls of the discharge tube, and a considerably larger voltage may be required to excite the discharge than that necessary to maintain it when once excited. The latter voltage depends mainly on the nature and pressure of the gas and the distance apart of the electrodes, but it also depends on the diameter of the tube, being greater for a narrow than for a wide tube. It also depends to some extent on the nature of the electrodes. The following figures, due to Townsend,[1] give the potential difference in volts necessary to maintain a current of 10^{-2} amperes between aluminium electrodes 11·5 centimetres apart in rarefied air at a pressure p:

p in mm. of Mercury.	v in Volts.	p in mm. of Mercury.	v in Volts.
4	650	·40	530
2·84	620	·29	590
1·65	500	·24	630
1·04	470	·17	740
·66	490	·13	800

The discharge tube in this example was 3 centimetres in diameter. There is therefore a definite pressure of gas at which the discharge takes place most easily. Either increasing or decreasing the pressure from this critical value increases the potential difference necessary to maintain the discharge.

If the potential is increased beyond the minimum necessary to maintain a discharge, the current through the tube increases. At the higher pressures a very slight increase in the voltage of only 1 per cent or 2 per cent may be sufficient to more than treble the current produced. This sensitiveness, however, becomes less marked as the pressure becomes lower.

§ (3) DISTRIBUTION OF ELECTRIC INTENSITY IN THE DISCHARGE.—The distribution of the

[1] Townsend, *Electricity in Gases*, p. 396.

electric intensity in various parts of the discharge has been investigated by many observers, including Hittorf,[1] Graham,[2] H. A. Wilson,[3] J. J. Thomson,[4] and Aston.[5] The method employed by the earlier observers was to insert in the tube a small subsidiary electrode consisting of a fine-pointed platinum wire connected to an electrometer. Assuming that there is an adequate supply of charged ions in the neighbourhood, any difference of potential between the point and the surrounding space will set up an electric field either towards or away from the point, and ions of the appropriate sign will thus be driven up to the wire until the difference of potential is neutralised. The potential of the wire will then be the same as that of the surrounding gas.

The accuracy of the method obviously depends on there being a sufficient supply of both positive and negative ions in the gas around the wire. Let us suppose, for example, that only negative ions are present, as is practically the case near the anode. The negative ions will collide with the wire, giving it a negative charge, and will continue to do so until the potential of the wire is so far below that of the surrounding gas that the resulting field is sufficiently strong to prevent any further negative ions reaching it. As the number of positive ions present is too small to neutralise the charge on the wire, the latter will thus acquire a potential which may be considerably lower than that in any part of the surrounding space before the introduction of the wire. Hence at points close to either electrode the results obtained by this method are almost certainly misleading. In the main part of the discharge they are probably sufficiently near the truth.

In applying the method it is desirable to use two exploring points situated at some small fixed distance apart, say 1 millimetre. Since the distance apart of the points is fixed, the difference of potential between the points will be directly proportional to the electric intensity in the portion of the discharge between the points. This method was employed by Graham (*loc. cit.*) and H. A. Wilson (*loc. cit.*). The two test points *e* and *f* (*Fig.* 4) were sealed into the walls of the discharge tube, while the main electrodes A and C between which the discharge was taking place were mounted on a movable frame which could be controlled magnetically from outside the tube. In this way the position of the test wires could be made to coincide with any desired part of the discharge.

[1] Hittorf, *Wied. Ann.*, 1883, xx. 705.
[2] Graham, *ibid.*, 1898, lxiv. 49.
[3] H. A. Wilson, *Phil. Mag.*, 1900, xlix. 505.
[4] J. J. Thomson, *ibid.*, 1909, xviii. 441.
[5] F. W. Aston, *Proc. Roy. Soc.* A, 1911, lxxxiv. 526; *ibid.* A, 1912, lxxxvii. 437.

An ingenious method, which would appear to be free from the uncertainties introduced by the test wire, was devised by Sir J. J. Thomson (*loc. cit.*), and is based on the fact

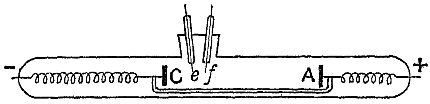


FIG. 4.

that a beam of cathode rays (see later, § (8)) is deflected by an electric field, the deflexion being directly proportional to the intensity of the field. A beam of cathode rays is generated in a side tube placed at right angles to the main discharge, and the rays, after crossing the main discharge, are received on a fluorescent screen placed in another side tube directly opposite to the first. The deflexion of the cathode rays from their normal position will then be a measure of the electric intensity in the portion of the discharge through which they have passed. This method was subsequently adopted by Aston (*loc. cit.*). Except in the vicinity of the electrodes, the results obtained by the two methods are in good agreement.

Starting at the cathode, it is found that the intensity of the electric field in the actual vicinity of the cathode itself is very high, much higher, in fact, than in any other part of the discharge. The intensity decreases rapidly as we pass through the Crookes dark space until on reaching the negative end of the negative glow its value is almost inappreciable. Aston (*loc. cit.*), who has made a special investigation of this part of the discharge, finds that the force at a point in the Crookes dark space is directly proportional to the distance of the point from the negative edge of the negative glow.

The electric intensity rises slightly in passing along the negative glow, falls again in the Faraday dark space, increasing once more as the positive column is approached. When the positive column is uniform the field in the column is also uniform. If, however, the column is striated the striations are accompanied by local variations in the intensity, the latter being a maximum at positions of maximum brightness. The intensity of the field in the positive glow decreases with the pressure. According to most observers, there is a small but rapid increase of potential, with corresponding increase in the field, in the direct vicinity of the anode.

The phenomena are, of course, modified if any parts of the discharge are not present. Graham's results for discharges of different types are given in *Figs.* 1-3, immediately below the diagrams illustrating the appearances of the discharge.

§ (4) Cathode Fall of Potential.—Since the field in the neighbourhood of the cathode, and through the greater part of the Crookes dark space, is much greater than that in any other part of the discharge, it is obvious that the difference of potential between the cathode and a point in the negative glow will in general be a very large fraction of the whole difference of potential across the tube. It is known as the cathode fall of potential. The cathode fall of potential is independent of the pressure of the gas in the tube, and also of the current density of the discharge if the latter is not too great. It is found that when the current through the tube is small the cathode glow only occupies the central part of the cathode, the area occupied by the glow increasing as the current is increased. As long as the cathode is not completely covered by the glow the cathode fall of potential remains constant,[1] and may be known as the normal cathode fall. If, however, after the glow has completely covered the electrode, the current is still further increased, the cathode fall increases with the current.

A considerable number of determinations of the normal cathode fall of potential have been made, and the results of the various observers have been summarised by Mey.[2] The fall of potential depends upon the nature of the gas. Using platinum electrodes, it is about 340 volts in air, 300 volts in hydrogen, 167 volts in argon, and 470 volts in water vapour. The value for a given gas is very nearly the same for electrodes of any of the less electropositive elements, such as platinum, silver, copper, or iron, but is considerably reduced when the electrodes are of magnesium, sodium, or potassium. With electrodes of the latter metal the normal cathode fall in hydrogen is reduced to 172 volts, and in argon to 69 volts. This is probably due to the great facility with which these metals emit electrons under the action of the radiations from the discharge.

§ (5) Conductivity in Various Parts of the Discharge.—The conductivity of various parts of the discharge has been investigated by H. A. Wilson [3] by measuring the current passing between two small platinum plates placed on opposite sides of the discharge, and parallel to its direction; a small potential difference (that of a single Clark cell) being maintained between the plates. For potential differences of this order it was found that the current was directly proportional to the applied potential difference, thus showing that the number of free ions in the main discharge was not appreciably affected by the field between the plates. Under such circumstances it can be shown that the current between the plates is proportional to $k_1n_1 + k_2n_2$, where n_1, n_2 are the numbers of positive and negative ions per unit volume, and k_1, k_2 their respective mobilities.

The matter has recently been re-investigated by van der Pol[4] by an ingenious method not involving the use of subsidiary electrodes. If stationary waves are induced in a pair of parallel wires, which have been adjusted to be in tune with the generator employed, the current in this Lecher system will reach a stationary condition in which the energy supplied by the generator is exactly balanced by the energy dissipated in the system. If, now, a conductor is introduced anywhere in the space between the wires, currents will be induced in the conductor and the rate of dissipation of the energy in the system will be increased. If the energy is supplied at a constant rate the current in the wires will therefore decrease, and the decrease in current affords a measure of the conductivity of the conductor. By bringing different parts of the discharge tube in turn into the plane of the wires the conductivity of the different parts of the discharge can be investigated.

The results obtained by the two methods, while agreeing in the main, exhibit some minor discrepancies. The conductivity and therefore the number of ions per unit volume is very small in the Crookes dark space (*Fig.* 5),

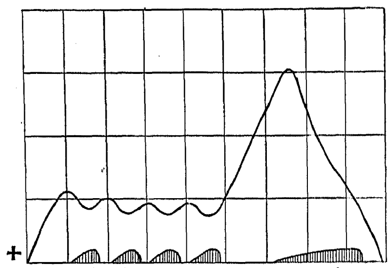


Fig. 5.

increases rapidly in passing along the negative glow, and attains a maximum value in the neighbourhood of the junction of the negative glow with the Faraday dark space. It decreases as the positive column is approached, and then, if the positive column is uniform, remains constant until the anode is approached, when it again decreases. If, however, the positive column is striated, variations in the conductivity are found corresponding with each of the striae. According to H. A. Wilson the maximum conductivity occurs at the brightest part of each striation; according to van der Pol the brightest part of the striation is a place of minimum conductivity. This is in

[1] Warburg, *Wied. Ann.*, 1887, xxxi. 545.
[2] K. Mey, *Ann. der Phys.*, 1903, iv. 11, p. 127.
[3] H. A. Wilson, *Phil. Mag.*, 1900, xlix. 505.
[4] B. van der Pol, *Phil. Mag.*, 1919, xxxviii. 352.

accordance with J. J. Thomson's observation that the field is most intense in the bright parts. Assuming that the velocity of the ions increases with the field, they should be removed more rapidly from places where the field is strong, and hence the number of free ions and the conductivity should be less in such regions. It is possible that in Wilson's experiments the radiations emitted by the luminous portions of the gas caused an actual emission of electrons from the surfaces of his test electrodes, thus increasing the apparent conductivity at these points. According to van der Pol, for a given current through the discharge tube the conductivity of the positive column increases as the pressure of the gas is decreased.

§ (6) The Crookes Dark Space.—The length of the Crookes dark space has been investigated by numerous observers, including Ebert[1] and Aston.[2] When the current through the tube is so small that the cathode glow does not cover the whole surface of the cathode the length occupied by the dark space depends only on the nature and pressure of the residual gas. According to J. J. Thomson,[3] Ebert's results show that under these circumstances the thickness d of the Crookes dark space may be expressed in the form $d=a+b/p$, where p is the pressure and a and b are constants. This may also be written in the form $d=a+\beta . \lambda$, where λ is the mean free path of an electron in the gas. The dark space, as measured from a distance a in front of the cathode, would thus be directly proportional to the mean free path in the gas. The distance a is of the order of ·25 millimetre.

When the current is sufficiently large to cause the negative glow to cover the cathode completely the phenomena are complicated by the action of the walls of the vessel. Under these circumstances Aston (*loc. cit.*) found that the length d of the Crookes dark space could be expressed by the formula

$$d=\frac{A}{p}+\frac{B}{\sqrt{I}},$$

where p is the pressure, I the current density, and A and B are constants for a given gas. Under the same circumstances the cathode fall of potential V is given by the formula

$$V=E+\frac{F\sqrt{I}}{p},$$

where E and F are constants for a given gas.

§ (7) General Theory of the Discharge.—Assuming that the discharge is conveyed across the tube by the motion of gaseous ions, then, in the absence of any external ionising agent, these ions must be produced from the molecules of the residual gas in the tube by the action of the ions already present. It is known that if a sufficiently strong electric field is applied to an ion, especially in a gas at low pressure, the ion acquires sufficient energy to ionise a neutral molecule with which it may collide. This ionisation by collision takes place more readily with negatively than with positively charged ions. It is obvious that for a continuous discharge to take place both the positive and the negative ions must be able to produce fresh ions by collisions at any rate in certain parts of the discharge. At low pressures such as those employed in the Geissler discharge the negative ion exists for the most part in the free or electronic condition. The positive ion, on the other hand, consists of a charged molecule or atom of the gas in the tube. The mobility of the negative ions will therefore be much greater than that of the positive.

Let us suppose that a discharge is passing through the tube and that conditions have become steady. We have seen that the electric field reaches its maximum value in the immediate neighbourhood of the cathode. The positive ions will therefore attain their maximum velocity, and hence their maximum power of producing fresh ions by collisions in this region. J. J. Thomson considers that the results of Ebert (*loc. cit.*) on the thickness of the Crookes dark space suggest that the negative ions are mainly produced in the region of gas lying about ·25 mm. in front of the cathode. On the other hand, the fact that the cathode fall of potential depends somewhat on the metal employed for the cathode would suggest that electrons may also be emitted by the cathode itself under the action of the rapidly moving positive ions. In either case the negative ions are produced within a region very close to the cathode, probably entirely within the cathode glow. Owing to the very strong field in this region the negative ions, which at this pressure consist of free electrons, will move away from the cathode with very high velocities, forming the cathode rays, which will be dealt with later. Both negative and positive ions will be rapidly removed from the space in front of the cathode, thus accounting for the small conductivity of this region, but the negative, being much more mobile than the positive, will be removed more quickly. There will thus be an excess of positive ions in the Crookes dark space. This accumulation of positive electricity in the neighbourhood of the negatively charged cathode accounts for the very large values of the electric field in this region.

The negative ions move from the cathode with sufficient velocity to enable them to

[1] Ebert, *Verhand. Deutsch. Physik. Ges.*, 1900, ii. 99.

[2] Aston, *Proc. Roy. Soc.* A, 1907, lxxvi. 80; Aston and Watson, *ibid.*, 1912, lxxxvi. 168.

[3] Thomson, *Conduction through Gases*, 1906.

produce fresh ions by collision. The probability of a collision will, however, not be very great until the ion has moved through a distance comparable with its mean free path in the gas. If we assume that the luminosity of the gas is associated with ionisation the Crookes dark space will represent the distance fallen through by the negative ions before collision with the molecules of the residual gas, that is to say it should be comparable with the mean free path of an electron. This is at any rate approximately the case, and the supposition is further supported by Sir J. J. Thomson's interpretation of Ebert's results.

The positive space charge in the Crookes dark space obviously reduces the field on the positive side of the space, which, therefore, falls to a very small value, in accordance with the experimental determinations. The negative ions formed in the negative glow do not acquire sufficient energy to produce fresh ions by collision. Ionisation, therefore, ceases as soon as the cathode rays have exhausted their original impetus, and we have the beginning of the Faraday dark space. The positive ions formed in the negative glow are attracted to the cathode and thus maintain the supply of negative ions.

The current across the Faraday dark space is conveyed mainly by negative ions drawn from the negative glow. There is thus an accumulation of negative electricity in this region which gradually increases the electric field until finally it attains a sufficiently high value to cause fresh ionisation by collision. This point marks the beginning of the positive column. If the field remains constant, ionisation by collision will occur throughout the rest of the tube with the production of a uniform column of light. The striations, when formed, are probably due to a repetition in a modified form of the phenomena at the negative end of the tube.

The mathematical treatment of the theory of the discharge tube presents great difficulties, owing largely to the disturbing effects of the walls of the discharge tube. These certainly exercise an important effect in determining the nature of the discharge, but very little is known about the exact part which they play. Theories have been advanced by J. J. Thomson [1] and Townsend,[2] but the questions raised cannot be regarded as yet settled.

The hypothesis that the discharge is maintained by the action of positive ions approaching the cathode from the negative glow is, however, demonstrated quite readily by experiments such as those of Villard.[3] In these experiments a metal diaphragm (*Fig.* 6) pierced with two small holes, *a*, *a*, was placed at a distance of 1·5 centimetres in front of the cathode C, which consisted of a large plane disc, and the tube was then gradually exhausted. As long as the Crookes dark space did not extend to the diaphragm the current flowed from the whole surface of the cathode as shown by the fact that the whole of its surface was covered by the negative glow. As soon, however, as the dark space had expanded so as to reach the diaphragm, and the negative glow ceased to occupy any part of the region between the diaphragm and the cathode, the luminosity was found to be confined to two spots, *b*, *b*, immediately opposite the two holes in the diaphragm. The position of these luminous spots could be deflected by placing a positively charged electrode in the dark space, the direction of the deflexion indicating that the particles producing the luminous spots carried a positive charge. The existence of these positive particles can be demonstrated by using a perforated cathode. The presence of particles passing through the perforation is evinced by the appearance of luminous streamers behind the cathode. They form, in fact, the well-known Kanahlstrahlen, or Positive rays. Recent experiments by Aston,[4] in which the charges conveyed by the particles were collected in a Faraday cylinder, suggest that about half the current in the tube is conveyed to the cathode by these positively charged particles.

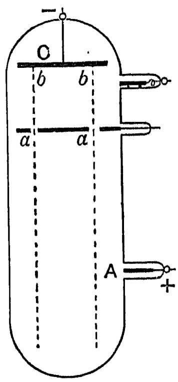


FIG. 6.

§ (8) THE CATHODE RAYS.—When the pressure in the discharge tube is very low the appearances presented are quite different from those of the Geissler discharges. The gas as a whole is non-luminous, except for the cathode glow, which also vanishes if the vacuum is high. At the same time a beam of light may be seen proceeding normally from the cathode, and penetrating to a greater or shorter distance along the tube as the pressure is comparatively low or high. At low pressures the beam may reach the further boundary to the tube, in which case it excites a vivid fluorescence on the portions on which it falls. The colour of this fluorescence depends on the nature of the substance on which the rays impinge. In ordinary German glass the fluorescence is bright green, possibly due to traces of manganese. With lithium glass the

[1] J. J. Thomson, *Conduction of Electricity through Gases*, 1906, pp. 497, 602. See also *Phil. Mag.*, 1921, xlii. 981.
[2] J. S. Townsend, *Electricity in Gases*, p. 440 *et seq.*
[3] P. Villard, *Journal de Physique*, 1899, iii. 8, p. 1.
[4] F. W. Aston, *Proc. Roy. Soc.* A, 1919, xcvii. 200.

fluorescence is blue, as is also the case with many samples of English glass.

These phenomena were first noticed by Hittorf.[1] The name cathode rays was given to them by Wiedemann. The discovery gave rise to much controversy, some holding that the rays consisted of vibrations in the ether, either transverse, as in the case of ordinary light, or longitudinal. Crookes, to whom many of the early experiments on the rays are due, suggested that they consisted of electrified particles of some very attenuated form of matter, projected with high velocities by the electric forces near the surface of the cathode.[2] This view is now universally accepted. The evidence in favour of this hypothesis will be gathered from the following summary of the properties of the rays.

An inspection of the trajectory of the rays, when the pressure is adjusted so as to make their path readily visible, shows that they travel in straight lines, proceeding normally from the surface of the cathode. If the cathode is plane, the beam is parallel and can be limited to a very narrow pencil, if desired, by placing a solid obstacle pierced with a small hole in front of the cathode. If the cathode is concave, the rays come to a focus at some point along the axis. Owing to the action of the strong electric field through which the rays are travelling, this focus is somewhat further from the cathode than its geometrical centre of curvature.

The cathode rays are deflected by a magnetic field, a discovery first made by Crookes. The direction of the deflexion is that which would occur in a flexible conductor coinciding with the path of the rays and carrying an electric current from the anode to the cathode. That the rays exert a pressure was proved by Crookes by placing a little windmill with mica vanes inside the discharge tube so that the rays impinged excentrically upon the vanes. On exciting the tube the mill revolved.

In order to explain the magnetic deflexion on the assumption that the rays consist of particles it is necessary to assume that the particles are negatively charged. The early experiments of Crookes and others to detect this charge were inconclusive, and sometimes gave quite unexpected results. This has been shown to be due to the very intense ionisation produced by the rays in the residual gas in the tube, and to the large emission of electrons which they produce when they impinge on a solid obstacle. The matter was placed beyond all doubt by the experiments of Perrin,[3] who passed the rays from a plane cathode C (*Fig.* 7) through a small aperture *a*, into a Faraday cylinder F connected to an electrometer. When the tube was excited so that the cathode rays entered the cylinder the latter acquired a negative charge of very

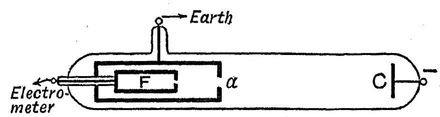


FIG. 7.

considerable amount. Using an induction coil to excite the discharge, the negative charge gained by the cylinder often reached as much as 100 electrostatic units for each interruption of the coil. If the rays were deflected by a magnet so that they did not strike the aperture of the chamber the cylinder remained uncharged. The experiments were repeated by J. J. Thomson in a slightly modified form with similar results.[4] These experiments were generally accepted as proving that the cathode rays consist of negatively charged particles.

The deflexion of the cathode beam by an electrostatic field, which is a necessary consequence of the hypothesis that they consist of negatively charged particles, had been indicated by experiments of Goldstein in 1876, and later by Perrin (*loc. cit.*), in which a subsidiary electrode which could be raised to a high potential was introduced into the path of the cathode stream. A negative charge repelled and a positive charge attracted the cathode beam, but very high potentials were required to produce an appreciable effect. Sir J. J. Thomson, however, by passing a narrow beam of the rays between a pair of parallel plate electrodes in a discharge tube at very low pressure was able to obtain a perceptible deflexion with a difference of potential of only two volts, thus placing the matter beyond all doubt. The difficulty experienced in obtaining the electrostatic deflexion is due to the fact that the rays produce intense ionisation in the residual gas of the discharge tube, and thus render it partially conducting. The rays are therefore screened to a considerable extent from external electric forces. This effect only becomes negligible when the pressure is very low.

The conductivity produced in a gas by the passage of the cathode rays was demonstrated by Lenard,[5] who allowed the rays to fall on a small aperture in the wall of the discharge tube covered with a very thin sheet of aluminium. The rays penetrate the aluminium leaf, passing into the air, which they also penetrate to a distance of some 8 or 10 centimetres. These Lenard rays, as they

[1] Hittorf, *Pogg. Ann.*, 1869, xiv. 136, p. 1.
[2] Crookes, *Phil. Trans.*, 1879, clxx. 135.
[3] Perrin, *Comptes Rendus*, 1895, cxxi. 1130; *Ann. de Chim. et de Phys.*, 1897, vii. 11, p. 496.
[4] Thomson, *Proc. Camb. Phil. Soc.*, 1897, ix. 243.
[5] P. Lenard, *Wied. Ann.*, 1897, lxiii. 253.

are called, produce very considerable ionisation in the gas through which they pass.

§ (9) Determination of the Ratio e/m for the Cathode Rays.—A measurement of the magnetic deflexion of the rays throws important light on the nature of the particles constituting the rays. The deviation produced in the path of a particle of mass m and charge e projected with a velocity v in a uniform magnetic field of strength H was calculated by J. J. Thomson.[1] He showed that such a particle would be acted upon by a mechanical force of $\mathrm{H}ev \sin \theta$, where θ is the angle between the velocity of the particle and the direction of the field, the direction of this force being at right angles to these two directions. For if dt be the time taken by the particle to traverse a distance ds, its effect is that of a current i where $e = i\delta t$, while $v = ds/dt$. Thus

$$\mathrm{H}ev \sin \theta = \mathrm{H}idt \sin \theta \frac{ds}{dt} = \mathrm{H}i \sin \theta \,.\, ds,$$

and this is the mechanical force[2] on the current element ids. The path of the particle, therefore, would in general be a spiral with its axis parallel to the lines of force, the radius of curvature ρ being given by the equation

$$\rho = \frac{mv}{e\mathrm{H} \sin \theta},$$

where θ is the angle between the direction of projection of the particle and the field. If the velocity of the particle remains constant the path is therefore a helix wound on a circular cylinder with its axis parallel to the field and of radius r given by

$$r = \rho \sin^2 \theta = \frac{mv \sin \theta}{e\mathrm{H}}.$$

If the particle is projected at right angles to the field, the helix contracts into a circle of radius $r = mv/e\mathrm{H}$. If the field is not uniform the particles will describe spirals of varying amplitudes about the lines of the magnetic field.

These equations obviously afford a method of determining the ratio of the charge to the mass on a cathode particle, if the velocity of the particles can be determined. Determinations of the ratio e/m for the cathode particles were made independently and almost simultaneously in 1897 by J. J. Thomson,[3] Kaufmann,[4] and Wiechert.[5] The experiments differ principally in the methods employed to estimate the velocity of the rays.

§ (10) Thomson's Method of determining e/m.—In Thomson's experiments the velocity v of the rays was determined by balancing the magnetic against the electrostatic deflexion of the rays. If a uniform electrostatic field of strength X is applied at right angles to the magnetic field and coterminous with it, and the cathode rays are directed across the fields in a direction at right angles to both, the electric force acting on the particle will be $\mathrm{X}e$ in the direction of the electric field, and the magnetic force will be $\mathrm{H}ev$ at right angles to the magnetic field, that is in the same direction as the electric force. If the two forces are adjusted so that they are equal and act in opposite directions the resultant force upon the particles will be nil, and the beam will be undeflected. Under these circumstances we have, obviously,

$$\mathrm{X}e = \mathrm{H}ev,$$
$$v = \mathrm{X}/\mathrm{H},$$

which determines the velocity v, since X and H can be measured.

The apparatus (*Fig.* 8) consists of a plane cathode E and an anode placed in a small

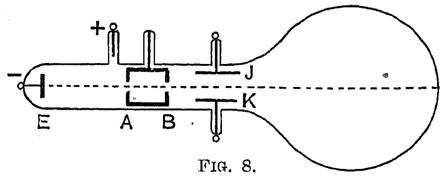


Fig. 8.

side tube to avoid interference with the rays. The cathode rays are limited to a narrow pencil by means of a pair of brass stops A and B pierced with a narrow slit. On emerging from the second stop the beam passes between two parallel plates J and K which can be charged to a difference of potential by a battery of cells. The magnetic field is applied by a small electromagnet, not shown in the figure, at right angles to the line joining the plates J, K. The strength of the two fields is adjusted until the position of the fluorescent spot produced where the beam strikes the further wall of the tube is unaffected by the joint action of the fields. The ratio X/H then gives the velocity of the rays. The electric field is removed, and the deflexion produced by the magnetic field alone is measured. From this and the dimensions of the apparatus the radius of curvature of the rays in the field can be calculated, thus giving a value for e/m. The value obtained by Thomson for this ratio was $1{\cdot}17 \times 10^7$ electromagnetic units per gm. This result is too low, probably owing to the screening action of the residual gas on the electric field. The ratio e/m was found to be constant, and independent of the nature and pressure of the gas in the tube. The velocity

[1] Thomson, *Recent Researches in Electricity and Magnetism*, 1893.
[2] See "Electromagnetic Theory," § (9).
[3] Thomson, *Phil. Mag.*, 1897, xliv. 293.
[4] Kaufmann, *Wied. Ann.*, 1897, lxii. 598.
[5] Wiechert, *ibid.*, Beiblatter, 1897, xxi. 443.

of the rays increased as the potential difference across the tube was increased.

§ (11) KAUFMANN'S METHOD OF DETERMINING e/m.—In Kaufmann's experiments (*loc. cit.*) a narrow pencil of rays from a plane cathode (*Fig.* 9) was passed through a small perforation in the anode of the tube, and it was assumed that the energy of the particles on passing through the hole was equal to that due to a free fall through the whole difference of potential V, between the anode and cathode. This assumption implies that the effect of the collisions between the rays and the molecules of the residual gas in the tube is negligible. This was apparently the case in Kaufmann's experiments. Thus we may write $\frac{1}{2}mv^2 = Ve$ or $v = \sqrt{2Ve/m}$. Combining this result with the magnetic deflexion of the rays we have two equations for determining e/m and v. In Kaufmann's apparatus the whole path of the rays from the aperture to the fluorescent screen was placed in a nearly uniform magnetic field produced by two coaxial solenoids. The actual distribution of the field was subsequently determined and corrections applied. Kaufmann's value for e/m was $1{\cdot}77 \times 10^7$, which is very near the mean of the most recent observations.

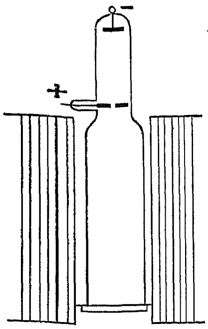


FIG. 9.

§ (12) WIECHERT'S DETERMINATION OF THE VELOCITY OF THE CATHODE RAYS.—The importance of Wiechert's experiments (*loc. cit.*) lies in the fact that the velocity of the rays was measured by direct experiment, as opposed to the indirect method of Thomson, and the assumption of Kaufmann. He found that the velocity of the cathode rays at the pressures he employed was of the order of one-tenth of that of light. His method consisted of comparing the time taken by the rays to cover a distance of about 20 centimetres with the period of the discharge of a condenser. The apparatus employed is indicated diagrammatically in *Fig.* 10. The cathode rays from the cathode C fall normally on a small hole in a screen A, and thence through a hole in a second screen A′ to a narrow fluorescent plate S. If a small magnet M is placed near the cathode the rays can be deflected so as to fall upon the solid part of the screen A, and the fluorescent screen will then be dark. A circuit *opqr* is placed near CA so that the magnetic field produced by a current passing round it is in the same direction as that due to the magnet. If a condenser of suitable capacity is now discharged round this circuit, a rapidly alternating magnetic field will be produced which will set the beam of rays swinging like a pendulum. If the oscillations are sufficiently large the rays will, in the course of their oscillations, fall intermittently upon the aperture in A, and the screen S will fluoresce. The fields are adjusted so

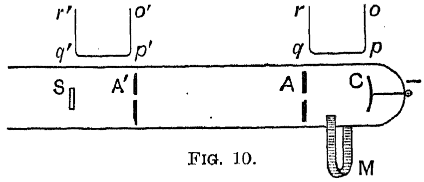


FIG. 10.

that the rays fall upon the aperture at the extreme end of their excursion.

A second circuit $o'p'q'r'$ is placed in the neighbourhood of A′S and connected in series with the circuit *opqr*. This will deflect the rays which pass through A′ exactly in the same manner as rays between C and A are deflected by *opqr*. The rays which emerge from A have been deflected to the maximum extent. If these rays reach A′ instantaneously they will again be deflected to the samé extent as between C and A, and will thus be thrown off the screen which will remain dark. If, however, the time taken by the rays to pass from A to A′ is such that the current has changed in phase by one-quarter period, there will be no current in $o'p'q'r'$ when the rays reach A′. For the moment there will be no magnetic field in the space, and the rays will pass through and reach the screen.

Thus the screen will be luminous if the time taken for the rays to describe the distance AS is equal to one quarter of the period of the condenser discharge. The latter can easily be determined experimentally, and thus the velocity of the cathode rays can be calculated.

The method is not susceptible of great accuracy, but the experiments are important, as providing a direct corroboration of the values obtained for the velocity of the cathode rays by less direct methods, and thus proving that their velocity is not that of light.

§ (13) THE MAGNETIC SPECTRUM.—The velocity of the cathode particles depends on the difference of potential between the ends of the tube and thus varies with the conditions of the experiment. Kaufmann's experiments may be regarded as proving that the maximum energy of the particles in the tube is equal to that which would be acquired if the particles fell freely through the full difference of potential between the electrodes. If, as is generally the case, the tube is excited by an induction coil, the cathode rays are far from being homogeneous, since the potential difference between the terminals of a coil is not constant.

The cathode beam will therefore contain particles having velocities corresponding to varying potentials between the minimum necessary to excite the discharge up to the maximum supplied by the coil. When a magnetic field is applied, the single spot of fluorescence produced by the undeflected beam will be drawn out into a band at right angles to the field, each point of which corresponds to rays of definite velocity. The effect is known as a magnetic spectrum, and the method is often employed to isolate a beam of cathode rays of definite uniform velocity.

The magnetic spectrum is generally not uniform, but consists of several bright lines, separated by more or less dark intervals. According to Strutt [1] the number and dispersion of these lines depend upon the peculiarities of the induction coil used to excite the tube. If a large electrostatic machine or a battery of cells is employed the rays are homogeneous and the spectrum reduces to a single line.

§ (14) NATURE OF THE CATHODE PARTICLES—THE ELECTRON.—The ratio e/m for the cathode particles is constant, being independent both of the nature and pressure of the gas in the tube and the nature of the electrodes. The most recent determinations of the ratio by different observers give values which do not differ by more than 1 per cent, and lie in most cases between $1 \cdot 76 \times 10^7$ and $1 \cdot 77 \times 10^7$ electromagnetic units per gram. The nature of the cathode particles is therefore independent of their mode of production, and of the nature of the substances from which they are produced. These particles are now known as ELECTRONS. They must obviously be present in all kinds of matter.

Since a charge of one electromagnetic unit in passing through a liquid electrolyte liberates $1 \cdot 0357 \times 10^{-4}$ grams of hydrogen, the ratio of the charge to the mass of a hydrogen atom in solution is $\cdot 96 \times 10^4$ electromagnetic units per gram. This is only about $\frac{1}{1850}$ of the value obtained for the cathode particles. If we assume that the charge is the same in the two cases the mass of the electron must be only $\frac{1}{1850}$ that of the hydrogen atom, the lightest particle hitherto known. There is now ample evidence in support of the assumption that the charges carried by an electron, and by a monovalent negative ion in solution are identical, perhaps the most direct being that supplied by the experiments of Rutherford [2] on the α-particles. We are therefore driven to the conclusion that the electron has a mass only $\frac{1}{1850}$ of that of the hydiogen atom.

§ (15) OTHER SOURCES OF ELECTRONS.—Electrons can be liberated in other ways than by the discharge in high vacua. If a metallic wire is raised to incandescence and charged to a small negative potential a very considerable emission of negative electricity takes place from the wire, which increases rapidly with increase in temperature. This emission can be shown to be due to the wire giving off negatively charged particles. The effect is enormously increased if the wire is coated with certain oxides, notably those of calcium and barium, in which case the emission of negative electricity may amount to several amperes per square centimetre of the surface. A platinum strip having a speck of barium or calcium oxide on its surface, and raised to incandescence by the current from a battery, forms a very convenient point source of electrons, and is known as a Wehnelt cathode. Cathodes coated with barium oxide are employed in the thermionic valves which are now employed so extensively in wireless telegraphy. A full account of the phenomena is given by O. W. Richardson,[3] to whom much of our knowledge of the subject is due. The negative particles emitted by hot bodies are often referred to as thermions.

Negative particles are also given off when a clean metal surface is illuminated by ultraviolet light. These are spoken of as photoelectrons. The effect is particularly well marked in the case of the alkali metals; so much so, in fact, that in the case of rubidium it has been actually employed to detect small quantities of light radiation. A well-constructed rubidium cell will easily detect the light radiation from a single candle at a distance of three miles, and is thus hardly inferior in sensitiveness to the human eye. This photo-emission takes place, though to a much smaller degree, with all metals, and also with non-metals if the wave-length of the illumination employed is sufficiently small.

Negative electricity in a corpuscular form is also given off when X-rays fall upon any material obstacle, and when the α-particles from radioactive substances, or the positive rays in a discharge tube, are stopped. The value of e/m for all these particles is identical with that obtained for the particles in the cathode rays. In other words, the negative emission consists in every case of electrons, which differ from each other only in the velocity with which they are emitted.

§ (16) DETERMINATION OF e/m FOR PHOTO-ELECTRONS AND THERMIONS.—The velocity of emission of the photo-electrons is very small compared with that of the cathode rays, corresponding generally to a fall through a potential difference of a few volts, that is to say it is of the order of 10^8 cm./sec. The velocity of the thermions is even less. As,

[1] R. J. Strutt, *Phil. Mag.*, 1899, xlviii. 478.

[2] Rutherford and Geiger, *Proc. Roy. Soc.* A, 1908, lxxxi. 141.

[3] O. W. Richardson, "Emission of Electricity from Hot Bodies," 1921. See also article "Thermionics."

however, they are all negatively charged, their velocity can be increased to any desired value by applying a suitable electrostatic field. Thus if the ordinary cathode in Kaufmann's experiment is replaced by a Wehnelt cathode the value of e/m for the electrons emitted can be determined in exactly the same way as for the ordinary cathode rays, by measuring the deflexion produced in a uniform magnetic field. Since the electrons are emitted with practically negligible velocities, the velocity attained by the electrons is due entirely to the field between the electrodes and can be calculated from the applied difference of potential. The value of e/m for the photo-electrons can be determined in a similar way, using a cathode of some suitable metal, illuminated by ultra-violet light.

The ratio can also be determined by a somewhat different method due to J. J. Thomson,[1] and applicable to all cases in which the initial velocity of the particles is small. A uniform electric field X is established between two plane parallel plates, and a uniform magnetic field H is applied at right angles to the electric field and, therefore, parallel to the plates. If the negative plate is now caused to emit electrons, say by illuminating it with ultra-violet light, these electrons, being negatively charged, will begin to move away from the plate with increasing velocity. As soon, however, as they have acquired a velocity they will be deflected by the magnetic field in a plane at right angles to the lines of the magnetic field, and will thus eventually return to the plate from which they started. It can easily be shown that, under these circumstances, the path of the particles is a cycloid, the curve traced out by a point on the circumference of a circle when the latter rolls along a straight line. The particles, therefore, cannot penetrate to more than a certain distance, equal to the diameter of this circle, from the negative plate. This maximum distance can be shown to equal $2X/H^2 . m/e$ where X and H are the electric and magnetic fields. If the distance apart of the plates is greater than this value no charge will reach the positive plate. If the distance is less than this critical value the whole of the negative electricity emitted by the negative plate will be conveyed to the other. The fields are adjusted until the positive electrode just begins to receive a negative charge. The value of e/m is then given by the equation

$$\frac{e}{m}=\frac{2X}{H^2d},$$

where d is the distance between the plates.

The same principle can be applied to the case of the thermionic emission. In this case the negative electrode takes the form of a straight incandescent wire, the positive electrode being a cool concentric cylinder. The magnetic field is applied at right angles to the electrostatic field, and parallel to the electrodes. The value of e/m is then given by the equation

$$\frac{e}{m}=\frac{2V}{H^2d^2},$$

where V is the difference of potential between the electrodes and d their distance apart.

§ (17) Recent Determinations of e/m for Electrons from Different Sources.—The table on the following page gives the values obtained for e/m for electrons from different sources by recent observers.

The close agreement between the values is remarkable and leaves no doubt as to the identity of electrons from different sources. The most generally accepted value is that given by Bucherer for the slow moving β-particles from radium. These experiments are dealt with later. The values calculated from the Zeeman effect for the ratio e/m for the negative particles emitting the bright lines in the spectrum of an element are also included in the table for comparison.

§ (18) The Nature of the Electron.—The question immediately presents itself as to what is the nature of these particles, which are so much lighter than the lightest known atom, and which can be produced in so many different ways from matter of every kind. The question has now been satisfactorily answered: the electron consists simply of a highly concentrated electrical charge, quite unassociated with anything which can be called matter in the ordinary sense of the word. In other words, it has been shown that the whole mass of the electron is due to the charge which it carries.

The fact that a moving charge will act as if it possessed inertia was first demonstrated by J. J. Thomson.[2] It can be shown that a point charge of strength e moving with a velocity v is equivalent to a current element ids coinciding with the path of the particle, and of strength given by the equation $ids=ev$. The moving particle will thus produce a magnetic field in its vicinity equal to $ev \sin\theta/r^2$, where r is the distance of the point considered from the charged particle, and θ the angle made by the line joining the two with the direction of the velocity. Now the energy in a magnetic field of strength H is $\mu H^2/8\pi$ per unit volume where μ is the magnetic permeability. Hence if du is a small element of volume in the neighbourhood of the point considered the magnetic energy in this element of volume will be $\mu(ev\sin\theta/r^2)^2\,du/8\pi$.

[1] J. J. Thomson, *Phil. Mag.*, 1899, xlviii. 547.

[2] J. J. Thomson, *Phil. Mag.*, 1881, ix. 229.

RECENT DETERMINATIONS OF e/m FOR ELECTRONS FROM DIFFERENT SOURCES

Source of Electrons.	Method employed.	e/m in e.m.u. per gm.	Reference.
Cathode rays . . .	Magnetic deflexion and potential difference between electrodes	$1{\cdot}77 \times 10^7$	Kaufmann, *Wied. Ann.*, 1897, lxii. 598.
,,	,,	$1{\cdot}769 \times 10^7$	Malassez, *Ann. de Chem. et de Phys.*, 1911, viii. 23, p. 231
,,	Magnetic and electrostatic deflexions	$1{\cdot}72 \times 10^7$	Bestelmeyer, *Ann. der Phys.*, 1907, iv. 22, p. 429
Wehnelt cathode . .	Magnetic deflexion and potential difference	$1{\cdot}776 \times 10^7$	Classen, *Phys. Zeit.*, 1908, ix. p. 762.
,,	,,	$1{\cdot}766 \times 10^7$	Bestelmeyer, *Ann. der Phys.* 1911, iv. 35, p. 909
Photo-electric effect .	,,	$1{\cdot}766 \times 10^7$	Alberti, *Ann. der Phys.*, 1912, iv. 35, p. 1133
Slow-moving β-rays from radium	Magnetic and electrostatic deflexions	$1{\cdot}763 \times 10^7$	Bucherer, *Ann. der Phys.*, 1909, iv. 28, p. 513
,,	,,	$1{\cdot}765 \times 10^7$	Neumann, *Verh. d. D. Phys. Ges.*, 1913, xv. 935
Electrons emitting spectral lines	Deduced from Zeeman effect	$1{\cdot}767 \times 10^7$	Weiss and Cotton, *Journ. de Phys.*, 1907, iv. 6, p. 429
,,	,,	$1{\cdot}771 \times 10^7$	Gmelin, *Ann. der Phys.*, 1909, iv. 28, p. 1079

The total magnetic energy in the space around the charged particle can be obtained by integrating this quantity throughout the whole of the field, from the surface of the particle to infinity. If the charge is supposed to act as if it were concentrated at the centre of a small sphere of radius a in air, for which $\mu=1$, it can easily be shown that the total magnetic energy in the space is equal to $\frac{1}{3}e^2v^2/a$.

For $\qquad du=r^2 \sin \theta dr d\theta d\phi.$

Thus the energy

$$=\frac{1}{8\pi}\cdot e^2\cdot v^2\cdot \int_a^a\int_v^\pi\int_v^{2\pi}\frac{\sin^3\theta}{r^2}dr d\theta d\phi,$$

and this when integrated gives the value above.

This magnetic energy is due to the fact that the charged particle is moving with a velocity v. It must therefore be given to the particle when the latter is set in motion. Thus if the particle has a mechanical mass M when uncharged, the work done in giving it a velocity v when it carries a charge e is obviously

$$\tfrac{1}{2}\mathrm{M}v^2+\tfrac{1}{3}\frac{e^2}{a}v^2=\tfrac{1}{2}\left(\mathrm{M}+\tfrac{2}{3}\frac{e^2}{a}\right)v^2.$$

The particle thus behaves as if its mass had been increased by $\frac{2}{3}e^2/a$. This may be called the electromagnetic mass of the particle. Thus even if M is zero the particle will behave as if it had a mass $\frac{2}{3}e^2/a$ owing to the fact that it carries a charge e.

Other assumptions as to the distribution of the charge on the electron give values differing from that of Thomson only by a numerical factor. Abraham,[1] on the assumption that the charge was distributed on the surface of a perfectly conducting sphere of radius a obtained the value $1/6\pi e^2/a$. The large values obtained for e/m for the electron compared with that for the hydrogen atom made it probable that at any rate part of its mass might be due to the charge which it carried.

A very close analogy to electromagnetic mass is to be found in hydrodynamics. When any body is moving through a fluid it sets the surrounding fluid in motion, and when the body is moving with a uniform velocity, this fluid motion travels along with the body as if it were rigidly attached to it. In order to set the body in motion with a definite velocity we have, therefore, to supply energy not only to the body itself, but also to the fluid. The body, in fact, behaves as if its mass had been increased by some fraction of the mass of fluid which it sets in motion.

The actual mass of an electron can be determined by experiments to be described later. As, however, we have no means of measuring the radius a of the electron, we cannot immediately determine whether the charge e is sufficient to account for the whole of the mass. The question has, however, been settled in another way.

It can be shown from electromagnetic theory that the expression obtained for the electromagnetic mass of a charged particle is only valid if the velocity of the particle is small compared with that of light, in practice if it is less than one-tenth that of light. If the velocity approximates to that of light the distribution of the electric field around the particle is altered in such a way as to increase the electromagnetic energy in the field, and thus the electromagnetic mass of the particle.

[1] Abraham, *Ann. der Phys.*, 1903, iv. 10, p. 105.

The electromagnetic mass is, therefore, not a constant, but increases rapidly as the velocity approaches that of light. The analysis is complicated, but an interesting illustration of the principles involved has been given by J. J. Thomson, founded on the hydrodynamic analogy already discussed. If the body moving through a fluid is unsymmetrical the apparent mass depends on the direction in which the body is moving. Thus a long thin circular cylinder moving through a fluid in a direction at right angles to its length has its mass increased by that of a volume of fluid equal to the volume of the cylinder. If the cylinder is moving in the direction of its axis the additional mass is comparatively negligible. It can also be shown that the cylinder will always tend to turn so that its axis is at right angles to the direction of motion, that is to say in such a way as to make the additional mass as large as possible.

Suppose, now, that the field of electric force round the charged particle is mapped out by a series of Faraday tubes, each of which is regarded as a long thin cylinder. When the particle is at rest the field is uniform and the tubes will be uniformly distributed. If we regard this system of Faraday tubes moving through space as equivalent to a similar system of cylinders moving through a fluid, the tubes which are directed parallel to the direction of motion will contribute practically nothing to the additional or electromagnetic mass, while those at right angles to this direction will contribute the maximum amount.

Now in a mechanical system such as that we have imagined the cylinders which were moving end on would be in unstable equilibrium, and if free to move would turn into the equatorial plane, where their effect on the apparent mass would be the greatest. On this analogy we should expect the Faraday tubes to tend to crowd together in a plane at right angles to the direction of motion of the particle; that is to say, the field would be strengthened in this plane and weakened in the direction of motion. This is what actually occurs. The mutual repulsion of neighbouring Faraday tubes, however, tends to maintain a uniform distribution, and it is only when the velocity is comparable with that of light that the distribution is seriously affected. When such velocities are attained the hydrodynamic analogy would lead us to expect that the electromagnetic mass would increase.

The theoretical relation between the electromagnetic mass and the velocity depends on the assumptions made as to the behaviour of the electron. The problem was attacked from two different standpoints by Abraham (*loc. cit.*) and Lorentz.[1] The former assumed that the electron was rigid, its dimensions being the same at all velocities. Lorentz, however, assumed that the electron contracted in the direction of motion in accordance with the hypothesis put forward by Fitzgerald[2] to account for the results of the Michelson-Morley experiment. On either hypothesis the electromagnetic mass of a particle for accelerations at right angles to the direction of motion differs from that for accelerations along the line of motion. The former is known as the transverse mass, the latter as the longitudinal mass. The two coincide when the velocity of the particle is small compared with the velocity of light. In determinations of e/m by measurements on the magnetic and electric deflexions it is obviously the transverse mass with which we are concerned, as the forces are applied at right angles to the direction of motion of the particles.

According to Lorentz's hypothesis the transverse mass m_t is given by the equation

$$m_t = \frac{m_0}{(1-\beta^2)^{\frac{1}{2}}},$$

where m_0 is the electromagnetic mass of the charged particle for velocities, small compared with that of light, and β is the ratio of the velocity of the particle to the velocity of light. The longitudinal mass m_l is given by the relation

$$m_l = \frac{m_0}{(1-\beta^2)^{\frac{3}{2}}}.$$

The same expressions can also be deduced very simply on the Principle of Relativity. The relations on the hypothesis of the rigid electron are far more complicated, the expression obtained by Abraham for the transverse mass being of the form

$$m_t = \frac{3m_0}{\beta^2}\left[\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right].$$

§ (19) Experimental Determinations of the Variation of the Mass of an Electron with the Velocity.—The experiments on the value of e/m for the cathode rays were made at speeds too small to produce any sensible effect on the electromagnetic mass. The β-rays from radioactive substances, however, furnish a source of electrons moving with much higher velocities than those usually obtained in the discharge tube, approaching in fact in some cases within about 2 per cent of the velocity of light. The value of e/m for these particles was investigated by Kaufmann,[3] by the method of magnetic and electrostatic deflexions. The two fields were applied in the same direction (instead of at right angles to each other as in Thomson's experiments on the cathode rays), and the deflexions produced by the two fields were thus at right angles to each other. The displacement of the rays from their undeflected

[1] Lorentz, *Theory of Electrons*, p. 210.

[2] Fitzgerald, *Nature*, June 16, 1892.

[3] Kaufmann, *Gottingen Nach.*, 1901.

position in a direction parallel to the fields measured the electrostatic deflexion, while the deflexion at right angles to this gave the magnetic deflexion. The two deflexions could thus be measured simultaneously. On making the experiments it was found that the single spot, given by the undeflected beam of rays, was drawn out into a continuous line, thus indicating that the value of e/m was a continuous function of the velocity. Kaufmann found that the effective mass of the particles increased with the velocity. For the slowest particles the value obtained for e/m ($1{\cdot}77 \times 10^7$) agreed with that for the cathode rays. The ratio, however, diminished to $1{\cdot}31 \times 10^7$ with a velocity of $2{\cdot}36 \times 10^{10}$, and to $\cdot 63 \times 10^7$ when the velocity was $2{\cdot}83 \times 10^{10}$ cm./sec.

If the whole mass of the electron is electromagnetic it should vary with the velocity in accordance with one or other of the relations given in the preceding section. If any portion of the mass is mechanical, this portion would be constant and the variation should be less rapid than that given by the formulae. Kaufmann considered that his results agreed more closely with the formula of Abraham than with that of Lorentz, but it is doubtful whether his experiments were sufficiently accurate to distinguish between the two theories. On either assumption it was clear that practically the whole of the mass of the β-particle was of electrical origin.

The question was definitely settled by the experiments of Bucherer [1] by a very ingenious method. The source of β-radiation, a small speck of radium fluoride R (*Fig.* 11), was

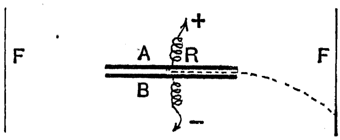


FIG. 11.

placed at the centre of a pair of plane parallel circular plates A, B, which could be charged to a suitable difference of potential. A uniform magnetic field was applied at right angles to the electric field, that is parallel to the plates, and extending over the whole of the path of the particles. The plates were placed so close together that only particles travelling parallel to their surface could escape from between them. These particles on emerging from the plates were acted upon only by the magnetic field, and after moving a certain distance were received upon a photographic film F, F, which was bent into a cylinder coaxial with the plates.

It is obvious from the arrangement of the apparatus that particles will only be able to escape from the space between the plates if the electric and magnetic forces upon them are exactly equal and opposite. Otherwise they will be deflected by whichever of the fields is the stronger, and will strike one or other of the plates. The electric force upon all the particles is the same and is equal to Xe where X is the field. The mechanical force due to the magnetic field, however, depends on the direction of projection of the particles, being given by H$ev \sin\theta$, where H is the magnetic field, v the velocity of the particle, and θ the angle between the direction in which the rays are travelling and the field. Thus the particles emerging from the plates at an angle θ with the direction of the magnetic field will have a velocity given by

$$Xe = Hev \sin\theta.$$

Hence the particles escaping from the plates along any given radius all have the same velocity, which can be calculated when the angle between the radius and the direction of the magnetic field is known. After leaving the plates the particles are deflected by the magnetic field. The magnetic field is reversed during the experiment and the distance between the two traces thus obtained on the photographic film at any point is twice the deviation experienced by the particles travelling in this direction. From this measured deviation the value of e/m for the corresponding particles can be determined. Thus v and e/m are known for all the particles emitted by the source.

Bucherer found that his results agreed very closely with the theoretical values given by the formula of Lorentz, the maximum discrepancy being less than 1 per cent. The agreement with the formula of Abraham was much less satisfactory. The values of m_v/m_0 as calculated from the Lorentz formula are given in the table, where m_v is the transverse electromagnetic mass at a velocity v, m_0 the value for slowly moving electrons, and β the ratio of the velocity of the particle to the velocity of light. The value of e/m_0 as deduced from Bucherer's experiments is $1{\cdot}763 \times 10^7$ e.m.u. per gm. These experiments have been accepted as proving the validity of the Lorentz formula, and the assumption that the whole mass of the electron is electromagnetic in origin.

TRANSVERSE MASS OF AN ELECTRON AT DIFFERENT VELOCITIES

β.	m_v/m_0.	β.	m_v/m_0.
01	1·000	·80	1·667
·10	1·005	·90	2·294
·30	1·048	·95	3·203
·50	1·115	·98	5·025
·60	1·250	1·00	∞
·70	1·400		

[1] Bucherer, *Ann. der. Phys.*, 1909, iv. 28, p. 513.

§ (20) DETERMINATION OF THE CHARGE ON AN ELECTRON.—It has been shown in the preceding section that the electron consists simply of a discrete negative charge, dissociated from anything which can be called matter. It may be described as an atom of electricity, since all electrons have the same charge, which is also the same as that carried by a monovalent ion in solution. There is no evidence of the existence of any charge smaller than that of the electron, while all charges sufficiently small to be compared directly with that of the electron are found to be exact multiples of this charge. The determination of the magnitude of this electronic, or "atomic" charge is therefore of some importance.

The earliest determinations were based on the discovery of C. T. R. Wilson,[1] that a charged ion was capable of serving as a nucleus for the condensation of vapour in a supersaturated space.

It had been shown by Aitken,[2] that the clouds which form in a gas which is supersaturated with water vapour are produced by the condensation of the vapour around small dust particles or other nuclei already présent in the gas. If these nuclei are removed by filtering the gas through cotton-wool, a very considerable degree of supersaturation may exist in the space without the formation of any drops. If, however, the pressure of aqueous vapour exceeds about eight times its saturation pressure for the temperature of the experiment a fine drizzle sets in, even in the absence of all nuclei.

Wilson showed that, in a dust-free space, condensation could be produced on charged ions when the supersaturation was appreciably less than this eight-fold value. The method consisted in making a very rapid expansion in air contained in a closed space saturated with water vapour. The rapid expansion of the gas causes an adiabatic cooling of the space, the lowest temperature reached being given by the relation[3]

$$\log_e \frac{\theta_1}{\theta_2} = (\gamma - 1) \log_e \frac{v_2}{v_1},$$

where θ_1, v_1 and θ_2, v_2 are the temperatures and volumes of the gas before and after expansion, and γ is the ratio of the specific heats of the gas. The saturated pressure of aqueous vapour at the temperatures θ_1 and θ_2 could be obtained from tables, and hence the degree of supersaturation could be calculated.

In this way Wilson showed that in an ordinary dust-free space no cloud was formed with values of v_2/v_1 less than 1·375. If, however, the gas was ionised by passing a beam of X-rays through it, a dense cloud was formed with an expansion of only 1·26, thus showing that fresh nuclei capable of condensing the water vapour were formed in the gas by the action of the rays If these nuclei are charged ions they should be removable by an electric field. This was found to be the case. The negative ions were found to be more efficient in producing condensation than the positive. If only positive ions were present condensation did not commence until an expansion of 1·30 was reached. Between the limits 1·26 and 1·30 condensation took place only on the negative ions.

These results can readily be applied to determine the charge on a single ion. The small drops making up the cloud fall under the action of gravity with a uniform limiting velocity v given by Stokes' law for the fall of a small sphere through a viscous fluid.[4] If r is the radius of the drop, η the viscosity of the air, ρ the density of the drop, and g the acceleration due to gravity

$$v = \frac{2}{9} \frac{g\rho r^2}{\eta}.$$

This velocity can be determined by observing the rate at which the surface of the cloud settles down, and the value of r can thus be calculated. Knowing the radius of the drop, the mass of a single drop can be calculated. If M is the mass of water vapour saturating the vessel at the initial temperature, and M′ the mass required to saturate it at the lower temperature, the mass of water condensed in the cloud is M − M′. This, when divided by the mass of a single drop, gives us the number of drops in the cloud. The total charge brought down by the cloud can be measured by allowing the cloud to settle on a horizontal surface connected to an electrometer. The total charge divided by the number of drops in the cloud gives the charge on each nucleus, that is, on each negative ion. The experiment was first performed by J. J. Thomson,[5] who showed that the value of the charge was independent of the mode of production of the ions, the same result being obtained for the ions produced by a beam of X-rays as for the ions set free from a metal plate illuminated by ultra-violet light. The latter are, however, emitted as electrons. The method therefore gives us the value of the electronic, or "atomic" charge.

The method was subsequently improved by H. A. Wilson.[6] The clouds were formed in the space between a pair of horizontal plates at a distance of about one centimetre apart, and the radius of the drops determined

[1] C. T. R. Wilson, *Phil. Trans.* A, 1897, clxxxix. 265; A, 1899, cxcii. 403.
[2] Aitken. *Trans. Roy. Soc. Edin.*, 1880, xxx. 337.
[3] See "Thermodynamics," Vol. I.
[4] See "Friction," Vol. I.
[5] J. J. Thomson, *Phil. Mag.*, 1898, xlv. 125.
[6] H. A. Wilson, *ibid.*, 1903, v. 429.

by the application of Stokes' law. An electric field X was then applied between the plates so as to attract the drops towards the upper electrode, the expansion being adjusted so that the drops were all negatively charged. The electric force Xe thus acts in the opposite direction to gravity, and by properly adjusting the value of X the cloud can be made to remain stationary, neither rising nor falling. Under these circumstances the electric force must be equal to that of gravity, that is,

$$Xe = mg.$$

The mass m is calculated from the radius of the drop, and so e can be determined. Both forms of the experiment are subject to the difficulty that the drops begin to evaporate as soon as they are formed, since the gas after expansion rapidly warms up again by conduction and radiation from the walls of the vessel. This defect has been overcome in more recent experiments, due to Millikan.

§ (21) MILLIKAN'S DETERMINATION OF THE CHARGE e.—Instead of condensing drops around the ions, Millikan[1] formed his drops mechanically by some sort of sprayer, and allowed them to fall into a chamber (*Fig.* 12) saturated with

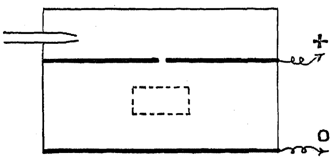


FIG. 12.

the liquid. The chamber was closed top and bottom by a pair of parallel plates, which could be charged to any required difference of potential, thus producing a vertical electric field across the space, or could be connected together, thus allowing the drop to fall under gravity alone. The drops showed no tendency to evaporate, and by suitably adjusting the field a single drop could be kept under observation for several hours.

The drops are not initially charged, but if the air is ionised they acquire charges by collision with the charged ions. Since the diameter of the drop is large compared with that of an ion, its potential rises comparatively slowly, and the drop may accumulate several charges of the same sign before its potential becomes sufficiently high to prevent other like charges from reaching it. If, however, all ions carry the electronic charge e the actual charge E on the drop at any instant should be some exact integral multiple of this charge. E was determined, as in Wilson's experiment, by observing the rate of fall under the action of gravity, and the field required to keep the drop stationary. The drops were observed by means of a long focus microscope.

A very large number of charges were thus measured by Millikan using drops of various sizes and of various liquids, and ions produced by different ionising agents. The charge was found in every case to be an integral multiple of the number $4{\cdot}774 \times 10^{-10}$ electrostatic units, to a very considerable degree of accuracy. The experiments of Millikan bring out very clearly the atomic nature of electricity.

§ (22) DETERMINATION OF e FROM EXPERIMENTS ON THE α-RAYS.—The value of e can also be deduced from experiments on the charges carried by the α-particles emitted by radioactive substances. Since negative electricity is atomic in structure a particle cannot lose less than one electron, and hence every positive charge must also be, numerically, an exact integral multiple of the charge e. The α-rays consist of atoms of helium carrying a positive charge. Owing to the energy of the particles each particle produces a distinct scintillation when it falls on a suitable fluorescent screen. It is thus possible actually to count the number of α-particles passing per second through a given aperture. If a Faraday cylinder connected to an electrometer is then placed behind the aperture the charge carried through the aperture per second by the particles can be measured. The current entering the Faraday cylinder divided by the number of particles entering per second gives the charge on a single particle.

Experiments on these lines were carried out by Rutherford and Geiger,[2] and by Regener.[3] In Rutherford's experiments the trustworthiness of the method of counting the α-particles by use of the fluorescent screen was tested by an electrical method of counting. The two methods gave the same results. According to Rutherford's experiments the charge on the α-particle is $9{\cdot}3 \times 10^{-10}$ electrostatic units. This is obviously twice the electronic charge. The α-particle thus consists of an atom which has lost two charges. The value of e as deduced from these experiments is therefore $4{\cdot}65 \times 10^{-10}$ e.s.u. Regener's experiments give a value for e of $4{\cdot}79 \times 10^{-10}$ e.s.u.

§ (23) DETERMINATION OF e FROM THE NUMBER OF MOLECULES IN A GRAM-MOLECULE OF GAS.—Since the charge e is identical with that carried by a monovalent ion in electrolysis, the value of the electronic charge can be determined from the phenomena of electrolysis if the number of molecules in a gram-molecule of gas is known. The mass of hydrogen liberated by the passage

[1] Millikan, *Phil. Mag.*, 1910, vi. 19, p. 209; *Physical Rev.*, 1911, xxxii. 349; *ibid.*, 1913.

[2] Rutherford and Geiger, *Proc. Roy. Soc.* A, 1908, lxxxi. 141.

[3] Regener, *Sitz.-Ber. der K. Preuss. Akad. der Wiss.*, 1909, xxxviii. 948.

of one electromagnetic unit of electricity is $1{\cdot}04 \times 10^{-4}$ grams; or in other words, the atoms in one gram of hydrogen convey a charge of $1/1{\cdot}04 \times 10^{-4}$ or $\cdot 96 \times 10^4$ e.m.u. Assuming that the hydrogen ion in solution consists of a single atom carrying a charge e, we have $ne = \cdot 96 \times 10^4$, where n is the number of atoms in a gram of hydrogen. Since there are two atoms of hydrogen in a molecule, and a gram-molecule weighs two grams, n is obviously also equal to the number of molecules in a gram-molecule of the gas, an important number known as Avogadro's constant. If n is known we can immediately determine e.

Numerous determinations of n, based on various atomic properties, have been made. The only method capable of any accuracy is that of Perrin,[1] based on observations of the Brownian movements. He showed that the particles in suspensions which showed the Brownian movements behaved like the molecules of a very dilute gas of very large atomic weight. It was possible, therefore, to apply the kinetic theory of gases to such suspensions, and so, from observations on these visible particles, to determine the various constants in the gas equations with far more certainty than in the indirect experiments made on actual gases. In this way Perrin obtained a value for n of $68{\cdot}2 \times 10^{22}$. Substituting this value in the above equation $ne = \cdot 96 \times 10^4$, he obtained a value for e of $1{\cdot}40 \times 10^{-20}$ e.m.u. or $4{\cdot}2 \times 10^{-10}$ e.s.u.

§ (24) Determination of e from Planck's Theory of Radiation.—The "quantum" theory of radiation, as developed by Planck,[2] provides a method of determining the number of molecules in a given mass of gas from observations on the energy of radiation, and thus affords another method of deducing the value of the elementary electronic charge. The theory assumes that the emission and absorption of radiant energy takes place not continuously, but in definite bundles, or "quanta," the magnitude of which is equal to $h\nu$, where ν is the frequency of the radiation, and h is a universal constant, known as Planck's constant, and has the value $6{\cdot}548 \times 10^{-27}$ erg. sec. On this assumption it can be shown that if L_λ is the proportion of radiant energy of wave-length λ contained in the full radiation from a black body at an absolute temperature T,

$$L_\lambda d\lambda = \frac{8\pi ch\lambda^{-5}}{\epsilon^{ch/RT\lambda} - 1} d\lambda,$$

where c is the velocity of light, h is Planck's constant, and R is the constant in the gas equation $pv = RT$, considering a single molecule of the gas. By differentiating the expression for L_λ with respect to λ and equating to zero we obtain the equation

$$5\left(1 - \epsilon^{\frac{ch}{RT\lambda}}\right) + \frac{ch}{RT\lambda}\epsilon^{\frac{ch}{RT\lambda}} = 0.$$

Solving this for the quantity $ch/RT\lambda$, we find that there are two real roots, viz. 0 and 4·965, and it can easily be shown that the latter corresponds to a maximum. The value of the wave-length λ_m for which the intensity of the radiation is a maximum is thus given by the expression

$$\frac{ch}{RT\lambda_m} = 4{\cdot}965$$

or

$$\lambda_m T = \frac{ch}{4{\cdot}965R} = \text{constant}.$$

This is the well-known displacement law of Wien, for which there is ample experimental evidence. The value of the constant is, according to the measurements of Lummer and Pringsheim,[3] 0·294 cm. deg. Substituting the known values of c and h, we have

$$R = 1{\cdot}346 \times 10^{-16} \text{ erg. deg.}^{-1}.$$

This value for R is, as we have mentioned, for a single molecule. We may therefore write the gas equation in the form $pv = NRT$, where N is the number of molecules present in the volume of gas considered and R is the constant which has been just determined. Now the volume of a gram-molecule of gas at normal temperature and pressure is 22·21 litres. Thus if N is the number of molecules in a gram-molecule of gas at 0° C. (273° abs.) and 76 cm. pressure, we have

$$(76 \times 13{\cdot}6 \times 981) \times (22{\cdot}21 \times 10^3) = N \times 1{\cdot}346 \times 10^{-16} \times \cdot 273,$$

reducing the pressure to dynes per sq. cm. and the volume to cubic centimetres. Thus N, the number of molecules in a gram-molecule, is equal to $61{\cdot}3 \times 10^{22}$. Substituting this value in the relation $Ne = 0{\cdot}96 \times 10^4$, obtained from the electrolysis of hydrogen, we have e, the elementary electronic charge, $= 1{\cdot}563 \times 10^{-20}$ e.m.u. or $4{\cdot}69 \times 10^{-10}$ e.s.u. The excellent agreement of this value with those of Millikan, Rutherford, and others by more direct methods furnishes very strong support both for the fundamental assumption of Planck and for the experimental values of e obtained by other methods.

§ (25) Values of e.—The various recent determinations of e are summarised in the table on the following page. The agreement is very good, except for the determinations of Perrin from the Brownian movement. The latter, of course, refers to the charge carried by a monovalent ion in solution. It is, however, sufficiently close to the other values

[1] Perrin, *Comptes Rendus*, 1908, cxlv. 967 ; 1911, clii. 1380.
[2] See "Quantum," Vol. III.
[3] Lummer and Pringsheim, *Verhand. der Deutsch. Phys. Ges.* i. 230.

to make it practically certain that the two charges are the same, but the agreement is not as close as might have been expected. Millikan claims an accuracy of one part in a thousand for his work, while Perrin considers that his results cannot be in error by more than three per cent. Perrin suggests that Stokes' law does not apply accurately to the drops used by Millikan, and that the allowance made by Millikan for the variation from Stokes' law is insufficient. On the other hand Millikan's value agrees closely with those obtained from the determinations of the charge on the α-particles. The value obtained by Millikan is the one generally accepted at the moment, but it is very desirable that the discrepancy should be cleared up.

Observer.	Method.	e in e.s.u.
Begeman .	Water drops (Thomson's method)	$4{\cdot}67 \times 10^{-10}$
Millikan .	Oil and mercury drops	$4{\cdot}774 \times 10^{-10}$
Perrin .	Brownian movements	$4{\cdot}2 \times 10^{-10}$
Rutherford and Geiger	Charge on α-particle	$4{\cdot}65 \times 10^{-10}$
Regener .	Charge on α-particle	$4{\cdot}79 \times 10^{-10}$
Planck .	Deduction from theory of radiation	$4{\cdot}69 \times 10^{-10}$

Determinations of the electronic charge, e.

§ (26) Mass and Radius of an Electron.—The determination of the value of the electronic charge enables us at once to calculate several other important constants. Assuming that e is $4{\cdot}77 \times 10^{-10}$ e.s.u. or $1{\cdot}59 \times 10^{-20}$ electromagnetic units, then since e/m for an electron is $1{\cdot}764 \times 10^7$ e.m.u. per gram, the mass of an electron is $(1{\cdot}59 \times 10^{-20})/(1{\cdot}764 \times 10^7)$ or about 9×10^{-28} grams. As the mass is electromagnetic we can further deduce the radius of the electron. Assuming Thomson's formula, the mass of an electron is given by $\frac{2}{3}\, e^2/a$ where a is the radius. The radius a is, therefore, of the order of $1{\cdot}9 \times 10^{-13}$ cm. This is about $\frac{1}{100,000}$ part of the value generally assumed for the radius of an atom. To use a comparison suggested by Lodge, the size of an electron bears to that of an atom very much the same relation as that of a pea to a cathedral.

The determination of e also gives us the actual mass of a hydrogen atom. Since the ratio of the mass to the charge for a hydrogen ion in solution, *i.e.* the electro-chemical equivalent of hydrogen, is $1{\cdot}04 \times 10^{-4}$ e.m.u., the mass of the hydrogen atom is given by the expression $(1{\cdot}04 \times 10^{-4}) \times (1{\cdot}59 \times 10^{-20})$ or $1{\cdot}65 \times 10^{-24}$ gm. The masses of the atoms of other elements can be obtained by multiplying this number by the corresponding atomic weight. The values given in this section all depend on the value assumed for e, the electronic charge.

§ (27) Electron Theory of Metallic Conduction.—Since electricity is to be regarded as made up of electrons the passage of an electric current implies the flow of electrons from one end of the conductor to the other. A conductor, therefore, is a substance which contains electrons in such a state that they are free to move under the action of an electric field.

According to the most prevalent theories, a metallic conductor consists of atoms of the metal, some neutral and some positively charged, surrounded by an atmosphere of electrons in a free state. This electronic atmosphere is supposed to be produced by the emission of negative electrons from the neutral atoms. All metals are electropositive and therefore emit electrons readily, though the cause of the emission in the case of an ordinary conductor is perhaps not obvious. The positively charged atoms will, of course, attract electrons from the surrounding space, and the number of free electrons will accumulate until the number of atoms dissociating per second is equal to the number recombining. The conditions of equilibrium are thus similar to those of a liquid evaporating into a closed space, and we may regard the electronic atmosphere as being due to an evaporation of electrons from the atoms of the metal.

Since the electrons are small compared with the spaces between the atoms they will behave like the molecules of a tenuous gas, and we may apply the principles of the kinetic theory of gases to the electronic atmosphere. They will exhibit the usual velocities of thermal agitation, and these velocities will be distributed according to the Maxwell-Boltzmann law. It is interesting to note that these suppositions have been verified experimentally by O. W. Richardson,[1] who has measured the distribution of velocities among the electrons emitted from the surface of an incandescent metal. Since the velocity of thermal agitation is inversely proportional to the square root of the mass of the particle, and the mass of an electron is $\frac{1}{1850}$ that of a hydrogen atom or $\frac{1}{3700}$ that of the hydrogen molecule, the mean velocity of the electron will be $\sqrt{3700}$ that of the hydrogen molecule or about 11×10^6 cm. per sec. at 0° C.

Since these velocities will be equally distributed in all directions in the metal they will produce no transference of electricity as a whole in any direction. If, however, an electric field is applied to the metal, this will produce a steady drift of electrons in the direction of the field, that is to say, it will set up an electric current. Since there is ample experimental evidence that the passage of an electric current through a metal does not

[1] O. W. Richardson, *Phil. Mag.*, 1908, xvi. 890; 1909, xviii. 681.

produce any transference of the material of the conductor we must suppose that the positively charged atoms are not free to move, and thus take no part in the conduction of the current. The complete analysis of the motion of the electrons under the field is very complicated, and has not yet been satisfactorily solved. The theory can, however, be illustrated by making simplifications analogous to those made in the elementary kinetic theory of gases.

Let us assume that all the electrons are moving with the mean velocity of thermal agitation v, and that each travels a constant distance λ between two successive collisions with the atoms of the conductor. This distance, which we may call the mean free path, will be equal to vt, where t is the time which elapses between two collisions. We will also assume that the motion of an electron after collision is entirely independent of its previous history, and that the time taken up by a collision is negligible in comparison with the interval between two collisions.

Now if e is the charge on the electron and X the applied electric field, the force acting upon it is Xe, and the acceleration in the direction of the field is Xe/m. Thus in the interval t which elapses between two collisions the electron acquires a velocity $\mathrm{X}(e/m)t$, and the average velocity produced by the field between two collisions is thus $\frac{1}{2}\mathrm{X}(e/m)t$. As we have assumed that the effect of a collision obliterates the previous history of the electron, this will obviously be the average velocity of the electron through the conductor under the influence of the field. As the velocity acquired under the field is very small compared with the velocity of thermal agitation v it will not materially affect the free time t, and we may therefore write $t=\lambda/v$. The velocity of the electrons in the directions of the field is thus

$$\frac{1}{2}\frac{e}{m}\frac{\lambda}{v}\mathrm{X}.$$

Now if n is the number of electrons per unit volume, the charge conveyed per second across any cross-section of the conductor of area A at right angles to the direction of flow is Aneu, where u is the velocity of the electrons, that is

$$\mathrm{A}\cdot\frac{1}{2}n\frac{e^2}{m}\frac{\lambda}{v}\mathrm{X}.$$

Thus the current i is equal to

$$\frac{1}{2}n\frac{e^2}{m}\frac{\lambda}{v}\mathrm{X}\cdot\mathrm{A}.$$

But by the kinetic theory of gases we may write $\frac{1}{2}mv^2=\alpha\mathrm{T}$, where T is the absolute temperature, and α is Boltzmann's constant. Thus

$$i=\mathrm{A}\frac{ne^2\lambda v}{4\alpha\mathrm{T}}\cdot\mathrm{X}.$$

If the field is uniform, $\mathrm{X}=\mathrm{E}/d$, where E is the potential difference between the ends of the conductor and d is its length. Thus

$$i=\frac{\mathrm{A}}{d}\left(\frac{ne^2\lambda v}{4\alpha\mathrm{T}}\right)\mathrm{E},$$

which gives us Ohm's law since the quantities within the bracket are constant for a given temperature and substance. The quantity in the bracket is obviously the specific conductivity of the substance. It is constant for a given substance at a given temperature. The only quantities which vary from substance to substance are λ and n. As it does not seem probable that there will be any great variations in λ, we must suppose that differences in conductivity are due mainly to differences in the number of free electrons in the different conductors. For most pure metals the specific conductivity is inversely proportional to the absolute temperature. According to our analysis this will be the case if $n\lambda v$ is independent of the temperature.

Since the best conductors of electricity are also the best conductors of heat, it seems natural to suppose that the two phenomena are attributable to the same cause. Assuming that the atmosphere of electrons conducts heat in the same way as a gas, its thermal conductivity k will be given by the equation $k=(\pi/8)nv\alpha\lambda$, where n, v, α, and λ have the same significance as in the previous equation. As insulators conduct heat to a certain extent, there will also be a certain amount of conductivity in the metal apart from its electron atmosphere, but as the thermal conductivity of electrical insulators is very much smaller than that of metals, we may expect that this effect will be a small fraction of the whole. If we neglect it altogether and assume that the thermal conductivity of the metal is entirely due to the electrons, the ratio of the thermal to the electrical conductivity of a conductor will be given by

$$\left(\frac{\pi}{8}nv\alpha\lambda\right)\Big/\left(\frac{ne^2\lambda v}{4\alpha\mathrm{T}}\right)=\frac{\pi}{2}\left(\frac{\alpha}{e}\right)^2\cdot\mathrm{T}.$$

Thus the ratio of the thermal and electrical conductivities at a given temperature is independent of the nature of the conductor. This is the well-known law of Wiedemann and Franz. Moreover, the expression for the ratio consists only of known constants. The constant α is approximately 2×10^{-16} and e is $1{\cdot}59\times10^{-20}$ e.m.u. Thus at 18° C. or 291° absolute the ratio should have the value

$$\frac{\pi}{2}\left(\frac{2\times10^{-16}}{1{\cdot}59\times10^{-20}}\right)^2 291 \text{ or } 7{\cdot}3\times10^{10}.$$

The experimentally determined values for the ratio vary from $6{\cdot}7\times10^{10}$ for copper to 8×10^{10} for iron. Considering the assumptions made during the argument the agreement is very

remarkable. According to the formula, the ratio of the thermal to the electrical conductivity is directly proportional to the absolute temperature. This is also in close agreement with the experimental results for pure metals.

The electron theory also gives at any rate a qualitative explanation of other effects associated with the passage of a current through a conductor. As the number of free electrons per unit volume is different in different metals the pressure of the electronic atmosphere will also be different. Thus if two dissimilar metals are brought into metallic contact there will be a flow of electrons from the one to the other until the difference of potential produced by the transference is sufficient to stop any further flow of electricity between them. There will thus be a permanent difference of potential established between the two metals. Again, if a current is passed from the metal of lower to the metal with the higher electronic pressure, work must be done against the electronic pressure, and this will involve an absorption of heat at the junction between the metals. Conversely the passage of a current in the opposite direction will result in the transference of electrons from a higher to a lower pressure, and heat will be liberated. This is, of course, the Peltier effect. Regarding the electrons again as behaving like the molecules of a gas, it is easy to show that the coefficient Π of the Peltier effect should be given by

$$\Pi = \frac{2}{3}\frac{\alpha}{e}\theta \log_e \frac{N_1}{N_2},$$

where N_1, N_2 are the number of electrons per unit volume in the two conductors. This result is due to Sir J. J. Thomson (Thomson, *Corpuscular Theory of Matter*).

Again, since a moving electron is deflected by a magnetic field, if a magnetic field is applied to a conductor at right angles to the direction of the current flowing through it the electrons will be deflected towards one side of the conductor and a transverse electric field will thus be set up at right angles both to the current and the field. This is of course the Hall effect. Similar qualitative explanations can also be given of other known effects, such as the Nernst and Ettinghausen effect, and the Leduc effect. The explanations, however, are not very satisfactory when applied numerically. In particular the Hall effect is in some metals of the opposite sign to that predicted by the simple electron theory, and so far no satisfactory explanation of the discrepancy has been proposed. The success of the theory in explaining electrical and thermal conductivities, however, would appear to show that the theory is fundamentally sound, and further developments may be hoped for. For a fuller account of the theory, Lorenz's *Theory of Electrons*, Bohr's *Studies over metallernes Elektrotheori*, or O. W. Richardson's *Electron Theory of Matter* may be consulted.

§ (28) THE ELECTRON THEORY OF THE ATOM.—The fact that matter of all kinds is capable of emitting electrons under suitable stimulus furnishes presumptive evidence that these electrons form an integral part of the atom of every element. The further proof that the mass of the electron is entirely electrical naturally suggested the inquiry as to whether all mass might not be of this kind, the mass of an atom being merely due to the electromagnetic mass of the charges contained in it. This view is now very generally held, though the proof is not yet complete. The mass of an electron is about $\frac{1}{1850}$ of that of a hydrogen atom. It would thus require 1850 such electrons to make up the mass of the lightest atom. Considering the smallness of the electron, there is no inherent impossibility in this suggestion, and speculations along these lines were freely indulged in for a time. Actual experiments, however, have proved fairly conclusively that the number of electrons in an atom is of the same order as its atomic weight, and is probably equal to its atomic number. The atomic number of an element is its number in the table obtained by arranging the elements in the order of their ascending atomic weights. Thus hydrogen has the atomic number one, helium two, lithium three, and so on. In the later parts of the series of elements the periodic classification indicates the possibility of the existence of elements as yet undiscovered. These gaps are taken into account in assigning atomic numbers to the still heavier elements. The atomic number can be determined directly by measurements on the characteristic X-ray spectrum of the element. It is now practically certain that the hydrogen atom only contains a single negative electron. If the mass of the hydrogen atom is electrical it must therefore be associated with the positive electricity in the atom, and not with the negative.

Since the atom as a whole is neutral, it must contain sufficient positive electricity to neutralise exactly the charges on the negative electrons. Again, as the most obvious feature of atoms in general is their great stability, the electrical system must be a stable one. Kelvin[1] suggested that the electrons were embedded in a uniform sphere of positive electricity of the dimensions of the atom itself. The possible stable arrangements of electrons inside such a sphere have been worked out in great detail by J. J. Thomson,[2]

[1] Kelvin, *Phil. Mag.*, 1902, iii. 257.
[2] J. J. Thomson, *Phil. Mag.*, 1904, vii. 237 ; *Corpuscular Theory of Matter*, 1907, cap. 6.

who obtained some exceedingly interesting and suggestive results.

The experiments of Geiger and Marsden,[1] and of Geiger [2] on the scattering of α-particles during their passage through thin metal foil, have, however, shown that some of these particles undergo deflexions much greater than could be produced by collision with an atom of the kind suggested by Kelvin. To produce deflexions of the order actually obtained (some of the particles are deflected through more than a right angle) it is necessary to suppose that the mass and the positive charge of the atom are concentrated on a nucleus of dimensions much smaller than the radius of an atom. More recent experiments by Rutherford [3] on the collisions between α-particles and the molecules of a gas indicate that the size of this positively charged nucleus is of the same order as that of an electron.

Rutherford, therefore, suggested [4] that the atom consists of a small positively charged nucleus in which practically the whole of the mass of the atom is concentrated, surrounded by an atmosphere of sufficient negative electrons to make the system as a whole electrically neutral. The resultant charge on the nucleus is supposed to be equal to the atomic number of the element, and the constitution of the nucleus thus determines the mass of the atom, that is its atomic weight. The other chemical and physical properties of the atom are determined by the number and arrangement of the negative electrons around the nucleus. This nuclear atom is the only structure so far proposed which seems capable of providing an adequate explanation of the numerous investigations which have recently been made of the internal structure of the atom.

On the ordinary laws of electricity and classical dynamics such a system as has been proposed by Rutherford could only be stable, if at all, if the electrons were in rapid rotation. On the ordinary principles of electrodynamics, however, an electron moving in a closed orbit would be continually radiating energy. There would thus be a continual drain on the energy of the atom, which would therefore eventually collapse. The nuclear theory thus apparently fails to account for the most fundamental property of an atom, namely, its great stability. To overcome this difficulty Bohr [5] has frankly discarded the classical principles by introducing the following additional hypotheses :

(i.) That the electrons are rotating about the central nucleus in closed orbits, and that these motions are stable when the angular momentum of the electron is an integral multiple of $h/2\pi$, where h is Planck's constant, a universal constant which can be deduced from the laws of radiation and other phenomena, and has the value $6{\cdot}55 \times 10^{-27}$ erg. sec.

(ii.) That instead of radiating continuously in accordance with electromagnetic theory, the electrons only radiate energy when they pass from one stable orbit to another. The radiation thus emitted is assumed to be monochromatic, corresponding to a single line in the spectrum of the element, and its frequency is assumed to be determined by the relation $E_1 - E_2 = h\nu$, where E_1, E_2 are the energies of the electron in the two states, and ν is the frequency of the radiation emitted.

Bohr's theory was specially directed to providing a model which would account numerically for the lines in the spectrum of the atom. The theory has been eminently successful in the case of the hydrogen atom, which is assumed to consist of a positively charged nucleus and a single negative electron. In this simple case Bohr's theory enables him to calculate the constant K in Balmer's series for hydrogen in terms of e, m, and h. The calculated value is $3{\cdot}26 \times 10^{15}$. That obtained from spectroscopic observation is $3{\cdot}29 \times 10^{15}$. The agreement is exact, within the limits of accuracy with which the various quantities have been determined. Some success has also been obtained with the case of helium.

In view of the close agreement between the nucleus theory of the atom and the recent experimental work, the fact that the theory would seem to necessitate a modification of the principles of classical dynamics cannot be made a valid objection. That some such modification is required when dealing with atomic phenomena had previously been indicated by Planck [6] from a study of the phenomena of thermal radiation. The "Quantum" theory of Planck indicates at any rate the general nature of the modifications which will be required, and it seems likely that for some time mathematical attempts to solve the question of the structure of the atoms will take the form of more or less empirical mixtures of "quantum" theory with ordinary electrodynamics.

More recently J. J. Thomson [7] has suggested that the stability of the nuclear arrangement might be secured, by assuming that the ordinary inverse square law for the force between two electric charges was modified when the distance between the charges was less than the radius of the atom. Rutherford's experiments on the collision of α-particles with light atoms [8] seem to afford some support for this hypothesis.

[1] Geiger and Marsden, *Proc. Roy. Soc.* A, 1909, lxxxii. 495.
[2] Geiger, *Proc. Roy. Soc.* A, 1910, lxxxiii. 492.
[3] Rutherford, *Phil. Mag.*, 1919, xxxvii. 581.
[4] *Ibid.*, 1911, xxi. 669.
[5] Bohr, *Phil. Mag.*, 1913, xxvi. 476, 857; 1914, xxvii. 506; 1915, xxx. 394.
[6] Planck, *Ann. der Phys.*, 1912, xxxvi. 642.
[7] Thomson, *Phil. Mag.*, 1919, xxxvii. 419.
[8] Rutherford, *Proc. Roy. Soc.* A, 1920, xcvii. 374.

§ (29) THE NUMBER OF ELECTRONS IN THE ATOM.—The number of electrons surrounding the nucleus of the atom can be determined by at least three independent methods. The most certain results are those given by experiments on the scattering of α-particles already alluded to.[1] In this case the deflexions of the α-particles are due to collisions between the α-particle and the positive nucleus of the deflecting atom, the deflexions produced by the electrons also present in the atom being, on account of their relatively small mass, almost negligible. A measurement of the most probable angle of deflexion makes it possible to calculate the magnitude of the charge on the deflecting nucleus. Since the atom is neutral, however, this must be equal to the sum of the charges on the surrounding electrons, and hence the number of the latter can be determined. The numbers obtained in this way were of the order of one-half the atomic weight, that is to say, they were approximately equal to the atomic number.

The number of electrons in the atom can also be estimated from experiments on the scattering of the β-particles by matter.[2] In this case the scattering is mainly due to collisions between the β-particles and the electrons in the atom. The effect of the positive charge, however, cannot be ignored, and the results obtained depend on the assumptions made on this point. Adopting a theory due to J. J. Thomson,[3] with which the experimental results seemed to be in good agreement, Crowther calculated that the number of electrons in an atom was almost exactly three times its atomic weight. A recalculation of the results by Rutherford,[4] starting from different assumptions, gave values agreeing fairly closely with those obtained from the experiments on the α-rays.

Further evidence on the point is furnished by experiments on the scattering of X-rays. The mechanism of the scattering is entirely different from that of the scattering of α- or β-rays. It is assumed that the electrons in the material through which the X-rays are passing are thrown into forced vibrations by the electric field in the rays, and thus in turn become radiators of energy of the same type as that by which they are excited. The intensity of the radiation will obviously be proportional to the number of electrons affected. A mathematical theory of the effect has been given by J. J. Thomson,[5] and experimental determinations of the scattering have been made by numerous observers. The results of Barkla and Dunlop[6] give values for the number of electrons in the atom very nearly equal to one half the atomic weight. Later experiments by Auren[7] however, indicate considerably smaller values for the scattering of the rays than those obtained by Barkla, and it is suggested that only the outermost of the electrons in the atom are actually affected by the rays.

On the whole there is a general agreement between the results obtained by these different methods of approach, and though each may be open to criticism, the cumulative evidence is very strong. Van der Broek[8] seems to have been the first to suggest that the number of electrons in an atom, or, in other words, the charge on the positive nucleus, was equal to the atomic number of the element. This suggestion is now generally accepted, and has been confirmed by more accurate experiments on the scattering of α-particles recently made by Chadwick.[9]

§ (30) THE CONSTITUTION OF THE POSITIVE NUCLEUS.—If Van der Broek's suggestion be accepted the hydrogen atom consists of a single electron associated with a single positive nucleus of equal but positive charge. As no positive charge has been discovered associated with a mass less than the mass of a hydrogen atom, it seems reasonable to assume that mass is a function of the positive charge. The electromagnetic mass of a charge is inversely proportional to its radius. Since the hydrogen nucleus has a mass of about 1850 times that of an electron while the charges are numerically equal, the radius of the positive nucleus must be $\frac{1}{1850}$ that of an electron, if the whole of the mass is electrical, that is to say, its radius must be about 10^{-16} cm. This value has not, of course, been experimentally confirmed. The experiments of Rutherford on the direct collision of the α-particles with hydrogen have, however, shown that the centres of the hydrogen and the helium nuclei must approach within a distance of about 3×10^{-13} cm. of each other, in order to account for the high velocity with which the hydrogen atom is driven forward by the impact. As the helium nucleus is certainly complex, and contains electrons, the hydrogen nucleus must be of exceedingly small dimensions. It seems reasonable to suppose that the hydrogen nucleus consists only of positive electricity concentrated on a sphere so small that its electromagnetic mass is sufficient to account for the whole mass of the atom. The hydrogen atom thus consists of one positive and one negative electron.

[1] Geiger, *loc. cit.*
[2] Crowther, *Proc. Roy. Soc.* A, 1910, lxxxiv. 226.
[3] Thomson, *Proc. Camb. Phil. Soc.*, 1910, xv. 465.
[4] Rutherford, *Phil. Mag.*, 1911, xxi. 669.
[5] Thomson, *Conduction through Gases*, 1906.
[6] Barkla and Dunlop, *Phil. Mag.*, 1916, xxxi. 222.
[7] Auren, *Phil. Mag.*, 1919, xxxvii. 165.
[8] Van der Broek, *Phys. Zeit.*, 1913, xiv. p. 32.
[9] Chadwick, *Phil. Mag.*, 1920, xl. 734.

The next element, helium, has an atomic weight approximately four times that of hydrogen, and its atomic number is of course two. Now the number of electrons in the atom, and hence the nuclear charge (taking the electronic charge as unity) is equal to the atomic number. The charge on the helium nucleus is therefore $2e$. But since the mass is four times that of the hydrogen nucleus, we must either suppose that in the helium nucleus the positive electricity is in a different state of concentration from that in the hydrogen nucleus, or otherwise that the helium nucleus consists of four hydrogen nuclei cemented together by two negative electrons, thus giving a mass of four times that of the hydrogen atom, with a resultant charge of $2e$. This seems the more probable assumption. The suggestion that the nuclei of elements of higher atomic weight than hydrogen are complex has recently been confirmed by Rutherford.[1] He found that on passing rapidly moving α-particles from Radium C through a tube containing pure nitrogen, particles were projected forward by the impact of the α-particles on the nitrogen atoms, which not only by their range, but also by the values obtained for e/m could be identified as charged atoms of hydrogen. The hydrogen nucleus is therefore a constituent part of the nitrogen atom. Similar results were also obtained with pure oxygen. In a later paper,[2] he showed that, in addition to this emission of hydrogen nuclei, a much larger number of particles were emitted carrying twice the electronic charge and having a mass three times that of the hydrogen atom, that is to say, having an atomic weight of three. As these particles are very freely emitted from both nitrogen and oxygen when bombarded by α-rays it seems probable that they play an important part in the structure of the nuclei of most of the elements. Rutherford suggests that these new particles consist of three hydrogen nuclei cemented together by a single negative electron, and that the hydrogen atoms obtained by collision are due to the break-up of these complex nuclei by particularly violent impacts. On the other hand, the fact that the α-particles emitted during the spontaneous decomposition of the radioactive elements consist entirely of helium seems to make it clear that, in the case of these heavy elements the helium nucleus is also present in their nuclei. The emission of high velocity β-particles during radioactive decomposition may be taken as evidence for the presence of negative electrons within the nucleus, as it is difficult to see how such extremely high velocities could be produced by the comparatively weak electric fields in the outer part of the atom. The most probable hypothesis at present is that the nuclei of the atoms consist of hydrogen nuclei, bound together by a certain number of negative electrons, and that these hydrogen nuclei tend to arrange themselves within the complex nucleus in groups of three or four.

This brings us back again to Prout's hypothesis that all matter is built up from atoms of hydrogen. The objection which proved fatal to that hypothesis, however, namely that the atomic weights of many of the elements were not integral multiples of the atomic weight of hydrogen, is no longer valid. Aston's recent work on the positive rays [3] has demonstrated quite clearly that the elements whose atomic weights are not exact whole numbers (when oxygen is taken as 16), consist of mixtures of two or more substances, differing only infinitesimally in their chemical and physical properties, but of different atomic mass. Thus chlorine, atomic weight 35·4, was found to be a mixture of two gases of atomic weights 35 and 37. Such substance, differing in atomic weight, but identical in their chemical properties are known as isotopes. Aston found that, within the limits of experimental error, every atom had a mass which was an exact whole number when that of oxygen was taken as 16. The masses of the different atoms can thus be expressed by a series of whole numbers. The only exception to this rule is hydrogen itself, which has a mass of 1·008 when oxygen is taken as 16. Thus the mass of the helium atom (atomic weight 4) is somewhat less than the mass of four hydrogen atoms, *i.e.* 4·032. There is some evidence that the energy of the helium nucleus is less than that of four hydrogen nuclei, that is to say, that energy is given out when the helium atom is formed from four hydrogen nuclei. According to Einstein's theory all energy possesses mass. The helium nucleus should therefore have a smaller mass than that of its separated constituents by the mass of the energy emitted from the system during formation. It is possibly simpler to suppose that the electromagnetic mass is slightly affected by the very close interaction of the electrical fields inside the nucleus. Aston's results would indicate that the more complex nuclei are constructed from the nuclei of helium and the new substance rather than from individual hydrogen nuclei, as the atomic masses are not exact integral multiples of that of the hydrogen atom.

The possibility of the formation of isotopes is clearly indicated on the nuclear theory of the atom. The chemical and physical properties of an atom are determined by the arrangement and number of the external electrons. This is fixed by the resultant charge on the nucleus, but is independent,

[1] Rutherford, *Phil. Mag.*, 1919, xxxvii. 581.
[2] Rutherford, *Proc. Roy. Soc.* A, 1920, xcvii. 374.
[3] Aston, *Phil. Mag.*, 1919, xxxviii. 707; 1920, xxxix. 611.

save to a very slight extent, on the way in which that resultant charge is made up. The mass of the atom, on the other hand, depends entirely on the number of hydrogen nuclei contained in the nucleus, irrespective of whether their charges are neutralised by the presence of electrons. Thus a nucleus consisting of three hydrogen nuclei and one electron would have the same resultant charge as a nucleus of four hydrogen nuclei and two electrons, and the two corresponding atoms would, therefore, each have two outer electrons and exhibit the same chemical properties. The former would, however, have a mass of three, the latter of four. Rutherford suggests that the particles of mass three ejected by the collision of α-particles with the atoms of nitrogen and oxygen have this structure, and are, therefore, isotopic with helium.

§ (31) The Arrangement of the Electrons in the Atom.—Although it is generally agreed that almost the whole of the chemical and physical properties of the atom are determined by the number and arrangement of the negative electrons outside the nucleus, very little is known as to their actual arrangement. It is generally agreed that they are arranged in series of rings, but the question as to whether the rings are coplanar, or even as to whether the electrons are in motion or at rest, does not appear to be definitely settled. The general problem of the arrangement of a number of electrons around a central positively charged nucleus is very complex, and has not yet been solved.

The problem is very much simpler in the case of the Kelvin atom, and a complete solution has been given by J. J. Thomson [1] for the case where the electrons are supposed to be confined to a single plane. He showed that a ring system consisting of a given number of electrons will in general only be stable if it contains a certain minimum number of other electrons within its orbit. Thus a ring of six electrons is unstable and would break up into a ring of five with one electron in the centre. The ring of six, however, could be made stable by placing another electron within it. A ring of nine required three electrons within it to make it stable, and an outer ring of sixteen no less than twenty. These inner electrons arrange themselves according to the same laws, and thus break up into a whole series of ring systems. It is found that if the stable systems are arranged in the ascending order of the number of electrons they contain, certain groupings tend to recur from point to point in the series. Thus the grouping, 12, 7, 1, which represents the arrangement of 20 electrons on the Thomson theory, recurs again with an outer ring of 16 when the total number of electrons is increased to 36, the whole group, 16, 12, 7, 1, being repeated further down the series with an additional outer ring of 18. Thus if the chemical and physical properties of the atom are to be referred to the grouping of the electrons around it, these properties will tend to recur periodically as we pass down the table of the elements arranged in order of their ascending atomic weights. This is, of course, the well-known periodic law of Mendeléeff.

A study of the periodic classification of the elements shows that, at any rate for the lighter elements, the characteristic properties repeat themselves for every increase of eight in the atomic number. That is to say, the addition of eight electrons (with, of course, an appropriate increase in the nuclear charge) leaves the general structure of the atom very much unaltered. It would seem, therefore, that the maximum number of electrons in a stable ring, at any rate for the lighter atoms, is eight. This is not indicated by Thomson's figures. A theory of atomic structure on these lines has been worked out by Langmuir.[2]

On the assumption that the equilibrium in the plane of the electron rings is given by the quantum relation, while equilibrium at right angles to this plane is governed by the ordinary laws of electrodynamics, Bohr (*loc. cit.*) has calculated the arrangement of the electrons in some of the lighter atoms. Though the assumptions may be open to objection, the calculated arrangements agree closely with the results to be expected from chemical considerations. Thus lithium is assigned the grouping 2, 1; sodium the grouping 8, 2, 1; and potassium the grouping 8, 8, 2, 1. These figures bring out very clearly the similarity of this group of elements, and similar results are obtained for other chemical groups.

§ (32) Summary of Atomic Dimensions.—Neglecting the more speculative points we may briefly summarise those facts about the structure of the atom which appear reasonably certain. As all the numbers concerned are exceedingly minute it will assist in forming a mental picture of the atom if we multiply all the distances by 10^{13}. On this scale one centimetre becomes equal to about two-thirds of the distance from the sun to the earth. The atom, then, if magnified to these dimensions, would be seen to consist of a central cluster or nucleus, which in the case of a light atom such as oxygen would have a diameter of about 5 cm. The nucleus itself would be found to be constructed of a certain number of negative electrons, having radii of about 1·9 cm. together with a larger number of positively charged particles of radius only

[1] J. J. Thomson, *Corpuscular Theory of Matter.*

[2] Langmuir, *Journ. Amer. Chem. Soc.*, 1919; *Phys. Rev.*, March 1921.

about 1/100th of a millimetre. Thus even with the large magnification imagined, the positive particles would hardly be visible. They account, however, for practically the whole mass of the structure. The nucleus of the oxygen atom would consist of 16 of these positive particles, together with 8 electrons. It is probable that the particles might be found to be grouped together within the cluster in groups of three or four.

Outside this closely packed nucleus the atom would be empty save for a few negatively charged electrons, probably arranged in concentric circles. The outermost of these rings would be about 1 kilometre from the centre of the nucleus, the innermost probably not more than 10 metres. The total number of these electrons is equal to the atomic number of the element. Some of the more important data are given in the following table:

Table of Atomic Data

Electronic charge, e .	$4{\cdot}77^4 \times 10^{-10}$ e.s.u. $1{\cdot}59 \times 10^{-20}$ e.m.u.
e/m for an electron .	$1{\cdot}764 \times 10^7$ e.m.u. per gm.
Mass of an electron .	9×10^{-28} gm.
Radius of electron .	$1{\cdot}9 \times 10^{-13}$ cm.
Mass of hydrogen atom	$1{\cdot}65 \times 10^{-24}$ gm.
Radius of hydrogen nucleus (positive electron)	10^{-16} cm.
Radius of nucleus of a light element	*circ.* 5×10^{-13} cm.
Number of molecules in a gram-molecule	61×10^{22}

J. A. C.

Electroplating of Gold, Silver, Nickel, Copper, Zinc, Brass. See "Electrolysis, Technical Applications of," V. §§ (8)-(13).

ELECTROSTATIC FIELD,[1] PROPERTIES OF THE

§ (1) General Considerations. — The properties of the potential, equipotential surfaces, lines, and tubes of force are described in the article "Potential," and their applications to electrostatics is there explained. It will be useful to recapitulate here and extend some of the results arrived at.

The field in the neighbourhood of any system of charged bodies or of magnets can be mapped out by a series of equipotential surfaces and tubes of force which cut them at right angles.

If these surfaces be drawn so that the difference of potential between any two consecutive surfaces is constant, the resultant force at any point will be inversely proportional to the distance between the surfaces, for if V and $V+\delta V$ be the potential and δn the distance between them, R the force in direction δn, then

$$R\delta n = \text{work in going from } V \text{ to } V+\delta V$$
$$= -\delta V = \text{constant}.$$

Thus $$R = \frac{\text{constant}}{\delta n},$$

i.e. it is inversely proportional to δn. The force is large when the surfaces are close together.

Again, consider a tube of force cutting any of the equipotential surfaces in an area δS. Then $R\delta S$ is constant[1] along the tube, so that R is also inversely proportional to δS. Thus the force is large where the tubes are contracted in area.

The lines of force terminate in charges of electricity.

In some cases they pass away beyond the boundaries of the field, and the charges in which they terminate are so far removed as to exert no influence on the field.

A charged conductor is an equipotential surface; for if a potential difference existed between two points on a conductor a current would flow and the electricity would not be in equilibrium.

There is no force inside an equipotential surface unless the surface surround a charge of electricity. Since there is no charge within the surface there can be no lines of force which terminate within the surface: any lines of force within it must pass from one point A to another point B of the surface; but this is impossible, for A and B are at the same potential; thus there are no lines of force within the surface; thus the potential is everywhere the same and there is no force within the surface.

§ (2) Gauss's Theorem.—If R be the value of the resultant force at any point P of any closed surface in an electrostatic field, δS an element of the surface at the point P, ϵ the angle between the direction of R and that of the outward-drawn normal at P, and Q_1 that part of the charge to which the field is due which lies within the surface S, then

$$\iint KR\cos\epsilon\, ds = 4\pi Q_1,$$

where K is the specific inductive capacity. If Q be the total charge in the field equal, say, to Q_1+Q_2, the force R may be divided into two parts, R_1, due to the charge Q_1 within S, and R_2 arising from Q_2, the charge exterior to S, and we have

$$R\cos\epsilon = R_1\cos\epsilon_1 + R_2\cos\epsilon_2,$$

where ϵ_1, ϵ_2 represent the angles between the

[1] Many of the results given in this article are direct consequences of the fact that the forces dealt with obey the inverse square law. They are true, therefore, *mutatis mutandis* for a magnetic as well as for an electrostatic field of force.

[1] See § (3).

outward-drawn normals and the directions of R_1, R_2 respectively.

Consider now the force R_1 and imagine the whole charge Q_1 concentrated at a point O

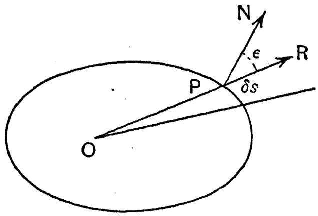


FIG. 1.

(*Fig.* 1) at a distance r from P. Let $\delta\omega$ be the solid angle subtended at O by δS. Then

$$R_1 = \frac{Q_1}{Kr^2},$$

$$\delta S \cos \epsilon_1 = r^2 \delta\omega.$$

Hence $\quad KR_1 \cos \epsilon_1 \delta S = Q_1 \delta\omega,$

and summing up for the whole surface

$$\iint KR_1 \cos \epsilon_1 dS = 4\pi Q_1. \quad . \quad . \quad (1)$$

Now let O (*Fig.* 2) be outside the surface. Any line such as OP drawn from O cuts the

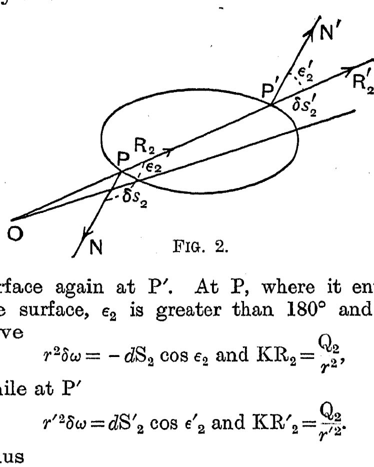


FIG. 2.

surface again at P′. At P, where it enters the surface, ϵ_2 is greater than 180° and we have

$$r^2\delta\omega = -dS_2 \cos \epsilon_2 \text{ and } KR_2 = \frac{Q_2}{r^2},$$

while at P′

$$r'^2\delta\omega = dS'_2 \cos \epsilon'_2 \text{ and } KR'_2 = \frac{Q_2}{r'^2}.$$

Thus

$$KR_2 \cos \epsilon_2 dS_2 = -Q_2\delta\omega = -KR'_2 \cos \epsilon'_2 dS'_2.$$

Thus the portions which the elements at P and P′ respectively contribute to the sum are equal and opposite and their total contribution is zero.

The whole area of S can be divided thus into a series of pairs of elements which when combined contribute nothing to the sum of quantities such as R cos ϵdS. Thus we have

$$\iint KR_2 \cos \epsilon_2 dS = 0. \quad . \quad . \quad (2)$$

Hence combining

$$4\pi Q_1 = \iint K(R_1 \cos \epsilon_1 + R_2 \cos \epsilon_2)dS$$
$$= \iint KR \cos \epsilon dS. \quad . \quad (3)$$

A similar proof holds when the closed surface is more complex in shape, so that a radius such as OP cuts it more often.

If the charge Q_1 is not concentrated at a point consider the various elementary charges of which it is composed. The theorem is true for each of these, and therefore for the whole charge Q_1.

§ (3) APPLICATION TO A TUBE OF FORCE.—Let the surface S consist of the curved surface of a tube of force and two sections dS_1, dS_2 at right angles to the tube; over the surface of the tube ϵ is a right angle and cos ϵ is zero; at one end ϵ is zero, cos ϵ is 1; at the other, ϵ is 180°, cos ϵ is -1; while within the surface the charge is zero. Hence

$$0 = \iint K(R_1 dS_1 - R_2 dS_2),$$

or $\quad \iint R_1 dS_1 = \iint R_2 dS_2.$

Thus, if the sections of the tube be small,

$$R_1 dS_1 = R_2 dS_2, \quad . \quad . \quad . \quad (4)$$

or the value of the product of the force and the area of the cross-section of a tube of force is constant along a tube as stated in § (1).

§ (4) COULOMB'S THEOREM.—If R and R′ are the resultant forces due to any electrostatic system on two sides of a surface electrified to density σ, and ϵ, ϵ' the angles between the outward-drawn normal and the directions of R and R′ respectively, then

$$K(R \cos \epsilon + R' \cos \epsilon') = 4\pi\sigma.$$

This follows from Gauss's Theorem; for imagine the element dS which carries a charge σdS to be surrounded by a surface drawn very close to it in comparison with its own dimensions; then within this surface there is a charge Q_1 given by σdS.

The surface integral is composed of the two terms R cos ϵdS and R′ cos $\epsilon' d$S due to the two opposite surfaces of the element, for that part which is due to the perimeter can be made as small as we please. Hence

$$K(R \cos \epsilon + R' \cos \epsilon') = 4\pi\sigma. \quad . \quad (5)$$

If S be a closed equipotential surface, such as the surface of a conductor, then the force is zero on one side and normal to the surface on the other; in this case we have $R' = 0$, $\epsilon = 0$. Hence over any conductor or other closed equipotential surface

$$KR = 4\pi\sigma,$$

$$R = \frac{4\pi\sigma}{K}. \quad . \quad . \quad . \quad . \quad (6)$$

Or, again, let the surface be a plane of infinite extent; it is clear from symmetry that the force must be the same on either side of the plane. Hence R cos ϵ = R′ cos ϵ', and in this case we have

$$KR \cos \epsilon = 2\pi\sigma.$$

If the plane be equipotential, such as the surface of a conductor, $\cos\epsilon$ is unity and

$$R=\frac{2\pi\sigma}{K}. \quad . \quad . \quad . \quad . \quad (7)$$

This may be proved directly thus. Let r be the distance PO between the element dS of a plane electrified to constant surface density σ and a point O (*Fig.* 3), dR the force at O due to σdS resolved

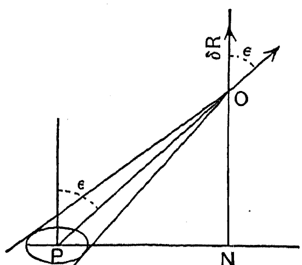


Fig. 3.

along NO, the normal to the plane ω, the solid angle which the element dS subtends at O; then

$$K\delta R=\frac{\sigma dS\cos\epsilon}{r^2}.$$

But $\qquad dS\cos\epsilon=r^2\delta\omega.$

Hence $\qquad K\delta R=\sigma\delta\omega.$

Thus

$KR=\sigma\times$ solid angle subtended by plane.

But when the point O is very close to the plane the solid angle subtended is 2π. Hence

$$KR=2\pi\sigma. \quad . \quad . \quad . \quad . \quad (8)$$

§ (5) Mechanical Force on a Conductor.—We have seen that the electrical force just outside a charged conductor is $4\pi\sigma/K$. Consider a small element of the conductor near the point; if the point be sufficiently close to the surface this element is practically part of an infinite plane and the force it exerts on the point is $2\pi\sigma/K$. Thus, of the whole force $4\pi\sigma/K$, we see that $2\pi\sigma/K$ is due to the neighbouring element and $2\pi\sigma/K$ to the electricity on the rest of the conductor excluding the element. This last force will also act on the electricity on the element, and if δS be its area its charge is $\sigma\delta S$. Hence the force on the element due to the rest of the charge is $2\pi\sigma\times\sigma\delta S/K$ or $2\pi\sigma^2\delta S/K$. We may represent this as a tension pulling the element outwards from the surface and of amount equal to $2\pi\sigma^2/K$ per unit of area. Moreover, if R be the electric intensity we have $\sigma=KR/4\pi$. Hence F, the mechanical force outwards per unit of area, which is equal to $2\pi\sigma^2/K$, is given also by the equation

$$F=\frac{KR^2}{8\pi}. \quad . \quad . \quad . \quad . \quad (9)$$

§ (6) Energy in the Electrical Field.—A charged system of conductors evidently possesses energy. We can show that this may be represented by supposing it distributed throughout the field with an amount $KR^2/8\pi$ per unit of volume.

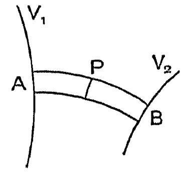


Fig. 4.

For consider a tube of force AB (*Fig.* 4) between two conductors at potentials V_1 and V_2. Let dS be the section of this tube at a point P at which the intensity is R, dS_1, dS_2 areas at A and B respectively, and let σ_1, σ_2 be the densities, e and $-e$ the charges at A and B—the tube is supposed to run from A to B. Then we have seen that

$$KRdS=KR_1dS_1=KR_2dS_2=4\pi\sigma_1dS_1=4\pi e.$$

Hence, if ds be an element of the length of the tube at P and the energy per unit volume be $KR^2/8\pi$, we find

$$\text{Energy in tube}=\frac{1}{8\pi}\int KR^2dSds,$$
$$=\frac{KR_1dS_1}{8\pi}\int Rds=\tfrac{1}{2}e\int Rds,$$
$$=\tfrac{1}{2}e(V_1-V_2),$$

for $\int Rds$ is the work done in carrying unit charge from A to B, and this is V_1-V_2. But $\frac{1}{2}e(V_1-V_2)$ is the energy of the condenser formed by the two ends of the tube at A and B and the intervening dielectric. Thus the assumption that the energy of the tube may be represented as an amount $KR^2/8\pi$ per unit of volume is verified for this tube; it can be verified similarly for all the other tubes and hence for the whole field.

Thus the quantity $KR^2/8\pi$ represents the tension along a line of force and also the energy per unit volume of the field, and we have for the energy of the field the expression

$$\text{Energy}=\iiint\frac{KR^2}{8\pi}dv, \quad . \quad . \quad . \quad (10)$$

where dv represents an element of volume at the point considered.[1]

Electrostatic Induction is measured by the quantity $KR/4\pi$—the electric displacement—R being the electric intensity and K the inductive capacity. See "Units of Electrical Measurement," § (14).

Electrostatic System of Units. A system of units originally due to Weber, developed by the B.A. Committee on Electrical Standards, 1862–63, based on the assumption that the inductive capacity of a vacuum, in practice air, is unity, so that the force

[1] In the magnetic field the mechanical force per unit area $=\mu H^2/8\pi=B^2/8\mu\pi$, and the energy per unit volume is given by the same expressions.

— F —

— G —

Vibration: a detecting instrument used in inductance and capacity measurements. See "Inductance, The Measurement of," § (30); also "Vibration Galvanometer."

GALVANOMETER CONSTANTS:

Experimental determination of. See "Vibration Galvanometers," § (31).

Deduction of intrinsic constants. See *ibid.* § (32).

GALVANOMETER NEEDLES, ageing of. See "Galvanometers," § (5) (iv.).

GALVANOMETER RESISTANCE, value of, for greatest sensitivity. See "Galvanometers," § (4) (iii.).

GALVANOMETERS

§ (1) INSTRUMENTS for measuring and detecting currents may be divided into three classes:

(i.) Absolute instruments such as current balances, the Helmholtz galvanometer, etc. These are described in the article on "Electric Measurements, Systems of," Part II. §§ (23)-(37).

(ii.) *Galvanometers.*—These are based on the forces between two conductors conveying currents, as in electrodynamometers; on electrothermal phenomena as in thermal ammeters, and on electrochemical and on electro-optical principles; but, in general, galvanometers are based on electromagnetic principles, *i.e.* on the mechanical forces between a magnet and a conductor conveying the current to be measured or detected.

(iii.) *Ammeters.*—The principles are the same as those of galvanometers, but ammeters are built so that the current strength in amperes can be read directly. Ammeters are described in the article on "Direct Current Instruments."

§ (2) TYPES OF GALVANOMETERS BASED ON ELECTROMAGNETIC ACTION.[1]—There are a large number of types of galvanometers based on the electromagnetic forces between conductors carrying current or between a conductor and a magnet, and in many of these the conductor, the magnet, and the control differ appreciably. The conductor may be

(i.) A straight wire or tube, as in the Einthoven galvanometer.

(ii.) A fixed coil, as in the Thomson and Paschen galvanometers.

(iii.) A moving coil, as in the Ayrton-Mather galvanometer.

The current in the conductor produces a magnetic field, and in the cases of (i.) and (iii.) there is an additional magnetic field, which is fixed, with the result that a mechanical force acts on the conductor. The additional magnetic field may be due to

[1] FitzGerald, *The Electrician*, xxxviii.

(*a*) A permanent magnet, as in most Ayrton-Mather galvanometers.

(*b*) An electromagnet, as in the Einthoven galvanometer and Duddell oscillograph.

When the conductor conveying the current is in the form of a fixed coil, there is

(*c*) A movable magnet system, as in the Thomson, Broca, and Paschen galvanometers, which is deflected by the action of the current.

The movable system, whether it is a wire, a coil, or a magnet, is subject to control, and such control may be

(*α*) Torsional or elastic, as in a perfectly astatic Paschen galvanometer, and in suspended coil galvanometers.

(*β*) Magnetic, as in most Thomson galvanometers.

(*γ*) Gravitational, as in the Kelvin current balance.

(*δ*) Due to inertia, as in the ballistic galvanometer and fluxmeter.

§ (3) GALVANOMETER PARTS AND CONSTANTS.—Galvanometers are at times used as measuring devices for very small currents, or changes in current, as when employed in the measurement of insulation resistance. At other times they are used to detect uniformity of potential between two points, as in bridge methods for the measurement of resistance. In other cases very small quantities of electricity are measured or compared, as in some methods for the comparison of capacities, and galvanometers are used also to determine when two small quantities of electricity are equal in amount, as in bridge methods involving inductances and capacities. In order to choose the best type of galvanometer for such measurements it is necessary to have a general knowledge of—

(*a*) Arrangement of and resistance of coils.
(*b*) Deflected magnet systems.
(*c*) Shielded systems.
(*d*) Deflected conductor systems.
(*e*) Field magnets.
(*f*) Suspensions.
(*g*) Period and damping conditions.
(*h*) Sensitiveness and constancy of rest point.

These are considered in the order given.

§ (4) ARRANGEMENT OF AND RESISTANCE OF COILS.—With galvanometers of the Thomson and other patterns in which the conductor conveying the current is in the form of a fixed coil, the windings of the coil should be so disposed as to give the maximum intensity of magnetic field in the neighbourhood of the movable magnet system. For example, suppose that in a Thomson astatic galvanometer a double coil system of many turns is wound about one set of magnets; then the outer layers will be at a considerable distance from the magnets and the effect of the current in the layers will be correspondingly small. If, instead, the

raised to the power 2/5 than to the resistance raised to the power 1/2.

(iv.) *Best Resistance for a Galvanometer.*—It is shown elsewhere that the best galvanometer resistance is equal to that of the circuit of which the galvanometer forms part excluding the galvanometer itself. The galvanometer coil should therefore be wound of high-conductivity material, such as copper or silver, and of a gauge appropriate to the channel in which the coil is to be wound, and such that the completed coil has the requisite resistance. In some cases, such as that of testing insulation resistance, the external galvanometer circuit may be many megohms, and since in practice the resistance of a galvanometer coil of very fine wire is not more than a few thousand ohms, the departure from the "best galvanometer resistance" is considerable.

However, the sensitiveness is not greatly reduced if the resistance of the galvanometer is appreciably different from that giving the best results. Suppose that R_G is the best galvanometer resistance; then the resistance of the galvanometer circuit is $2R_G$ and the deflection of the galvanometer is proportional to $i\sqrt{R_G}$, where i is the current. If, instead, NR_G is the resistance of the galvanometer the new current is $2i/(N+1)$ and the deflection is proportional to $2i\sqrt{NR_G}/(N+1)$. The ratio of the new sensitiveness to the maximum sensitiveness is therefore $2\sqrt{N}/(N+1)$. The following table, showing the relation between N and $2\sqrt{N}/(N+1)$, was first given by Schuster:[1]

N.	$2\sqrt{N}/(N+1)$.	N.	$2\sqrt{N}/(N+1)$.
1	1·000	11	0·553
2	·943	12	·533
3	·866	13	·515
4	·800	14	·499
5	·745	15	·484
6	·700	16	·471
7	·661	17	·458
8	·629	18	·447
9	·600	19	·436
10	·575	20	·426

In practice, except for the measurement of insulation resistance, there is a tendency to choose a galvanometer of too great a resistance. In a Thomson or Broca galvanometer having two coils and intended for general measurements it is of advantage to choose each coil to be of 100 ohms resistance. When the coils are in series the galvanometer resistance is 200 ohms, and if so used in measurements when the external galvanometer circuit has a resistance of 10,000 ohms the sensitiveness is 0·27 time the maximum possible. When the coils are in parallel, the galvanometer resistance is 50 ohms, and the reduction in sensitiveness when the external circuit is 1 ohm in resistance is the same as before.

In the case of moving-coil instruments, the resistance of the suspension strip, when the strip conveys the current, must be counted as belonging to the external galvanometer circuit.

§ (5) Deflected Magnet Systems.—In the Thomson, Broca, Paschen, and other sensitive galvanometers the magnet systems are astatic, that is, they are such that when placed in a uniform magnetic field no turning moment results, whatever be the orientation of the system. In general, perfect astaticism is not obtained, but the directive force is always very small.

(i.) *Thomson Galvanometer.*—The most commonly used astatic combination consists of two similar groups of magnets, A and B, *Fig.* 2, the axes of which are horizontal and in the same vertical plane. The effective magnetic moments of the groups are equal, and the north poles of the upper group point in the opposite direction to those of the lower group. In the Thomson galvanometer there are from three to five magnets in each group, each magnet being about 4 millimetres long. The groups are usually not less than 5 cm. apart and are connected together by a fine wire or strip; a small reflecting mirror M is attached to the system. The fixed coils are sometimes two in number, but occasionally four; in the former case the A group lies centrally with respect to the two coils and the B group is often attached to the back of the mirror. When four coils are used the A group is central with respect to two of the coils and the B group is in the centre of the lower coils. The suspension may be of silk or quartz. In general a carefully constructed system of this kind is sufficiently astatic to necessitate the employment of a control magnet. Such a magnet is often fixed above the galvanometer and creates a magnetic field which is stronger, or weaker, in the neighbourhood of the upper group of magnets than in the vicinity of the lower group. Adjustment of the position of the magnet alters the intensity of the magnetic field near the magnets and so changes the sensitiveness and period.

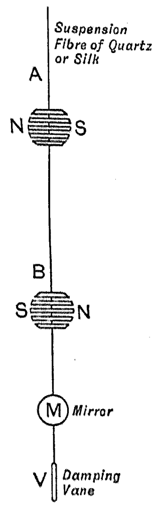


Fig. 2.

The relation between the length of a magnet

[1] *Phil. Mag.*, 1895, xxxix. 175. See also "Resistance, Standards of," § (5) (ii.)

in a group and the diameter of the smallest turn in a coil is of some importance. If the magnets swing in a cylindrical chamber which runs right through the coils, the diameter of the smallest turn must be greater than the length of a magnet, and in such case every turn in the coil produces a beneficial effect. In the case when the magnet is longer than the smallest diameter the inner turns may produce an effect opposite to that of the outer turns.[1]

In regions where the earth's magnetic field changes frequently and suddenly in direction and intensity the Thomson system is not sufficiently astatic, unless very great care is taken, to be of use in precision measurements. In the neighbourhood of electric traction circuits the changes in magnetic intensity are so sudden and of such magnitude that an unprotected Thomson system is in general in constant motion. The difficulty may be greatly reduced by making the system more astatic, but a better remedy is to shield the galvanometer with soft-iron screens.

(ii.) *Paschen Galvanometer.*—In principle this is the same as the Thomson galvanometer but in design it is a great improvement. The magnet system consists of two groups each with about thirteen magnets, the magnets being arranged alternately on opposite sides of a fine glass stem which supports also a very thin glass mirror. The complete moving system of a Paschen galvanometer made by the Cambridge and Paul Instrument Company weighs about 30 milligrams. A special feature of the galvanometer is the winding of the coils which are graded and have the theoretically best cross-section. A horizontal cross-section of the coil system is shown in *Fig.* 1. Four coils are provided, each with two terminals so that various combinations may be used.

(iii.) *Vertical Magnet Systems. The Broca Galvanometer.*—In the Broca galvanometer the magnet system consists of two steel wires or tubes, about 0·5 mm. diameter and 45 mm. long placed vertically and parallel. Each of the magnets is so magnetised that its two ends are of like polarity, with a consequent pole in the middle. The suspended system of a Broca galvanometer is shown in *Fig.* 3, and the galvanometer itself in *Fig.* 3A. The system is astatic if the magnets are symmetrically magnetised, and if not the system becomes astatic when the magnets are parallel to the axis of rotation. The steel wires or tubes (which should be of tungsten steel if possible) are best magnetised by insertion in a small coil wound as a right-handed helix for half its length, and as a left-handed helix for the remaining half; the inner diameter of the coil is slightly greater than the diameter of a wire. After magnetisation the wires are placed in two parallel grooves, about 4 mm. apart, cut in brass sheet, and clamped in position, while the aluminium foil connecting pieces AA are connected to the magnets by wax or cement. The mirror M and damping vane V are rigidly connected by a fine wire to the lower piece of aluminium foil and a quartz or silk fibre is connected to the upper piece of aluminium. By bending the aluminium tongue it is easy to ensure that the magnets hang vertically.

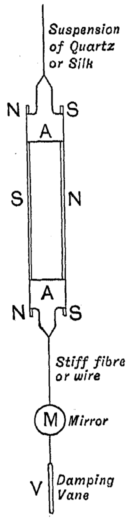


FIG. 3.

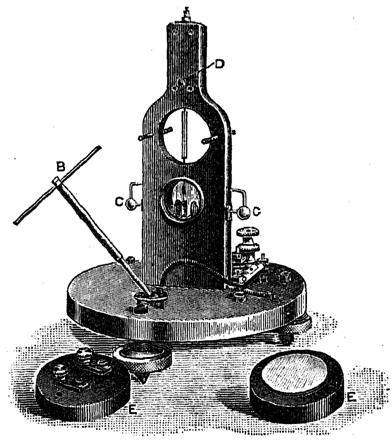


FIG. 3A.

In practice it is not difficult to make the Broca system very astatic; at times systems have been made so astatic that the main control was due to the fibre. In general, however, a small control magnet is used to direct the system. Because of its excellent astatic qualities the Broca galvanometer has replaced the Thomson in laboratories subject to sudden fluctuations in the earth's magnetic field.

(iv.) *Ageing of Galvanometer Needles.*—The magnets used in the Thomson, Broca, and Paschen galvanometers should be as strong as possible, for the period of such instruments is inversely proportional to the strength of the magnets. Very little quantitative information has been obtained of the loss of strength with time of such needles, but that obtained by Ayrton, Mather, and Sumpner in 1890 shows the importance of remagnetising the systems after the lapse of a few years.

The data is as follows, D being the deflection in scale divisions at a constant distance.

[1] Ayrton, Mather, and Sumpner, *Phil. Mag.*, 1890, xxx. 64.

1. Double-coil astatic reflecting galvanometer of 686 ohms resistance. Period = 10 seconds. Current = 1 microampere.

	D.
(*a*) Needles magnetised and tested the same day	1600
(*b*) After about two years' use .	366
(*c*) Needles remagnetised and then used for about one year .	826

2. Single-coil astatic reflecting galvanometer of 26 ohms resistance. Period = 10 seconds. Current = 1 microampere.

	D.
(*a*) As obtained from the makers .	198
(*b*) After about three years' use .	170
(*c*) Needles remagnetised and tested the same day	243

3. Double-coil astatic reflecting galvanometer. Resistance 9744 ohms. Period = 10 seconds. Current = 1 microampere.

	D.
(*a*) As obtained from the makers .	4200
(*b*) After about three years' use .	3000
(*c*) Needles remagnetised and tested the same day	4280

In the case of a Broca galvanometer which had been in use for four years the writer found the magnetisation of the magnets had fallen to one-quarter of the initial value. When the magnets of galvanometers are made with properly hardened tungsten steel and artificially aged the fall of magnetisation with time is very small.

§ (6) Shielded Systems. *Du Bois-Rubens Galvanometer.*—Owing to sudden fluctuations in the earth's magnetic field, and because of disturbances in laboratories, it is often necessary to shield the suspended systems of galvanometers from the influence of external fields. The shields are of iron or steel, spherical or cylindrical, and may consist of one, two, three, or more nearly complete shells. The Du Bois-Rubens galvanometer usually has two spherical shells. Very complete data on shielding are given by E. F. Nichols and S. R. Williams,[1] who made experiments on shields of cast silicon steel, before and after annealing, and on others made from ordinary soft-iron water-pipes. The results obtained with the latter are given in the following table :

Shield No.	Inner Radius.	Outer Radius.	Weight in Grams.	Shield used.	Shielding Ratio.
	cm.				
1	2·7	3·0	1530	1	19
2	3·9	4·5	3120	1+2	104
3	5·2	5·7	4260	1+2+3	252
4	6·5	7·1	6130	1+2+3+4	723
5	9·0	9·7	9650	1+2+3+4+5	2725

It is apparent from these figures that a Thomson galvanometer can be efficiently protected from local variations in the magnetic field by the provision of shields of soft iron.

The magnetic shield of the Paschen galvanometer is of great importance; the instrument is extremely sensitive to external fields and also to temperature changes. Because of the latter Paschen was in the habit of filling the space between his magnetic shields with cotton-wool, and had a large covering of cotton-wool over his external shield.

§ (7) Deflected Conductor Systems.—Deflected conductor systems may be divided into three classes: (1) those consisting of a single straight conductor, as in the Einthoven galvanometer; (2) those consisting of a single loop of wire, as in a thermo-galvanometer; (3) those consisting of a coil of many turns, as in a D'Arsonval galvanometer. Instruments of the second class consisting of Boys' radiomicrometer and Duddell's thermo-galvanometer are described in the articles on the "Radio Micrometer," Vol. III., and on "Radio-frequency Measurements," § (18) (v.), § (19) (ii.).

(i.) *Einthoven Galvanometer.*[2]—In this, the movable element is either a fine glass or quartz fibre or tube, coated with a conducting layer of silver, gold, or platinum, or a fine metal wire of platinum, tungsten, silver, gold, copper, phosphor bronze, or aluminium. The fibre is placed in an intense magnetic field due to an electromagnet and is kept in tension by a weak spring. Adjustment of the tension alters the frequency and also the damping. In some instruments two strings are used, and in others as many as twelve have been placed in the magnetic field and operated on by separate electromotive forces in separate circuits.

When a current passes through a fibre the latter moves at right angles to the magnetic field, and its motion is observed by means of a microscope with micrometer eyepiece or by projection of an image of the string on to a screen or photographic film. When used in conjunction with a photographic recording apparatus, alternating or pulsating currents such as are received in wireless telegraphy may be recorded. The high-resistance instrument has been used extensively in physiological investigations, especially for recording the electrical currents produced during the beating of a heart.

The resistance of a silvered glass fibre 0·0025 mm. in diameter and of length suitable for a galvanometer may be as great as 12,000 ohms, but copper wires having a resistance as small as 6 ohms have been used.

The tension of the string can be altered within limits. The upper limit is when the tension is so great that the elastic limit is approached; the lower limit is the total absence of tension. In the latter case, how-

[1] *Physical Review*, 1908, xxvii.

[2] A. H. Crebore, *Phil. Mag.*, 1914, xxviii.

ever, the string still has a frequency depending on its length, diameter, density, and elasticity.[1] The frequency is given by

$$f=\frac{2m^2r}{8\pi l^2}\sqrt{\frac{E}{\rho}},$$

where l and r are the length and radius of the wire respectively, E is the elasticity, and ρ the density: m is a constant equal to 4·73. A copper wire 0·01 mm. in diameter and 10 cm. long has a natural frequency of about 3 per second.

The electromagnet of the Einthoven instrument is shaped so as to concentrate the field on the air-gap in which the fibre moves. The iron is magnetised to saturation, and consequently small changes in the magnetising current have little effect on the intensity of the field in the gap.

(ii.) *Deflected Coil Systems.*—The suspended coil principle was used first by Sir William Thomson in 1870, and the use of such coils for galvanometers was dealt with subsequently by Maxwell. However, no considerable use was made of such deflected systems until attention was recalled to them by D'Arsonval in 1882.

The deflected coil system consists of a circular or rectangular coil of many turns of fine wire suspended between the poles of a permanent magnet or an electromagnet. Often inside the coil a solid core of soft iron is fixed. Connections to the coil are made either through two wires or strips constituting a bifilar suspension or through one suspending wire and a fine loosely coiled metallic spiral leading to the lower end of the coil. Ayrton, Mather, and Sumpner have pointed out that suspended coils should be long and narrow, and that there should be no internal core. Subsequently Mather[2] determined the best shape of the section of the coil perpendicular to the axis about which it turns.

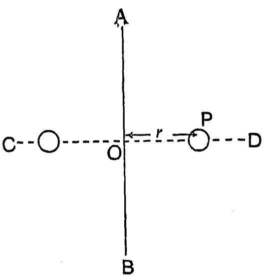


FIG. 4.

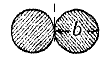
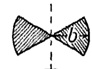
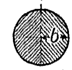
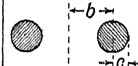
Let O (*Fig.* 4) represent in plan the axis about which the coil turns, and let it be placed in a magnetic field parallel to CD. Let P be an element of the section of the coil, the length of wire being at right angles to the paper. Then the deflecting moment exerted by unit length is Hiar sin θ, where H is the intensity of the magnetic field, i the current density, a the area of the element, and r its distance from the axis. The element has a moment of inertia about O equal to mar^2, where m is the mass per unit cube. Mather points out that it is important for the period of oscillation to be not inconveniently long and for the power consumed by the instrument to be as small as possible; then, since for a constant period the controlling moment at unit angle must be proportional to the moment of inertia, the best section of coil will be such that the total deflecting moment for a given total moment of inertia is a maximum.

For the element considered the ratio of the deflecting moment to the moment of inertia is

$$\frac{\mathrm{H}iar\sin\theta}{mar^2}=\mathrm{C}\frac{\sin\theta}{r},$$

where C is a constant depending on H, i, and m. Hence sin θ/r is a measure of the efficacy of the element and its position. If, now, a curve is drawn whose polar equation is $r=x_1\sin\theta$, then it may be shown that a given length of wire wound within this space is more efficient than if wound outside it. Hence the curve $r=x_1\sin\theta$, *i.e.* a circle tangential to AB at O is the best form of the section of the moving coil. The complete section consists of two circles touching at O.

The kind of improvement obtained by winding coils of this shape is shown in the following table, which is a portion of one given by Mather:

No.	Shape of Section.	Deflecting Moment / Moment of Inertia	Proportions in Particular Cases.	Deflecting Moment per Unit Moment of Inertia.
1		$\frac{4}{3b}$	..	1·02
2		$\frac{4\sin\theta}{3b\theta}$	$\theta=30°$ $\theta=45°$	0·91 0·95
3		$\frac{8}{3\pi b}$	..	0·80
4		$\frac{b}{b^2+(c^2/2)}$	$c=\frac{1}{11}b$	0·40

Shapes 1, 2, and 3 are not quite possible in practice, because insulation must be allowed for. Usually in Ayrton-Mather galvanometers the coil is of a narrow shuttle-shaped form and moves between the poles of a powerful permanent magnet with a narrow air-gap. The coil is wound with special non-magnetic wire, and, after winding, it is treated by a chemical process to eliminate any traces of magnetic material in the silk or covering of

[1] Wertheim Salomonson, Konink. Akad. van Wetenschappen te Amsterdam.
[2] *Phil. Mag.*, 1890, xxix. 434.

the wire. The whole of the suspended system is enclosed in a dust-tight tube to prevent contamination with magnetic matter; an excellent coil may be ruined by a slight amount of careless handling or a few minutes' exposure to a dusty atmosphere.

§ (8) Field Magnets.—In the Einthoven and suspended coil galvanometers the conductor conveying the current moves in the magnetic field of a powerful magnet, and the sensitiveness of the galvanometer is directly proportional to the intensity of the field. It is well known that permanent magnets lose their strength with time, and to ensure comparative constancy of strength it is usual to slightly demagnetise the magnet. Some makers do this by heating the hardened steel in a steam bath for one day, after which it is magnetised strongly and afterwards heated again in the steam bath for six hours. Another and better procedure is to demagnetise the magnet to 80 per cent of its initial magnetisation; in such cases, if the magnet is of properly hardened tungsten steel, the constancy is satisfactory.

Electromagnets are used for Einthoven galvanometers and many oscillographs, and the question of loss of magnetisation with time does not then arise.

§ (9) Suspensions.—The usual fibres employed for suspending the movable systems of galvanometers are of silk, phosphor bronze, silver, platinum-silver, and quartz. In the best types of Thomson galvanometer—including the Paschen—quartz fibres are used; in those used in less accurate measurements the suspensions are of silk. Wires and strips of phosphor bronze and silver are used for suspended coil galvanometers, and platinum-silver strips and wires are used in twisted strip ammeters. If special leads are provided for the leading in wires of suspended coils the latter may be supported by quartz fibres

In the Thomson type of galvanometer the suspensions may be regarded as being cylindrical wires. If the length is l, the radius r, the twisting couple due to the current in the coils u, and, as a result of this couple the magnet system is deflected through an angle θ, then

$$\theta = \frac{2lu}{\pi n r^4},$$

where n is the coefficient of torsional rigidity of the fibre. The values of n for some solids are given below.

Steel	$8{\cdot}2 \times 10^{11}$ dynes/cm.^{2}
Brass	3·8 ,,
Quartz (fused)	2·9 ,,
Glass	2·4 ,,

After deformation by twisting, a fibre does not always completely recover its initial shape, and as a result the rest point of the galvanometer is variable. Of all known substances, quartz recovers its original shape best;[1] in other words, it exhibits elastic fatigue less than any other known body, and it is because of this it is so largely used for suspensions. Quartz fibres were first made by C. V. Boys,[2] and when drawn fine the breaking strength exceeds that of steel. Unspun silk is used in galvanometers for measurements of small precision; they are, however, troublesome, for they are much affected by changes in the humidity of the atmosphere, the result being a shift of the rest point.

If, in any galvanometer, the control is entirely due to a single cylindrical wire or quartz fibre, the deflection produced is inversely proportional to the fourth power of the radius of the wire; it is of great importance, therefore, that the fibre should be of as small a radius as possible consistent with the necessary mechanical strength. Quartz fibres can be made exceedingly fine; the diameters of quartz fibres used in galvanometers vary from 0·003 mm. to 0·025 mm.

§ (10) Period and Damping Conditions.—When the angle of deflection of a galvanometer is small, the deflecting moment may be regarded as proportional to the value of the current. It may be represented by Gi, where G is a constant depending on the galvanometer and i is the current. The restoring moment also is proportional to the angle of deflection and is equal to $c\theta$, where θ is the angle of deflection and c is the restoring moment for unit angle of deflection.

In its motion the system is retarded by air currents and by induced currents, and for small velocities the retardation is proportional to the angular velocity of the system. The angular velocity is $d\theta/dt$, and the retarding moment may be written $bd\theta/dt$, where b is a constant known as the coefficient of damping; it is the ratio of the retarding torque on the moving system to the rate of displacement.

The angular acceleration is $d^2\theta/dt^2$, and if K is the moment of inertia of the rotating system, the moment tending to change the angular velocity is $Kd^2\theta/dt^2$. This must be equal to the sum of the deflecting moment Gi, the restoring moment $c\theta$, and the retarding moment $bd\theta/dt$. Hence, when there is no applied voltage,

$$K\frac{d^2\theta}{dt^2} = -c\theta - b\frac{d\theta}{dt},$$

or

$$K\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = 0,$$

which equation represents the motion of the moving system of a galvanometer. The form

[1] Threlfall, *Phil. Mag.*, 1890, xxx. 99.

[2] C. V. Boys, *Phil. Mag.*, 1890, xxx. 116. See also "Radio Micrometer," Vol. III.

of the solution of this equation depends upon whether the roots of the quadratic equation

$$Km^2 + bm + c = 0$$

are real or imaginary.

(i.) *Oscillatory Condition of Galvanometer System.*—When

$$\frac{b^2}{c} < 4K$$

the roots are imaginary, and the solution is

$$\theta = A\epsilon^{-(b/2K)t} \cos\left\{\left(\frac{c}{K} - \frac{b^2}{4K^2}\right)^{\frac{1}{2}} t + \alpha\right\},$$

where A and α are arbitrary constants. Hence θ is alternately positive and negative, the period being

$$\frac{2\pi}{\sqrt{(c/K) - (b^2/4K^2)}}.$$

Therefore the galvanometer system oscillates.

(ii.) *Non-oscillatory Condition of Galvanometer.*—When

$$\frac{b^2}{c} > 4K$$

the solution of the differential equation changes its character. In this case we have

$$\theta = A\epsilon^{-m_1 t} + B\epsilon^{-m_2 t},$$

where $-m_1$, $-m_2$ are the roots of the quadratic equation

$$Km^2 + bm + c = 0.$$

Hence
$$m_1 = \frac{b}{2K} + \sqrt{\frac{b^2}{4K^2} - \frac{c}{K}},$$

and
$$m_2 = \frac{b}{2K} - \sqrt{\frac{b^2}{4K^2} - \frac{c}{K}}.$$

$d\theta/dt$ vanishes when $t=0$, and if θ_0 is the value of θ when $t=0$,

$$\theta = \frac{\theta_0}{m_1 - m_2}(m_1\epsilon^{-m_2 t} - m_2\epsilon^{-m_1 t}),$$

$$\frac{d\theta}{dt} = \frac{m_1 m_2}{m_1 - m_2}\theta_0(\epsilon^{-m_1 t} - \epsilon^{-m_2 t}).$$

Hence $d\theta/dt$ never vanishes except when $t=0$ and when t is infinity, so that θ never changes sign but continually diminishes. The galvanometer system is non-oscillatory and is overdamped.

(iii.) *Critical Damping.*—A galvanometer system is critically damped when the motion of the needle is just becoming non-oscillatory. The system returns to the rest point in the minimum time, but the velocity never changes in direction. In this case

$$\frac{b^2}{c} = 4K.$$

When a current i flows through the galvanometer and the ratio of the displacing torque to the current is G, the mechanical power is $Gi d\theta/dt$, *i.e.* the torque Gi multiplied by the angular velocity, and this product must be equal to i times the back voltage.[1] Consequently, if $(r + R_G)$ is the resistance of the circuit and e is the impressed voltage,

$$i = \frac{e}{r + R_G} - \frac{G}{r + R_G} \cdot \frac{d\theta}{dt}.$$

Hence the equation of motion of the moving system may be written

$$K\frac{d^2\theta}{dt^2} + \left(b' + \frac{G^2}{r + R_G}\right)\frac{d\theta}{dt} + c\theta = \frac{Ge}{r + R_G},$$

where R_G is the resistance of the galvanometer and r is the resistance of the remaining portion of the circuit. The damping factor consists of two parts, one of which $(G^2/(r + R_G))$ represents the extra retarding torque due to induced currents when the circuit is closed, and the other (b') is the damping constant with open circuit. In galvanometers of the Thomson type the damping due to induced currents is small, and if critical damping is desired the retardation by air friction must be capable of adjustment.

(iv.) *Critical Damping with Suspended Coil Galvanometers.*—With suspended coil galvanometers critical damping is achieved by adjustment of the magnitude of the induced currents by alteration of the resistance. Thus the condition for critical damping is

$$\left(b' + \frac{G^2}{r + R_G}\right)^2 = 4Kc,$$

and the particular value of r which satisfies this equation is the resistance external to the galvanometer requisite to ensure critical damping. If the resistance of the apparatus with which the galvanometer is used is less than the external critical resistance, additional resistance is added in series with the galvanometer. On the other hand, if the resistance of the circuit is too great, critical damping may usually be obtained by connecting a suitable resistance in parallel with the galvanometer coil. Sometimes the system is damped by short-circuited turns of the coil.

(v.) *Critical Damping with Thomson Type of Galvanometer.*—In galvanometers of the suspended magnet type the damping due to induced currents is small, and if critical damping is desired, the moment of inertia must not be too great and the retardation due to air friction must be capable of adjustment. In the Paschen galvanometer no means of adjusting the damping is provided; however, the moment of inertia is very small, and the air retardation reduces the number of complete oscillations to a reasonable amount. In modern Thomson and Broca galvanometers the damping effect due to air is adjustable; a light aluminium vane about 80 sq. mm. in area is attached to the lower end of the suspended system and swings between two parallel damping plates, of somewhat greater

[1] Wenner, *Bureau of Standards, Bull.*, 1916, xiii.

surface, fitted to the frame of the galvanometer. The distances of the damping plates from the vane are adjustable, and the damping increases with decrease of the distances. It is easy in this way to make the system critically damped, over-damped, or oscillatory. A similar damping device has been used with magnetometers. It is necessary for the surfaces of the vane and damping plates to be conducting, as otherwise electrostatic effects are often troublesome after the vane has made contact with and slightly rubbed the plates. The surfaces must not, therefore, be lacquered; coating with platinum black is recommended.

(vi.) *Critical Damping with Einthoven Galvanometer.*—The damping of an Einthoven galvanometer with a given conducting string may be adjusted by altering either the frequency of the string, the intensity of the magnetic field in which the string moves, or the external resistance. Critical damping results when

$$4\pi f=\frac{8\mathrm{H}^2 r}{\pi^2\rho\delta\mathrm{R}},$$

where f is the frequency of the string, H the intensity of the magnetic field, r the resistance of the effective portion of the string, ρ the resistivity of the string, δ its density, and R the resistance of the circuit, all in consistent units. In the type of Einthoven galvanometer used in sound-ranging and in ballistic work for the measurement of the velocity of projectiles, the strings are of tungsten, copper, silver, or phosphor bronze, and the frequency of the strings is adjusted by altering the tension on them. When the frequency is fixed it is usually possible to alter the resistance of the external circuit until critical damping results. In the most sensitive instruments a string is a very fine glass fibre or tube coated with silver and weighs about 10^{-6} grams; the moment of inertia is in consequence very small, and with a string frequency of about 200 critical damping is usually obtained.

Wertheim-Salomonson has pointed out that the most suitable metal for the string is aluminium. From the equation for critical damping it is apparent that the frequency of the string will vary inversely as the product of the density and resistivity. A small product is therefore of considerable advantage. Wertheim - Salomonson gives the following data for platinum, gold, silver, copper, and aluminium:

Material.	δ.	ρ.	$\delta\rho\times10^4$.
Platinum	21·2	$0{\cdot}094\times10^{-4}$	1·99
Gold	19·3	·022 ,,	0·425
Silver	10·6	·0175 ,,	0·185
Copper	8·9	·0162 ,,	0·144
Aluminium	2·7	·0287 ,,	0·0774

Wires of aluminium 0·030 mm. thickness have been used, and wires of platinum and copper of 0·010 mm. thickness have been employed. When small frequencies are required the wires should be annealed *in situ* by the passage of an electric current.

(vii.) *Galvanometers with Small Damping. Ballistic Galvanometer.*—A ballistic galvanometer is designed to measure the total quantity of electricity which passes through it in a very short interval of time. The damping must be very small and the moment of inertia must be great; as a result the period is great and the whole quantity of electricity it is desired to measure passes through the galvanometer before the system has made any appreciable movement from its initial position. The ballistic galvanometer is described in the article on "Magnetic Measurements," § (4).

§ (11) Sensitiveness.—The sensitiveness of a galvanometer has been expressed in a variety of ways. The current sensitivity has been defined (*a*) as the *current* required to produce a deflection of 1 millimetre on a scale placed 1 metre from the galvanometer mirror; and (*b*) as the *deflection* on the scale produced by unit current. If the first were 10^{-10} ampere, the latter would be 10^{10} divisions. The more modern practice is to measure current sensitivity as the deflection in millimetres produced on a scale 1 metre away by a current of 1 microampere. The *voltage* sensitivity is usually defined as the deflection obtained at the distance stated when the voltage applied to the instrument is one microvolt. The voltage sensitivity is therefore the current sensitivity divided by the resistance of the galvanometer.

It is clearly unfair to compare galvanometers with different periods and different resistances unless allowance is made for these differences. It has already been shown that the sensitiveness of a galvanometer varies as the square root of the resistance if the insulation is negligible and the channel in which the coil is wound is constant; in practice the insulation modifies this relationship so that the sensitiveness is more nearly proportional to the power 2/5 of the resistance. When in a Thomson galvanometer the damping is small the relation between the sensitiveness and the period is given by

$$\mathrm{T}=2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{MH}}},$$

where T is the period, I the moment of inertia of the suspended system, M the moment of the magnet, and H the intensity of the magnetic field. As the deflection due to a current through the galvanometer is inversely proportional to H, it follows that the sensitiveness is proportional to the square of the period.

If the current in amperes is i and the deflection in millimetres at 1 metre is D, then

$$i=\frac{10\mathrm{H}}{\mathrm{G}}\cdot\frac{\mathrm{D}}{2000},$$

or $$\mathrm{D}=\frac{200i\mathrm{MT}^2\mathrm{G}}{4\pi^2\mathrm{I}},$$

where G is the constant of the galvanometer and varies with $\sqrt{R_G}$ or $(R_G)^{\frac{2}{5}}$, R_G being the resistance of the galvanometer.

(i.) *Factor of Merit.*—Since D varies with the period and the resistance of the galvanometer, it is desirable when comparing galvanometers to reduce the sensitivities to the values they would possess if all the galvanometers had the same period and resistance. This suggestion was made first by Ayrton, Mather, and Sumpner in 1890. Later they suggested 10 seconds as the standard period and 1 ohm as the standard resistance of the galvanometer. The question as to whether the relation "sensitiveness varies as $(R_G)^{\frac{1}{2}}$" or "sensitiveness varies as $(R_G)^{\frac{2}{5}}$" depends largely on the galvanometer. Some experimenters assume the former to be true, but many makers of instruments prefer the latter as being closer to practice. When the sensitiveness has been reduced to allow for period and galvanometer resistance it is called either "Normal Sensitiveness" or "Factor of Merit." The accepted period being 10 seconds, it follows that for any galvanometer the Factor of Merit is

$$\frac{100\mathrm{D}}{\mathrm{T}^2(R_G)^{\frac{1}{2}}}\text{ or }\frac{100\mathrm{D}}{\mathrm{T}^2(R_G)^{\frac{2}{5}}},$$

T being the period in seconds, R_G the resistance of the galvanometer, and D the deflection in millimetres at 1 metre. For current sensitivity the deflection should be taken as that due to one ampere or one microampere through the galvanometer coils. For voltage sensitivity the Factor of Merit is the deflection obtained when the voltage applied to the galvanometer coils is one volt or one microvolt. Usually the microampere and microvolt are used as standards.

In 1890, and again in 1896, Ayrton, Mather, and Sumpner[1] gave Factors of Merit for a number of galvanometers. Since then there have been many improvements in galvanometer construction; the following Factors are from a catalogue by The Cambridge and Paul Scientific Instrument Company:

Type of Galvanometer.	Period in Seconds (T).	Deflection for 1 Microampere (D).	Resistance of Galvanometer (R_G).	Factor of Merit $100\mathrm{D}/\mathrm{T}^2(R_G)^{\frac{2}{5}}$.
Thomson . . .	14·4	6830	1965	158
Broca . . .	10	350	8·8	147
	10	2200	860	148
Paschen . . .	6	8560	12·2	8,720
	6	2080	0·75	6,500
Ayrton-Mather .	8·2	245	20	110
	8·3	757	400	100
	3·0	18	7	90
	2·8	550	275	744
Einthoven (silvered glass)	0·004	7·3	8720	120,000
Einthoven (aluminium wire)	0·039	13·2	6·9	40,000

In the case of the Einthoven instrument angular deflections are not observed, but the displacements of the fibre are always magnified by optical means. The deflections given in the table are those obtained after a magnification of 600.

(ii.) *Zero-keeping Qualities.*—The sensitivity of a galvanometer as calculated by the method described is not entirely satisfactory, as no account is taken of the wandering of the rest point. Clearly, if, on an average, after a small deflection, the rest point wanders by an amount d, this limits the precision with which the deflection should be read. Even in null methods of using galvanometers, as in bridge measurements of resistance, reversals of the current must be made, and the galvanometer system is in general deflected through a small angle for every change. If, when no current passes through the galvanometer, the rest point changes while measurement is made, the effective sensitiveness is reduced. For example, if two galvanometers have the same Factor of Merit, and the position of each galvanometer spot can be read with ease within 0·1 mm., but the rest point of one galvanometer changes during a measurement by 1 mm. while the other is constant, the useful sensitiveness of the latter instrument, in null methods, is 10 times that of the former.

The change in the rest point of a galvanometer is due to several causes. In the Thomson type of instrument variations in the earth's magnetic field are often responsible for such changes, but this difficulty may be overcome by shielding the magnet system. The suspension when of silk is affected by changes in the humidity of the atmosphere, and markedly better results have been obtained by enclosing a galvanometer to prevent any interchange of air or moisture. Electrostatic disturbances

[1] *Phil. Mag.*, 1890, xxx. 58; *ibid.*, 1898, xlvi. 349.

are not infrequent, and may be caused by electrostatic attractions between the suspended system and the near surfaces of the fixed galvanometer frame. The surfaces of the galvanometer coils should be covered with tinfoil and the sheets of foil connected together. If an air damping system is provided, the surfaces should be coated with platinum black. Small quantities of radioactive compounds are sometimes inserted in the space between the coils and the suspended system.

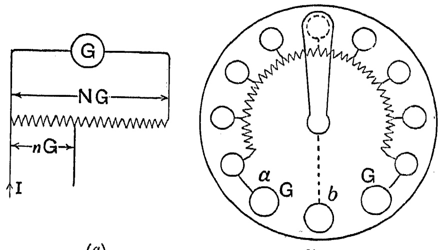
In suspended coil galvanometers the most frequent cause of change is due to the metallic suspending fibre. If the fibre is too coarse the galvanometer is insensitive; if too fine, elastic fatigue is observed. In suspended coil galvanometers one end of the coil should be attached to the frame and magnet; this reduces electrostatic troubles.

If a suspended coil is associated with magnetic matter (dust is often sufficient) the controlling force may be many times that due to the suspension, and the sensitiveness is reduced in proportion. Instrument-makers of the first class have special dust-proof rooms for the assembly of the coil parts and of the suspension; the wire is made from electrolytic copper or silver and is drawn through diamond dies; the silk is of exceptional purity, and after completion of the coil it is treated with chemicals to remove all traces of iron.

(iii.) *Galvanometer Shunts.*—Resistances are often placed in parallel with suspended coil galvanometers to make them aperiodic, and, at times, with all kinds of galvanometers, to reduce the sensitiveness.

Let R_G be the resistance of the galvanometer and R that of the shunt; then if I is the current in the main circuit, that in the galvanometer is $IR/(R_G+R)$. The maker often provides a set of shunt coils for use with a particular galvanometer.

A universal shunt system has been designed by Ayrton and Mather which is capable of being applied to *any* galvanometer. This is shown diagrammatically in *Fig.* 5 (*a*). R_G is the resistance of the galvanometer, and across its terminals there is a high resistance of NR_G ohms. The current I in the main circuit enters and leaves as shown in *Fig.* 5 (*a*) and the current through the galvanometer is $In/(N+1)$, that is, the fraction of the current which flows through the galvanometer is not dependent on the galvanometer resistance. By having a number of tapping points so that $n/(N+1)$ is equal to 0·1, 0·01, etc., the effect of the shunt may be varied. The value of the main current I varies with the value of n owing to the change in the effective resistance of the circuit, and such a change naturally affects the real sensitiveness. In order that the sensitiveness may not be unduly reduced R must be much larger than the resistance of the galvanometer.



(*a*) (*b*)

FIG. 5.

Fig. 5 (*b*) shows an arrangement of coils often adopted in the Ayrton and Mather shunt.

§ (12) CHOICE OF A GALVANOMETER.—Experience is the only safe guide to the choice of a galvanometer for any particular purpose. It is useful, however, to tabulate the points which must be considered.

(i.) *Resistance.*—The resistance should be appropriate for the measurement in order to secure good sensitivity. If a variety of measurements are to be made and the circuits vary greatly in resistance it is of advantage to choose as a suspended magnet system a Thomson type having at least two coils, each of 100 ohms resistance. If a suspended coil galvanometer is chosen, the resistance of the coil should be about 100 ohms.

(ii.) *Period.*—For general use a short-period galvanometer is most advantageous. A convenient period is 10 seconds.

(iii.) *Damping.*—Except for the measurement of a quantity of electricity, *i.e.* except when the galvanometer is required to be used ballistically, the damping should be critical. In choosing a Thomson or Broca galvanometer preference should be given to one having variable air damping. In the suspended coil type the damping can be adjusted by adding resistance in series or in parallel with the coil.

(iv.) *Constancy of Rest Point.*—If the place where the experiments are to be made is subject to large magnetic disturbances and a suspended magnet system must be chosen, a Broca galvanometer is probably best. If any type of galvanometer will serve, a suspended coil type like the Ayrton-Mather galvanometer is very suitable. In general a Broca galvanometer is not much disturbed by leakage effects due to electric trams and trains.

(v.) *Suspension.*—The constancy of rest point depends also on the suspension; if the type permits, a quartz fibre is best. For very sensitive work a silver strip suspension may be used in a suspended coil instrument except when ratios of deflections have to be measured: in such cases a phosphor bronze suspension should be used. This also is best for general work.

(vi.) *Immunity to Mechanical Disturbances.*—When a system is subject to much vibration, such as is common in many London laboratories, a marked advantage is obtained by mounting the suspended system so that there is symmetry about the axis of rotation. A trial of the instrument is needed to see if this condition is sufficiently well fulfilled.

(vii.) *Optical System.*—The definition of the light spot used in reading deflections and rest points should be as perfect as possible in order that time may be saved and fatigue of the eye prevented. The galvanometer should therefore be provided with a good mirror, which should not be so thin as to change in radius of curvature with time. Plane mirrors are frequently employed with a lens mounted in front of the galvanometer; in this way the focal length of the optical system can be readily altered.

(viii.) *Visibility of Parts.*—The parts should be sufficiently accessible to ensure that the clearances are ample and to enable reasonable adjustments, such as damping, to be made with ease.

F. E. S.

GASES, DIELECTRIC CONSTANTS OF. See "Capacity and its Measurement," § (19).

GAUSS. The name given to the unit of magnetic force or the intensity of a magnetic field on the C.G.S. practical system of units.

1 Gauss = 1 unit of magnetic force.

See "Units of Electrical Measurement," § (27).

GAUSS THEOREM. See "Electrostatic Field, Properties of," § (2).

GEISSLER TUBES, for exhibition of discharge through gases. See "Electrons and Discharge Tube," § (1).

GIEBE, E., air condensers of. See "Capacity and its Measurement," § (32).

GLASS, tabulated values of dielectric constants of various kinds of. See "Capacity and its Measurement," § (24).

GLASS PLATE CONDENSERS. See "Capacity and its Measurement," § (31).

GLAZEBROOK'S REVOLVING COMMUTATOR, use of, for the measurement of capacity. See "Capacity and its Measurement," § (41).

GOLD:
Electroplating. See "Electrolysis, Technical Applications of," § (12).
Extraction and refining of. See *ibid.* §§ (18), (19).

GOLDSCHMIDT ALTERNATOR: a machine for the production of radio-frequency oscillations. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (5).

GOTT'S COMPARISON METHOD, for the measurement of capacity. See "Capacity and its Measurement," § (47).

GRAY'S MAGNETOMETER. See "Magnetic Measurements and Properties of Materials," § (2) (iv.).

GRID: the control electrode of a thermionic valve. See "Thermionic Valves," § (1).
Action of, in thermionic valves. See *ibid.* § (2).

GROVER, F. W., work of, on the measurement of the capacity and power factor of a condenser. See "Capacity and its Measurement," § (48).

GUARD RING: a device for checking distortion of the electric field at the edges of condenser plates, etc. See "Capacity and its Measurement," § (7).

GUARD WIRES: earthed wires employed to protect overhead telegraph circuits from power circuits. See "Telegraph, The Electric," § (14).

GUMLICH YOKE AND ISTHMUS METHOD, for high magnetisation tests. See "Magnetic Measurements and Properties of Materials," § (42).

GUMLICH'S MAGNETOMETER. See "Magnetic Measurements and Properties of Materials," § (2) (iii.).

H

HALLWACHS' EFFECT: another name for photoelectric fatigue. See "Photoelectricity," § (2).

HAMMERS, ELECTROMAGNETIC, for chiselling and riveting. See "Electromagnet," § (5).

HARMONIC ANALYSIS, by vibration galvanometers. See "Vibration Galvanometers," § (47).

HARTMANN-KÄMPF ALTERNATOR, for audio-frequency work. See "Inductance, The Measurement of," § (8).

HAY'S C.B. DUPLEX SYSTEM: a system of telegraphy employing a central battery, in which each circuit carries simultaneously a message in each direction. See "Telegraphy, Central Battery System of," § (3).

HEAT TREATMENT, application of, to test specimens used in magnetic work. See "Magnetic Measurements and Properties of Materials," § (19).

HEAT TREATMENT OF STEELS: procedure with small specimens for magnetic work.

See "Magnetic Measurements and Properties of Materials," § (54).

HEAVISIDE LAYER: a layer of ionised air in the upper atmosphere. Effect of, on electromagnetic waves, and hence on directional wireless telegraphy. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (13).

HEAVY CURRENTS, measurement of, at radio frequencies. See "Radio-frequency Measurements," § (20).

HEMATITE, magnetic properties of. See "Magnetic Measurements and Properties of Materials," § (73).

HENRY: the practical unit of inductance.

$$1 \text{ henry} = 10^9 \text{ centimetres.}$$

See "Inductance, The Measurement of," § (1).

HERTZ RADIATOR, THE: a device for producing electric waves. Frequency and damping of. See "Wireless Telegraphy," § (6).

HETERODYNE METHODS, use of, for the detection of continuous waves in wireless telegraphy. See "Wireless Telegraphy," § (25).

HETERODYNE WAVEMETERS, use of, in radiotelegraphic work. See "Radio-frequency Measurements," § (16).

HIBBERT MAGNETIC STANDARD: a standard of magnetic flux. See "Magnetic Measurements and Properties of Materials," § (3).

HIGH-FREQUENCY EXPERIMENTS, on the magnetic properties of iron. See "Magnetic Measurements and Properties of Materials," §§ (65), (66), and (67).

HISSING ARC. See "Arc Lamps," § (4).

HOLDEN'S PERMEABILITY BRIDGE. See "Magnetic Measurements and Properties of Materials," § (28).

HONDA AND OKUBO, theory of magnetism. See "Magnetism, Modern Theories of," § (1).

HOPKINSON BAR AND YOKE, use of, in magnetic testing. See "Magnetic Measurements and Properties of Materials," § (24).

HOT WIRE AMMETERS, use of, at radio frequencies. See "Radio-frequency Measurements," § (19).

HOYT-TAYLOR BALANCE: an arrangement of loop aerial and underground wire devised to balance out atmospherics. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (10).

HUGHES SYSTEM: a low-speed system of telegraphy in which the receiving instrument prints the message in roman type on a paper slip. See "Telegraphs, Type-printing," § (2).

HYDROGEN, ELECTROLYTIC PREPARATION OF. HYDROGEN GAS-MAKING. See "Electrolysis, Technical Applications of," § (29).

HYDROGEN ATOM, determination of mass of, given value of e. See "Electrons and the Discharge Tube," § (25).

Structure of. See *ibid.* § (28).

HYPOCHLORITES, ELECTROLYSIS OF: Kellner cell and Haas-Oettel cell. See "Electrolysis, Technical Applications of," § (25).

HYSTERESIS: the tendency shown by iron and steel to retain the magnetism which has been imparted to them by any magnetising force. See "Magnetic Hysteresis"; "Magnetic Measurements and Properties of Materials," § (1); also "Magnetic Hysteresis."

In the magnetic circuits of transformers. See "Electromagnet," § (6)

HYSTERESIS LOSS, determination of, from the hysteresis loop. See "Magnetic Measurements and Properties of Materials," § (22); also "Magnetic Hysteresis."

HYSTERESIS OF DIELECTRICS. See "Dielectrics," § (6) (ii.).

HYSTERETIC CONSTANT: a magnetic constant for each material, which is a measure of the loss of energy occurring in that material due to hysteresis. See "Electromagnet," § (6).

— I —

ILIOVICI PERMEAMETER. See "Magnetic Measurements and Properties of Materials," § (35).

IMPEDANCE. If a simple harmonic electromotive force of amplitude E and frequency p acts on a circuit of self-inductance L and resistance R, the current I is given by the equation

$$I = \frac{E}{\sqrt{R^2 + L^2p^2}} \sin\left(pt - \tan^{-1}\frac{Lp}{R}\right).$$

The quantity Lp is the reactance of the circuit and $\sqrt{R^2+L^2p^2}$ is the impedance. If the circuit has capacity C and no inductance the current is

$$\frac{E}{\sqrt{[R^2+(1/C^2p^2)]}} \sin\left(pt + \tan^{-1}\frac{1}{CRp}\right).$$

The reactance is then $1/Cp$, and the impedance

$$\sqrt{\left(R^2 + \frac{1}{C^2p^2}\right)}.$$

If there are both inductance and capacity

in the circuit the reactance is $Lp - (1/Cp)$, and the impedance

$$\sqrt{\left\{R^2 + \left(Lp - \frac{1}{Cp}\right)^2\right\}}.$$

See "Inductance, Measurement of," § (3).

Impedance Measurements, determination of capacity by means of. See "Capacity and its Measurement," § (38).

Impurities in Surrounding Gas, Effect of, on the thermionic emission from palladium. See "Thermionics," § (4) (iv.) (*b*).

On the thermionic emission from platinum. See *ibid.* § (4) (iv.) (*a*).

On the thermionic emission from tungsten. See *ibid.* § (4) (iv.) (*c*).

INCANDESCENCE LAMPS

§ (1) Historical.—The earliest work on incandescent filament lamps was carried out by various experimenters from as far back as 1841, when J. W. Starr, an American, carried out much experimental work and eventually took out patents for "a metallic or carbon conductor intensely heated by the passage of electricity for the purpose of illumination." These early stages in the evolution of the electric incandescent lamp were terminated in 1878 by the work of T. A. Edison and J. W. Swan, who, working independently, produced between 1878 and 1880 the first practical carbon filament lamps. Edison aimed at producing filaments from natural fibres of grasses or bamboo, whilst Swan developed the parchmentised cotton thread and eventually the squirted thread of cellulose, which was destined to become the universal process. The carbon filament lamps of the types developed by Edison and Swan, and improved by Sawyer's process of "flashing" in an atmosphere of hydrocarbon gas, held their own with but slight modifications for some twenty years, and it was not until 1904 that the General Electric Company of America produced the so-called metallised carbon filament lamp. Metallised or graphitised carbon filaments were produced by treatment of the ordinary filaments at a higher temperature. The result was a more refractory material which stood, without disintegration, a temperature which permitted the efficiency to be increased from 0·25 to approximately 0·3 average candle[1] per watt.

The Nernst lamp introduced about 1897 never really promised to attain popularity, in spite of the great amount of ingenuity expended in its development. Its overall efficiency approximated to that of the metallised carbon filament lamp. In 1898 the first of the practical metal filament lamps appeared in the form of the Osmium Lamp; to be followed in 1902 and 1903 by the Tantalum Lamp with an efficiency of approximately 0·45 candle per watt.

From 1904 the highly refractory properties of tungsten filaments began to be realised, and it became obvious that the future of the incandescent lamp for some time to come would be with the tungsten filament lamp. For several years no method was found of producing tungsten in the form of metallic wire, and all the earlier filaments were made by squirting threads of tungsten powder held together by a binder which had eventually to be eliminated, except in the case when colloidal tungsten was the binder. So perfect had this process become that when, in 1906 drawn tungsten wire was produced in the laboratories of the General Electric Company of America, it was only the greater convenience of manipulation during manufacture which commended this form of wire for practical adoption by lamp makers generally. The use of drawn tungsten wire is now almost universal, and the vacuum lamp has for a time settled down to a form which yields an efficiency of from 0·6 to 0·7 average candle per watt over sizes ranging from 20 to 200 watts and voltages up to 250. The fine filaments required for low-watt high-voltage lamps necessitate running at an efficiency which is some 15 per cent lower than for the larger watt low-voltage types.

§ (2) General.—The chief limiting factor in the improvement of tungsten filament lamps is not the melting-point of the tungsten but the temperature at which it disintegrates or volatilises. The melting-point of tungsten is in the neighbourhood of 3600° K., but the practical operating temperature in a vacuum is approximately 2300° K. If run at a higher temperature than this the volatilisation of the tungsten soon blackens the bulb and also increases the resistance of the filament itself. Filaments will burn for very short periods at temperatures which yield efficiencies of the order of three candles per watt, and if it were possible to prevent volatilisation, efficiencies of this order would undoubtedly be possible in practice. The apparent impossibility of achieving this result has led to the adoption of the device of surrounding the filament with an inert gas, the function of which is to suppress the tendency for the tungsten particles to leave the filament owing to their collision with the molecules of the gas. In such gas-filled or "half-watt" lamps the filament wire must be compressed into as small a space as possible so as to diminish the cooling effect of the gas. Even though the filaments are run at a temperature of 2800° K. this cooling effect renders the gain in efficiency relatively small in all but the larger sizes of lamps.

[1] The term "average candles" is synonymous with "mean spherical candles."

§ (3) The Tungsten Wire.—Tungsten wire is made from finely divided tungsten powder. The powder is made by two methods, according as wolframite or scheelite ore is employed.

Starting from wolframite, the ore is crushed to a coarse powder, mixed with soda-ash, and roasted in a reverberatory furnace at a temperature just high enough to keep the mass in a pasty condition. After cooling, the product is crushed, and the sodium tungstate leached out. The solution of sodium tungstate is heated to boiling, and calcium tungstate is precipitated by the addition of a solution of calcium chloride. After washing the calcium tungstate to remove all excess chlorides, it is decomposed with hot hydrochloric acid, the tungstic oxide is allowed to settle, the solution decanted off, and the oxide washed on a suction filter. The tungstic oxide so obtained is purified by dissolving in ammonia, evaporating the filtrate consisting of a solution of ammonia paratungstate to the consistency of a white mud, and decomposing the mud with nitric acid. The yellow tungstic oxide so obtained is washed with distilled water on suction filters, and dried.

If scheelite ore be employed, it is crushed and decomposed by heating with strong hydrochloric acid. The residue of tungstic acid is subjected to a second extraction with hydrochloric acid, and after washing in the usual way is taken up with ammonia and the resulting solution of ammonium tungstate decanted through a filter. The solution of ammonium tungstate is then run into hot hydrochloric acid when the tungstic acid is precipitated in a pure form. After washing with distilled water it is dried at 200°-300° C.

The oxide formed by either of these methods is fired in Battersea crucibles to about 1000° C. for several hours, crushed, and then reduced at a temperature of about 950° with hydrogen. The resulting metal is practically pure except for slight impurities which the oxide has taken up from the Battersea crucible during firing. These impurities have in the past been considered necessary to control grain growth in the filament during the life of the lamp.

If desired, a small percentage of thoria can be mixed with the tungsten oxide in order to produce the same effect as the impurities derived by firing in the Battersea crucible.

The powder so formed is weighed out into batches sufficient to make slugs of about $\frac{1}{4}$ inch square section and 6 inches long. These are formed by pressing the powder into dies under hydraulic pressure, after which they are sintered at a temperature of about 1100° C. The slugs are then brought up near to their melting-point by passing a current of about 1400 amperes through them in an atmosphere of hydrogen. During this process they contract, and the metal is given a definite crystalline structure. The slugs are then swaged at a temperature which may lie between 1600° and 2000° K. Swaging consists in passing the metal repeatedly between rapidly moving hard-steel machine-operated hammers in such a way that it becomes formed into a long wire of final diameter about 0·8 mm. In this condition it has considerable tensile strength, and a microscopic examination shows that the original crystals have become elongated in the direction of the length of the wire. The wire can now be drawn through diamond dies, provided it is heated at the point of passing through the dies. The smallest sizes of wire employed in practice are of the order of 0·02 mm. diameter. Wire so formed has physical properties which enable it to be easily handled. It has both high tensile strength and pliability, induced by the fibrous structure given to the wire in the drawing process. These properties are lost when the filament is heated in the lamps, due to a change in crystal structure known as equiaxing. The fibrous structure is no longer maintained but gives place to small equiaxed crystals which tend afterwards to grow at the temperatures of operation. The wire in this condition is brittle and fragile, the degree of which depends presumably on the ultimate size and shape of the grains and nature of the grain boundaries which may sometimes extend right across the filament. The condition of the filament after equiaxing has taken place is largely governed by the presence of small quantities of impurity in the metal. The function of these impurities is probably to act at the intercrystal boundaries, regulating the sizes of the crystals and preventing subsequent crystal growth.

The firm of Pintsch in Berlin produced in 1914–1916 a "single crystal" wire. A thread of sintered tungsten powder was passed slowly through a heater in which the temperature gradient was adjusted to give the most favourable conditions for crystal growth. A crystal commences to grow and, spreading across the whole cross-section of the wire, incorporates within itself each new grain which it approaches in its growth. If the rate of travel of the metal corresponds with the rate of crystal growth there results a wire which, being made up of one crystal, has within itself no crystal boundaries; hence, it is claimed, it is not subject to the disadvantage introduced by grain boundaries.

§ (4) Construction of Lamp.—The production of the mount for the filament, the winding of the filament, and the sealing in of the completed mount to the glass bulb are operations which call for much skill and organisation as processes of routine manufacture and glass working, but do not make any special demands on the physicist.

The production of satisfactory vacuum tight "sealing-in" wires has entailed in the past a large amount of experimental work. In the early days a short length of platinum wire was welded between two lengths of copper wire and sealed into the glass "pinch" or "press" in such a way that the platinum was buried in the glass and effectually stopped any leakage path there might be along the surface of the copper wires. It is commonly considered that platinum is the only possible metal to use for sealing into glass, because its coefficient of expansion is approximately the same as that of glass. The correct coefficient of expansion, however, does not seem to be the only quality requisite for a good sealing-in wire. The quality of "wetting" between the glass and the metal appears to be equally important, but the exact physical nature of this quality is not easy to define. Glasses vary in their coefficient of linear expansion from 8×10^{-6} to 10×10^{-6}, so that a satisfactory platinum substitute must therefore "wet" well and also have a coefficient of expansion similar to that of the glass for which it is intended. There are two common substitutes, in both of which nickel steel forms the core, because the coefficient of expansion of this alloy can be adjusted by varying the nickel content. In the one a nickel steel rod of suitable coefficient of expansion is coated with copper, after which a thin sheath of platinum is drawn over. The whole is taken up to the melting-point of copper, which causes the coating between the platinum and the steel to make a tight joint with both, and permits the rod to be drawn down in dies like ordinary wire. Short lengths of this are used as described above. The saving of platinum by this process is considerable, being about two-thirds. In the other platinum substitute, the nickel steel wire with copper coating is used without any platinum, but the proper "wetting" action is achieved by a coating of borax on the surface of the copper. Such wire can be employed without any joints and has come into very wide use.

§ (5) Evacuation.—Evacuation is one of the chief problems in electric lamp manufacture. The vacuum in a tungsten lamp should be of the order of 0·0001 mm. If the pressure is as high as 0·005 mm., but below about 1 mm., space currents will pass sufficient for the resulting discharge within the bulb to destroy the lamp. It has also to be remembered that a lamp must not only start with a high vacuum but must retain it throughout its life. The presence of any trace of water vapour cannot be tolerated, as it acts as a catalyst, becoming dissociated at the high temperatures and oxidising the filament, whereupon the tungsten oxide becomes reduced again with the formation of water vapour and the process repeats itself, thus leading to a transference of tungsten from the filament to the glass walls of the bulb. To ensure a permanent vacuum, all possible occluded gas must be removed from the glass before the lamp is removed from the pumps. The glass forms the main reservoir of gas, there being too small a volume of metal present to add much to the total occluded quantity. The gas is partially removed from the glass by heating the lamps to a temperature approximating 400° C. during evacuation. Sometimes the filaments are also glowed during evacuation so as to render all internal parts as hot as possible before the lamps are sealed off from the pumps. The really high vacua achieved in modern lamps are secured automatically by the use of phosphorus within the bulb. This material has the property, in the presence of an electric discharge, of being capable of "cleaning up" a considerable quantity of residual gas. Other materials also have a similar action, but phosphorus is one of the most reliable in this respect. The mechanism of this action is not altogether clear and certain phases of it are quite obscure. The use of such an agent, however, has the practical result that much of the work of producing a high vacuum may be left to its action, and that it is only necessary to ensure, when exhausting lamps, that more gas is not left in the bulb and occluded on the walls than the phosphorus is capable of dealing with.

§ (6) The Gas-filled Lamp.—The filaments of gas-filled lamps are coiled into close spirals so that conduction of heat through the gas and convection by it may be a minimum. In order to obtain the requisite candle-power it is necessary to have a certain area of incandescent metal free to radiate into space, but owing to the presence of gas this area should be located within as small a volume as possible. Other limiting factors, however, come in to prevent the achievement of the ideal arrangement. Firstly, the diameter of wire which is necessary for commercial voltages can only be made up into spirals of a certain size, otherwise they will sag and open out at the temperatures at which they must operate. Secondly, the very high temperature conditions prevailing in the gas immediately surrounding the incandescent wire reduce its electrical resistance, which increases the tendency for an electrical discharge to take place. If this transpires, the lamp is quickly destroyed and sufficient gas clearance is thus essential between the points of highest potential difference.

The two principal inert gases employed are nitrogen and argon; both these gases fulfil to a fair extent the conditions necessary to make an efficient lamp. These conditions are: (1) That the gas should be inactive, *i.e.* it

should have no action on the filament. (2) That the thermal conductivity, specific heat, and density of the gas should be as low as possible. (3) That the gas should not dissociate when raised to the temperature of the filament, thus causing loss of heat of the filament by the heat absorbed on decomposition. (4) That the gas should be of such a nature that the tendency for an electrical discharge to take place should be a minimum.

Nitrogen, which is the cheaper of the two, has a slight action on the filament, forming a nitride of tungsten; this action is only noticeable in types of lamps having a thin filament, when also its higher conductivity is more marked. In these lamps argon is used to good advantage, but it conducts the electricity more easily, and if used quite pure a discharge tends to take place. To prevent this a few per cent of nitrogen are added.

§ (7) Physical Characteristics of Lamps.—The governing factor in the design, production, and operation of incandescent filament lamps is the relatively large changes in the characteristics of a lamp which are produced by a small change in filament temperature, combined with the sensitiveness of that temperature to changes in the electrical input to the lamp, or the dimensions of the filament. Consider, for instance, a standard vacuum lamp with an efficiency of 0·8 average candle per watt (1·25 watts per M.S.C.P.). The true filament temperature will be approximately 2275° K.

A 1 per cent increase in volts or a 2 per cent increase in filament diameter will produce a change of approximately 3·6 per cent in candle power, 2 per cent in efficiency, and 14 per cent in life.

The following table is useful as showing how a 1 per cent change in electrical input or in filament dimensions affects the performance of a lamp as regards candle power, efficiency, and approximate life:

Vacuum Lamp—Tungsten Filament, Approximate Temperature 2300° K. Rate of Variation of Performance with Change of Electrical Input and Filament Dimensions.

	Filament Temperature.	Candle Power.	Candles per Watt.	Approximate Life.
	per cent.	per cent.	per cent.	per cent.
For constant filament size:				
1 per cent increase in volts	+0·34	+3·6	+2·1	−14
1 per cent increase in amps. . . .	+0·55	+5·9	+3·3	−22
1 per cent increase in watts . . .	+0·21	+2·2	+1·3	− 8_5
At constant volts:				
1 per cent increase in filament diameter .	+0·17	+1·8	+1·0	− 7
1 per cent increase in filament length .	−0·33	−3·5	−2·0	$+13_5$

The figures in this table show that individual incandescent lamps, although given the standard rating, must necessarily be liable to vary somewhat from the ideal lamp which the manufacturer sets out to make. Lamps as made tend to group themselves round that ideal, but it is necessary to settle certain limits within which it is reasonable to require any lamp to fall which purports to comply with the rating. Diagrams showing these limits have candle power plotted as

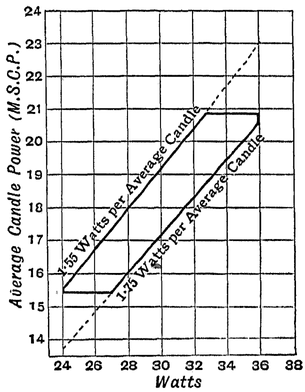


Fig. 1.

ordinates and watts as abscissae. Every lamp having a definite efficiency will lie on a straight line which runs diagonally across the diagram. As it is usual in commerce to rate incandescent lamps in watts per candle rather than in candles per watt, these lines run from the lower left-hand part of the diagram to the upper right-hand side. A limit (or target) diagram prescribed by the British Engineering Standards Association for 220-volt 30-watt vacuum lamps is shown in *Fig.* 1.

It will be seen by the proximity of the diagonal boundaries that the narrowest limits are placed upon watts per candle. The temperature of the filament corresponds closely with the watts per candle, and as life is largely a function of filament temperature the effect of narrowing the limits of watts per candle is to secure lamps which are relatively uniform as regards life. By life is here meant both maintenance of candle power and avoidance of filament breakage. The testing of the life of incandescent lamps re-

quires the greatest care if trustworthy results are desired. The life of a lamp varies roughly as the sixth power of the watts per candle. As the watts per candle vary inversely as about the square of the voltage, the life will be seen to be very sensitive to voltage adjustments and variations. Furthermore, even over the narrow range of watts per candle permitted within the limits of the diagram there will be a variation of life of about ± 40 per cent. It is thus clear that a criterion as to the life quality of any batch of lamps can only be obtained by adjusting the voltages of the individual life test lamps to values which will bring them exactly to the standard or rated efficiency (*i.e.* to a uniform filament temperature).

§ (8) Filament Temperatures.—The determination of the true efficiency of a lamp entails the measurement of its average candle power and its watts. A direct measure of efficiency may be secured by the measurement of filament temperature, using a Lummer Brodhun contrast photometer. The hue of the illuminated field in such a photometer changes according to the temperature of the tungsten filament which is illuminating it, and by using a suitable certified colour identity standard a simple filament temperature gauge is obtained, employing only the photometric apparatus commonly found in a works photometric laboratory. Such an arrangement is of special use when the temperature of gas-filled lamp filaments is to be estimated, because with such lamps there are other agencies besides radiation for removal of heat energy from the filament, so that watts per candle are no longer an accurate measure of filament temperature. Ordinary optical pyrometry at these temperatures is not a very exact art, partly owing to the differences in the quality of the radiation coming from different points on the filament convolutions.

G. E. C.

Indicating Instruments. Instruments in which the value of the quantity measured is deduced from the deflection of a moving part, against the force of gravity or of a spring. See "Alternating Current Instruments," § (7).

Inductance:

Campbell Standard of Mutual: an absolute standard of mutual inductance which is constructed so that an accurate knowledge of the dimensions of the coils is not required. A coil of square winding section is placed coaxial with and midway between two cylindrical helices which are connected in series and whose dimensions may be determined with high precision. The mean diameter of the coil is arranged so that the mutual inductance between it and the two helices has a maximum value. See "Inductance, Calculation of Coefficients of (Mutual and Self)," § (3).

Calculation of. See "Radio-frequency Measurements," § (31).

Coefficients of. See "Inductance, The Measurement of," § (1).

Comparison of Self and Mutual. See *ibid.* §§ (88)-(96).

General properties of, in radio-frequency circuits. See "Radio-frequency Measurements," § (32).

Mutual, measurement of, at radio frequencies. See *ibid.* § (39).

Mutual, between two circuits, Neumann's formula for. See "Inductance, Calculation of Coefficients of (Mutual and Self)," § (1).

Mutual, between two coaxial circles, derived from Neumann's formula. See *ibid.* § (2) (i.).

Mutual, between two coaxial circles, formulae for. See *ibid.* § (2) (ii.).

Mutual, of long cylinder and internal concentric coaxial short cylinder, employed in the construction of the standard field used in the calibration of ballistic galvanometers. See *ibid.* § (5).

Mutual, of rectilinear circuits of thin wire, calculated from Neumann's formula. See *ibid.* § (7).

Mutual, of short coaxial cylinders; determination of, by replacement by equivalent circles. See *ibid.* § (4).

(At radio frequencies), references to original papers on. See "Radio-frequency Measurements," end of Section V.

Self, measurement of, at radio frequencies. See *ibid.* § (37).

Standards of, design and construction of coils for use as. See "Inductance, The Measurement of," § (51) *et seq.*

Use of, in radio-frequency circuits. See "Radio-frequency Measurements," Section V.

Variable, use of, in radio-frequency work. See *ibid.* § (34).

With iron cores, the measurement of. See "Inductance, The Measurement of," §§ (122), (123).

INDUCTANCE (MUTUAL AND SELF), CALCULATION OF COEFFICIENTS OF, WITH FORMULAE AND TABLES

The calculation of the mutual inductance between two coils from their linear dimensions is of primary importance in the absolute determination of the ohm and the ampere, and consequently the development of suitable formulae for this purpose has received much

attention. A very complete collection of the formulae which have proved useful has been made by Rosa and Grover of the U.S. Bureau of Standards and published by them in Circulars S. 169 and S. 320. The footnotes of these circulars also constitute a complete bibliography of the subject.

The mathematical methods employed are treated in detail in Gray's *Absolute Measurement in Electricity and Magnetism.*

§ (1) Fundamental Formula.—The mutual inductance between two circuits A and B is measured by the magnetic flux threading B when unit current flows round A. The obvious method of calculation is, therefore, to divide the contour of A into line elements; then, using the well-known formula for the field due to a current element, to calculate the flux through B for each element by surface integration over B, and finally to obtain the total flux through B by contour integration round A. However, by making use of Stokes' theorem, the surface integral over B may be converted into a contour integral round B, and therefore the mutual inductance may be obtained by double contour integration [1] round A and B.

Thus the field H at a point r, ϕ due to a current element Idl is $Idl \sin \phi/r^2$ directed normally to the plane r, Idl, and such that I, r, H form a right-handed system. Now H is the curl of a vector G of magnitude Idl/r parallel to the current element Idl, so that by Stokes' theorem the surface integral of H over any surface is equal to the contour integral of G round that surface. Hence if dl, dl' are line elements of A and B respectively, the mutual inductance between A and B is given by

$$\int dl \int dl' \frac{\cos \theta}{r}, \quad . \quad . \quad . \quad (1)$$

in which θ is the angle between dl and dl' and the integrations are round the contours of A and B.

The above expression is known as Neumann's formula.

§ (2) Coaxial Circles. (i.) *Maxwell's Formula.*—The mutual inductance between two coaxial circles is readily derived from Neumann's formula.

If A, a are the radii of the two circles, and b the distance between their planes, the distance between two elements which are inclined at an angle θ is

$$r = \sqrt{A^2 + a^2 + b^2 - 2Aa \cos \theta},$$

and since $dl = A d\theta$ the mutual inductance M is given by

$$M = 2\pi A a \int_0^{2\pi} \frac{\cos \theta d\theta}{\sqrt{A^2 + a^2 + b^2 - 2Aa \cos \theta}},$$

which by the substitution $\theta = \pi - 2\psi$ yields

$$M = 4\pi \sqrt{Aa} \left\{ \left(\frac{2}{k} - k \right) K - \frac{2}{k} . E \right\}, \quad . \quad (2)$$

[1] See also "Electromagnetic Theory," § (4).

in which

$$K = \int_0^{\frac{\pi}{2}} \frac{d\psi}{(1 - k^2 \sin^2 \psi)^{\frac{1}{2}}}, \quad E = \int_0^{\frac{\pi}{2}} (1 - k^2 \sin^2 \psi)^{\frac{1}{2}} d\psi,$$

and

$$k^2 = \frac{4Aa}{(A + a)^2 + b^2}.$$

Now K and E are complete elliptic integrals of the first and second kinds respectively and have been tabulated in terms of the angle $\phi = \sin^{-1} k$ by Legendre. Moreover, Maxwell, to whom the above formulae is due,[2] has computed a table of values of $\log (M/4\pi \sqrt{Aa})$ for values of ϕ ranging from 60° to 90°. By means of this table it is possible to obtain M to seven-figure accuracy, if the value of ϕ falls within the above limits. Table I. on the two following pages may be used if an accuracy of 1 in 1000 is all that is required.

(ii.) *Other Formulae.*—By suitable manipulation of formula (2) a large number of alternative formulae may be obtained. These formulae have a twofold use. Some may be used for computation in cases where formula (2) ceases to be accurate, while others form starting-points for the derivation of formulae for more complex cases. Thus by applying Landen's transformation to (2) Maxwell obtained the formula

$$M = 8\pi \sqrt{Aa/k_1}(K_1 - E_1), \quad . \quad . \quad (3)$$

in which K_1, E_1 are complete elliptic integrals to modulus k_1, and $k_1 = (1 - k')/(1 + k')$, $k'^2 = 1 - k^2$. This formula is more accurate than (2) when ϕ is nearly 90°.

Again, by differentiation of (2) it may be shown that M satisfies the equation

$$k^2(1 - k^2)\frac{d^2M}{dk^2} - k(1 + k^2)\frac{dM}{dk} - 3M = 0,$$

and by appropriate changes of the independent variable five series formulae for M may be obtained from this equation, two converging most rapidly for circles far apart and three for circles close together.[3] Certain of these formulae may be employed so as to obtain the mutual inductance between coaxial cylinders, or again for determining the mutual inductance between eccentric parallel circles. The method employed in this latter case is based on the fact that as one of the circles moves from its concentric position the value of the mutual inductance varies as a potential function of the position of the centre of the moving circle.[4]

Finally, Nagaoka [5] has made use of Jacobi's q-series to develop from (2) three formulae of wide range and remarkable convergency.

[2] *Electricity and Magnetism*, § 701.
[3] Butterworth, *Phil. Mag.*, 1916, xxxi. 276.
[4] *Ibid.*, 1916, xxxi. 443.
[5] *Tokyo Math. Phys. Soc.*, 1903, vi. 10; 1911, vi. 10.

TABLE I

Mutual Inductance between Coaxial Circles

The table gives the values of log $M/\sqrt{Aa}$ for various values of the variable $\sigma^2=\dfrac{(A-a)^2+b^2}{(A+a)^2+b^2}$, where A, a are the radii of the circles and b their distance apart.

σ^2.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	σ^2.
0·00	Inf.	1·5530	4970	4606	4330	4102	3909	3739	3586	3448	0·00
0·01	1·3320	3201	3091	2986	2888	2794	2705	2620	2539	2460	0·01
0·02	1·2384	2310	2239	2171	2105	2041	1979	1918	1858	1800	0·02
0·03	1·1743	1687	1633	1580	1527	1476	1425	1376	1328	1280	0·03
0·04	1·1233	1187	1142	1097	1053	1010	0967	0925	0883	0842	0·04
0·05	1·0801	0761	0722	0683	0644	0606	0568	0531	0494	0457	0·05
0·06	1·0421	0385	0349	0314	0279	0245	0211	0177	0143	0110	0·06
0·07	1·0077	0044	0012	9980	9948	9916	9884	9853	9822	9791	0·07
0·08	0·9761	9731	9701	9671	9641	9611	9582	9553	9524	9495	0·08
0·09	0·9466	9438	9410	9382	9354	9326	9298	9270	9243	9216	0·09
0·10	0·9189	9162	9136	9109	9082	9056	9030	9004	8978	8952	0·10
0·11	0·8926	8901	8875	8850	8825	8800	8775	8750	8725	8700	0·11
0·12	0·8675	8651	8626	8602	8578	8554	8530	8506	8482	8458	0·12
0·13	0·8435	8411	8388	8364	8341	8318	8294	8271	8248	8226	0·13
0·14	0·8203	8180	8157	8134	8112	8089	8067	8045	8022	8000	0·14
0·15	0·7978	7956	7934	7912	7890	7868	7846	7825	7803	7781	0·15
0·16	0·7760	7738	7717	7696	7674	7653	7632	7611	7590	7568	0·16
0·17	0·7548	7527	7506	7485	7464	7443	7422	7402	7381	7361	0·17
0·18	0·7340	7320	7299	7279	7258	7238	7218	7198	7177	7157	0·18
0·19	0·7137	7117	7097	7077	7057	7037	7017	6997	6978	6958	0·19
0·20	0·6938	6918	6899	6879	6859	6840	6820	6801	6781	6762	0·20
0·21	0·6742	6723	6704	6684	6665	6646	6626	6607	6588	6569	0·21
0·22	0·6550	6531	6512	6492	6473	6454	6435	6416	6398	6379	0·22
0·23	0·6360	6341	6322	6303	6285	6266	6247	6228	6210	6191	0·23
0·24	0·6172	6154	6135	6116	6098	6079	6061	6042	6024	6005	0·24
0·25	0·5987	5968	5949	5931	5912	5894	5876	5857	5839	5821	0·25
0·26	0·5803	5784	5766	5748	5730	5711	5693	5675	5657	5639	0·26
0·27	0·5621	5603	5585	5567	5548	5530	5512	5494	5476	5458	0·27
0·28	0·5440	5422	5404	5386	5368	5350	5332	5315	5297	5279	0·28
0·29	0·5261	5243	5225	5207	5190	5172	5154	5137	5119	5101	0·29
0·30	0·5083	5065	5047	5030	5012	4994	4977	4959	4941	4924	0·30
0·31	0·4907	4889	4871	4854	4836	4819	4801	4784	4766	4749	0·31
0·32	0·4731	4713	4696	4678	4661	4643	4626	4608	4590	4572	0·32
0·33	0·4554	4537	4519	4502	4484	4467	4449	4431	4414	4396	0·33
0·34	0·4378	4361	4343	4325	4308	4290	4272	4255	4237	4219	0·34
0·35	0·4202	4184	4166	4149	4131	4114	4096	4079	4061	4044	0·35
0·36	0·4027	4009	3991	3974	3956	3939	3921	3904	3886	3869	0·36
0·37	0·3851	3834	3816	3799	3781	3764	3746	3729	3711	3694	0·37
0·38	0·3676	3659	3641	3624	3606	3589	3571	3554	3536	3519	0·38
0·39	0·3501	3484	3466	3448	3431	3413	3396	3378	3360	3343	0·39
0·40	0·3325	3307	3290	3272	3254	3237	3219	3203	3184	3166	0·40
0·41	0·3148	3130	3112	3095	3077	3059	3041	3021	3006	2988	0·41
0·42	0·2970	2952	2935	2917	2899	2882	2864	2846	2829	2811	0·42
0·43	0·2793	2775	2757	2740	2722	2704	2686	2669	2651	2633	0·43
0·44	0·2615	2597	2579	2561	2543	2526	2508	2490	2472	2454	0·44

TABLE I—*continued*

σ^2.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	σ^2.
0·45	0·2436	2418	2400	2382	2364	2346	2327	2309	2291	2273	0·45
0·46	0·2255	2237	2219	2200	2182	2164	2145	2127	2109	2090	0·46
0·47	0·2072	2054	2036	2017	1999	1980	1962	1943	1925	1906	0·47
0·48	0·1888	1869	1851	1832	1814	1795	1777	1758	1740	1721	0·48
0·49	0·1703	1684	1666	1647	1629	1610	1591	1573	1554	1536	0·49
0·50	0·1517	1498	1480	1461	1442	1424	1405	1386	1367	1349	0·50
0·51	0·1330	1311	1292	1273	1254	1235	1216	1197	1178	1159	0·51
0·52	0·1140	1121	1102	1083	1064	1044	1025	1006	0987	0968	0·52
0·53	0·0949	0930	0910	0891	0871	0852	0833	0813	0794	0775	0·53
0·54	0·0755	0735	0716	0696	0677	0657	0637	0618	0598	0579	0·54
0·55	0·0559	0539	0519	0500	0480	0460	0440	0420	0401	0381	0·55
0·56	0·0361	0341	0321	0300	0280	0260	0240	0220	0199	0179	0·56
0·57	0·0159	0139	0118	0098	0077	0057	0037	0016	9996	9975	0·57
0·58	$\bar{1}$·9955	9934	9914	9893	9872	9852	9831	9810	9789	9769	0·58
0·59	$\bar{1}$·9748	9727	9706	9685	9664	9643	9622	9601	9580	9559	0·59
0·60	$\bar{1}$·9538	9517	9495	9474	9452	9431	9410	9388	9367	9345	0·60
0·61	$\bar{1}$·9324	9302	9280	9259	9237	9215	9193	9171	9150	9128	0·61
0·62	$\bar{1}$·9106	9084	9062	9040	9018	8996	8974	8952	8929	8907	0·62
0·63	$\bar{1}$·8885	8863	8840	8818	8795	8773	8750	8727	8705	8682	0·63
0·64	$\bar{1}$·8659	8636	8613	8591	8568	8545	8522	8499	8475	8452	0·64
0·65	$\bar{1}$·8429	8406	8382	8359	8336	8312	8289	8265	8241	8218	0·65
0·66	$\bar{1}$·8194	8170	8146	8122	8099	8075	8051	8026	8002	7978	0·66
0·67	$\bar{1}$·7954	7930	7905	7881	7857	7832	7808	7783	7758	7734	0·67
0·68	$\bar{1}$·7709	7684	7659	7634	7609	7584	7559	7534	7509	7483	0·68
0·69	$\bar{1}$·7458	7432	7407	7381	7356	7330	7304	7278	7252	7226	0·69
0·70	$\bar{1}$·7200	7174	7148	7122	7095	7069	7042	7016	6989	6963	0·70
0·71	$\bar{1}$·6936	6909	6882	6856	6829	6802	6774	6747	6720	6692	0·71
0·72	$\bar{1}$·6665	6637	6610	6582	6554	6527	6499	6471	6442	6414	0·72
0·73	$\bar{1}$·6386	6358	6329	6300	6272	6243	6214	6185	6156	6127	0·73
0·74	$\bar{1}$·6099	6070	6041	6011	5982	5952	5923	5893	5863	5833	0·74
0·75	$\bar{1}$·5803	5773	5743	5712	5681	5651	5620	5589	5558	5527	0·75
0·76	$\bar{1}$·5496	5465	5433	5402	5370	5338	5306	5274	5242	5210	0·76
0·77	$\bar{1}$·5178	5146	5113	5081	5048	5015	4982	4949	4916	4882	0·77
0·78	$\bar{1}$·4849	4815	4782	4748	4714	4680	4646	4611	4577	4542	0·78
0·79	$\bar{1}$·4507	4472	4437	4402	4366	4331	4295	4259	4223	4187	0·79
0·80	$\bar{1}$·4150	4113	4076	4040	4002	3965	3928	3890	3852	3815	0·80
0·81	$\bar{1}$·3776	3738	3700	3661	3622	3584	3544	3505	3466	3426	0·81
0·82	$\bar{1}$·3386	3346	3306	3265	3224	3183	3142	3101	3059	3017	0·82
0·83	$\bar{1}$·2975	2933	2890	2848	2805	2762	2718	2675	2631	2586	0·83
0·84	$\bar{1}$·2542	2498	2453	2408	2362	2317	2271	2225	2178	2131	0·84
0·85	$\bar{1}$·2084	2036	1989	1941	1892	1844	1795	1746	1696	1646	0·85
0·86	$\bar{1}$·1596	1546	1496	1445	1394	1342	1291	1238	1186	1132	0·86
0·87	$\bar{1}$·1079	1025	0971	0917	0862	0807	0751	0694	0637	0580	0·87
0·88	$\bar{1}$·0522	0464	0405	0346	0286	0226	0165	0104	0042	9980	0·88
0·89	$\bar{2}$·9917	9855	9792	9728	9663	9598	9532	9465	9398	9330	0·89
0·90	$\bar{2}$·9262	9193	9123	9053	8982	8910	8838	8765	8691	8616	0·90
0·91	$\bar{2}$·8540	8465	8388	8309	8230	8151	8071	7989	7906	7823	0·91
0·92	$\bar{2}$·7738	7653	7567	7479	7391	7301	7210	7118	7024	6930	0·92
0·93	$\bar{2}$·6834	6737	6638	6538	6437	6335	6230	6124	6016	5906	0·93
0·94	$\bar{2}$·5795	5683	5568	5451	5332	5211	5088	4964	4837	4706	0·94
0·95	$\bar{2}$·4574	4438	4300	4160	4017	3871	3720	3567	3410	3250	0·95
0·96	$\bar{2}$·3086	2917	2744	2567	2386	2199	2006	1808	1605	1394	0·96
0·97	$\bar{2}$·1178	0953	0721	0481	0232	$\bar{3}$·9973	9704	9424	9131	8825	0·97
0·98	$\bar{3}$·8503	8166	7811	7435	7036	6613	6160	5674	5149	4580	0·98
0·99	$\bar{3}$·3955	3265	2495	1622	0614	$\bar{4}$·9423	7966	6089	3444	$\bar{5}$·8959	0·99

§ (3) Uniform Cylindrical Helix and Coaxial Circle.—The use of coils approximating to coaxial circles is not advisable in the construction of absolute standards of mutual inductance on account of the difficulty of measuring their dimensions with sufficient precision.

However, by placing a coil of square winding section coaxial with and midway between two cylindrical helices which are connected in series, and arranging the mean diameter of the coil so that the mutual inductance between it and the two helices has a maximum value, an accurate knowledge of the dimensions of the coil is no longer necessary. Moreover, the dimensions of the helices may be determined with high precision. This is the system adopted in the standard of mutual inductance constructed by Campbell, to whom the above principle is due.[1] Use has been made of the same principle in proportioning the coils and discs of the Lorenz apparatus at the National Physical Laboratory.[2]

Accurate and rapid methods for calculating the mutual inductance between a helix and coaxial circle are therefore of the first importance.

An exact (elliptic integral) formula for the case in which the circle is an end plane of the helix was obtained by J. Viriamu Jones [3] by integration from Neumann's formula (1). Greenhill[4] has shown that the same formula may be obtained more simply from Maxwell's formula (2) for the mutual inductance between coaxial circles.

Jones's formula is as follows :

$$M=\Theta(A+a)ck\left\{\frac{K-E}{k^2}+\frac{1-c^2}{c^2}(K-\Pi)\right\}, \quad (4)$$

in which Θ = angular length of helix,

A = radius of helix,

a = radius of circle,

d = axial length of helix,

$c^2=4Aa/(A+a)^2$,

$k^2=4Aa/\{(A+a)^2+d^2\}$.

K, E, Π are complete elliptic integrals of the first, second, and third kinds respectively, K and E to modulus k, and

$$\Pi=\int_0^{\frac{\pi}{2}}\frac{d\psi}{(1-c^2\sin^2\psi)(1-k^2\sin^2\psi)^{\frac{1}{2}}},$$

which may be expressed in terms of tabulated functions.

If the radius of the circle is equal to that of the helix, $c=1$ so that (4) reduces to the simpler form

$$M=2\Theta A(K-E)/k. \quad . \quad . \quad (5)$$

Numerical computation from Jones's formula is naturally very laborious, while if the simplified form (5) is adopted, Campbell's principle can no longer be utilised.

Another formula for the case treated by Jones is a series formula due to Rosa.[5] When the helix is not too short this is simpler for computation than (4), but its slow convergency in the case of short helices involves great arithmetical labour if the utmost accuracy is desired. If the circle is not in the end plane of the helix the case may be treated by the method of differences, but here again when the helix is short precision is difficult to obtain.

In these cases it is perhaps better to adopt some method of approximate integration. It may be shown that so far as its mutual inductance on a coaxial circle is concerned, a helix may be replaced by a current sheet having the same number of turns. The calculation then consists in dividing the cylinder into circular filaments and integrating along the axis. If an accurate table of mutual inductances between circles is available the integration may be carried out numerically, say by the method of equidistant ordinates. The following method is, however, to be preferred, as it requires fewer circles than the equidistant ordinate method for the same degree of accuracy.

Replace the current sheet by two or more equivalent circles (of the same radius as the current sheet) situated at the positions indicated in column 2 of the table. Distribute the total turns (N) among the circles as in column 3, and calculate the mutual inductances of these circles on the given circle. Their sum then gives the approximate mutual inductances of the helix on the circle, the accuracy of course increasing as the number of replacing circles increases.

$2t$ = axial length of Helix.

No. of Replacing Circles.	Displacement of Circles from Mean Plane of Helix.	Turns associated with Replacing Circle.
2	$\pm t/\sqrt{3}$	N/2
3	$\pm\sqrt{3/5}t$	5N/18
	0	8N/18
4	$\pm 0{\cdot}86113632t$	0·17392742N
	$\pm 0{\cdot}33998104t$	0·32607258N
	0	0.284444444N
5	$\pm 0{\cdot}906179846t$	0·118463442N
	$\pm 0{\cdot}538469320t$	0·239314335N

[1] *Proc. Roy. Soc.*, 1907, lxxix. 428.
[2] F. E. Smith, *Roy. Soc. Phil. Trans.*, A., 1914, ccxiv. 27.
[3] *Roy. Soc. Proc.*, 1898, lxiii. 198.
[4] *Roy. Soc. Phil. Trans.*, 1919, ccxx. 35.
[5] *Bureau of Standards Bulletin*, 1907, iii. 209.

That the above method is capable of yielding results of great accuracy is shown by the following examples:

A.	a.	$2t$.	Distance of Circle from Mean Plane of Helix.	N.	M.	Method.
16	10	16	16	160	6900·15 6901·0 6900·3 6900·14_9	Jones 3 Circles 6 Circles (3+3) 10 Circles (5+5)
14·5	10	5	7·5	100	9175·9 9175·9_4	Jones 5 Circles

The dimensions in the first example are approximately those of the Ayrton-Jones current balance, and in the second those of the Campbell Standard of Mutual Inductance.

§ (4) Short Coaxial Cylinders.—The above method of replacement by equivalent circles may also be applied to determine the mutual inductance between two short cylindrical coils. For ordinary purposes only two, or at most three, circles are required. Thus in the case where three circles are used, if the circles replacing the first coil are denoted by A, B, C, and those replacing the second coil by a, b, c, the mutual inductance between the coils is given by

$$\frac{N_1N_2}{384}\Big\{25(M_{Aa}+M_{Cc}+M_{Ac}+M_{Ca}) + 40(M_{Ab}+M_{Ba}+M_{Bc}+M_{Cb})+64M_{Bb}\Big\},$$

N_1, N_2 being the total turns in either coil.

§ (5) Long Cylinder and Internal Concentric Coaxial Short Cylinder.—This case has some importance, as it is the form often employed in the construction of the standard field used in the calibration of ballistic galvanometers.

Let the dimensions of the long cylinder be as follows: Length $= 2x$, radius $=$ A, number of turns $= N_1$. For the short cylinder, length $= 2l$, radius $= a$, number of turns $= N_2$.

Then as a first approximation the mutual inductance is given by multiplying the field at the centre of the long coil by the product of the number of turns into the area of the short coil. This gives

$$M_0 = 2\pi^2 N_1 N_2 a^2/d,$$

in which $d^2 = A^2 + x^2$.

This must be modified by two corrections:

(a) For the radial variation of the field,
(b) For the axial variation of the field.

When the correction (a) is applied we obtain the mutual inductance if the length l is negligible, and correction (b) takes into account the length of the short coils. Correction (a) is obtained by a simple application of Rosa's circle-cylinder formula already referred to. Using three terms of Rosa's series, M_0 must be multiplied by

$$1+\tfrac{3}{8}\frac{A^2a^2}{d^4}+\tfrac{5}{64}\frac{A^4a^4}{d^8}\Big(3-4\frac{x^2}{A^2}\Big).$$

Correction (b) is obtained by replacing the short coil by three equivalent circles. The mutual inductance between the long coil and either of the two outer circles is less than that between the long coil and central circle because of the diminution in the axial field as we proceed from the mid-point of the long coil. The diminution may be calculated with the help of a table of mutual inductance between coaxial circles; for the effect is as if an annulus of width $\sqrt{3/5}\,.\,l$ were transferred from one end of the long coil to the other. The method gives for correction (b)

$$-\tfrac{5}{9}\sqrt{\tfrac{3}{5}}\,.\,\frac{l}{x}N_1N_2\{m(x-\tfrac{1}{2}\sqrt{\tfrac{3}{5}}l)-m(x+\tfrac{1}{2}\sqrt{\tfrac{3}{5}}l)\},$$

in which the notation $m(d)$ indicates the mutual inductance between two circles of radii A and a at distance d.

In illustration of this method let $A=5$, $a=4$, $2x=30$, $2l=5$, $N_1=300$, $N_2=200$.

The first approximation gives

$$M_0 = 1{\cdot}19848 \text{ millihenries.}$$

After applying correction (a)—

$$M_1 = 1{\cdot}20123 \text{ millihenries.}$$

After applying correction (b)—

$$M = 1{\cdot}19990 \text{ millihenries.}$$

Complicated series formulae for this case have been given by Roiti,[1] Searle and Airey,[2] and Russell,[3] and the above example has been treated by Rosa using each of the above series. He finds by Roiti's formula

$$M = 1{\cdot}1998950,$$

by Searle and Airey's formula

$$M = 1{\cdot}1998957,$$

by Russell's formula

$$M = 1{\cdot}19989.$$

Rosa states that the convergence is most satisfactory in the case of Searle and Airey's formula.

It is important to notice the balancing effect of the radial and axial corrections. In fact Gray [4] shows that if the ratio of length to

[1] *Bureau of Standards Bulletin*, iii. 309.
[2] *Electrician*, 1905, lvi. 318.
[3] *Phil. Mag.*, 1907, xiii. 420.
[4] *Absolute Measurements*, ii. Part I. 274.

radius in either coil is $\sqrt{3}$ to 1 and a/A is small, the two corrections cancel each other and the simple form M_0 is sufficient.

The case when the short coil is external to the long coil is obtained by interchanging A and a in the above method.

§ (6) SELF INDUCTANCE OF CIRCULAR COILS.—The self inductance of circular coils of various sections may be obtained by integration, treating the problem as one for the mutual inductance between two coincident coils. The more important results obtained by this method are as follows :

(i.) *Circular Section.*[1]—

$$L=4\pi N^2a\left\{\left(1+\frac{\rho^2}{8a^2}\right)\log_e 8a/\rho+\rho^2/24a^2-1\cdot 75\right\}, \quad (6)$$

in which a is the radius of the coil, ρ that of the section, and N is the total number of turns. When N=1, this gives the self inductance of a ring of circular wire when the current is uniformly distributed over the section. It is thus only suitable for steady or low frequency alternating currents. In the case of very high frequency currents the current distributes itself only on the skin of the conductor, and the appropriate formula in this case [2] is

$$L=4\pi a\left\{\left(1-\frac{\rho^2}{4a^2}\right)\log_e 8a/\rho-2\right\}. \quad (7)$$

(ii.) *Single layer cylindrical coil of N turns.*[3]—

$$L=\tfrac{8}{3}\cdot\frac{\pi N^2a^3}{b^2k^3}\left\{(1-k^2)K-(1-2k^2)E-k^3\right\}, \quad (8)$$

in which a is the coil radius, b the coil length, $k^2=4a^2/(4a^2+b^2)$. K and E are complete elliptic integrals of the first and second kind with k as modulus.

Eight series formulae may be developed from the above exact formula[4] one or the other of which is suitable for any ratio of a to b. For practical purposes it is useful to write

$$L=N^2aQ, \quad (9)$$

and to tabulate Q as a function of a/b. See Table II.

(iii.) *Single-layer flat coil of* n *turns per centimetre inner radius* r, *outer radius R.*—Spielrein[5] has developed two formulae suitable for this case which will cover all possible values of r/R. Computation is facilitated by writing

$$L=n^2R^3Q', \quad (10)$$

in which Q' is a tabulated function of r/R. See Table III.

[1] Rayleigh's *Collected Papers*, ii. 15.
[2] Grover, *Phys. Rev.*, 1901, xxx. 787.
[3] Lorenz, *Wied. Annal.*, 1879, vii. 161.
[4] Bromwich, *Quarterly Journal of Pure and Applied Mathematics*, N., 1913, clxxvi. 363, 381; Butterworth, *Phil. Mag.*, 1916, xxxi. 276.
[5] *Archiv. f. Electrot.*, 1915, iii. 187.

TABLE II

Self Inductance of a Single Layer Solenoid

$L=N^2aQ$

$\frac{a}{b}$.	Q.	$\frac{a}{b}$.	Q.
0·10	3·6324	0·90	19·5794
0·15	5·2337	1·00	20·7463
0·20	6·7102	1·10	21·8205
0·25	8·0747	1·20	22·8150
0·30	9·3389	1·30	23·7401
0·35	10·5135	1·40	24·6048
0·40	11·6079	1·50	25·4161
0·45	12·6306	1·60	26·1801
0·50	13·5889	1·70	26·9018
0·60	15·3380	1·80	27·5855
0·70	16·8984	1·90	28·2349
0·80	18·3035	2·00	28·8534

L = self inductance of solenoid. b = length.
a = radius. N = total turns.

TABLE III

Self Inductance of Flat Coil

$L=n^2R^3Q^1$

$\frac{r}{R}$.	Q^1.	$\frac{r}{R}$.	Q^1.	$\frac{r}{R}$.	Q^1.
0·00	6·970	0·35	5·996	0·70	2·528
0·05	6·964	0·40	5·632	0·75	1·946
0·10	6·930	0·45	5·213	0·80	1·397
0·15	6·845	0·50	4·743	0·85	0·8892
0·20	6·728	0·55	4·231	0·90	0·4574
0·25	6·544	0·60	3·682	0·95	0·1394
0·30	6·300	0·65	3·105	1·00	0·0000

L = self inductance of coil.
n = turns per unit length.
R = outer radius of coil.
r = inner radius of coil.

(iv.) *Rectangular Section.*—This case has been treated by Weinstein,[6] Stefan[7] and Lyle.[8]

Lyle's formula is the most accurate one and is as follows :

$$L=4\pi aN^2\left\{\left(1+m_1\frac{d^2}{a^2}+m_2\cdot\frac{d^4}{a^4}+m_3\frac{d^6}{a^6}\right)\log_e\frac{8a}{d}-l_0+l_1\frac{d^2}{a^2}+l_2\frac{d^4}{a^4}+l_3\frac{d^6}{a^6}\right\}, \quad (11)$$

in which a is the mean radius of the coil,
N the number of turns,
$d^2=b^2+c^2$ where
b = the axial width of the coil,
c = the radial depth of the coil,

and m_1, m_2, m_3, l_0, l_1, l_2, l_3 are functions of b/c, which are given in Tables IV. and V. on following page:

[6] *Wied. Annal.*, 1884, xxi. 329.
[7] *Ibid.*, 1884, xxii. 113.
[8] *Roy. Soc. Phil. Trans.*, 1914, ccxiii. A, 421-435.

TABLE IV

Coefficients m_1, m_2, m_3, l_0, l_1, l_2, l_3 *in Formula* (11)

$b>c$

c/b.	$100\,m_1$.	$10^4\,m_2$.	$10^6\,m_3$.	l_0.	$100\,l_1$.	$10^4\,l_2$.	$10^6\,l_3$.	c/b.
0·00	3·125000	−9·7656	76·29	0·5000000	0·781250	6·5104	−69·30	0·00
·025	3·123699	−9·7463	76·01	·5252663	·783689	6·4896	−68·94	·025
·05	3·119805	−9·6886	75·14	·5489951	·790984	6·4274	−67·88	·05
·10	3·104373	−9·4613	71·79	·5924342	·819830	6·1838	−63·77	·10
·15	3·079157	−9·0942	66·50	·6310248	·866769	5·7944	−57·35	·15
·20	3·044872	−8·6040	59·67	·6652018	·930230	5·2827	−49·20	·20
·25	3·002451	−8·0115	51·78	·6953236	1·008207	4·6774	−39·99	·25
·30	2·952982	−7·3402	43·36	·7217163	1·098406	4·0102	−30·44	·30
·35	2·897643	−6·6144	34·88	·7446891	1·198386	3·3128	−21·17	·35
·40	2·837644	−5·8573	26·77	·7645392	1·305696	2·6145	−12·70	·40
·45	2·774168	−5·0903	19·34	·7815523	1·417987	1·9404	− 5·39	·45
·50	2·708333	−4·3316	12·82	·7960019	1·533097	1·3107	+ ·55	·50
·55	2·641155	−3·5961	7·32	·8081473	1·649113	·7400	+ 5·05	·55
·60	2·573529	−2·8951	+ 2·89	·8182324	1·764399	+ ·2378	8·17	·60
·65	2·506224	−2·2366	− ·51	·8264842	1·877606	− ·1912	10·03	·65
·70	2·439877	−1·6260	− 2·94	·8331124	1·987664	− ·5460	10·82	·70
·75	2·375000	−1·0656	− 4·52	·8383088	2·093763	− ·8287	10·73	·75
·80	2·311992	− ·5563	− 5·37	·8422476	2·195318	−1·0437	9·97	·80
·85	2·251149	− ·0970	− 5·61	·8450864	2·291944	−1·1966	8·73	·85
·90	2·192680	+ ·3141	− 5·37	·8469663	2·383421	−1·2939	7·16	·90
·95	2·136717	+ ·6800	− 4·75	·8480134	2·469663	−1·3425	5·42	·95
1·00	2·083333	+1·0037	− 3·85	·8483397	2·550686	−1·3490	3·62	1·00
1·05	2·032551	+1·2888	− 2·75	·8480444	2·626593	−1·3199	1·83	1·05
1·10	1·984351	+1·5387	− 1·53	·8472152	2·697542	−1·2613	+ ·13	1·10

TABLE V

$c>b$

b/c.	$100\,m_1$.	$10^4\,m_2$.	$10^6\,m_3$.	l_0.	$100\,l_1$.	$10^4\,l_2$.	$10^6\,l_3$.	b/c.
0·00	1·041667	2·3872	14·97	0·5000000	3·732639	4·1667	17·05	0·00
·025	1·042978	2·3913	15·01	·5252663	3·732506	4·1614	17·00	·025
·05	1·046862	2·4035	15·13	·5489951	3·731810	4·1434	16·81	·05
·10	1·062294	2·4508	15·59	·5924342	3·727159	4·0584	15·88	·10
·15	1·087510	2·5237	16·25	·6310248	3·716052	3·8971	14·13	·15
·20	1·121795	2·6140	16·96	·6652018	3·696644	3·6550	11·60	·20
·25	1·164216	2·7115	17·56	·6953236	3·667845	3·3359	8·45	·25
·30	1·213685	2·8057	17·91	·7217163	3·629250	2·9510	+4·98	·30
·35	1·269024	2·8859	17·89	·7446891	3·581036	2·5161	+1·49	·35
·40	1·329023	2·9430	17·42	·7645392	3·523847	2·0499	−1·73	·40
·45	1·392498	2·9694	16·49	·7815523	3·458662	1·5715	−4·42	·45
·50	1·458333	2·9601	15·11	·7960019	3·386676	1·0990	−6·44	·50
·55	1·525512	2·9119	13·36	·8081473	3·309190	+ ·6479	−7·68	·55
·60	1·593137	2·8239	11·33	·8182324	3·227522	+ ·2308	−8·16	·60
·65	1·660442	2·6971	9·11	·8264842	3·142942	− ·1431	−7·93	·65
·70	1·726790	2·5337	6·83	·8331124	3·056619	− ·4677	−7·09	·70
·75	1·791667	2·3372	4·58	·8383088	2·969599	− ·7400	−5·77	·75
·80	1·854675	2·1114	+2·44	·8422476	2·882783	− ·9592	−4·09	·80
·85	1·915518	1·8608	+ ·48	·8450864	2·796929	−1·1265	−2·21	·85
·90	1·973987	1·5898	−1·24	·8469663	2·712655	−1·2448	− ·23	·90
·95	2·029950	1·3028	−2·69	·8480134	2·630449	−1·3175	+1·74	·95
1·00	2·083333	1·0037	−3·85	·8483397	2·550686	−1·3490	+3·62	1·00
1·05	2·134116	·6963	−4·71	·8480444	2·473638	−1·3437	+5·34	1·05
1·10	2·182315	·3839	−5·28	·8472152	2·399492	−1·3062	+6·86	1·10

This formula fails when d/a becomes large. For long coils ($b/a>4$) the following formula [1] will give an accuracy of 1 in 10,000:

$$L=L_0\left(1-\alpha\frac{b}{R}+\beta\frac{b^2}{R^2}-\gamma\frac{b^4}{R^4}\right), \quad . \quad (12)$$

in which

$$L_0=\tfrac{2}{3}\pi^2\frac{R^2}{b}\left(1+2\frac{r}{R}+3\frac{r^2}{R}\right)N^2.$$

N is the total number of turns,
b is the coil length,
r the inner radius,
R the outer radius,

and α, β, γ are functions of r/R given in Table VI.

TABLE VI

Coefficients α, β, γ in Formula (12)

$\frac{r}{R}$.	α.	β.	γ.
0·0	0·73238	0·33333	0·0953
0·2	0·73699	0·33719	0·0973
0·4	0·75574	0·35579	0·1071
0·6	0·78447	0·39042	0·1306
0·8	0·81718	0·43906	0·1701
1·0	0·84883	0·50000	0·2306

In the intermediate region for which neither of the above two formulae apply, reference should be made to the method of Rosa.[2] It should also be noted that the current is assumed to be distributed uniformly over the rectangular section, *i.e.* the coil is supposed wound with square wire of negligible insulation space. A correction is therefore required when using circular wire. The method of obtaining this correction is given by Maxwell,[3] and involves the use of tables (for which see Rosa, *loc. cit.* p. 140).

§ (7) RECTILINEAR CIRCUITS.—The mutual inductance between rectilinear circuits of thin wire may be calculated from Neumann's formula (1), and usually only elementary integrations are involved. The simplest and most important case is that of two parallel filaments of length l and separation d. The mutual inductance for this case is

$$M=2\left(l\ \log_e\frac{l+\sqrt{l^2+d^2}}{d}-\sqrt{l^2+d^2}+d\right). \ (13)$$

In many cases d/l is small, and then

$$M=2l\left(\log_e\frac{l}{d}-1\right) \text{ approximately.} \quad (14)$$

Formula (14) may be applied to determine the self inductance of long straight conductors of various sections by integration over the section of the conductors. The result is the same as if the conductor were replaced by two equivalent filaments at distance apart D such that

$$\log_e D=\lim_{n\to\infty}\frac{1}{n}\Sigma\log_e d,$$

in which the summation is between every possible pair of elementary filaments in the section.

D is called the "geometric mean distance" (G.M.D.) of the section.

When D is known the self inductance of the conductor in question is given by

$$L=2l\left(\log_e\frac{l}{D}-1\right). \quad . \quad . \quad (15)$$

The same method will apply to the mutual inductance between two parallel conducting bars provided the current distribution is uniform. For the details of calculation of geometric mean distances reference should be made to Maxwell's *Electricity and Magnetism* (vol. ii. §§ 691-693), and to a paper by Rosa (*Bureau of Standards Bulletin,* 1907, iv. 325).[4]

The more important geometrical mean distances are given below.

(A) For self inductances.

(1) Rectangular area ($a\times b$) from itself:

$$\log_e D=\log_e\sqrt{a^2+b^2}-\tfrac{1}{6}\frac{a^2}{b^2}\log_e\sqrt{1+\frac{b^2}{a^2}}-\tfrac{1}{6}\frac{b^2}{a^2}$$
$$\log_e\sqrt{1+\frac{a^2}{b^2}}+\tfrac{2}{3}\frac{a}{b}\tan^{-1}\frac{b}{a}+\tfrac{2}{3}\frac{b}{a}\tan^{-1}\frac{a}{b}-\tfrac{25}{12}.$$

If $b=0$, the above reduces to

$$D=0{\cdot}22313a.$$

If $b=a$,

$$D=0{\cdot}22352(2a),$$

while for any values of a and b, if we write

$$D=0{\cdot}2235(a+b),$$

the error is always less than 2 in 1000.

(2) Circular area of radius a:

$$D=ae^{-\frac{1}{4}}=0{\cdot}7788a.$$

(3) Annular area, external radius a_1, internal radius a_2.

The exact formula is

$$\log_e D=\log a_1-\log x/(x^2-1)^2+(3-x^2)/4(x^2-1),$$

in which $x=a_1/a_2$.

Computation is, however, simplified by expanding in powers of $(x-1)/x$ ($=t$ say).

Then

$$\log_e D=\log_e a_1-t/3+t^3/30+t^4/40+\ldots$$

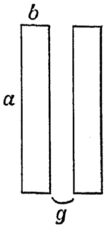
(B) For mutual inductances between parallel bars.

[1] Butterworth, *Phil. Mag.*, 1915, xxix. 591.
[2] *Bureau of Standards Bulletin*, 1912, viii. 138.
[3] *Elect. and Mag.* ii. § 693.

[4] See also a paper by Silsbee (*Bureau of Standards Scientific Paper*, No. 281, 1916, p. 375).

(1) Two circles whose centres are at a distance d apart:

$$D=d.$$



(2) Two rectangular bars (each $a \times b$) separated by insulation of width g perpendicular to the side a. The general formula is very complicated, but in the important case where b/a and g/a are small and the bars form a go and return circuit of length l, the total self inductance is given by

$$L=4l\left\{\frac{\pi}{3}(3\beta-\delta)-\tfrac{25}{12}\beta^2-\tfrac{1}{12}\beta^4-\tfrac{1}{12}\beta^2\delta^2\right.$$
$$\left.+\frac{1}{12\delta^2}(\alpha^4\log\alpha-2\beta^4\log\beta+\gamma^4\log\gamma-2\delta^4\log\delta)\right\},$$

in which

$$\alpha=(2b+g)/a,\ \beta=(b+g)/a,\ \gamma=g/a,\ \delta=b/a.$$

(3) Circular annular area and internal area of *any* shape.

If a_1, a_2 are the external and internal radii of the enveloping annulus,

$$\log D=\frac{a_1{}^2\log_e a_1-a_2{}^2\log_e a_2}{a_1{}^2-a_2{}^2}-\tfrac{1}{2}$$
$$=\log_e a_1-t/2-t^2/12+t^4/60\ \ldots$$

in which $t=(a_1-a_2)/a_1$.

S. B.

INDUCTANCE, THE MEASUREMENT OF

I. Introductory

§ (1) Self and Mutual Inductance.—The effects of inductance (whether mutual or self) only occur when the current is varied, this being expressed mathematically in the equations

$$e_1=L\frac{di_1}{dt}\text{ and }e_2=M\frac{di_1}{dt},$$

where e_1 and e_2 are the voltages induced (at any moment) in a primary and secondary circuit respectively by variation of i_1, the current in the primary, L being the self inductance of the primary, and M the mutual inductance between primary and secondary. In most practical work the unit in which L and M are expressed is the *henry*. With an inductance of 1 henry the induced voltage is 1 volt when the current is changing at the rate of 1 ampere per second.

With an ironless circuit possessing self inductance, the current produces a magnetic field whose lines interlink the circuit, 1 henry giving 10^8 line-turns per ampere of current, and similarly for the flux-turns in a secondary circuit when mutual inductance is present. Thus if a current i produces a flux Φ in a circuit of N turns, we may usually write

$$L=10^{-8}N\Phi\text{ henries},$$

remembering, however, that if ferromagnetic material is present the conditions are more complicated. In what follows we shall in general express all inductances in henries and the convenient submultiples, millihenries (mH or mhen) and microhenries (μH or μhen). To convert to absolute units the relation is

$$1\text{ henry}=10^9\text{ abhenries},$$

an abhenry being the absolute electromagnetic C.G.S. unit of inductance. As the dimensions of inductance (when expressed in the electromagnetic system) are merely length, 1 abhenry is sometimes written as 1 cm.; in this notation

$$1\text{ microhenry}=1000\text{ cm.}$$

§ (2) Effective Resistance and Inductance.—If a direct current I passing through a circuit dissipates power of the amount RI^2, then R is called the *resistance* of the circuit. If, however, the current is an alternating one of effective value I, and if the circuit contains inductive coils with iron cores, condensers with absorptive dielectrics, vibrators, or other apparatus which consume alternating power, then in general more power will be dissipated by the alternating current than by the direct. If the alternating current I spends $R'I^2$ watts, R' is called the *effective resistance* of the circuit. Usually it varies with the frequency of the alternating current, sometimes also with the value of the current (*e.g.* when magnetic hysteresis is present). Sometimes the direct current resistance is called the ohmic resistance, but a more suitable name is much to be desired.

The apparent self inductance of a coil as measured at its terminals is altered if iron or closed secondary circuits are brought near it, and the term *effective self inductance* is convenient to describe the modified value.

§ (3) Impedance and Reactance.—In a circuit of resistance R and self inductance L, let

v be the instantaneous terminal voltage,
i the instantaneous current,
V and I being the corresponding effective values.

Also let the *pulsatance* $\omega=2\pi n$, where n is the frequency, and let $j=\sqrt{-1}$.

Then in symbolic notation

$$z=v/i=R+Lj\omega,\quad .\quad .\quad .\quad (1)$$

where z may be called the symbolic or vectorial *impedance*. Taking effective (or square root of mean square) values, we have

$$Z^2=V^2/I^2=R^2+L^2\omega^2.\quad .\quad .\quad .\quad (2)$$

Z is called the *impedance*, and $L\omega$ the *reactance* of the circuit; the reciprocal of the reactance is called the *admittance*. If there

is series *capacitance* K also in the circuit, instead of equation (1) we have

$$z = R + \left(Lj\omega - \frac{1}{Kj\omega}\right) . \quad . \quad . \quad (3)$$

and $$Z = R^2 + \left(L - \frac{1}{K\omega^2}\right)^2 \omega^2. \quad . \quad . \quad (4)$$

The total reactance is now $\left(L\omega - \frac{1}{K\omega}\right)$. It will be noticed that a series condenser of capacitance K is equivalent to a negative self inductance of value $1/K\omega^2$. This equivalence furnishes us with a very useful practical device, by which any method of measurement involving self inductances may give a corresponding method involving one or more condensers instead. It is only necessary to write the equivalent $1/K\omega^2$ for each L to be changed in the original equations and then to examine the new equations to see that they are not practically impossible. For example, they must not require negative values of resistance—a condition which at once makes a method impossible.

When $LK\omega^2 = 1$, the reactance is zero, and the circuit behaves as if it were non-inductive, the impedance reaching its minimum value R. The circuit is then said to be electrically tuned for pulsatance ω (or frequency n).

II. Sources of Current

§ (4) Introductory.—For all measurements of inductance a source of current that can be varied in some systematic manner is required. In the older days this variation usually consisted of a single make and break or reversal of a steady direct current, but in modern times the use of periodically interrupted or alternating currents has become general; as the frequency of such current is increased the inductive effects become more and more prominent, and following on the introduction of suitable detecting and measuring instruments for alternating currents the modern methods give very much greater sensitivity and accuracy of observation than the earlier ones. When direct current is used all that is necessary as a source is an ordinary battery, preferably of steady voltage; for alternating or interrupted current a very wide range of frequency has to be dealt with, and a great variety of arrangements have been used, some of them extremely simple and cheap, and others much more elaborate and often requiring considerable power to drive them. We proceed to describe a number of different types, giving their capabilities and ranges of frequency, so that the experimenter may be able to select a type meeting his requirements and adapted to the power available.

§ (5) Use of Transformer.—First it should be remarked that in nearly all cases it is best to connect working apparatus to the source through a small transformer having a number of separate windings, so as to afford a wide choice of working voltage. This transformer must have a well-closed magnetic circuit, for any magnetic leakage in the vicinity of the testing apparatus is almost certain to give trouble and cause errors that may not easily be detected. (For the same reason it may often be necessary to arrange the source at a long distance from the test table.) It is of great advantage to have a conducting screen[1] interposed between the primary and secondary windings of the transformer and connected to earth. It must of course be of such a form as not to allow any eddy-current effect. The function of such a screen is to minimise the effects of leakage and more particularly of capacity to earth in parts of the testing circuit.

It is always desirable to repeat readings with the connections to the secondary of the transformer reversed. If any difference is observed, it nearly always indicates prejudicial effects of capacity to earth (or magnetic leakage).

The following data of a type of small transformer much used in the National Physical Laboratory will indicate a construction found by long experience to work satisfactorily. The core consists of an anchor ring of inner diameter 12 cm., outer 20 cm., and about 16 sq. cm. cross-section, formed of silicon-iron (Stalloy) stampings insulated from one another. The windings are insulated from one another by silk ribbon, each set of turns (except the 1s) being evenly distributed round the ring to avoid magnetic leakage. After winding the whole is immersed in hot melted paraffin wax. The numbers of turns in the sections are as follows: 100, 100, 100 for primary; 100 for earth screen; 100, 100, 100, 100, five of 10, and five of 1 as secondaries. The earth screen here consists of a set of turns completely covering the ring, one end being earthed and the other left free. Sometimes it is advantageous to have also an outer winding as part of the earth screen.

§ (6) Alternator.—An alternator forms a convenient source, especially if it gives approximately sine wave voltage and can be run at constant speed. Current from an alternating lighting supply is sometimes quite suitable. When the alternator is driven by a direct current motor run from an unsteady source the speed can be kept wonderfully steady by means of a Helmholtz automatic regulator mounted on the shaft of the alternator. The principle on which it acts is simple. When the speed rises slightly above the normal a weight controlled by a spring is slightly displaced by the increased centrifugal action, and closes an electric contact which short-circuits a portion of the regulating resistance in the shunt circuit of the motor, and thus lowers the speed. With Giebe's

[1] G. A. Campbell, *Electrical World and Engineer*, 1904, xliii. 647.

modification[1] constancy of speed to 1 part in 1000 is in general easily obtained.

When extreme constancy of speed is required the best plan is to mount a suitable commutator on the shaft of the alternator and connect it up with a good mica condenser as if measuring the capacity by Maxwell's method. (See article on "Capacity.") The movement of the galvanometer light-spot to one side or the other of the scale gives an extremely sensitive indication of whether the speed is above or below the normal desired value, and thus enables an auxiliary observer to control the speed with the greatest nicety by the application of some form of variable brake (*e.g.* by simple hand pressure on the rim of a fly-wheel).

For low frequencies (10 up to 200 ~ per sec.) a quite ordinary alternator is sufficient. With motor driving the very slow speeds can be best obtained by separately exciting the motor field with normal voltage and running the armature from a few storage cells. For audio frequencies (200 up to 10,000 ~ per sec.), which include the range used in telephonic work, special alternators have been designed. The simplest of these machines are of the inductor type, in which a toothed iron disc is rotated close to the poles of an electromagnet magnetised by a winding carrying direct current. Secondary coils are wound on the magnet near the poles, and as the teeth pass the poles the resulting variation of magnetic flux induces an alternating voltage in these polar coils, from which alternating current can be obtained.

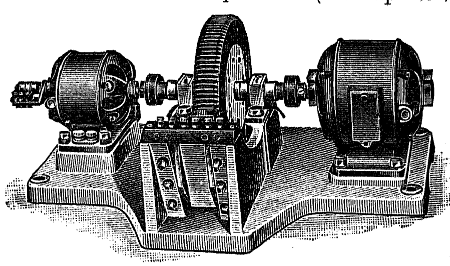


FIG. 1.—Dolezalek High-frequency Alternator.

A machine of this type designed by Dolezalek,[2] and supplied by Messrs. Siemens Bros. & Co., is shown in *Fig.* 1. The toothed disc has 100 teeth and is built up of several hundred thin iron sheets. It can be run at a speed of 3000 revs. per min., which gives a frequency of 5000 ~ per sec. The output at 3600 ~ per sec. is about 7 watts with 200 ohms in the external circuit and resonance. The machine is run by a direct-coupled motor, and in order to keep the speed more steady a small dynamo is coupled to the other end of the shaft. By loading this dynamo much greater steadiness of speed is obtained when the bearing friction is made only a fraction of the total power used.

A smaller type, shown in *Fig.* 2, goes up to 3600 ~ per sec. with a disc of 60 teeth, and has an output of 1·7 watts at 3000 ~ per sec. (for 200 ohms and resonance). The motor has two slip rings connected with the armature winding for connecting up a brake resistance applying a variable load for controlling the speed. The wave form is not free from harmonics, but these may be largely suppressed by inserting a variable condenser in the working circuit and adjusting it to give resonance for the fundamental frequency. The tuning is best done by having a thermal or other suitable ammeter in the working circuit and altering the capacity until the ammeter shows maximum current. This electrical tuning is of general application to almost any source of alternating current. It not only tends to suppress harmonics, but also, by reducing the total impedance of the circuit, often allows a much greater output to be obtained from the source.

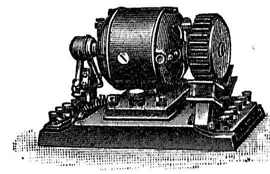


FIG. 2.—Small Dolezalek High-frequency Alternator.

In G. A. Campbell's Wave Filter,[3] by means of a multiple system of condensers and inductances, the harmonics are very effectively suppressed.

§ (7).—The Duddell alternator[4] shown in *Fig.* 3 is a much more elaborate and powerful machine and is designed to give very nearly a pure sine wave form. For this reason the stator consists of a well-laminated smooth

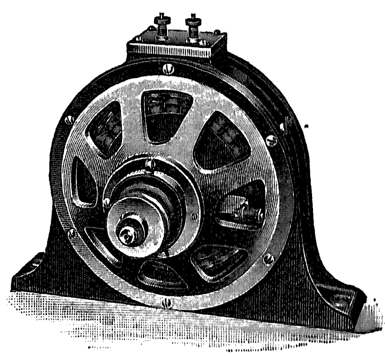


FIG. 3.—Duddell Alternator for Audio Frequencies.

[1] E. Giebe, *Zeits. Instrumentenk.*, 1909, xxix. 205; *Electrician*, 1909–10, lxiv. 509.

[2] F. Dolezalek, *Zeits. Instrumentenk.*, 1903, xxiii. 240.

[3] G. A. Campbell, *Phil. Mag.*, 1903, p. 313 (5).

[4] W. Duddell, *Phys. Soc. Proc.*, 1912, xxiv. 172.

ring wound like a gramme ring, the comparatively long air-gap between it and the rotor ensuring that the armature reaction is kept low, and hence the wave form only slightly distorted when the machine is loaded. The rotor, of 20 cm. diameter and with 30 salient poles, carries the field winding and is run up to a maximum speed of 8000 revs. per min. The working range of frequency is from 100 up to 2000 ~ per sec. As an output of 10 watts can be obtained at any frequency of this large range, the output at the highest speed is considerable, being about 500 watts (5 amperes at 100 volts). The field excitation is from a battery at 35 volts. The wave form is not absolutely pure, but contains a slight third harmonic.

§ (8).—Distinct in type from either of the above machines is the high-frequency alternator described by Hartmann-Kämpf.[1] It requires no exciting current, as permanent magnets are used. These are of horseshoe form, 24 of them being mounted in ring form with their poles facing inwards and surrounding a slot-wound armature. The ring of magnets and the armature are rotated in opposite directions, each with a maximum speed of 4000 revs. per sec. The minimum output is 500 watts. By passing the current through an iron-cored choking coil a fair approximation to sine wave form is obtained.

§ (9).—When only a very small amount of power is required, a minute inductor alternator can be easily constructed by taking the magnet (with its polar windings) from a two-pole telephone receiver and fixing it with its poles very close to the rim of a finely toothed disc of well-laminated iron. The disc can be mounted directly on the spindle of a small motor or else on a separate spindle with belt drive. The current is taken from the polar windings.

§ (10) Interrupters.—Very often a quite satisfactory source consists of an interrupter of some kind by which the current from a battery (or other direct current source) is made and broken periodically. Sometimes this intermittent current is applied directly to the measuring apparatus, but more often it is turned into alternating current by interposing a transformer. There are many types of such interrupters for various purposes other than that of measurement. For example, under the name of "buzzers" they are very largely used in signalling of all kinds, and many of the types evolved for this purpose are quite useful for inductance testing. It will suffice, however, to describe one or two types, but before proceeding to details we shall discuss a trouble which is common to nearly all such apparatus, namely, the sparking at the contacts.

Suppression of Contact Sparking.—When a circuit is broken by separating two metallic conductors the resulting spark is often troublesome, especially when the circuit has considerable inductance (or, to be more precise, a high time constant), which is a quite usual case in inductance testing. In the majority of interrupters the contact surfaces or points are made of platinum because of its high melting-point and resistance to oxidation; in some instances it has been found possible to replace it by solid tungsten.[2]

But even with the best material for the contact pieces there is usually trouble of sticking, uncertainty of contact, etc., unless means are used to lessen or suppress the spark. Either of the two following methods will in general reduce and often practically suppress the objectionable sparking. When, however, the interrupter has to deal with considerable amounts of power, the problem of spark suppression does not admit of such easy solution.

Method A.—Across the contact break are connected (as shown in *Fig.* 4) a condenser and a non-inductive resistance in parallel, the resistance being adjusted to give minimum sparking. The disadvantage of this method is that a certain amount of current always remains flowing even when the contact is left broken. This does not occur in the following method.

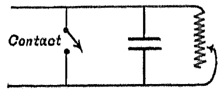


Fig. 4.

Method B.—As shown in *Fig.* 5, a condenser and a resistance in seriesare connected across the break. When the interrupter is working on any actual load the resistance can usually be adjusted so as to make the sparking negligible. (This system is also of great value for relays and other instruments with delicate contacts.)

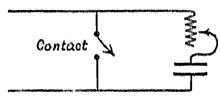


Fig. 5.

For either method an ordinary paraffin-paper condenser of 1 or 2 microfarads capacity is generally sufficient. In Method A the resistance may have to be high (thousands of ohms), while in Method B it may range from 1 ohm up to several hundred ohms. In certain cases a condenser alone may be formed best, and sometimes (as used by Helmholtz, 1872) a resistance without any condenser will suffice.

§ (11) Wire Interrupter.—For frequencies from 30 or 40 up to 300 ~ per sec. the wire interrupter forms a steady but easily adjustable source. It was used by Gray in 1875 and by Niemöller[3] in 1879, but more fully discussed by M. Wien later,[4] and a very elaborate and accurate type has been described by Hartmann-Kämpf.[5]

In most of the types a steel wire (under

[1] R. Hartmann-Kämpf, *Phys. Zeits.*, 1909, x. 1018.

[2] W. D. Coolidge, *Am.I.E.E. Proc.*, 1010, xxix. 953.

[3] F. Niemöller, *Wied. Ann.*, 1879, vi. 302.

[4] M. Wien, *Wied. Ann.*, 1891, xlii. 593.

[5] R. Hartmann-Kämpf, *Elektro-akustische Untersuchungen*, 1903.

tension), maintained in steady vibration by an electric circuit, interrupts periodically another circuit from the same or another battery. The following description refers to the form used at the National Physical Laboratory. The general arrangement is shown in *Fig.* 6, and the connections of the circuits in *Fig.* 7. On a long iron bed AB are mounted the two lathe-heads C and D, having screws of coarse and fine pitch respectively, and a wire GH of piano steel is stretched between them, the two screws giving coarse

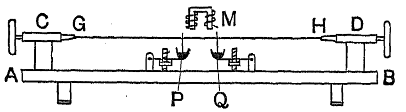


FIG. 6.

and fine adjustment of the tension. Two short pieces of nickel wire are soldered to the wire GH near its middle point and project downwards into mercury cups (P and Q), each consisting of a porcelain crucible mounted on a pivoted support which can be raised or lowered by a nut on a vertical screw in order to keep the mercury surface at just the right level. An electromagnet M is supported with its poles directly over the middle of the wire, and its height is also adjustable.

The driving circuit (*Fig.* 7) is from G through contact P, magnet M, and resistance r, the contact being shunted merely by a resistance S. These resistances r and S are

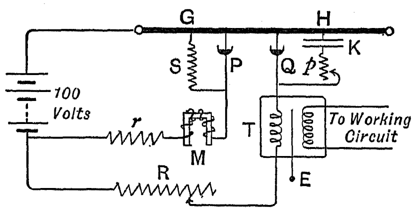


FIG. 7.

ordinary 200-volt glow-lamps. If the height of P is properly adjusted and the wire is set into vibration (by slightly twanging it) this electrical circuit will maintain it in steady vibration at its natural frequency, which can be set to a desired value by altering the tension and tuning by the method of beats against a tuning-fork or other standard of pitch.

The second circuit is from G through contact Q, primary winding of transformer T, and resistance R, the working circuit being taken from the secondary of the transformer.

To prevent the oxidation of the mercury by the spark the mercury surface is covered with a layer of paraffin oil 5 or 6 mm. deep. This gradually forms a sludge with the mercury. To keep the contacts in good order the sludge should be skimmed off from time to time and a little fresh paraffin added. The paraffin oil is preferable to alcohol, which is commonly used, as the oxidation of the alcohol forms aldehyde which makes the air of the room both unpleasant and unhealthy.

For the lower frequencies it is sometimes convenient to clamp a small weight to the middle of the wire. For the higher frequencies two sliding bridges can be placed under the wire to reduce the working length.

§ (12).—The wire interrupter of Arons [1] works on a different principle. The wire is of non-magnetic material and, near its middle point, passes through the transverse field of a powerful electromagnet. In series with the wire is a mercury contact which is opened and closed by a stirrup carried by the wire. The exciting current of the magnet is kept constant (from a separate source) and the current through the wire only is interrupted.

The great advantage of the wire interrupter is that it gives a very steady (and easily adjustable) frequency. The current in the working circuit, however, is by no means of sine wave form, unless the harmonics are suppressed by electrical tuning.

A wire interrupter may be set up at no great distance from the testing table, but care must be taken to avoid disturbances from the alternating magnetic field of the electromagnet (of the first type). The field at a distance can be considerably reduced by specially designing the magnet. To detect the presence of errors due to such stray fields, the magnet circuit should be reversed and the measurement repeated, a change in the result usually indicating disturbance of this kind. Similarly with nearly all the other sources here described, the observer must always be on his guard against stray alternating magnetic fields.

§ (13) TUNING-FORK INTERRUPTER.—A tuning-fork may be maintained in vibration by the same method as the wire, the contact breaker being a platinum or nickel wire dipping into mercury. This system works quite well with a large fork for frequencies up to 200 or perhaps 250 ~ per sec. For higher frequencies the contact breaker often consists of a slightly springy platinum wire or strip [2] working against a flat surface of platinum mounted on the end of a screw by which the contact can be adjusted. As in the case of the wire interrupter a second contact can be arranged so as to keep the driving circuit independent of the working load. A tuning-fork thus maintained gives a tolerably steady frequency. Variations of frequency of several parts in 1000 may, however, sometimes occur, due to alteration of voltage of the driving battery or change of adjustment of the contact break. The working current is not of sine wave form. If only a quite small

[1] L. Arons, *Wied. Ann.*, 1898, lxvi. 1177, and E. Orlich, *Elekt. Zeits.*, 1903, xxiv. 502.

[2] A. Guillet (*Lum. élect.*, October 16, 1909, p. 81) uses a contact against a stretched wire.

amount of power is required in the working circuit, a much nearer approach to sine wave form may be obtained by A. Campbell's method,[1] in which a fork, driven by the ordinary contact method, carries on one of its limbs a flat coil, which is thus vibrated in the non-uniform field of a powerful magnet and generates alternating voltage for the working circuit. This system suits forks of 5 to 30 ~ per sec. having large amplitudes.

§ (14) ELECTRIC TRUMPET OR BUZZER.—An ordinary signalling electric trumpet or buzzer often forms a very convenient source of current. The General Electric Company's type is illustrated in *Fig.* 8, the three standard sizes having frequencies lying between 300 and 1000 ~ per sec. The working parts are shown in diagram in *Fig.* 9.

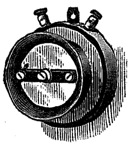


FIG. 8.—Trumpet Buzzer (General Electric Co.).

A platinum point carried by a setting screw A just touches a small platinum plate fixed to the centre of the telephone diaphragm D, behind which there is a bipolar electromagnet M with solid iron cores. The buzzer may be used as supplied by merely putting a transformer into its circuit along with a 6-volt battery, at the same time checking the spark with a condenser and resistance (K and *r*). But in general a more efficient way is to use the system suggested by Mr. D. W. Dye as shown in *Fig.* 9. The electromagnet is re-

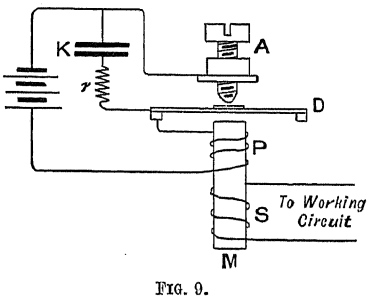


FIG. 9.

wound (with silk-covered wire) with two windings P and S, and from the latter the working circuit is taken. Between P and S is an earth-screen winding. The secondary winding S can be designed so that its resistance suits the apparatus used in the working circuit.

A buzzer of this kind with a battery of accumulators of 6 or 8 volts gives sufficient power for most testing purposes. The frequency of the current is not constant, but alters with change of adjustment of the contact, variation of battery voltage, and working load ; it must be determined independently at the time of test, if a knowledge of its exact value is required. It remains, however, sufficiently constant for most purposes for hours at a time ; indeed the arrangement has often been run continuously for several days, and when in good working order can be switched on and off as readily as an ordinary lighting supply. The wave form obtained is not sinoidal, but, if desired, tuning can be applied by a series condenser in the working circuit. The buzzer (with 6 volts) unfortunately gives out a very loud note; it is necessary, therefore, to enclose it in at least two boxes well padded with wadding and, if possible, place it in a distant room, bringing the switch leads and the working circuit leads only to the testing table. Distance from the testing apparatus has also the advantage of getting rid of trouble due to stray magnetic fields.

§ (15) MICROPHONE HUMMER.—When constancy of frequency and a tolerably pure wave form are required, a microphone hummer is sometimes a convenient source. In the interrupters, since the circuit is completely broken

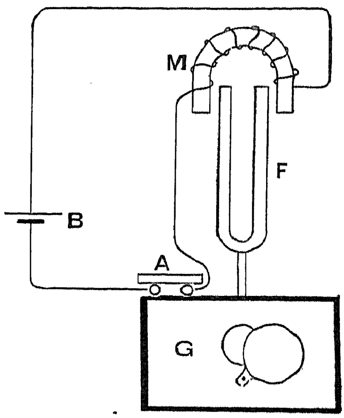


FIG. 10.

in each oscillation, the wave form of the current is naturally far from pure. In a microphone, on the other hand, we have an apparatus which when vibrated causes periodic variations in the current without any complete break of contact ; these variations have usually a wave form not far from sinoidal. In 1890 Appleyard[2] showed how a tuning-fork could be electrically maintained by the help of a microphone. His apparatus, shown in diagram in *Fig.* 10, was very simple, consisting of a tuning-fork F mounted on a resonator box G, on the top of which rested a microphone A (three carbon rods). A battery B of one cell was connected through the microphone to an electromagnet M whose two poles were close to the ends of the prongs of the fork. If the fork is set vibrating, the shaking of the microphone causes ripples in the battery current, which, acting through the magnet

[1] A. Campbell, *Phys. Soc. Proc.*, 1919, xxxi. 85.

[2] R. Appleyard, *El. Review*, 1890, xxvi. 57 and 656.

M, keep the fork in vibration. Since that time various experimenters, using this principle, have developed a number of different types of microphone hummer for use as a source of small alternating current. Taylor [1] used the system with (*a*) a tuning-fork and (*b*) a telephone diaphragm as vibrator. The electrical connections, which are much the same for both types, are shown in *Fig.* 11. A battery B is connected through a microphone capsule A carried by a telephone diaphragm D to the primary of a transformer T. The secondary of the transformer is connected in series with adjustable self inductance L and capacity K to a polar winding Q on the permanent magnet M, one or both of the poles of M being close to the diaphragm. A portable form of this type designed by Dolezalek [2] and supplied by Messrs. Siemens Bros., giving a frequency about 550 ~ per sec., is shown in *Fig.* 12.

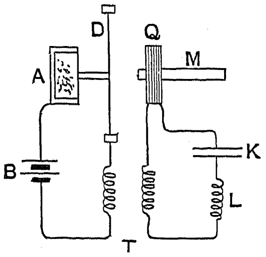


FIG. 11.

§ (16).—A tuning-fork hummer, however, gives a far more constant frequency than the type with a diaphragm. Forks of frequencies from 250 up to 500 ~ per sec. can be run without difficulty, using an ordinary capsule

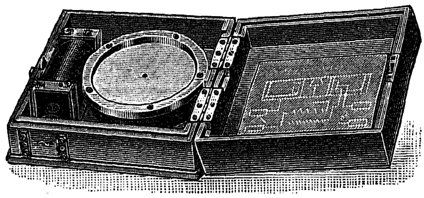


FIG. 12.—Dolezalek's Diaphragm Microphone Hummer.

microphone such as is used for the transmission of speech, but special precautions [3] must be taken for those of lower frequency (50 to 100 ~ per sec.). A capsule microphone (*e.g.* the "solid-back" type, which has a mica diaphragm) has a natural frequency of its own, usually of the order of 1000 ~ per sec. For the lower frequencies it should be loaded with a considerable added mass (of the order of 100 to 200 gm.) in order to bring the natural frequency nearer that which is to be generated. Also the circuit containing the secondary of the transformer should be tuned so as to have the effective electrical frequency near the frequency of the fork. For the lower frequencies condensers of the order of 20 to 30 microfarads are sometimes required. It should also be remembered that ordinary granule microphones usually work best when the plane of the diaphragm is not vertical but tilted at a certain angle (sometimes about 40°) to the vertical. For this reason it is best to mount the microphone capsule in such a way as to allow the angle of tilt to be varied in order to obtain by trial the best position.

§ (17).—The higher frequencies (1000 up to 4000 or 5000 ~ per sec.) are best obtained by using a straight steel bar as vibrator, as in Campbell's microphone hummer [4] (*Fig.* 13). In this a mild-steel bar of 25 mm.

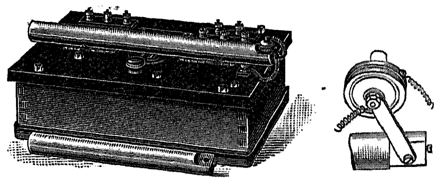


FIG. 13.—Campbell Microphone Hummer.

diameter is supported by pieces of cork or knife-edges at two nodal points. For such a bar the length l in cm. to give any desired frequency n can be found from the formula $l \doteqdot 1075/\sqrt{n}$. The magnet poles are very close to the bar, the distance being adjustable by a screw. Either a permanent magnet or an electromagnet may be used. In the latter case the magnet should be polarised by a direct current in a separate winding. This is most simply achieved by the system shown in *Fig.* 13A, in which the current through the

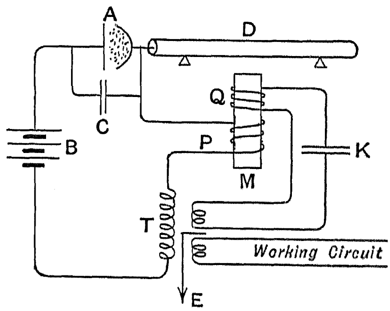


FIG. 13A.

microphone keeps the magnet polarised. This composite system also works well with tuning-forks. Care must be taken to connect the ends of the polar winding in the right direction

[1] J. E. Taylor, *J. Inst. El. Eng.*, 1901, xxxi. 396.
[2] F. Dolezalek, *Zeits. Instrumentenk.*, 1903, xxiii. 240.
[3] A. Campbell, *Phys. Soc. Proc.*, 1919, xxxi. 84.
[4] A. Campbell, *Roy. Soc. Proc.* A, 1906, lxxviii. 209.

to the transformer terminals. The working current is taken from a third winding on the transformer, and this method applies to other types as well. The advantages of a microphone hummer of this type are the great constancy of frequency (to 1 or 2 parts in 1000) and good wave form, but in many cases these are outweighed by the fact that the output, especially above 1500 ~ per sec., is very small and sometimes not sufficient to give high accuracy.

§ (18) Humming Telephone.—In 1890 Hibbard[1] discovered that if a telephone and microphone are connected through a transformer as for speech (*Fig.* 14), and the telephone is held against or near to the diaphragm of the microphone, a musical note is given out and continues as long as the instruments remain in that position. The pitch of the note is usually rather unstable, altering if the relative positions of A and B are changed. Many years later Professor Larsen,[2] of Copenhagen, found that the pitch depended on the dimensions of the air spaces in the telephone and microphone and between them, and starting from this discovery he constructed a simple alternating current generator giving steady frequency, which can also be regulated between wide limits. *Fig.* 15 shows the general

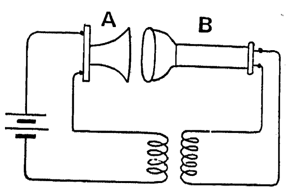


Fig. 14.

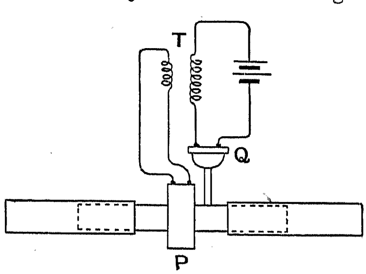


Fig. 15.

arrangement of the apparatus. A watch-shaped telephone P has two equal open brass tubes fixed to it, one in front of and one behind the diaphragm. Over each of them slides another tube, by which the length of the vibrating column of air facing the diaphragm can be adjusted at will. The microphone Q is connected by a narrow tube with the air in one of the fixed tubes at a point near the telephone. By adjustment of the two sliding tubes the frequency can be altered continuously from 600 up to 1100 ~ per sec., for this range the tube lengths being changed from 8 cm. to 21 cm. After switching on the battery the frequency sometimes gradually changes by perhaps 1 per cent, but after that remains constant to about 2 parts in 1000 for hours. The generator only requires two dry cells as battery power. The working circuit is taken from the terminals of the secondary coil of the transformer.

§ (19) Musical Arc.—Duddell's[3] musical arc forms an alternating current generator giving a very large range of frequency and a considerable amount of power. Professor C. L. Fortescue finds that, instead of an ordinary carbon arc, a "Pointolite" lamp used as a musical arc works very well as a small-power source of alternating current. In the Pointolite lamp of the Edison-Swan Company a small electric arc is formed between electrodes consisting of a tungsten wire and small sphere enclosed in a gas-filled glass bulb. The arc is started by a special arrangement, but after starting, the regulation is quite automatic and

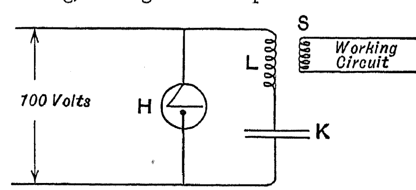


Fig. 16.

the lamp can be run without much attention from any supply circuit of 100 volts or higher. The 100 c.p. size takes 7 amperes at starting and 1·5 amperes when running. *Fig.* 16 shows the connections for the musical arc. A circuit consisting of a coil of self inductance L and a condenser of capacity K is connected across the arc terminals of the Pointolite lamp. The working current may be taken from a coil S placed as a secondary to L. The frequency n is obtained approximately by the formula $n = 159{\cdot}2/\sqrt{KL}$, K being in microfarads and L in henries. It is desirable that the coil L should be of large time constant (*i.e.* L/Resistance should be large). The wave form is not free from harmonics.

§ (20) Vreeland Oscillator.—The Vreeland sine wave oscillator[4] works by the action of an oscillating electric circuit upon a double mercury vapour arc. The arrangement is shown in diagram in *Fig.* 17. A large exhausted glass bulb A has a mercury cathode at B and two carbon anodes at P and Q. A battery F (or other direct current source) is

[1] A. S. Hibbard, *El. World*, Sept. 19, 1890. Also see F. Gill, *J. Inst. El. Eng.*, 1901, xxxi. 388.
[2] A. Larsen, *Elekt. Zeits.*, 1911, xxxii. 284.
[3] W. Duddell, *J. Inst. El. Eng.*, 1900, xxx. 232.
[4] F. K. Vreeland, *Phys. Rev.*, 1908, xxvii 286.

connected as shown, the positive terminal being joined through regulating resistances and reactance coils to the anodes P and Q. The side electrode D is only used to start the arc, which is done by switching on at S and tilting up B until the mercury surfaces at B

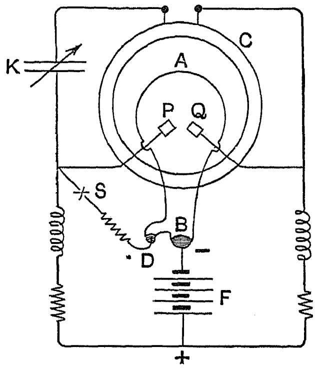


FIG. 17.

and D come into contact. Across the terminals of P and Q is connected an oscillatory circuit consisting of an adjustable condenser K and two (inductive) deflecting coils, represented in the diagram by C, in front of and behind the globe. If the symmetry is so exact that the arcs at P and Q are equal, no current will flow round the oscillatory circuit KC; but if they are not quite equal the potential at one end of the oscillatory circuit will be slightly higher than at the other end, and a current will flow in KC, the deflecting coils, these coils now increasing the arc at the side of higher potential until the condenser K is fully charged. Then the condenser begins to discharge and the reverse process takes place, the arc at the other side becoming the larger, and so on periodically, an oscillatory current being maintained in the KC circuit. The working circuit is taken from a coil closely coupled to the deflecting coils. A steady frequency can be obtained at any desired value from 160 up to 4000 ~ per sec. by adjusting the condenser K. The current given is of exact sine wave form. In working care must be taken to avoid the stray field of the large deflecting coils.

§ (21) TRIODE VACUUM TUBE GENERATOR.—The audion or triode electric valve, which is now so widely used in radio-telegraphy, furnishes without doubt one of the very best sources of alternating current. Such a valve consists essentially of a highly exhausted glass globe containing at one side a tungsten filament (kept glowing by current from a low-voltage battery) and at the other side a metal plate. Between the filament and the plate there is another electrode usually called the "grid," which has various forms in the different types, commonly being constructed of wire gauze or perforated metallic foil. In *Fig.* 18 are shown the connections of a triode when used as a generator. The filament F is kept glowing by current from a 4- or 6-volt battery D, the current being regulated by a rheostat so as to bring the filament to the proper working temperature. (It is well to have an ammeter in this circuit.) A battery B is connected to the filament as cathode and the plate P as anode, the latter connection being through the self inductance L_1 and the capacity K in parallel. From the filament to the grid there is another circuit similarly having inductance L_2 and capacity C in

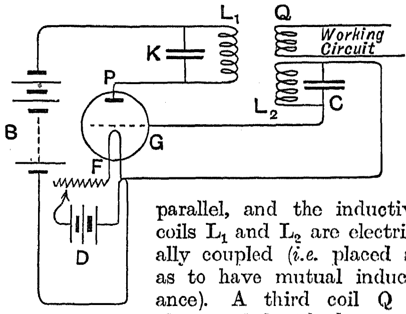


FIG. 18.

parallel, and the inductive coils L_1 and L_2 are electrically coupled (*i.e.* placed so as to have mutual inductance). A third coil Q is also coupled with them, and from it the working circuit is taken. The action of the combination is as follows. A small oscillation set up in the L_2C circuit causes variations of potential difference between the filament and the grid. By the amplifying properties of the valve these variations produce magnified variations of voltage between the filament and the plate, which cause the circuit L_1K to oscillate. This circuit reacts inductively on the L_2C circuit, and thus the oscillations are increased and a steady state maintained. Various other triode generators have been devised, but the system above described gives sufficient illustration of the principles involved.

Without much difficulty an enormous range of frequency is obtained—from 1 cycle in 5 seconds up to several million cycles per second. The adjustment of the frequency is made by varying the condensers or inductances. The higher frequencies (say from 300 ~ per sec. upwards) are easily obtained; for the lower range there is more difficulty, as the inductances and capacitances have to be large. In the latter case the coupling should be close, and, in order to keep the time constants of the coils L_1 and L_2 high, these coils should be of large size. To obtain power of 20 watts, coils having a total weight of 20 kg. (of copper) are suitable, with a large triode tube and a 500-volt battery at B. At the higher

frequencies, however, much larger amounts of power are easily obtained. For testing purposes at audio frequencies and lower, K and C may be subdivided paraffin-paper condensers of good quality, giving capacitance up to 40 or 50 μF. The frequency may be set by tuning the note given out with a tuning-

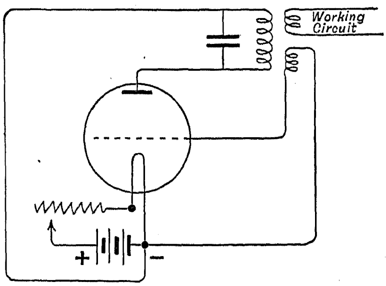


FIG. 18A.

fork or other standard of pitch, or by measuring it electrically as described below (equation 85). When very accurate adjustment is required a variable air condenser should be put in parallel with one of the two condensers and (or) a continuously variable self inductance should form a small part of either L_1 or L_2. The wave form of the current obtained is a good approximation to a sine wave, and the frequency remains remarkably steady if the batteries are in good condition and sufficient for their load.

When only very small amounts of power are required the high-voltage battery B may be omitted and the circuits connected as in *Fig.* 18A.

§ (22) TUNING-FORK MAINTAINED BY TRIODE.—Eccles[1] has shown that a tuning-fork can be maintained in vibration by means of a

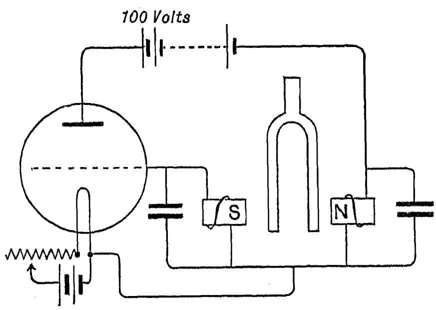


FIG. 18B.

triode valve instead of by contacts or a microphone. In one form two electromagnets act on the prongs of the fork, their windings being respectively in the grid and plate circuits of the tube. The vibration of the fork induces alternating voltage in the grid circuit, which controls the current in the plate circuit and its magnet, and the vibration of the fork is thus maintained. To ensure that the fork shall respond readily to its excitation it is well to put condensers across each of the magnet coils, as shown in *Fig.* 18B, so as to get approximate tuning in the electrical circuits.[2] When the electrical conditions are right a very constant frequency is obtained.

III. MEASURING AND DETECTING INSTRUMENTS

§ (23) INTRODUCTORY.—In the measurement of inductance various instruments are used for measuring voltage or current or merely detecting these, some of them being in common use for electrical measurements in general, and others being more specially designed for this purpose. As general instruments may be mentioned ammeters, voltmeters, and wattmeters, all of which should be suitable for alternating currents and not affected by variations in frequency or wave form. Thermal ammeters and electrostatic voltmeters fulfil these conditions, and electrodynamometers of various types are also useful. Ordinary galvanometers are sometimes wanted, but ballistic galvanometers are more specially used as detectors in some of the null methods. In the majority of modern methods, however, the detecting instruments are either vibration galvanometers or telephones.

§ (24) BALLISTIC GALVANOMETER.—A ballistic galvanometer is one designed for the measurement of the quantity of electricity sent through it in a short interval of time, this quantity being measured by the "throw" of the pointer (or light spot), that is to say, the distance on the scale which the pointer moves until it first comes to rest before swinging back again. In order that this may be a correct measure of the quantity passing, it is an essential condition that the whole discharge shall be complete before the pointer has moved appreciably. To ensure this, the galvanometer is constructed so as to have a relatively long time of swing (say from 8 to 20 seconds complete period); this is usually achieved by increasing the moment of inertia of the moving part and using a weak control. Both moving needle and moving coil types of ballistic galvanometer are used, but the latter is more common in modern practice. Sometimes the damping is small, and the swings of the pointer or light spot die down slowly, but it is generally better to work with considerable damping so that the pointer only just passes the zero point on returning from its first throw. A fuller discussion of the ballistic

[1] W. H. Eccles, *Phys. Soc. Proc.*, 1919, xxxi. 269.

[2] S. Butterworth, *Phys. Soc. Proc.*, 1920, xxxii. 354.

galvanometer will be found in the article on "Galvanometers."

§ (25) ELECTRODYNAMOMETER.—Various experimenters [1] have used electrodynamometers as detectors in inductance measurements, not connecting the moving coil and fixed coil circuits in series (as is done for the ordinary measurement of current), but separating them and combining them in a variety of ways with the other circuits. An electrodynamometer for this purpose should be as sensitive as possible, and to this end a light moving coil with a long and thin suspension will be found advantageous. In an instrument of this kind used at the National Physical Laboratory it was found that with the necessarily weak control considerable disturbances were caused by electrostatic forces acting on the moving coil system. To get rid of these as far as possible the moving coil was screened from the fixed coils by means of tinfoil (with slits in the proper directions to avoid eddy currents) connected to one end of the moving coil which had comparatively few turns. The mica vane for air damping was similarly covered with tinfoil.

§ (26).—In many methods it is desirable that the effective inductance of the moving coil circuit should be zero, and for this purpose Rosa used Sumpner's method [2] of compensating for self inductance, which consists in putting in series with the inductive circuit a condenser shunted by a resistance.

In *Fig.* 19 let L be the self inductance and R the resistance of the circuit in question, K a condenser

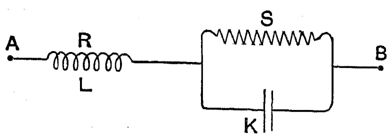


FIG. 19.

and S a resistance, ω being written for $2\pi \times$frequency. Then the (vectorial) impedance of the circuit AB is equal to

$$R+Lj\omega+\frac{S/Kj\omega}{S+1/Kj\omega},$$

which reduces to

$$R+\frac{S}{1+S^2K^2\omega^2}+\frac{L-S^2K}{1+S^2K^2\omega^2}\cdot j\omega.$$

The last term of this expression is the reactance, and hence the reactance of the circuit is zero when $S^2K=L$, a condition which can easily be carried out by suitable choice of K and S. It should be noticed that the compensation is correct at all frequencies, thus differing totally from the case in which reactance is made zero by electrical tuning, where $LK\omega^2=0$ and the circuit is only non-inductive for one frequency. With the Sumpner compensation the circuit is non-inductive, but the total effective resistance is

$$R+S/(1+S^2K^2\omega^2),$$

which is dependent on the frequency. In most ordinary cases $S^2K^2\omega^2$ can be neglected in comparison with 1, in which case the total resistance is practically equal to R+S, and the compensation becomes complete. Rosa adjusted the compensation experimentally by connecting A and B, sending current through the fixed coils, and setting K and S until the deflection was zero.

§ (27) ALTERNATING CURRENT GALVANOMETERS.—By putting an iron core into the fixed coils of an electrodynamometer the magnetic field, and hence the sensitivity, can be largely increased. The instrument is then known as an electromagnet moving coil galvanometer or merely as an alternating current galvanometer. Stroud and Oates [3] introduced such a galvanometer and showed that it gave high sensitivity when used in measurements of inductance and capacity. Abraham [4] a few years later brought out an electromagnet moving coil galvanometer and investigated how its behaviour depended on the reactance of the moving coil circuit. He greatly improved it by adding the capacitance resistance compensation to that circuit. Working independently of these observers, Sumpner and Phillips [5] introduced a number of electromagnet instruments, and of these the Sumpner electrodynamometer is fitted for highly sensitive measurements of inductance and capacitance. It is shown in *Fig.* 20. The electromagnet is built up of thin laminations of alloyed iron sheet of high resistivity and having small hysteresis loss,

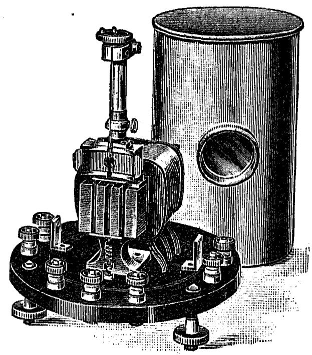


FIG. 20.—Sumpner Electrodynamometer.

[1] H. A. Rowland, *Physical Papers*, 1897, pp. 294 and 314; E. B. Rosa, *Bulletin of Bureau of Standards*, 1907, iii. 43.

[2] W. E. Sumpner, *Journ. Soc. Tel. Eng.*, 1887, xvi. 344.

[3] W. Stroud and J. H. Oates, *Phil. Mag.*, 1903, vi. 707.

[4] H. Abraham, *Comptes Rendus*, 1906, cxlii. 993.

[5] W. E. Sumpner and W. C. S. Phillips, *Phys. Soc. Proc.*, 1908, xxii. 395, and 1910, xxii. 395; W. E. Sumpner, *J. Inst. Elec. Eng.*, 1906, xxxvi. 421, and 1908, xli. 227; *Phil. Mag.*, 1910, xx. 309.

and the magnetic circuit is designed so that the air-gap flux density is accurately proportional to the applied voltage and in quadrature with it. Thus the deflection of the moving coil is only due to that component of the current through it which is in quadrature with the voltage applied to the field magnet. The magnet is wound with three coils, two having 2000 turns each, and one having 200 turns. With the first two coils in series the windings will bear a voltage up to 200 volts at 50 ~ per sec., but in normal working 40 volts is sufficient. The moving coil, which has only a few turns (to keep the inductance low), is suspended in two air gaps of the magnet, and a vane in a closed chamber gives air damping.

§ (28).—Weibel [1] has described several highly sensitive electromagnet galvanometers designed for use in inductance and capacity tests. He has also given a thorough investigation of the mathematical theory of such instruments, and has derived the equation of motion of the moving coil. He shows that the damping and the period of swing both depend on the circuit external to the moving coil, the period being shortened by inductance and lengthened by capacitance. In his instruments, as in the most sensitive direct current galvanometers, the moving coils had to be treated with potassium copper chloride to remove the magnetic dirt. To minimise electrostatic disturbances the moving coil was screened. The galvanometers worked well at low frequencies; one of them showed high sensitivity at a frequency of 2100 ~ per sec., but for satisfactory working required many precautions to be taken. In all instruments of this class the results in most cases are affected by the harmonics present in the current from the source.

§ (29).—This remark also applies to another type of sensitive detector of alternating current, Bellati's so-called electrodynamometer,[2] in which a soft-iron needle is at an angle of 45° to the axis of a surrounding coil. An ordinary moving magnet mirror galvanometer will work on this principle if the magnet system is turned so as to make even a small angle with the faces of the coils when in its position of rest.

§ (30) Vibration Galvanometer.—In certain respects the best detecting instrument for inductance testing is the vibration galvanometer. It might be more appropriately named a resonance galvanometer, for the fundamental principle of its working is that the moving system (magnet, moving coil, etc.) is tuned so as to have its natural frequency the same as that of the alternating source used. When current of this frequency is sent through the galvanometer the moving part is set into resonance, and a very much larger deflection is obtained than if the same current had any other frequency. Thus two very important properties are attained, namely, (1) great increase of sensitivity (sometimes as much as 500-fold), and (2) high selectivity, the harmonics in the wave form having practically no effect. This selective property is of great value, for even with a source of impure wave form the results obtained correspond exactly with the condition of pure sine wave form. As the frequency is lowered, the sensitivity tends to approach that of a good direct current galvanometer. At the lower frequencies (from 150 ~ per sec. downwards) the vibration galvanometer is much more sensitive than a telephone, while at the higher audio frequencies (1000 ~ per sec. and upwards) the telephone is much better than the galvanometer. For further information on vibration galvanometers the reader is referred to the separate article on the subject.

§ (31) Telephone.—The ordinary Bell telephone is the most widely used detecting instrument for alternating current. It consists essentially of three simple elements—a permanent magnet, polar coil or coils, and a thin iron plate mounted with rim clamping close up to one or both poles of the magnet. It has been altered very little in form for many years, but it is still open to improvement in detail. For example, as Wagner [3] has pointed out, by making the pole pieces of silicon iron and replacing the usual ferrotype diaphragm by one of silicon iron there is considerable gain in efficiency. Although the ordinary type can be used over a wide range of frequency, the sensitivity is by no means constant at different frequencies. As M. Wien [4] showed, there are several frequencies at which the diaphragm resonates, and at these frequencies the current sensitivity rises very considerably. For example, in a certain Bell telephone points of resonance were observed at frequencies of 1100, 2800, and 6500 ~ per sec., and in a Siemens telephone at 720, 2100, and 5000 ~ per sec. The very large increase in the motion of the diaphragm for a given current as the frequency is brought to the point of resonance is shown in the curve of *Fig.* 21, which is drawn from the results of observations of Kennelly and Affel [5] on a certain telephone receiver. The amplitude of oscillation of a point near the centre of the diaphragm is plotted against the frequency. The actual sensitivity, however, does not depend only on the motion of

[1] E. Weibel, *Bureau of Standards Bulletin*, 1918, xiv. 23.
[2] Bellati-Giltay Galvanometer. See M. Wien, *Ann. d. Physik*, 1901, iv. 445.
[3] K. W. Wagner, *Elekt. Zeits.*, 1911, xxxii. 110.
[4] M. Wien, *Ann. d. Physik*, 1901, iv. 450.
[5] A. E. Kennelly and H. A. Affel, *Amer. Acad. Proc.*, 1915, li. 421.

the diaphragm, but also on the sensitivity of the ear, which latter factor varies greatly

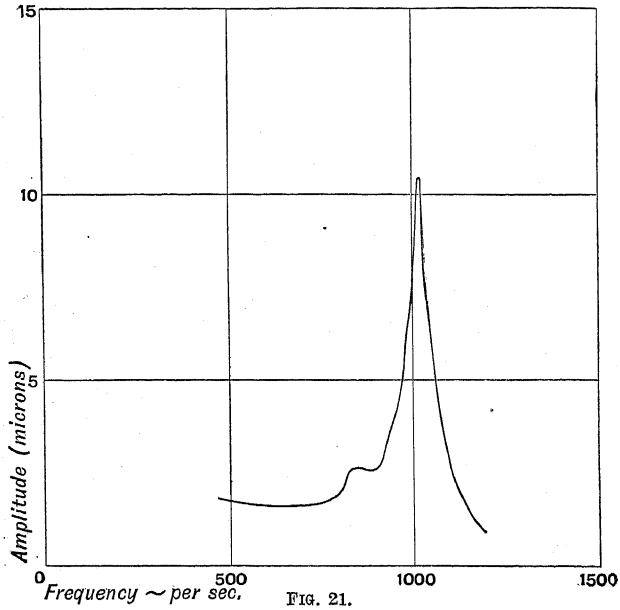


FIG. 21.

with change of frequency (and differently in different individuals).

Lord Rayleigh[1] and others have investigated the working sensitivity of the telephone at various frequencies by observing the minimum current necessary to give an audible sound in the telephone.

The results obtained by M. Wien for the Siemens telephone mentioned above are given in Table I.

TABLE I

Frequency.	Current.
~ per sec.	microamperes.
64	18·0
128	2·2
256	0·26
512	0·017
720	0·015
1 024	0·030
1 500	0·060
2 030	0·008
2 400	0·020
4 000	0·50
8 000	7·0
16 000	22·0

From this example it is clear that a telephone behaves much better at certain frequencies than at others, and for this reason, when working at any particular frequency, it is desirable to choose a telephone suited to the frequency. For the higher frequencies it is often of advantage to replace the ferrotype diaphragm of the ordinary speech telephone by a thicker one of tinplate. Rough tuning may be carried out by slackening the ear-piece (*i.e.* the cap which clamps the plate) and gradually tightening it again till maximum sensitivity is observed.

[1] *Phil. Mag.*, 1894, xxxviii. 294.

§ (32) TUNED TELEPHONES.—A. Campbell found that an ordinary telephone can be tuned over a considerable range by means of a small screw working through the ear-piece and pressing against the plate at an excentric point. Other experimenters have introduced more accurately tunable telephones. The monophonic receiver of Abraham[2] is more sensitive than an ordinary telephone. The diagram in *Fig.* 22 shows the system of working, A being the general arrangement of parts, and B showing the diaphragm in plan. The diaphragm D is mounted on two flattened wires (or strips) SS. These are fastened at one end, and at the other are fixed to the wheel W turning on a spindle and carrying an arm through which the tension of the wires is adjusted by means of the milled-head screw P.

Telephones tuned for only one fixed frequency have also been found useful. Mercadier's monotelephone,[3] in which a large iron plate (one or two millimetres thick) rests horizontally on three points with a telephone magnet underneath it, was used with success by Dongier[4] for inductance measurements, but in general it is not sufficiently sensitive.

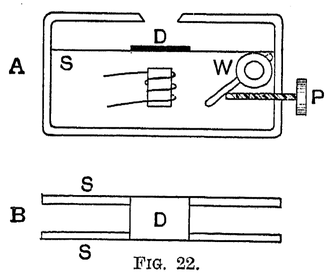


FIG. 22.

R. W. Paul introduced highly sensitive ordinary telephones tuned for fixed frequencies by proper choice of the dimensions of the diaphragm.

S. G. Brown has brought out a telephone in which the ordinary diaphragm is replaced by an iron reed to which is screwed a slightly conical aluminium

[2] H. Abraham, *Comptes Rendus*, 1907, cxliv. 906 and 1154; *Soc. Française de Phys. Bull.*, 1908, p. 703.
[3] E. Mercadier, *Comptes Rendus*, 1900, cxxx. 1382.
[4] R. Dongier, *Comptes Rendus*, 1903, cxxxvii. 115.

diaphragm, the rim of which is sometimes attached to the case by a flexible ring of tissue paper.

§ (33) TELEPHONE WITH ACOUSTICAL RESONANCE.—M. Wien[1] found that by adding a suitable acoustic resonator to a telephone the sensitivity for a fixed frequency could be increased. It appears probable that this method could be used with advantage in inductance testing where rapidity of working may not be important. (It cannot be used for quick signalling purposes, as the resonance takes an appreciable time to build up the full amplitude.)

§ (34) MICROPHONE-TELEPHONE AS DETECTING INSTRUMENT.—As already stated, the sensitivity of a telephone becomes small as the frequency is brought down to the lower values (100 ~ per sec. and lower), and below a certain limit (about 30 ~ per sec.) the ear no longer hears a musical note. To detect currents at these lower frequencies and even down to zero frequency (direct current) Kennelly, Laws, and Pierce[2] combined a

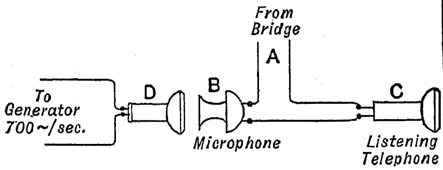


FIG. 23.

telephone with a microphone to work on a kind of relay system, as in *Fig.* 23.

By the leads A the bridge (or other testing circuit) is connected to the observer's telephone C in series with a microphone B. Close up to the microphone is fixed another telephone D which is kept sounding a loud note by current from a source of higher frequency (*e.g.* 700 ~ per sec.). As long as no current passes from A round the microphone circuit nothing will be heard in the listening telephone, but if any current passes (whether low frequency or direct) the maintained vibration of the microphone causes ripples in it and the high note is heard in the telephone. The stimulation of the microphone could be effected by a loud high note produced by other means, such as a siren or whistle. In all cases it is essential to have the microphone and source of sound in a silencing box at a distance from the observer.

§ (35) WORKING CONDITIONS FOR TELEPHONE.—The telephone for any particular measurement should be chosen not only to suit the frequency but also to have resistance of the proper order for the bridge or other arrangement used. Ordinary bipolar telephones can be obtained having any resistance from 4000 ohms down to a fraction of an ohm. Sometimes when only a high resistance telephone is available, while a low resistance bridge is required, better results are obtained by interposing a transformer between the bridge and the telephone, few turns being connected to the bridge and many to the telephone. The converse arrangement can also be used with a low resistance telephone, and the same system is also applicable to a vibration galvanometer. In using a telephone trouble may arise from capacitance to earth through the observer's body. If a hand telephone is used it should never be held directly in the hand but always by a handle of ebonite or other good insulating material. If the telephone is brought near the testing apparatus errors are sometimes caused by stray magnetic fields, the presence of which may be detected by moving the telephone into various positions while listening. Sometimes this trouble is avoided by mounting the telephone on a stand which can be turned into a position at which the effect of stray field is zero, and connecting it to the observer's ear by a stethoscope tube.

§ (36) VIBRATION ELECTROMETER.—There are certain cases where the test circuits have high impedance and hence require a detecting instrument whose impedance is also high. A vibration electrometer has this property. Greinacher[3] uses for this purpose a modified form of the Wulf electrometer, of which the essential parts are shown in *Fig.* 24. The moving part is a very thin (Wollaston) platinum wire P mounted near a metal strip S. The upper end of P is fixed, and the lower end is fastened to the middle of a quartz fibre Q, by which it is kept in tension. When the highest sensitivity is required, S is maintained at a potential of 40 volts above that of the case by means of a battery. The instrument is connected to the bridge through a transformer, whose secondary terminals are connected to the wire and the case respectively.

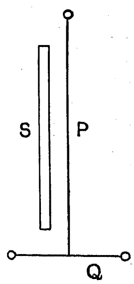


FIG. 24.

§ (37).—A different type of vibration electrometer has been introduced by Curtis.[4] In this a light rectangular aluminium vane (2 cm. × 1 cm.) is mounted vertically on a tunable bifilar suspension between four vertical metal plates (each 1 cm. square) which correspond to the quadrants of an ordinary electrometer, and are connected up in pairs diagonally. The vane is free to vibrate about a vertical axis, and its natural

[1] M. Wien, *Phys. Zeits.*, 1912, xiii. 1034.
[2] A. E. Kennelly, F. A. Laws, and P. H. Pierce, *Am.I.E.E. Proc.*, August 1915, p. 1749.
[3] H. Greinacher, *Phys. Zeits.*, 1912, xiii. 388, and *Elektrotechn. u. Maschinenbau*, 1914, xxxii. 415.
[4] H. L. Curtis, *Bureau of Standards Bulletin*, 1915, xi. 535.

frequency can be tuned up to about a maximum of 100 ~ per sec. The working is heterostatic, the vane being maintained at a higher potential than its surroundings by a battery. The plates are normally from 1 to 2 mm. apart. With the 1 mm. separation a battery voltage as high as 170 volts may be used. The volt-sensitivity increases as the potential of the vane is raised, and, with the other adjustments constant, the resonance frequency is lowered at the same time. As the energy losses are almost entirely due to air damping, the instrument is worked in an enclosure that can be exhausted. The sensitivity increases as the pressure is lowered, and even at a pressure as low as 0·005 mm. of mercury the fraction of the damping due to the suspension is small. As the damping is diminished, however, the frequency range becomes more and more limited, and hence the lowering of the pressure must not be carried too far.

In the paper mentioned above Curtis gives the mathematical theory of the instrument and shows experimentally that the behaviour of the instrument is in agreement with this. The following statement indicates the order of sensitivity obtained. At a pressure of 0·005 mm. of mercury with 140 volts on the vane and at a frequency of 50 ~ per sec., 1 volt on the plates (2 mm. apart) gave a deflection of 50 mm. at 1 m. scale distance. Since the current taken by the electrometer is very small (the impedance being so high), the current sensitivity is very high, being of the order of 10,000 mm. at 1 m. per microampere.

IV. Resistances for Use in Inductance Measurements

§ (38) Introductory. — In nearly all methods of measuring, inductance standard resistances, whether in the form of single coils, resistance boxes, or rheostats, form an essential part of the apparatus. In most cases it is desirable that such resistances should be as free as possible from self inductance and self capacitance (which may be considered as negative inductance). As the inductiveness of a circuit is most appropriately measured by the value of its "*time constant*," we shall consider the origin and definition of this quantity before proceeding to describe the construction of non-inductive resistances.

§ (39) Time Constant.—If a steady voltage V is suddenly applied to the ends of a circuit having resistance R and self inductance L, the current produced in the circuit does not attain its full value immediately, but rises from zero to a practically steady value in an appreciable time, which depends on R and L. To find the law by which it rises, let i be the current at time t from the moment of switching on. Then we have

$$V = Ri + L\frac{di}{dt}, \quad . \quad . \quad . \quad (5)$$

and the solution of this equation is

$$i = \frac{V}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right). \quad . \quad . \quad . \quad (6)$$

When $t=0$, $i=0$ and $\frac{di}{dt}=V/L$;

when $t=\infty$, $i=V/R$ and $\frac{di}{dt}=0$.

Thus we see that the current starts from zero value with an initial rate of rise equal to V/L and reaches its steady value asymptotically, the time taken to reach any given fraction of its steady value depending only on L/R, which is called the *time constant* of the circuit. It is defined as the time taken by the current to rise until it only differs from its steady value by $1/\epsilon$th part of this value. Thus since $\epsilon=3$ approximately the current always rises to approximately two-thirds of its full value in L/R seconds. As a simple example the curve in *Fig.* 25 shows the rise of current when R=1 ohm, L=2 henries, and V=10 volts.

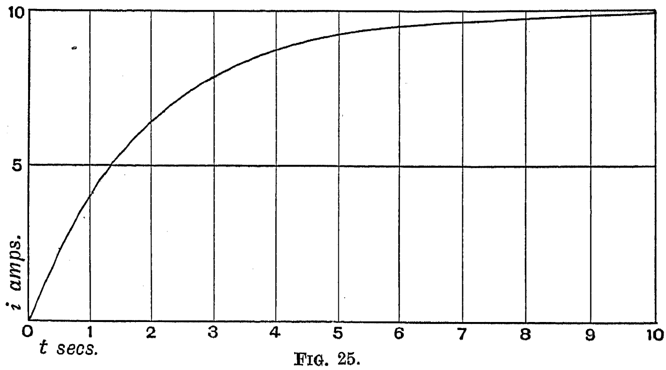


Fig. 25.

The time constant is here 2 seconds; in 10 seconds the current is within less than $\frac{1}{100}$th part of its full value. This is a case of a large time constant, such as would occur with an iron-cored coil. In ordinary standard inductance coils the time constant is of the order of 0·2 millisecond.

If ϕ be the angle of lag of the current behind the terminal voltage, then $\tan\phi = (\text{time constant}) \times \omega$, and the *power factor* is $\cos\phi$.

§ (40) NON-INDUCTIVE RESISTANCE COILS.—A resistance coil of which the self inductance is negligible is commonly called "non-inductive," but this term can only be taken as relative, for in even the best coils it is nearly always possible to detect and measure the small residual inductance. In the older coils and resistance boxes it was customary to attempt to annul the self inductance by using bifilar winding, in which the whole length of wire was doubled back on itself from the middle point and the two wires together were wound on to the bobbin. This system gives considerable self capacity in the coil if the wire is long, for points of the wire having the full applied potential difference between them are very close together (near the terminals), having only thin insulation between them. The result is that in such boxes the lower coils have slight inductance, the middle coils (say 100 ohms) may be nearly non-inductive, while in the 1000 and 10000 ohm coils there may be considerable negative effective self inductance, as the capacity predominates. The capacitance is here distributed all along the winding.

To calculate its amount from the dimensions of the bifilar loop and the inductive capacity of the insulating material, to a first approximation, we may consider the loop as a long twin cable connected across at the distant end. If K be the capacity when the circuit at the end is open, then the distributed capacity (when the end is on closed circuit) may be replaced by an equivalent capacity K/3 connected across the terminals. To find the effect of this capacity upon the effective inductance of a coil, in *Fig.* 26 let AB be the terminals of a con-

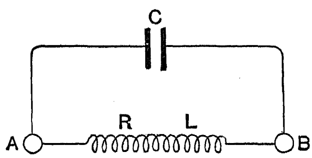


FIG. 26.

ductor of resistance R and self inductance L with capacitance C connected in parallel. If R′ and L′ be the effective resistance and inductance of the combination, we have

$$\left.\begin{matrix} R+Lj\omega \\ 1/Cj\omega \end{matrix}\right\} \text{ in parallel equivalent to } R'+L'j\omega,$$

or $$R'+L'j\omega=\frac{(R+Lj\omega)/Cj\omega}{R+Lj\omega+1/Cj\omega}$$

$$=\frac{R+[L(1-\omega^2LC)-R^2C]j\omega}{(1-\omega^2LC)^2+R^2C^2\omega^2}.$$

Separating the real and imaginary terms, we have

$$R'=\frac{R}{(1-\omega^2LC)^2+R^2C^2\omega^2}, \quad . \quad . \quad . \quad (7)$$

and $$L'=\frac{L(1-\omega^2LC)-R^2C}{(1-\omega^2LC)^2+R^2C^2\omega^2}. \quad . \quad . \quad . \quad (8)$$

In many cases in practice the term $R^2C^2\omega^2$ may be neglected and then

$$R'=R/(1-\omega^2LC)^2, \quad . \quad . \quad . \quad (9)$$

and $$L'=\frac{L(1-\omega^2LC)-R^2C}{(1-\omega^2LC)^2}. \quad . \quad . \quad . \quad (10)$$

At moderate frequencies, for inductance coils usually ω^2LC is small compared to 1 and R^2C may be neglected. Then the equations become

$$R' \doteqdot R(1+2\omega^2LC), \quad . \quad . \quad . \quad (11)$$

and $$L' \doteqdot L(1+\omega^2LC). \quad . \quad . \quad . \quad (12)$$

These last are Dolezalek's formulas.[1]

For resistance coils used with frequencies up to at least 2000 ~ per sec. the terms in $LC\omega^2$ may be neglected, and we have

$$L' \doteqdot L-R^2C, \quad . \quad . \quad . \quad (13)$$

and $$\text{time constant}=L/R-CR. \quad . \quad (14)$$

In this last equation R is in ohms, L in henries, and C in farads.

From equation (13) it will be seen that to a close approximation the effect of the self capacitance is independent of frequency.

§ (41).—If $R^2C=L$, then the effective inductance L′ is zero. So, if the self capacitance have the proper value (L/R^2), the inductance L will be exactly counterbalanced and the resistance coil will be non-inductive. Table II. gives the values of the compensating C required for some typical values of R and L. The examples in the table show that for the lower resistances the compensating capacities are relatively large, while for the high resistances they are very minute.

TABLE II

R.	L.	Compensating K.
ohms.	microhenries.	microfarad.
1	0·1	0·1
10	0·3	0·003
100	1·0	0·0001
1 000	10	0·00001
10 000	100	0·000001

G. A. Campbell[2] in 1904 suggested the above system of compensation by self capacity and investigated the design of compensated coils. A little later various observers (Curtis and Grover[3] at the Bureau of Standards, A. Campbell at the National Physical Laboratory, Orlich[4] at the Reichsanstalt, Wagner and Wertheimer[5] at the Kaiserliches Telegraphen-Versuchsamt) independently designed and constructed coils on this compensation prin-

[1] F. Dolezalek, *Elekt. Zeits.*, 1904, xxv. 152.
[2] G. A. Campbell, *El. World*, 1904, xliv. 728.
[3] H. L. Curtis and F. W. Grover, *Bureau of Standards Bulletin*, 1912, viii. 455.
[4] E. Orlich, *Verhandl. der Deutsch. Phys. Ges.*, 1910, xii. 949.
[5] K. W. Wagner and A. Wertheimer, *Elekt. Zeits.*, 1913, xxxiv. 616 and 649.

ciple, and nearly all of them used the same devices for the large and small resistances. The coils of lower resistance (1 ohm and 10 ohms, for example) are wound bifilar with thin manganin strip, the two halves of the loop being separated from one another by a thin strip of silk or mica. At the N.P.L. special silk-covered strip was used. Time constants as low as 0·05 and 0·015 microsecond for 1 ohm and 10 ohms respectively can be attained. For 100 ohms, if the proper length be measured off of double silk-covered manganin wire of 0·19 mm. diameter (No. 36 S.W.G.), doubled back as a loop and the two wires twisted together, the effective inductance usually is extremely small. The double wire can be wound in single layer on a bobbin or a piece of mica and the whole paraffined or shellacked. The effective self inductance comes out about 0·2 or 0·1 microhenry (or less) and the time constant of the order of 0·0015 microsecond. For 1000-ohm coils ten sections (in series) of 100 ohms each may be used (or five sections of 200 ohms each); in the latter case Curtis and Grover use manganin wire of 0·10 mm. diameter. When only small current-carrying capability is required, a 1000-ohm coil can be wound without sections by using a much shorter length of thinner wire. 10,000-ohm coils can be built up from thousands.

§ (42) Very Thin Wire Resistances.—For use with very small currents the lower resistances of the order of 1 to 10 ohms can be made of bifilar loops of very thin constantan wire up to a few centimetres in length, and this system has the advantage that it is applicable to radio frequencies as it avoids increase of effective resistance due to skin effect. These thin wire resistances should be immersed in oil to give better current capability. Consider, for example, constantan wire of 0·04 mm. diameter, which has a resistance of about 3·6 ohms per cm. If the distance apart of the two wires of the bifilar loop is 0·4 mm., the self inductance will be roughly 0·0065 microhenry per cm. of total length, and hence the time constant will be about 0·002 microsecond per cm. The difficulty of adjusting such short loops is a distinct disadvantage in this type of resistance.

§ (43).—Orlich's method of construction is applicable to coils of resistance from 3000 up to 20,000 ohms. It consists of unifilar winding in two layers with opposite directions of winding (heterochiral), one being directly over the other but separated from it by a considerable distance (2 to 5 mm.). A coil of 25,000 ohms on this system gave a time constant of −0·45 microsecond.

An older system of reducing the capacitance in the coils of higher resistance is that of Chaperon,[1] in which the winding is unifilar in an even number of layers, the direction of winding being reversed in alternate layers. It is much better than the old bifilar system, which gives in certain cases time constants ten times as great.

When considerable power has to be dissipated in the resistance the type introduced by Duddell and Mather[2] is convenient for high resistances. In it fine silk-covered resistance wire is woven into a warp of silk threads. The spacing of the wire is arranged so as to make the capacitance compensate the inductance as nearly as possible.

High resistances of very small inductance can be made from rods of suitable mixtures of graphite and clay baked at a high temperature.[3] Good connection is made to the ends by copper-plating them.

Another system is that devised by Kundt, in which a film of platinum is deposited on a porcelain tube, and then by means of screw thread tracks divided into two parallel spiral strips joined at one end.

§ (44).—For the higher resistances tubes containing poorly-conducting liquids are sometimes employed. A good solution for the purpose is that suggested by Nernst[4] (from the results of Magnanini's experiments). It[5] consists of a $\frac{5}{8}$ normal solution of mannite and boric acid (121·1 gm. of mannite and 41·2 gm. of boric acid made up to one litre in water). Its resistance temperature coefficient is negative below 15° C. and positive above it. The addition of 0·04 gm. of KCl per litre raises the point of minimum resistivity to about 20° C. and gives smaller temperature coefficients. A tube 50 cm. long of cross-section about 0·75 sq. cm. gives a resistance of about 50000 ohms. The electrodes are flat discs of platinum foil. Schering and Schmidt found that the apparent self capacity of a tube of 10000 ohms resistance falls off with increase of frequency as shown in Table III.

Table III

Frequency.	Capacitance.	Frequency.	Capacitance.
~ per sec.	$\mu\mu$F.	~ per sec.	$\mu\mu$F.
25·3	67	500	2·1
49·4	21	997	1·6
91·6	7·9	2025	0·1
261	2·5		

§ (45).—For the winding of large air-cooled resistance frames capable of dissipating 100 to 150 watts, a system due to A. Campbell has

[1] Chaperon, *Comptes Rendus*, 1889, cviii. 799.

[2] W. Duddell and T. Mather, *Electrician*, 1905, lvi. 54.

[3] J. A. Fleming, *Electrician*, 1899, xliii. 492 and 497.

[4] *Zeits. Phys. Chem.*, 1894, xiv. 622, and 1890, vi. 58.

[5] H. Schering and R. Schmidt, *Archiv f. Elektrotech.*, 1913, i. 423.

been in use in the N.P.L. since about 1902; it is convenient for values from 50 ohms up to 1000 ohms and even higher. On slotted rails at the top and bottom of a wooden frame two insulated wires (of equal resistance) are wound in opposite directions, as shown diagrammatically in *Fig.* 27.

There are fifteen to twenty slots and the wires are tied together at many points to ensure

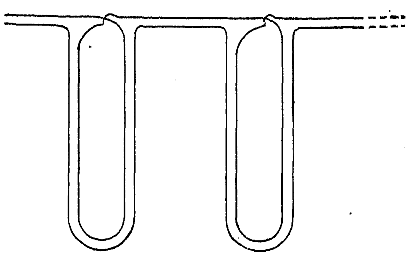


FIG. 27.

closeness. The two windings are connected in parallel. Some years later this system of winding was applied by Ruhstrat to non-inductive rheostats in which the wires, insulated merely by a film of oxide, are wound heterochirally on a porcelain cylinder.

With regard to all the systems described it should be kept in mind that, when the highest accuracy is required, the residual self inductances of all coils used should be tested by actual measurement; this is particularly advisable during the construction of new coils. The permanence of the compensation of inductance by capacitance depends on the constancy of these quantities, and care must be taken to avoid variation in capacitance due to changes in the insulating material, as for example when moisture is absorbed by shellacked silk.

§ (46) LOW RESISTANCES.—Resistances from 0·1 ohm downwards are usually constructed of thin sheet metal (such as manganin), and should have four terminals, one pair being for the current and the other for the potential difference. By doubling the strip of sheet metal back on itself and bringing the two halves as close as possible (with insulation between) the self inductance can be made very small. Ayrton and Mather[1] on this system constructed non-inductive resistances using varnished silk ribbon between long metal strips. Orlich[2] investigated this system for small resistances and constructed a series going down to 0·001 ohm, using manganin strips clamped together with thin mica (0·1 to 0·3 mm. thick) between them. The 0·01 ohm had a time constant of 1·3 microseconds, and the 0·001 ohm one of 5·1 microseconds.

In *Fig.* 28 is shown a diagrammatic end view of the 0·003 ohm, which is oil-cooled. The strip, which has a total length of 50 cm., is in three pieces, A, B, C, bent as shown. A and C are hard-soldered at P and Q to thick copper strips, and the upper ends of these at H and J are connected to massive terminals. Copper strips are hard-soldered to the four lower edges of A, B, and C, and when the whole is fitted together these copper strips are sweated together with soft solder at D and E, being drawn tightly together with screws when the solder is fluid. The dotted lines indicate insulating material, which is shellacked mica between the sheets and thin ebonite between the copper strips. The case, which has a vane stirrer at the bottom, is immersed in an oil bath. If the bath is water-cooled the resistance will bear a load of 100 watts.

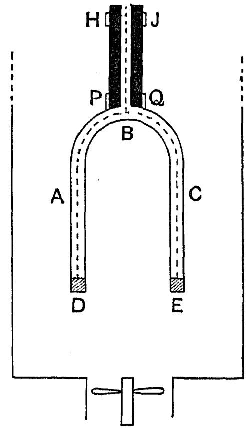


FIG. 28.

§ (47). A quite different system was suggested by A. Campbell,[3] having been foreshadowed by Lichtenstein.[4] In this the self inductance was compensated for by arranging the leads to the potential terminals in a special manner. In *Fig.* 29 let ACB represent a resistance

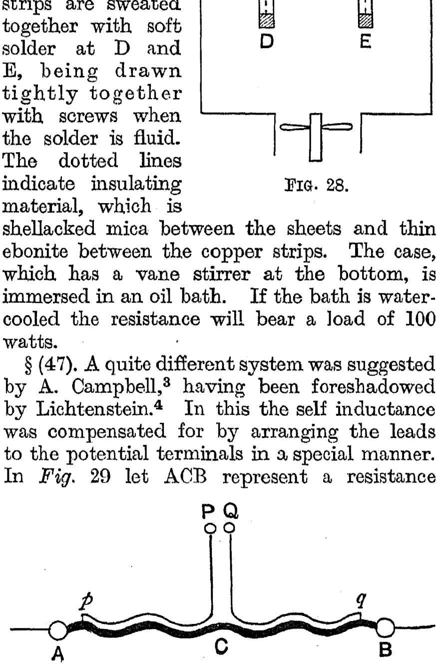


FIG. 29.

having self inductance, A and B being the current terminals, and p and q the potential points. If the leads from p and q to the potential terminals P, Q be of similar form to ACB and brought as close to it as possible (with insulation between), then the effective self inductance will be very much reduced, since the closeness of the potential leads to ACB excludes from the voltage circuit the greater part of the magnetic field round ACB which causes the self inductance. This system was carried out by Paterson and Rayner[5] in

[1] W. E. Ayrton and T. Mather, *Phys. Soc. Proc.*, 1892, xi. 269.
[2] E. Orlich, *Zeits. Instrumentenk.*, 1909, xxix. 241.
[3] A. Campbell, *Electrician*, 1908, lxi. 1000.
[4] Lichtenstein, *Dingler's Polytechn. Journ.*, 1906, vii. 321.
[5] C. C. Paterson and E. H. Rayner, *Inst. El. Eng.*, 1909, xlii. 455.

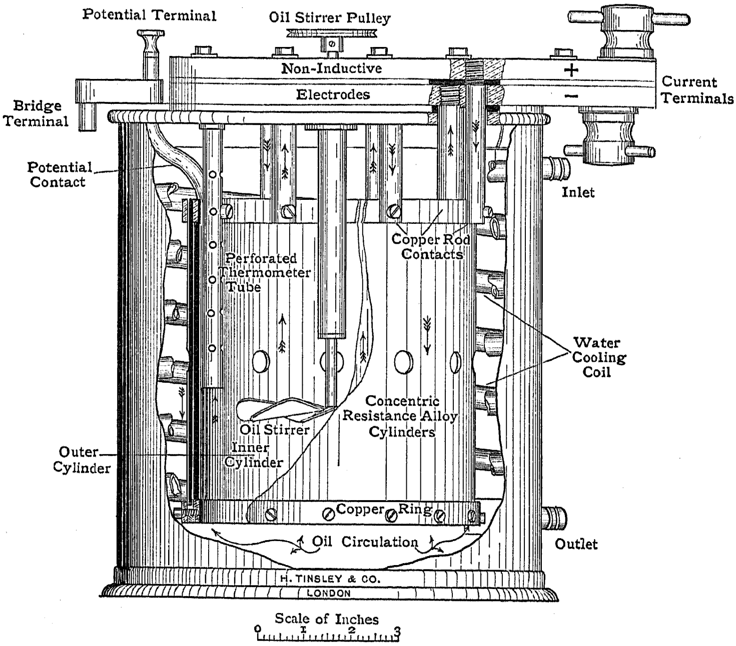


FIG. 31.

the construction of a series of water-cooled tubular resistances at the National Physical Laboratory. In these the leads to the potential terminals consist of two tubes of thin sheet metal concentric with the main tube and separated from it by a layer of varnished cloth 0·2 mm. thick. The time constants varied from 0·16 microsecond for 0·04 ohm up to 3·0 microseconds for 0·001 ohm. The two resistances, which are shown in *Fig.* 30, have current capabilities of 50 and 2000 amperes respectively.

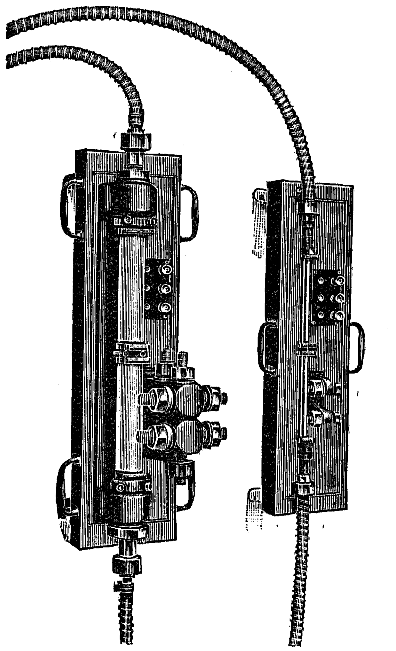


FIG. 30.—Paterson and Rayner's Water-cooled Resistances of 0·001 and 0·04 ohm at the National Physical Laboratory.

§ (48).—Silsbee [1] has discussed the theory of tubular resistances and has suggested a variety of interesting forms. One of these (0·001 ohm) gave a time constant as low as 0·5 microsecond. This was constructed like a concentric main with an inner tube of resistance material and an outer of copper forming the return circuit.

For resistances from 0·001 down to 0·0002 ohm Drysdale [2] used a pair of concentric cylinders close together and joined at the lower end. The current passes down one of them and returns by the other. In this system, as in Orlich's and Silsbee's, the external field produced by the current is very small and hence not liable to cause disturbance in other circuits. An oil-cooled resistance of this type is illustrated in *Fig.* 31.

It is provided with an oil stirrer, and an outer spiral tube traversed by running water keeps the oil cool.

[1] F. B. Silsbee, *Bureau of Standards Bulletin*, 1916, xiii. 375.

[2] C. V. Drysdale, *Electrician*, 1910, lxvi. 341.

§ (49) Rheostats.—Along with non-inductive resistance boxes it is often necessary to use a continuously adjustable rheostat to give sufficiently fine setting of the resistance. A rheostat for this purpose does not always require to be non-inductive; it is sometimes quite sufficient that it should have constant inductance over its whole range of resistance.

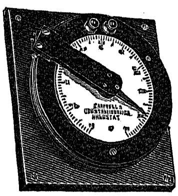


Fig. 32.—Campbell Constant Inductance Rheostat.

Campbell's constant inductance rheostat is illustrated in *Fig.* 32. It consists of a marble or slate disc having a manganin and a copper wire of equal diameters mounted parallel to one another on its rim, with turning radial arm carrying sliding contact pieces which make connection across from one wire to the other. The principle of working is made clearer in *Fig.* 32A, in which *a* and *b* are the terminals, *p* and *r* the manganin and copper wires respectively, *q* the return lead running between them, and *s* the cross contact piece carried by the turning arm. As *s* is moved from left to right the resistance is increased as more of the manganin is brought into circuit while an equal length of copper is cut out. As the wires are equal in diameter the self inductance remains constant to a close

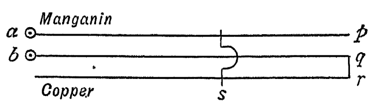


Fig. 32A.

approximation. The most usual range is 1 ohm. The turning arm carries a pointer indicating the approximate resistance on a scale. As the contact resistances come into the total resistance, the contacts should be kept good by applying a trace of paraffin oil to the wires from time to time. When very fine adjustment is required, in series with this rheostat may be put a double channel containing mercury with a sliding copper piece connecting across from the one channel to the other. This may consist of a piece of ebonite having two deep grooves cut in it, say 10 cm. long and 3 mm. wide, with a very thin wall (1 to 2 mm.) left between them. The total inductance is small, and only small movements of the slider are required if the rheostat is first set with care.

§ (50).—Another rheostat of practically constant inductance is that of Wenner and Dellinger,[1] and it is particularly adapted for very low resistance adjustment. It is shown in section in *Fig.* 33.

A vertical ebonite tube *a*, with copper terminal blocks *c*, *c′* at the top and bottom, contains mercury (*b*), into which dips a copper rod *d* held in position by a spring clip *e*. The rod and terminal blocks are amalgamated where they touch the mercury. As the rod is moved up or down it short-circuits less or more of the mercury resistance, and very smooth and fine adjustment is thus obtained. By using tubes of different-sized bores a variety of ranges can be got, but it is not desirable to go below 1 mm. diameter of bore. With this minimum size of bore a tube 12 cm. long gives a range of about 0·1 ohm.

Another type of Wenner and Dellinger's rheostat is shown in *Fig.* 33A in the form made by R. W. Paul. It consists of a U-tube with one limb wide and the other narrow, thus giving both a fine and a coarse adjustment.

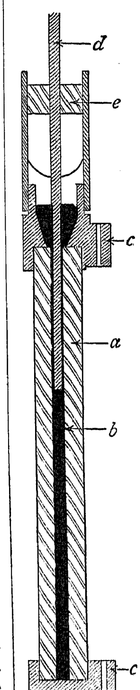


Fig. 33.

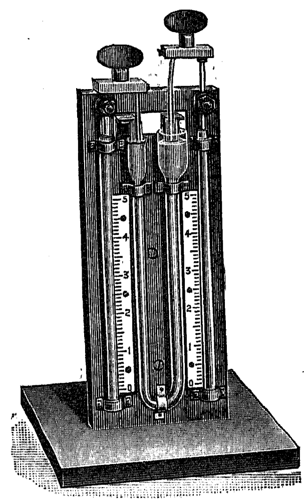


Fig. 33A.—Wenner and Dellinger's U-Tube Mercury Rheostat (R. W. Paul's Pattern).

V. Construction of Standard Inductances

§ (51) Mutual Inductance, Fixed Standards.—Of primary standards of mutual inductance the most commonly used form has been a long solenoid with a secondary coil near the middle of its length. The solenoid (which forms the primary circuit) is wound as evenly as possible on a supporting tube, and the secondary coil is either (1) outside the primary and as close as possible to it, or

[1] F. Wenner and J. H. Dellinger, *Phys. Review*, 1911, xxxii. 614, and 1911, xxxiii. 215; also *Bureau of Standards Bulletin*, 1912, viii. 584.

(2) inside it and wound on a separate supporting tube or bobbin. In case (1) the mean area of the secondary coil is taken as equal to that of the solenoid, while in case (2) the mean area of the secondary coil must be carefully measured: its supporting tube or bobbin should be made of some permanent material (*e.g.* stabilit or marble) constructed so as to allow of accurate measurement. It is a good plan to cut holes or radial slots in the flanges of the bobbin permitting the winding to be seen and the diameters measured by a micrometer microscope. Care should also be taken to mount the bobbin with its mean plane perpendicular to the axis of the solenoid. Two sources of uncertainty may be mentioned: the measurement of the small secondary coil is liable to error and there is great difficulty in winding the long solenoid uniformly enough.[1] The supporting tube should be of insulating material such as ebonite or (better) stabilit, and the winding should be in a screw thread if possible.

§ (52).—Let the solenoid be l centimetres long, with N_1 turns. If H is the magnetic field at the centre due to a current of i amperes, then

$$H \doteqdot \frac{4}{10}\,\frac{\pi N_1 i}{l}. \quad . \quad . \quad . \quad (15)$$

More nearly $$H \doteqdot \frac{4}{10}\,\frac{\pi N_1 i \cos\phi}{l}, \quad . \quad . \quad (16)$$

where $\tan\phi = (\text{solenoid diameter})/l$.

If $\tan\phi$ is a small fraction,

$$\cos\phi \doteqdot 1 - \tfrac{1}{2}\tan^2\phi.$$

As an example of this correcting factor, let the diameter be 8 cm. and the length 100 cm. Then $\cos\phi \doteqdot 0{\cdot}003$, a correction of 3 parts in 1000. If the secondary coil have N_2 turns and mean area s, and if M be the mutual inductance, then

$$M \doteqdot \frac{4\pi N_1 N_2 s \cos\phi}{l} \text{ in abhenries} \quad . \quad . \quad (17)$$

$$\doteqdot 4\pi N_1 N_2 \left(\frac{s}{l}\right) \cos\phi \times 10^{-9} \text{ in henries.} \quad (17\text{A})$$

§ (53).—Searle and Airey[2] have given a much more accurate formula for the calculation of the mutual inductance of a system of this kind with single layer coils, when the dimensions are known with accuracy. It is not easy, however, to get a conveniently large inductance with this type of construction, as will be seen from the following example:

Primary Coil: length 30 cm., diameter 10 cm., turns 300.

Secondary Coil: length 5 cm., diameter 8 cm., turns 200, giving a mutual inductance of 1·1999 millihenries.

[1] See E. Gehrcke and M. v. Wogau, *Verhandl. Deutsch. Phys. Gesellschaft*, 1909, xi. 664, and 1911, xiii. 448.

[2] G. F. C. Searle and J. R. Airey, *Electrician*, 1905, lvi. 318.

The standard solenoid used in the Physikalisch-Technische Reichsanstalt[3] belongs to the first type, in which the secondary coil is outside the solenoid. It consists of a solid marble cylinder a little more than 80 cm. long, on which the primary coil of bare copper wire is wound in a screw thread of 1 mm. pitch, the diameter of the cylinder being about 4·8 mm. The secondary coil has a winding of 1290 turns.

§ (54) Campbell Standard Mutual Inductance.—In a primary standard of mutual inductance, as the value is calculated from the dimensions, the design must be such as to allow of accurate measurement of all the important dimensions. Also the nature of the materials and the construction must be such as to ensure permanency. There is always considerable uncertainty in determining the dimensions of many-layered coils, and modern experience has shown that a single-layered coil wound with bare wire in a screw thread on a marble cylinder gives the nearest approach to ideal conditions. If, however, both primary and secondary circuits are single-layer coils, it is not easy to obtain a value of the mutual inductance (M) large enough for practical requirements, since with given dimensions M is proportional to N_1N_2, the product of the primary and secondary turns. In Campbell's type[4] of fixed mutual inductance the primary only is single-layered, while the secondary is a many-layered coil of many turns, but the geometrical design of the system is such that the dimensions of the secondary coil do not need to be determined with high accuracy, as the accuracy of the calculated value of M is made to depend very much more on the measurement of the

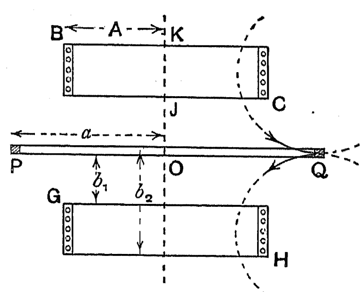


Fig. 34.

dimensions of the primary coil. The arrangement of the coils is shown in the diagram of *Fig.* 34. The primary circuit consists of two equal single-layer coils BC and GH in series, wound on the same cylinder with a gap

[3] E. Gumlich, *Wissenschaftl. Abhandl. der P.T.R.*, 1918, iv. 276.

[4] A. Campbell, *Roy. Soc. Proc.*, 1907, lxxix. 428, and 1912, lxxxvii. 391.

between them. The axial length of this gap is equal to the axial length of one of the coils. The secondary consists of a channel-wound coil PQ concentric and coaxial with the primary coils and placed midway between them. Its mean radius a is made about 1·46 times that of the primary coils (A), and for this size the mutual inductance M is a maximum. Thus the central winding of the secondary coil lies in a region throughout which the magnetic field (due to current in the primary) is zero.

As shown in the figure, the magnetic lines of force due to the upper and lower primary coils are tangential to one another at Q and in opposite directions, and hence the resultant field at that point is zero. Table IV. has been calculated for the following dimensions: $A=10$ cm., $b_1=5$ cm., $b_2=10$ cm., and $N_1N_2=100{,}000$. It will be seen from the values given that near the value $a=14·6$ cm. a considerable change

TABLE IV

a.	M.	$\delta M/\delta a$.
cm.	millihenries.	
14·1	9·1631	+0·05595
14·3	9·1722	+0·03182
14·5	9·1759	+0·009109
14·7	9·1754	−0·012401
14·9	9·1696	−0·032148

in a makes very little change in M. In this position the M is a minimum for axial displacements. Thus the value of M is not much affected (1) by slight variations in a, or (2) by small radial or axial displacements of the whole coil.

Result (1) allows a many-layered coil to be used, and result (2) ensures that very exact adjustment of the position of the secondary coil relative to the primary is not necessary.

The value of M is calculated by Viriamu Jones's formula.[1]

Fig. 35 shows the actual standard of this type constructed at the National Physical Laboratory.

The dimensions are approximately as follows:

Primary Coils (75 turns each): diameter 30 cm., distance between inner ends of coils 15 cm., between outer ends 30 cm.

Secondary Coil (485 turns): mean diameter $43·73_7$ cm., axial depth 1·00 cm., radial depth 0·86 cm.

The actual value of M is 10·0178 millihenries.

The secondary coil is wound in a channel in a thick marble ring, which is supported on a ring tray of mahogany with ebonite feet and levelling screws. By these screws and the other horizontal ones shown in the figure the secondary coil is brought into its correct position before the standard is used. The vertical adjustment is made as follows: An alternating current is sent through the primary coils in series but with the connections of one of them reversed (to make their magnetic fields oppose one another), a telephone or vibration galvanometer is connected to the secondary terminals and the height of the coil is adjusted until the telephone or galvanometer shows that no voltage is induced in the coil. The chief disadvantage of this type of standard is that the secondary coil possesses considerable self-capacity, arising from the necessary crowding

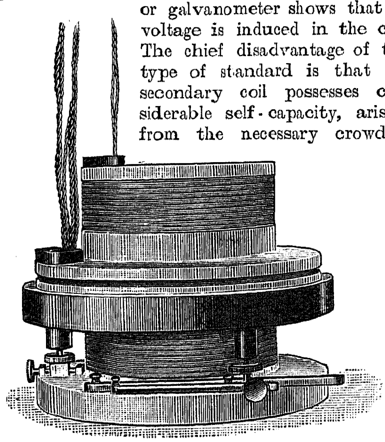


FIG. 35.—Campbell Primary Standard of Mutual Inductance at the National Physical Laboratory.

of its turns into a small channel. There is, however, very little intercapacity between the primary and secondary coils. As the primary coils are wound with bare hard copper wire under considerable tension the temperature coefficient of the mutual inductance may be taken as that of the expansion of the marble, namely, about +3·5 parts in 1,000,000 per degree C. Careful remeasurements in 1914 gave $M=10·0177_8$ millihenries, which is almost identical with the value ($10·0178_5$) obtained in 1908.

§ (55) SECONDARY STANDARDS.—When an accurate primary standard of mutual inductance has been set up, it is easy, by methods to be described below, to derive from it secondary standards having the same magnitude or being multiples or fractions of its value. These can be designed of a much more portable and

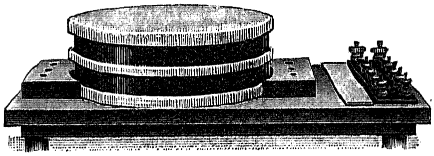


FIG. 36.—Secondary Standard of Mutual Inductance at the National Physical Laboratory.

convenient form than the primary standard. For ordinary and large values a good type consists of two many-layered coils wound on a double bobbin made from a single piece of marble or well-paraffined wood. In *Fig.* 36 is shown a standard of this kind which was constructed in 1905 at the National Physical Laboratory.[2]

The bobbin is of white marble and has two winding

[1] J. Viriamu Jones, *Roy. Soc. Proc.*, 1898, lxii. 247.

[2] A. Campbell, *Phys. Soc. Proc.*, 1907, xx. 626, and *Phil. Mag.*, 1907, xiv. 494.

spaces of 2·5 cm. axial length each, separated by a flange 1 cm. thick. The inner and outer diameters of the winding spaces are about 19 and 24 cm. respectively. The primary winding fills one of the spaces, and in the other space there are three coils (superimposed) giving mutual inductances of 50, 10, and 1 millihenries respectively. Double silk-covered wire was used, and, after winding, the whole bobbin was heated for several hours in melted paraffin. In a standard of this type the various sections can be brought very near to their correct values by small adjusting coils fixed underneath the large bobbin. An external adjustment of this kind is particularly necessary for coils of small number of turns, whether for self inductance or mutual. When the adjustment has been made correct to within one turn, the final setting can often be made by bringing one end of the wire out along one flange of the bobbin (in a groove or otherwise) so as to form the equivalent of a fraction of a turn either positive or negative in value. This system of adjustment is quite applicable to coils that are inaccessible due to the super-position of other windings.

§ (56).—A more modern standard of this type at the National Physical Laboratory consists of a much smaller double marble bobbin wound with well-stranded double silk-covered copper wire and kept immersed in paraffin oil. It has a temperature coefficient of about 10^{-5} per degree C., and has shown very satisfactory constancy. In a standard wound in this way the variation with temperature is mainly due to the expansion of the copper wire.

Campbell has found that temperature compensation can be to some extent applied by using a copper coil overwound with an aluminium one. With proper choice of their relative dimensions complete compensation is theoretically possible. If both coils had the same coefficient of expansion the M would increase with rise of temperature, but as the aluminium is more expansive than the copper, the mean distance between the coils is more than proportionately increased, and this causes a diminution of the M, which can be arranged to balance the increase.

The small temperature coefficients of mutual inductance standards is of more importance than may appear at first sight, for the physical embodiment of mutual inductance has greater permanence in accuracy than that of any of the other electrical quantities, as it depends so entirely on geometrical configuration and so little on the materials of which the standard is constructed. Also for this reason mutual inductance forms the most usual basis for the absolute determination of the units of voltage and resistance.

§ (57).—A commoner form of secondary standard consists of a single bobbin with two windings one over the other. In order to avoid intercapacitance between the two circuits, which always gives troublesome effects, the coils should not be close up to one another, but should be separated by a good thickness of insulating material such as a number of layers of silk ribbon afterwards paraffined. In cases where the self-capacitance of the coils is objectionable, it can be reduced by using one of the special methods of winding to be described further on.

To obtain a number of values of M with one bobbin, the secondary coil may be wound in sections. Their separate values will add correctly when they are put in series and all have the same direction of winding. If any are connected up in the reverse direction, their values will count as negative and have to be subtracted from the sum of the others.

§ (58) Astatic Coils.—When using standards such as have been already described, the external magnetic fields produced by them may cause disturbance in the other working circuits, or the other circuits may in the same way induce disturbing voltage in them. This trouble can generally be avoided by care in the arrangement of the circuits. It can, if necessary, be more completely cured by winding the coils astatically, so that their currents produce no external field, and that a uniform alternating magnetic field induces no voltage in them. For complete astaticism the primary and secondary coils should both be evenly wound one over the other on an anchor ring of insulating material, and every layer of each circuit must go exactly round the ring. Such a system is difficult to construct and adjust. Less perfect astaticism is obtained when only the primary winding completely fits

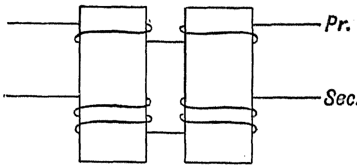


Fig. 37.

the ring, but sometimes this is sufficient. A simpler but less effective way is to wind two equal bobbins each with an identical primary and secondary winding, fix them close together and connect the windings in the directions shown in *Fig.* 37.

Astatic coils have the disadvantage that they require considerably more wire (for a given value of M) than coils with ordinary winding.

§ (59) General Conditions.—For the winding of secondary standards the wire should always be well stranded to minimise *skin effect*, by which in solid wire an alternating current tends to distribute itself un-uniformly through the section of the wire, having its greatest density at the surface of the wire. As the frequency is raised the skin effect increases, and the value of the mutual inductance will usually alter also. The strands

should be of thin wire well insulated. For a like reason, in the construction of the coils no metal parts (such as terminals) should be very near the windings, as induced eddy currents must be avoided. The insulation between the primary and secondary circuits should be extremely good, and all the materials of construction should be non-magnetic.

§ (60) VARIABLE MUTUAL INDUCTANCES.—For a great many measurements it is desirable to use a mutual inductance standard whose value can be varied continuously over a long range if possible. A standard of variable self inductance was named an *inductometer* by Heaviside,[1] and the name *mutual inductometer* is sufficiently distinctive to fit the corresponding mutual standard.

The more modern term *mutual inductor* is perhaps preferable, as it is more descriptive of the actual function of the instrument. The name *variometer*, introduced by M. Wien to designate a variable self inductance, seems too indefinite to be adopted with advantage. Mutual inductometers may be divided into classes according as their ranges are short or long.

For short-range standards it is sufficient to arrange that one of the two circuits (primary or secondary) can be moved relatively to the other, and that its position can be read off on a scale, which can be marked so as to read the inductance directly.

The mutual inductometer used by Lord Rayleigh [2] consisted of two narrow circular coils, of which the secondary was the larger and was fixed, while the smaller was inside it and could be rotated round a common diameter. The whole positive range is comprised within 90° of turning, the position for zero M being when the coils are at right angles; after passing through this position the M is negative and rises to a maximum at 90° on the other side. Lord Rayleigh showed that if the ratio of the mean diameters of the coils is $\sqrt{0{\cdot}3}/1$, *i.e.* 0·548, the mutual inductance is very nearly proportional to the angle through which the turning coil is displaced from the zero position up to about 70° of range. Thus for the greater part of the range the scale can be made almost uniform; it is also approximately symmetrical on the positive and negative sides of the zero line.

When two coils or circuits are so placed that there is no mutual inductance between them, the one is said to be in a *conjugate* position with respect to the other. In the Rayleigh inductometer there are two conjugate positions 180° apart.

§ (61) CAMPBELL MUTUAL INDUCTOMETERS.—Although a uniform scale may be useful in some cases, in general it is far better that the scale should open out very much as the readings diminish down to zero, for this allows the smaller values to be read with much greater accuracy, and thus extends the useful working range. If two coils P and Q be so mounted that one (P) is kept fixed and the other can be rotated about an axis such that the mean plane of Q is always parallel to that of P, then by proper choice of dimensions the mutual inductance scale can be made very open towards the zero reading and a range of nearly 180° can be obtained. This system is used in Campbell's mutual inductometers,[3] of which there are two types, one having a number of short ranges and the other having one very long range with proportionately increased accuracy.

The general arrangement of the coils for the short range type is shown diagrammatically in *Figs.* 38 and 39, which give plan and

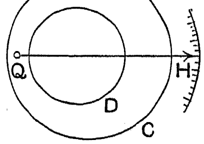


FIG. 38.

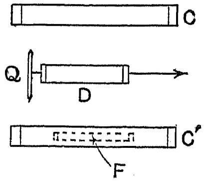


FIG. 39.

vertical section respectively. One of the circuits, which we may call the primary, consists of the two equal coils C and C′, which are coaxial and connected in series, with their windings in the same direction of turning. The secondary circuit consists of the coil D which can turn (midway between C and C′) about the excentric axis Q. The movable coil carries a pointer H which indicates the mutual inductance on a scale either directly or by the help of a multiplier. The character of the scale depends on a number of variables, namely, the diameters of the coils, their distance apart, and the distances between the axis Q and the axes of the coils. A sample scale is shown in *Fig.* 40; it will be noticed that it

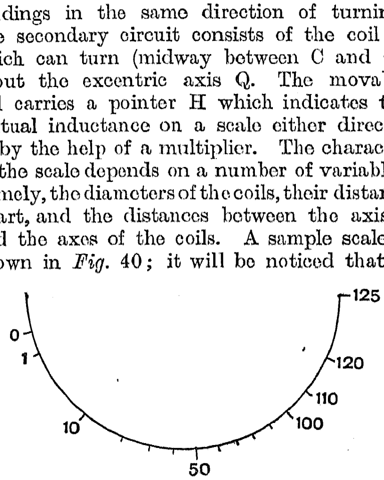


FIG. 40.

opens out at the higher readings as at the lower end, and includes a small negative part. This last portion is often useful, as it enables us to pass continuously through the zero

[1] O. Heaviside, *Phil. Mag.*, 1887, xxiii. 173.
[2] Lord Rayleigh, *Phil. Mag.*, 1886, xxii. 469.

[3] A. Campbell, *Phys. Soc. Proc.*, 1908, xxi. 69, and *Phil. Mag.*, 1908, xv. 155.

value. In *Fig.* 41 is given a picture of a short-range inductometer. The instrument also includes ratio arms to form a bridge. The scale is not long, but there are seven ranges corresponding to multipliers 1, 2, 5, 10, 20, 50, and 100, the lowest range being

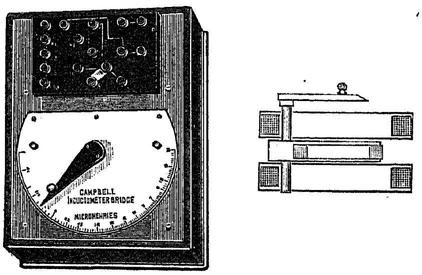


FIG. 41.—Campbell Inductometer Bridge.

from 0 to 10 microhenries. In this way a reasonably large reading can always be had for any inductance from 2 up to 1000 microhenries. The variety of ranges is got by a device which we proceed to describe.

§ (62) SUBDIVISION OF MUTUAL INDUCTANCES BY STRANDED WIRE COILS.—If the secondary coil of a mutual inductor be wound with well-stranded wire (of insulated strands) the mutual inductances between the primary and each of the strands of the secondary will be all practically equal, and these inductances can be added or subtracted by connecting the strands in series, the positive or negative sign being taken according to the respective directions of winding. For example, if ten strands are put in series, all in the same direction, the total mutual inductance will be almost exactly ten times that of one strand. To ensure close proportionality, the stranding must be as symmetrical as possible, every strand coming in turn to a similar position in the section. When sufficient care is taken in the stranding, and if the secondary coil consists of at least several turns not too close up to the primary coil, the accuracy of subdivision is usually within 1 part in 10,000. The equality of the strands can be best tested by opposing each in turn to any selected one, which should give almost zero mutual inductance. The same system can be applied to the primary winding, and in both cases it is only necessary to adjust the winding so as to make the total M correct, or to set one of the strands to the correct value. Thus we can either step up to multiples or down to submultiples of any M to which the adjustment is made. By this method the six ranges of the inductometer bridge (*Fig.* 41) are obtained. Caution must be used, however, in applying this system of subdivision, for the stranding introduces considerable distributed capacity which may cause error in certain cases. As already shown (equation (12)), the effective self inductance of a coil is given by the equation

$$L' \doteqdot L(1+\omega^2 LC),$$

and the self capacity C causes very little error when $\omega^2 LC$ is very small compared with 1. The conditions therefore are favourable when the frequency and L are small, for C is, in general, small. The effects of self capacity in the primary and secondary coils are more complicated and depend on the circuit in which the secondary is included. However, it will be found that, when the frequency is not high or the mutual and self inductances are small, no large error is introduced by the stranding system.

§ (63).—The Campbell mutual inductometer [1] of long range is illustrated in *Fig.* 42, and the

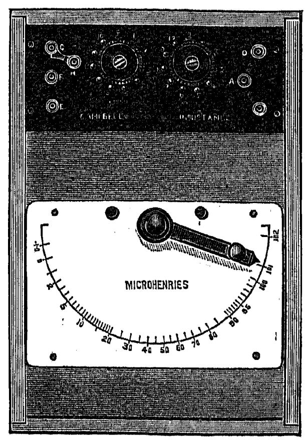


FIG. 42.—Campbell Mutual Inductometer (Long Range).

internal connections and the arrangement of the coils are shown in *Fig.* 43. One circuit, which for convenience of description we may call the *primary*, consists of two equal coils P and P_1, on fixed bobbins. For a purpose to be explained later, the exact centre point of the primary is brought out to the terminal *a*.

The secondary circuit consists of the movable coil S and the multiple coils BB and A, which can all be put in series, and the terminals are EF. By altering the link GH the connections of S are reversed and it can be checked against the first step on the lower dial. The moving coil S is mounted similarly to that of the inductometer bridge. As its plane is always midway between the two primary bobbins and not very near them, small variations in its vertical position have very little effect on the mutual inductance. The pointer

[1] A. Campbell, *Phys. Soc. Proc.*, 1908, xxi. 69, and *Phil. Mag.*, 1908, xv. 155.

and scale belong to this coil. The coils BB and A form the fixed part of the secondary and are on the same bobbins as P and P_1. They are each divided into ten equal sections which are brought out to two stud-switch

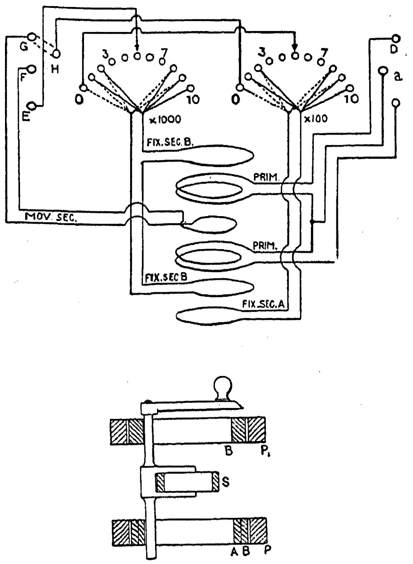


FIG. 43.—Arrangement of Coils and Connections in Campbell Inductometer.

dials, the first reading in steps from 100 up to 1000 microhenries, and the second from 1000 up to 10,000 microhenries. The higher sections are all adjusted separately. The total range extends from $-0{\cdot}2$ up to 11,000 microhenries, and at the higher part of the range the accuracy of reading is to 1 or 2 parts in 100,000. (In a slightly different type this instrument has a range up to 110 millihenries.) In the large inductometer at the National Physical Laboratory great permanence has been secured by winding all the coils on white marble bobbins which are themselves supported by a marble framework. It has a temperature coefficient of $+10$ parts in 1,000,000 per degree C. The mutual inductance increases by about 3 parts in 1000 when the frequency is raised from zero to 2000 ~ per sec.

§ (64). Other experimenters [1] have introduced mutual inductometers which are made astatic by duplicating the fixed or the moving coils or both. Toroidal coil mutual inductors have been constructed by Hanson [2] and by Fortescue; [3] in Fortescue's form the coils are wound on marble cores.

The system of subdivision by means of insulated strands forms a very simple method of constructing accurate mutual inductance standards having very small values. It is easy, for example, to build an inductometer reading to thousandths of a microhenry.

§ (65) SELF INDUCTANCE, FIXED STANDARD.—For primary standards of self inductance calculable from the dimensions much the best form consists of a single-layer coil of bare wire wound in a screw thread on a marble cylinder. The wire should be of copper, hard drawn to ensure elasticity. As it is wound on under considerable tension, it remains in close contact with the marble at all ordinary temperatures, and the temperature coefficient of the dimensions is practically that of the marble. The coefficient of expansion of marble varies with the thermal treatment, and lies between 2×10^{-6} and 12×10^{-6}.

As an example of a primary standard of this kind we may take the 10-millihenry coil constructed at the Physikalisch-Technische Reichsanstalt.[4] The marble cylinder has an outer diameter of 354 mm. and an axial length of 185 mm. It is wound with 162 turns of bare copper wire 0·5 mm. thick in a screw thread of 1 mm. pitch.

§ (66) SECONDARY FIXED STANDARDS OF SELF INDUCTANCE.—The best modern secondary standards of self inductance (fixed) usually consist of multiple-layered coils wound on white marble bobbins. To obtain the maximum self inductance with a given length and thickness of wire the coil should be of square cross-section and wound on a bobbin of the relative dimensions [5] shown in *Fig.* 44.

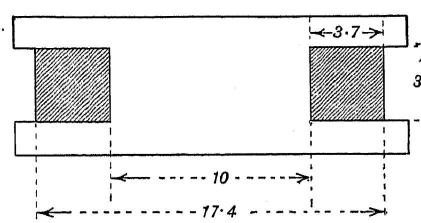


FIG. 44.

Here the outer diameter $=1{\cdot}74\times$ (inner diameter) and depth of winding $\doteq 0{\cdot}37\times$ (inner diameter). For these proportions the self inductance is given approximately by the formula

$$L \doteq 6\pi N^2 a \times 10^{-9} \text{ henries}, \quad . \quad . \quad (18)$$

where N is the total number of turns, and a the mean radius of the coil.

[1] A. Larsen, *Elekt. Zeits.*, 1910, xxxi. 1039; C. H. Sharp and W. W. Crawford, *Am.I.E.E. Proc.*, 1910, xxix. 1207; H. B. Brooks and F. C. Weaver, *Bureau of Standards Bulletin*, 1917, xiii. 369.

[2] A. E. Hanson, Thesis at Mass. Inst. of Technology, Sept. 1916. See also Kennelly and Velander, *Journ. of Franklin Inst.*, July 1919.

[3] C. Fortescue, *Am.I.E.E. Proc.*, 1915, xxxiv. 1199.

[4] Grüneisen and Giebe, *Zeits. Instrumentenk.*, 1911, xxxi. 152, and 1912, xxxii. 160.

[5] See Maxwell, *Electricity and Magnetism*, 1881, ii. (2nd ed.), 316, 706.

Professor T. Mather has shown that the time constant L/R is equal to $5{\cdot}4 \times 10^{-4}a^2$, when the wire is of copper of resistivity 1·630 microhm-cm. From this formula he has calculated the values given in Table V.:

Table V

a.	L/R.	Power Factor.
cm.	secs.	at 100 ~ per sec.
5	0·0135	0·117
10	0·054	0·0295
15	0·122	0·0131
20	0·216	0·00737
30	0·486	0·00328
40	0·864	0·00184
50	1·35	0·00118

To find the weight of copper wire required for a given time constant, H. Armagnat has given the following formula:

$$\text{Mass in kgm} = q(100\text{L/R})^{\frac{3}{2}}, \quad . \quad . \quad (19)$$

where q lies between 2 and 3.

§ (67). To minimise variation of inductance with frequency, due to skin effect, the coils should be wound with well-stranded wire. In the most perfect system of stranding the wires are first stranded in groups of 3 wires each, these groups being again combined three at a time, and so on. When very thin strands are used there is great risk of some of them being broken. Enamel insulation has the advantage of occupying little space, but silk gives more certain insulation and at least should be used on the complete stranded wire. After winding the coils should be kept in hot melted paraffin wax for several hours and allowed to cool slowly before being taken out. The process is best carried out in an air-tight enclosure partially exhausted by a filter pump.

The series of standard coils of this type at the National Physical Laboratory have the

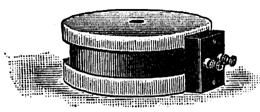


Fig. 45.—Self Inductance Coil (Secondary Standard) at National Physical Laboratory.

form shown in *Fig.* 45. The terminals are mounted at the side for two reasons, (*a*) in order to keep them away from the stronger part of the magnetic field of the coil and so avoid errors due to eddy currents, and (*b*) to allow of one coil being placed immediately over the top of another when it is desired to use them together. The coils are wound with wire stranded from double silk-covered copper wire of 0·19 mm. diameter with 7 strands for the larger coils and 19 for the smaller. The amount of space occupied by the insulation in such a case makes formula (18) inapplicable, and this must always be borne in mind when designing coils with stranded wire. For the 7-strand wire, with thin paper between each layer, about twice the winding space is required.

Giebe[1] has wound somewhat similar coils with much more highly stranded wire; in two of his coils, for example, the wire has $4 \times 3 \times 3 \times 3$—*i.e.* 108 strands of enamelled wire of 0·07 mm. diameter. He finds that for well-paraffined coils with various kinds of stranded wire the temperature coefficient of the self inductance varies from -23 to $+13$ parts in 1,000,000 per degree C.

§ (68) Distributed Capacitance in Multiple-layer Coils.—In closely wound multiple-layered coils the effects of distributed capacitance become more and more pronounced as the self inductance becomes greater. The following data given by Orlich[2] for two standard coils will indicate the order of the self capacitance to be expected:

Table VI

Thickness of Wire.	Number of Turns.	Mean Diameter.	Resistance.	Self inductance.	Self Capacitance.
mm.		cm.	ohms.	henry.	$\mu\mu$F.
0·5	1130	7·8	24	0·1	40
0·5	2894	12·2	94	1·0	150

From the values given it will be found (by equations (11) and (12)) that the self inductance of the 1 henry coil is 2·4 per cent less at 2000 ~ per second than at zero frequency, while its effective resistance is 4·8 per cent higher at the higher frequency.

For self capacities of single layer coils see § (115).

§ (69) Coils of very small Self Capacitance.—As the frequency is raised the self capacitance has more and more effect, and, especially when radio frequencies are reached, it becomes necessary to make the standard coils of such design as will give the smallest possible self capacitance. For the smaller values (up to about 1 millihenry) single layer coils can be used, but for the larger values special systems of winding are required. Various devices are employed, of which the two following will serve as examples.

(1) *H. Rein's System.*[3]—In this type the insulated wire is wound, as shown in *Fig.* 46, on a circle of spokes of insulating material fixed radially in a central disc. The single element thus formed is sometimes termed a pancake coil, and the larger inductances are constructed by assembling a number of these

[1] E. Giebe, *Zeits. Instrumentenk.*, 1911, xxxi. 33.
[2] E. Orlich, *Elekt. Zeits.*, 1903, xxiv. 504.
[3] H. Rein, *Radiotelegraphisches Practicum*, 2nd ed., Springer, Berlin, 1912.

flat coils one above the other with a small space between each coil and the next.

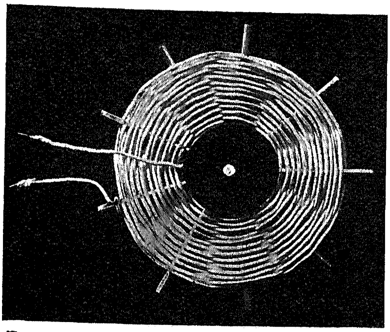


FIG. 46.—Single Coil of Rein Self Inductance.

(2) *A. Campbell's System.*—This method of winding is illustrated in *Fig.* 47. Two rectangular pieces of ebonite P and Q are fixed together at right angles as in the end view B. P and Q have at each of their outer edges a number of deep slots, each slot being just

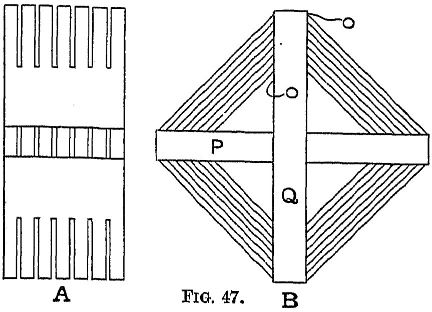


FIG. 47.

slightly wider than the diameter of the insulated wire to be used. The wire is wound into the slots so as to fill one after another, and thus the complete winding is formed of a number of pancake coils separated from one another by small spaces and held rigidly in their relative positions.

The system of winding in pancake coils ("slice winding") has long been used in another way by makers of large spark coils, and in that application it affords protection from breakdown, as it prevents points of the secondary circuit which are at very different potentials from being near each other.

§ (70) INSULATION OF INDUCTIVE COILS.—It is very important that in all standard inductance coils the insulation shall be very good. The effect of leakance on the effective resistance and inductance was first investigated by Rayleigh,[1] and later for a more general case by Campbell and Eckersley.[2]

[1] Lord Rayleigh, *Collected Papers*, ii. 566.
[2] A. Campbell and T. L. Eckersley, *Electrician*, Dec. 10, 1909.

In the more general case, let a leaky inductance coil be represented as in *Fig.* 48, R and L being resistance and self-inductance, K the self capacitance and S the leakage resistance. Let G be the leakance, where $G=1/S$, and let $Z=\sqrt{R^2+L^2\omega^2}$, where the pulsatance $\omega=2\pi\times$frequency.

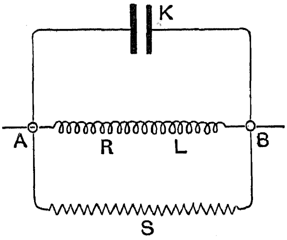


FIG. 48.

If R′ and L′ are the effective resistance and self inductance of the coil, then

$$R'=\frac{Z^2(R+GZ^2)}{(R+GZ^2)^2+\omega^2(KZ^2-L)^2} \quad . \quad (20)$$

and

$$L'=\frac{Z^2(L-KZ^2)}{(R+GZ^2)^2+\omega^2(KZ^2-L)^2}. \quad . \quad (21)$$

When $K=0$ we have the case treated by Lord Rayleigh, where

$$R'=R+\frac{G(Z^2-2R^2-RGZ^2)}{1+2RG+G^2Z^2}. \quad . \quad (22)$$

and

$$L'=\frac{L}{1+2RG+G^2Z^2}. \quad . \quad . \quad (23)$$

For a highly inductive coil, where R is small compared with the impedance Z, if the leakance G is relatively small (S very large compared with R), the equations become

$$R'\doteqdot R+GZ^2 \quad . \quad . \quad . \quad . \quad (24)$$

and

$$L'\doteqdot L(1-2RG-G^2Z^2). \quad . \quad . \quad (25)$$

An examination of equations (20) to (23) shows that a shunt (leakance) across the terminals of an inductive coil always lowers the effective self inductance, but may either raise or lower the effective resistance. For example, if $R=20$ ohms, $L=0{\cdot}1$ henry, at a frequency of 1000 ~ per sec. a shunt of 100,000 ohms put across the terminals increases the effective resistance to 24 ohms. The important practical fact is that, as the effective leakance includes the energy loss due to dielectric hysteresis in the insulation, its effect is not usually constant for different frequencies, as in nearly all insulating materials the energy loss increases with increase of frequency. For this reason if an inductance coil is wound with cotton-covered wire (and even boiled well in paraffin wax) it will usually be found useless as a standard owing to the presence of large dielectric hysteresis. Here, as in many other cases, the trouble arises largely from absorbed moisture, and the greatest care should be taken to avoid all insulating materials of this nature.

§ (71) EDDY CURRENTS.—In the construction of inductance coils metal parts of any kind should be avoided, particularly in positions

near the winding. If screws have to be used they can be of bone, ivory, or stabilit. If metal screws are necessary they should be of a high resistance material such as constantan. When these conditions are not observed, eddy currents, which are proportional to the square of the frequency, may be set up in the metal parts and cause both the effective resistance and the self (or mutual) inductance to vary with frequency.

A piece of metal anywhere near a coil (of resistance R and self inductance L) may be considered as a closed secondary circuit of resistance Q and self inductance N with mutual inductance M to the coil. Then the alterations in R and L due to the presence of the metal are given by the equations

$$R'-R=\frac{QM^2\omega^2}{Q^2+N^2\omega^2} \quad . \quad . \quad . \quad (26)$$

and

$$L'-L=-\frac{NM^2\omega^2}{Q^2+N^2\omega^2}. \quad . \quad . \quad (27)$$

§ (72) STANDARDS OF SMALL SELF INDUCTANCE.—When standards of small self inductance with high resistance are required, it is a common practice to use a pair of equal very thin wires stretched parallel to one another and joined at one end, since the self inductance of such a system is calculable with fair accuracy from the formula

$$L_0=\left[4l\log_e\left(\frac{2b}{d}\right)+l\right]10^{-9}\text{ henries,} \quad (28)$$

where l is the length of one wire, d the diameter, b the distance from wire to wire, and L_0 the low frequency self inductance. The effective self inductance L' is got from L_0 by correcting for the distributed capacity as already shown in equation (13)

$$L'=L_0-R^2C,$$

where $C=K/3$, K being the capacity between the wires when the ends are disconnected. When b is small compared with the distance from earth, K may be got from the formula [1]

$$K\doteq\frac{l\times10^{-6}}{3\cdot6\ \log_e(2b/d)}\text{ microfarads} \quad . \quad (29)$$

This formula assumes the wires to be in air. Sometimes they are used immersed in oil for cooling purposes, and then the above value of K must be multiplied by the specific inductive capacity of the oil.

§ (73) VARIABLE SELF INDUCTANCE STANDARDS.—Self inductometers (or inductors) have very much the same forms as those already described for mutual inductance. A long range is not conveniently obtained with self as it is with mutual inductance, for self inductances do not add algebraically in the simple way that mutual inductances do. The types to be described have all short ranges.

[1] A. Russell, *Alternating Currents*, i. 135, and O. Heaviside, *Phil. Mag.*, 1887, xxiv. 81.

Ayrton and Perry Inductometer.—One of the earliest inductometers is that of Ayrton and Perry [2] (*Fig.* 49). The fixed and moving coils are both wound on spherical surfaces of mahogany built up of sheets with the grains crossed to ensure permanence of form. It has a single range (from 5 to 50 millihenries) and its resistance is about 13 ohms, giving a time constant of 3·8 milliseconds at the top

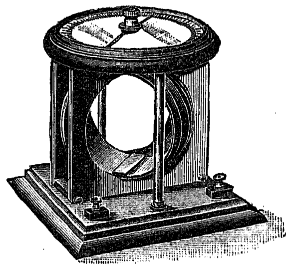


FIG. 49.—Ayrton and Perry Self Inductometer.

reading. Probably owing to the use of metal parts in the construction, the effective resistance at the higher readings is about 13 per cent greater at 1000 ~ per second than at low frequency, while the self inductance is less at the higher frequency by about 8 in 1000. The large areas of the coils cause the magnetic field to produce disturbance at considerable distances.

The coils are always used in series, and the range of angular motion of the inner coil (whose axle carries the indicating pointer) is about 165°. The scale can be graduated to read directly.

If two coils (fixed and movable) of self inductances L_1 and L_2 are in series and so placed that their mutual inductance is M, then M, being either positive or negative, may either help or oppose the self inductances. In one position of the movable coil the total self inductance is L_1+L_2+2M; if that coil be now turned round a diameter through 180°, the M will have the same value but of opposite sign, and the total self inductance will now be L_1+L_2-2M. This turning is equivalent to reversing the connections of the coil. Thus it is seen that in this type of inductometer the whole range lies within the limits L_1+L_2-2M and L_1+L_2+2M. Since it can be shown that L_1+L_2 is always greater than M, it is impossible to make the minimum reading zero. It is not difficult, however, by making the coils fit close to one another, with very little clearance between, to have the minimum reading not more than 1/10th of the maximum.

§ (74) WIEN INDUCTOMETER.—M. Wien [3] introduced an inductometer of the same type,

[2] W. E. Ayrton and J. Perry, *Electrician*, 1895, xxxiv. 546.
[3] M. Wien, *Wied. Ann.*, 1896, lvii. 249.

but of very much greater capability, for it has a large number of overlapping ranges from 0·6 up to 120 millihenries. To furnish so many ranges the fixed coil has four separate windings consisting of 2, 4, 8, and 16 layers respectively, and the turning coil has two windings of 2 and 4 layers. By the help of a small plug board the successive ranges are obtained by suitable combinations of the windings. The scale is in degrees and unfortunately requires a different calibration curve or table for each range.

The inductometers made by C. Lorenz of Berlin are very similar in form to the Ayrton and Perry instrument, but are wound with highly stranded wire on ebonite bobbins, so that they can be used at radio frequencies.

§ (75) Parallel Coil Self Inductors.—The system in which a movable coil turns round an axis perpendicular to the mean planes of the other coils has already been described for mutual inductors, and any of them could be calibrated so as to be used for self inductance also. A number of experimenters have brought out self inductors of this type, one or two of which may be noticed here.

In the Mansbridge inductometer, shown in *Fig.* 50, an ebonite disc carrying two "D"

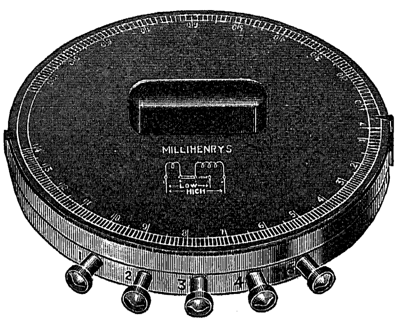


Fig. 50.—Mansbridge Self Inductometer.

shaped coils can rotate over another disc carrying a second pair of similar coils.

The coils are placed thus, ᗡD, and are embedded in the discs, this construction allowing the two circuits to be brought very close together, which gives relatively large mutual inductance and hence greater range of scale. There are two ranges with a direct-reading scale for each. All the coils are used in series for the higher range (9 to 105 millihenries), but only portions of each in series for the lower range (0·7 to 12 millihenries). The arrangement of the coils is astatic and the time constant is about 1·6 milliseconds at the maximum.

In an inductor of this type with only four coils, any slight accidental tilt of the moving disc may cause error. A better plan is to use two pairs of fixed coils above and below the moving system. Inductors of this kind were constructed some years ago at the National Physical Laboratory by D. W. Dye, using thin flat coils embedded in paraffin wax. The fixed and moving systems are moderately close together, and the coils are all made as nearly identical as possible in order to use Heaviside's device[1] of paralleling to give extra ranges. By means of two small switches the coils can be altered from series connection to parallel, giving two lower ranges of $\frac{1}{2}$ and $\frac{1}{4}$ of the maximum range.

At the Bureau of Standards, Brooks and

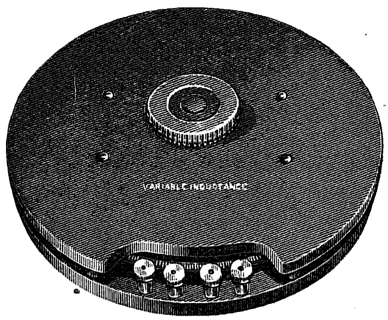


Fig. 50a.—Brooks and Weaver Variable Inductor.

Weaver[2] have built an inductor with three pairs of link-shaped coils designed to give a uniform scale over the greater part of its range. The dimensions of the coils were also chosen so as to approach the conditions for maximum self inductance (such as given by Maxwell for circular coils) at the highest reading. The complete instrument is illustrated in *Fig.* 50a, and consists of three ebonite discs (35·5 cm. diameter), each carrying a pair of coils, two of the discs being fixed together parallel to one another, while the third is between them and can be rotated about a central spindle. The shape of the coils and their relative positions and dimensions are shown in the illustration of the inner disc given in *Fig.* 50b. The coils are all connected in series when the instrument is used for self inductance, but by separating the circuits of the fixed and movable coils it can also be used for mutual inductance. As the scale reads directly for self inductance (from 125 to 1225 microhenries), when used for mutual inductance M is obtained from the formula

$$M = \tfrac{1}{2}(\text{Scale Reading} - 669).$$

The scale is practically uniform from a reading of 325 up to 1025. For the exact dimensions

[1] O. Heaviside, *Phil. Mag.*, 1887, xxiii. 186.
[2] H. B. Brooks and F. C. Weaver, *Bureau of Standards Bulletin*, 1917, xiii. 369.

of the coils which give this result the reader is referred to the original paper. The coils are wound with stranded wire (7 strands of

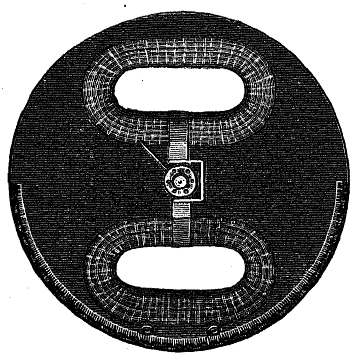


FIG. 50B.—Inner Disc of Variable Inductor.

0·8 mm. diameter each), and the time constant is 3·4 milliseconds at the reading for maximum self inductance.

VI. THE MEASUREMENT OF MUTUAL INDUCTANCE

§ (76) METHOD OF AMMETER AND VOLTMETER.—If no standard of inductance is available, an unknown mutual inductance can sometimes be measured by means of an ammeter and an electrostatic voltmeter. It is only necessary to send a known alternating current I of pure sine wave form through the primary coil and measure by an electrostatic voltmeter the voltage V_2 at the terminals of the secondary; the frequency n should be determined at the same time.

In *Fig.* 51 let M be the unknown mutual inductance, A an ammeter, and B an electrostatic voltmeter. Let i be the instantaneous value of the primary current, where $i=i_{\max}\sin\omega t$, and v_2 the instantaneous value of the secondary

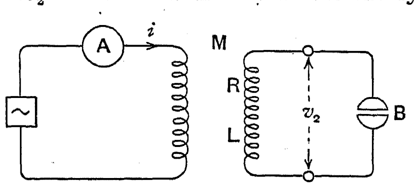


FIG. 51.

terminal voltage, I and V_2 being the corresponding effective values. Then

$$v_2=\mathrm{M}\frac{di}{dt}=\mathrm{M}\omega i_{\max}\cos\omega t. \quad . \quad . \quad (30)$$

Therefore $$\mathrm{V}_2=\mathrm{M}\omega\mathrm{I}, \quad . \quad . \quad . \quad . \quad (31)$$

or $$\mathrm{M}=\mathrm{V}_2/\omega\mathrm{I}.$$

For example, with mutual inductance of 10 millihenries a current of 1 ampere at a frequency of 100 ~ per sec. will give a voltage of about 6·28 volts at the secondary terminals.

If the wave form of i is not pure the harmonics will cause error, particularly as they induce components in v_2 proportional to their frequencies. The electrostatic voltmeter always has capacitance; let this be K. Then if R be the resistance and L the self inductance of the secondary circuit, it can be shown that

$$\mathrm{V}_2=\frac{\mathrm{M}\omega\mathrm{I}}{[\mathrm{R}^2\mathrm{K}^2\omega^2+(1-\mathrm{LK}\omega^2)^2]^{\frac{1}{2}}}. \quad . \quad (32)$$

In nearly all cases $\mathrm{R}^2\mathrm{K}^2\omega^2$ may be neglected, and then we have

$$\mathrm{M}\doteqdot\frac{\mathrm{V}_2}{\omega\mathrm{I}}(1-\mathrm{LK}\omega^2). \quad . \quad . \quad . \quad (33)$$

The capacity of an electrostatic voltmeter varies with the deflection; it may be as large as 0·0001 μF. For moderate values of L and ω this capacity will be found by equation (33) to introduce very little error. Care must be taken, however, to avoid the use of long twisted leads (particularly of flexible cord) in the voltmeter circuit.

§ (77) METHOD OF SIMPLE OPPOSITION.—For the measurement of mutual inductance

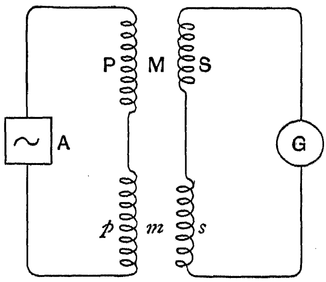


FIG. 52.

the simplest method is that of direct opposition as shown in *Fig.* 52.

If M is the mutual inductance to be measured, it is connected to a mutual inductometer, so that their primary coils P and p are in series with each other and an alternating source A, while the secondaries are in series and connected to a detector G, such as a vibration galvanometer or a telephone. The secondaries must be connected so that their induced voltages are in opposition; the right direction is found by trying which way gives the smaller want of balance in the detecting instrument. The inductometer (m) is then adjusted until a balance is obtained, the galvanometer or telephone indicating that the secondary current is zero. Then $\mathrm{M}=m$.

In this test, as in most others also, if a vibration galvanometer is used, it is well to have a turning switch with a graduated series of shunts connected across the leads to the

galvanometer, which should be kept shunted until an approximate balance has been obtained. With a long-range inductometer and at the high part of its range it is easy to *read* to an accuracy of 1 part in 100,000. This method is only limited by the condition that the value of the unknown inductance must lie within the range of the inductometer. If a multiple-range inductor be used the greatest accuracy will be got by reading on the lowest range on which a balance can be obtained.

Sometimes it will be found that an exact balance cannot be got, but only a minimum in the galvanometer or telephone. This is generally due to distributed capacitance or eddy currents in one or both of the inductances (M and m). Let us consider capacitance first.

In *Fig.* 53 for simplicity let only the secondary coil of M have self capacitance K, the inductometer

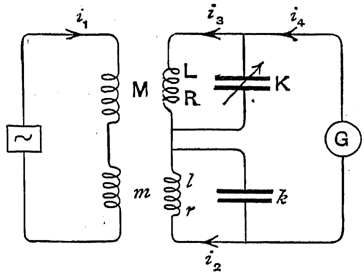


FIG. 53.

being supposed to have none. Then if an adjustable condenser k be put across the secondary of m, by adjusting k as well as m a perfect balance can be obtained.

Let the resistances and self inductances of the secondary circuits be R, r and L, l respectively, and let the instantaneous currents be i_1, i_2, i_3, and i_4 as shown, ω being the pulsatance $2\pi n$.

Then, when a balance has been obtained, $i_4=0$, and the voltages across K and k must be equal and opposite. Thus we have

$$i_2/j\omega k=i_3/j\omega \mathrm{K},$$

also
$$\left(r+j\omega l+\frac{1}{j\omega k}\right)i_2=j\omega m i_1$$

and
$$\left(\mathrm{R}+j\omega \mathrm{L}+\frac{1}{j\omega \mathrm{K}}\right)i_3=j\omega \mathrm{M} i_1.$$

Hence
$$\frac{\mathrm{M}}{m}=\frac{1-\omega^2\mathrm{LK}+j\omega\mathrm{RK}}{1-\omega^2 lk+j\omega rk}$$

Separating the real and imaginary parts we obtain

$$\frac{\mathrm{M}}{m}=\frac{\mathrm{RK}}{rk}=\frac{1-\omega^2\mathrm{LK}}{1-\omega^2 lk}. \quad . \quad . \quad (34)$$

If $\omega^2 lk$ and $\omega^2\mathrm{LK}$ are both small compared with 1, then $\mathrm{M} \doteqdot m$, and

$$\mathrm{M}/m \doteqdot 1-\omega^2 k(\mathrm{L}r/\mathrm{R}-l). \quad . \quad . \quad (35)$$

Thus we can find the correct value of M in spite of the presence of self capacity.

At the National Physical Laboratory this method is used in comparing secondary standards (inductometers) with the primary standard already described. The secondary coil of this latter has a self induction of 0·26 henry and a self capacity of about 0·0026 μF. Both of these values would of course be excessive in any working secondary standard.

The method can be carried out without alternating current by replacing the alternating source by a battery with a reversing switch, and using a ballistic galvanometer as the detector. The inductometer is then adjusted until reversal of the primary current gives no throw of the ballistic galvanometer. This system avoids the effects of self capacity, but is in general much less sensitive than when alternating current is used.

§ (78).—In an ideal mutual inductance ϕ, the angle of lag between the primary current and the secondary induced voltage is exactly 90°, and when this is the case the inductance is termed "pure" by Silsbee;[1] where self capacitance is present, we have a case of "impure" mutual inductance, and if $\theta=\frac{\pi}{2}-\phi$, then θ is called the "phase defect."

For an impure mutual inductance in symbolic rotation

$$e_2=(\mathrm{P}+j\omega\mathrm{M})i, \quad . \quad . \quad . \quad (36)$$

and for a pure one $\quad e_2=j\omega\mathrm{M}i. \quad . \quad . \quad . \quad . \quad (37)$

Then $$\mathrm{E}_2{}^2=(\mathrm{P}^2+\omega^2\mathrm{M}^2)\mathrm{I}^2 \quad . \quad . \quad . \quad (38)$$

and $$\tan\theta=\cot\phi=\mathrm{P}/\omega\mathrm{M}. \quad . \quad . \quad . \quad (39)$$

The term P is of the order of a resistance, and its presence indicates expenditure of power in the system. The angle θ is usually very small, ϕ being nearly a right angle.

§ (79) EFFECT OF EDDY CURRENTS.—If there are pieces of metal or other closed conducting circuits near the coils of the mutual inductance M, then eddy currents may be induced in them which will alter the effective mutual inductance and, owing to the presence of the P term, make it impossible to obtain a balance by a pure mutual inductance m.

Campbell[2] has shown how a balance can be got by associating with m a resistance r as in *Fig.* 54. Here the eddy current system is represented by a closed circuit of resistance R and self inductance L, having mutual inductances F and G to the primary and secondary circuits respectively of the unknown mutual inductance M.

As r is usually small it should consist in whole or in part of a slide wire. Let the instantaneous values of the currents in the primary, secondary, and tertiary

[1] F. B. Silsbee, *Bureau of Standards Bulletin*, 1916, xiii. 380.

[2] A. Campbell, *N.P.L. Report for 1908* (March 1909), *Phys. Soc. Proc.*, 1910, xxii. 214, and *Phil. Mag.*, 1910, xix. 503.

circuits of M be i_1, i_2, and i_3 respectively, their effective values being I_1, I_2, and I_3.

By adjusting m and r a balance can be got, when i_2,

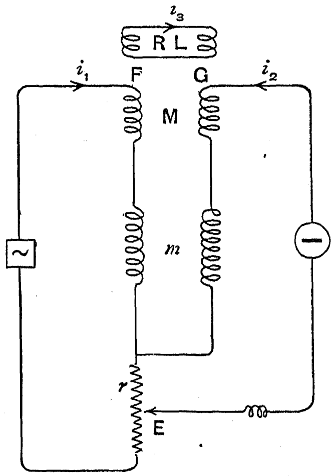


FIG. 54.

the current in the vibration galvanometer or telephone, is zero.

Then we have

$$(R+j\omega L)i_3=j\omega F i_1,$$

and therefore

$$(R^2+\omega^2L^2)I_3^2=\omega^2F^2I_1^2. \quad . \quad . \quad (40)$$

Also

$$\begin{aligned} j\omega(M-m)i_1 &= -ri_1-j\omega Gi_3 \\ &= -ri_1+\frac{\omega^2FGi_1}{R+j\omega L} \\ &= -ri_1+\frac{\omega^2FGRi_1}{R^2+\omega^2L^2}-\frac{j\omega^3FGLi_1}{R^2+\omega^2L^2}. \end{aligned}$$

Hence, separating real and imaginary parts,

$$M=m-\frac{\omega^2FGL}{R^2+\omega^2L^2} \quad . \quad . \quad . \quad (41)$$

and

$$r=\frac{\omega^2FGR}{R^2+\omega^2L^2}. \quad . \quad . \quad . \quad (42)$$

Also

$$M=m-rL/R. \quad . \quad . \quad . \quad (43)$$

From this investigation we see how an impure mutual inductance of the form $P+j\omega M'$ can be measured by being balanced against a known $r+j\omega m$, the r and m being connected as in *Fig.* 55, in which they form an elementary pair with current terminals AB and potential terminals CD. (Silsbee has pointed out the close analogy here to a four-terminal resistance with current and potential leads. AB and CD are interchangeable, as in the case of the resistance.)

The unknown mutual inductance may have an iron core, in which case the resistance P corresponds to power loss arising from both hysteresis and eddy currents. When this total iron loss is relatively small we can determine it from the above equations; when the primary current i_1 is of sine wave form and the iron is in the form of a uniform ring wound with

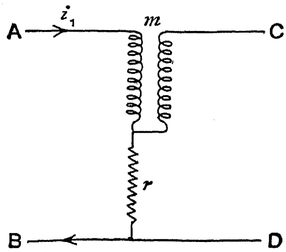


FIG. 55.

superimposed primary and secondary coils of N_1 and N_2 turns respectively,

then the power loss $=RI_3^2=rI_1^2 \cdot N_2/N_1$. . . (44)

The method also gives a convenient way of testing a current transformer (under load). Let part of the load be a known resistance S.

Then

$$\tan\phi=\omega m/r \text{ and } I_1^2/I_2^2=S^2/(r^2+\omega^2m^2). \quad (45)$$

When ϕ is small we have

$$\begin{aligned} I_1/I_2 &\doteqdot \frac{S}{r}(1-\omega^2m^2/r^2)^{\frac{1}{2}} \doteqdot \frac{S}{r}(1-\tfrac{1}{2}\tan^2\phi) \\ &\doteqdot \frac{S}{r}\left(1-\frac{\phi^2}{2}\right). \quad (46) \end{aligned}$$

In practice ϕ is generally so small that we may take

$$I_1/I_2=S/r. \quad . \quad . \quad . \quad . \quad (47)$$

The elementary pair $(r+j\omega m)i_1$ of *Fig.* 55 has been used in other methods of testing transformers by various experimenters.[1] Larsen[2] has used it as the basis of his alternating current potentiometer, as by adjusting r and m any unknown voltage can be balanced and so resolved into two components in phase and in quadrature with a given current i_1.

§ (80) MAXWELL'S METHOD OF COMPARING MUTUAL INDUCTANCES.—For the comparison of unequal mutual inductances Maxwell's[3] method is probably the best. It can be worked either with a battery and reversing key as source and a ballistic galvanometer as detector, or with alternating current and a vibration galvanometer or telephone. The latter case is illustrated in *Fig.* 56. The primaries of the two inductances and the source A are connected in series, while the secondaries are connected to two resistances, Q_1 and Q_2 respectively, and form

[1] C. H. Sharp (in *Discussion*, June 30, 1909), *Am. I.E.E. Trans.*, 1910, xxviii. (2), 1040; Sharp and Crawford, *ibid.*, 1911, xxix. (2), 1517; Agnew and Silsbee, *ibid.*, 1912, xxxi. 1635.

[2] A. Larsen, "Der komplexe Kompensator," *Elekt. Zeits.*, 1910, xli. 1039; and *Electrician*, 1911, lxvi. 738.

[3] Maxwell, *Electricity and Magnetism*, 2nd ed. ii. § 755.

two branch circuits bridged over by the galvanometer G.

Case 1.—When a battery and ballistic galvanometer are used, one or both of the resistances Q_1 and Q_2 are adjusted until the

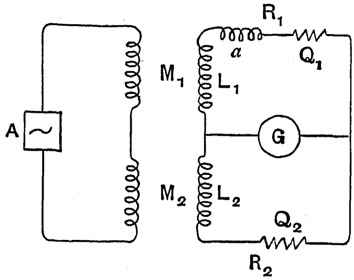


FIG. 56.

galvanometer shows no throw on reversal of the primary current.

Then $$M_1/M_2 = R_1/R_2, \quad . \quad . \quad . \quad (48)$$

where R_1 and R_2 are the resistances of the two branch circuits and include the resistances of the secondary coils. (If no balance can be obtained, the connections of one of the secondary coils should be reversed.) This is the simplest way of using the method and gives excellent results if a sufficiently sensitive galvanometer is available. When by gradually altering Q_1 or Q_2 the point of balance is passed the throws of the ballistic galvanometer are in the reverse direction, which makes the adjustment easier than with alternating current when the detecting instrument has not this power of discrimination. If possible the resistances of the copper-wound secondary coils should be swamped by Q_1 and Q_2 to prevent uncertainty of temperature causing error. In comparing an unknown mutual inductance against a fixed standard whose value is a power of 10 it is best to set the total resistance in the standard branch to a power of 10 (say 1000 ohms), and then the method becomes almost direct reading. The method is applicable to the comparison of two secondary coils with one primary, and is useful for checking the proportionality at different readings on an inductometer.

In using a ballistic galvanometer it is important to set the needle in the mean plane of the coils, or the moving coil facing symmetrically with respect to the magnet. If this is not done the galvanometer may respond somewhat to alternating current, and give a throw even though the total quantity of electricity passing is zero. If the throw will not come quite to zero, it is advisable to carry out the second condition of Case 2 here as well.

Case 2.—With alternating currents the conditions for a balance are not so simple as in Case 1. Two conditions must be fulfilled, namely,

$$M_1/M_2 = R_1/R_2$$

and $$L_1/L_2 = R_1/R_2. \quad . \quad . \quad . \quad (49)$$

To ensure the fulfilment of this last conditiou an adjustable self inductance a is introduced into one or other of the secondary branches. By alternate adjustments of R_1/R_2 and a a balance is finally reached.

It should be noted that in this case, when two secondary circuits have the same primary, instead of equation (49) the condition is

$$\frac{L_1 \pm m}{L_2 \pm m} = \frac{R_1}{R_2}, \quad . \quad . \quad . \quad (50)$$

where m is the mutual inductance between the two secondaries.

It is evident from equation (49) that the method can be used to compare two self inductances. It will be discussed further in the chapter on the measurement of self inductance.

The equations still hold if the alternating source A and the detecting instrument G have their positions interchanged.

§ (81) CAMPBELL'S COMPARISON METHOD FOR UNEQUAL MUTUAL INDUCTANCES.—If a mutual inductance is beyond the range of a given inductometer, it can be tested by the following method.[1] Let the primary circuits of the inductometer (L_1) and the unknown inductance (L_2) be connected up in a bridge with ratio arms R and S as in *Fig.* 57, adding to L_2,

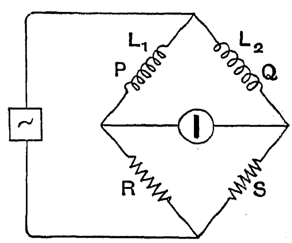


FIG. 57.

if necessary, a coil c of sufficient self inductance to make L_2 considerably greater than L_1. By altering R or S and P or Q let a balance be obtained.

Then $$PS = RQ$$

and $$L_2R = L_1S.$$

Now let the secondary circuits be introduced (in opposition) into the galvanometer

[1] A. Campbell, *Phys. Soc. Proc.*, 1907, xxi. 69, and *Phil. Mag.*, Jan. 1908, xv. 155.

circuit as in *Fig.* 58, and let the balance be restored by adjusting the inductometer M_1.

Then $$M_2 = M_1 S/R.$$

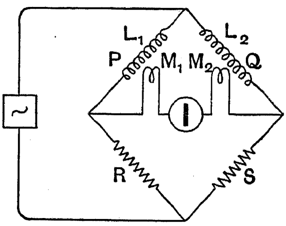


FIG. 58.

§ (82) METHODS OF MEASURING MUTUAL INDUCTANCE IN TERMS OF RESISTANCE.—A standard mutual inductance, being calculable from the dimensions of the coils, is much more easily constructed than a standard of resistance, and in the most accurate methods the standard resistance is derived from a known mutual inductance. Since, however, accurately calibrated resistance coils and boxes are in much more general use than mutual inductances, it is sometimes convenient to determine mutual inductance in terms of resistance. The two following methods will serve as examples.

Method with Ballistic Galvanometer.[1]—The secondary coil of the unknown mutual inductance M is connected through added resistance to a ballistic galvanometer, the whole secondary circuit having a total resistance R. A current i_1 is now reversed in the primary circuit, which causes a quantity of electricity $2Mi_1/R$ to be sent through the galvanometer, giving a throw α.

Then $$\frac{2Mi_1}{R} = k\frac{T}{\pi}\left(1+\frac{x}{2}\right)\sin\frac{\alpha}{2}, \quad . \quad . \quad (51)$$

where k is the galvanometer constant, T the time (in secs.) of a complete period of the galvanometer, and x the logarithmic decrement of its oscillations (for half period). Then k is determined by measuring the current i_1 on the same galvanometer shunted so that a fraction $1/b$ of the current i_1 passing through it gives a steady deflection θ.

Thus $$i_1 = bk\tan\theta,$$

and hence $$M = R\frac{T}{2b\pi}\left(1+\frac{x}{2}\right)\frac{\sin\frac{\alpha}{2}}{\tan\theta}. \quad . \quad . \quad (52)$$

If R is in ohms, M will be in henries. It is best to choose b so that θ and α are approximately equal. T, the time in seconds between successive passages of the light spot through zero in the same direction, is found by counting a number of these and taking the time with a chronograph or stop-watch. If the lengths $a_1, a_2, a_3 \ldots$ of successive swings are observed, then

$$x = \frac{2}{n-m}\log_e\left(\frac{a_m}{a_n}\right). \quad . \quad . \quad (53)$$

§ (83) CAMPBELL'S TWO-PHASE METHOD.—When two-phase alternating current of nearly pure sine wave form is available Campbell's method[2] may be used. The connections are shown in *Fig.* 59.

Let M be the mutual inductance to be measured and R a non-inductive resistance. The secondary coil of M is connected in series with R to a vibration galvanometer G. Currents in exact quadrature are sent through

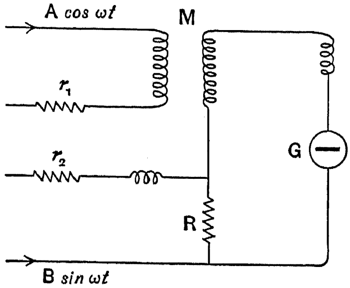


FIG. 59.

the primary coil and R respectively. By adjusting the ratio of these currents or the value of R the deflection of the galvanometer is reduced to zero. The ratio A/B of the currents is now determined by the readings of an electrostatic voltmeter put across the resistances r_1 and r_2 in turn, and the frequency is accurately observed.

Then $$M = \frac{B}{A}\cdot\frac{R}{\omega}, \quad . \quad . \quad . \quad (54)$$

where $\omega = 2\pi \times$ frequency.

§ (84) MEASUREMENT OF MUTUAL INDUCTANCE IN TERMS OF CAPACITANCE.—There are a number of methods by which mutual inductance can be determined in terms of capacitance. These will be discussed in detail in the article on the measurement of capacity. For convenience of reference, however, brief descriptions of two of them are given here.

Campbell's Sifter Method.—The simplest method is that of Campbell,[3] which is shown in *Fig.* 60.

Let the mutual inductance be m and the condenser k. One terminal of the primary coil of m is connected to one of the secondary at F. Alternating current (of frequency n)

[1] R. T. Glazebrook, *Roy. Soc. Phil. Trans.*, 1883, clxxiv. 223.

[2] *Roy. Soc. Proc.* A, 1908, lxxxi. 450, and 1912, lxxxvii. 398.

[3] A. Campbell, *Phys. Soc. Proc.*, 1908, xxi. 69, and *Phil. Mag.*, 1908, xliv. 155; also *Phys. Soc. Proc.*, 1917, xxix. 350.

is sent through the primary coil in series with k, a vibration galvanometer (or telephone) G being connected across k and the secondary

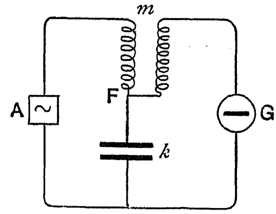


FIG. 60.

coil. When by adjusting k a balance is obtained, we have

$$\omega^2 mk = 1, \quad . \quad . \quad . \quad (55)$$

where the pulsatance $\omega = 2\pi n$.

The proper ends of the primary and secondary coils to connect must be found by trial. In order to get a good balance the condenser should be as free as possible from absorption.

§ (85) CAREY FOSTER'S METHOD WITH HEYDWEILLER'S MODIFICATION.—The connections for Carey Foster's [1] method are shown in *Fig.* 61, M being the mutual inductance and K the condenser. R and S are non-inductive

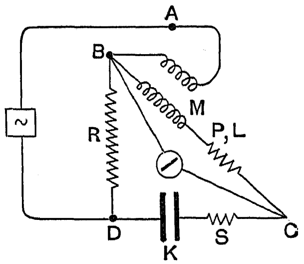


FIG. 61.

resistances, and the arm BC consists of the secondary coil of M in series with an adjustable resistance. The detector (telephone or vibration galvanometer) is connected across BC. A balance is obtained by adjusting P and S. Then, if P is the total resistance of the branch BC, and L its self inductance, we have

$$M = 10^{-6} KPR, \quad . \quad . \quad . \quad (56)$$

where K is in microfarads;

also $$S + s = R(L - M)/M, \quad . \quad . \quad . \quad (57)$$

where s represents the dielectric losses in the condenser K.

Since $S + s$ cannot be less than zero, M must not be greater than L. Sometimes to ensure this condition an additional self inductance has to be inserted in the branch BC. In the original form of the method single reversal of direct current was used, with a ballistic galvanometer, and the resistance S in the condenser branch was absent. Except in the accidental case when $s = R(L - M)/M$, the method did not work with alternating current until the resistance S was introduced by Heydweiller.[2] In connecting the primary and secondary coils at B, the right direction must be found by trial.

[1] G. Carey Foster, *Phil. Mag.*, 1887, xxiii. 121.

§ (86) MUTUAL INDUCTANCE MEASURED AS SELF INDUCTANCE.—A mutual inductance can always be determined by any method for measuring self inductance (for example, by resonance at high frequencies) merely by the device of connecting the primary and secondary into one circuit. Let the self inductances of the two coils be L and N respectively and their mutual inductance M. The coils are connected in series and the total self inductance measured. Then the connections of one of them are reversed and the self inductance again measured. Let A and B be the two values found, A being the greater.

Then $$A = L + N + 2M. \quad . \quad . \quad (58)$$

and $$B = L + N - 2M. \quad . \quad . \quad (58\text{A})$$

Hence $$M = \tfrac{1}{4}(A - B). \quad . \quad . \quad (58\text{B})$$

If self-capacity is not present, B cannot be negative, and therefore 2M is never greater than L + N; but M may be either greater than L or greater than N.

VII. THE MEASUREMENT OF SELF INDUCTANCE

§ (87) CLASSIFICATION OF METHODS.—The various methods of measuring self inductance may be classified according to the chief electrical property in terms of which the unknown self inductance is determined. The determination may be in terms of:

(A) another self inductance,
(B) mutual inductance,
(C) resistance (including current and voltage), and
(D) capacitance.

The following descriptions of a number of typical methods will be set forth in the order of this classification, and some special cases will be discussed by themselves.

(A) *Comparison with Known Self Inductance*

§ (88) MAXWELL'S BRIDGE METHOD.—The comparison of two self inductances may be made by Maxwell's [3] bridge method as shown in *Fig.* 62, which refers to the most general case. The unknown self inductance L (of resistance P) is connected with a standard

[2] A. Heydweiller, *Ann. d. Physik*, 1894, liii. 499.
[3] J. C. Maxwell, *Electricity and Magnetism*, 2nd ed. ii. § 757.

inductance N (of resistance Q) in a Wheatstone's bridge with proportional arms R and S,

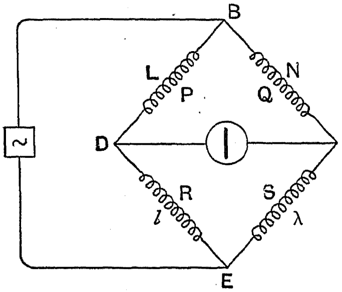


FIG. 62.

whose residual self inductances are l and λ respectively.

In the original arrangement a single reversal of direct current was used in the source, and the detector was a ballistic galvanometer, but the modern system with alternating current and a telephone or vibration galvanometer as shown in the figure is easier to use and much more sensitive. When a balance is obtained

$$\frac{P+j\omega L}{Q+j\omega N}=\frac{R+j\omega l}{S+j\omega\lambda}.$$

Hence $\quad PS-QR=\omega^2(L\lambda-Nl)$

and $\quad LS-NR=Ql-P\lambda,$

or $$\frac{P}{Q}=\frac{R}{S}+\omega^2\left(\frac{L\lambda-Nl}{QS}\right) \quad . \quad . \quad (59)$$

and $$\frac{L}{N}=\frac{R}{S}+\frac{Ql-P\lambda}{SN}. \quad . \quad . \quad (60)$$

In practice l and λ are usually very small. When they can be neglected, we have

$$P/Q=R/S \quad . \quad . \quad . \quad (61)$$

and $$L/N=R/S, \quad . \quad . \quad . \quad (62)$$

which give the effective resistance P and the self inductance L in terms of Q, L, and R/S.

In the most accurate work, since l and λ cannot be neglected, it is best, if possible, to adjust the ratio coils R and S by the addition of a small self inductance to one of them so as to make

$$l/\lambda=R/S. \quad . \quad . \quad . \quad (63)$$

Combining this condition with (59) and (60) and eliminating l and λ we obtain

$$-(PS-QR)^2=\omega^2(LS-NR)^2. \quad . \quad (64)$$

Hence $\quad PS-QR=0=LS-NR,$

or $\quad P/Q=R/S=L/N,$

and the balance is independent of the frequency.

Condition (63) is particularly important when the ratio arms have to be unequal, as, for example, in stepping up from a 10-millihenry coil to 100 millihenries.

In nearly all bridge measurements, however, the best conditions exist when the ratio arms R and S are approximately equal. Their self inductances should also be nearly equal. To ensure this, their positions in the bridge should be interchangeable, and repeated adjustments should be made until the interchange does not alter the balance. (Heaviside's system may be used, in which the wires forming R and S are first twisted together, then doubled back to form a loop and wound on to a single bobbin.) In any case each reading should be repeated with the ratio arms interchanged and a mean taken.

§ (89). For measurements of the highest precision Giebe[1] used the arrangement shown in *Fig.* 63 (with the source and galvanometer omitted). The ratio arms R and S each consisted of a bifilar loop of two bare manganin wires stretched parallel to one another,

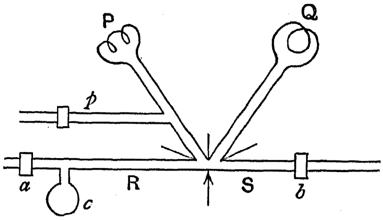


FIG. 63.

with short-circuiting sliders a and b, by which the resistances could be set. The self inductances of these loops were small and were calculated from their dimensions.

A small circular loop, also of calculable inductance, could be inserted at c, so as to make the inductances of the loops proportional to their resistances. An adjustable resistance p in the L arm was also a bare bifilar. Between R and S was a very short slide wire for fine adjustment. After each balance with alternating current the ratio R/S was accurately determined by switching over to a direct current bridge.

§ (90) CAPACITY EFFECTS.—In the ordinary use of the Maxwell bridge Q (*Fig.* 62) consists of a self inductometer, and a non-inductive resistance box and rheostat are inserted in either the P or the Q arm. The balance is obtained by alternate adjustments of the inductometer and the added resistance. Repeated adjustments are necessary, since the two different adjustments are not independent in their effects. The range of the inductometer should include the value of the unknown L, so that equal ratio arms may be used. To minimise disturbing capacity effects, the detecting instrument should *always* be connected across the ratio arms (as in *Fig.* 62).

Especially at the higher audio frequencies the capacities of the coils to earth may introduce error. To protect against this

[1] E. Giebe, *Zeits. Instrumentenk.*, 1911, xxxi. 6.

Giebe put screens round the coils L and N (*Fig.* 62) and connected the screens to the point B, while the point E was earthed. A screen enclosing the telephone was connected to the point D.

A self inductance less than the minimum reading of the inductometer can be approximately determined by finding the change of reading produced when it is added (in series) to a larger coil already balanced.

Giebe measures small self inductances accurately by obtaining a balance first, and then introducing the small unknown inductance x into the S ratio arm. If ΔP be the change in P necessary to restore balance, then

$$(x+\lambda)-\frac{N}{L}l=\frac{S\Delta P}{\omega^2 L}. \quad . \quad . \quad (65)$$

As λ and l are known, x can be determined.

§ (91) Secohmmeter Methods.—Before the introduction of vibration galvanometers Brillouin [1] in the comparison of mutual inductances made a great improvement in the sensitivity and accuracy of the measurements by using a double rotating commutator in the battery and galvanometer circuits. Some years later Ayrton and Perry,[2] using the same system, brought out an instrument which they named a Secohmmeter, and they showed how it could be applied in a number of ways to the measurement of inductance and capacity. The hand-driven form of this is illustrated in *Fig.* 64; motor-driven types are also in use.

It consists of two rotary reversing commutators mounted on the same spindle, which

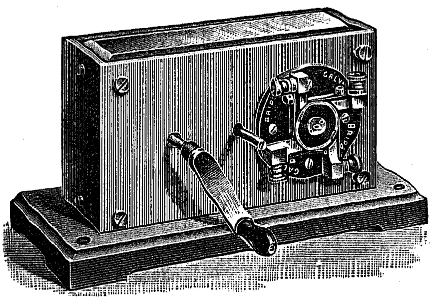


Fig. 64.—Ayrton and Perry's Secohmmeter.

can be driven at various speeds (giving from 3 up to 100 reversals per second). Each commutator has four fixed brushes, and one set of brushes can be set relatively to the other so that both series of reversals can be made at the same moment or one before or after the other. The commutators are connected with a Maxwell bridge as shown in *Fig.* 65, but the system is applicable to many other null methods. It will be seen from the figure that the battery with its commutator is equivalent to a source of alternating current,

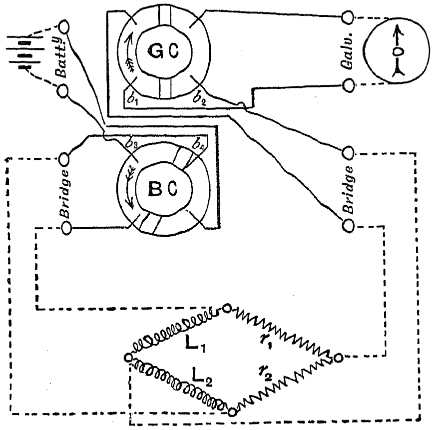


Fig. 65.

while the other commutator rectifies the alternating voltage in the galvanometer circuit and allows a direct-current galvanometer to be used. The speed must not be so great that the currents do not reach their steady states between consecutive reversals of the battery.

§ (92) Synchronous Reversing Key.—Instead of using a battery and reversing commutator as in the secohmmeter, ordinary alternating current may be used as the source, the galvanometer connections being periodically reversed at the right instants by a commutator driven on the alternator shaft or by a synchronous motor run on the circuit. Sharp and Crawford,[3] finding difficulties with the brush contacts of a rotary commutator, introduced a rectifier consisting of a reversing key with platinum contacts worked at synchronous speed by means of a synchronous motor and a cam. By adjusting the angular position of the contacts with respect to the poles of the motor, the reversal can be made to occur at any desired phase of the current. By properly setting the position the galvanometer may be made to respond to any given component of the current while it is insensitive to the component in quadrature with it. The double adjustments necessary in most bridge methods are made more easily by setting the rectifier contacts for sensitivity either to the resistance or reactance components. A detecting circuit formed thus of a synchronous commutator and a direct-current galvanometer is applicable to nearly all the null methods

[1] M. Brillouin, *Thèses présentées à la Faculté des Sciences de Paris*, 1882.

[2] W. E. Ayrton and J. Perry, *J. Inst. El. Eng.*, 1887, p. 292. See also S. R. Milner, *Phil. Mag.*, 1906, xii. 297.

[3] C. H. Sharp and W. W. Crawford, *Am.I.E.E. Proc.*, 1910, xxix. 1207.

used in testing inductances and capacities. It may be remarked, however, that it should be used with caution when harmonics are present in the wave form, for it is not a selective device like a vibration galvanometer.

§ (93) Electromagnet Galvanometer as Detecting Instrument.—The various iron-cored electrodynamometers already discussed can be used as detecting instruments for many inductance measurements. For the Maxwell bridge the connections are as in *Fig.* 66.

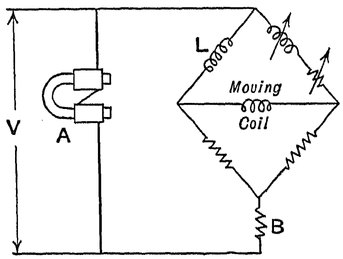


Fig. 66.

The field magnet A is connected directly across the supply voltage V, while the bridge is connected in, as shown, with a resistance B in series, sufficiently large to bring the current entering the bridge approximately into phase with V. As the field of the magnet is in quadrature with V, the moving coil will be particularly sensitive to reactance components of current—a condition favourable for the measurement of L. For measuring effective resistance, on the other hand, the magnet field and the bridge current should rather be nearly in phase, and in using the instrument for any particular null method, it should always be connected up with due regard to the quantity that is to be measured.

§ (94) Comparison of Self Inductances by Differential Transformer.—The differential transformer method of comparing self inductances is really a development of

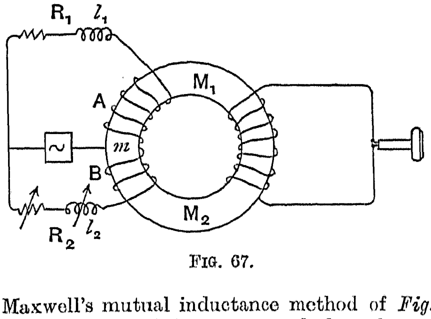


Fig. 67.

Maxwell's mutual inductance method of *Fig.* 56, Case (2). If the source and the galvanometer be interchanged and the two secondaries united in a single coil, we get the connections as in *Fig.* 67, in which also the three coils are shown wound on an iron ring core. The figure is a diagrammatic representation of a differential transformer. In practice, however, the three coils are each wound with uniform spacing over the whole of the iron ring. Two cases arise according as the coils A and B have unequal or equal numbers of turns.

Case 1. Let the turns of A and B be unequal, which is the system used by Hausrath.[1] If the respective mutual inductances are M_1, M_2 and m as in the figure, then, for a balance,

$$\frac{L_1 \pm m}{L_2 \pm m} = \frac{R_1}{R_2} = \frac{M_1}{M_2}, \quad . \quad . \quad (66)$$

where L_1, L_2, R_1 and R_2 refer to the total circuits of A and B. To measure a self-inductance l_1 in terms of a known variable l_2, a balance is obtained without l_1 and l_2 (*Fig.* 67) by adjusting R_1, R_2, L_1 and L_2. Then l_1 and l_2 are inserted as in the figure, and the balance restored by adjusting the variable l_2 and R_1 or R_2. Then

$$\frac{L_1 + l_1 \pm m}{L_2 + l_2 \pm m} = \frac{M_1}{M_2} = \frac{L_1 \pm m}{L_2 \pm m},$$

and hence

$$l_1/l_2 = M_1/M_2 = q \text{ (say)}. \quad . \quad . \quad (67)$$

Now q is equal to the ratio of the numbers of turns of the coils A and B, and so by proper choice of q a very small inductance can be measured in terms of a much larger standard or *vice versâ*. This arrangement gives a method of stepping up or down without the use of resistance ratio arms.

Case 2. When the coils A and B have equal numbers of turns, the system is much simpler. It has been used in the differential transformers of various experimenters. Elsas[2] and others used equal coil inductors for the measurement of electrolytic resistance, and Trowbridge[3] used a similar transformer for measuring self inductances and capacities. In this transformer the equality of the coils A and B was ensured by twisting the two wires together before winding them; with care in winding their self inductances were equal to within 1 part in 20,000. In this case when m is not zero, but $M_2 = M_1$, the conditions become independent of m, and we have

$$M_1/M_2 = R_1/R_2 = L_1/L_2 = 1. \quad . \quad (68)$$

The procedure for measuring l_1 against l_2 is the same as in Case 1, but l now must be within the range of l_2.

§ (95) Differential Transformer for Radio Frequencies.—On the same system Hund[4] used an equal-coil differential trans-

[1] H. Hausrath, *Die Untersuchung elektrischer Systeme*, J. Springer, 1907.
[2] A. Elsas, *Wied. Ann.*, 1888, xxxv. 828, and 1891, xlii. 165.
[3] A. Trowbridge, *Phys. Rev.*, 1905, xx. 56.
[4] A. Hund, *El. World*, May 22, 1915.

former (without an iron core) for measuring the effective resistances of coils at radio frequencies. Kennelly and Affel[1] employed his method in their tests of the skin effect in conductors. The arrangement of the connections as used by them is given in *Fig.* 68.

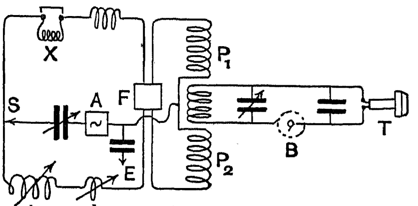


FIG. 68.

The equal primary coils are P_1 and P_2, and their connections can be interchanged by the reversing switch F. The source of current is a radio frequency alternator A, and the detecting instrument a telephone T. As the frequencies used were above the limits of hearing, a rotary interrupter B running at 1000 ~ per sec. is put in series with the telephone and, by the periodic breaking of the high-frequency current, gives audibility in the telephone. The coil under test is at X, and the balance is obtained by adjustments of L, l and the slide wire S.

§ (96) DIFFERENTIAL TELEPHONE. — An earlier method involving much the same principle is that of the differential telephone, which was introduced by Chrystal.[2] The working procedure is the same as with the equal-coil transformer.

(B) *Self Inductance determined in Terms of Mutual Inductance*[3]

§ (97) HISTORICAL INTRODUCTION.—In a Wheatstone bridge there are six circuits. When used with alternating currents if mutual inductance be introduced between two or more of these circuits, it is possible in certain cases to obtain a balance by which self inductance can be compared with mutual. For example, Maxwell[4] gave two bridge methods of making this comparison. In 1886 Hughes[5] introduced another bridge method, the correct theory of which was pointed out by H. F. Weber, Rayleigh and Heaviside.[6] Then Heaviside[7] discussed the problem thoroughly, examining all the cases in which mutual inductance is introduced between any pair of the circuits, and he pointed out the advantages of several of the combinations. (In more recent times other experimenters, unaware of his work, have rediscovered several of these methods.) The best of the methods is the following:

§ (98) HEAVISIDE MUTUAL INDUCTANCE BRIDGE.—In the Heaviside bridge the mutual inductance M is introduced between the alternator circuit and one of the bridge arms as shown in *Fig.* 69, which is for the general

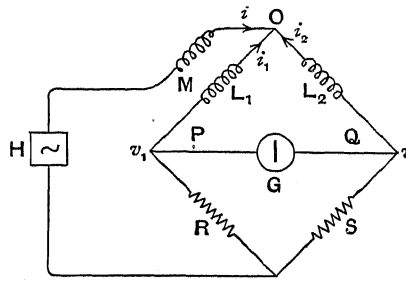


FIG. 69.

case (R not necessarily equal to S). The galvanometer circuit might have been used instead of the alternator branch, as in networks of this kind the alternator and galvanometer are interchangeable. Let the resistances of the bridge arms be P, Q, R and S, L_1 and L_2 being the self inductances of P and Q. Let the instantaneous potentials of the upper three corners be v_1, o and v_2 respectively, and the instantaneous values of the currents into the upper corner be i_1, i_2 and i as in the figure. When the vibration galvanometer shows a balance, the current through it is zero at every instant,

and hence $$v_1 = v_2.$$

Also $$i = -i_1 - i_2.$$

Hence
$$(P + j\omega L_1)i_1 - j\omega Mi = (Q + j\omega L_2)i_2,$$

and therefore
$$[P + j\omega(L_1 + M)]i_1 = [Q + j\omega(L_2 - M)]i_2 ;$$

also $$Ri_1 = Si_2.$$

Hence
$$S[P + j\omega(L_1 + M_2)] = R[Q + j\omega(L_2 - M)].$$

Separating the real and imaginary parts, we find the two conditions necessary for balance to be

$$SP = RQ, \quad . \quad . \quad . \quad (69)$$

and $$S(L_1 + M) = R(L_2 - M).^8 \quad . \quad . \quad (70)$$

[1] A. E. Kennelly and H. A. Affel, *Radio Engs. Proc.*, 1916, iv. 523.
[2] G. Chrystal, *Roy. Soc. Edin. Trans.*, 1880, xxix. 609.
[3] See also "Electrical Capacity and its Measurement," §§ (56)-(62).
[4] J. C. Maxwell, *Electricity and Magnetism*, 2nd ed. ii. § 756.
[5] D. E. Hughes, *J. Inst. El. Engs.*, 1886, xv. 6.
[6] H. F. Weber, *El. Review*, 1886, xviii. 321, and 1886, xix. 30; Lord Rayleigh, *Phil. Mag.*, 1886, xxii. 469; O. Heaviside, *Electrician*, 1885-86, xvi. 489.
[7] O. Heaviside, *Phil. Mag.*, 1887, xxiii. 173.

[8] If $L_2 = 0$ we have Maxwell's method of comparing the M between two coils with the L of one of them, and equation (70) becomes $L/M = -(1 + R/S)$. The method requires unequal ratio arms.

If the connections of the inducing coil be reversed, the second condition for balance becomes

$$S(L_1 - M) = R(L_2 + M). \quad . \quad . \quad (71)$$

It will be noticed that the balance is independent of the frequency. The most useful case is when the ratio arms are made equal, *i.e.* $S = R$.

Then (69) and (70) become

$$P = Q, \quad . \quad . \quad . \quad (72)$$

and $$L_2 - L_1 = 2M. \quad . \quad . \quad . \quad (73)$$

§ (99) Campbell[1] has shown how the method can be used, with the help of a mutual inductometer, for the measurement of a wide range of self inductances, errors due to leads and connections being eliminated by differential reading, which is of great importance for small self inductances.

The simplest arrangement is given in *Fig.* 70.

The ratio arms are equal (R, R). The arm AB consists of the secondary coil *a* of the inductometer in series with *r* which is a non-inductive resistance box with a constant inductance rheostat for fine adjustment. In the arm AC there is a " balancing coil " *b*

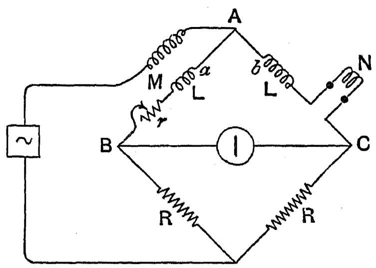


FIG. 70.

having the same self inductance as coil *a* and slightly greater in resistance; the self inductance N to be measured is also in this arm.

The coil N is first short-circuited across its terminals and a balance obtained by adjusting *r* and M. Let the readings be r_0 and M_0, where M_0 is almost zero (being due to the leads of N, etc.). Then the short-circuit is removed from N and a new balance obtained with readings r_1 and M_1. Then by (72) and (73)

$$N = 2(M_1 - M_0); \quad . \quad . \quad . \quad (74)$$

and the effective resistance $= r_1 - r_0$.

In this way any self inductance can be read directly, and the range of values that can be measured runs from 0 up to twice the maximum reading of the inductometer. For example, with a long range 10-millihenry inductometer self inductances from 0·2 microhenry up to 20,000 microhenries can be directly determined.

In the arrangement shown in *Fig.* 70 the addition of the balancing coil lowers the sensitivity.

The more modern inductometers are designed so as to avoid this by the arrangement shown in *Fig.* 71. Here the inducing circuit acts on both the P and Q

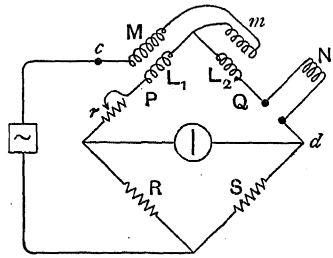


FIG. 71.

arms, L_1 and L_2 being the upper and lower fixed coils in the inductometer.

Case 1.—When $R = S$, the conditions for balance are

$$P = Q$$

and $$L_2 - L_1 + N = 2(m + M).$$

It is best to have L_2 set permanently equal to L_1 in the inductometer. Then, since $(m + M)$ is the reading of the instrument,

$$N = 2 \times \text{Reading}.$$

The balance in an equal arm bridge is independent of the mutual inductance between the P and Q arms.

Case 2.—When N is beyond the range of the inductometer, unequal ratio arms must be used. Let $R = \sigma S$, and let y be the mutual inductance between the coils L_1 and L_2. Then it can be shown that, for a balance,

$$P = \sigma Q \quad . \quad . \quad . \quad . \quad (75)$$

and $$\sigma(L_2 + N) - L_1 = (m + M)(\sigma + 1) + y(\sigma - 1). \quad (76)$$

Thus with unequal arms the balance depends on y. It is possible, but not easy, to arrange by a tapping from one of the inductometer coils to make $(\sigma L_2 - L_1)$ permanently equal to $y(\sigma - 1)$. It is better, however, to arrange the bridge as in *Fig.* 72, in which the unknown N is introduced at the left-hand side and the inducing coil of M reversed. The balancing coil L_2 is made equal to L_1/σ.

For a balance, by equation (71)

$$N = (\sigma + 1)M + \sigma L_2 - L_1$$

or $$N = (\sigma + 1)M. \quad . \quad . \quad . \quad (77)$$

The effective resistance of (N) is equal to the change in r required when the coil is inserted in the bridge arm.

[1] A. Campbell, *Phys. Soc. Proc.*, 1908, xxi. 69, and 1910, xxii. 207; also *Phil. Mag.*, 1908, xlix. 155, and 1910, xlvi. 497.

If the ratio R/S is given values of $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, then N will equal 10 M, 100 M, 1000 M, and so on,

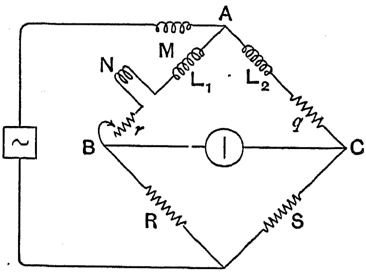


FIG. 72.

assuming that the M reading is zero when the bridge is in balance before (N) is inserted.

§ (100) EFFECT OF RESIDUAL INDUCTANCE IN THE RATIO ARMS.—Although nominally non-inductive, the ratio arms in general have appreciable residual self inductance. In measurements of high precision the effect of this must be taken into account or errors will come in, particularly at the higher frequencies. In *Fig.* 73 let the four arms have resistances P, Q,

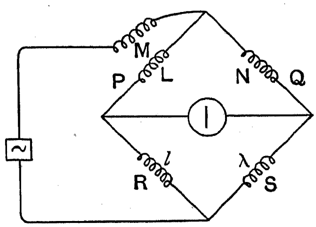


FIG. 73.

R, and S, and self inductances L, N, l, and λ respectively. It can be shown that the conditions for a balance are

$$PS-QR=\omega^2[(L-M)\lambda-(N+M)l] \quad . \quad (78)$$

and $$SL-RN=(S+R)M-P\lambda+Ql. \quad . \quad . \quad (79)$$

When M=0 these reduce to equations (59) and (60), given first by Giebe for the Maxwell bridge.

Case 1. *R not equal to S.*—If l and λ are known, equations (78) and (79) can be used to give accurate results. It will be found that in general the effective resistance result is much more affected than the self inductance by the presence of l and λ. Owing to the factor ω^2 in equation (78) the resistance effect increases rapidly as the frequency is raised.

Note.—The occurrence of ω in any equation giving a condition for balance indicates that the balance is dependent on the frequency, being different for different frequencies. If the frequency is not steady, this condition gives trouble; if the source has not a pure sine wave form, it makes the use of a telephone as detector difficult, for when the fundamental is balanced the harmonics are not.

With unequal arms, however, the best plan is to construct them so that their self inductances are proportional to their resistances. Then $l/\lambda=R/S=\sigma$ (say), and the equations reduce to

$$S(P-\sigma Q)=\omega^2\lambda[L-\sigma N-M(1+\sigma)]$$

and $$S(L-\sigma N)=SM(1+\sigma)-\lambda(P-\sigma Q).$$

Hence

$$S^2[L-\sigma N-(1+\sigma)M]=-\omega^2\lambda^2[L-\sigma N-(1+\sigma)M].$$

Now S^2 cannot be equal to $-\omega^2\lambda^2$, and therefore

$$L-\sigma N=(1+\sigma)M, \quad . \quad . \quad . \quad (80)$$

and hence $$P/Q=R/S. \quad . \quad . \quad . \quad . \quad (81)$$

So when the time constants of R and S are equal the corrections disappear, and the balance is independent of frequency.

Case 2. *Equal Arms.*—When R=S, but $l\neq\lambda$, we can find l and λ by the following method. After obtaining a balance, let R and S be interchanged and new balances found by altering (1) only P and M, and (2) only Q and M. If the changes required are p and m, and q and m', then

$$l=R\cdot\frac{2\omega^2(m-m')^2-q(p+q)}{\omega^2(m-m')(p-q)}, \quad . \quad . \quad (82)$$

and $$\lambda=R\cdot\frac{-2\omega^2(m-m')^2+q(p+q)}{\omega^2(m-m')(p-q)}, \quad . \quad (83)$$

The equations for balance are

$$(P-Q)R=\omega^2[(L-M)\lambda-(N+M)l]$$

and $$(L-N-2M)R=Ql-P\lambda,$$

showing that the balance is not independent of frequency. So, as in the case of the Maxwell bridge, it is best to make $\lambda=l$ once for all, and then the corrections vanish.

The coil under test should be placed at some little distance from the bridge and it should be set up with its mean plane passing approximately through the axis of the inducing coil of the inductometer. Each test should be repeated with the connections reversed at the terminals of the unknown coil, and also with the connections to the source reversed.

§ (101) HUGHES'S METHOD OF COMPARING A SELF INDUCTANCE WITH A MUTUAL.—*Fig.* 74 gives the connections for the Hughes method

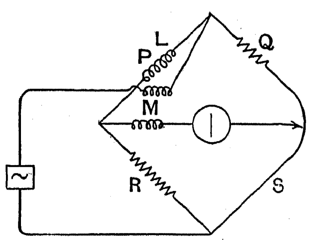


FIG. 74.

of comparing L with M. The mutual inductance is here between the alternator branch and the galvanometer circuit, while L is in

the arm P, the other arms being non-inductive. The conditions for a balance are

$$QR - SP = \omega^2 ML . \quad . \quad . \quad (84)$$

and $$M(P+Q+R+S) = SL. \quad . \quad . \quad (85)$$

Since ω^2, L, and $(P+Q+R+S)$ are all positive, M must also be positive, and hence QR must always be greater than SP. Also from (85) L must be greater than M. Equation (85) gives L in terms of M and all the resistances.

More usually, however, L and P are the unknown quantities. In that case

$$P = \frac{SQR - \omega^2 M^2 (Q+R+S)}{S^2 + \omega^2 M^2}, \quad (86)$$

and L can be got by substituting this value in equation (85).

It is evident from equation (84) that the balance is essentially dependent on the frequency. As Campbell[1] has shown, the method can be readily adapted to the measurement of frequency. For this purpose L is kept constant, while M is made variable. The total $(P+Q+R+S)$ is kept constant, and also P, R, and $(Q+S)$. S consists of a slide wire which the slider w divides in any desired ratio between S and Q.

Let $$P+Q+R+S = a$$

and $$Q+S = b.$$

Then $$\omega^2 = \frac{a}{L^2}\left[\frac{bR}{S} - (P+R)\right]. \quad (87)$$

Accordingly the frequency n can be expressed in terms of S and constants, and the slide wire can be graduated to read n directly. Double adjustment, however, is required, M being altered as well as S. The chief difficulty in this and similar methods occurs when the wave form of the current is not pure.

The following example gives values that have been found convenient in practice. With L=0·1 henry, P=25 ohms, R=5 ohms, Q+S=4 ohms, for a range of frequency from 10 up to 120 ~ per second, S varies from about 0·60 down to 0·10 ohm, and M runs from about 1·7 down to 0·28 millihenry.

§ (102) Butterworth's Method.—Butterworth[2] has shown that another of Heaviside's methods is convenient for frequency measurement. It is illustrated in *Fig.* 75, and the conditions for balance are

$$QR - PS = \omega^2(M - L)N \quad . \quad . \quad (88)$$

and $$LS + PN = M(S+R). \quad . \quad . \quad (89)$$

If L=N,

$$QR - PS = \omega^2(M - L)L \quad . \quad . \quad (90)$$

and $$L(S+P) = M(S+R). \quad . \quad . \quad (91)$$

[1] A. Campbell, *Phys. Soc. Proc.*, 1907, xx. 626, and *Phil. Mag.*, 1907, xliii. 494.
[2] S. Butterworth, *Phys. Soc. Proc.*, 1912, xxiv. 86.

Now, if L, N, M, P, R, and S are all kept constant and satisfy equation (91), a value of Q can always be found to satisfy equation (90), whatever the value of ω may be. Thus

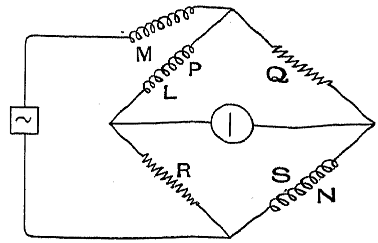


Fig. 75.

a balance can always be got at any particular frequency by adjustment of the single variable Q, and a table can be drawn up giving n in terms of Q. When $n=0$, $Q=PS/R$, and when $n=\infty$, $Q=\infty$. Clearly M must be greater than L.

Very similar to this method is one used by Campbell, in which the arm R is inductive instead of S (*i.e.* resistances P, Q, R, S, and self inductances L, 0, X, and 0 respectively). For this case

$$PS - QR = \omega^2 MX \quad . \quad . \quad . \quad (91\text{A})$$

and $$XQ - LS = M(R+S). \quad . \quad . \quad (91\text{B})$$

If X=L,

$$PS - QR = \omega^2 ML$$

and $$(Q-S)L = M(R+S).$$

(C) *Self Inductance determined in Terms of Resistance*

§ (103) Impedance Methods.—If a current I of sine wave form and frequency n is sent through a coil whose self inductance L is to be determined, the potential difference V across its terminals can be measured by an electrostatic voltmeter, while the current is measured by an accurate ammeter (*e.g.* a Kelvin balance). If R is the resistance of the coil, then

Impedance $Z = V/I = \sqrt{R^2 + \omega^2 L^2}$,

and therefore

$$L = \sqrt{(Z^2 - R^2)}/\omega$$

$$= \frac{1}{2\pi n}\sqrt{\left\{\frac{V^2}{I^2} - R^2\right\}}. \quad . \quad (92)$$

The resistance R can be measured with direct current and an ordinary bridge, or by ammeter and voltmeter and direct current. It should be determined between two readings with the alternating current. The frequency may be read on a frequency meter or deduced from the speed of the alternator.

The impedance method was first proposed

by Joubert and has been improved by later experimenters. As carried out by Gray,[1] an alternating current is sent through the coil in series with a known non-inductive resistance r, and by means of an electrometer the terminal voltages of each are measured. Let these be V and U.

Then $$L=\frac{R}{\omega}\left(\frac{r^2V^2}{R^2U^2}-1\right)^{\frac{1}{2}}. \qquad (93)$$

In Fleming's [2] method an alternating current of frequency n is first passed through the coil and the terminal voltage V measured on an electrostatic voltmeter. Then direct current of the same amount is passed and the voltage U observed. If $\omega=2\pi n$, we have

$$L=\frac{R}{\omega}\sqrt{\frac{V^2-U^2}{U^2}}. \qquad (94)$$

Rosa and Grover [3] made the method still more accurate by using a series resistance as Gray did, but adjusting it also until the electrometer showed the same potential difference on this resistance and on the coil. This gives

$$r^2=\sqrt{R^2+\omega^2L^2}$$

or $$L=\frac{1}{\omega}\sqrt{r^2-R^2}, \qquad (95)$$

assuming the current to have an exact sine wave form. They showed that for a wave form containing harmonics this value of L must be multiplied by a correcting factor f, where

$$f=\sqrt{\frac{I_1^2+I_2^2+I_3^2+\ \ldots}{I_1^2+9I_2^2+25I_3^2+\ \ldots}}, \qquad (96)$$

and the current I consists of harmonic components I_1, I_3, I_5 . . . of relative frequencies 1, 3, 5 . . ., so that

$$I^2=I_1^2+I_3^2+I_5^2+\ \ldots \qquad (97)$$

To apply this correction, the wave form has to be determined and analysed. From equation (96) it will be seen that the higher harmonics have a much greater effect on the correction factor than the lower harmonics have.

§ (104) M. Wien's Bridge Method.—The connections for Wien's [4] method are shown in *Fig.* 76. Two circuits having self inductances L_1 and L_2 are placed in adjacent arms of the bridge. L_1 (of resistance P) is shunted by a non-inductive resistance S, while in series with L_2, which is variable, there is a rheostat q giving a total resistance Q in that arm. The arms R and R_2 are non-inductive. In the original experiments the detecting instrument was a resonance optical telephone, which in modern work is replaced by a vibration galvanometer. By varying L_2, Q, and the ratio R_1/R_2, a balance is found, then

$$L_1L_2=\frac{S+P}{\omega^2}\left(Q-\frac{\sigma SP}{S+P}\right). \qquad (98)$$

and $$\frac{L_1}{L_2}=\frac{S+P}{\sigma S-Q}, \qquad (99)$$

where $\sigma=R/S$.

Hence

$$L_1=\frac{S+P}{\omega}\sqrt{\left(Q-\frac{\sigma SP}{S+P}\right)\Big/(\sigma S-Q)} \qquad (100)$$

and $$L_2=\frac{1}{\omega}\sqrt{\left(Q-\frac{\sigma SP}{S+P}\right)(\sigma S-Q)}. \qquad (101)$$

Thus each of the self inductances is measured in terms of resistances and frequency. The evaluation of L_1 and L_2 by these formulas is made easier by switching over to direct current after the balance with alternating current has been made. In this way $\sigma SP/(S+P)$ can be directly determined by altering q to restore the balance. Owing to the dependence on frequency it is not easy to attain such high accuracy with this method as with some others. Orlich [5] gives the maximum accuracy as about 1 part in 1000.

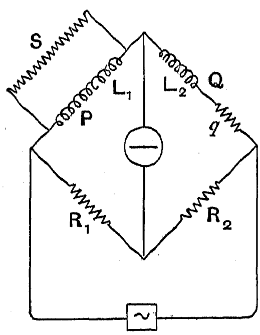


Fig. 76.

§ (105) Maxwell's Ballistic Galvanometer Method (Self Inductance in Terms of Resistance).—A self inductance can be determined in terms of resistance with the help of a ballistic galvanometer by the method due to Maxwell,[6] which was used by Rayleigh [7] in his determination of the B.A. unit of resistance in absolute measure, in which it was necessary to determine L as a correction to the main observed quantity. The coil of self inductance L to be measured is connected, as shown in *Fig.* 77, with three non-inductive resistances Q, R, and S, to form an ordinary Wheatstone bridge with a ballistic galvanometer. The bridge is first balanced for steady currents, so that on closing the battery key b first, and shortly afterwards the galvanometer key a, no deflection of the galvanometer occurs. Then a is closed first, and the throw θ caused

[1] See A. Gray, *Absolute Measurements in Elec. and Mag.*, ii. Part 2, 487.
[2] J. A. Fleming, *Handbook for the Elec. Laboratory and Test Room*, ii. 205.
[3] E. B. Rosa and F. W. Grover, *Bureau of Standards Bulletin*, 1905, i. 125.
[4] M. Wien, *Wied. Ann.*, 1891, xlii. 681.
[5] E. Orlich, *Kapazität u. Induktivität*, 1909, p. 245.
[6] J. C. Maxwell, *Roy. Soc. Phil. Trans.*, 1865, clv., or Maxwell's *Collected Papers*, i. 547.
[7] Lord Rayleigh, *Roy. Soc. Phil. Trans.*, 1882, Part 2.

by closing key b is observed. The bridge is now thrown out of balance for steady currents by altering the resistance of the arm Q to $Q+q$. Let this change produce a steady deflection a in the galvanometer when both

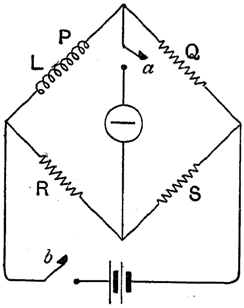


FIG. 77.

keys are kept closed. From a knowledge of the resistances of all the six branches of the network the ratio of the currents now flowing in the P and Q arms respectively can be calculated If this ratio is h, then

$$L=qh.\frac{T}{\pi}.\frac{\sin\frac{1}{2}\theta}{\tan a}\left(1+\frac{x}{2}\right), \quad . \quad (102)$$

where T is the complete periodic time of oscillation of the galvanometer and x the logarithmic decrement for half period. (If q is in ohms, L will be in henries.)

(D) *Determination of Self Inductance in Terms of Capacitance* [1]

§ (106) MAXWELL'S METHOD.—Maxwell's [2] bridge for determining a self inductance L in terms of a condenser K (and resistances) is shown in *Fig.* 78.

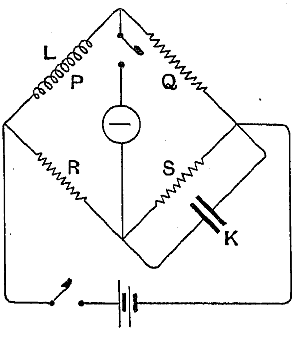


FIG. 78.

The condition for balance when the currents are steady is

$$SP=QR, \quad . \quad . \quad . \quad (103)$$

[1] See also "Electrical Capacity and its Measurement," § (63).

[2] J. C. Maxwell, *Electricity and Magnetism*, 2nd ed. ii. § 778.

and the additional condition for no throw of the galvanometer on making or breaking the battery circuit is

$$L/P=SK, \quad [L \text{ henries, } K \text{ farads}]. \quad (104)$$

Accordingly the balance is independent of frequency. To carry out the test, the resistances should first be set so as to give the steady-current balance, and then the value of K adjusted until the throw at make or break is reduced to zero. This can only be done with accuracy when a very highly subdivided condenser is available. The difficulty may be got over by finding two values of K which give small throws on opposite sides of the galvanometer zero point and nding by interpolation the value of K for which the throw would be zero.

The bridge can equally well be used with alternating currents, the two conditions for balance being still the same. If only a fixed condenser is available, troublesome adjustments of the resistances have to be made.

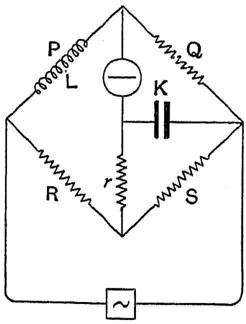


FIG. 79.

To avoid this, the method has been successfully developed into other methods by various experimenters.[3] The most widely used of these methods is Anderson's.

§ (107) ANDERSON'S METHOD.—In Anderson's method the condenser is not directly across one of the bridge arms, but is connected as shown in *Fig.* 79. In the original practice of the method single reversal of a battery was used and a ballistic galvanometer. Stroud interchanged the source and the galvanometer and used alternating current and an alternating current galvanometer.[4] Fleming and Clinton [5] used Anderson's method with a battery and secohmmeter, and afterwards Fleming [6] used an interrupted current

[3] E. C. Rimington, *Phil. Mag.*, 1887, xxiv. 54; C. Niven, *Phil. Mag.*, 1887, xxiv. 225; A. Anderson, *Phil. Mag.*, 1891, vol. xxxi. 329; Iliovici, *Comptes Rendus*, 1904, cxxxviii. 1141; S. Butterworth, *Phys. Soc. Proc.*, 1912, xxiv. 210.

[4] W. Stroud and J. H. Oates, *Phil. Mag.*, 1903, vi. 707.

[5] J. A. Fleming and W. C. Clinton, *Phys. Soc. Proc.*, 1903, xviii. 386, and *Phil. Mag.*, 1903, v. 493

[6] J. A. Fleming, *Phil. Mag.*, 1904, vii. 586.

with a telephone as detector. Rosa and Grover [1] employed alternating current and a vibration galvanometer.

The conditions for balance are

$$SP=QR \quad . \quad . \quad . \quad (105)$$

and $$L=K[r(P+Q)+RQ]. \quad . \quad . \quad (106)$$

If the bridge is balanced for steady currents by adjusting the resistance of one of the arms so that SP = QR, the balance for alternating (or transient) currents can be got by altering r without interfering with the steady current balance. The independence of these two adjustments gives the method a great advantage over the simple Maxwell bridge. As in other bridges it is best to make R and S equal and to arrange a reversing key to interchange them as Rosa and Grover did.

When R = S the conditions for balance become

$$P=Q \quad . \quad . \quad . \quad (107)$$

and $$L=KQ[2r+R]. \quad . \quad . \quad (108)$$

For an investigation of the effects of residual inductance in the bridge arms and of absorption in the condenser the reader is referred to Rosa and Grover's paper already cited.

Note.—In all the formulas here given, when K is expressed in *farads*, L will be in *henries*; when K is in *microfarads*, L is in *microhenries*.

§ (108) BUTTERWORTH'S METHOD.—This is a combination of the Anderson and Iliovici

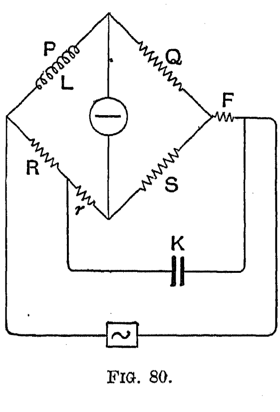


FIG. 80.

methods. The connections are as in *Fig.* 80. The conditions of balance are

$$SP=Q(R+r). \quad . \quad . \quad (109)$$

and $$L=K\frac{R}{S}[F(Q+S)+Q(S+r)]. \quad . \quad (110)$$

By a proper choice of R any self inductance can be measured with a single condenser; and by adjusting F the inductive balance can be got independently of the resistance balance. The method is particularly well adapted for the measurement of small inductances, since R can be made as small as we please, keeping $R+r$ constant, without disturbing the resistance balance.

§ (109) HAY'S METHOD.—In Hay's [2] method the condenser K in series with a resistance S is placed, as shown in *Fig.* 81, in the bridge arm opposite to L, the self inductance to be

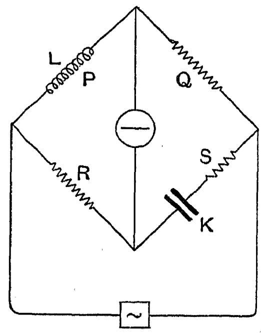


FIG. 81.

measured. A balance is got by adjusting K and S. Then

$$L=\frac{RQK}{1+\omega^2S^2K^2}. \quad . \quad . \quad (111)$$

and $$P=\frac{RQK^2S\omega^2}{1+\omega^2S^2K^2} \quad . \quad . \quad (112)$$

Hence the time constant $L/P=1/KS\omega^2$. (113)

In most practical cases $\omega^2S^2K^2$ is very small compared to 1, and hence

$$L \doteq RQK \quad . \quad . \quad . \quad (114)$$

and $$P \doteq RQKS\omega^2. \quad . \quad . \quad (115)$$

A highly subdivided condenser is used.

§ (110) DONGIER'S METHOD.—In Dongier's [3] method of determining a self inductance L in terms of a capacitance K two distinct measurements are made, in order that the result may be independent of the frequency. (a) L and K are connected in the bridge shown in *Fig.* 82, and a balance is obtained by adjusting r and the ratio arms R and S.

Then $$L=Kr^2/(1+\omega^2K^2r^2), \quad . \quad . \quad (116)$$

where $\omega=2\pi\times$ frequency.

[1] E. B. Rosa and F. W. Grover, *Bureau of Standards Bulletin*, 1905, i. 291.

[2] C. E. Hay, *Inst. of Post Office Elec. Engs. Proc.*, Nov. 1912.

[3] R. Dongier, *Comptes Rendus*, 1903, cxxxvii. 115.

(*b*) The condenser is then put in shunt across the whole arm containing L, as in

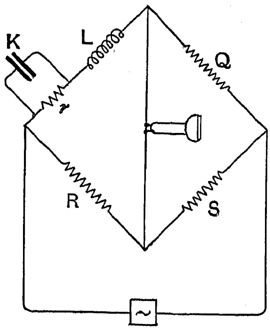


FIG. 82.

Fig. 83, and the resistance of the arm P adjusted as well as the ratio arms until a balance is again obtained.

Then $$L=K(P^2+\omega^2L^2). \quad . \quad . \quad . \quad (117)$$

If the frequency has been kept constant, ω

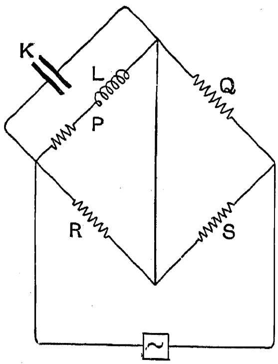


FIG. 83.

can be eliminated from (116) and (117), and then

$$L=KPr. \quad . \quad . \quad . \quad (118)$$

If the frequency is known, either of the equations (116) or (117) will give L in terms of K.

§ (111) GRÜNEISEN AND GIEBE'S METHOD.—The connections for Grüneisen and Giebe's[1] method are shown in *Fig.* 84. The inductive coil L and the condenser K are in series in one bridge arm, while all the other arms are non-inductive. A balance is found when

$$SP=QR \quad . \quad . \quad . \quad (119)$$

and $$\omega^2LK=1, \quad . \quad . \quad . \quad (120)$$

where the pulsatance $\omega=2\pi\times$frequency.

Equation (120) is the condition for zero reactance in the P branch for the given frequency. The method has been used at the Physikalisch-Technische Reichsanstalt for the comparison of a primary standard self inductance and an air condenser in connection

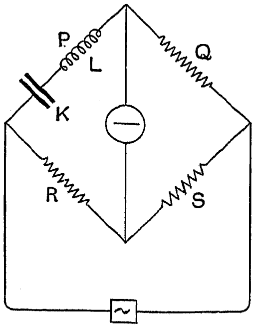


FIG. 84.

with a determination of the ohm in absolute measure. The frequency is held extremely steady by means of Giebe's speed regulator, and a vibration galvanometer is used as the detecting instrument.

§ (112) OWEN'S METHOD.—In Owen's[2] method two condensers K and C are connected in a bridge with the self inductance L, as shown in *Fig.* 85.

When the bridge is in balance the vector impedances of the arms must be in proportion,

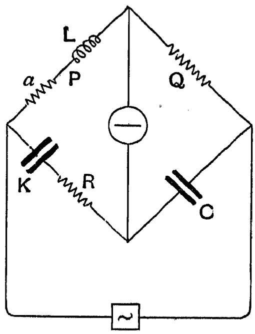


FIG. 85.

just as the resistances in an ordinary Wheatstone's bridge are.

Thus $$\frac{P+j\omega L}{Q}=\frac{R+1/j\omega K}{1/j\omega C},$$

and therefore

$$j\omega PK-\omega^2LK=j\omega QC-\omega^2QRCK.$$

Separating the real and imaginary terms, we have, as the two conditions of balance,

$$P=QC/K \quad . \quad . \quad . \quad (121)$$

and $$L=QRC. \quad . \quad . \quad . \quad (122)$$

[1] Grüneisen and E. Giebe, *Zeits Instrumentenk.*, 1910, xxx. 147.

[2] D. Owen, *Phys. Soc. Proc.*, 1915, xxvii. 39.

These equations show that the balance is most easily obtained by altering P and R alternately, for these adjustments are independent of one another.

Since L is proportional to R, and R can have any value from 0 upwards, all values of L can be measured with the same pair of condensers.

The balance is independent of frequency, unlike the more general case investigated by Rosa and Grover[1] for the measurement of the power factor of a condenser.

In Owen's paper he investigates the errors introduced by (1) the residual inductances of R, Q and the added resistance a, and (2) dielectric losses in the two condensers (which may be represented as resistances in series with the capacitances). By making a second experiment with L cut out the errors can be largely eliminated. Let a in the first experiment have residual inductance l, and let its new value a_0 in the second experiment have l_0, the corresponding values of R being R and R_0. Then in most cases it is sufficiently accurate to take

$$L = QC(R - R_0) - (l - l_0). \quad . \quad . \quad (123)$$

If good mica condensers are used, it is only important to make the auxiliary test when L is very small. The method works well with condensers of about $0{\cdot}3\mu F$ each.

§ (113) Resonance Methods.—If a constant alternating voltage is induced in a closed circuit having self inductance and capacitance, the resulting current will vary inversely as the impedance of the circuit; from observations of this current as the capacitance is varied,

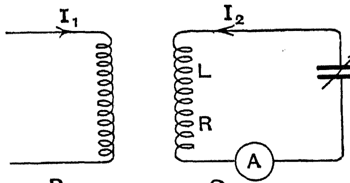


Fig. 86.

the inductance can be found in terms of the capacitance. Let the circuit P carrying a current I, of sine wave form, induce a constant total voltage E_2 in the closed circuit Q, consisting of a coil of self inductance L, a variable condenser K, and a non-inductive ammeter A. Let R be the total resistance of this secondary circuit including the ammeter, and let $\omega = 2\pi n$, where n is the frequency. Then if the induced current is I_2,

$$I_2 = E_2 / \sqrt{R^2 + (L\omega - 1/K\omega)^2}.$$

If K be continuously variable the current I_2 will be a maximum when $(L\omega - I/K\omega) = 0$, *i.e.* when

$$\omega^2 LK = 1. \quad . \quad . \quad . \quad (124)$$

[1] E. B. Rosa and F. W. Grover, *Bureau of Standards Bull.*, 1907, iii. 390.

When this occurs the circuit is said to be in *electrical resonance* for the frequency n, and L can be found in terms of K and the frequency by equation (124). The ammeter by which the maximum of I_2 is observed must be suitable for alternating current of the frequencies used, and its residual self inductance, l, should be constant and known. If l is comparable with L, then instead of equation (124) we have

$$\omega^2(L + l)K = 1. \quad . \quad . \quad (125)$$

A vibration galvanometer with a low-resistance shunt, a thermoammeter, or a heater and thermopile connected to a galvanometer are examples of the kind of ammeter that may be suitable. In order that E_2 may be constant, the mutual inductance between P and Q must be kept so small that Q does not react appreciably on P. This can be tested by increasing the distance between them and trying if the same resonance value of K is found. The method is applicable with high audio frequencies and is most valuable at radio frequencies.

At the higher frequencies the capacitance K will usually be small and the self-capacity of the coil L will not be negligible compared with K. This is the case for ordinary radio wavemeters, in which this resonance method is used for the measurement of frequency.

§ (114).—In *Fig.* 87, let the inductive coil have resistance R, self inductance L, and self capacitance represented by the condenser c

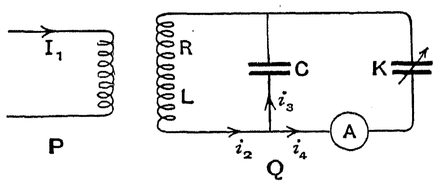


Fig. 87.

across its terminals. Let K be the external adjustable condenser, and let the ammeter A measure the current through K. Let the circuit P induce a constant total voltage E_2 in the inductive coil (L). Then if i_2, i_3, and i_4 are the instantaneous values of the currents in the branches as marked in the figure, and I_2, I_3, and I_4 their effective values respectively, we have, neglecting the resistance and inductance of the ammeter,

$$i_2 = i_3 + i_4 \text{ and } I_4 = \frac{K}{K + c} I_2,$$

and hence

$$\frac{E_2^2}{I_4^2} = \left(1 + \frac{c}{K}\right)^2 \frac{E_2^2}{I_2^2}$$

$$= \left(1 + \frac{c}{K}\right)^2 \left[R^2 + \left(L\omega - \frac{1}{(K + c)\omega}\right)^2\right]. \quad (126)$$

When I_4 is a maximum, this expression is a minimum, in which case

$$\frac{1}{K}=\frac{L(1-Lc\omega^2)-cR^2}{1/\omega^2-2cL+c^2(R^2+L^2\omega^2)},$$

or

$$L^2(K+c)c\omega^2-[L(K+2c)-R^2(K+c)c]\omega^2+1=0. \quad (127)$$

When the term in R^2 is nearly negligible the equation becomes

$$\left(\omega^2-\frac{1}{Lc}+\frac{R^2}{L^2}\cdot\frac{K+c}{K}\right)\left(\omega^2-\frac{1}{L(c+K)}-\frac{R^2}{L^2}\cdot\frac{c}{K}\right)\doteq 0.$$

Except when the first factor happens to be zero (in self-resonance),

$$\frac{1}{L(K+c)}\doteq\omega^2-\frac{R^2}{L^2}\cdot\frac{c}{K}. \quad . \quad (128)$$

Usually (for higher frequencies) the last term can be neglected, and then

$$L(K+c)\omega^2\doteq 1. \quad . \quad . \quad (128A)$$

If l, the self inductance of the ammeter, is not quite negligible, we have

$$\omega^2(L+l)(K+c)\doteq 1. \quad . \quad . \quad (129)$$

To find L and c, let resonance be obtained at another known frequency, giving

$$\omega_1^2(L+l)(K_1+c)\doteq 1.$$

Then
$$c\doteq\frac{K_1\omega_1^2-K\omega^2}{\omega^2-\omega_1^2} \quad . \quad . \quad (130)$$

and
$$L\doteq\frac{\omega^2-\omega_1^2}{\omega^2\omega_1^2(K_1-K)}. \quad . \quad . \quad (131)$$

In the above investigation it is assumed that the coil is such that skin effect is negligible compared to the effect of self capacitance.

§ (115) Measurement of Self Inductance and Self Capacity of Single-layer Coils, using Drop Chronograph.—In equations (130) and (131) it is assumed that the two frequencies can be determined accurately. At the higher frequencies this would be done by a standard wavemeter. Hubbard[1] measured the frequencies absolutely by tracing the wave form of the high-frequency current by means of a drop chronograph capable of measuring accurately very minute intervals of time. In this way he showed that for single-layer coils, whose axial length b is equal to the diameter, the effective self capacity in micromicrofarads is approximately equal to $1\cdot 11\times$(radius of coil). This value increases to $1\cdot 38\times$radius, when $b=4$ times the diameter.

§ (116) Resonance Values of L and K for various Frequencies.—As the resonance condition $\omega^2LK=1$ is often wanted in practice, it is convenient to be able to refer to a table giving for a series of given frequencies values of L and K which will give resonance. These can be, of course, only sample values, but the product LK is definite for each frequency. In the resonance condition $\omega^2LK=1$, L is in henries and K in farads. If they are expressed in henries and microfarads respectively, we have

$$n\doteq 159\cdot 16/\sqrt{LK} \quad . \quad . \quad (132)$$

and
$$LK\doteq 25300/n^2. \quad . \quad . \quad (132A)$$

In Table VII. are given approximate values of LK corresponding to a number of different frequencies, and also convenient values of L and K in each case. For the higher frequencies the corresponding wave-lengths (λ) are also given.

Table VII

Resonance Values of L and K

n	λ	LK	L	K
~ per sec.	Meters.	Henries and μF.	Henries.	μF.
10		253	6·3	40
20		63	1·6	40
40		16	1·6	10
50		10	1·0	10
80		4	0·4	10
		mH and μF.	mH.	
100		2 530	126	20
200		630	63	10
400		160	16	10
500		100	10	10
800		40	40	1
1 000	300 000	25·3	25·3	1
2 000	150 000	6·3	6·3	1
4 000	75 000	1·6	1·6	1
5 000	60 000	1·00	1·00	1
8 000	37 500	0·40	0·40	1
10 000	30 000	0·253	0·253	1
		μH and μF.	μH.	
10 000	30 000	253	2530	0·1
20 000	15 000	63	630	0·1
50 000	6 000	10	1000	0·01
100 000	3 000	2·53	253	0·01
		μH and $\mu\mu$F.		$\mu\mu$F.
200 000	1 500	630 000	630	1000
500 000	600	100 000	100	1000
1 000 000	300	25 300	25·3	1000
2 000 000	150	6 300	6·3	1000
5 000 000	60	1 000	1·00	1000
10 000 000	30	253	0·25	1000

[1] J. C. Hubbard, *Phys. Rev.*, 1917, ix. 529.

Special Cases of Self Inductance Measurement

§ (117) SELF INDUCTANCE OF LOW RESISTANCE SHUNTS.—For the determination of the self inductances of four-terminal low resistances, such as are used for shunts to carry large currents, rather special methods have to be employed. It will be sufficient to describe one or two of these methods.

Orlich's Electrometer Method.—In Orlich's[1] method the measuring instrument used is a highly sensitive electrometer, such as that of Dolezalek. *Fig.* 88 gives the connections

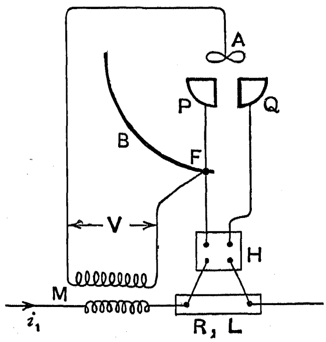


FIG. 88.

diagrammatically, two of the quadrants of the electrometer being omitted for the sake of clearness. A known mutual inductance M has one end of its secondary coil connected to the needle A and the other to one pair of quadrants P and the case B. The primary circuit is put in series with the resistance R of unknown self inductance L, and the potential terminals of R are connected to the two pairs of quadrants through the reversing switch H.

Let V be the potential of the needle, and C the electrometer constant for this potential.

Here

$$C \times \text{deflection} = (U_1 - U_2)\left(U - \frac{U_1 + U_2}{2}\right),$$

where U, U_1, and U_2 are the potential of the needle and the two sets of quadrants respectively.

Now let an alternating current I_1 (of instantaneous value i) be sent through the primary circuit and the resistance R.

Then

$$C\alpha = \frac{1}{T}\int_0^T M\frac{di}{dt}\left(Ri + L\frac{di}{dt}\right)dt$$

$$= \frac{ML}{T}\int_0^T \left(\frac{di}{dt}\right)^2 dt, \quad . \quad . \quad (133)$$

where α is the deflection on reversing the switch H, and $T = 1/\text{frequency}$.

But $$V^2 = \frac{M^2}{T}\int_0^T \left(\frac{di}{dt}\right)^2 dt. \quad . \quad . \quad (134)$$

Hence $$L = \frac{MC\alpha}{V^2}. \quad . \quad . \quad . \quad (135)$$

V, the voltage given by the secondary of the mutual inductance, can be measured on the same voltmeter used idiostatically or on a separate electrostatic voltmeter. C is found by earthing the point F, disconnecting the secondary coil of M, sending a direct current through R, and applying a known direct voltage V between F and A. If α is the deflection on reversing,

$$C\alpha = 2VRI. \quad . \quad . \quad . \quad (136)$$

§ (118).—E. Wilson and W. H. Wilson[2] used a very similar method, but connected the two pairs of quadrants to the two potential terminals, and the point F to the middle of R. They used current of sine wave form, and, by measuring I_1, eliminated M by the help of the relation $V = MI_1\omega$. If θ is the deflection with alternating current I_1,

then $$L = \frac{2C\theta}{\omega V I_1}. \quad . \quad . \quad . \quad (137)$$

§ (119) CAMPBELL'S METHODS.—For the measurement of the self inductances of low resistance shunts Campbell[3] has used two methods, both of them depending on mutual inductance. For this purpose a mutual inductometer of very low range is required, having a maximum of, say, 1 -henry. There is no difficulty in constructing this and calibrating the scale by one of the stepping-down methods. Method 1 is arranged as shown in *Fig.* 89, in which r is the resistance with

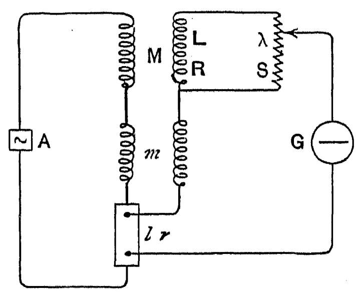


FIG. 89.

potential terminals, whose self inductance l is to be found. It is connected with a low-reading inductometer giving mutual inductance m, and a pair of coils whose mutual inductance M can be varied. The second of these coils has resistance R and self inductance

[1] E. Orlich, *Zeits. Instrumentenk.*, 1905, xxv. 114.

[2] *Electrician*, Jan. 1906, p. 464.

[3] A. Campbell, *Phys. Soc. Proc.*, 1917, xxix. 345.

L, and forms a closed circuit with a resistance S having small self inductance λ. If S is adjustable by a slide-wire, R and L must include the part above the slider. Then a balance on the vibration galvanometer can be obtained by adjusting m, S, and M, in which case we have

$$MS = r(L+\lambda)+(R+S)(l+m) \quad . \quad (138)$$

and $$(L+\lambda)(l+m)\omega^2 - \lambda M\omega^2 = r(R+S), \quad (139)$$

where ω is the pulsatance.

Let L/R be relatively large, λ/S small, and λM negligible compared with $L(l+m)$, and then equation (139) becomes

$$L(l+m)\omega^2 = r(R+S). \quad . \quad . \quad (139\text{A})$$

From this equation l can be conveniently obtained without exact knowledge of M and λ.

In this method the conditions are not easily obtained unless the frequency is fairly high (say 800 ~ per sec.), but this is quite allowable, for Bethenod and Orlich have shown that the resistance and self inductance of a well-designed shunt remain practically constant up to frequencies of this order.

Method 2.—The method illustrated in *Fig.* 90 is of more general application. The low resistance shunt to be tested is, as before, r, l, and m is a low-reading inductometer. In

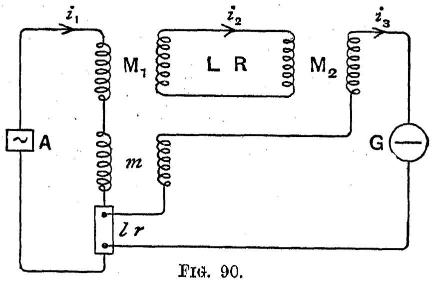


Fig. 90.

addition to this the primary circuit is linked to the galvanometer circuit by an intermediary closed circuit of resistance R and self inductance L. The coupling mutual inductances should be variable, but usually their values need not be known. There must be no direct mutual inductance between the primary circuit and that of the galvanometer. To check this condition, r is cut out and the loop (L, R) circuit is opened; then m should read zero, and if it does not, the positions of the coils should be altered until it does. This is best ensured by placing the four coils forming M_1 and M_2 at a fair distance from the inductometer and turning them so as to be conjugate to each other pair and to the inductometer coils.

Let i_1, i_2, and i_3 be the instantaneous values of the currents in the three circuits.

Then, when $i_3=0$,

$$(R+j\omega L)i_2 + j\omega M_1 i_1 = 0$$

and $$[r+j\omega(l+m)]i_1 + j\omega M_2 i_2 = 0,$$

and hence $$R(l+m)+rL=0 \quad . \quad . \quad (140)$$

and $$Rr=(l+m)L\omega^2 - M_1M_2\omega^2. \quad . \quad (141)$$

Let the signs of M_1 and m be changed, which is always allowable with mutual inductance, and let m be greater than l.

Then $$\frac{m-l}{r}=\frac{L}{R} \quad . \quad . \quad . \quad (142)$$

and $$Rr=[M_1M_2-(m-l)L]\omega^2. \quad . \quad (143)$$

Equation (142) gives l without requiring a knowledge of the values of M_1 and M_2. We also have

$$r=\frac{RM_1M_2\omega^2}{R^2+L^2\omega^2} \quad . \quad . \quad . \quad (144)$$

and $$m-l=\frac{LM_1M_2\omega^2}{R^2+L^2\omega^2}, \quad . \quad . \quad (145)$$

and when $L\omega/R$ is small these become

$$r \doteq M_1M_2\omega^2/R \text{ and } m-l \doteq LM_1M_2\omega^2/R^2. \quad (146)$$

Equation (142) shows that $(m-l)$ must always be positive. Also M_1M_2 must always be greater than L^2/R, since

$$Rr=[M_1M_2 - L^2r/R]\omega^2. \quad . \quad . \quad (147)$$

§ (120) Thomson Double-bridge Method.—Sharp and Crawford [1] used the Thomson double bridge for the comparison of self inductances of low resistance shunts. The method has also been used by Wenner,[2] to whose paper the reader is referred for further information. The detecting instrument in Sharp and Crawford's experiments was a sensitive electrodynamometer whose field coils were separately excited by current in quadrature with the bridge current.

§ (121) Measurement of Residual Self Inductances.—The measurement of very small self inductances associated with moderate or high resistance always presents considerable difficulty. Giebe's method has already been described (equation (65)), and Campbell's method with constant inductance rheostat is applicable in some cases. For the higher resistances nearly every method resolves itself into the comparison of the unknown inductance with another of similar resistance whose inductance can be calculated. Usually this latter consists of two very fine wires supported in a permanent way parallel to one another as already described. With resistances of over 1000 ohms the effect of

[1] C. H. Sharp and W. W. Crawford, *Am.I.E.E. Trans.*, 1910, xxix. 1540.

[2] F. Wenner, *Bureau of Standards Bulletin*, 1912, viii. 559.

the capacity of the leads is very disturbing, and direct substitution methods are almost essential. Capacity to earth also comes in. Its effects have been thoroughly discussed by Curtis and Grover.[1]

§ (122) Measurement of Self Inductance at High Frequencies.—Watson [2] measures small self inductances at radio frequencies by the bridge shown in *Fig.* 91, in which the

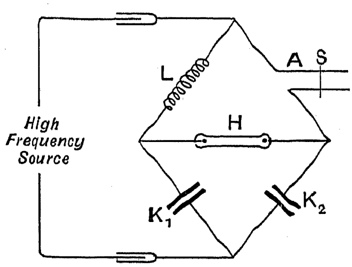


Fig. 91.

detecting instrument is a helium tube. The standard inductance is formed of a pair of parallel wires A with a short-circuiting slider S. By observing the distance S has to be moved to restore balance when unknown inductance L is introduced into the adjacent arm, L can be determined from the calculated value of the equivalent portion of A.

Taylor [3] had made use of this system of parallel wires for measuring small self inductances by a resonance method of simple substitution.

§ (123) Measurement of Inductances with Iron Cores.—When inductive circuits have iron or other magnetic material in their magnetic fields the measurements often become more difficult, for both the self inductance and the mutual then involve μ, the magnetic permeability of the material; and μ is not in general constant, but is a function of magnetic field and thus depends on the currents in the circuits. Also when μ is not constant another difficulty comes in, by the distortion of the wave form. If the current wave form is sinoidal, then the voltage wave forms are not so, and *vice versa*. In this way strong harmonics may be introduced which give trouble in some of the methods. When μ is not constant, a tuned instrument such as a vibration galvanometer should be used as detector, the bridge current should be kept very constant during a test, and the current I through the unknown inductive branch should be known, for the values thus found for mutual inductance, self inductance, and effective resistance only hold good for the particular current value I.

[1] F. W. Grover and H. L. Curtis, *Bureau of Standards Bulletin*, 1912, viii. 455.
[2] C. J. Watson, *Electrician*, March 5, 1909, p. 809.
[3] A. H. Taylor, *Phys. Rev.*, 1904, xix. 273.

In certain cases, however, the permeability is practically constant. This occurs, for example, with some samples of soft iron or silicon-iron for low values of the magnetising field H, and in some practical applications the fields used are so small as to permit of the assumption of almost constant permeability, *e.g.* in iron-clad telephone cables. In such cases the testing current must be kept very small—a condition which very much reduces the sensitivity. In order that the effective resistance found in the test shall measure correctly the power used in the inductive circuit, the primary current (and not necessarily the voltage) should be sinoidal.

§ (124) Inductance of Iron Ring with Uniform Windings.—If an iron ring of uniform section s (sq. cm.) and mean circumference l is wound uniformly (all round) with primary and secondary coils of turns N_1 and N_2 respectively, then if L is the self inductance of the primary coil and M the mutual inductance between the primary and secondary coils, we have

$$L \doteq 1{\cdot}257\frac{\mu s}{l}N_1^2 \times 10^{-8} \text{ henries} \quad (148)$$

and $$M \doteq 1{\cdot}257\frac{\mu s}{l}N_1N_2 \times 10^{-8} \text{ henries}, \quad (149)$$

where μ is the effective permeability.

Hence μ can be found by measuring either L or M. M. Wien [4] employed the self inductance method and Campbell [5] that of mutual inductance for the measurement of permeability and hysteresis by alternating current null methods. A. C.

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INDUCTANCE COILS:
Design of, for radio-frequency work. See "Radio-frequency Measurements," § (33).
Design of, for valve generating sets for radio-frequency work. See *ibid.* § (36).
Electromagnetic, in telephone circuits. See "Telephony," § (35).
Electrostatic, in telephone circuits. See *ibid.* § (35).
Magnetic (**B**): the number of lines of magnetic force per unit area at any point in a magnetic circuit. See "Magnetic Measurements and Properties of Materials," § (1); "Electromagnetic Theory," § (2).
Magnetic circuit of. See "Electromagnet," § (6).
Measurement of effective capacity of, at radio frequencies. See *ibid.* § (29).

INDUCTION MOTOR: an asynchronous alternating current motor with alternating magnetic field. See "Dynamo Electric Machinery," § (9).
Description of. See *ibid.* § (13).

INDUCTION WATT-HOUR METERS:
Accuracy of. See "Alternating Current Instruments," § (42).
Energy meters dependent upon the interaction of alternating magnetic fields, and eddy currents in a conducting disc. See *ibid.* § (36).
Phase relations of fluxes and currents in. See *ibid.* § (38).
Quadrature adjustment of devices by means of which the fluxes produced by the two magnetic systems of the meter are caused to have exact time quadrature when the circuit power factor is unity. See *ibid.* § (39).
Types of, for two-phase and three-phase circuits. See *ibid.* § (44).

INDUCTIVE CAPACITY. The force in F dynes between two electrical charges e, e' concentrated at two points at a distance of r centimetres apart, is given by the expression

$$F=\frac{e\,.\,e'}{Kr^2},$$

where K is a constant depending on the medium in which the charges are placed. This constant is known as the inductive capacity of the medium.

On the electrostatic system of measurement the inductive capacity of air—vacuum—is assumed to be unity. See "Units of Electrical Measurement," §§ (2), (3); "Capacity and its Measurement," § (6).

INDUCTIVE INTERFERENCE: in telephony, a disturbance introduced by induction from other sources of electrical energy. See "Telephony," § (35).

INDUCTOMETERS: variable standards of inductance, self or mutual. See "Inductance, The Measurement of," §§ (60), (61), and (73)-(75).

INSULATING MATERIALS:
Change of Resistance with temperature. See "Resistance, Measurement of Insulation," § (1) (iv.).
Effect of temperature on. See *ibid.* § (1) (iv.).

INSULATION, in static transformers. See "Transformers, Static," § (9).

INSULATION RESISTANCE. See "Resistance Measurement of Insulation."

INSULATION RESISTANCE, measurement of, by portable instruments. See "Measurement of Insulation Resistance," § (3).

INSULATION TESTS ON ELECTRIC PLANT:
A.C. Tests. See "Dielectrics," § (14).
D.C. Tests up to 1000 Volts. See *ibid.* § (15).
D.C. Tests above 1000 Volts. See *ibid.* § (16).

INSULATORS, for telegraph lines. See "Telegraph, The Electric," § (14).
Pillar Type, description of. See "Switchgear," § (9).
Pillar Type: insulators used to support conductors or parts which are under pressure. See *ibid.* § (9).
Pin Type, description of. See *ibid.* § (9).

INTERNATIONAL AMPERE. The unvarying current which when passed through a solution of nitrate of silver in water, in accordance with a certain definite specification, deposits silver at the rate of 0·00111800 gramme per second.

1 International Ampere = 0·9999 ampere.

See "Units of Electrical Measurement," §§ (31), (32); "Electrical Measurements, Systems of," § (40).

INTERNATIONAL CONFERENCE ON ELECTRICAL UNITS. Last conference 1908. For resolutions defining the International ohm, ampere, volt, and watt, see "Electrical Measurements," § (38); "Units of Electrical Measurement," §§ (31), (32).

INTERNATIONAL OHM. The resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of constant cross-sectional area and of a length of 106·300 centimetres.

1 International Ohm = $1{\cdot}0005_2$ ohms.

See "Units of Electrical Measurement," §§ (31), (32); "Electrical Measurements, Systems of," § (39).

INTERNATIONAL TECHNICAL COMMITTEE (Bureau of Standards, Washington, 1910). Investigations on the silver voltameter. See "Electrical Measurements," § (40).

— J —

— K —

KOHLRAUSCH: MEASUREMENT OF ELECTROLYTIC RESISTANCE by a method in which the effects of polarisation are overcome by the use of alternating currents. Assuming that the polarisation arises in accordance with the equation $E = Ri + P\int idt$, and that the applied E.M.F. alternates in accordance with the equation $E = E_0 \sin pt$, the final value of the current through the electrolyte is

$$i = \frac{E_0 \sin (pt + \epsilon)}{R\sqrt{1 + P^2/p^2R^2}},$$

where $\tan \epsilon = P/pR$: the corresponding values of e, the back E.M.F., are given by

$$e = -\frac{PE_0 \cos (pt + \epsilon)}{pR\sqrt{1 + P^2/p^2R^2}}.$$

If P/pR can be made negligibly small, the phase difference between E and i, and the existence of e, can be ignored, and the value of R can be obtained by a bridge method if a suitable indicating instrument be employed. See "Electrolysis and Electrolytic Conduction," § (17).

— L —

LAMP, THE GAS-FILLED. See "Incandescence Lamps," § (6).

LAMP FILAMENTS, TEMPERATURES OF. See "Incandescence Lamps," § (8).

LAMPS, EVACUATION OF. See "Incandescence Lamps," § (5).

LEAD, REFINING OF. See "Electrolysis, Technical Applications of," § (23).

LEAKAGE: from negatively charged body under action of ultra-violet light. See "Photoelectricity," § (1).

LEAKAGE INDICATORS, use in electrical supply systems. See "Resistance, Measurement of Insulation," § (4).

LEAKAGE RESISTANCE OF DIELECTRICS. See "Dielectrics," § (6) (i.).

LEVEL ERROR: Effect of, on Meters. See "Watt-hour and other Meters for Direct Current. II. Watt-hour Meters," § (42).

LEYDEN JAR: a form of condenser with glass dielectric. See "Capacity and its Measurement," § (31).

LIFTING MAGNETS: description, design, and use of. See "Electromagnet," § (4).

LIGHT, CHEMICAL CHANGES PRODUCED BY. See "Photoelectricity," § (7).

LIGHTNING, effects of, on telegraph circuits. See "Telegraph, The Electric," § (14).

LIMITING DEVICES: arrangements for enabling desired signals to be read through strong disturbances. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (10).

LINE CONSTRUCTION, TELEGRAPH. See "Telegraph, The Electric," § (14).

LINE OF FORCE. A curve the tangent to which at any point of its length gives the direction of the resultant force—electric or magnetic—at that point.

A line of electric force issues from a positive charge and terminates on a negative charge.

A line of magnetic force due to a permanent magnet commences at the positive pole and terminates at the negative pole.

A line of magnetic force due to a current is a closed curve encircling the current.

See "Units of Electrical Measurement," § (14).

LINE OF INDUCTION. A curve the tangent to which at each point of its course gives the direction of the magnetic or electrostatic induction at that point.

For measurement purposes unit induction is represented by one line per square centimetre of a surface at right angles to the lines.

See "Units of Electrical Measurement," § (14).

LINE WIRES, TELEGRAPH, gauge of. See "Telegraph, The Electric," § (14).
Jointing of. See *ibid.* § (14).

LINES, TELEPHONE, TYPES OF. See "Telephony," § (2).

LIQUID STARTER: a motor starting device consisting of a variable liquid rheostat. See "Switchgear," § (11).

LIQUIDS AND GASES, dielectric constants of. See "Capacity and its Measurement," § (18).

LLOYD SQUARE, for the measurement of power losses in iron. See "Magnetic Measurements and Properties of Materials," § (60).

LOADING COILS: iron-cored inductance coils employed to increase the inductance of telephone circuits. See "Telephony," § (25).

LOADING OF TELEPHONE CABLES, PARTICULARS OF. See "Telephony," § (23).

LOCAL ACTION: a term used, in electricity, to denote the solvent action of an acid electrolyte upon the commercial zinc plate forming an electrode of a voltaic cell, irrespective of whether the cell is supplying a current or not. See "Batteries, Primary," § (6).

LOCAL BATTERY EXCHANGES: telephone exchanges for systems in which the D.C. power for the subscriber's transmitter is supplied from a local source. See "Telephony," § (4).

LODGE COHERER, THE: use of, as detector in wireless telegraphy. See "Wireless Telegraphy," § (19).

— M —

the magnetic intensity or strength of field at the point, and if F be the force experienced by a pole m, H the intensity, then $H = F/m$. If the force is due to a current flowing in a wire bent into the form of a long spiral or solenoid the magnetic force at any point within the spiral is $4\pi ni$, where n is the number of turns per unit length, i the current in C.G.S. units. If I be the current in amperes this becomes

$$H = \frac{4\pi}{10} nI$$

$$= \frac{4\pi}{10} \times \text{ampere turns per unit of length.}$$

See "Units of Electrical Measurement," § (15).

MAGNETIC HYSTERESIS

If a piece of unmagnetised steel or iron be subject to a gradually increasing magnetic force H, the magnetic induction B increases with the force up to a saturation or maximum value, and the curve connecting the induction and the force has a form such as OAC in *Fig.* 1. If the magnetisation be stopped at a point such as C on the curve and the force reversed and gradually reduced to zero the induction falls also but not so fast as it rose, the curve

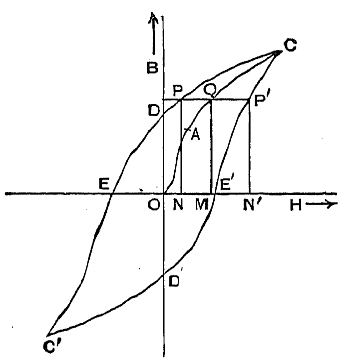


Fig. 1.

of demagnetisation takes the form CPD and when the magnetising force has become reduced to zero an amount of induction represented by OD remains. This behaviour is said to be due to magnetic hysteresis.

If the force be still further reduced, becoming negative, the induction falls along the curve DEC′ where C′ is the point on the curve at which the magnetising force is $-H$ equal and opposite to that at C, and a straight line joining C and C′ passes through O. At E the induction is zero, the magnetising force being negative and equal to OE.

On reversing the force at C′ and gradually increasing it through zero to its original value at C the magnetisation curve is shown by C′D′E′C, thus forming a closed loop, and on continually taking the material through this series of changes $+H$ to $-H$ and back to $+H$ the loop is continually retraced.

If the material is not magnetically neutral before commencing or has not been previously subject to this cyclic series of changes it will be necessary to repeat the operations a number of times before the loop is retraced.

In the figure OD measures the Residual induction, that is the amount of induction left when the magnetising force is reduced from some specified value to zero. OE measures the coercive force, that is the reversed magnetising force which must be applied to reduce the induction to zero.

Now let PQP′ be a line parallel to the axis of H cutting the curves shown in P_1Q_1P'. Draw QM, PN, P′N′ perpendicular to that axis.

The induction is the same at each of these points. On the original curve it corresponds to a magnetising force OM. On the descending curve CP this value of the induction is not reached until the force has passed through OM and arrived at the value ON. It corresponds to a value of the force on the original curve, which is reached earlier in the process than ON. The induction lags behind the force. The same is true, *mutatis mutandis*, at P′. The induction produced by the force ON′ is the same as that due to a force ON on the original curve, which again in the process has been reached earlier than ON′. The induction again lags behind the force. Hence the name "Hysteresis," a lagging behind.

Hysteresis takes place in the course of many other physical changes besides magnetisation. It is due in general to the fact that the molecules of the substance acted upon resist a change imposed on them by some external agency. Work has to be done and the energy expended appears as heat in the body acted upon. In the case of magnetic hysteresis we can calculate the loss of energy thus:

Let the iron be in the form of a bar l centimetres long and s square centimetres in cross-section closely surrounded by a long solenoid of n turns per unit of length in which a current i is flowing.

The magnetising force H at any point inside is equal to $4\pi ni$—

$$i = \frac{H}{4\pi n}.$$

Again if Φ be the total magnetic flux through the circuit, R its resistance, and E the electromotive force other than that due to the magnetic field, we have

$$E = Ri + \frac{d\Phi}{dt}.$$

The work done in time dt by the current is $Eidt$ or $(Ri^2 + id\Phi/dt)dt$.

But Ri^2dt is the work done in heating the

wire, hence the energy spent in changing the magnetisation of the iron is $id\Phi/dt(dt)$ or $id\Phi$.

Now there are ln turns in the circuit and its area is S. Hence if B be the number of lines of induction per unit area we have $\Phi = ln\mathrm{SB}$.

Thus $$id\Phi = \frac{\mathrm{H}}{4\pi} . l\mathrm{S}d\mathrm{B}.$$

Now PS is the volume of the iron, hence the work done per unit volume in producing a change of induction dB is HdB/4π, and the total work in going from H_1 to H_2 is

$$\frac{1}{4\pi}\int_{\mathrm{H}_1}^{\mathrm{H}_2} \mathrm{H}d\mathrm{B}.$$

Now in *Fig.* 2 if P, Q be two adjacent points on the curve and PP[1]M, QQ[1]N be parallel to

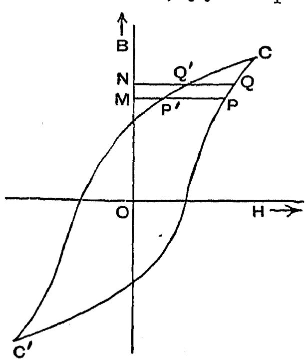


FIG. 2.

OH, MN is δB and the area PMNQ is equal to HB.

If the iron be taken through the complete series of changes from $-$H to $+$H and back then the value of $\int_{\mathrm{H}_1}^{\mathrm{H}_2}$ HdB is the area of the loop CEC′E′C. Thus the work done in taking the iron through the complete series of changes is found by dividing the area of the loop by 4π.

The work [1] so done appears as heat in the iron, and when constructing electromagnetic machinery, steps have to be taken to reduce this heat as much as possible and to get rid of it and so prevent too great a rise of temperature of the machine.

MAGNETIC INDUCTION. The value of the quantity μH, where H is the magnetic intensity and μ the permeability, is known as the magnetic induction. See "Units of Electrical Measurement," §§ (16) and (28).

MAGNETIC MEASUREMENTS AND PROPERTIES OF MATERIALS

INTRODUCTION.—The classification of magnetic measurements is a matter of some difficulty, where it is necessary to preserve the greatest clarity and conciseness, so that any particular kind of test may be immediately presented, and at the same time the test on a particular shape of specimen or kind of material may also be equally accessible.

The following scheme of headings has therefore been adopted as affording the greatest ease of accessibility with the requisite conciseness:

I. General remarks on the quantities to be measured and on the apparatus used in the magnetic measurements.
II. Measurements on ring-shaped specimens.
III. Measurements on round rods, flat bars, and strips.
   A. In moderate fields (**H** to 500).
   B. In strong fields (above **H** = 500).
IV. Measurements on magnet steels and on permanent magnets.
V. Alternating current magnetic tests.
VI. Measurements on so-called non-magnetic steels and on feebly magnetic materials.

I. QUANTITIES TO BE MEASURED AND THEIR DEFINITIONS

§ (1) (i.) *Magnetic Pole* [2] (m).—This is a quantity of magnetism which may be conceived of as composed of an imaginary magnetic matter and which can be represented as concentrated at a point. If two equal magnetic poles, each concentrated at a point, repel one another with a force of 1 dyne when placed 1 cm. apart in air—strictly in a vacuum—they are defined as unit magnetic poles.

Law of attraction or repulsion of magnetic poles is F (dynes) $= m_1m_2/r^2$, m_1 and m_2 being the strengths of the magnetic poles, and r their distance apart.

(ii.) *Pole Strength.*—The strength of a magnetic pole is equal to the force in dynes between it and a unit magnetic pole in air when 1 cm. separates them.

(iii.) *Magnetic Moment* (M).—The magnetic moment of a magnet is equal to the product of its pole strength and the distance between its poles.

(iv.) *Magnetic Force or Field Strength* (**H**).—A magnetic field may be looked upon as a state, produced in any space considered, whether by magnets or electric currents, of such a nature that a magnetic pole is acted upon by a mechanical force when introduced into the space. Considering, for instance, a permanent magnet having poles N and S, the force exerted by N on a unit pole at P is equal to m/NP^2 in the direction NP; that due to S is equal to m/SP^2 in the direction PS. The resultant force PQ is then the measure in magnitude and direction of the magnetic field at the point P due to the magnet NS. Conventionally the magnetic field is thought

[1] If H and B are measured in C.G.S. units the work is given in ergs per cycle.

[2] See "Units of Electrical Measurement," §§ (1), (2).

of as consisting of "lines or tubes of force," these lines being the paths along which the force on a magnetic pole is directed at any point in the field. The number of lines per sq. cm. is equal numerically to the field strength.

The unit of magnetic field or force has been conveniently chosen such that a unit magnetic pole placed in it experiences a force of 1 dyne.

Hence from this it follows, from the consideration of a sphere of unit radius surrounding a unit magnetic pole, that there are 4π lines of force radiating from the pole.

(v.) *Magnetic Potential* [1] of a magnet at any point is the work expended in bringing up a unit magnetic pole from infinity to the point considered. The magnetic potential at a point P distant r (*Fig.* 1) due to a magnetic pole m is equal to m/r.

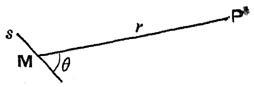


FIG. 1.

For a small magnet of moment M the magnetic potential due to the action of the two poles

$$=\frac{M}{r^2}\cos\theta.$$

(vi.) *Magnetic Flux* (Φ).—This is best defined as a particular "sheaf" or "tube" of lines of force which is complete and closed on itself, the number of lines of force being constant throughout. If a section be taken anywhere across the tube the constant number of lines of force will be a defined amount of magnetic flux.

(vii.) *Magnetic flux density* (**B**) is the amount of flux per unit area across a small section at the point considered, and is therefore spoken of as so many lines per sq. cm. This quantity is usually expressed by the symbol **B**.

(viii.) *Intensity of Magnetisation* (**I**).—Considering any magnetised body having a magnetic moment M and uniform magnetisation throughout, any portion of it will have a moment proportional to the volume. The magnetic moment per cubic centimetre is equal to the intensity of magnetisation.

(ix.) *Permeability* (μ).—The permeability of a magnetic material is the ratio of the flux density **B** induced in it to the magnetising field **H**. It must be clearly understood that **B** represents the total number of lines of force induced per sq. cm., and that **H** represents the actual magnetising field operating within the material.

(x.) *Magnetic Susceptibility* (k).—This is another means of representing the magnetisability of a material in which the ratio of intensity **I** to the magnetising field **H** is the measure so that $\mathbf{I}=k\mathbf{H}$. When k is positive the material is paramagnetic and when negative the material is diamagnetic.

[1] See also "Electromagnetic Theory," § (1).

(xi.) *Relations between the Various Magnetic Quantities.*—

$$\mathbf{B}=\mathbf{H}+4\pi\mathbf{I}.$$

This is a vector equation because each of the quantities involved has direction as well as magnitude, and although, usually, the direction of the quantities **H** and **B** is either the same or opposite, this is not necessarily the case.

By definition $k=\mathbf{I}/\mathbf{H}$ and $\mu=\mathbf{B}/\mathbf{H}$, hence $\mu=1+4\pi k$.

Note.—The equations and quantities given above only have the meanings given, subject to the following conditions:

(*a*) The material must not be already magnetised nor must it be subjected to any other magnetising field not included in the quantity **H**.

(*b*) The material must have been in a magnetically neutral condition before being brought into the condition under observation, or it must be in a truly cyclic condition.

(xii.) *Hysteresis Loss.*—If a magnetic body is carried round a cycle from one state of magnetisation to another, then the magnetisation will lag behind the magnetic force, so that the curve connecting, say, **B** and **H** will be quite different according to the direction of change of field.

If a complete curve be obtained connecting **B** with **H** for a whole cycle from one value of **H** to any other and then back to the original value of **H**, the curve connecting **B** and **H** will form a closed loop, the area of which is a measure of the energy absorbed in carrying the magnetisation through the cycle. This energy is known as the hysteresis loss [2]; it is usually expressed in ergs per c.c. per cycle, by the symbol h.

In the diagram (*Fig.* 2) the loop PC_1P_1C represents the magnetic condition **B** as a function of **H** when the cycle of magnetisation is carried round in the direction of the arrows. The energy dissipated in doing this is proportional to the area PCP_1C_1.

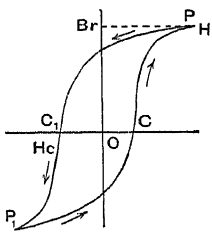


FIG. 2.—Hysteresis Loop.

Numerically the energy is equal to the area PCP_1C_1 (in units of $\mathbf{H}\times\mathbf{B}$) divided by 4π ergs per c.c. per cycle, and, in general, for any cycle between limits of $\mathbf{H}_1$ and $\mathbf{H}_2$ the hysteresis loss in ergs per c.c. per cycle is equal to

$$h=\frac{1}{4\pi}\left[\int_{\mathbf{H}_1}^{\mathbf{H}_2}\mathbf{H}d\mathbf{B}-\int_{\mathbf{H}_2}^{\mathbf{H}_1}\mathbf{H}d\mathbf{B}\right].$$

[2] See "Magnetic Hysteresis."

The following definitions are important:

(xiii.) *Remanence* ($\mathbf{B}_{rem.}$).—The remanence is the magnetisation (expressed in terms of **B** or **I**) which exists in the specimen when, whilst in a cyclic condition, the magnetising field is reduced to zero from some maximum value of **H**.

(xiv.) *Coercive Field* ($\mathbf{H}_c$).—The coercive field[1] is the reversed magnetic field $\mathbf{H}_c$ necessary to reduce the induction **B** to zero from any specified value.

According to some writers,[2] $\mathbf{H}_c$ is that value of **H** required to reduce **I** to zero.

Coercivity is the reversed **H** necessary to reduce **B** to zero from its saturation value.

(xv.) *Steinmetz Coefficient* η.—An examination of a number of specimens of iron to determine the relation between hysteresis and the maximum value of the magnetic induction reached in the cycle gave the empirical relation $h=\eta\mathbf{B}^n$, where η and n are constants.

For a limited range of magnetisation the value of n is approximately 1·6 or (according to Ewing)[3] more nearly 1·59. This relation holds not only for iron but also for nickel, cobalt, and materials such as silicon- and aluminium-iron. In cast-iron the value is much greater.

The Steinmetz constant η varies widely for various materials (from 0·0007 in good quality silicon-iron to 0·025 in hard cast-steel).

If at any point on a main hysteresis loop the direction of changing **H** be reversed, the magnetisation will not retrace the path it had just before reversal, but will proceed along some other line lying within the main loop. If now the original **H** be arrived at, the curve will again be a different one, so forming a subsidiary loop lying within the main loop. Such a loop is indicated in the accompanying diagram (*Fig.* 2A), the area of this subsidiary loop also represents the energy spent in carrying the magnetisation round the subsidiary cycle PQ. Such cycles occur in many cases in practice such as with permanent magnets on magnetos, telephone pole-pieces, telephone transformers, triode valve transformers. The investigation of such loops is of some importance.

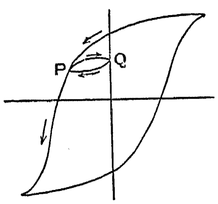


FIG. 2A.—Subsidiary Hysteresis Loop.

(xvi.) *Total Loss.* — The above remarks regarding hysteresis losses refer to a cycle of magnetisation, which is so slow that no circulating currents arise due to the lines of force cutting the electrically conducting metal. If, however, the cycle of magnetisation is performed several times a second or more quickly, the rapid cutting of the lines of induction through the metal gives rise to induced eddy currents which waste energy, so that the "total losses," as they are called, are greater than the hysteresis losses.

The eddy current losses are proportional to the square of the induced currents, and hence are proportional to n^2, $\mathbf{B}^2$, and f^2; they may therefore be written

$$\text{Eddy currents} = \xi n^2 f^2 \mathbf{B}^2.$$

Where n=frequency and f=form factor of secondary induced voltage, this is given by the ratio ($V_{RMS.}/V$ mean), which for a sine wave is $\pi/2\sqrt{2}$ or 1,1107. The quantities η and ξ are constants for a particular specimen within a certain range of **B**.

We then have for the total losses

$$W = \eta n\mathbf{B}^{1\cdot 6} + \xi n^2 f^2 \mathbf{B}^2.$$

The methods of separating the losses in actual tests are given in Part V. § (55) on alternating current magnetic tests.

The eddy current losses are greatly reduced by alloying certain materials with iron so as to increase the resistivity and thereby reduce the actual currents induced in the material. The most successful alloys are those with silicon or with aluminium. A further advantage in the use of such alloyed materials is that the hysteresis losses are also materially reduced. The ageing effects (increase in hysteresis losses with time) are also smaller in these alloyed materials than in pure iron or mild steel.

§ (2) APPARATUS USED IN MAGNETIC MEASUREMENTS.—The apparatus used in magnetic measurements may be divided into two classes: that actually used in the tests; auxiliary and standard instruments used in calibrating the testing apparatus.

Treating first the standard apparatus. We have apparatus for providing a known magnetic field, such apparatus used in conjunction with a coil of known area and number of turns provides a standard number of line-turns for calibrating ballistic galvanometer, fluxmeters, etc.

(i.) *Solenoid.*—A standard solenoid is one of the most useful pieces of apparatus for general standardising purposes. It consists of a long coil of one or more layers of wire wound as uniformly as possible on a tube, preferably, of insulating material. A thick-walled ebonite tube 5 or 6 cm. internal diameter and 1 metre long is of suitable size.

It should be wound with such wire and

[1] J. Hopkinson, *Phil. Trans.*, 1885, clxxvi. (2), 455.
[2] *Elec. Rev.*, 1899, xliv. 40 (Curie).
[3] J. A. Ewing, "Magnetic Induction in Iron and other Metals," 1900, p. 111. See also "Magnetic Hysteresis."

number of turns per cm. that a field of 100 may be readily obtained without undue heating.

The formula[1] for **H** in an infinitely long solenoid is, of course, $\mathbf{H}=4\pi NI/10$, where N = turns per cm. and I = current in amperes.

Owing to the finite length, the open ends subtend solid angles which are not zero at any point along the axis, and the field is therefore reduced by an amount dependent upon the solid angles subtended at the ends by the point.

The field for any point along the axis becomes

$$\mathbf{H}=2\pi NI\left[\frac{l+x}{\{a^2+(l+x)^2\}^{\frac{1}{2}}}+\frac{l-x}{\{a^2+(l-x)^2\}^{\frac{1}{2}}}\right],$$

where l = half the length of the solenoid, x = distance along axis of point considered from centre of solenoid, and a = radius of solenoid.

Such a solenoid should be provided with one or more secondary coils. If the coil is outside the solenoid the mean diameter of the solenoid will need to be known; if it is inside, then the diameter of the coil will need to be known in order to calculate the mutual inductance from the dimensions. A suitable number of turns in the secondary coil is 500.

The line-turns cutting the secondary in such a mutual inductance are of course

$$\frac{8\pi}{10}\cdot N_1N_2sI,$$

where I = *reversed* current in primary in amperes,
N_1 = turns per cm. of solenoid,
N_2 = secondary turns,
s = area of, either, search coil for case (*a*), or solenoid for case (*b*).

S. J. Barnett[2] has described an improved form of standard solenoid, which avoids the uncertainty of the long leads connecting various layers and yet allows the layers to be wound, each in the groove of the layer below.

This is accomplished by providing two solenoids one to slip inside the other. One is a right-hand spiral and the other a left-hand one. They are each wound on a tube of bakelite, which has a screw thread cut along it. The successive layers are wound in the same direction so that the pitch of each layer is precisely the same.

In connecting up, short connections are used from any layer in solenoid A to any layer in solenoid B. These short connections are at the ends, where their effects on the field at the middle portion are negligible.

Other types of solenoids, not wholly cylindrical or uniform, have been devised with the object of providing a uniform field over a considerable region without necessitating the large and long coil which would have to be used if it were cylindrical and uniform.

One such type makes use of the principle of tapering the end portions.[3] The mathematical treatment is given in the article referred to, and a coil was constructed having the following proportions (*Fig.* 3):

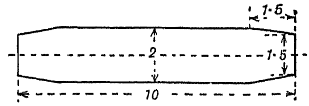


FIG. 3.—Short Solenoid with Tapered Ends giving Uniform Field.

The lengths are expressed in terms of the radius r of the central cylindrical part. The field was found to be uniform to about 1 in 1000 over a length, along the axis of $2r$ on each side of the mid-point.

Another type[4] (*Fig.* 4) has two smaller short cylindrical coils placed axially and with their mid-planes approximately in the planes of the ends of the main coil. The proportions are approximately as shown in the diagram.

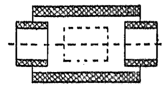


FIG. 4.—Compound Coils giving Uniform Field.

Taking the inside radius of the main coil as unity, the following are the proportions:

Main Coil—	Inside diameter	2·0
	Outside diameter	2·4
	Length	5·2
Distance between centres of end coils		5·0
End Coils—	Inside diameter	1·4
	Outside diameter	1·75
	Length	0·9

The field over a central cylindrical space 1·5 radius in length and 1·2 radius in diameter was very uniform (about 1 in 1000).

(ii.) *Magnetometer.*—The magnetometer is one of the simplest and most fundamental pieces of apparatus for measuring magnetic quantities.

The form which the instrument takes varies considerably according to the nature of the measurements required.

Essentially all magnetometers measure a magnetic field. Usually it is the component in a horizontal plane which is measured, excluding magnetometers for measurements on the earth's magnetic field.

In the most usual form of magnetometer one or more small magnetised pieces of steel called hereafter "the needle" is suspended by a suspension of good torsional elastic

[1] See "Electromagnetic Theory," § (13).
[2] "A Double Solenoid for the Production of Uniform Magnetic Fields," *Phil. Mag.*, 1920, xl. 519.
[3] "Axial Field of Bobbin in the Form of a Truncated Cone," L. Moullevigue, *J. de Phys.*, 1898, vii. 466.
[4] "Production of a Homogeneous Magnetic Field," A. Bestelmeyer, *Phys. Zeit.*, 1911, xii. 1107.

properties. The torsional control of the suspension, however, is usually small compared to the torsional control exerted on the needle by the field in which it swings.

The magnetometer may be used to measure (1) intensity of magnetic field; (2) direction of magnetic field; (3) intensity and direction together.

Case (1) usually applies when the direction of the field to be measured is known and can be suited, if desired, to the magnetometer itself.

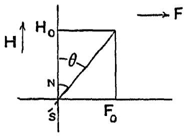


FIG. 5.

If H (*Fig.* 5) be the direction of the control field and H_0 its magnitude, then if an unknown field F_0 in magnitude and having the direction F at right angles of H be applied at the same time, a deflection of the needle will be produced equal to θ, where

$$\tan\theta = \frac{F_0}{H_0}.$$

If F_0 is produced by a specimen, such as a bar magnet or piece of magnetic material of suitable shape, then, from a knowledge of the distances involved and a measurement of the deflection, the magnetic properties of the material may be measured.

In a case such as that represented in *Fig.* 6, where the poles of the specimen tested may be

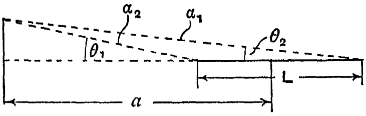


FIG. 6.

considered as localised near its ends, the following is the full formula for the moment M of the rod:

V = volume of specimen,

L = length between poles of specimen,

a_1, a_2, θ_1, θ_2, and a are as shown in the diagram,

ϕ = deflection produced.

Then $$M = \frac{LH\tan\phi}{\left[\frac{a-L/2}{a_2^{\,2}} - \frac{a+L/2}{a_1^{\,2}}\right]}$$

For a long thin specimen or for an ellipsoid we can immediately determine **I** or **B** by the ordinary substitution.

The relation connecting the various constants of the magnetometer system, when swinging freely in a magnetic field, is for any angular displacement θ, MH $\sin\theta = U$, where U is the couple acting on the needle of magnetic moment M when displaced through an angle θ from the direction of the control field H. For small angles the restoring couple is proportional to the displacement, the needle will therefore perform simple harmonic oscillations if released from any deflected position. The periodic time T is given by

$$T = 2\pi\sqrt{\frac{K}{MH}},$$

where K is the moment of inertia of the needle about the axis of suspension. If the torsional control cannot be neglected, a correction must be applied. This torsional control may be considered as equivalent to an increase in the field H. The value of this correction may very easily be found by turning the torsion head through any known angle, say 1 revolution or 360°. If the deflection of the needle thus produced $= \epsilon°$, then the correction to be applied to H is $+H\epsilon/360$.

(iii.) *Gumlich's Magnetometer.* — A typical magnetometer is that described by E. Gumlich,[1] being very suitable for general use where a torsion free instrument is required.

The design is illustrated diagrammatically in *Fig.* 7. The needle is in the form of a small ring with consequent poles as shown. It is mounted on an aluminium rod A to which is attached the mirror X. This unit hangs by the quartz fibre Q. The adjustment is such that the ring-shaped magnet hangs with small clearance in a partly spherical cavity within a mass C consisting of two fitted blocks of iron-free copper; by this means very good electromagnetic damping is provided.

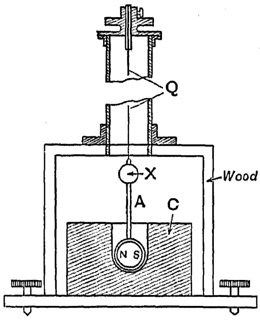


FIG. 7.—Gumlich Magnetometer.

If it is desired to alter the sensitivity of the instrument, it can be placed within the wide gap of a large wooden cored toroid, as shown in *Fig.* 8.

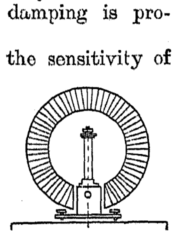


FIG. 8.—Control Field for Magnetometer.

The ring has an open single layer winding of pitch about 1 turn per cm. The plane of the ring is in the magnetic meridian. By sending a small steady current through the winding, a field, uniform throughout a considerable space, can be superposed upon the earth's field. By applying the field in this manner there is practi-

[1] *Magnetische Messungen*, p. 19.

cally no leakage outside the apparatus to affect neighbouring instruments.

Where the magnetometer is used in connection with tests involving solenoids or other magnetising windings around the specimen being tested, there will, in general, be large effects on the magnetometer due to leakage fields from these magnetising windings. It is necessary, therefore, to compensate these effects by means of other coils carrying the same current as the magnetising coils and so placed as to leave the control field, in which the magnetometer needle swings quite undisturbed when magnetising current is switched on and no specimen is present.

(iv.) *Gray's Magnetometer.*—A very convenient set-up of the magnetometer including the compensating coils has been described by J. G. Gray and A. D. Ross.[1] The lay-out is as shown in *Fig.* 9.

A wooden "bed" is provided with two "ways" intersecting at right angles in the

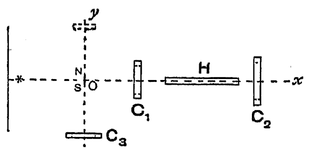


FIG. 9.—Lay-out for Magnetic Measurements with Magnetometer.

form of a cross. The magnetometer is at the point of intersection. Along the ways the various pieces of apparatus can slide smoothly, and can be clamped without moving at the moment of clamping.

H is the main magnetising coil or solenoid. C_1 and C_2 are the main compensating coils, C_2 being a fine adjustment.

C_3 is an adjustable compensating coil to compensate for any small component in the direction Oy due to the coils H_1, C_1, and C_2 when they have been adjusted to produce an exactly zero field in the direction Ox. This latter adjustment would not be necessary if the magnetometer needle kept the direction Oy, but as soon as it deflects, any residual field due to the magnetising circuit and having a component along Oy will exert a torque on the needle and so vitiate the readings.

A small deflector magnet (shown dotted) is used to give the needle a deflection when making the adjustment with C_3.

When the compensation is perfect, no effect should be produced when the full magnetising current is switched on. This should be the case for all positions of the needle.

The above effect is known as the Erhard Effect.[2]

(v.) *F. E. Smith's Magnetometer.*—Another form of magnetometer which was devised mainly for recording variations in the horizontal intensity of the earth's magnetic field[3] has also other applications on account of its great sensitiveness, quick period, and deadbeatness.

The moving system consists of small magnets mounted with a damping vane of aluminium. The suspension is a quartz fibre. Instead of swinging freely in the earth's field the needle is brought into a position nearly at right angles thereto by applying torsion through the suspension. If the torsional constant of the fibre is very small, a large number of turns will have to be applied to the torsion head to bring the needle nearly east and west, in this case the sensitivity will be very great.

The equation for such an instrument is

$$H=\frac{T\phi}{M \sin \theta},$$

where T = torsional control per unit angle of twist of the suspension,

ϕ = angle of torsion of suspension,

and θ = angle between the magnet and the meridian.

Also $$\frac{d\phi}{dH}=\frac{\phi}{H(1+\phi \cot \theta)}.$$

This shows that the sensitivity increases with ϕ, but it also shows that the sensitivity alters with the deflection.

To convert deflections into changes of H the deflections must be measured from a known base angle θ_0. If the direction of H changes account must be taken of this independently.

For a change in H equal to h (h may, of course, be a small quite independently applied field) and parallel thereto we have

$$\frac{h}{H}=\left(\frac{1}{\cos \alpha}-1\right)+\frac{\alpha}{\phi \cos \alpha},$$

where α = deflection produced measured from $\theta_0 = 90$ as datum angle.

If $(1/\cos \alpha)-1=y$ is plotted with α as abscissa and y as ordinate, a curve (*Fig.* 10) will be obtained. Similarly, if $\alpha/\phi \cos \alpha = y'$ is plotted against α, a family of curves (*Fig.* 11) will be obtained for different values of ϕ. For any initial position of the

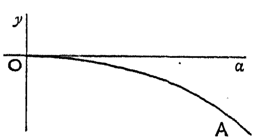


FIG. 10.—Appertaining to Smith Magnetometer.

needle θ_0 where $\theta_0 = 90° - \alpha$ the sensitivity h/H can be read off as the sum of the ordinates on curve A, and the appropriate curve B for the particular value of ϕ.

To take account of change in direction of H a second instrument is set up as an

[1] *Proc. Roy. Soc. Ed.*, 1908–9, xxix. 182.

[2] *Ann. d. Phys.*, 1902, ix. 724.

[3] F. E. Smith, *Phys. Soc. Proc.*, 1914, xxvi. 279. See also "Magnetism, Terrestrial, Electromagnetic Methods of Measurement."

ordinary declinometer (free from torsional control). If the change in declination is β, the formula for h/H becomes

$$\frac{h}{\mathrm{H}}=\left[\frac{1}{\cos\ (\alpha+\beta)}-1\right]+\frac{\alpha}{\alpha+\beta}\left[\frac{\alpha+\beta}{\phi\cos\ (\alpha+\beta)}\right].$$

In a good instrument set not to be too near the unstable point (say, $\theta_0=87^\circ$) a deflection of 3 mm. can be obtained for an applied h of 1×10^{-5} C.G.S. unit, the scale distance being 2 meters.

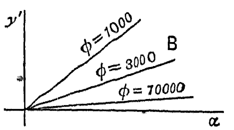


FIG. 11.

Although no experiments appear to have been made to use the instrument for measurements where a Curie balance would ordinarily be used, it seems probable that such use could very well be made of such an instrument. Extraordinary precautions would have to be taken in compensating for any solenoids or magnetising windings used.

§ (3) BALLISTIC GALVANOMETER.—The ballistic galvanometer as used in magnetic testing measures a quantity of electricity equal to $(n\Phi/\mathrm{R})\times10^{-8}$ coulombs, where n = number of turns in search coil, Φ = total flux interlinking each turn of the search coil.

Note.—If the flux is reversed (for instance, by reversing a current) then, of course, the interlinkage is $2\times\Phi$.

R = whole resistance of the circuit round which the quantity flows.

The galvanometer only functions ballistically when the time taken for the quantity to flow through it is short compared to the time of swing, *i.e.* the galvanometer receives a "blow."

In magnetic testing the time of the flux change varies over very wide limits, but for most test purposes a time of swing of 15 seconds is suitable. In testing very massive specimens such as dynamo poles, frames, etc., the steady magnetic state is not reached till many seconds after applying the magnetising field. For such purposes a galvanometer of exceptionally long period or a fluxmeter should be used. Galvanometers of 10 minutes period have been used successfully.

It is not usually possible to secure all the necessary qualities in a ballistic galvanometer without adding inertia, in the form of a small solid cylinder, flywheel, or weights on the ends of arms, or in some cases by the use of a very large moving coil.

(i.) *Suitable Conditions of Using, and Precautions.*—The highest class of workmanship is required in a ballistic galvanometer since it is used almost entirely deflectionally.

The conditions to be satisfied are:

The zero should be stable.

Only small zero creep should be observed when the galvanometer is held deflected at full scale and then slowly allowed to return without overshooting the zero.

The deflections should be proportional to current or quantity. Hence in the D'Arsonval type of galvanometer the moving coil should be free from magnetic material. The suspension should be carefully chosen and mounted (in some very high-class galvanometers the suspension does not carry the current and may therefore be non-metallic).

The coil should be carefully centred in the gap of the magnet. The angle of swing should not be too large. A maximum deflection of the light spot of 300 mm., with a scale distance of 2 metres, is satisfactory.

Copper terminals throughout the apparatus are desirable so as to avoid troublesome zero shift due to thermoelectric effects.

In using the galvanometer, the throws should always be taken towards one side only of the scale, and on the return journey the light spot should not travel very far through zero on the other side.

If, accidentally, a reversed throw is given to the galvanometer, a further full-scale throw should be given in the right direction and the zero reset if necessary before taking any further observations.

It is sometimes stated that a ballistic galvanometer should be used with very small damping, but in practice the reverse is the case; the most convenient degree of damping is when the galvanometer is not quite dead-beat but passes through zero with a reverse swing of roughly $\frac{1}{20}$ of the original swing. In this case the damping must be accurately known.

To obtain the utmost speed of working a key can be provided to short-circuit the galvanometer, and with a little practice the galvanometer can be allowed to swing back quickly and then be suddenly checked at zero.

(ii.) *Calibration.*—As shown below in the theory of the galvanometer, so long as the quantity Q passes through it in a short time the deflection is proportional to the quantity $\mathrm{Q}=\mathrm{K}\theta$; K is not a fixed constant for the galvanometer but depends on the damping and the shunting, but for any fixed set of conditions K is a constant; for this reason any apparatus introduced into the galvanometer circuit must be left connected during an experiment or its equivalent resistance inserted when such apparatus is removed.

The constant K can, of course, be determined from the fundamental equation for the galvanometer by inserting all the quantities concerned, using a stop-watch and a known current as fundamental measuring tools. In practice, however, it is much better to calibrate

directly by sending a known quantity of electricity through the galvanometer and observing the throw obtained.

The following are the usual methods of doing this:

(*a*) *By the Use of a Hibbert Magnetic Standard.*—This apparatus is self-contained, requires no auxiliary current- or voltage-measuring devices, but gives only definite quantities of invariable amount and cannot be set to give the calibration corresponding to a definite number of flux-turns. It is very constant and reliable, and is probably the best method to use in 90 per cent of the cases where magnetic testing is done.

The construction of the magnet of the Hibbert standard is as shown in *Fig.* 12. Here M is a permanent round bar magnet accurately and firmly bolted to the base of a cast iron yoke C of cup form. The magnet carries at its upper end a circular plate P, also of cast iron. The dimensions are such that a narrow annular air-gap of about 2 mm. width is left between P and C.

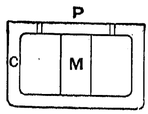


FIG. 12.—Hibbert Magnetic Standard.

A brass support on P carries a coil of about 1 cm. length of winding in a single layer on a shallow groove in a brass tube which can slide up and down freely on the support; the tube passes freely through the annular space between P and C, and so the coil on it cuts the flux in the narrow air-gap.

Various coils of different numbers of turns may be used, and so a variety of line-turns are available for calibrating purposes. The coil is released by a trigger and falls quickly through the annulus, being brought to rest on a soft pad inside the cast-iron container.

A usual type has two coils on separate brass tubes—one coil has 100 turns with a tapping brought out giving 30, 70, and 100 turns; the other coil is similar, having 3, 7, and 10 turns.

The total flux cut in such an apparatus is of the order of 20,000 lines. The apparatus, therefore, gives standard line-turns of approximately 60,000, 140,000, 200,000, 600,000, 1,400,000, and 2,000,000.

It is a secondary standard and has, of course, to be calibrated by some more fundamental method.

(*b*) *By the Use of a Duddell Inductor.*—This apparatus, used for calibrating the ballistic galvanometer, consists of a mutual inductance with movable secondary as shown in the accompanying photograph (*Fig.* 13). The primary is fixed and carries a steady current, which may be anything up to 10 amperes. The change in line-turns is produced by moving the secondary from one coplanar position with respect to the primary, through 180 degrees to the other coplanar position. This is effected by mounting the secondary coils on a vertical spindle working smoothly without shake in bearings above and below. The upper end of the spindle projects outside the apparatus and carries an arm. A control spring normally tends to turn the coils in an anti-clockwise direction. A detent is provided at both right- and left-hand sides; to operate the apparatus the arm is turned against the control spring and engaged in the left-hand detent, which locates the coil accurately. When it is desired to make the observation the detent is pulled out of engagement and the arm and coils fly quickly round and are caught by the right-hand detent, which also locates them accurately in the second position.

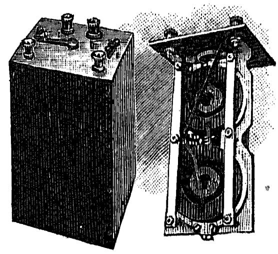


FIG. 13.—Duddell Inductor.

The mutual inductance thus quickly changes from a certain positive value quickly through zero to an approximately equal negative value. The flux-turns thus obtained are $2I(M_1 + M_2) \times 10^{+8}$, where I = primary current in amperes, M_1 and M_2 are the values in henries of the mutual inductance in the two positions.

The coils are duplicated above and below, so as to render the apparatus astatic with regard to the earth's or any stray magnetic field.

Any value (up to the limit corresponding to a current of 10 amperes) of line-turns may be obtained by setting the current to the appropriate value.

(*c*) *By the Use of a Mutual Inductometer.*—One of the most convenient and elastic methods of calibrating a ballistic galvanometer is by reversing a known steady current through the primary of a variable mutual inductometer,[1] for instance of the Campbell type. A useful range of mutual inductance is from 0 to 1 or 2 millihenries. With this range a reversed current of 1 ampere is convenient.

The simplest and most accurate method of obtaining the current of 1 ampere is by making a four-terminal resistance of 1·0186 ohms and connecting the potential terminals, in series with an opposing Weston cell and resistance of about 500 ohms to a galvanometer. Of course such a shunt must be placed in the battery circuit in front of the reversing switch so that the current is not reversed through it.

[1] See "Inductance, The Measurement of," § 60.

A more usual form of mutual inductance used in calibrating the ballistic galvanometer is to wind a long solenoid on a tube of known diameter and provide a secondary coil outside the middle of it. The advantage of this form is that the mutual inductance can be calculated without reference to any other standard than that of length.

The secondary coil can, of course, be inside the solenoid if desired; in this case it is the mean area of the secondary and not that of the solenoid which must be determined. There is some difficulty in this type of secondary in that, if a single layer winding, it must be of very thin wire in order to obtain the necessary turns in a reasonable length. In this case it will have rather a high resistance. On the other hand, if it is an overwound coil, its mean diameter cannot be accurately measured.

For a suitable design of mutual of the solenoid pattern see "Solenoid," § (2) (i).

If the secondary coil is outside the solenoid it should lie fairly close against the outer surface, in the middle of the solenoid, otherwise there will be leakage correction due to the negative field outside the solenoid produced by its poles.

This effect can be approximately allowed for by deducting an amount of flux in the annular space between the solenoid and the secondary (outer coil) equal to the area of the annulus multiplied by the difference in **H** between that in an infinite solenoid and that in the actual solenoid. Hence the self-demagnetising field at the central plane of a solenoid due to its poles $\doteqdot \frac{1}{2}a^2/l^2 \cdot \mathbf{H}$, where **H** is the field due to an infinite solenoid.

The mutual inductance may also very conveniently take the form of a toroid with primary inside, of known section and turns per cm. The secondary of known number of turns should be very well insulated with paraffined silk. Both coils should be uniformly wound. The core may conveniently be a turned built-up ring of paraffined mahogany of square section. With careful construction and measurement a standard can be obtained from first principles in this form accurate to about 1 in 1000.

(*d*) *By the Aid of a Condenser.*—A fourth method sometimes used for calibrating a ballistic galvanometer is to discharge through it a condenser previously charged from a standard cell. This method, however, cannot be considered accurate unless the conditions of charge and discharge can be reproduced closely and the capacity of the condenser is known under these conditions. In general the damping of the galvanometer is absent or different when calibrating from what it will be in use. The proper damping correction must be applied in this case, by multiplying the condenser deflection by the fractional quantity $(1-\lambda/2)$. λ is the logarithmic decrement per complete period.

(iii.) *Standard Magnetic Field of High Value.*[1]—R. Gans and P. Gmelin have developed the following method of obtaining a standard field in the region **H**(3000-7000) using a massive iron core and narrow air-gap.

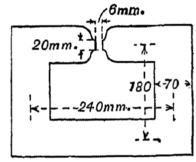


FIG. 14.—Standard Magnetic Field of Gans and Gmelin.

The standard is shown in the accompanying diagram (*Fig.* 14). The principle consists in operating the apparatus over that part of the curve where the effective permeability, measured in the gap, is a maximum. Thus, in *Fig.* 14A, OP is the tangent to the permeability curve and the part of the curve near P is utilised. We have

$$\mathbf{H}=\frac{4\pi NI}{10(l_i/\mu+l_a)},$$

where l_i = mean length of iron path, l_a = length of air-gap.

In the actual apparatus the core is of specially pure cast steel of the dimensions shown (*Fig.* 14). The winding consists of 1700 turns of wire and fills up the part where the pole tips are tapered down, so that the outside of the winding presents a uniform appearance.

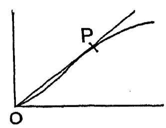


FIG. 14A.—Appertaining to *Fig.* 14.

The proportionality was found to hold to 0·5 per cent over a range of **H** from 3000 to 7000.

In using such a standard the current must be carefully applied and brought up steadily to the required value. If, by accident, it overshoots the reading or a smaller value is required the current must be reversed and cyclically reduced to zero and then brought to the required value.

The standard, being a derived one, must be calibrated. This is best done by making use of a standard search coil wound with a single layer of fine wire on a flat marble cylinder whose dimensions may be accurately determined.

§ (4) THEORY AND EQUATIONS FOR BALLISTIC GALVANOMETER.[2]—Assume a steady deflection to be proportional to steady current.

Case 1. *Damping Negligible.*—Dealing with d'Arsonval galvanometer.

[1] "Magnetic Standard," *Electrician*, November 15, 1901.

[2] Theory of Ballistic Galvanometer taken mainly from F. A. Law's *Electrical Measurements*, chap. ii.

If I = steady current through galvanometer (amperes),
τ = control torque per unit angle (radian) of deflection of coil (dyne, cm.),
θ = deflection (radians),
Q = total quantity in the discharge (coulombs),
$\mathbf{H}$ = magnetic field in which coil moves (gauss),
A = area of coil (sq. cm.) and n = number of turns,
Moment of inertia of coil about axis of suspension = K (dyne cm.^{2}),

then $$I = \frac{10\tau\theta}{\mathbf{H}An} = C\tau\theta.$$

Equation of motion is
$$K\frac{d^2\theta}{dt^2} = \frac{i}{C} - \tau\theta.$$

For an electrical "blow," the duration of which is so short that it is over before motion takes place ($\tau\theta = 0$, during discharge),
$$K\int\frac{d^2\theta}{dt^2}\,.\,dt = \frac{1}{C}\int i\,.\,dt,$$
whence $$\frac{1}{C}Q = K\frac{d\theta}{dt}_{\theta=0}.$$

The energy in the system is
$$E = \tfrac{1}{2}K\left(\frac{d\theta}{dt}\right)^2_{\theta=0} = \frac{\tfrac{1}{2}Q^2}{KC^2}.$$

Since the damping is zero all this energy is transferred to the suspension when the coil comes to rest at the end of its first swing θ_1, so that
$$\frac{Q^2}{KC^2} = \tau(\theta_1)^2,$$
whence $$Q = \theta_1 C\sqrt{K\tau}.$$
The quantities K, τ, and C are constants of the galvanometer. Introducing the time of free vibration of the galvanometer and the current sensitivity I/θ we have $T_0 = 2\pi\sqrt{K/\tau}$ and $C\tau = I/\theta$. Therefore
$$Q = \theta_1\left(\frac{I}{\theta}\right)\left(\frac{T_0}{2\pi}\right) \quad . \quad . \quad . \quad . \quad (1)$$

Ratio of quantity sensitivity to current sensitivity is equal to
$$\frac{2\pi}{T_0}.$$

Case 2. *Damped Galvanometer.*—The fundamental equation is
$$K\frac{d^2\theta}{dt^2} + \left(k_0 + \frac{1}{C^2R}\right)\frac{d\theta}{dt} + \tau\theta = \frac{1}{CR}\left(e - L\frac{di}{dt}\right), \quad (2)$$
where k_0 = damping coefficient on open circuit, e = E.M.F. acting on galvanometer, $1/C^2R$ = damping due to current induced in galvanometer circuit of resistance R by its own motion.

Let $k = k_0 + 1/C^2R$ and assume $L\,di/dt$ negligible.

The solution of (2) is
$$\theta = A_1\epsilon^{p_1t} + A_2\epsilon^{p_2t} + \frac{1}{CRK(p_1 - p_2)}\,.$$
$$[\epsilon^{p_1t}\textstyle\int e\epsilon^{-p_1t}\,.\,dt - \epsilon^{p_2t}\int e\epsilon^{-p_2t}\,.\,dt], \quad (3)$$
where p_1 and p_2 are the roots of the equation
$$p^2K + kp + \tau = 0.$$

If $t = 0$, $\theta = 0$, and $d\theta/dt = 0$ represent initial conditions when $e = 0$, equation (3) becomes
$$\theta = \frac{1}{CKR(p_1 - p_2)}\left[\epsilon^{p_1t}\int_0^t e\epsilon^{-p_1t}\,.\,dt - \epsilon^{p_2t}\int_0^t e\epsilon^{-p_2t}\,.\,dt\right]. \quad (4)$$

Equations (3) and (4) are general and do not assume that the impulse given to the galvanometer is instantaneous.

If e becomes sensibly zero in a time $t' < t$, the time t being that for the galvanometer to reach its first maximum deflection θ_1, then equation (4) may be written
$$\theta_1 = \frac{1}{CKR(p_1 - p_2)}\left[\int_0^{t\overline{>}t'} e\,.\,dt\right]\left\{\left(\frac{p_2}{p_1}\right)^{\frac{p_1}{p_1-p_2}} - \left(\frac{p_2}{p_1}\right)^{\frac{p_2}{p_1-p_2}}\right\}N^{\frac{p_1}{p_1-p_2}}\,.\,M^{\frac{p_2}{p_2-p_1}}, \quad (5)$$
where
$$M = \frac{\int_0^{t\overline{>}t'} e\epsilon^{-p_1t}\,.\,dt}{\int_0^{t>t'} e\,.\,dt} \text{ and } N = \frac{\int_0^{t>t'} e\epsilon^{-p_2t}\,.\,dt}{\int_0^{t>t'} e\,.\,dt}$$

M and N are determined by plotting the curve for e and t and obtaining therefrom the curves for $e\epsilon^{-p_1t}$ and $e\epsilon^{-p_2t}$. The three curves are then integrated by planimeter.

The ratio between θ_1, the actual deflection, to θ_1', the deflection that would have been obtained if the same displacement were instantaneous, is
$$\frac{\theta_1}{\theta_1'} = N^{\frac{p_1}{p_1-p_2}}\,.\,M^{\frac{p_2}{p_2-p_1}} \quad . \quad (6)$$

This equation gives the error produced by reason of the prolongation of the discharge through the galvanometer when the system is non-periodic.

Equations (5) and (6) only refer to the case where p_1 and p_2 are real, *i.e.* when the galvanometer is overdamped.

For the case where the galvanometer system is oscillatory, p_1 and p_2 are complex; $p_1 = -a + jb$ and $p_2 = -a - jb$, where $a = k/2K = 2\lambda/T$, and $b = 2\pi/T$.
$$\theta_1 = \frac{T}{2CRK}\epsilon^{-\frac{\lambda}{\pi}\tan^{-1}\frac{\lambda S + \pi R}{\lambda R - \pi S}}\left[\int_0^{t>t'} e\,.\,dt\right]\frac{\sqrt{R^2+S^2}}{\sqrt{\pi^2+\lambda^2}}, \quad (7)$$

where $$R=\frac{\int_0^{t\overline{\gtrless}t'} e\epsilon^{at}\cos bt\,.\,dt}{\int_0^{t\overline{\gtrless}t'} e\,.\,dt}$$

and $$S=\frac{\int_0^{t\overline{\gtrless}t'} e\epsilon^{at}\sin bt\,.\,dt}{\int_0^{t\overline{\gtrless}t'} e\,.\,dt}.$$

If the discharge is instantaneous,

$$\frac{\theta_1}{\theta_1'}=\frac{\sqrt{R^2+S^2}}{\epsilon^{\lambda/\pi\tan^{-1}S/R}}. \quad . \quad . \quad (8)$$

This quantity is a measure of the error produced in the deflection due to prolongation of the discharge.

The formula commonly used for the ballistic galvanometer when the discharge is instantaneous is

$$Q=\left(\frac{T}{2\pi}\right)\left(\frac{I}{\theta}\right)\left[\frac{\pi}{\sqrt{\pi^2+\lambda^2}}\epsilon^{\lambda/\pi\tan^{-1}\pi/\lambda}\right]\theta_1,$$

where λ is the logarithmic decrement per half period, and I_g/θ = current sensitivity. Expanding, we obtain by Maclaurin's theorem

$$\frac{\pi}{\sqrt{\pi^2+\lambda^2}}\epsilon^{\lambda/\pi\tan^{-1}\pi/\lambda}=1+0{\cdot}5\lambda-0{\cdot}026\lambda^2 -0{\cdot}055\lambda^3\ .\ .\ .$$

When λ is small

$$Q=\left(\frac{T}{2\pi}\right)\left(\frac{I_g}{\theta}\right)\left\{1+\frac{\lambda}{2}\right\}\theta_1'.$$

Assuming T is unaffected by the damping,

$$Q=G\left(1+\frac{\lambda}{2}\right)\theta,$$

where G is the constant that would be obtained, for instance, from the discharge of a condenser where the galvanometer is open-circuited and λ is nearly zero.

§ (5) Critically Damped Galvanometer.—

$$p_1=p_2=\frac{-k}{2K},$$

$$\theta=(C_1+C_2t)\epsilon^{\frac{-kt}{2K}}+\frac{1}{CRK}\left[t\epsilon^{\frac{-kt}{2K}}\int e\epsilon^{\frac{kt}{2K}}\,.\,dt -\epsilon^{\frac{-kt}{2K}}\int te\epsilon^{\frac{kt}{2K}}\,.\,dt\right],$$

where $$C_1=\frac{1}{CRK}\left[\int te\epsilon^{\frac{kt}{2K}}\,.\,dt\right]_{t=0}$$

and $$C_2=-\frac{1}{CRK}\left[\int e\epsilon^{\frac{kt}{2K}}\,.\,dt\right]_{t=0}.$$

If initially $t=0$, $\theta=0$, and $d\theta/dt=0$,

$$\theta=\frac{1}{CRK}\left[\int_0^t edt\right]t\epsilon^{\frac{-kt}{2K}}.$$

The first throw occurs when

$$d\theta/dt=0=1-kt/2K,$$

$$t_1=\frac{2K}{k}=\sqrt{\frac{K}{\tau}}=\frac{T}{2\pi},$$

whence

$$\left[\int_0^t edt\right]=\left(\frac{T}{2\pi}\right)\left(\frac{I}{\theta}\right)R\epsilon\theta_1=k\theta_1.$$

From this it is seen that the quantity sensitivity is $1/\epsilon$ or 37 per cent of what it is for the undamped case, also the time to reach the maximum deflection is $2/\pi$ or 64 per cent that of the undamped instrument.

§ (6) The Fluxmeter.—The fluxmeter[1,2] (*Fig.* 15) is a special case of the ballistic galvanometer in which the control torque and the air damping are made as small as possible.

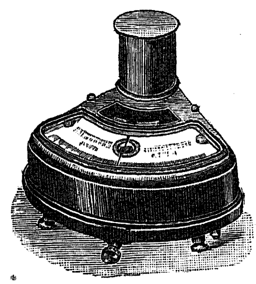


Fig. 15.—Grassot Fluxmeter.

The instrument (*Fig.* 15A) as usually constructed consists of a fairly large open suspended coil of small cross-section. The moving coil B is hung from a spring support by a silk fibre C and the current is led into it by delicate spirals of annealed silver strip rr_1', which exert practically no control over the moving system. The accompanying diagram (*Fig.* 15A) shows the proportions of the moving system.

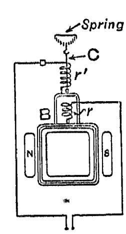


Fig. 15A. — Constructional View of Grassot Fluxmeter.

Great care is needed in the construction of these instruments; one of the chief troubles is to get such satisfactory working that the pointer remains approximately stationary at its deflection, as it should do according to theory, after a flux linkage has been made through it.

Before choosing an instrument, its behaviour when on open circuit should be noted, to see what stability of zero it has and what kind of control—the period should not be quicker than 30 seconds. If tested with, say, a bar magnet, by pulling off a search coil connected to the

[1] "An Electric Quantometer," R. Beallie, *Electrician*, 1903, l. 383.
[2] "Fluxmètre," M. E. Grassot, *Journ. de Physique*, 1904, 4e Ser. iii. 696.

fluxmeter the deflection should be very definite, the pointer moving quickly to its position and remaining stationary, or creeping only very slowly.

If the search coil is pulled off the magnet slowly the deflection obtained should be identical with that obtained when the coil was pulled off quickly. If this occurs when 10 seconds are taken in pulling off the search coil the instrument may be counted a good one.

The external resistance, namely, that of the search coil or other device on which the fluxmeter is being used, must not be too large or the damping will be too small, and, as indicated in the theory below, a term will become important which involves the resistance of the circuit.

(i.) *The Main Features of a Fluxmeter.*—

(1) The readings are independent of the time occupied by the flux change being measured.

(2) The deflection is independent of the resistance of the search coil provided this is less than a certain value (approximately 10 times the resistance of the instrument).

(3) The instrument is portable and quickly set up.

As ordinarily constructed it is not so accurate or sensitive as a ballistic galvanometer and its scale must be calibrated experimentally.

By careful design and as a reflecting instrument, however, the fluxmeter becomes a very valuable piece of apparatus.

(ii.) *Theory.*—When the torsional control τ is zero the roots p_1 and p_2 of the equation $Kp^2+kp+\tau=0$ become $p_1=0$, $p_2=-k/K$.

Under these conditions (see § (4) equation (5)) $M=1$, $N=$definite number, a constant.

The factor

$$N^{\frac{p_1}{p_1-p_2}}\times M^{\frac{p_2}{p_2-p_1}}$$

necessarily becomes 1.

Equation (5) becomes

$$\theta_1 \doteq \frac{n}{CRk},$$

since

$$k=\frac{1}{C^2R}+k', \quad n=\left(\frac{1}{C}+Ck'R\right)\theta_1,$$

where $n=$flux linkages.

The second term $Ck'R$ shows why the air damping and the total resistance R must be small if this term is to be negligible.

To a close approximation under these conditions

$$\Phi=\frac{1}{NC}\theta_1,$$

where $\Phi=$total flux linked with the coil, $N=$turns in the exploring coil.

§ (7) Search Coils.—Search coils are of two kinds:

(*a*) for measuring flux in solid materials, *i.e.* for measuring **B**;

(*b*) for measuring flux in air, *i.e.* for measuring **H**.

Search coils of type *a* call for no special comment. They are usually merely a few turns of thin wire wound so as to embrace the specimen being tested. For instance, in the case of bars of material they are simply an appropriate number of turns wound on top of some paraffined silk tape which is first wound directly on the specimen. For round bars of standard diameter the **B** coil is conveniently wound on a thin brass tube which fits snugly over the rod.

The brass tube must be tested first to see if it is quite non-magnetic. The coil is then wound on the tube with a layer of well-paraffined silk ribbon between. The ends of the **B** coil should be brought out through paraffined silk braid or tube so as to insulate the **B** coil thoroughly from the specimen and hence, in general, from the rest of the magnetising apparatus. If the measurements of **B** are to be made in strong fields it is necessary to make a correction on account of the air space included between the specimen and the search coil. If $s=$area of cross-section of specimen and $s'=$mean area of cross-section of **B** search coil, then the correction to be applied to **B** is $\mathbf{H}(s'-s)/s$. In the case of small specimens in strong fields this correction may amount to 20 per cent of **B**. The area of cross-section of the **B** coil is conveniently determined either by careful measurement or by placing it in the standard solenoid, determining its product of turns and area and hence its area.

Search coils to determine uniformity of **B** in a test rod are very useful. They may consist of two separate coils of 50 turns each wound on a short length of thin-walled brass tube. The two coils are placed, say, 4 or 5 cm. apart, one on each side of the measuring search coils for **H** and **B**. The two coils are connected in opposition, and a zero indication on the galvanometer with reversal of the magnetising current indicates uniformity of magnetisation. It is, of course, most essential that the two coils have an exactly equal number of turns.

§ (8) Variable Mutual Inductometer.—This is a most useful piece of apparatus both for calibrating the ballistic galvanometer and for calibrating the various search coils used in magnetic testing.

A reasonably sensitive ballistic galvanometer should require 2000 ohms in series with it to give a throw of 100 divisions when a **B** of 10,000 is reversed in a search coil of 10 turns on a specimen 1 cm. diameter. Under

these conditions a suitable inductometer for calibrating purposes is one reading up to 1 millihenry.

(i.) *Calibrating Magnetic Apparatus.*—The most accurate standard method of calibrating magnetic apparatus consists in reversing a current of exactly 1 ampere through the primary of such a variable mutual inductometer. The 1 ampere is obtained most accurately by balancing the potential drop along a resistance of 1·0186 ohm, against a standard cell. The arrangement is shown in *Fig.* 16. By setting the mutual inductance

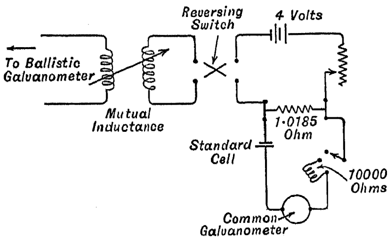


FIG. 16.—Calibration of Ballistic Galvanometer by Mutual Inductometer.

to the requisite value any standard number of line-turns may be obtained to an accuracy of about 1 in 10 000 for values of M near the top reading of the mutual inductometer. This is the method in use at the National Physical Laboratory for calibrating Hibbert's Magnetic Standards, Duddell Inductors, and other secondary standards used for magnetic purposes.

The standard cell circuit should be opened before operating the reversing switch in the mutual inductometer circuit. If, however, the reversing switch is thrown over very quickly the switch in the standard cell circuit may be left on the 10 000 ohm stud. One can then observe immediately whether the current is keeping steady and whether it has the same value after as before reversing.

(ii.) *Calibrating Search Coils.* — Another valuable use of the mutual inductometer is

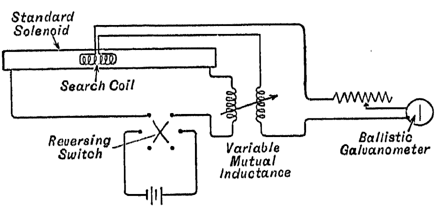


FIG. 17.—Calibration of Search Coils.

in calibrating search coils in the standard solenoid.

The connections are as in the accompanying diagram (*Fig.* 17).

This is a null method in which the line-turns obtained from the search coil on reversing the current in the standard solenoid are balanced by the line-turns obtained on reversing the same current through the mutual inductometer. The balance is obtained by trial and error by repeated reversals of the current.

$$Ns = M \times \frac{10^8}{b},$$

where b = constant of solenoid, *i.e.* the value of **H** for a current of one ampere.

Ns = area-turns of unknown search coil.

M = mutual inductance in henries.

§ (9) STANDARD SEARCH COIL. — This is a very valuable piece of apparatus for checking solenoids and exploring their uniformity of field.

Such a coil consists of an accurately turned small marble cylinder about 3 cm. diameter and 2 or 2·5 cm. long. On this is wound uniformly a single layer of fine-silk-covered wire (100 turns of No. 40 S.W.G. double-silk-covered wire is a convenient winding). The diameter of the wire over the insulation should be measured at a number of places when winding. The marble cylinder should be measured both before and after winding at a number of different diameters in different planes.

The marble should be tested beforehand to see that it is non-magnetic.

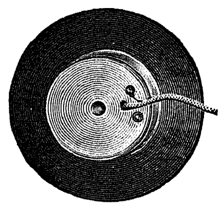


FIG. 18.—Standard Search Coil.

The accompanying photograph (*Fig.* 18) shows such a standard as used at the National Physical Laboratory.

It can be used to calibrate any magnetising winding into which it will go.

When used in connection with a standard solenoid, if the field along the axis is explored over a length of 20 cm., the average **H** so obtained should agree with that obtained by the ordinary solenoid formula (corrected for its ends) to about 1 in 2000.

The number of area-turns of the search coil can be determined with care to an accuracy of 1 in 2000.

§ (10) MEASUREMENT OF MAGNETIC FIELD. —The provision of a uniform magnetic field and the measurement of it are usually the most difficult parts of magnetic testing, on any form of specimen other than a ring, or one in which the section is small compared to the length.

In the case of the ring specimen the average **H** is, of course, given by the formula

$$\mathbf{H}=\frac{4\pi\mathrm{NI}}{10l},$$

where N = total magnetising turns,
I = magnetising current in amperes,
l = length of the ring in cm.

The **H** as thus determined assumes uniformity of winding and uniformity of specimen both in regard to cross-sectional area and to its magnetic properties in different parts.

The uniformity of winding presents no special difficulties, and in the case of machined and annealed rings magnetic uniformity, *i.e.* freedom from poles, is also closely approximated to.

In the case of long rods the **H** can be ascertained by carrying out the tests in a long solenoid (somewhat longer than the rod). The approximate **H** is given by the ordinary solenoid formula and a correction made for the demagnetising field at the centre of the rod due to its poles. (This correction factor F may be approximately obtained by assuming the rod to be an ellipsoid of revolution.[1]) In this case

$$\mathbf{H} \doteq \mathbf{H}_0 - \mathrm{F}\cdot\frac{\mathbf{B}}{4\pi},$$

where $\mathrm{F}=4\pi/p^2(\log_e 2p-1)$; p = length/diam. of the rod. For the case of a rod 100 diameters long F = 0·0054.

If $\mathbf{B}=10{,}000$, $\mathbf{H}=\mathbf{H}_0-4{\cdot}3$. The correction in such a case may be as large as the effective **H**. Excepting the cases where the permeability curve is required in the region of high permeability, this method of determining **H** is trustworthy on thin rods up to 5 or 6 mm. diameter and 100 diameters long.

§ (11) Search Coils for the Measurement of **H**.—These have been used at the National Physical Laboratory for a number of years and have proved themselves to be accurate, powerful, and elastic means of measuring **H**.

Search coils can be constructed to meet nearly every case which otherwise presents great difficulties in the measurement of **H**. The following are a few typical cases and types of suitable coils:

(i.) *Round Rods.*—The form of coil in use at the National Physical Laboratory for the measurement of **H** in rods is an annular one. It consists of a brass tube having a shallow and narrow channel first turned at its centre to accommodate the **B** coil. The ends of the **B** coil are brought out along a groove cut along the tube, as in sketch (*Fig.* 19).

The number of turns of this **B** coil depend, of course, on the section of the specimen and on the sensitivity of the galvanometer. With a reasonably sensitive galvanometer 10 turns are ample on a search coil for 1 cm. diameter specimens. Longer search coils with the **B** coil distributed along the length have also been constructed and are probably better for average **B** and **H** along the bar.

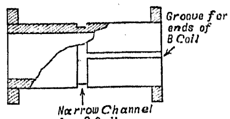


Fig. 19.

The grooves are filled up with paraffin wax so as to present a smooth surface for the **H** coils. The **H** coils consist of at least two windings on top of one another, and if it is desired to explore the field radially from the specimen a third and fourth may be added. The coils must all have *exactly* the same number of turns. There must be no doubt about the equality of the number of turns. The first coil is wound on and the ends brought out through one of the ebonite cheeks. A thin layer of silk separates each coil from the one next it; there may conveniently be an even number of layers per coil so that the ends come out together. In the one cm. single coil under consideration a suitable number of turns of wire per coil would be 500 of No. 42 S.W.G. wire, the inner pair when connected in opposition in the normal way would have a product (N × s) of about 400 (turn × sq. cm.). The length of the coil may be conveniently 3 to 5 cm.

With this length of coil the mean annulus measured by the first two coils in opposition will not be more than 1 or 1·5 mm. wide. The chief drawback to such a search coil is that it cannot easily be made much more sensitive than to give a throw of 100 divisions on the galvanometer for $\mathbf{H}=100$. This sensitivity is not sufficient if, for instance, the hysteresis loop on a specimen of annealed soft iron or silicon steel is required.

Another type of search coil suitable for round rods is what may be termed a saddle coil. It consists of a half circle of thin brass tube or a piece of sheet brass bent to form a saddle thus—

Fig. 19a.—Search Coils for measuring **H** on Round Rods.

This is then wound with thin wire as shown. The wire is wound over the outer convex surface and then wound on to the inner concave surface; each turn must be stuck down with wax or cellulose acetate solution so that it may adhere to the concave surface. Such a coil is very troublesome to wind and takes a long time; it has the advantage over the circular differential coil that the question of exact equality of turns in the two coils in the former case does not arise with the saddle

[1] Maxwell, *Electricity and Magnetism*, ii. 66.

coil; another advantage is that it can be applied more readily to the specimen and can approach very near to the surface of it. By suitable arrangements it can also be quickly withdrawn from the surface, out of the magnetic field without in any way altering the magnetic condition of the specimen. It can therefore measure **H** absolutely instead of measuring changes of **H** only, as in the case of the annular differential coil. Accidental short circuiting of one or a few turns will not render a saddle coil worthless, as would be the case with a differential coil; the saddle type, however, does not seem to keep so constant with time as the more solidly constructed and wound annular coils.

(ii.) *Flat Bars.*—For flat bars the search coils take the form of flat strips of glass or other non-magnetic rigid and permanent material. They may vary in size from minute coils only 5 mm. long and 5 mm. wide × 2 mm. thickness, for use in high magnetisation tests, up to large sizes such as 5 cm. × 5 cm. × 1 mm. thick, for tests on wide flat magnet steel bars. Glass forms a very suitable material, and in many cases selected microscope slide glasses are convenient, or smaller strips cut therefrom.

Flat search coils have also been used in the Epstein apparatus for permeability measurements on sheet materials.[1]

Another type of search coil for flat bars which is very useful is the turning coil. This consists of a small coil wound on a piece of glass about 1·5 cm. × 1·5 cm. The coil is fixed in a recess turned in a small pulley of ebonite which is housed in a frame in such a way that the coil can be turned in its own plane by threads round the pulley; stops locate the position of the coil so that its axis turns through exactly 180°. By such a coil the statical field at any point (for example, on a bar magnet) can be determined without changing the field. The coil measures the component of the field in the plane of the coil and in the direction of its axis. The accompanying *Fig.* 20 shows such a coil. Coils which could be suddenly changed in position have been used by Ayrton and Mather[2] in the form of trigger coils for measurement of fields in the air gaps of electrical machines.

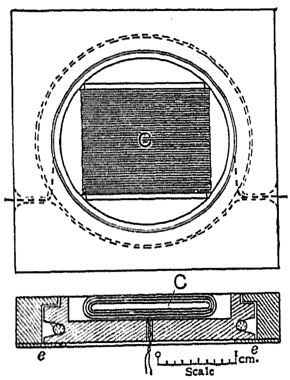


FIG. 20.—Turning Search Coil.

For bent specimens, such as magneto magnets, a successful form of search coil consists of a number of coils wound on flat glass strips 4 cm. long and 1 cm. wide. These coils are then sewn between silk ribbon so as to form a flexible set of, say, 10 coils; these will then adapt themselves to a wide range of curvatures; one set is placed round the outside, and a similar set round the inside, of the bend of the magnet as in sketch (*Fig.* 21). The mean surfaces explored by the coils will be parallel to the surfaces of the magnet and about 1 mm. therefrom. They are calibrated by laying them out straight in a long solenoid.

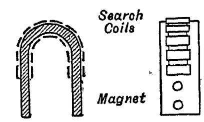


FIG. 21.—Ladder Search Coil for Magnets.

Another form, which is simpler to construct, is to wind a strip of thin ebonite or presspahn so as to form a flexible coil. Such a coil will fit closer to the magnet but will not keep so constant, and there might be some doubt about the value of (area × turns) being the same when in the straight form (as calibrated) and when bent into an arch.

§ (12) OSCILLATION METHOD.—One of the simplest ways of measuring magnetic field strength is by means of the oscillations of a small magnetic needle. This method can, in general, only be used where there is sufficient space to accommodate the needle and to observe its oscillations.

The time of free oscillation of a small magnet when placed in a uniform magnetic field is given by

$$T = 2\pi\sqrt{\frac{K}{MH}},$$

where K = moment of inertia about axis of oscillation,
M = moment of magnet,
H = magnetic field.

If the magnet is first calibrated by observing its period of oscillation in a known field (for example, the earth's field) then the unknown field is obtained directly

$$\mathbf{H}_1 = \frac{T_0^2}{T_1^2} \cdot \mathbf{H}_0,$$

where $\mathbf{H}_0$ is the known field and T_0 = time of swing in it.

This method is quite useful for estimating leakage fields, and stray fields of the order up to **H** = 10; it is not a reliable method for strong fields unless a correction is made for the alteration of moment of the magnet due to the field.

[1] Gumlich and Rogowski, *Elek. Zeit.*, 1911, p. 613; also 1912, p. 262.

[2] *Electrician*, 1895, xxxv. 674.

Unless it is known that the field is fairly uniform the magnet should be short, so that no large variation of field may exist in the space occupied by the needle.

§ (13) FORCE ON CONDUCTOR CARRYING CURRENT.—Since any conductor carrying a current and placed in a magnetic field is subject to a force at right angles to the field,[1] and to itself, this force may be used as a means of measuring the field when the current is known.

The arrangement for carrying this out [2] is as follows (*Fig.* 22):

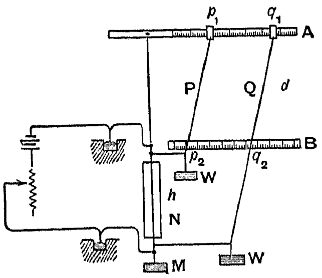


FIG. 22.—Pendulum Method of measuring Magnetic Field.

In this system of measuring the magnetic field (between the poles of a magnet, for instance) a wire h of sufficient stiffness not to distort from straightness is suspended vertically by a thread and weighted with a weight M.

Sighting marks behind the threads above and below the wire enable it to be accurately located in the position it has normally with no current or no field acting.

Two fixed horizontal scales A and B, at a known distance d apart, are provided, and from the upper one, two threads P and Q are suspended on sliders p_1 and q_1. These threads have equal weights W hung on them; they are connected to wire h by thin horizontal threads as shown.

To make a determination of the field **H**, observations on A and B are first made with no current in h. The sliders p_1 and q_1 are moved so as to effect this; they are moved sufficiently to the left to cause the horizontal connecting threads to be quite slack, all three strings being now, of course, vertical. Current i is then switched on through h. The sliders p_1, q_1 are then independently moved to the right until h is in its zero position. The four readings on A and B are then taken. Assume the scales are set so as to give the same readings p_0 and q_0 on A and B when in the zero position.

Then the total horizontal force maintaining the wire h vertical and balancing the force on the conductor is given by

$$\mathbf{H}li = 10\frac{Wg}{d}(p_1 - p_2 + q_1 - q_2),$$

where W is in grams,
i in amperes,
g = acceleration due to gravity = 981,
l = length in cm. of wire h acted upon by field.

In this method it is desirable to make one set of observations after bringing wire h to its zero position from a deflection to the right, and then another independent set should be made bringing h to zero from a deflection to the left. The difference, if any, between corresponding readings forms a valuable check on the accuracy of setting and indicates the amount of uncertainty due to stiffness of the threads, control from the leading-in wires, etc.

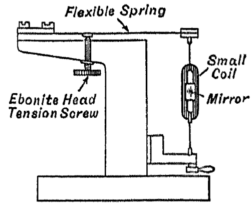


FIG. 23.—Deflecting Coil for Measurement of Magnetic Field.

Another method of utilising the force on a conductor carrying a current is to observe the torque produced on a narrow coil placed with its plane parallel with the field. The coil may be only 1 or 2 mm. wide and about 1·5 cm. long suspended in a small frame by strained strip suspension as in *Fig.* 23.

Such a mounted coil forms a very convenient device for measuring the flux in air gaps of magnets and especially in narrow gaps.

Such a device is best calibrated by using a magnet of known gap flux density as determined by the previous method. When such a known magnet is available, the apparatus can be set to read gap flux density directly using a known current and adjusting the tension on the suspension.

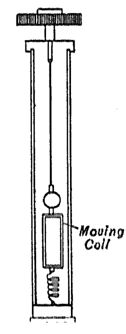


FIG. 24.—Damped Moving Coil for Measurement of Magnetic Field by Damping.

The damping effect has been used to measure magnetic fields by P. E. Klopsteg.[3] The moving coil (*Fig.* 24) is an ordinary galvanometer type of narrow coil mounted in a housing for convenient insertion into the field

[1] See "Electromagnetic Theory," § (9).
[2] A. Gray, *Absolute Measurements in Elec. and Mag.* ii. part 2, p. 700.
[3] "The Measurement of Magnetic Fields by their Damping Effect upon a Vibrating Coil," P. E. Klopsteg, *Phys. Rev.*, 1913, ii. Ser. 2, p. 390.

to be measured. A coil resistance of 120 ohms is suitable.

If H = magnetic field,

l, d, and n = length, breadth, and number of turns of coil,

T_0 = free complete undamped period,

T_1 = complete period on closed circuit,

R = total resistance in circuit when motion of coil is periodic (ohms),

R_0 = total resistance necessary for critical damping,

λ_0 = log dec. (complete period) on open circuit,

λ = log dec. (complete period) on closed circuit,

then $$H = G\sqrt{\frac{R}{T_0}(\lambda - \lambda_0)},$$

where $G = 2\sqrt{10^9 K}/nld$, K = moment of inertia of coil.

For critical damping,

$$H = G\sqrt{\frac{R_0}{T_0}(\pi - \lambda_0)}.$$

The displacements may be made by means of a condenser discharge and times of oscillation by scale and stop-watch. K is calculated from observations using an added inertia mass of known constants.

§ (14) THE MAGNETIC POTENTIOMETER.—One of the most useful devices for exploring magnetic fields is that known as the magnetic potentiometer, due originally to Chattock.[1] It measures directly the difference of magnetic potential between any two accessible points. The theory of the device is as follows: Let P and Q (*Fig.* 25) be two points in a

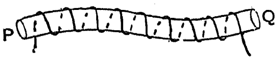


FIG. 25.—Chattock Potentiometer.

magnetic field connected by any line (straight or otherwise) of length l; and let $\mathbf{H}$ represent magnetic force resolved along l. Then if γ be the difference of magnetic potential between P and Q

$$\gamma = \int \mathbf{H} \,.\, dl.$$

If, instead of points, P and Q represent two equal plane surfaces of area S, and γ be their average difference of potential,

$$S\gamma = \int \gamma \,.\, dS = \int \mathbf{H} \,.\, dU,$$

where U = volume of a tube of constant cross-sectional area S, connecting P and Q by *any* path.

[1] *Phys. Soc. Proc.*, 1888, ix. 23; *Phil. Mag.*, 1887, xxiv. 94.

Let a uniform helix be wound upon such a tube, N being the turns per cm. length. Then if the field $\mathbf{H}$ varies with the time t, and the permeability within the tube is unity, the electromotive force e set up will be

$$e = \frac{d}{dt}\int\int \mathbf{H} \,.\, dS \,.\, N dl = \frac{d}{dt} N \int \mathbf{H} \,.\, dU = NS\frac{d\gamma}{dt},$$

i.e. the E.M.F. is proportional to $d\gamma/dt$ only, if N and S are constant.

Hence if the ends of the helix are connected to a ballistic galvanometer and γ changed quickly from γ_1 to γ_2 there will be an impulsive throw on the galvanometer coil such that $\gamma_1 - \gamma_2 = k\theta$ when θ is small.

The combination, therefore, forms a magnetic potentiometer.

Such a helix has many valuable uses. Thus if one end is held fixed at any point and the other end moved quickly from the same point to another position in a magnetic field, a throw will be obtained proportional to $\int \mathbf{H} \,.\, dl$ along the line joining the two ends.

A very convenient form for such a helix is to wind a strip of presspahn about 1·5 cm. to 3 cm. wide and 30 cm. long with one or more layers of thin wire and a fine silk thread wound together. This will form a flat helix whose turns per cm. and cross-sectional area remain constant and uniform along the length so that the relation $dS \,.\, N dl = N \,.\, dU$ may hold whether the strip is straight or bent.

Another form of the potentiometer as used by Chattock is to take a piece of thick-walled rubber tubing and wind it in a screw-cutting lathe with wire, leaving a small space between the uniformly distributed turns.

R. Goldschmidt's[2] type of the apparatus uses an iron core divided at the middle, leaving an air gap in which a small compass needle is pivoted. Near each end of the core a coil of known number of turns is wound, so that by sending a measured current I through them any desired magnetomotive force may be applied. If the ends of the core are at any two points P and Q and the current I is adjusted until the magnetic needle shows no deflection, then the magnetic difference of potential between P and Q is equal to the magnetomotive force applied by I.

If $lN\Phi$ be the total flux-turns in the helix,

then $$lN\Phi = NS\int_P^Q \mathbf{H} \,.\, dl,$$

where P and Q represent the ends of the helix.

The quantity NS representing the constant of the helix may be readily determined by calibration in a standard solenoid.

A valuable check on this calibration and on the uniformity of the helix is to thread it through a coil of known number of turns and

[2] R. Goldschmidt, *Electrician*, 1904, liv. 207.

bring the ends together so as to form a closed toroidal coil as in *Fig.* 26.

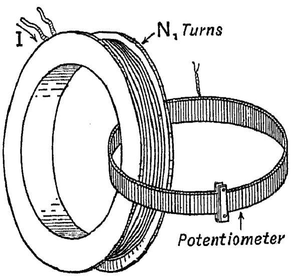


FIG. 26.—Use of Magnetic Potentiometer for Determination of Turns in a Coil or for checking Uniformity of the Potentiometer.

If a current I be started in the coil the total magnetomotive force due to it will be

$$0{\cdot}4\pi N'I,$$

and hence $\quad 0{\cdot}4\pi N'I = NSG\theta,$

where θ = galvanometer throw and G is a constant.

The linked potentiometer is connected up to a mutual inductometer and a balance obtained for reversal of the current or when

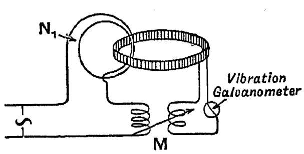


FIG. 26A.

alternating current is used, as in the diagram (*Fig.* 26A).

For a balance we have

$$NS = \frac{10^8 M}{4\pi N_1}.$$

M = Mutual inductance (henries).

N_1 = Turns in coil threading helix.

By moving the helix round so that various parts are brought within the coil, a test of uniformity of winding is made; there should be no change in the value of M.

Applications.—(*a*) Finding the number of turns in a coil.

This use follows immediately from the method of testing the potentiometer described above.

(*b*) Measuring reluctance of a magnetic joint, as for instance between frame and pole of a dynamo or motor, between keeper and magnet when testing permanent magnets, between magnet and rail in track brake, etc. If calibration is made using a coil of known number of turns and a known current, the ampere-turns used up in the joint can be immediately determined.

In using the potentiometer for this purpose the two ends are placed close together on one side of the joint and one of them is then quickly moved across the joint to the iron on the other side and the resulting throw taken.

(*c*) Leakage fields may be measured, and, by using a vibration galvanometer, alternating magnetic fields may be explored.

(*d*) Tests may be made on bar magnets and magnet steel by stepping out from one point to another; in general the same tests can be carried out as with the rigid search coils on glass, but the potentiometer cannot be considered so permanent in its calibration as the glass search coils.[1]

§ (15) REVOLVING DISC FLUXMETER (K. T. Fischer).[2]—In this apparatus for measuring magnetic fields, a small disc is driven at a speed from 50 to 100 revs. per second. The E.M.F. induced between rim and spindle of the disc is used as a means of measuring magnetic field. It is possible to measure fields as small as the earth's horizontal field, using a mirror galvanometer. The apparatus as made by Hartmann & Braun consists of a thin disc (of area = 8 sq. cm.) having a groove on the edge. A silver wire passes round the disc and also round a similar disc driven by clockwork. The wire serves to make contact with the rim. A platinum iridium brush makes the inner contact. The speed is measured either by counter or frequency meter. The flux is read directly on the scale of a millivoltmeter.

§ (16) MEASUREMENT OF MAGNETIC FIELD BY BISMUTH SPIRAL.—The effect of a magnetic field upon the electrical conductivity of metals, first discovered by Lord Kelvin in 1856 and known as the Hall[3] effect, is very marked in the case of bismuth wire or plates. A great number of experimenters have investigated the effect in bismuth, notably Righi,[4] Leduc,[5] Ettinghausen,[6] and Lenard.[7]

More recently Houllevigue[8] and Jewett[9] have shown that the effect is closely connected with the crystalline structure of the metal, since thin films formed by cathodic spluttering do not show the effect at all when deposited cold. Becker,[10] Richtmyer and Curtiss,[11] have

[1] A large number of applications of these flexible coils are given in the *Archiv für Elek.*, 1912, i. 141; 1913, i. 511, and 1914, ii. 303 (W. Rogowski and W. Steinhaus).
[2] *Deutsch. Phys. G.U.*, Nov. 1905, 7, xxii. 434-9.
[3] Hall, *Phil. Mag.*, 1880, (5), ix. 225.
[4] Righi, *Jour. de Phys.*, 1884, (2), iii. 355.
[5] Leduc, *C.R.*, 1886, xcviii. 673, cii. 358.
[6] v. Ettinghausen and Nernst, *Wien. Ber.*, 1886, xciv. II. 560.
[7] Lenard and Howard, *Elek. Zeit.*, 1888, ix. 340; and Lenard, *Wied. Ann.*, 1890, xxxix. 619.
[8] Houllevigue, *C.R.*, cxxxv. 626.
[9] Jewett, *Phys. Rev.*, 1903, xvi. 51.
[10] Becker and Curtiss, *Phys. Rev.*, 1920, (2), xv. 457.
[11] Richtmyer and Curtiss, *Phys. Rev.*, 1920, (2), xv. 465.

investigated this effect quite recently and have obtained a number of interesting results.

From the practical point of view of measuring magnetic fields, the most convenient form of the wire is that of a thin flat bifilar spiral. These spirals have been made for a number of years by Messrs. Hartmann & Braun.

A typical case indicating the order of the change in resistance at ordinary temperatures is given in the annexed table from Lenard:[1]

Field (Gauss).	Resistance.
0	1
2,000	1·049
5,000	1·171
10,000	1·420
15,000	1·687

For accurate measurements it is essential to take account of temperature because of the large temperature coefficient of bismuth wire (+0·0042 at 18° C.). For this reason it has been proposed to wind a spiral of copper or platinum side by side with the bismuth for the purpose of making accurate temperature measurements. In the case of the films referred to above, it was found that the temperature coefficient was negative and of the order −0·002 ohms per ohm over a range 0° to 200° C. By successive heatings to a temperature near the melting-point, the Hall effect was again obtained with the sputtered films, but was in the best cases only about 1/20 of the value for wires, *i.e.* $\Delta R/R = 0{\cdot}015$ with $\mathbf{H} = 10{,}000$.

The large negative temperature coefficient of the films, however, indicates the possibility that they might be used in conjunction with a wire spiral to produce a combination having zero temperature coefficient.

II. Forms of Specimen and Preparation of the Material

(i.) *Forms of Specimen.*—The form of specimen to be tested will vary widely with the nature of the information desired, partly on account of the limitations of the various methods of testing and partly on account of the nature of the material, rapidity with which tests can be made, accuracy required, etc.

(ii.) *Treatment of Material.*—The treatment of the material, both thermal and mechanical, is of considerable importance in testing, depending on the particular qualities desired, and on the form in which the material will be used.

These two factors are dealt with in general terms below, but no fixed laws can be laid down which will serve for every case. New materials may require special treatment of their own which can only be found by experiment. This will react immediately on the nature of the tests required.

The preparation and heat treatment of magnet steels is dealt with in §§ (19), (20).

§ (17) Forms of Specimen. (i.) *Rings.*—Ring specimens may be used for both solid material and sheet material, in some cases also with wires.

(*a*) *Solid Material.*—The rings should, in general, be machined from a representative piece. The most suitable form of cross-section, from the point of view of ease and accuracy of production, is rectangular. For high accuracy the ratio of radial thickness to outside diameter should be not greater than 1/15. If results accurate to 1 per cent are sufficient this ratio may be increased to 1/10. Corrections due to variations of **H** from inside to outside of the ring are given in the section on the testing of rings.

(*b*) *Sheet Material.*—The same remarks regarding ratio of radial thickness to diameter apply as in the case of solid rings. Since, however, the effects of cutting or stamping the rings penetrate to a considerable distance inwards from the edge sheared, it is desirable to have a fair radial width of specimen unless it is to be annealed after cutting; for this reason the diameter of the rings may with advantage be large, say 17 cm. outside diameter and 14 cm. inside diameter. If alternating magnetic tests are to be made, paper insulation should be used between the rings to avoid eddy currents across from ring to ring.

To represent the average permeability of the material, the rings should be assembled with the direction of rolling distributed, so as to keep the reluctance uniform round the sample.

Ring specimens may with advantage be used in the following cases:

(*a*) Where very accurate results are desired and the value of $\mathbf{H}_{\text{max.}}$ does not exceed 200.

(*b*) Where reliable results are desired on the magnetic properties of materials in the region where the permeability is a maximum. In some apparatus the material is used in a specially annealed condition at a region of **B** where μ is very high (greater than 6000). The remanence and hysteresis losses may also require to be determined. In such cases it is practically essential to carry out the tests on rings.

(*c*) Ring specimens should be used as standards of comparison when setting up an apparatus for testing bars and strips. In such cases the sampling should be done with care, so that the rods and ring or strips and rings have as nearly as possible the same magnetic properties. At least two rods should be prepared, one cut with its length in

[1] Lenard, *Wied. Ann.*, xxxix. 619

the direction of rolling in the case of the rolled material, and one at right angles to it. If a double yoke method is to be used it will be preferable to prepare two rods in each direction, forming a square, and to cut the ring from the central portion thus:

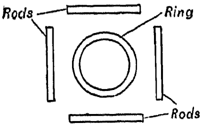


FIG. 27.—Sampling of Magnetic Material.

In the case of sheet material, strips may be cut distributed round the ring stampings.

In cases where samples are to be used as standards of reference, it is desirable that they should be slowly annealed and then aged.

§ (18) RODS AND STRIPS.—A machined rod is such a very convenient form of specimen to produce accurately, and the tests on it can be carried out with such expedition, that it is very widely used commercially as a test specimen.

For good accuracy the rod should be fairly long compared to its diameter. A suitable proportion is 1 cm. diameter and 30 or even 35 cm. long. The rod should be uniform in diameter and chosen, if possible, to be uniform in its magnetic qualities. It is specially desirable to have the end portions accurate in diameter for a length of 5 cm. at each end to ensure accuracy of fit in yokes.

Strips may be cut about 1 to 1·5 cm. wide and 25 to 30 cm. in length to suit the particular apparatus on which they will be tested. Equal numbers should be taken with and at right angles to the direction of rolling. They should be cut of uniform width so that the section may be uniform along the length.

With samples of the dimensions given above, tests up to $\mathbf{H}_{\text{max.}}$ of about 500 may be made, and with difficulty up to $\mathbf{H}=1000$.

For tests in strong fields (above $\mathbf{H}=1000$) it is necessary to reduce the size of specimen owing to the difficulty of producing the magnetic field throughout a length of more than a centimetre or two.

A convenient size of specimen for tests up to $\mathbf{H}=5000$ is a small turned rod from 3 to 5 mm. diameter and 70 mm. long. No particular care beyond obtaining uniformity of diameter need be taken, since the magnetic properties of materials in strong fields are not very dependent on mechanical treatment.

For strips, a convenient size is a small bundle having a cross-section of 5 mm. × 5 mm. and 70 mm. long. The effects of cutting or bending are usually small or negligible in such tests.

§ (19) HEAT TREATMENT.—In many cases it is desirable or necessary to obtain information regarding the properties of magnetic materials in the slowly annealed state. Sometimes also, in order to wipe out the previous heat treatment of the specimen, it is necessary to raise the temperature of the material to a point beyond the magnetic transformation temperature.

Rods and strips, or wires, may be conveniently slowly annealed in a tubular electric furnace; the material should be protected from oxidation by embedding the specimens in a material like magnesium oxide after packing them tightly in a metal tube filled with air-excluding inert substance. A more troublesome but more effective method of protecting the specimen is to maintain a slow steady stream of nitrogen or hydrogen round it. This method can be used in the case of electric furnaces, but is difficult with muffle or gas-heated furnaces.

The temperature of the specimen should be raised to about 850° C. and then allowed to fall slowly. The rate of cooling should be specially slow through the first 200° C. This may be secured by reducing the heating current or gas gradually so as to occupy, say, two or three hours over this stage of the cooling.

In tests of total losses with alternating magnetisation it is sometimes desirable to determine the ageing effect, *i.e.* the alteration (usually an increase) in the hysteresis losses with time. This can be carried out in an artificial manner by prolonged heating at 100° C. For this purpose the sheet material is placed in a self-regulating oven at 100° C. and kept at this temperature for three or four weeks. The losses are then again measured, and may be taken to represent the permanent properties of the material.

§ (20) DEMAGNETISATION.—In practically all magnetic tests, and in particular for tests within the region $\mathbf{B}=0$ to $\mathbf{B}=15,000$, it is necessary first to get the material into a reproducible cyclic condition. It is found that the properties of the material under any given magnetising force are different if the material has previously been subjected to a magnetising force of greater value, unless the effect is carefully removed by gradual diminution of the magnetising field from a sufficiently high value. The magnetising force is continually reversed during the diminution.

In the majority of cases when tests are made on the softer magnetic materials, the following procedure is sufficient and necessary to bring the specimen into a steady cyclic condition.

A magnetisation of about $\mathbf{H}=15$ is applied. Reversals of current are continuously made at the rate of about 1 per second; the magnetising current is slowly and smoothly reduced down to a value not greater than

corresponding to the smallest value of **H** at which measurements will be made. The number of reversals should be not less than 100 and they should, strictly, be made in a way that causes $\mathbf{B}_{\text{max.}}$ to decrease uniformly and not $\mathbf{H}_{\text{max.}}$. This, however, requires some idea of the shape of the permeability curve.

Alternating current may be also used for demagnetising test specimens. An apparatus for this purpose designed by E. Gumlich and Rogowski [1] consists in a transformer having a primary winding in the form of a cylindrical coil 50 cm. long and 6 cm. clear internal diameter. The secondary winding — on a laminated iron core—is guided in a vertical frame and can pass completely into the primary winding.

By means of a rope and wheels the secondary winding can be slowly and smoothly withdrawn from the primary, and in this manner the secondary voltage can be gradually diminished to zero from any desired value.

The specimen is placed inside a magnetising coil connected to the secondary of the variable transformer; the current is adjusted to carry the magnetisation up to a point above the knee of the permeability curve ($\mathbf{B}=15{,}000$). The secondary winding is now slowly and continuously withdrawn from the primary coil which carries a constant alternating current. In this way a very smooth demagnetisation can be secured.[2]

In carrying out the tests a number of reversals (about 20) should be made at each point observed, and, if any doubt exists as to whether the demagnetisation has been thorough enough, an observation of **B** should be made at, say, $\mathbf{H}=3$. Twenty reversals should then be made at a slow rate and another observation taken. If the second value of **B** is the same as the first, the demagnetisation may be considered satisfactory. If the second value is smaller than the first, another 20 reversals should be slowly made and a further observation of **B** taken. When constancy of deflection for reversal of the same **H** has been obtained, the material may be considered in a cyclic condition.

§ (21) Measurements on Ring Specimens.—The measurements usually made on ring specimens are (*a*) tip-point permeability curve up to $\mathbf{H}_{\text{max.}}$ not exceeding about 200; (*b*) hysteresis loop with a particular value of $\mathbf{B}_{\text{max.}}$ or of $\mathbf{H}_{\text{max.}}$. A suitable value of $\mathbf{B}_{\text{max.}}$ is 10,000 because the index 1·6 is fairly accurately true in this region of $\mathbf{B}_{\text{max.}}$, and hence the value of η in the formula $h=\eta\mathbf{B}^{1\cdot6}$ can be determined and compared with the known value of this constant for good-quality similar material and for various other materials.

[1] E. Gumlich, *Magnetische Messungen*, pp. 28-29.

[2] A very complete investigation of the various methods of demagnetisation is given in a paper by C. W. Burrows, *Bull. Bureau Stds.*, 1907–8, iv. 205.

In the case of solid rings the cross-sectional area is most accurately determined from micrometer measurements of width and depth at a number of places round the ring. Check measurements should be made by weighing and calculation from the density if this is known or can be assumed.

In the case of sheet stampings the thickness of the rings, and hence the cross-sectional area, cannot be measured directly by gauge to sufficient accuracy; the density should be determined if an accuracy greater than 1 per cent is desired. It is common to assume a density for the sheet materials, for instance 7·80 for soft sheet-iron and low carbon steels, and values varying from 7·75 to 7·55 for silicon alloys according to silicon content.

The measurements completed, a layer of soft tape is wound on the ring. The search coil for determining **B** is next wound uniformly spaced round the ring; for a ring of approximately 1 sq. cm. section it may consist of 10 turns of thin wire in paraffined silk tube or braid. The high insulation of the **B** search coil is essential when using a ballistic galvanometer; well-paraffined silk may be considered perfect insulation for the purpose.

The search coil having been wound on, it is covered with a further layer of soft tape, for protection against damage, and the magnetising winding put on.

Where many rings have to be tested, much time can be saved by making a stranded flexible cable consisting of, say, 6 strands of triply-cotton-covered flexible conductor twisted together and taped up. A 5-pin plug and socket at the ends connects the strands in series. By this means a winding of 240 effective turns can be applied to a ring in 5 minutes.

For carrying out the tests, the following set-up of apparatus has been found very convenient and flexible in working:

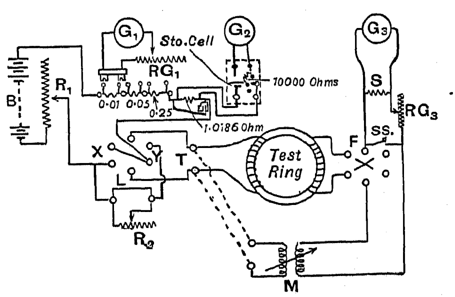


Fig. 28.—Set-up for Tests on Ring Specimens.

The battery B is a 24-volt battery capable of giving 30 amperes if necessary. R_1 is a series of graded resistances giving smooth

regulation of current from 30 amperes down to 0·1 ampere. For smaller currents a lower voltage is used. The current measuring shunts are permanently connected in circuit, but the 0·05 ohm and the 0·25 ohm shunts can be cut out when large currents are used. The resistance marked 1·0186 ohm is for calibrating purposes, enabling 1 ampere to be accurately obtained with the aid of the standard cell and balancing galvanometer.

The main mercury reversing switch is arranged so that when in the Y position the additional set of resistances R_2 can be included in the circuit by removal of the short-circuiting link L. The resistances R_2 should be graded to carry from, say, 10 amperes to 0·1 ampere on the full 24 volts.

The connections to the mercury switch are so arranged that when in the X position the current traverses the diagonal connecting link; by this means the resistance of the circuit can be kept exactly the same when the switch is in either position.

The magnetising winding of the test ring is connected to the main terminals T. The secondary winding is connected through the small reversing switch to the ballistic galvanometer.

In this circuit are included the adjustable resistance box RG_3, the galvanometer shunt S, and the secondary winding of the calibrating variable mutual inductometer M, or standard solenoid, as the case may be.

The current or **H** measuring galvanometer G need not be highly sensitive, but should be permanent in its calibration and free from zero creep. Of course, if desired, an ordinary ammeter may be used, but the galvanometer is more accurate and much more convenient to work with. The resistance box RG_1 is included in this circuit, and by adjustment of this in conjunction with the standard 1 ampere the galvanometer G may be calibrated to read **H** directly or to read some exact and simple multiple or submultiple of it.

Calibration.—It is a great convenience to have the galvanometers G_1 and G_3 so calibrated as to read **H** and **B** directly.

(i.) *Calibration for* **H**.—

If N_1 = total magnetising turns,
N_2 = turns on search coil,
l = mean length of ring—cm.,
s = cross section of ring—sq. cm.,
I = magnetising current—amperes,
$\mathbf{H}/\mathbf{I} = 4\pi N_1/10l$.

The current is set to 1 ampere and galvanometer G_1 made to give a reading equal to $4\pi N_1/10l$ by adjusting res. RG_1. In a normal case this will be when the galvanometer is on the 0·05 ohm shunt.

(ii.) *Calibration for* **B**.—Using a mutual inductometer and 1 ampere reversed in primary. M is set to be equal to $N_2 \times s \times 100$ microhenries. The resistances S and RG_3 are adjusted so that on reversal of 1 ampere in primary of the mutual inductometer when set to the value of $N_2 \times s \times 100$ (microhenries) a throw of 100 divisions is obtained. The ballistic galvanometer will then read so that a throw of 100 divisions corresponds to a **B** in the ring sample of 10,000. If a solenoid and secondary coil are used as standard for calibration purposes, then ballistic galvanometer throw should be made equal to $M/N_2 s$ divisions for 1 ampere reversed. M = mutual inductance of solenoid (microhenries).

In general it is not necessary to make a correction for the air space included in the search coil for **B** unless the section of the ring is very small. The correction to be applied to **B** is $(s_1 - s)\mathbf{H}/s$, where s = sectional area of ring, s_1 = cross-sectional area of **B** search coil. In a small ring the correction may amount to -1 per cent of **B** at $\mathbf{H} = 100$.

§ (22) Procedure for Testing. (i.) *Cyclic Permeability Curve.*—Demagnetisation is first carried out as previously described, an initial magnetising field of about 20 being applied as indicated on G_1, R_1 being smoothly and slowly increased until the cyclic **H** is a little smaller than the lowest value of **H** at which observations are to be made. The magnetising current is now set so that G_1 reads a definite value of **H**, say $\mathbf{H} = 1$. The reversing switch is now operated twenty or more times, the short-circuiting switch SS being meanwhile closed. An observation of **B** is now made. A further set of about 20 reversals is now made and then again observed. If the value obtained is the same in the second case as in the first observation, it may be assumed that the demagnetisation is satisfactory. If, however, the second observation gives a different value for **B** than the first, it is a fairly certain indication that the demagnetisation was imperfect, and should be carried out again more carefully. Having satisfactorily obtained the first point, the current is raised until another exact value of **H** is obtained, 20 reversals are again made and the corresponding **B** observed. In this way the curve connecting **H** and **B** is obtained point by point, reversing the magnetisation at each stage until a cyclic condition is indicated. Eight or nine points, if suitably chosen, will delineate a curve with sufficient accuracy between $\mathbf{H} = 0$ and $\mathbf{H} = 150$.

(ii.) *Hysteresis Loop.*—The procedure is as follows. If the loop is required for a definite value of $\mathbf{B}_{max.}$ the corresponding value of $\mathbf{H}_{max.}$ is found from the previously determined permeability curve. If the loop is required for a definite value of $\mathbf{H}_{max.}$ this can, of course, be immediately set.

The magnetising current is set so that G_1 reads, on an extended scale, the necessary value of $\mathbf{H}_{max.}$. A number of reversals are performed and the $\mathbf{B}_{max.}$ observed to see that the actual $\mathbf{B}_{max.}$ desired is obtained.

The switch in the X position is now brought to the "off" position and the corresponding throw obtained. This throw must be multi-

plied by 2 to give the change in **B**, and a second column should be provided headed **B′**. The values set down in this column are $2 \times \mathbf{B}$ (throw) except for $\mathbf{B}_{max.}$, which is set down at its actual value.

The reason for this is that in the original calibration the ballistic galvanometer has been made to read directly when the **B** has been reversed from its maximum positive to the same negative value, and hence the (**B** change) has been $2\mathbf{B}_{max.}$. Consequently, when a **B** change from say $\mathbf{B}_{max.}$ to $\mathbf{B}_{rem.}$ is made, the galvanometer will only read half the change.

The value of **B′** being obtained when **H** is changed from $+\mathbf{H}_{max.}$ to 0 is subtracted from $\mathbf{B}_{max.}$ and gives $\mathbf{B}_{rem.}$.

To obtain other points on the hysteresis loop the link L is used in conjunction with the rheostat R_2.

Thus, to obtain any point P (*Fig.* 29) corresponding to an **H** of $-$ON the current-reversing switch is placed in the Y position, the link L is removed and R_2 adjusted, until the desired **H** is obtained. The link is now replaced and a number of cycles given to the specimen, finishing up with the switch in the X position. The link L is now removed and the switch thrown over to the Y position. The throw obtained multiplied by 2 gives the **B** change in passing from T to P. This value subtracted from $\mathbf{B}_{max.}$ gives **B** at the point P.

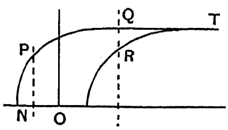


FIG. 29.

It is sometimes easier to observe flux changes from the remanence point because the full magnetising current corresponding to $\mathbf{B}_{max.}$ need not then be kept on longer than a momentary time.

To determine a point Q on the descending portion of the loop corresponding to a positive **H**; the switch is placed in the Y position, link L is removed and the current adjusted by R_2 to give the desired **H**.

The link is now replaced and a number of cycles with $\mathbf{B}_{max.}$ given, ending with the switch in the Y position.

The ballistic galvanometer reversing switch F is now reversed. To make an observation; the current switch being in the Y position and current at $\mathbf{H}_{max.}$, link L is suddenly removed and the throw observed. Twice this throw gives the **B** change from $\mathbf{B}_{max.}$ to Q, and hence gives **B** at Q.

To observe the **B** at a point R, where **H** is the same as at Q but on the ascending portion of the loop; the initial procedure is as before except that the current switch must be held at X, then the link L removed before throwing over to the Y position, which is now done. The throw is then observed when link L is suddenly inserted. Twice this throw gives the **B** change from R to T.

Several precautions must be observed when making observations on hysteresis loops and on permeability tests.

(*a*) If the magnetisation is accidentally taken above the point under observation, demagnetisation by repeated reversals must be carried out until the field is reduced to a value somewhat below that being observed, before proceeding.

(*b*) In performing any operation on a hysteresis loop where the cycle is interrupted at any point such as P, for the purposes of observation or adjustment, after this has been made, the cycle should be continued on from that point, in the proper direction to $\mathbf{B}_{max.}$ and then to $\mathbf{H}_0$ when switching off.

(*c*) The temperature of the ring must not be allowed to rise appreciably during a test; for this reason, when observing a hysteresis loop with $\mathbf{H}_{max.} = 100$, as many observations as can with accuracy be made should be made, using the remanence point as the point of reference. The currents need only be switched on and cycled for a second or two and then switched off so as to bring the ring to $\mathbf{B}_{rem.}$ ready for an observation to be taken onwards from that point.

(*d*) As a check on the measurements it is desirable to observe one or two points on the loop by arriving at them in different ways. This will show whether viscosity effects are occurring, scale errors, time effects on the ballistic galvanometer, etc.

When observing an hysteresis loop on soft material with $\mathbf{H}_{max.}$ say 100, it is essential to change the shunt reading **H** on G_1 to a low value when determining that part of the loop between $\mathbf{B}_{rem.}$ and **H**, say -5, in order to obtain accurate readings of **H**, hence the cycle must be interrupted at $\mathbf{B}_{rem.}$, *i.e.* $\mathbf{H} = 0$, and the $\mathbf{B}_{rem.}$ used as a point of reference. Care must be taken to short-circuit the low-reading shunts before completing the cycle to $-\mathbf{H}_{max.}$ immediately before switching off.

(iii.) *Determination of Hysteresis Loss h and Steinmetz Coefficient η.*—For soft materials and with $\mathbf{B}_{max.} = 10{,}000$ the curve may be plotted as in *Fig.* 30 and the area of the half loop measured by planimeter. The planimeter should be calibrated on the same squared paper as the curve is plotted, and not by any markings in sq. cm. on it unless the paper is of exceptional quality.

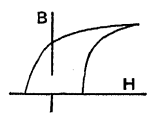


FIG. 30.

The hysteresis loss is given by the equation $h = \eta \mathbf{B}^{1.6} =$ area whole **HB** loop/4π, ergs per c.c., the area being expressed in **HB** units.

The coefficient η is obtained from the expression $h = \eta \mathbf{B}^{1.6}$. (*Note.* $10\,000^{1.6} = 2.512 \times 10^6$.)

In measuring the hysteresis loss for a loop with $\mathbf{H}_{max.} = 100$ it is necessary to plot in two parts as in *Fig.* 31. If this is not done the loop will be so narrow that it is impossible to measure the area accurately.

In a very well annealed soft iron the H_c may be only 0·8 with $\mathbf{H}_{\max.}=100$.

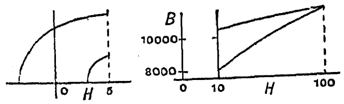


FIG. 31.

(iv.) *Möllinger's Quick-winding Apparatus for Ring Specimens.*—For rapid tests on rings of sheet material, more particularly in connection with total loss measurements, Möllinger has developed a system of 100 turns for the magnetising winding, each turn consisting of a hinged conductor with a conical plug end fitting into a corresponding socket on the next one. They are connected in groups of ten to an excentric device below by means of which they can at one operation all be connected in series forming a uniform winding over the ring.

The mean diameter of the rings so tested is about 27 cm. so that the magnetising force for 1 ampere is only about 1. Such a winding is very suitable for tests of total losses in the region where **H** is not greater than 10—15, but the method is not suitable for permeability tests except in low fields, owing to the large current required.

III. Magnetic Measurements on Rods and Bars

(A) Measurements in Moderate Fields (H 0 to 500)

§ (23) Magnetometer Method.—This is one of the oldest methods and gives quite reliable results for certain kinds of tests. For such a method the specimens should be either in the form of an ellipsoid, in which the demagnetisation field can be calculated, or the rod should be long compared to its diameter.

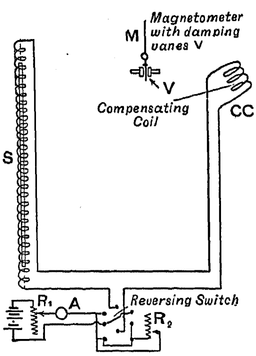


FIG. 32.—Magnetometer Method for Tests on Rods.

For general purposes the magnetometer should be a suspended needle with a mirror, and it may be used swinging freely under the control of the earth's horizontal magnetic field only.

(i.) *Single-pole Method.*— One arrangement of apparatus is shown in the accompanying diagram (*Fig.* 32).

In this "one-pole" arrangement, as it may be termed, the specimen is in a vertical position inside the solenoid S, which is longer than the rod by a few cm. to give a reasonably uniform field over the whole length of the rod (apart from its own demagnetising field).

The magnetometer is set up at the same level as the upper end of the rod and at a distance from it so that a convenient deflection is obtained for the range of magnetisation being investigated. This distance should not be more than about one-quarter the length of the rod.

A compensating coil CC is also set up and adjusted in position so as to annul the effect of the empty solenoid on the magnetometer.

In making this adjustment the iron is removed from the solenoid and a fairly large current switched on. Coil CC is now gradually moved until no effect is observed on the magnetometer when the current is reversed.

The set-up is in such a direction that the line joining the end of the bar to the magnetometer is at right angles to the needle itself.

Since the rod is in the vertical component of the earth's magnetic field it may be necessary to allow for this in some tests, or to compensate by a second winding on the solenoid carrying a suitably adjusted current.

Additional compensation for the action of this on the magnetometer may be required in the form of an additional coil corresponding to CC.

The reversing key and ammeter for reading **H** with the associated resistances are similar to those used in the ring test (*Fig.* 28).

In calibrating the magnetometer it is, of course, necessary to know or determine the controlling field and to see that no extended masses of iron are near enough to cause variable effects on the magnetometer needle. The measurement of the force due to the earth's magnetism may be made by any of the usual methods of measuring the intensity of its horizontal component.

A direct calibration of the magnetometer may also be made by providing a single turn of wire 50 cm. in diameter, set up so that the magnetometer needle is at its centre and the plane of the coil is in the plane of the needle.

If a current of 0·5 ampere is sent through this a suitable deflection will be obtained. Readings should be taken with the current in each direction.

Another method of calibration is to use a well-aged bar magnet, which can be placed in a cradle at a standard distance from the magnetometer. The magnet should be turned end for end and the mean of the two deflections taken. The moment of the magnet will be determined by some absolute method such as the previous method.

Calibration of Set-up.—Let N (*Fig.* 33) be the position of the magnetometer and A B the poles of

the magnet. Then if S=cross-sectional area of test rod, A B positions of poles, and H the horizontal component of the earth's field, we have

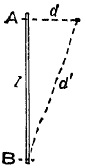


FIG. 33.

$$\mathbf{I}=\frac{\overline{AN}^2 H \tan\theta}{S\{1-(AN/BN)^3\}}.$$

Hence

$$\mathbf{B}=\frac{4\pi d^2}{S}\cdot\frac{1}{1-(d/d_1)^3}H\tan\theta+\mathbf{H}.$$

If the length of the test rod is 4 times the distance d it will be sufficiently accurate to take $(d/l)^3$ as the correcting factor in the denominator.

For the calibration of the magnetometer we have, using the 1 turn circle of wire of radius a carrying current I amperes and producing a deflection θ_0,

$$H=\frac{\pi I}{5a\tan\theta_0}.$$

If a bar magnet is used to calibrate the magnetometer

$$H=\frac{2M}{a^3\tan\theta_0},$$

where M=moment of magnet and a=distance of its centre from magnetometer.

In some cases the rod or wire being tested may have so large a cross-sectional area S that the deflection cannot be kept on the scale without making d too large. In this case a more powerful field than the earth's magnetic field may be applied by means of a bar magnet placed under the magnetometer.

Convenient maximum dimensions to suit this method of testing are

Length of specimen, 1 metre.

Diameter of specimen, 3 mm.

Distance of magnetometer from end of specimen, 25 cm.

(ii.) *Variations of this Method.*—A variation on this method is to place the specimen in

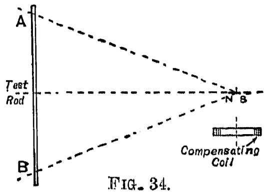


FIG. 34.

the position shown in plan in the diagram (*Fig.* 34) with respect to the magnetometer. For this case

$$\mathbf{B}=\frac{4\pi}{S\,.\,AB}H\tan\theta+\mathbf{H}.$$

The distance AB between the "poles" of the bar is a quantity depending on its shape and distribution of magnetism.

For an ellipsoid length l

$$AB=\frac{2}{3}l.$$

Another position of the specimen with regard to the magnetometer is as in *Fig.* 35.

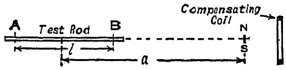


FIG. 35.

When the magnetometer needle is very small compared to a the expression for **B** becomes

$$\mathbf{B}=\frac{2\pi(a^2-l^2/4)^2}{aV}H\tan\theta+\mathbf{H},$$

where a=distance from centre of rod to needle,
l=length of ellipsoid,
V=volume of ellipsoid.

(iii.) *End Corrections.*—For the determination of **H** the correction for the ends may become important in the case of materials of high permeability. The formula for this correction is given in § (10). It is of the form $\mathbf{H}=\mathbf{H}_0-\alpha\mathbf{B}$.

The following table gives the value of α for various values of the ratio length/diameter in the case of ellipsoidal and cylindrical rods:

Ratio $\frac{\text{Length}}{\text{Diameter}}$.	α (Ellipsoid).	α (Cylinder).*
20	0·00675	0·00714
50	0·001446	0·001456
100	0·000430	0·000410
200	0·000125	0·000118
300	0·000060	0·000056
500	0·000024	0·000022

* C. R. Mann, "Über Entmagnetisierungfaktoren kreiszylindrischer Stabe," *Inaug. Diss.*, Berlin, 1895.

In deducing the results, a convenient way of applying the end correction is to draw a line through the origin whose slope is $1/\alpha$ as in the diagram (*Fig.* 36). Here OX is the line giving a new axis for measuring the true effective **H** acting on the bar.

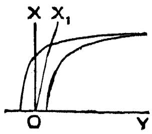


FIG. 36.

The magnetometer methods allow more possibilities in certain measurements than methods using a ballistic galvanometer because the magnetometer measures the static **B** directly whereas ballistic instruments measure *change* in **B**.

The magnetometer method is useful for experiments on the viscosity effects when a magnetic condition is attained very slowly or by small increments. In the study of effects depending on time the method is specially useful.

The "one pole" method, in which the specimen is vertical, may be considered as the best method from most points of view. It does not necessitate an accurate estimation of the position of the effective poles of the rod in the way that the two other positions require.

The method is suited to investigation on the effects due to tension on the specimen since weights can be hung directly on the rods or wires being tested.

(iv.) *Ballistic Galvanometer*.—Tests on long rods can also be carried out using a search coil to measure **B** instead of the magnetometer. This method with the ballistic galvanometer is more flexible in regard to calibration (*i.e.* obtaining a suitable deflection with various sizes of specimens) on account of ease both of adjusting sensitivity and of calibration.

The same corrections to **H** will, of course, apply as in the case of the magnetometer.

In using a ballistic galvanometer care must be taken when testing long rods to see that there is no direct action on the galvanometer due to the poles of the rod. This is easily tested by disconnecting the **B** search coil and short-circuiting the galvanometer leads going to it. No deflection should be observable when a considerable magnetisation of the rod is reversed in direction.

§ (24) Hopkinson Bar and Yoke.[1]—Hopkinson was the inventor of the system of embedding the ends of a test bar into a massive yoke for the purpose of securing approximate endlessness. In his original yoke, the bar was in two parts abutting against one another near the middle of the yoke. The search coil D could be slipped in and out between the two sections of the magnetising winding as in the sketch (*Fig.* 37). The search coil D was arranged with a spring so that when one of the test rods T was suddenly pulled away from T′, the search coil jumped out of the field.

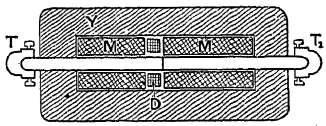


Fig. 37.—Hopkinson Bar and Yoke.

This allowed the flux in the bars to be measured on a ballistic galvanometer.

The main objection to the method is the joint between the ends of the bars. The leakage is large and occurs at the very place where the search coil is.

§ (25) Ewing Double Bar Two-length Test.—This method of testing bars is a yoke method in which two similar bars of the material being tested are provided.

The apparatus is shown in the accompanying photograph (*Fig.* 38a) and consists of two pairs of magnetising coils. The shorter pair is exactly half the length of the longer pair and contains half the number of magnetising turns. Both pairs are provided with search coils for measuring.

The bars should be accurately turned to standard dimensions and should be uniform. The yokes (*Fig.* 38) consist of a pair of similar

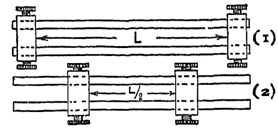


Fig. 38.—Ewing Two-length Test.

blocks of annealed soft iron with accurate and parallel holes to fit the bars with a clearance of a few hundredths of a mm. on the diameter.

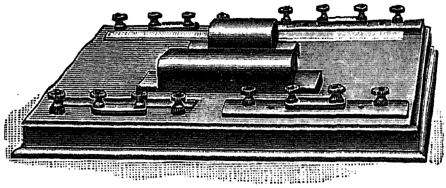


Fig. 38a.—Ewing Double Bar Two-length Apparatus.

Let L be the length between the yokes in test (1), L/2 the length in test (2); I_1, I_2 the currents for the same value of **B**; $\mathbf{H}_1$, $\mathbf{H}_2$ the apparent values of the field, neglecting the air gaps; N the number of turns on the lower magnetising coils, and n the ampere turns used in the yokes and air gap; **H** the true value of the field. Then

$$\mathbf{H}_1=\frac{4\pi}{10}\frac{\mathrm{NI}_1}{\mathrm{L}}.\ \mathbf{H}_2=\frac{4\pi}{10}\frac{\mathrm{NI}_2}{\mathrm{L}},$$

$$\frac{4\pi}{10\mathrm{L}}(\mathrm{NI}_1-n)=\mathbf{H}=\frac{8\pi}{10\mathrm{L}}\left(\frac{\mathrm{NI}_2}{2}-n\right).$$

Thus $\qquad n=\mathrm{NI}_2-\mathrm{NI}_1$

and
$$\mathbf{H}=\frac{4\pi}{10\mathrm{L}}(2\mathrm{NI}_1-\mathrm{NI}_2)$$
$$=2\mathbf{H}_1-\mathbf{H}_2.$$

In carrying out the test the bars are first inserted in the long coils and the yokes fitted on, abutting against the ends of the bobbin on which the magnetising coils are wound. A series of observations of apparent **H** and **B** are taken in the ordinary way and plotted. The bars are now removed and a similar test made on them in the short magnetising coils. Careful demagnetisation must be carried out before carrying out this second test.

The two curves are plotted on the same sheet and will appear as shown in *Fig.* 39.

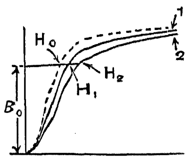


Fig. 39.

The two curves obtained are drawn in full and marked 1 and 2 respectively. The true curve (dotted) is then obtained by reading off $\mathbf{H}_1$ and $\mathbf{H}_2$ and setting back from $\mathbf{H}_1$ a

[1] "Magnetisation of Iron," *Phil. Trans.*, 1885 (2), p. 455.

distance equal to $\mathbf{H}_2 - \mathbf{H}_1$, thus giving the true value of the magnetising force corresponding to the induction $\mathbf{B}_0$.

To secure the most accurate results a second series of tests is made, interchanging the bars; certain differences between the bars and joints with the yokes will be eliminated.

The main use of this method is in standardising bars which are afterwards to be used in the Ewing permeability bridge. It is a somewhat laborious method of carrying out magnetic tests.

§ (26) Ewing Permeability Bridge.[1]—This is a method of testing in which a com-

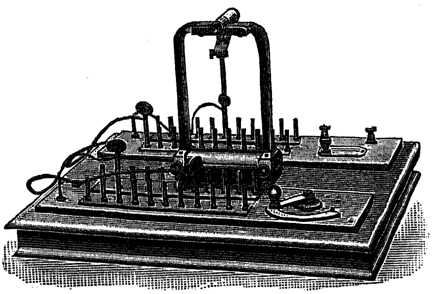


Fig. 40.—Ewing Permeability Bridge.

parison is made between the bar being tested and a standard bar whose curve is known. The bridge is shown in the accompanying photograph (*Fig.* 40) and diagram (*Fig.* 41).

The **HB** curve of the rod to be tested is determined by finding the ratio of the magnetising force which has to be applied to the specimen to that force which has to be applied to the standard rod in order that the **B** may be the same in both. The **HB** curve of the standard bar being known, that of the specimen is thus determined.

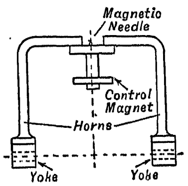


Fig. 41.

The equality of **B** in the two rods is obtained by varying the number of turns in the magnetising winding surrounding one or other of the rods whilst the same current flows through both windings.

This equality of **B** is determined by observing when the magnetic potential difference between the yokes is zero.

From the yokes two long curved horns project as shown in *Fig.* 41. In the gap between these is a narrow box containing a short compass needle and pointer. It indicates when there is no induction from one yoke to the other through the horns. This can only occur when the total induction in each rod is the same, and hence (if the rods are of the same cross-sectional area) when the **B** is the same.

As will be seen in the photograph, there are two sets of plugs. These enable any number (up to 200) of turns to be applied uniformly to the specimen and standard rod respectively; the resistance of the internal circuits is so arranged that the current remains constant when the number of turns on either magnetising coil is changed.

The procedure in testing consists in placing a standard bar in one magnetising coil and the test bar in the other coil and clamping up. The current is now switched on and the bars demagnetised down, using the reversing switch on the apparatus.

When the demagnetisation has been completed and no current is flowing, the compass needle is brought into the free and zero position by means of the control magnet underneath.

A magnetising current (preferably corresponding to an exact value of **B** in the standard rod) is now applied and repeatedly reversed. The number of turns on the test rod magnetising coil is now varied until the compass needle remains at zero after any reversal of current.

The ratio of turns thus determined gives the ratio of the applied **H** to each bar and hence gives the **H** required to produce the particular value of **B** under observation.

The apparatus does not lend itself readily to the determination of hysteresis loops.

If widely varying materials are to be tested a range of standard bars of corresponding qualities is desirable so as to avoid the comparison of very unequal materials such as cast iron against mild steel.

Rods having holes bored right through them[2] are valuable for producing, artificially, standard rods equivalent to less permeable material.

Greater accuracy can be obtained by making comparisons against two different standards, one slightly poorer and one slightly better in magnetic quality than the test bar.

The apparatus is very easy to work with and results can be quickly obtained. A battery, ammeter, and rheostat are the only auxiliary apparatus required.

§ (27) Du Bois Magnetic Balance.[3]—This method makes use of the tractive effect between two magnetised surfaces near together. In the Du Bois apparatus the tractive force is not measured on a section of the test piece itself but across a narrow air gap at a large surface in the yoke circuit. The apparatus is shown diagrammatically in *Fig.* 42.

The surfaces are well finished and plane; the air gap between the pole pieces and the pivoted bar yoke piece is about 0·25 mm. at each end. The length of specimen tested is

[1] *Electrician*, xxxvii. 41.

[2] A. Campbell, *N.P.L. Collected Researches*, ii. No. 10.

[3] H. du Bois, *Electrician*, 1892, xxix. 448.

about 30 cm., and a field of 500 gauss can be obtained in the magnetising coil.

The specimen is inserted in the yoke pieces and the weight slid along until the beam rocks over breaking contact with the locating stop I.

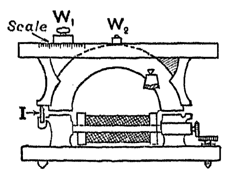


FIG. 42.—Du Bois Magnetic Balance.

The position of the weight gives a measure of the flux, which is approximately proportional to **B** in the specimen. The apparent **H** given by the current must be corrected for the leakage effects due to the gaps. This is a constant quantity for a particular position of the weight and for a particular instrument. It is determined by differences using a standard bar whose curve is known.

§ (28) HOLDEN'S PERMEABILITY BRIDGE.[1]—This bridge is similar to the Ewing bridge in that the test rod is compared against a standard rod by determining equality of induction in the two bars. In the Holden bridge, however, it is the current that is varied in the magnetising winding surrounding the test specimen, so that it is the ratio of two currents that gives the ratio of **H** in the test specimen to **H** in the standard rod.

The equality of induction is determined by the help of a small magnetic needle, as in the Ewing bridge, but no horns are used.

§ (29) YOKE METHOD USING SEARCH COILS TO MEASURE **H**.[2]—In this apparatus, shown in *Fig.* 43, the specimen (rod or flat bar) is clamped (by suitable split end-pieces EE bored or cut to suit the size of bar being tested) between circular yokes YY built up of ring stampings.

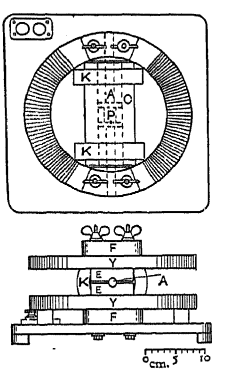


FIG. 43. — Ring Yoke for General Magnetic Testing on Rods and Bars.

The magnetising coil C is of sufficient weight to enable magnetising fields up to **H**=1000 to be obtained, and compensating coils KK can be included to correct for leakage if desirable.

In testing large bars (2·5 cm. diameter) the compensating coils are necessary on account of the large flux carried by the yokes; these latter also have turns wound spaced as shown.

On large bars of the size mentioned, accuracy can be obtained to 1 per cent for values of $\mathbf{H} > 30$, and on smaller bars, where the ratio of length to diameter is much greater, trustworthy results can be obtained in much smaller fields.

For round rods, annular search coils wound on brass tube are used for measuring **H** as described in the section on measurement of magnetic field. On thin rods (6 mm. diameter) it has been found quite satisfactory to use two or more thin long **H** search coils wound on glass slips about 5 mm. wide and 40 mm. long. These are lightly bound round the circumference of a thin brass tube fitting snugly over the rod and provided with a **B** coil in a very shallow groove at the middle. Search coils of this type are more reliable than the annular type but are rather troublesome to make.

For flat bars, flat search coils for **H** are used and fit very close against the surface; it is desirable to use one on each side of the bar and connect them in series.

The circuits and connections used in making tests with this apparatus are similar to those for ring samples (*Fig.* 28, § (21)) as far as the application of the magnetising current is concerned, except that the galvanometer G_1 is calibrated to measure current and is used merely as a rough guide.

The circuits for the ballistic galvanometer are conveniently arranged as shown in the following diagram (*Fig.* 44):

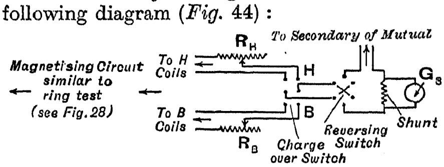


FIG. 44.—Ballistic Galvanometer Circuits for Ring Yoke Tests.

The calibration for **H** is made by adjusting R_H with the change-over switch in the **H** position.

> If $N_2 . s$ = area-turns of search coil for **H**;
> Throw desired for (**H**=100) is, say, x;
> then mutual inductance must be set so that 1 ampere reversed through it gives, say, 100 divisions.

In this case $M = N_2 . s \times 100/x$ microhenries.

The **B** calibration is exactly similar to that for the ring test; the change-over switch being in the **B** position and R_B being adjusted until a throw of 100 divisions is obtained corresponding to **B**=10 000.

The method is slower to operate than the ring method, since observations have to be made alternately on **H** and **B** at any given

[1] Parshall and Hobart, *Engineering*, 1898, lxv. 2.

[2] Gumlich, *Magnetische Messungen*, p. 140; Campbell and Dye, "The Magnetic Testing of Bars of Straight or Curved Form," *Journ. I.E.E.*, Dec. 1915, liv. 35.

point on the curve; that on **H** by reversing a steady current I in the magnetising winding whilst the galvanometer switch is in the **H** position. The **B** throw is then taken by again reversing the *same* steady current I, the ballistic galvanometer switch being in the **B** position.

In carrying out hysteresis tests, or when observing coercive field and remanence, care must be taken to observe the deflections with accuracy, since, in general, it cannot be assumed that **H** is zero when the current has been reduced to zero.

Thus, in the diagram (*Fig.* 45) assume that the material under test is at a tip point P of an hysteresis loop in a cyclic condition by repeated reversals of the current $I_{max.}$. On switching off the magnetising current the induction will fall along the upper part of the loop to some point Q which corresponds to a small magnetising force OB, and may be either positive or negative, according to whether the coercive magnetomotive force of the yoke is greater or less than that due to the specimen.

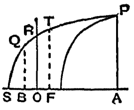


FIG. 45.

This point Q is determined by observing the **H** throw when switching from $\mathbf{H}_{max.}$ to zero current, then observing the throw from zero to $-\mathbf{H}_{max.}$. The difference between these throws gives OB.

To observe the induction at the point Q the throw for a reversal of $\mathbf{B}_{max.}$ is first taken and then the throw from $\mathbf{B}_{max.}$ to the switching-off position. Twice this latter throw subtracted from $\mathbf{B}_{max.}$ gives BQ.

The point Q should be determined with some care as it is a valuable point to use as a datum for further observations on remanence and coercive force.

To determine points on the right of Q on the hysteresis loop the following procedure is adopted. Determine Q as mentioned above, now switch the magnetising current to some small positive value. The magnetising reversing switch is in the Y position (see *Fig.* 28) for this operation, L being out. Having cycled, arrived at P, and then removed link L, the cycle of magnetisation is arrested at some point T.

Now switch magnetising current off and observe **H** throw in passing from T to Q. Repeat for **B**, observing TF – QB or observe PA – TF directly in switching from $\mathbf{H}_{max.}$ to T.

Even when only remanence and coercive force are required it is desirable to observe one or two points Q, R, T in the neighbourhood of R, and similarly at S to observe a point above and below this point.

Uniformity of **B** along the specimen can be tested by using a differential **B** coil consisting of two similar windings in opposition of say 20 turns, each close over the rod and spaced 7 or 8 cm. apart with the **H** and **B** search coils between.

Some differential **H** search coils show considerable divergence of **H** readings when turned end for end on the specimen, the mean plane of the coil occupying the same position in each case. This discrepancy in the readings of the apparent **H** is due to want of uniformity in the induction in the specimen, and, at the same time, want of uniformity in the distribution of windings of the search coil.

If precautions are taken to secure uniformity of **B** in the specimen, and the search coil is a uniform one, the reliability of the readings for **H** is to about 1 gauss, except in the case of specimens of very permeable material in the region of their maximum permeability.

There is considerable difficulty in constructing differential search coils for **H** to give a greater sensitiveness on the ballistic galvanometer than 1 division for **H** reversed = 1 gauss.

§ (30) MORRIS AND LANGFORD.—Method of Magnetic Testing by Uniform Rate of Change of Flux.[1] In this interesting method the constant electromotive force induced in a secondary winding over the specimen connected to any galvanometer, when the flux is made to vary at a uniform rate, is used as a means of measuring the permeability and of determining the hysteresis losses in magnetic materials.

If there is a total flux Φ in a ring, produced by current I in the primary winding, when the current I is varied in any manner there will be an E.M.F. induced in the secondary equal to $n\,.\,d\Phi/dt$, where n = turns in secondary. The total change of flux in time $t_2 - t_1$

$$\Phi_2 - \Phi_1 = n\int_{t_1}^{t_2} \frac{d\Phi}{dt}dt.$$

If now $d\Phi/dt$ = constant b, then

$$n\int_{t_1}^{t_2} \frac{d\Phi}{dt}dt = bn(t_2 - t_1).$$

If E = constant E.M.F. induced, then

$$b = \frac{\mathrm{E} \times 10^8}{n},$$

and total change of flux

$$\Phi_2 - \Phi_1 = b(t_2 - t_1) = \frac{\mathrm{E} \times 10^8}{n}(t_2 - t_1).$$

Hence if E is known, a magnetisation curve can be obtained from a knowledge of current and time.

The method of carrying out such a procedure is given in *Fig.* 46.

The voltage induced in S is balanced by the fall of potential along r_1 carrying a known current. A zero reading of the galvanometer indicates when this is true. The manipulation consists, therefore, in varying the flux in the

[1] *Proc. Phys. Soc.*, 1911.

ring specimen in such a manner as to keep the galvanometer at zero. This can be done with some practice with considerable accuracy.

The specially constructed resistance A carries two contacts a, b, whereby a smooth

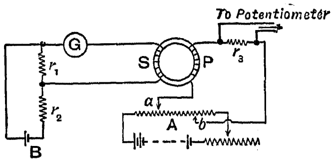


FIG. 46.—Uniform Rate of Change Method of measuring **B**.

continuous adjustment can be made. The battery circuit is closed, and, at the same instant, sliders a and b are moved from opposite ends of the resistance A at such a rate as to keep the average deflection of the galvanometer zero. The primary current is measured on a potentiometer, and the exact times at which it has convenient values is recorded on a chronograph.

The circuits can be simplified if a deflection is used instead of a potentiometer method for determining E, but in this case there is difficulty in interpreting the results whilst the galvanometer is attaining its steady deflection.

The chief value of the method lies in its accuracy of determination of **B** for very slow changes in **H**, whereby viscosity effects may be studied. Accurate measurements of change in **B** can be made when the change occupies as long as five minutes.

The method has been used by Stroud in an investigation on the Steinmetz coefficient of transformer iron, stalloy, and cast-iron.[1]

§ (31) BUREAU OF STANDARDS METHOD FOR BARS.[2]—The underlying principle of this method is to approximate to the condition of applying at every part of the magnetic circuit a magnetomotive force proportional to the reluctance at that point. By this means the lines of magnetic induction are

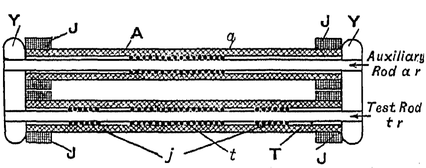


FIG. 47.—Bureau of Standards Method for Standard Bars.

entirely confined to the test bar and yokes so that the leakage is zero.

The magnetic circuit and windings are as shown in *Fig.* 47.

[1] F. Stroud, *Phys. Soc. Proc.*, 1911–12, xxiv. 238.
[2] "Determination of Magnetic Induction in Straight Bars," *Bull. Bur. Stds.*, 1909–10, vi. 31.

tr is the test rod; *ar* is an auxiliary rod and may be one already standardised.

A and T are the main magnetising windings and consist of 10 layers of insulated wire wound very carefully. The commencing layer rests in a screw-cut groove, and succeeding layers are wound in the same direction so as to lie in the groove formed by adjacent turns of the layer underneath.

The constant of these windings is $\mathbf{H} \doteq 100\,\mathrm{I}$, where I is measured in amperes. Fields of 350 may be obtained for short periods. JJ are windings to compensate for the reluctance of the joints; they are in series on an independent circuit.

The search coils for measuring **B** are disposed as shown, symmetrically, on the test rod, t centrally and jj in two halves at each end, but not too near the yokes; a test coil a similar to t is applied to bar *ar*.

These search coils are each distributed over a considerable length of the rod so as to smooth out local variations in the magnetic properties of the specimen.

(i.) *Calibration.* (a) For **H**.—The calibration for **H** is obtained from the dimensions of the various magnetising and compensating windings, each of which may be carrying different currents.

A calculation for the central point of the test magnetising coil shows that the correction to be applied to the ordinary formula for an infinite solenoid is only about 0·1 per cent in the case of a solenoid 30 cm. long.

In the actual case considered the constant of the solenoid is $\mathbf{H}=100{\cdot}53$ I. The measurement of **H** is made by determining, on a potentiometer, the fall of potential E over a standard resistance of 1·0053 ohms in series with the magnetising coil so that $\mathbf{H}=100\mathrm{E}$.

(b) For **B**.—The calibration for **B** is made in the ordinary way from the knowledge of sectional areas of the rod and number of turns N_2 in the **B** search coil.

The calibration of the ballistic galvanometer is made by means of a mutual inductometer of special design consisting of two concentric cylindrical coils, one of which can be smoothly withdrawn from the other without rotation, by means of a screwed spindle.

It is indicated that the **B** can be measured by a null method if the current through the mutual is reversed simultaneously with the magnetising current.

If the mutual carries the same primary current as the magnetising coil, then for zero deflection

$$\mathrm{M}=\frac{\mathbf{B}\mathrm{N}_2 s}{\mathrm{I}}\times 10^{-8}\text{ henries},$$

and $\mathbf{H}=\mathrm{D}\times\mathrm{I}$, where D=constant.

Hence $$\mu=\frac{\mathrm{M}\times 10^8}{\mathrm{N}_2 s\times \mathrm{D}}.$$

For a particular set-up $\mu=\text{constant}\times\mathrm{M}$. No mention is made of the double throw which results on account of eddy currents and viscosity in the specimen, unless the galvanometer is of exceptionally long period and some compensating eddy currents are incorporated into the mutual inductance, in the

form of a closed tertiary winding or solid metal circuit; such double "kick" on the galvanometer is very troublesome and greatly detracts from the accuracy which otherwise appertains to a null method.

The complete circuits in the set-up are as shown in *Fig.* 48 below.

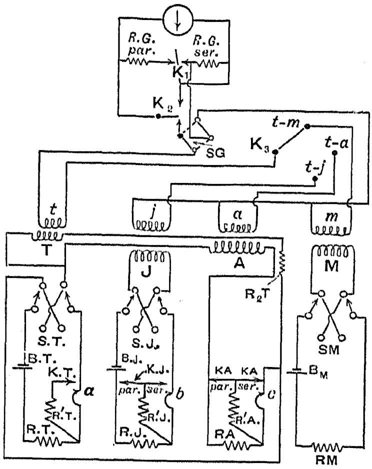


FIG. 48.—Circuit for Bureau of Standards Method for Bars.

The letters have the same reference as in *Fig.* 47, illustrating the bars and yokes.

Three independent batteries are used, one for the test and auxiliary magnetising circuits, one for the compensating circuits, and one for the mutual inductance circuit.

The current reversing switches and the five keys K are all conveniently mounted together on a board for operation by the fingers in any desired manner. The procedure is as follows:

The specimen having been carefully demagnetised, an appropriate magnetising current is applied. Switches ST and SJ are repeatedly reversed and resistances RA and RJ adjusted until the three test coils t, j, and a indicate the same change in flux. With the key K_3 on the point $t-a$ equality of flux in the test and auxiliary rods is first secured. The key is now changed to $t-j$, and a balance obtained indicating uniformity of magnetisation of the test rod.

The throw is then taken from the test coil t or a balance obtained using the mutual with its opposed secondary m.

The hysteresis loop corresponding to any desired value of **B** (maximum) or **H** (maximum) may also be determined. The guiding principle is that the induction for the steady cyclic state is first adjusted to uniformity along the bar, and then, when a desired point on the loop has been obtained for the test rod, uniformity of induction at this point is again obtained by adjustment of resistances R_1J and R_1A.

§ (32) GENERAL REMARKS ON YOKE MAGNETIC CIRCUITS.—Yoke methods in general include the following general principles as shown in the accompanying typical diagram (*Fig.* 49).

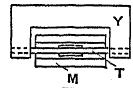


FIG. 49.

N = total number of turns on magnetising coil,
L_0 = length of magnetising coil,
$\mathbf{H}_1$ = calculated **H** for magnetising winding,
l = length of specimen between yoke and magnetising coil,
s = section of specimen,
L_1 = mean length of yoke,
S = section of yoke,
λ = effective length of the two air gaps (between specimen and yoke),
σ = section of the two air gaps,
μ = permeability of specimen,
μ_1 = permeability of yoke,
$\mathbf{B}$ = flux density in specimen,
Φ = total flux.

Then

$$\Phi=\frac{0{\cdot}4\pi NI}{L_0/s\mu+l/s\mu+\lambda/\sigma+L_1/S\mu_1}$$

$$=\frac{0{\cdot}4\pi NI}{L_0(1+l/L_0+\lambda s\mu/L_0\sigma+L_1 s\mu/S\mu_1 L_0)/s\mu},$$

also $\qquad \Phi=\mathbf{B}s$ and $\mu=\dfrac{\mathbf{B}}{\mathbf{H}_0}$.

Hence

$$\mathbf{H}_0 \,\dot{\because}\, \frac{0{\cdot}4\pi NI}{L_0}\left(1-\left[\frac{l}{L_0}+\frac{s\lambda\mu}{L_0\sigma}+\frac{L_1 s\mu}{L_0 S\mu_1}\right]\right).$$

This is approximately equal to

$$\mathbf{H}_1-\mathbf{B}\left(\frac{l}{\mu L_0}+\frac{s\lambda}{\sigma L_0}+\frac{sL_1}{SL_0\mu_1}\right),$$

where $\mathbf{H}_1$ = calculated **H** for the magnetising winding alone, and $\mathbf{H}_0$ = true **H**.

From this it is seen that the effect of the yoke and air gaps, etc., is to shear the permeability curve as shown in *Fig.* 50.

In some cases it is possible to apply these corrections for **H** from a knowledge of the quantities involved. Dealing with the three terms in the bracket in order, l can be made quite small by designing the magnetising coil to come right up to the inner faces of the yoke. This correction is strictly proportional to **H** and can be made about 1 per cent to 1·5 per cent of **H** for a rod with free length of 30 cm. The correction for the air gaps is not so certainly determinable, but with well-fitting rods in accurately bored yokes might be of the order of 3 per cent or 4 per cent in

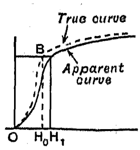


FIG. 50. — Shearing Effect of Reluctance of External Magnetic Circuit.

the worst part of the curve, *i.e.* where μ is a maximum; this correction is proportional to **B**.

The correction term for the yoke can be determined by first obtaining the permeability of the yoke. This is easily done in the case of a symmetrical yoke as in *Fig.* 51, since the yoke can be wound with a few magnetising turns and a search coil and tested as a closed magnetic circuit.

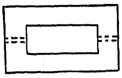


FIG. 51. Symmetrical Yoke.

In a good design of yoke, the section will be from 40 to 50 times that of the specimen, and hence the **B** will range from 0 to about 200, so that high permeability for very small values of **B** is a desirable quality in the yoke. For this reason very well annealed soft iron is desirable. A value of μ equal to 250 to 300 may be expected in a good sample. This μ will be nearly constant so that this correction is also approximately proportional to **H**. It may reach a value of 10 per cent of **H** if the value of **B** in the specimen is about 10 000. It becomes almost negligible at high values of **H**.

The total effective correction to be applied is most accurately carried out by making measurements on a standard bar whose curve has been determined by a standard method.

§ (33) PERMEAMETERS.[1]—One of the earliest types of permeameter is that introduced by Köpsel in 1890, and improved by Kath, 1898.[2]

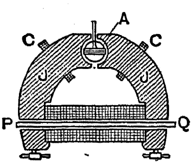


FIG. 52.—Köpsel Permeameter.

In this instrument the induction is measured by means of a suspended or pivoted coil in a narrow cylindrical gap in the yoke of the apparatus. A sectional plan is shown in *Fig.* 52.

The specimen PQ is clamped in the massive yokes J. These have a narrow annular air gap at A in which swings, under control, a moving coil, through which a constant small current is maintained.

The deflection of the coil is proportional to the gap flux, which is approximately proportional to the induction in the specimen.

Compensating windings C are provided on the yokes near the air gaps. The object of these is to neutralise, in the air gap, the magnetisation of the yokes by the ends of the main magnetising coil. They are shunted by a variable resistance, and the combination is connected in series in the magnetising circuit.

[1] Ewing, *Mag. Ind.* p. 373, and Burrows, *Bull. Bur. Stds.*, 1915, xi. 101.

[2] A. Köpsel, "Apparat zur Bestimmung der magnetischen Eigenschaften des Eisens in absoluten Mass und directer Ablesung," *E.T.Z.*, April 1894, xv. 214; Zath, *E.T.Z.*, 1898, xix. 411; Rohr, *E.T.Z.*, 1898, xix. 713; Gans and Goldschmidt, *E.T.Z.*, 1896, xvii. 372.

The adjustment is made by switching on a large magnetising current and adjusting the shunt resistance until a zero deflection is obtained. No test rod is in the apparatus for this adjustment.

The instrument is set up with the axis of the specimen at right angles to the magnetic meridian.

The winding and control force of the moving coil are so designed that in order to make the pointer read **B** directly on the scale the small current in the moving coil is set to a value equal to 50/S in milliamperes, S being the section of the test specimen in sq. cm.

The **H** given by the magnetising coil is equal to 100 I (I being measured in amperes).

To carry out a test, the specimen is inserted in the yokes and magnetising coil, and clamped up. It is then carefully demagnetised below the lowest value of induction at which the test is to be taken.

The current in the moving coil is set to the required value so as to give direct reading values of **B**. Since the apparatus reads **B** statically, readings may be taken by increasing **H** step by step if so desired, but more reliable results are obtained by successive reversals, taking the mean of readings to left and right of the zero for each point.

The hysteresis loop may also be determined by interrupting the cycle at any desired point and observing current **H** and deflection **B** corresponding to it.

The apparatus may be considered useful as a comparative method, but the results must be corrected by determination of the errors of the instrument with the help of standard bars whose magnetic properties are known otherwise. A series of standard bars and strip bundles of various sections and qualities of material should be provided, so that a standard may be used of similar properties to any of the various types of magnetic materials it is desired to test.

With care in making the tests and in applying the corrections appropriate to the quality and size of specimen tested, results accurate to about 5 per cent in **H** may be obtained.

The apparatus lends itself readily to tests on strips cut from sheet material.

§ (34) THE FAHY PERMEAMETER.[3] *Theory.*—The following diagram (*Fig.* 53) indicates the principle of this permeameter.

In it M is a magnetising coil wound on a solid iron core which is divided in the centre and contains an adjustable air gap. There are also end yoke-pieces bolted to the core. The magnetic circuit external to the test rods thus forms an I with the core as an axis of symmetry.

The cross-pieces have clamps at their ends

[3] *Bull. Bureau Stds.* No. 306, Aug. 1917; Fahy, *El. World*, 1917, lxix. 315.

for clamping various types of specimens. S and T are test coils bridging the ends of the cross-pieces. Specimens may be clamped in either or both S and T. With no specimens in the clamps there will be equal magnetomotive forces along S and T. When the test specimen is inserted through T and clamped up, this equality of flux distribution is disturbed and there will be less magnetomotive force expended along T than along S, owing to the increased flux in the right-hand magnetic circuit containing the specimen. The magnetic potential differences between the ends of the coils can be again equalised by means of compensating coils CC, which are all in series and carry an independently adjustable current. The current through CC can, however, be reversed simultaneously with the main magnetising current. The equality of magnetic reluctance round the two circuits is determined with the help of search coils a_s and a_t by observing when the leakage between a_s and S equals that between a_t and T. When the adjustment has been made, the leakage paths outside the coils S and T are symmetrical and equal on both sides of the apparatus and hence the magnetomotive forces along T and S are also equal.

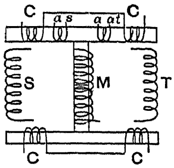


FIG. 53.—Principle of Fahy Permeameter.

The magnetomotive force along T is measured by means of another coil H (not shown in the diagram) of a relatively large number of turns wound uniformly along the length of S. The product (area × turns) of this coil is known. The induction **B** in the test specimen is measured by means of coil T.

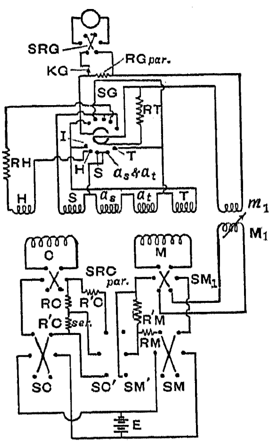


FIG. 54.—Circuits of Fahy Permeameter.

The four coils, S, T, a_s, a_t, are all of equal number of turns, and coils S and T are of equal cross-section also.

The complete circuit connections are shown in *Fig.* 54, where the letters refer to the same windings on the apparatus as in *Fig.* 53.

The additional circuits are M_1, m_1 primary and secondary of calibrating mutual inductance, SM and SC are reversing switches for their respective circuits, SM is a change-over switch. The various resistances RC, RM, etc., can be inserted in various ways to carry out the operations in measuring hysteresis loops, etc., with the help of the switches SC, SC′, SM, SM′.

In the galvanometer circuit a selector switch enables the following connections to be made to the galvanometer:

Stud 1. S, a_s, a_t, and T all in series for determining equality of magnetomotive forces round the circuits.
Stud 2. H is connected for determination of **H**.
Stud 3. S is connected for determination of **B** in the standard specimen when one is used.
Stud 4. Coils a_s and a_t are connected in series in opposition for reading differences in the fluxes in the two arms of the I when no specimen is in T.
Stud 5. T is connected for determination of **B** in test specimen.

The calibrations are made in the usual way with the help of the mutual inductance.

(*a*) For **B**.

$$MI = \theta\frac{NA}{10^8}\phi,$$

where θ = a convenient throw (say 100 divisions) on reversing I through M,
N = number of turns of search coil,
A = area of T or S (test specimen or standard),
ϕ = **B** desired for one division on the scale.

(*b*) For **H**.—When absolute measurements are being made, **H** is measured by means of a search coil (which is not shown in *Fig.* 33; it is wound along with S).

(i.) *Permeability* **B H** *Curve.* (*a*) *Absolute.*—The test rod is inserted and clamped up and carefully demagnetised. Magnetising current is switched on corresponding to the point on the curve being determined. After repeated reversals an observation is made with the ballistic galvanometer switch on Stud 1. The compensating current is applied and adjusted until a reversal of both currents gives zero deflection.

Observations of **H** and **B** are then made in succession, the switch being on Stud 2 for reading **H** and on Stud 5 for reading **B**.

The adjustment of compensating current through coils C must be made for each point determined when accurate results are required, especially in the region of $\mu_{max.}$.

(ii.) *Determination against a Standard Test Rod.*—This is inserted in the S side of the permeameter. The procedure is the same as

above for balancing and determining **B** in the test specimen, but since H cannot now be used for determining **H** the galvanometer switch is put on Stud 3 and the **B** in the standard specimen determined. The **H** corresponding to this is known.

(iii.) *Hysteresis Loop.*—The test rod alone is used, and is first got into a steady cyclic state at the desired value of $\mathbf{H}_{\text{max.}}$ or $\mathbf{B}_{\text{max.}}$. The specimen is now magnetically at D (*Fig.* 55). For determination of a point P the procedure is similar to that for ring ballistic tests in regard to manipulation of the magnetising current and its circuit. Special compensation must be made in the compensating circuit by appropriate adjustment of R′C series, so that on changing over SC′ and SM′ simultaneously the galvanometer, on Stud 1, shows no deflection. When this is effected $\mathbf{H}_p$ and $\mathbf{B}_p$ are separately determined by observation and reduction of the **H** (change) and **B** (change) throws obtained in passing from D to P.

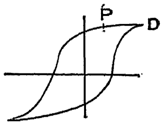


Fig. 55.

(iv.) *Precautions.*—The ordinary precautions regarding good magnetic joints between test and standard rods and the clamps must be taken if the highest accuracy is desired.

The specimens should be of the correct length to just span the clamps. Owing to the very considerable magnetic leakage from this permeameter, care must be taken that stray fields do not react on the mutual inductance, galvanometer, leads, etc.

With care very good accuracy can be obtained from this permeameter, which is a distinct improvement on many direct-reading instruments. An accuracy of 3 per cent in **H** for a given **B** may be obtained.

§ (35) Iliovici Permeameter.[1]—This permeameter operates on the principle of adjusting the magnetic potential between the two ends of the specimen to zero by the help of a magnetic potentiometer consisting of an iron-cored search coil spanning the ends of the specimen.

The apparatus is shown diagrammatically in *Fig.* 56. The yoke Y has a winding on it which is in parallel through an adjustable rheostat with the uniform magnetising winding M (shown divided in the diagram). This has also a rheostat and ammeter in circuit. The current through both windings can be reversed simultaneously by a reversing switch in the common battery circuit.

There is, on the opposite side of the specimen to the yoke, an exploring yoke-shaped piece of iron P having a winding on the straight portion.

This winding constitutes a search coil for determining when the magnetic potential between the points AB is zero. It is connected to one side of a throw-over switch with a ballistic galvanometer.

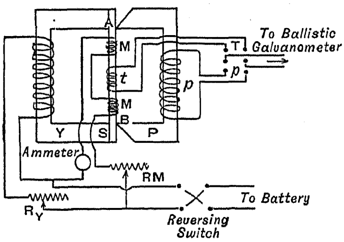


Fig. 56.—Iliovici Permeameter.

The test coil on the specimen for measuring **B** is connected to the other side of the throw-over switch.

This auxiliary yoke can be pressed close against the specimen and locked in position; at the same time it serves to clamp the specimen against the faces of the yoke proper.

The procedure in making a test (after clamping up and demagnetising) is to put galvanometer switch on *p* and reverse the magnetising current previously set to give the required **H.** A throw will, in general, result. Resistance RY is now adjusted until on reversal of the main reversing switch no throw is obtained. When this condition is realised the magnetomotive force of the magnetising coil is just sufficient to overcome the reluctance of the test piece over the length between the horns of the test yoke. The number of turns and length of magnetising coil are so adjusted that $\mathbf{H}=100\,\mathrm{I}$.

After the adjustment has been made the galvanometer switch is thrown over to T and a throw for **B** obtained.

By means of a commutating device in the galvanometer circuit the connections of it to the potentiometer coil P can be reversed simultaneously with the reversal of the main reversing switch, and so a cumulative effect can be obtained. The process of obtaining the balance then consists in continuously reversing and adjusting RY until no deflection is obtained on the galvanometer.

The advantages claimed for this apparatus are:

(1) It is very simple to manipulate. A zero balance is first obtained and then **B** is read directly.

(2) Any desired value of **H** can be set in advance; tests for permeability can be made at exact values of **H.**

(3) No special precautions need be taken; the reluctance of the joints is of no particular consequence.

(4) Fields up to $\mathbf{H}=400$ can be obtained.

[1] *Bull. de la Soc. Int. des Électriciens*, 1913, iii. 581. *Sur un nouveau perméamètre universel*, M. A. Iliovici.

§ (36) PICOU PERMEAMETER.[1]—In this permeameter (*Fig.* 57) there are two equal yokes similar to the Iliovici permeameter, but they each carry equal windings (through which current can be sent), A on one yoke, B on the other. The specimen is also surrounded by a magnetising coil C.

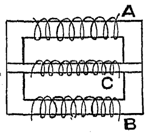


FIG. 57.—Picou Permeameter.

The principle of the operation is as follows.

Current is switched on through A and B in series in such a connection that the flux traverses the yokes and crosses the joints and the part of the specimen between the yokes, thus forming a closed magnetic circuit. By reversal, this flux is measured with the help of test coils on the yokes.

If now the current in B is reversed and magnetising current applied to C, this latter current can be adjusted until the flux in the yokes is the same as before. Under these conditions the whole of the magnetomotive force applied to C is used up in the specimen, so that **H** can be directly deduced. **B** is observed by the help of a test winding on C using a ballistic galvanometer.

§ (37) THE BAILY PERMEAMETER.[2]—This apparatus determines directly the ratio of **B** to **H**; hence when **H** is known, a simple multiplication suffices to give **B**, or the **H** μ curve may be plotted directly if desired.

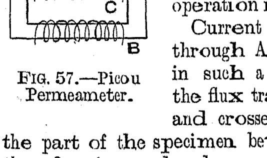


FIG. 58.—Baily Permeameter.

The principle of the apparatus is as set out in *Fig.* 58.

Two rods are used and are slipped through magnetising coils in the usual way; they are clamped to two soft iron blocks as in an ordinary yoke method. One of these blocks is cut across, at right angles to the flux through it, and a narrow crevasse is provided by the insertion of a piece of thin sheet brass or copper as at S in the figure.

Immediately above the gap two small bar magnets are supported side by side in a vertical position, with opposite poles above and below, in such a way that they can turn about a vertical axis; in a uniform field the system would be astatic.

The lower pair of poles can swing close above the upper surface of the split block of soft iron; it therefore forms a sensitive detector of the leakage from the two poles formed by the block, and hence experiences a torque proportional to the magnetic potential difference between the two parts of the block. This magnetic potential difference is proportional to the flux through the narrow air gap.

Surrounding the upper two poles of the small magnets is a coil in series with the magnetising windings. This coil can be rotated about an axis coincident with the axis of the small magnets. A torque variable from zero upwards and proportional to the magnetising current can, therefore, be applied to the upper poles of the small magnets.

The moving system of magnets carries a light arm which plays between stops; it is on a zero line when the two lower poles are exactly above the crevasse. In this position they experience the maximum torque from the leakage field. The movable coil is connected so that the torque due to it on the upper poles is in opposition to the torque on the lower pair. By rotating the upper coil a balance of torques can be obtained. A pointer attached to the movable coil indicates on a scale the angle through which it has been turned. When the axis of the coil coincides with the axis of the upper virtual magnet the torque is zero, and the pointer attached to the coil then indicates zero on the scale.

The scale can be calibrated to read μ directly by using a standard bar. The manipulation of the apparatus is as follows:

The bars are first got into a cyclic state at the desired **H** as read directly on the ammeter.

The small balancing movable coil is now turned until the magnet system floats with its pointer free between the stops.

The permeability is then read off directly on the scale. The scale reads directly to $\mu=800$, and by a shunt on the moving coil a multiplying factor of 5 can be introduced, allowing readings up to $\mu=4000$ to be obtained.

No attempt is made to render the flux uniform along the specimens nor to compensate for the magnetomotive force required at joints and in the gap of the yoke. The apparatus, therefore, would be subject to serious errors in the region of $\mu_{max.}$.

§ (38) DRYSDALE PERMEAMETER FOR TESTING MAGNETIC QUALITIES IN BULK.[3]—In this method a special drill is used which cuts a hole with tapering upper part and leaves a small projecting parallel pin of the metal in the axis as shown (*Fig.* 59). A special plug fits into the upper taper portion of the hole and closely fits the central pin also. This plug is split and so makes a very good joint outside and inside. The plug

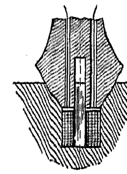


FIG. 59. Drysdale Plug Permeameter.

[1] "A Universal Permeameter," R. V. Picou, *Bull. Soc. Int. Elec.*, 1902, ii. 828.
[2] F. G. Baily, *Electrician*, 1901, xlviii. 172.
[3] C. V. Drysdale, *Journal I.E.E.*, 1901, xxxi. 283.

carries the magnetising coil and search coil for **B**. A suitable size of hole is one about 1 cm. diameter by 2 cm. deep, and the pin is about 2·5 mm. diameter.

The testing can be carried out by connecting the magnetising winding in series with a reversing switch and ammeter to a battery and the search coil to a ballistic galvanometer in the usual way.

A method using a secohmmeter and variable self inductance is described, and also a modification of this for producing direct readings of the permeability on an ohmmeter. The secohmmeter must be run at constant speed for this direct reading method. Curves are given of results obtained on mild steel, wrought iron, and cast iron, both for permeability and hysteresis.

The advantages of the method are quickness and ease of obtaining results.

The burden of the accuracy of the method is thrown on the drill and on the soft iron plug which completes the magnetic circuit. These must be accurately made to gauge. It would appear desirable to check the accuracy of the results by testing a ring cut from the same material to form a standard of reference, and to correct the results given by the plug method if necessary.

(B) Measurements in Strong Fields

§ (39) Ewing Isthmus Method.[1]—In this method a strong magnetic field is provided by means of a large electromagnet with conical pole pieces leaving a comparatively short gap. Between the faces of this gap the specimen is placed; it is in the form of a bobbin, being turned down quite small in the centre part and having sloping conical faces receding from this neck. These form a continuation of the main conical pole pieces as in diagram, *Fig.* 60.

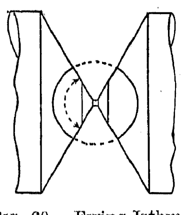


Fig. 60.—Ewing Isthmus Method for High Magnetisations.

The bobbin can be turned end for end through a semicircle, being in a frame with a cylindrical surface, bearing against a similar cylindrical surface on the poles.

The central neck is wound with a known number of turns for measuring **B**. Over this is a second coil separated from the first by a known air space (about 1·3 mm.). This coil also has its turns known, and by difference in the total flux measured by the two coils the magnetising force can be calculated. Correction for the air flux included in the inner **B** search coil can be made from a knowledge of **H** and of the mean cross-sectional area of the inner coil.

The form of cone to give the maximum field is one having a semi-angle of approximately 60°. For greatest uniformity of field the semi-angle of the cones should be about 40°.

A diagram of the form of apparatus used is given in *Fig.* 61 (p. 157, *Fig.* 74, Ewing). A brass piece *aa* having hollow conical recesses cut in it, to fit the pole pieces, has also a cylindrical hole bored at right angles to the axis of the poles. Into this hole fits, accurately, a plug containing the specimen. The parts *cc* are of brass, and screwed together by long screws passing clear of the bobbin *d*. A central hole in the axis of this plug and penetrating to the bobbin permits the leads from the search coils to be brought out.

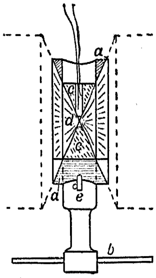


Fig. 61.

In one large electromagnet used by Ewing in which the pole pieces were about 10 cm. diameter the magnetising force was concentrated upon a neck only 2·6 mm. diameter and 3·5 mm. long. The maximum magnetising field obtained was $\mathbf{H}=24{,}500$.

In some experiments on tool steel, the neck part only was of the steel and was bedded into soft iron conical pieces which again fitted into the pole pieces as in sketch (*Fig.* 62). By removing one of the cones the search coil can be slipped off the specimen to determine the residual **B**. This can then be added to the **B** change produced when the main field is applied and then removed.

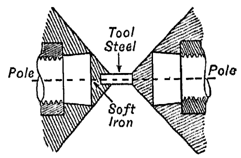


Fig. 62.—Pole Pieces for Parallel Rod Specimens.

The isthmus method when used with the system of turning round the specimen cannot easily be adapted to determine a hysteresis loop.

§ (40) N.P.L. Method for Fields up to $\mathbf{H}=4000$.[2]—This method makes use of a differential search coil for measuring **H**. The electromagnet is built up of laminations of ring stampings with a gap about 80 mm. wide.

The general disposition of the magnetic circuit is shown in *Fig.* 63, where *a*, *b* are the

[1] *Proc. R. Soc.*, 1887, xlii. 200; *Phil. Trans.*, 1889, clxxx. 221.

[2] "The Magnetic Testing of Bars of Straight or Curved Form," *J.I.E.E.*, Dec. 1915, liv. 35.

poles. Two blocks of soft iron, E, F, are held in a frame in such a manner that parallel holes through them of a diameter of 5 mm. are exactly in line. The outer faces of the blocks are 70 mm. apart, leaving 5 mm. air gap between these faces and those of the laminations; the inner faces of the blocks are about 1·4 cm. apart; this length thus forms the effective length of the specimen. The specimens are small turned rods 5 mm. diameter × 70 mm. long. The ends thus come just flush with the outer faces of the blocks.

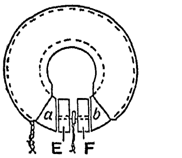


FIG. 63.—N.P.L. Magnet for High Magnetisation Tests.

The outer air gaps at the sides of the blocks serve the purpose of rendering the magnetic field more uniform in between them at the expense of lowering its value.

The search coil used for round rods consists of a narrow bobbin of ebonite chosen to be non-magnetic and wound first with a single layer of 40 turns to serve for measuring **B**. Over this are wound three successive coils, of 1200 turns each, of very thin enamelled copper wire. The ends of these search coils, *a*, *b*, and *c*, come to a switch which can connect either *b-a* on the ballistic galvanometer or *c-b*; in this way a check is obtained on the radial uniformity of **H**. The length of the bobbin (along the specimen) is about 6 mm.

For tests on strip materials a second pair of soft iron blocks is provided, having a slot cut radially down each to a distance about 2·5 mm. beyond the centre. Into these slots (which are 5 mm. wide) accurately fitting pieces of soft iron are fixed, and can be clamped down on the bundle of strips.

The sample consists of a bundle of strips 5 mm. wide × 5 mm. depth, and 70 mm. long.

The search coils for **H** in this case consist of two very small rectangular sectioned bobbins filed out of glass. The cores of the bobbins are only 1 mm. thick and about 4 mm. wide. On each is wound 550 turns of No. 47 enamelled copper wire. They are attached to a very light square brass tube on which the **B** test coil is wound.

The arrangement of the circuits is shown in the following diagram (*Fig.* 64).

In this diagram A is the laminated electromagnet of 1000 turns of No. 14 S.W.G. wire. The current-carrying parts of the circuit are similar to those in *Fig.* 28. In addition there are included the primaries of two variable mutual inductometers, M_1 and M_2. The secondary of M_1 is in the galvanometer circuit of the **H** coil. Its object is to balance the throw for **H** on reversal of the magnetising current. It is desirable to be able to do this, in part at least, so that sufficient sensitiveness for **H** may be secured in measuring coercive force; and yet when measuring $\mathbf{H}_{\text{max.}}$ the throw may not go right off the scale.

M_2 is in the **B** test coil circuit; its object is to compensate for the air space included in the **B** test coil. This correction becomes very large in measurements in high fields, for two reasons. (1) Owing to smallness of the section of specimen the air space necessarily bears a larger proportion to the total space than with larger specimens. (2) Owing to the smallness of the permeability the **H** is very much larger, and hence also the air flux, for a given **B**. This method of correction is only accurate so long as **H** is proportional to magnetising current. This is very nearly the case for moderate values of **H**, and the proportionality improves as **H** increases where

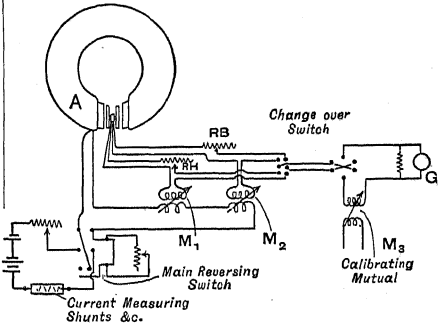


FIG. 64.—Circuits for High Magnetisation Tests.

the correction is of more importance. The correct value of mutual is given by the expression

$$M_2 = \frac{\mathbf{H}}{\mathrm{I}} N_3(s_0 - s) \times 10^{-2},$$

where M_2 is in microhenries, $\mathbf{H}/\mathrm{I}$ = constant for electromagnet, N_3 = turns in **B** test coil, s_0 and s are sections of **B** coil and of iron respectively.

A convenient mutual inductometer is a simple one consisting of two coils, one turning with respect to the other; a suitable range is 0 to 50 microhenries. The primary must be capable of carrying the magnetising current. The value of the mutual for balancing the **H** throw is given by

$$M_1 = \frac{\mathbf{H}}{\mathrm{I}} \cdot \frac{N_2 s}{100},$$

where $N_2 s$ is the area × turns of the **H** search coil, M_1 being in microhenries.

When this setting of M_1 has been made, then

$$\mathbf{H}_a - \mathbf{H}_b = \frac{M(I_a - I_b)}{N_2 s}.$$

The procedure in testing is similar to that

for rods in moderate fields by the **H** search coil method.

In determining hysteresis loops using the mutual inductance to balance the **H** throw, the ballistic galvanometer is calibrated so that a throw of one division on its scale corresponds to $\mathbf{H}=1$. The mutual M_1 is then set so that on reversal of $\mathbf{H}_{max.}$ no throw is obtained. When I—the magnetising current —is switched off from $+I_{max.}$ to 0 there will, in general, be a small throw which may be positive or negative according to whether the polarity of the magnet or of the specimen is the greater. This small **H** can then be used as a datum point from which to observe the part of the loop between $B_{rem.}$ and H_c. The mutual M_1 is then no longer required and the readings for **H** become independent of current. The main advantage of this method of measurement, in which the magnetising current is changed, over the isthmus method in which the specimen is rotated is, that hysteresis loops can be taken, and the coercive force and remanence of magnet steels determined under the conditions of high magnetisations. In some modern magnet steels saturation is not reached even with $\mathbf{H}_{max.}=4000$, and the evidence is, that until this saturation is reached or approximated to, the maximum values of remanence and coercivity are not attained.

It is therefore important to be able to investigate these properties in high fields.

It is, however, difficult to obtain a magnetisation greater than $\mathbf{H}=5000$ by a method involving reversing of the magnetising current, owing to the destructive sparking and to time lag in the magnetisation of the heavy masses of iron involved in the yokes and cores of the magnets.

§ (41) Apparatus of the Phys. Tech. Reichsanstalt.[1]—This method is a modification of the Ewing isthmus method. The alterations consist in replacing the small bobbin-shaped specimens by parallel turned rods which fit into holes in the axis of the conical pole pieces.

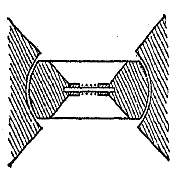


Fig. 65.—Gumlich Isthmus.

A diagram of the apparatus is given in *Fig.* 65.

The method of obtaining the flux change in the specimen by reversing it is retained. This operation is carried out by turning the handle through a half turn.

A very small air gap is maintained between the slightly conical plug portion and the correspondingly bored out pole pieces.

The search coils for **H** consist of two coils of 140 turns each in two layers on separate bobbins; the outer one is about 6 mm. diameter, and the inner one rather greater than 3 mm. diameter.

For measurements between $\mathbf{H}=130$ and $\mathbf{H}=4500$ the specimen is 20 mm. free length × 3 mm. diameter. For higher fields, small cylindrical hollow pieces of soft iron, about 6 mm. diameter and 6 mm. long, reduce the free length of specimen to about 8 mm., and thereby a field of $\mathbf{H}=6500$ can be attained (in the small Du Bois Half Ring Electromagnet).

For the determination of **B** the inner of the two search coils is used, suitably calibrated. Corrections for the air-space flux are made in the usual way.

For this correction it is necessary to know the mean area of the search coil with considerable accuracy, since this correction may amount to 10 per cent of the throw obtained.

§ (42) Gumlich Method with Yoke and Isthmus.[2]—This is an arrangement of a yoke with pole pieces of soft iron projecting far into the magnetising coil, leaving only a narrow gap at the centre. The magnetic circuit is shown diagrammatically in *Fig.* 66.

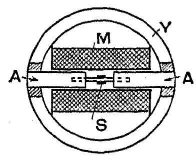


Fig. 66.—Yoke arranged for High Magnetisation Tests.

AA are two soft iron bars 25 mm. diameter with a 6 mm. hole bored centrally along them for the test specimen of 6 mm. diameter. The distance between the inner ends of the bars is 12 mm. M is the magnetising coil. Y is the laminated ring yoke.

The search coil S is constructed and arranged as in *Fig.* 67.

The coils are four in number and are shown by dots. Each coil consists of 40 turns of silk-covered wire, and they are separated by layers of paper.

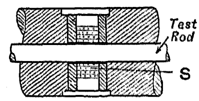


Fig. 67.—Detail of S in *Fig.* 66.

Numbering them from within outwards by 1, 2, 3, and 4, the field in successive zones can be explored by connecting 1 and 2 in opposition; 2 and 3; 3 and 4.

Experiments made at various values of the field from $\mathbf{H}=150$ to $\mathbf{H}=6000$ showed that by extrapolating back to the surface of the specimen the field can be determined to an accuracy of less than 0·5 per cent.

The curves connecting **H** and radial distance from the centre of specimen are convex, as shown in *Fig.* 68.

[1] E. Gumlich, *Magnetische Messungen*, p. 123; *Elek. Zeitsch.*, 1909, xxx. 1065.

[2] "Vorrichtung zur Messung höher Induktionen im Joch," *Arch. für Elek.*, 1914, ii. 465; *Verhand. der Deutsch. Phys. Ges.*, 1914, xvi. 395

The intersection X gives the true **H** at the surface of the specimen.

In making the air-space correction on **B**, however, the expression will be: Correction $= s_1 - s/s \cdot \mathbf{H}_1$, where $\mathbf{H}_1$ is the mean **H** in the annulus embraced by the first coil and the specimen, and not the **H** at the surface.

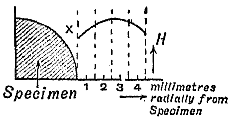


FIG. 68.—Radial Variation of **H** in Method of *Fig.* 66.

The determination of the correction on **H** by extrapolation to the surface need only be made once for all, and becomes, if one uses coils 3 and 1 in opposition, only $-0{\cdot}8$ per cent.

By using bars A with rectangular holes and a rectangular search coil, measurements on sheet materials can be made.

§ (43) HEAVY SOLENOID OF B. O. PEIRCE.[1]—In this most carefully carried out investigation the magnetising field was supplied by a very massive and accurate solenoid containing about 300 kgm. of copper. The solenoid was 106 cm. long and had approximately 14,000 turns of wire in two sections, the total resistance when in series being 17·5 ohms.

The windings when in series and connected to 550-volt mains allowed a field of $\mathbf{H}=3000$ to be obtained. When in parallel $\mathbf{H}=5000$ could be reached. Water circulation was maintained through the brass tube on which the solenoid was wound so as to maintain the specimen at constant temperature. A special switch was used to deal with the large energy stored in the solenoid. Owing to the heating of the winding when carrying large currents the current fell off during measurements. The value of current at the instant of reversal was determined by the charge on a standard condenser momentarily connected across a standard resistance in the main circuit. This charge could be read at leisure a short time after reversal.

The rate of rise of current being rather slow, a ballistic galvanometer of very long period (about 10 minutes) was used. It was calibrated by mutual inductance standards.

The corrections for the ends of the rod are small in such high fields, and with rods of 100 cm. long become negligible in fields above 2000 gauss.

The method constitutes probably the most accurate one for the determination of the saturation intensity in large rods. The experiments made with the apparatus were carried out with great care and elaboration.

§ (44) EXPERIMENTS OF DU BOIS.[2]—In these experiments on magnetic properties in strong fields a very novel method of making the measurements was used, *i.e.* the Kerr effect[3] was made use of. Kerr discovered that when plane polarised light is reflected from a polished, reflecting magnetised surface the plane is rotated through an angle which depends on the intensity of magnetisation.

A preliminary investigation by Du Bois[4] showed that the angle of rotation is proportional to the intensity of magnetisation. The method therefore measures **I**.

The means of carrying out the tests are shown in *Fig.* 69.

P_1P_2 are the conical pole pieces of the powerful magnet. P_1 has a central hole to allow the beam of polarised light to pass on to the polished specimen M and back.

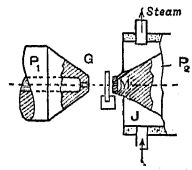


FIG. 69.—Du Bois Method of Measurement in Strong Fields.

The intensity of surface magnetism, and hence of the whole magnetism in such a thin sample, was directly read off from the angle of rotation of the plane of polarisation.

H was measured by making use of the formula $\mathbf{H}=\mathbf{B}-4\pi\mathbf{I}$. **B** is the induction normal to the surface of the specimen, and since the spreading of the lines of force is negligible near the surface, the **B** will also be equal to the normal air flux close to the surface.

This **B** was also measured optically by reflection from the polished glass plate G with a silvered back which was standardised by comparison with carbon disulphide whose coefficient of rotation is well known.

The method was checked by polishing small flats on ellipsoids.

Experiments were made on iron, nickel, and cobalt in fields up to 13,000 for nickel and to 4500 for steel. Further experiments were made on magnetite. The tests were made at 0° C. and 100° C.

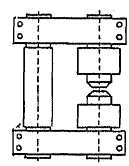


FIG. 70.—Heavy Electromagnet of B. O. Peirce.

§ (45) EXPERIMENTS OF B. O. PEIRCE.[5]—The form of yoke used by Peirce is shown in *Fig.* 70. It weighed about 300 kg. and was provided with about 3000 turns in the magnetising windings. The reversals were done by specially designed switches and, on account of the very consider-

[1] "The Maximum Value of the Magnetisation in Iron," *Proc. Am. Ac. of Arts and Sciences*, June 1913, xlix. No. 2.

[2] *Phil. Mag.*, 1890, xxix. 293.

[3] *Phil. Mag.*, 1877, vi. 321.

[4] *Ibid.*, 1890, xxix. 253.

[5] B. O. Peirce also carried out tests in high fields, using an exceptionally large magnet (*Am. Ac. Arts and Sciences, Proc.*, 1909, xliv. 354, and *Am. Journal of Sci.*, 1909, xxvii. 273, and 1909, xxviii. 1).

able time lag of the magnet, a ballistic galvanometer of exceptionally long period was used (10 minutes). **H** was measured by annular search coils.

Specimens in rod form 1·27 cm. diameter and 15 cm. long were used. The ends were tapered to fit accurately into corresponding sockets in the pole pieces.

For the highest fields the exposed part of the rod was turned down to a small diameter.

§ (46) P. Weiss.[1] — A very large electromagnet having poles 15 cm. in diameter with flat faces 35 mm. apart was used in these experiments.

In one method 4 mm. diameter rods were used. The induction was measured by withdrawing the rod through a hole in the pole. A fixed search coil connected to the ballistic galvanometer gave the throw proportional to **B**.

In the second method used by Weiss, small ellipsoids were made 4 mm. maximum diameter by 9 mm. long.

These could be withdrawn through a short fixed solenoid which was accurately wound and calibrated for the spacing of each turn, so that the flux threading it could be deduced from the known section of the ellipsoid at the plane of each.

With such short ellipsoids great accuracy of shape is necessary in ordinary fields, but at very high fields it is of less importance.

W. L. Chesney[2] has used the Du Bois type of magnet with holes bored through the poles in such a manner that long rods can be inserted and tested at various portions of their length.

Annular search coils for measuring **H** were used as in the N.P.L. apparatus. The power of the electromagnet was such that with a 2 cm. free length of specimen 6 mm. diameter a field of 3000 gauss could be obtained.

The search coil was such that **H** in annular zones at two different distances from the specimen could be measured to show the variation from uniformity of field. Various kinds of iron and steel were tested, including an exceptionally hard magnet steel containing cobalt.

IV. Measurements on Magnet Steels and on Permanent Magnets

These tests fall into two classes, those on the material before making up into magnets and those on the finished article.

The tests on the material consist in determination of the magnetic change point and measurements of the hysteresis loop with a fairly large value of $\mathbf{H}_{max.}$ (about 500).

[1] *Comptes Rendus*, 1907, cxlv. 1155.
[2] *Bull. Bur. Stds.*, Feb. 21, 1920, No. 361.

It is desirable to determine also the tip point permeability curve.

Bars of all sizes are used for making magnets, and the tests must adapt themselves, to some extent, to the shapes of the sections of these.

A large number are of rectangular section, in the form of flat bars about 1 cm. thick and from 2 to 8 cm. wide. In general it is not necessary to make tests on the full width of wide bars; 4 cm. × 1 cm. may be taken as the maximum size of section it is necessary to test.

The bars should be heated up to a temperature somewhat above the change point (say 50° C. above) and then quenched by allowing to fall into water kept stirred and at about 15°-20° C.

It is a useful preliminary test to measure the Brinell hardness, since for a particular steel this hardness number follows the coercive field roughly. An indication can thus be quickly obtained as to the quality of the steel before proceeding to the magnetic tests.

An excellent résumé of the work done on the correlation of the magnetic and mechanical properties of iron and steel has been made by C. W. Burrows.[3]

§ (47) Permanent Magnets. — Permanent magnets are of extremely various shapes and sizes, and the tests which are desirable greatly vary according to the purpose for which they are used.

Among the chief uses for which permanent magnets are required may be included the following: Magnetos (ignition and telephone-bell ringing), measuring instruments (ammeters and voltmeters, etc.), recording instruments (ampere-hour and watt-hour meters), telephone receivers (a great many varieties), relays, braking and damping magnets on meters and instruments, compasses, galvanometers.

The most important test on a large number of magnets is the determination of that part of the hysteresis loop between the remanence and coercive field points. The loop referred to should be that for a fairly high value of $\mathbf{H}_{max.}$ (at least 400).

On magnets used on some relays it is desirable to have the tests continued on to that part of the loop lying between $\mathbf{H}_{max.}$, $\mathbf{B}_{max.}$ and the remanence point.

On magnets for braking and damping purposes the air gap is usually narrow, and it is gap flux and the distribution of it which should be measured.

For measuring instruments, the permanence of the magnet with time is very important, also the temperature coefficient of the magnet as a whole and the effects of demagnetising fields.

[3] "The Correlation of the Magnetic and Mechanical Properties of Steel," C. W. Burrows, *Bull. Bur. Stds.*, 1916, xiii. 173.

From the heat treatment point of view it is most desirable, also, to make determinations of the magnetic change point.

In the case of some telephone magnets the resistivity of the steel is of importance.

§ (48) TESTS OF MAGNET STEELS. (i.) *N.P.L. Method.*—For flat bars in fields up to about $\mathbf{H}_{max.}=500$ the bars are placed in a ring yoke apparatus, and flat search coils about 8 cm. long and of width equal to that of the bars are used, one on either side in the middle portion of the bar. The **B** coil is wound directly on the bar (over silk tape) and occupies about 8 cm. length of the bar.

For **H** calibration the galvanometer is adjusted so as to read 100 divisions for an **H** change of 200.

The $\mathbf{H}_{max.}$ is determined as the sum of two throws; θ_1 when switching from $+\mathrm{I}_{max.}$ to 0, and θ_2 in switching from 0 to $-\mathrm{I}_{max.}$ These two throws in general are not equal; $(\theta_1-\theta_2)$ gives the **H** due to poles when the current has been switched off, from $+\mathrm{I}_{max.}$ to zero. This point, which usually is in the neighbourhood ($\mathbf{H}=0$ to -10), is used as a datum point in determining the important part of the hysteresis loop, *i.e.* that part lying between $\mathbf{B}_{rem.}$ and $\mathbf{H}_c$. The **B** corresponding to the point $\mathbf{H}=\theta_1-\theta_2$ is determined in the usual manner by first obtaining $\mathbf{B}_{max.}$ (switching from $+\mathrm{I}_{max.}$ to $-\mathbf{I}_{max.}$) and then subtracting the B change in switching from $+\mathrm{I}_{max.}$ to 0.

A turning coil can be used for measuring H if desired. It only measures the field very locally since its dimensions are necessarily very small.

It possesses the great advantage of measuring **H** absolutely and not merely change in **H**. Accurate measurements of coercive field can therefore be made by it without determining $\mathbf{H}_{max.}$ The method is thus applicable to measurements under magnetising forces of greater amounts than 400 or 500. A turning coil is shown in *Fig.* 20. The turning coil mounted in its frame is attached to the specimen in such a way that the face of the coil turning in its own plane just clears one side of the specimen.

Such a turning coil can be calibrated by placing it in the standard solenoid and turning through half a turn; the throw is compared with that obtained from a mutual inductance and reversed current.

A more accurate method of calibration, however, is to balance the mutual inductance between the coil and solenoid by a variable mutual inductance in the usual way with the turning coil first in one position and then in the other.

In a yoke apparatus as described in § (29), *Fig.* 43, bars up to 4 cm. × 1 cm. and 25 cm. long can be tested. The maximum magnetising field which can be applied is about 1000 gauss. This is for short intervals of time only.

(ii.) *Researches of Mme. Curie.*[1]—In this classical research experiments were made on square section bars 20 cm. long × 1 cm. square and on rings cut across a diameter with the surfaces accurately ground plane.

In the case of the bars, they were heated in either a bath of fused potassium and sodium chlorides or in an electric furnace.

The transformation point was determined with the help of a compass needle. The heating winding of the furnace served also to magnetise the specimen. The compass needle was placed near one end of the bar in such a position that it deflected at right angles to the meridian under the influence of the magnetisation of the bar. When the bar became non-magnetic at the transformation point T, the compass needle returned to the direction of the meridian.

The following effects were observed.

§ (49) PURE CARBON STEELS.—(*a*) With a rising temperature the temperature of transformation becomes lower as the percentage of carbon increases.

(*b*) The difference between the temperature at which the bar loses its magnetism on heating and the temperature at which it regains its magnetism on cooling is very small for low carbon steel or iron, but increases to 40°-50° for 0·8 per cent carbon steel.

(i.) *Tests on Bars.*—The bars were quenched in water from various temperatures and the following results obtained in the case of a bar containing 0·84 per cent of carbon:

Condition.	$\mathbf{H}_c$.	$\mathbf{I}_{rem.}$
Annealed	8	85
Quenched at 705° (above T_2 but below T_1)	14	130
Quenched 770° (above T_1)	52	410
Quenched 690° during slow cooling after heating above T_1	50	380

The magnetic tests made on the bars were:

(*a*) Magnetic moment, giving mean intensity of magnetisation.

(*b*) **I** in the central portion of the bar.

(*c*) Coercive force.

(*a*) was measured by magnetometer in the usual way, by comparison against a standard solenoid carrying a known current.

(*b*) was measured by quickly withdrawing a small coil from the centre of the bar and measuring the throw on a ballistic galvanometer.

(*c*) was determined by withdrawing a small coil from the middle of the bar when it was

[1] Magnet Steels. "Propriétés magnétiques des aciers trempés," *Comptes Rendus*, 1897, cxxv. 1165-1169; "Magnetic Properties of Tempered Steels," *Elec. Rev.*, 1899, xliv. 40-42, 75-76, and 112-113.

in a solenoid and subjected to a demagnetising field just sufficient to demagnetise it.

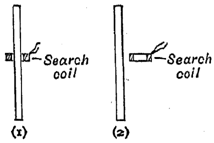


FIG. 71.—Curie Method of Determination of H, B, and I on Steel Bars.

I was determined from **B** with a knowledge of $\mathbf{H}_1$, the flux in the opposite direction just outside the bar.

Thus **B** is determined by withdrawing a small coil from position 1 in the diagram (*Fig.* 71), and $\mathbf{B}_1$ by withdrawing the same coil from position 2. Then

$$\mathbf{I}=\frac{\mathbf{B}+\mathbf{H}_1}{4\pi}.$$

(ii.) *Tests on Rings.*—The rings were in two halves and the faces ground as shown in *Fig.* 72.

On separation of the two halves, the search coil can be withdrawn and thus a measure of **B** made corresponding to any condition of magnetisation or point on the hysteresis loop.

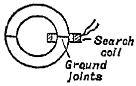


FIG. 72. — Split Steel Ring Tests of Mme. Curie.

The free poles introduce considerable error into the determination of **H** as calculated from the magnetising current. This is more important with soft materials than with hard steels.

Each new observation necessitates rejointing the halves of the ring and cycling the magnetisation, then arriving at the point desired before making an observation.

Corrections were made for the air flux embraced by the search coil, which must be large enough to be outside the magnetising winding.

The properties investigated were:

(*a*) Influence of chemical composition.
(*b*) Action of shock, blows and vibration.
(*c*) Action of temperature.
(*d*) Effect of external magnetic fields.
(*e*) Effect of time in producing spontaneous changes.

The steels experimented on were:

Tungsten.
Molybdenum.
Boreas (manganese).

The conclusions arrived at were:

To render a magnet as far as possible immune to disturbing actions without reducing its strength too much, it is best to bake it (after hardening) at a temperature of 60° to 70° for 48 hours and then, after strongly magnetising, to demagnetise by 10 per cent.

Only time can show beyond doubt the stability of a magnet.

§ (50) N.P.L. METHOD OF TESTING BENT (HORSESHOE) MAGNETS.—A large number of magnets shaped (*Fig.* 73) are used in electric measuring instruments and in magnetos.

The method used at the N.P.L. for testing these is by means of a search coil. Two sets of search coils of lattice form, as described in § (7), embrace the curved part of the magnet, one set on the outside and one set inside. For the straight portions or limbs of the magnet, flat search coils of suitable length, to come within a short distance of the ends of the magnet, are used. They have approximately the same product of (area × turns) per unit of their length as the lattice coils so as to give proportionate weight to the part of the magnet to which they are applied. The test coil for measuring **B** consists of 5 or 6 turns of thin wire distributed over the length embraced by the **H** coils. It is very important to ensure high insulation of these coils by the use of well-paraffined silk.

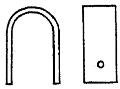


FIG. 73.—Typical Magneto Magnet.

In magnets having holes in the limbs a more uniform magnetisation can be secured by the use of plugs of hardened magnet steel.

The magnetising coils are very heavy and occupy practically all the space within the arch of the magnet. Straight coils with sloped tops (as in *Fig.* 74) are used on the limbs, and over the bend two or sometimes three coils of sector shape are used. These coils have their turns selected so as to provide approximately a uniform field all over the magnet. Short additional coils are provided, for use on extra long magnets.

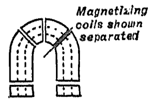


FIG. 74.—Magnetising Coils for testing Bent Permanent Magnets.

The magnetic circuit is closed by a keeper which is sometimes a short flat bar of annealed iron or series of bars about ½-inch square if the poles are not ground on their ends.

For extra short magnets a keeper of the shape shown in *Fig.* 75 is used. This is also annealed and the important faces are surfaced.

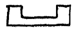


FIG. 75.—Keeper for Tests on Bent Magnets.

The circuit is exactly the same as for flat bars or for tests in the ring yoke apparatus previously described.

The minimum $\mathbf{H}_{max.}$ which it is desirable to employ in testing such magnets is 400. The magnetising coils previously described can provide this field by taking a current of 12 amperes. If the tests are carried out with the facility gained by practice, no appreciable heating occurs during a test with $\mathbf{H}_{max.}=400$.

The manipulation is the same as for flat bars or hysteresis tests on rods, etc., in the ring yoke apparatus. The observations re-

quiring most care are the two readings for the **H** (change) in passing (1) from $+\mathbf{H}_{max.}$ to current=0, and (2) current 0 to $-\mathbf{H}_{max.}$ These two throws obtained on the ballistic galvanometer will be, say, 204 and 196 respectively, thus giving for the point corresponding to I=0 in switching off from $\mathbf{H}_{max.}$, a value of **H** equal to −8. The accuracy of the subsequent part of the hysteresis loop between $\mathbf{H}_c$ and $\mathbf{B}_{rem.}$ depends on the accuracy of this observation. No special difficulty attends the determination of the corresponding value of **B**.

§ (51) COMMERCIAL APPARATUS FOR TESTING PERMANENT MAGNETS. (i.) *Betteridge Apparatus.*[1]—In the later form of this apparatus used by Betteridge the flux across a narrow air gap of small reluctance is measured by the electromotive force induced between the rim and axis of a thin rotating disc placed in the field.

The disc is of iron, coppered, and is rotated at constant speed by an electric motor.

A carbon brush bears on the edge of the disc and another on the spindle; these are connected to a millivoltmeter whose readings are proportional to the total flux in the air gap between the pole pieces. The upper parts of the pole pieces are surfaced off and form a platform on which the magnet to be tested and the magnetising windings rest, as shown in the diagram (*Fig.* 76).

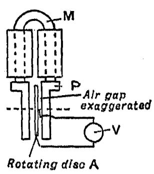


FIG. 76.—Betteridge Apparatus for Commercial Testing of Permanent (Magneto) Magnets.

The value of **H** corresponding to any given value of magnetisation is calculated from the ampere-turns applied to the windings.

Leakage will, of course, affect the results, but if the magnetising coils embrace the magnet fairly closely, the positive leakage from them will be small, and by the use of the iron disc and ball bearings for the spindle the clearances can be made very small. If desired the negative leakage due to the reluctance of the gaps and pole pieces can be allowed for.

By applying various magnetising forces as calculated from the magnetising current, and simultaneously observing the induced voltage as read on the millivoltmeter, any point on the hysteresis loop or the permeability curve may be determined.

In another type of magnet-testing apparatus the principle adopted is similar to the Köpsel permeameter, in which the flux is measured by the deflection of a moving coil mounted in a housing similar to the movement of a millivoltmeter.

(ii.) *The Magnet Meter.*—The instrument known as the "Magnet Meter" is provided with soft iron pole pieces forming an annular air gap, in which can turn a moving coil carrying a pointer similar to a voltmeter or ammeter. This moving coil is independently damped so as to be dead beat.

Adjustable magnetising coils—which only embrace the straight limbs of the magnet—are provided, and the magnet is slipped through these and its ends abut on the pole pieces.

The general disposition of the apparatus is as shown in the diagram (*Fig.* 77).

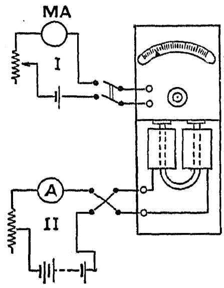


FIG. 77.—Magnet Meter.

The milliammeter MA indicates a steady small current flowing through the moving coil of the magnet meter. This current is usually held steady at one value during a test.

Magnetising or demagnetising current is applied and read through a current circuit II containing the necessary battery, ammeter, and regulating resistance. From a knowledge of the applied ampere-turns the approximate **H** is known, and from the reading of the pointer of the instrument and a knowledge of the small current through the moving coil, the gap flux can be determined. This flux is roughly proportional to the flux in the magnet.

The apparatus cannot be considered very accurate, since the external magnetic circuit has large reluctance, which will produce considerable shearing over of the loop or permeability curve.

The applied magnetising field is also very non-uniform owing to the magnetising coils being short and only embracing part of the magnet.

The apparatus is best calibrated by observing the readings given by it when a number of magnets whose characteristics are known are applied to it.

It is very rapid and simple to operate, the results would probably be reliable to 7 per cent in **H** or **B**.

(iii.) *Ericsson Magnet Tester.*—In this apparatus (*Fig.* 78), an improvement over the type previously described consists in providing a shunt path for the main part of the magnetic flux; only a small part traverses the air gaps and core of the moving coil part of the yoke; by this means the total reluctance of

[1] "Apparatus for the Commercial Testing of Permanent Magnets," Betteridge, *Electrician*, 1916, No. 17.

the yoke is greatly reduced, and hence results more nearly representing the condition of the magnet itself are obtained.

The main flux passes across the thin separating pieces P, and a part proportional thereto passes across the movement M. The face F is surfaced up and is sufficiently large to accommodate a number of different sizes of magnets.

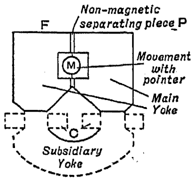


FIG. 78.—Ericsson Magnet Tester.

A subsidiary yoke can be used with additional magnetising coils, if desired, to compensate for the reluctance of the main yoke. A preliminary test is first made using this yoke and its magnetising coils. If a curve is obtained connecting deflection and magnetising current in coils C using the subsidiary yoke, then in testing the magnet, if the appropriate current is at the same time applied to the auxiliary yoke, corresponding to the deflection of the instrument, the reluctance of the main yoke will be compensated.

§ (52) ADDITIONAL TESTS ON BAR MAGNETS. (i.) *Moment.*—The moment of a bar magnet is measured by the help of a magnetometer. There are two main positions in which the magnet may be placed relatively to the magnetometer.

In position 1 (*Fig.* 79) the magnetometer magnet is at O and the magnet being tested is at M due magnetic north or south of O. The axis of the magnet at M is at right angles to OM. Then Moment of $M = r^3 H \tan\theta$, where r = distance shown, H = horizontal component of earth's magnetic field, and θ = deflection produced on O.

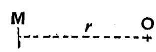


FIG. 79.
(Position 1.)

For position 2 (*Fig.* 80)

$$\text{Moment} = \frac{r^3 H \tan\theta}{2}.$$

For accuracy in the measurements r should be 10 times the length of the magnet and θ is preferably kept small. The magnet should be turned end for end and the deflection in the opposite direction noted; by this means error due to torsional control on the magnet when in the zero position is eliminated.

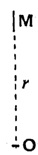


FIG. 80.
(Position 2.)

A very convenient set-up for measuring moments of bar magnets is to provide a turn-table with a flat top and a V groove across a diameter. The turn-table can be turned through a half circle located by stops so that the V groove is in line with the magnetometer axis. A magnetometer of the type described by F. E. Smith, used without torsion, is very convenient and rapid in use. For tests on similar magnets the scale can be calibrated to read the moment directly. Check calibrations should be taken at intervals, using a well-aged standard bar magnet of known moment.

(ii.) *Demagnetisation.*—If bar magnets are required to be constant with time they should be demagnetised by placing in a solenoid and applying a suitable demagnetising field. Tests should then be repeated on the magnet, and when by trial and error its moment has been reduced by 10 to 15 per cent of the moment obtained by magnetising to saturation, the magnet may be considered to be in a steady state with regard to time. Tests should, however, be repeated after, say, one month so as to eliminate faulty magnets, which show the defect of further decay in moment after demagnetisation.

The central flux in a bar magnet is a quantity which sometimes should be measured. In most cases it can very easily be determined by slipping off from the middle, a short test coil, of a few turns connected to a ballistic galvanometer.

§ (53) ADDITIONAL TESTS ON MAGNETS USED IN INDICATING AND RECORDING INSTRUMENTS.—In addition to the tests for coercive force and remanence on bent magnets, various other tests are desirable.

Magnets for Measuring and Recording Instruments. (i.) *Demagnetisation.*—The magnets for accurate measuring instruments should be demagnetised some 10 to 15 per cent of their initial strength. This can be conveniently measured by placing them in position on a standard movement and applying such a demagnetising field (with the help of fixed windings) that the initial deflection produced is reduced by the above percentage when the demagnetising field is applied and then removed, a steady small current being maintained through the moving coil of the movement.

(ii.) *Ageing.*—The magnets should be kept for a considerable time (about two or three months) after the above treatment and then tested again without remagnetising. The reduced or increased value of the effective gap flux gives a valuable indication of the constancy of the magnet with time. Heating for a comparatively short time at 100° C. can also be used to artificially age magnets. Carrying the magnet round a number of cycles of temperature between 100° and room temperature is a still quicker method of ageing (Mme. Curie).

(iii.) *Measurements of Gap Flux and Braking Power.*—Magnets used for braking purposes in measuring or recording instruments usually have a comparatively narrow air

gap and fairly large section of gap. The polar faces are usually well defined, but there is always a considerable amount of fringing, so that the measurements on gap flux density or on total gap flux, together with the area of the polar faces, cannot be considered a very accurate method of measuring the braking power.

For this reason it is very desirable to use a direct method, *i.e.* one in which the actual braking force can be determined either absolutely or relatively to a standard magnet.

In practice only a comparative method would be permissible. An excellent method here described by courtesy of the Metropolitan Vickers Electrical Co., Ltd., consists in the following:

A standard magnet with accurately ground parallel polar faces is carefully measured for magnetic and mechanical properties.

It is attached to the working element of a standard watt-hour meter having a large dial and pointer to record the number of revolutions made by the meter disc in a given time. The magnets under test are mounted on the same circuit on a number of similar watt-hour movements whose series coils all carry the same current as the standard instrument.

The volt coils are all in parallel across the same pair of terminals as that of the standard instrument. When current is switched on all the discs revolve and the revolutions they make in a given time are inversely proportional to the braking powers of the magnets.

All the dials are set at zero, current is then switched on for such a time that the standard instrument has recorded one revolution on its dial. The current is now switched off and the readings of the dials taken. These are inversely proportional to the braking powers of the magnets. The method is very quick, accurate, and independent of fluctuations in the applied voltage and current.

The accompanying photograph (*Fig.* 81) shows the general arrangement and the details.

§ (54) HEAT TREATMENT OF STEELS FOR MAGNETIC INVESTIGATIONS.—In general two different cases arise.

(*a*) Heat treatment applied whilst the magnetic tests are in progress and in which the material is at some definite temperature and condition.

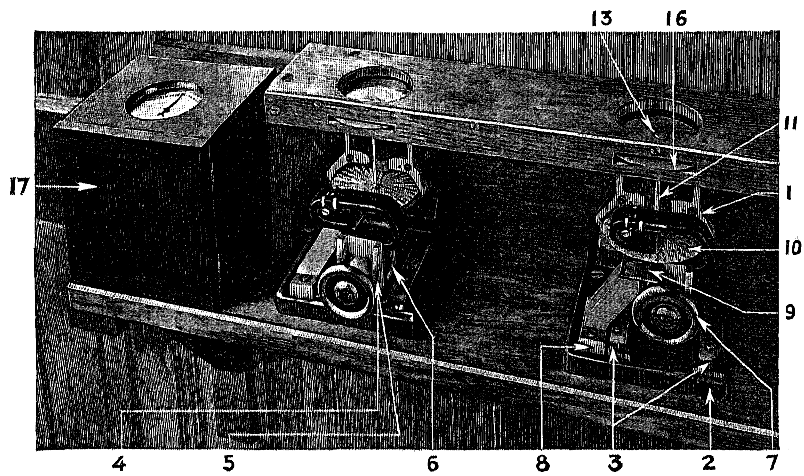


FIG. 81.—TESTING APPARATUS FOR BRAKE MAGNETS (Metropolitan Vickers Co.).

1, 2, the meter movement with rails 3; 4, 5, 6, and 7, a vice with parallel non-magnetic jaws sliding along 3; 8 and 9, a gauge for accurately locating the gap when clamping the magnet in the vice; 10, 11, 13, and 16 are the revolving disc, spindle, and dial; 17 is the standard shielded meter.

(*b*) Heat treatment which has been applied beforehand; the specimen is tested afterwards under some other set of conditions.

The investigations coming under class (*a*) are generally of such a specialised nature that each case must be dealt with according to the special requirements.

The determination of the magnetic change point is, however, of sufficient general interest to be described.

An arrangement, which has been successfully used at the National Physical Laboratory, for a number of investigations on magnet steels, is shown in the accompanying diagram (*Fig.* 82).

The specimen is a small turned rod, 5 mm. diameter and 70 mm. long, placed centrally in a long quartz tube as shown in the sectional diagram. A thermojunction touches each

end and serves to measure the temperature. The heating winding is of thin platinum strip wound on the small quartz tube and extending

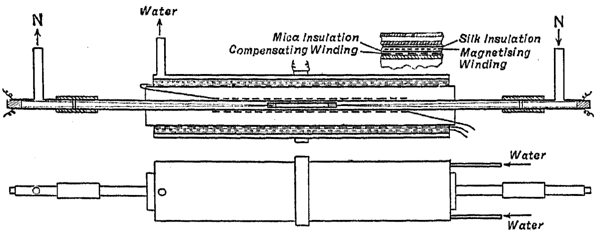


FIG. 82.—Electric Furnace for Determination of Change Points of Magnet Steels.

over a length of about 30 cm. Supported coaxially with the inner tube, by means of porcelain plugs, is a shorter quartz tube of about 3·5 cm. diameter. Immediately on this tube is a winding of bare copper strip of the same number of turns per cm. as the platinum heating spiral. It is connected in series with the heating winding and compensates the magnetic field due to the heating current. Mica insulation is provided over this copper winding and the magnetising winding comes next. Silk insulation is wound over this winding, and a water-jacket fits closely over the whole.

The test coil for measuring the magnetic condition of the specimen is outside the water-jacket and centrally located as shown.

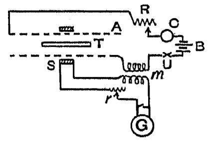


FIG. 83.—Circuit for Determination of Magnetic Change Points.

The electrical test circuits are as shown in *Fig.* 83, where T is the specimen, A the magnetising winding connected through variable rheostat R and primary of small variable mutual inductance M to battery B and ammeter C *via* reversing switch U.

The secondary circuit includes search coil S, secondary of variable mutual inductance M, and resistance *r*.

The temperature measuring circuits have been omitted. The variable mutual is adjusted so that on reversing the full magnetising current no throw is obtained on the ballistic galvanometer when no specimen T is present.

In determining the magnetic change point a small magnetising field should in general be used (**H** ⊁ 10); if throws are taken at intervals during steady heating of the specimen, using a constant value of applied **H**, a curve approximately as shown in *Fig.* 84 will be obtained, in which the permeability steadily rises as the temperature increases, frequently running up to a peak at a temperature in the neighbourhood of 800° C.; on further heating, the permeability falls very suddenly to a value nearly zero when the material changes to the non-magnetic state. On slow cooling a reverse change occurs, but in general there is a displacement corresponding to hysteresis. For successful quenching it is essential that the steel must have passed into the non-magnetic condition and must be in that condition when quenched.

A convenient apparatus for quenching bars consists of a vertical quartz tube electric furnace having a clear internal diameter of about 5 cm.

The bar to be quenched is suspended centrally within the tube and heated up to a temperature of 30° to 50° C. above the magnetic change point. When it is desired to quench the bar, the suspension is cut through or otherwise released and the bar allowed to fall into a fairly deep vessel of water.

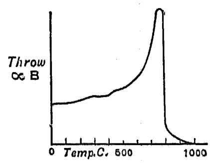


FIG. 84.—Curve showing Variation of **B** with Temperature for a Fixed Applied **H**.

The protection of the surface of specimens against oxidation and decarburisation is a matter of considerable difficulty. It is not very important in the case of large specimens, but with small rods the nature of the material may be seriously affected by prolonged heating if air is allowed free access to the surface. The best system of all is to heat the specimen in a vacuum. This can be carried out for the small-size rods such as are described above in connection with the change point apparatus. The next best system is to provide a steady stream of nitrogen through the tube containing the specimen.

The space must be closed as far as possible, and a definite leak through a water flask arranged so that it may be seen that the nitrogen is slowly passing through.

V. ALTERNATING CURRENT MAGNETIC TESTS

§ (55) MEASUREMENT OF LOSSES.—A very large proportion of magnetic material is used in the form of thin sheets and is subjected to alternating cycles of magnetisation.

When the cycles are very slow a certain amount of energy is spent in carrying the magnetisation through each cycle; this energy is of course the hysteresis loss. When, however, the cycles of magnetisation are quick the flux in cutting the material itself as it builds up and dies away again causes electromotive forces to be generated which

give rise to circulating currents, the so-called eddy currents. A further dissipation of energy thus occurs, known as the eddy current loss. The two losses together are known as the "total losses."

When the rate of magnetisation is in the region of commercial alternating current frequency, *i.e.* between 25 and 100 cycles per second, the hysteresis loss is sensibly the same as for slow cycles, and within a certain range of $\mathbf{B}_{max.}$ may be represented, therefore, by a formula of the type $h=\eta\mathbf{B}^{1\cdot6}$ per cycle. The energy thus expended per unit time, or power spent, in hysteresis becomes $W_h=\eta\,.\,n\,.\,\mathbf{B}^{1\cdot6}$ (ergs per sec. per c.c.), where n=number of cycles of magnetisation per second.

The eddy current losses can easily be seen to be approximately proportional to $\mathbf{B}^2$, n^2, and $1/\rho$, where ρ=resistivity of the material. In the case of sheets the eddy current losses are further proportional to the square of the thickness; for wires they are proportional to the square of the diameter.

The formula for eddy current losses, therefore, becomes, for a sine wave of secondary induced voltage (for sheets), $W_e=\xi(t^2\mathbf{B}^2n^2/\rho)$, and for wires $W_e=\psi(a^2\mathbf{B}^2n^2/\rho)$, ξ and ψ denoting functions of the quantities shown in the brackets.

In general it is not necessary to push the analysis of the losses to the degree given in these equations.

The total losses may be written, simply $W_t=\eta\mathbf{B}^x+\beta\mathbf{B}^y$, where x is not very different from 1·6 over a range of $\mathbf{B}$ between 5000 and 12,000, y is approximately 2 over the same range, and β is a constant for any given value of n.

In practice there are two ways of separating the losses corresponding to any particular value of $\mathbf{B}_{max.}$. The first of these methods depends on the fact that the hysteresis loss varies as n whilst the eddy current losses vary as n^2. If therefore for the same $\mathbf{B}_{max.}$ the losses are measured at a number of frequencies, and each of the values of W so obtained be divided by its appropriate frequency, a series of values of total energy loss per cycle will be obtained. These take the form $a+bn$, a and b being constants, hence plotting them against frequency will give a straight inclined line as in *Fig.* 85. This line intercepts the axis of y at a point R, and OR=hysteresis loss per cycle. The eddy current losses per cycle at any frequency ON are given by PQ. The power losses for the two components are given by $n\cdot\overline{OR}$ and $n\cdot\overline{PQ}$ respectively.

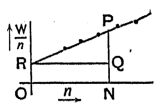


FIG. 85.—Separation of Hysteresis and Eddy Current Losses.

The second method of separating the losses is based on the fact that the eddy current losses depend on the manner in which the point $\mathbf{B}_{max.}$ of the cycle is reached with respect to time, *i.e.* they depend on the shape of the wave of $\mathbf{B}$ with respect to time. It can be shown[1] that the eddy losses are proportional to the square of the "root mean square" voltage which would be induced in a secondary winding close around the material; $\mathbf{B}_{max.}$ being constant, of course. The "mean" voltage in a secondary winding, when the wave of instantaneous E.M.F. is rectified, is proportional to $\mathbf{B}_{max.}$ If therefore, by the help of a synchronously rotating commutator, the wave of induced voltage is rectified and the mean value read on a galvanometer, which responds only to direct voltage, whilst, at the same time, the root mean square voltage is read on a voltmeter whose readings are proportional to root mean square voltage (such as an electrostatic instrument), the ratio of the readings of the two voltmeters will give the "form factor" of the voltage wave.

By means which are indicated below, the form factor can be varied over a wide range and a series of readings of total loss obtained.

Plotting these values of W against f^2, where f=(form factor), a sloping line will be obtained as in *Fig.* 85A, the frequency n being constant throughout.

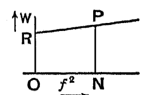


FIG. 85A.—Separation of Hysteresis and Eddy Current Losses.

In this case the intercept OR on the y axis will give the power losses due to hysteresis. To find the total losses corresponding to a secondary induced voltage of sine wave form, the ordinate PN is read off corresponding to the value of f^2 for a sine wave, *i.e.* 1·234.

For complete information regarding the total losses, it is necessary to measure them at two or three frequencies and two or three values of $\mathbf{B}_{max.}$.

The total losses are affected by temperature in the case of soft sheet iron or mild steel on account of the temperature coefficient of resistivity. In these materials the eddy current losses are a considerable proportion of the total losses. They may vary from 0·4 to 0·7 of the total losses within the range of commercial frequencies in the case of sheets 0·5 mm. thick. A correction for temperature, ranging from $-0\cdot2$ per cent to $-0\cdot4$ per cent per 1° C., may have to be applied to the total losses if it is desired to reduce them to a standard temperature. In the case of alloyed materials such as silicon iron no such correction is required, since not only are the eddy current losses much smaller, owing to the greatly increased resistivity

[1] G. Rössler, *Electrician*, 1895, xxxvi. 124.

resulting from the alloying, but the temperature coefficient of the resistivity is also practically zero.

In many uses of magnetic sheet materials the material is subjected to a steady magnetic field and a superimposed alternating magnetic field at the same time.

The hysteresis losses and the effective permeability for the alternating component of the magnetisation are largely affected by the steady magnetisation. The general effect appears to be to increase the hysteresis loss for the same range of ($\mathbf{B}_{max.\sim}$) in comparison with the value found by tests made in the ordinary way.

The effective permeability is altered in a way which depends on the amount of the superposed steady magnetisation $\mathbf{B}_0$.

For small superposed cycles the effective permeability increases as $\mathbf{B}_0$ is increased towards the region where μ is the maximum. Beyond this region $\mu_\sim$ rapidly diminishes as μ_0 falls. The two curves practically coincide for large values of $\mathbf{H}_0$.

In carrying out tests of this kind, care must be taken in applying the steady field. The simplest method is to include a battery or potentiometer source of steady voltage in series with the alternating source. Another method is to provide an independent winding on the sample. In this case there will be eddy current losses due to the alternating currents induced in this winding. They can be made small by including a high resistance in series and using a high voltage battery to provide the necessary steady magnetising current.

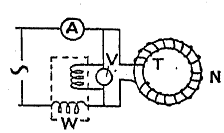


FIG. 86.—Simple Wattmeter Method for Measurement of Total Losses.

§ (56) GENERAL PRINCIPLES OF THE WATTMETER METHOD.[1]—In a number of earlier forms of apparatus for measuring total losses, only one winding was provided. *Fig.* 86 gives diagrammatically the connections.

T is the sample with winding of N turns.
A is an ammeter reading r.m.s. amperes.
V is a voltmeter reading r.m.s. volts.
W is the wattmeter.
Further:

Let I = current read by ammeter A,
I_1 = current in magnetising winding,
V = voltage read on voltmeter,
V_1 = reactance voltage between the ends of the winding,
r_v = resistance of voltmeter,
r_w = resistance of volt circuit of wattmeter,
R = resistance of magnetising coil,
W_t = power losses in the iron,
W = wattmeter reading,

[1] Gumlich, *Magnetische Messungen*, p. 150.

then
$$W_t = W - \left(\frac{V^2}{r_v} + \frac{V^2}{r_w} + I_1{}^2R\right),$$
where
$$I_1 = I - \left(\frac{V}{r_v} + \frac{V}{r_w}\right)\cos\theta + \tfrac{1}{2}\left(\frac{V/r_v + V/r_w}{I}\right)^2 \sin^2\theta,$$
in which $\cos\theta = W/VI$,
$$V_1 = V - I_1R\cos\theta_1,$$
where $\cos\theta_1 = (W_t + I_1{}^2R)/VI_1$. In many cases $\cos\theta$ is very nearly equal to $\cos\theta_1$.

The back electromotive force V_1 includes a part due to the air flux; this part is exactly in quadrature with the magnetising current, whereas the back E.M.F. due to the induction in the iron is not quite so. If E be that part of the self-induced electromotive force due to the iron, the expression connecting these quantities is (neglecting the phase difference)
$$E \doteq V_1 - 2\pi nLI_1,$$
where L is the coefficient of self induction of the magnetising winding excluding the space occupied by the iron.

Finally E is connected with the induction $\mathbf{B}_{max.}$ in the material by the expression
$$E = 4s\mathbf{B}_{max.}nf \times 10^{-8},$$
where s = section of material,
n = frequency (periods per second),
f = form factor of induced voltage wave.

Hence, if it is desired to measure the losses at a particular value of $\mathbf{B}_{max.}$, E must be first calculated, and then V, in order to set the reading on the voltmeter to correspond.

These calculations and corrections can be largely avoided by the introduction of other windings on the sample. The system of connections is as shown in *Fig.* 87.

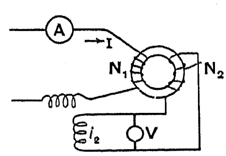


FIG. 87.—Ring with Separate Wattmeter Volt Circuit Winding.

In this arrangement the apparatus or ring is provided with a secondary winding, preferably inside, as close to the iron as possible. This winding may have the same number of turns as the magnetising winding or a fractional part thereof.

If $N_1 = N_2$ and the symbols have the same meaning as before, we have
$$W_t = \left(W - \left[\frac{V^2}{r_v} + \frac{V^2}{r_w}\right]\right)\left(1 + \frac{r_s}{r_w} + \frac{r_s}{r_v}\right),$$
r_s = resistance of secondary winding.

The correction terms in the second bracket are for the drop in volts on the secondary winding due to the current taken by the voltmeter and volt coil of the wattmeter. In a well-designed apparatus these two terms together only amount to 0·002 of V_s. To

obtain the $\mathbf{B}_{max.}$ we have, to a very close approximation,

$$E = 4sfn10^{-8}(\mathbf{B}_{max.} + a\mathbf{H}_{max.}),$$

where $$a = \frac{s_0 - s}{s},$$

(s_0 = area of cross-section of secondary),
(s = area of cross-section of sample),

and $$V - E \doteq V\frac{r_s(r_v + r_w)}{r_v r_w}.$$

If the wattmeter has a correction for inductance of the volt circuit, this will have to be applied after the other corrections have been made.

The most usual method of calibrating the wattmeter is to make use of a standard resistance of, say, 1 ohm, capable of carrying 1 or 2 amperes. This, in conjunction with a standard ammeter, potentiometer, or other means of measuring the current, forms the simplest and most accurate method of calibration.

The currents through the shunt and series coils of the wattmeter should be reversed when calibrating, since the earth's magnetic field exerts considerable torque on the shunt (moving) coil of a sensitive wattmeter.

It is desirable to be able to test the whole set up of apparatus for measurement of total loss. This is best done by the help of a standard ring sample of insulated stampings provided with various windings. The rings should be of old or aged material so as not to change their properties with time.

Accurate tests should be made on these with various values of $\mathbf{B}_{max.}$ and at two or three frequencies each. By the choice of convenient windings for the magnetisation and for the wattmeter volt circuits, almost any case arising can be imitated. The set up should be tested on the same multiplier of wattmeter and voltmeter and a similar reading on the wattmeter obtained.

A suitable design for such a standard ring is a set of stampings of 3 kgm. mass, inside diameter 20 cm., outside diameter 25 cm., thickness 0·5 mm. The stampings should have paper rings between and be bound up with paraffined silk.

The windings in order from within outwards which are convenient are 10, 20, 40, 80, 160, and 320 turns. Each wire should be well insulated from the others. The outer 160- and 320-turn windings should be of wire capable of carrying 3 or 4 amperes. The inner windings of 10 and 20 turns may be wound all in one layer so as to include as nearly as possible the same air space. This can be done by first winding the 10-turn coil in paraffined silk tube spaced round the ring and then winding on the 20-turn coil with each wire in between the 10-turn coil and going round the ring twice in this manner.

§ (57) Epstein Apparatus for testing Sheet Materials.[1] — The original Epstein apparatus consisted of four long magnetising windings on square sectioned formers. Each coil was 42 cm. long and the clear space inside just large enough to allow the bundle of strips 3 cm. wide and 50 cm. long to pass through. The coils had 100 turns each secondary winding and 400 each primary.

Fig. 88.—Method of insulating Iron Strips with Paper.

The sample consists of four bundles of strips 3 cm. × 50 cm., each bundle containing 2 to 2·5 kgm. of material. A thin sheet of paper is zigzagged between the strips, as in *Fig.* 88. They are arranged in the form of a closed square, as in *Fig.* 89, and are clamped up at the corners.

This square can be used with rough accuracy for determining effective permeability, using the 400-turn winding per coil for magnetising, but the leakage is very considerable. For wattmeter tests the magnetising winding is 100 turns per coil.

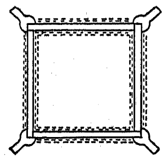


Fig. 89.

In a form of this apparatus as used at the N.P.L. a third winding of 5 turns per coil is used in connection with the determination of the form factor and $\mathbf{B}_{max.}$ The method is described in detail in § (61), dealing with the N.P.L. methods.

In the form used at the P.T.R.,[2] in which permeability tests are also made, four small flat search coils for **H** are provided on each coil; they lie close against the four sides of each bundle and are about 5 cm. long.

§ (58) Richter Apparatus.[3]—This apparatus was designed for the purpose of testing complete sheets as rolled without cutting them or damaging them in any way. The apparatus consists of a hollow drum with 120 turns of thick copper. The winding is in the form of a long narrow toroid. The drum is made up of wooden lattices between which the sheets are slid round. The overlapping edges are clamped between stout wooden clamps. Four sheets are used; in general the two ends of each sheet should lap directly on each other; this presents some difficulty in manipulation owing to the four long edges presented. The size of sheet tested is 100 cm. × 200 cm., the drum therefore being 100 cm. clear length and 63 cm. mean diameter. Sheets shorter than 200 cm. cannot be tested.

Owing to the fewness of the magnetising

[1] "Die magnetische Prüfung von Eisenblech," *E.T.Z.*, 1899, p. 590, and *E.T.Z.*, 1900, xxxi. 303-307.
[2] Gumlich and Rogowski, *E.T.Z.*, 1911, p. 613, and *E.T.A.*, 1912, p. 262.
[3] *E.T.Z.*, 1902, p. 491, and 1903, p. 341.

turns per unit length the magnetising current becomes rather large for values of $\mathbf{B}_{\text{max.}}$ above 12,000. The copper losses are also a considerable proportion of the total losses at high values of $\mathbf{B}_{\text{max.}}$

The air-space correction is considerable owing to the shape of the specimen, and this correction must be made for all values of $\mathbf{B}_{\text{max.}}$

A further disadvantage of this apparatus arises from the fact that the sheets are in a state of elastic strain when tested. As shown by Campbell and Booth,[1] large increase in hysteresis loss and reduction in permeability can be produced in sheet materials by subjecting them to an elastic strain.

The material is only tested in a direction parallel to the direction of rolling in this apparatus.

§ (59) Möllinger Apparatus.[2]—This apparatus tests the material in the form of stamped rings of standard dimensions (outer diameter 32 cm. and inner diameter 22 cm.). A weight of about 10 kgm. of stampings is used. They are separated with paper rings between and are bound up with insulating tape.

The magnetising winding is arranged so that each turn can be applied very quickly to the ring without threading any cable through it. 100 turns are provided: since this involves the same number of contacts owing to the special form of winding, these must be kept in order and the plugs properly driven home. In this single winding apparatus corrections for energy losses in the winding have to be made, so that it is important to keep the resistance low and constant.

The air-space correction is smaller than with the Richter apparatus, but must be taken account of in regions outside that where the permeability is very high.

An investigation of the respective merits of the three foregoing types of apparatus has been made by E. Gumlich and Rose.[3]

§ (60) Bureau of Standards Method.[4]—In this method, due to Lloyd and Fisher, the material to be tested is in the form of strips 25 cm. × 5 cm. These are assembled in the form of a square with the help of corner-pieces as in *Fig.* 90. The corner-pieces are bent sharply so as to localise the overstrained material to a small volume of the total material tested. Special clamps are provided at the corners to hold them together firmly with a good magnetic joint.

The overlap of the corners on the ends of the strips is a few millimetres. Each of the magnetising coils has a number of windings. First there are two windings in one layer of 45 turns each, close to the core of the bobbin, which has a clear space 5 cm. × 1 cm. Over these is wound a heavy winding of 250 turns No. 14 copper wire.

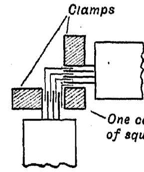


Fig. 90.—Corner and Clamps of Lloyd Square.

There are thus two secondaries of 180 turns each and a primary magnetising winding of 1000 turns.

One secondary is connected to a sensitive voltmeter for determining $\mathbf{B}_{\text{max.}}$ The other secondary is connected to the volt circuit of a reflecting wattmeter. The losses in the voltmeter and wattmeter volt circuit are small and known.

The magnetising winding is connected directly to the terminals of the alternator with no series resistance; the alternator has a very perfect wave form, and experiments have shown that the secondary induced voltage closely approximates to a sine wave.

The exact $\mathbf{B}_{\text{max.}}$ required is obtained by first setting the speed of the alternator with the help of a Hartmann and Braun frequency meter and then adjusting the alternator field excitation until the voltmeter connected to the secondary winding of the square indicates the correct voltage as given by the formula

$$V_{\text{RMS.}} = 4{\cdot}44 sn\mathbf{B}_{\text{max.}} \times 10^{-8}.$$

No allowance is necessary for the drop in voltage in the secondary winding.

For separation of the losses, tests are made at a number of frequencies and the line drawn as indicated in the first part of this section.

To commence with, corner-pieces must be cut for each sample, but when a sufficiently graded series of these has been accumulated, a set similar to the material under test can be chosen and small corrections made for the differences in quality.

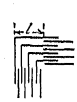


Fig. 91.

Corrections for the Corner-pieces.—Owing to the overlap at the corners the flux density is approximately halved in this part, hence the losses are reduced.

If M = mass of strips,
m = mass of corners,
l = free half-length of corners (*Fig.* 91),
L = length of strips, approximately 25 cm.,
W = measured loss,
$\mathbf{B}$ = average $\mathbf{B}_{\text{max.}}$,
$\mathbf{B}_1 = \mathbf{B}_{\text{max.}}$ near end of specimen,
then l/L = proportional increase in length of magnetic circuit due to corners,
$m/M - l/L = c$ = proportional mass of overlapping part of corners,
$2c$ = total material (corners and strips) in which flux density $= \mathbf{B}_1/2$.

[1] "On Errors in Magnetic Testing due to Elastic Strain," *Proc. Phys. Soc. London*, 1912, xxv. 192.
[2] Möllinger, *E.T.Z.*, 1901, p. 379.
[3] "Vergleichende magnetische Untersuchungen mit den Eisenprüfungapparaten von Epstein, Möllinger und Richter," *Elek. Zeits.*, 1905, p. 403.
[4] "The Testing of Transformer Steel," M. G. Lloyd and J. V. S. Fisher, *Bull. Bureau Stds.*, 1908–9, v. 453.

The correction to W for lap thus becomes $2cW[1-(\mathbf{B}_1/2\mathbf{B})^x]=2cWk$, where x expresses the law of variation of total loss with variation of $\mathbf{B}$, k may be taken as 0·70 for most purposes.

The total losses, per unit mass, for the sample then become

$$W_0=\frac{W}{(M+m)(1-1\cdot4c)}\doteq\frac{W}{(M+m)(1-1\cdot1y/100)},$$

where y = overlap in mm.

If the corner-pieces have a different total loss per unit mass, or a different cross-sectional area, an additional correction will be necessary; this will be given, to a sufficient approximation, by applying to the determined value of W_0 for the material under test, a correction equal to

$$(w_0-W_0)\left(\frac{m}{M}-0\cdot005y\right),$$

where w_0 = total losses per unit mass for the corners when subjected to the actual $\mathbf{B}_{max.}$ in them, and y = the overlap in millimetres.

The flux density in the corners will be t_s/t_c $\mathbf{B}_{max.}$ where t_s = thickness of sample and t_c = thickness of corners. If the losses in the corners are only known at standard $\mathbf{B}_{max.}$ of test, the actual losses per unit mass in them will be, for moderate differences in t, approximately $=(t_s/t_c)^{1\cdot8}$ times standard loss.

§ (61) N.P.L. Methods.—Both the Epstein and the Lloyd (Bureau of Standards) squares are used at the National Physical Laboratory. The set up is somewhat different in regard to the measurement of the quantities involved.

With the Lloyd square a greater variety of windings is provided and the strips used are 7 cm. wide instead of 5 cm. in order to reduce still further the effect (at the edges) of the cutting.

The actual windings provided in order from within, outwards, are 20, 40, 80, 160, 160, and 640 total turns. The outer 160- and 640-turn windings are magnetising coils.

For normal tests the 160-turn magnetising winding is used, and either the 160- or 80-turn windings for the wattmeter volt circuit.

The 20- or 40-turn coils are used for determination of $\mathbf{B}_{max.}$ and for measuring the form factor.

The $V_{RMS.}$ is read on a 100-volt Kelvin electrostatic reflecting voltmeter. The voltage is obtained from the 20-turn coil by the help of a 100 : 1 ratio transformer having a great many turns in the secondary. The actual maximum numbers of turns in primary and secondary are 400 and 40,000 respectively.

Each winding of the transformer is subdivided into a number of sections so that a great variety of ratios can be obtained if desired. They are all uniformly distributed round a closed core of iron ring stampings.

The current and power taken by the transformer are entirely negligible, owing to the very low flux density at which it is worked.

The form factor is determined in conjunction with the electrostatic voltmeter reading by the help of a synchronously running commutator and moving coil reflecting voltmeter which reads V_{mean}.

The pair of brushes which reverse the voltmeter connections at each half of the secondary induced voltage wave, can be slowly rotated together and locked in any position. In operation, with the magnetising current on and the instruments all deflecting, the voltmeter reading V_{mean} is watched, and the commutating brushes are slowly turned until a maximum is obtained on the voltmeter. At the same time the magnetising current is varied until the maximum V_{mean} has the value, as given by calculation, which corresponds to the $\mathbf{B}_{max.}$ desired.

The value of V_{mean} is,

$$V_{mean}=4\times N\times n\times s\times\mathbf{B}_{max.}\times10^{-8},$$

where N = 20 or 40 turns, n = frequency, and s = section of iron.

The circuits are as in *Fig.* 92:

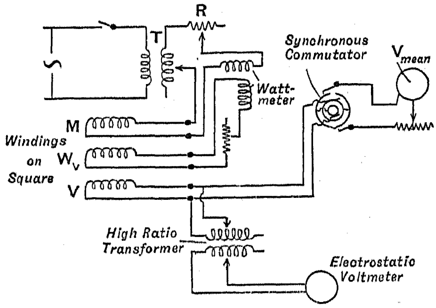


Fig. 92.—Circuits for Measurement of Total Losses in Sheet Material.

The wattmeter is a very sensitive instrument and is free from any solid metal parts. Normally, when carrying 1 ampere in the current winding and with 1 volt on the volt circuit, a series resistance of 5000 ohms is required to give 100 divisions for 1 watt.

The wattmeter is calibrated by sending the standard ampere through a standard rèsistance of 1 or 2 ohms. The volt circuit is arranged with multipliers on a dial, enabling readings of 100 on the scale to correspond to 0·5, 1, 2, etc., watts.

The voltmeter for reading V_{mean} is a bifilar reflecting instrument free from zero creep and has similar multipliers to the wattmeter volt circuit.

In carrying out the tests (the square having been carefully assembled from the weighed and bundled strips) the voltmeter is made to read the calculated V_{mean} as described above by the regulation of the magnetising current with the help of transformer T and rhéostat R. When this has been obtained, the watt-

meter and electrostatic voltmeter are read, care being taken to see that the frequency is keeping constant and exact. The form factor is determined immediately from the ratio

$$f=\frac{V_{RMS.}}{V_{mean}}.$$

The corresponding values of W_0 and f^2 are plotted, and from this the total losses at $f=1{\cdot}234$ corresponding to the sine wave are read off.

The variations in form factor are obtained by varying the secondary turns of the power transformer T and the rheostat R. When R is zero the form factor corresponding to a sine wave is closely approximated to. Increasing R and the necessary secondary turns on T gives increasing values to f. By this means f may be varied from 1·11 to 1·5 without difficulty.

A series of samples [1] tested at the Bureau of Standards, the Physicalische Technische Reichsanstalt, and the National Physical Laboratory by the various methods described above gave very consistent results.

The agreement in total losses was to about 1 per cent, and in the separated hysteresis losses to about 2 per cent for various materials and values of $\mathbf{B}_{max.}$.

§ (62) Bridge Method.—A. Campbell [2] has described a very simple bridge method for measuring total losses and effective permeability of sheets or wires.

The sample, preferably in ring form, is provided with superimposed primary and secondary turns N_1 and N_2 respectively, thus forming a mutual inductance m. The windings are connected to a mutual inductometer M as shown in *Fig.* 93. A slide wire or other resistance with a sliding potential contact is provided and connected as shown. A vibration galvanometer at G indicates when a balance has been obtained for the sine wave component of the secondary voltage. The current is read by an ammeter A. This current should be a sine wave as nearly as possible, either by tuning, or by including a resistance R large compared to the resistance of the primary winding of the ring, and using a sine wave alternator.

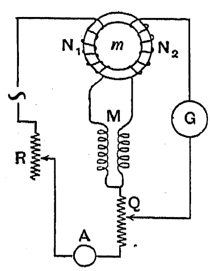


Fig. 93.—Campbell Mutual Method of measuring Total Loss.

Under these conditions we have to a close approximation

$$m=M,$$

and the total iron losses

$$W=\frac{N_1}{N_2}QI^2.$$

The mutual m includes the air flux common to both windings

$$m=4\pi\times10^{-9}\times N_1\times N_2\frac{(\mu s+s_1)}{l},$$

where μ=permeability, s=section of iron, and s_1=air section outside the iron and common to both windings, l=mean circumference of ring.

The value of $\mathbf{H}_{max.}$ is obtained from the current I, and is equal to

$$\frac{4\pi N_1 I_1\sqrt{2}}{10l}$$

(I_1=effective value of sine wave current).

Substituting $\mathbf{B}_{max.}$ and I_1 in the expression for m we get

$$m=\frac{10^{-8}\times N_2(s\,.\,\mathbf{B}_{max.}+s_1\,.\,\mathbf{H}_{max.})}{I_1}.$$

This is the simplest form of the equation to use when it is desired to adjust the current for a particular value of $\mathbf{B}_{max.}$.

When it is required to make tests at telephonic frequencies, a telephone replaces the vibration galvanometer. The source is then preferably a three-electrode valve generator.

The method is extraordinarily sensitive and adaptable to various conditions. By winding, say, 1000 turns on the secondary (N_2), measurements under the influence of magnetising fields as low as $\mathbf{H}_{max.}=0{\cdot}001$ can be made with good accuracy.

For tests with superimposed steady magnetisation the circuit shown in *Fig.* 94 is convenient.[3] It is self-explanatory.

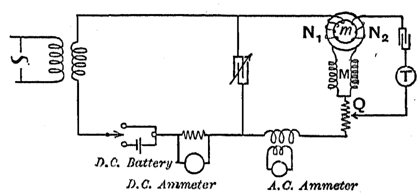


Fig. 94.—Method of *Fig.* 93 adapted to Measurements under Conditions of Superimposed Steady Magnetisation.

The d.c. ammeter reads only direct current and the a.c. ammeter is connected through a current transformer so as to read only the alternating component.

§ (63) Mutual Inductance. Bridge Method.—This is suitable for measuring

[1] "Total Loss and Hysteresis of Magnetic Sheet Materials," A. Campbell, H. C. H. Booth, and D. W. Dye, *Journal I.E.E.*, 1912, xlviii. 269.
[2] *Proc. Phys. Soc.*, 1910, xxii. 24.
[3] *Proc. Phys. Soc.*, 1920, xxxii. 232.

losses and effective permeability in telephone loading coils and iron lapped telephone cables.

P and L (*Fig.* 95) are the effective resistance and inductance of the wound sample, Q is a variable resistance of negligible inductance, N and T are the inductance and resistance, respectively, of the other arm, R and S are non-inductive ratio arms.

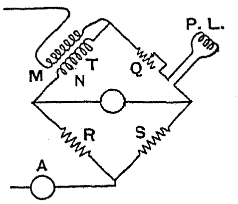


FIG. 95.—Mutual Bridge for Measurement of Total Losses.

The balance is first obtained with the sample short-circuited. Let the readings of M and Q be M_0 and Q_0 respectively.

P, L are now inserted and a new balance M_1 and Q_1 obtained. Under these conditions

$$L=(M_1-M_0)\left(\frac{R+S}{S}\right),$$

$$P=Q_0-Q_1.$$

From a knowledge of L and P the effective permeability and the total losses may be calculated as in § (62), putting P = Q and $N_1=N_2$, allowing for copper losses.

§ (64). MAGNETISATION ON MAXWELL BRIDGE BY ALTERNATING CURRENTS.[1] — A valuable series of experiments were made at various frequencies on different kinds of iron wire.

The samples were in the form of toroids wound up of the wire. These were provided with a uniform winding, thus forming a self-inductance coil.

The effective inductance and resistance of these coils was measured on a Maxwell's bridge, an optical telephone being used as detector of the balance. The alternating source was a siren and frequencies of 128, 256, and 512 ~ per second were used. The current was made approximately a sine wave by tuning.

The effect of raising the frequency is to make the loops rounder and the permeability smaller for a given $\mathbf{H}_{\max}$.

It was found that some of the energy is spent in the iron in the form of eddy currents of higher frequency than the fundamental.

This energy is not reduced by insulation of the separate iron wires. It is due to the non-linear relation between **H** and **B**, and since **H** varies approximately as a sine wave there are necessarily harmonics in the analysis of the curve connecting **B** and time.

§ (65) MEASUREMENTS IN HIGH-FREQUENCY FIELDS.—A number of investigations have been made in alternating magnetic fields of radio frequency. These measurements present great difficulties peculiar to the use of currents of radio frequency. The effects of capacity of various parts of the circuits to each other and of eddy currents other than in the magnetic material must be carefully watched and guarded against as far as possible.

In the experiments of Jouaust[2] the iron was in thin sheet form, each sheet insulated and in ring form. They were provided with a winding connected to an electrometer. A central copper tube formed the magnetising conductor, it was fed from a Poulsen arc generator, the frequency of the oscillations being 10^5 ∿ per second. On the same tube was threaded an exact replica of the wound ring but provided with a wooden core.

The ratio of electrometer readings was taken in the two cases and the effective permeability calculated therefrom.

Steel and silicon steel were tested in fields from 1 to 3 gauss. An effective permeability of about 150 was found for both materials. The losses were also measured and found to be 30 watts per kgm. in the steel and 150 watts per kgm. in the silicon steel.

Alexanderson[3] has carried out a fairly complete series of experiments on wires and strips of very thin iron. A high frequency alternator was used as source of radio frequency current. The arrangements for making the measurements are given in *Fig.* 96.

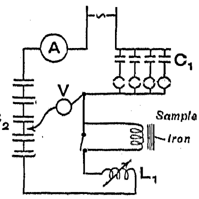


FIG. 96. — Alexanderson Method of measuring Magnetic Properties at Radio Frequencies.

The specimen is made up in a ring form and provided with a single winding which is inserted in the circuit as shown in series with a variable self inductance. The adjustments are made as follows for loss measurements. The alternator is held at constant speed, condensers C_1 are adjusted until the current is a maximum as read at A. The voltmeter contact V is then moved along the bank of condensers C_2 until it reads an approximate minimum, the variable inductance L_1 is then adjusted to give an exact minimum. When this condition has been realised the voltmeter reads the resistance component of the voltage across the variable inductance and the ring. This reading multiplied by the current gives the total energy spent in the two inductances and in the condensers C_2 which are included up to the point of contact of the voltmeter.

A blank experiment is then made with the ring short-circuited.

Corrections must be made for the losses

[1] M. Wien, *Wied. Annalen*, 1898, lxvi. 851-953.

[2] R. Jouaust, "Permeability of Iron at High Frequencies," *Soc. Int. Elect. Bull.*, 1911, Ser. 3, i. 9-57, and *Comptes Rendus*, 1910, cli. 984.

[3] *Proc. Am. I.E.E.*, 1911, p. 2433.

in the copper winding of the ring, and any change in resistance of the inductometer due to change of inductance, also change in the tapping off of C, must be allowed for.

The method can be checked partly by replacing the wound specimen with a resistance whose value is known at radio frequency.

In the experiments made by Alexanderson, tests were carried out at a number of different frequencies between 40,000 and 200,000, also at values of $\mathbf{B}_{max.}$ ranging from 450 to 1500.

The effective permeability was measured by the help of an electrostatic voltmeter connected across the ends of the wound specimen.

The conclusions arrived at were, that after allowing for eddy current effects, the true permeability is not very different at radio frequencies from that at low frequencies.

§ (66) Silsbee Method.[1]—The specimens had primary and secondary windings. The flux was determined by the voltage induced in the secondary winding. This measurement was made by the help of a modified form of potentiometer using a thermojunction and galvanometer as a detector of the balance. A variable inductance and resistance in series in the primary circuit was arranged as in *Fig.* 97 so that the voltage and phase angle across the combination could be adjusted exactly equal and opposite to that induced in the secondary winding. From the known L, R, primary current and frequency, both the magnitude and phase angle can be computed, and hence the effective permeability and the losses calculated.

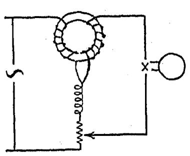


Fig. 97.—Silsbee's Method.

A Poulsen arc giving 2 amperes at 350,000 ~ was used. Four specimens of fine iron wire 0·009″ to 0·002″ diameter were tested. They were wound up (after shellacking) into small rings of circular section.

The effective permeability, after correcting for skin effect, was found to be independent of frequency.

The formula is given for the skin effect based on the assumption that the hysteresis loop is of the form

$$\left(\frac{\mu\mathbf{H}-\mathbf{B}}{\nu\mathbf{B}_0}\right)^2+\left(\frac{\mathbf{H}}{\mathbf{H}_0}\right)^2=1,$$

where $\mathbf{H}$ = magnetomotive force per cm. at any point;
$\mathbf{B}$ = corresponding flux density produced;
$\mu=\mathbf{B}_0/\mathbf{H}_0$;
$\nu=(\mathbf{B}\text{ at }\mathbf{H}=0)/(\mathbf{B}\text{ at }\mathbf{H}=\mathbf{H}_0)$;
and $\mathbf{H}_0$ = maximum value of $\mathbf{H}$;
$\mathbf{B}_0$ = corresponding value of $\mathbf{B}$.

[1] Discussion to Alexanderson's paper above, *A.I.E.E.*, 1911, Part III. p. 2452.

§ (67) Experiments by A. Battelli and L. Magri.[2]—These experiments were made on thin well-insulated wires with the help of a Braun cathode-ray tube to delineate the form of the hysteresis loop.

The original paper should be consulted for details of the method of obtaining the loops and deducing the results.

Steel wires 0·005 cm. diameter and iron wires 0·01 cm. diameter were tested. The hysteresis loop was found to be practically the same at 10,000 ∿ per second as at 50 ∿ per second. At higher frequencies the area of the loop was slightly less.

With iron wire 0·03 cm. diameter the curve becomes greatly modified and tends towards the form of an ellipse due to eddy currents.

Further experiments were made by L. Schames.[3]

An arc was employed for producing undamped oscillations using carbon rods in air for frequencies up to 20,000 ∿ per second and with a Bunsen flame surrounding the arc for higher frequencies.

The permeability was calculated from the ratio of flux produced in a coil of 251 turns wound on a 10 mm. diameter glass tube, first with no core in the tube and then with a core consisting of 100 wires of soft iron 15 cm. long and 0·005 cm. diameter.

The corrections to the voltmeter used to determine $\mathbf{B}$ were small, and the accuracy was considered to be to about 3 per cent.

There is, however, a considerable correction to be applied on account of the shielding effect of the outer wires in the bundle on the inner ones.

This correction was calculated and found to vary from 2·5 per cent at a frequency of 88×10^3 to 13 per cent at a frequency of 220×10^3.

The curves connecting $\mathbf{H}$ and μ for various frequencies show that while from 2400 to 3600 ~ per second μ is fairly high (700 when $\mathbf{H}=20$ and 350 at $\mathbf{H}=60$), at higher frequencies μ changes much less with change in $\mathbf{H}$, having a mean value of 425 at frequency 62,500, 385 at 110,000, and 360 at 152,000.

VI. Measurements on so-called Non-magnetic Steels and on Feebly Magnetic Materials

§ (68) Non-magnetic Steels.—A class of materials in between the strongly magnetic metals and their alloys and the very feeble paramagnetic and diamagnetic bodies are the so-called non-magnetic steels, the tests on which call for special treatment since they are in general of too small permeability to be measured in the ordinary form of apparatus

[2] *Accad. Lincei Atti*, 1906, xv. 485-492.
[3] *Ann. d. Physik*, 1908, xxvii. I. 64-82.

as previously described, but at the same time they are far too magnetic to be satisfactorily measured in the magnetic balances to be described below.

In some respects the tests are simplified because the leakage from the ends of the specimens is so small that **H** may be considered to be unaffected by the presence of the material, and may therefore be taken as accurately given by the constant of the solenoid or other magnetising coil used for providing the field.

The method in use at the National Physical Laboratory is as follows (*Fig.* 98): The

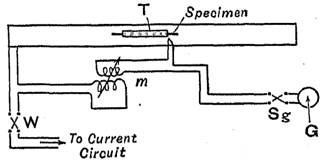


FIG. 98.—Method of measuring Properties of Non-magnetic Steels.

specimen, usually in the form of turned rod or flat strip, is provided with a search coil T of known large number of turns (about 2000). The specimen and search coil are placed centrally in a solenoid of known constant; they are mounted in such a way that the specimen can be withdrawn without disturbing the search coil.

A variable mutual inductometer is included in the circuit (range about 1 millihenry). This inductometer is connected up and adjusted so that on reversing any current through the circuit no throw is obtained when the search coil is in the solenoid but with no specimen through it. If now the rod is inserted in position in the solenoid and search coil, a throw will be obtained on reversal proportional to $\mathbf{B}-\mathbf{H}$. By this means the accuracy is greatly increased, because the ballistic galvanometer calibration can be made so much more sensitive for reading flux density, also there is no necessity to observe **H** with very great accuracy nor to hold the current steady.

Hysteresis loops can be determined with good accuracy even at low values of $\mathbf{H}_{\max}$. In carrying out the operations of observing a loop, the **H** change made must be added (with due regard to sign) to twice the **B** throw.

§ (69) MAGNETIC BALANCES.—The various types of magnetic balances form the most sensitive means of determining the magnetic properties of the feebly paramagnetic and diamagnetic materials.

A most successful balance of this type is the magnetic balance of P. Curie used in his classical researches on magnetic properties of various materials at different temperatures.[1] The torsion balance operates on the principle of the force exerted on any material when placed in a non-uniform magnetic field (*Fig.* 99). Consider a field at a point on the axis Ox having the direction Oy and varying in intensity along Ox according to some law given by the curve $\mathbf{H}_y$.

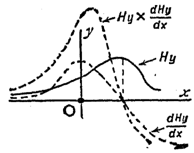


FIG. 99.—Appertaining to Curie Balance.

Then a body placed at O will experience a force in the direction Ox and equal to $m\,.\,\mathbf{I}\,.\,d(\mathbf{H}_y)/dx$, where $\mathbf{I}$ = intensity of specific magnetisation = magnetic moment/mass, m = mass of body.

Calling k the coefficient of specific magnetisation where $\mathbf{I}=k\mathbf{H}_y$, we have

$$f=m\,.\,k\,.\,\mathbf{H}_y\frac{d(\mathbf{H}_y)}{dx}.$$

In the original apparatus the non-uniform field was provided by means of two electromagnets with inclined axes as in diagram (*Fig.* 100). The field $\mathbf{H}_y$ was measured by reversing a known small coil placed at successive positions along Ox. The slope $d(\mathbf{H}_y)/dx$ can best be measured by making use of the law for any magnetic field that

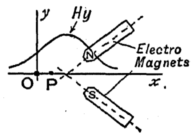


FIG. 100.—Principle of Curie Balance.

$$\frac{d}{dx}\mathbf{H}_y=\frac{d}{dy}\mathbf{H}_x.$$

If a known search coil is placed at any point P with its plane parallel to the field, *i.e.* parallel to Oy, and a small known displacement Δy be given it in direction Oy, we have

$$\Delta\Phi'=s\frac{d}{dy}\mathbf{H}_x\Delta y,$$

$$\frac{\Delta\Phi'}{\Delta y}\text{ will be equal to }\frac{\Delta\Phi}{\Delta x},$$

where the limit of

$$\frac{\Delta\Phi}{\Delta x}=s\frac{d}{dx}\mathbf{H}_y.$$

The advantage of calibrating by this method is that the actual flux Φ linking the coil is zero when in the plane Oy, and hence vibration and small variations in the magnetising current are unimportant.

The non-uniform field provided in the original apparatus by means of two electromagnets with inclined axes is replaced in the apparatus

[1] P. Curie, *Journal de Physique*, 1895, iv. 197 and 263; also P. Curie, *Journal de Physique*, 1903, ii. 796-802; *Sc. Abs.*, 1904, p. 374; also P. Curie and C. Cheneveau, *Phys. Soc. Proc.*, 1909–10, xxii. 343.

described in the *Proc. Phys. Soc.*, 1909–10, xxii. 343, by a permanent magnet in a more portable and conveniently operated arrangement as indicated in *Fig.* 101. The material to be tested is placed in a glass tube suspended by the hook *c* from one end of an arm TT, which is suspended by a silk or quartz fibre J. The other end of the arm TT carries a copper sector swinging in the gap of the damping magnet A. Balance weights B and *p* serve to keep the arm T horizontal. The deflections are observed with the help of the mirror *m*.

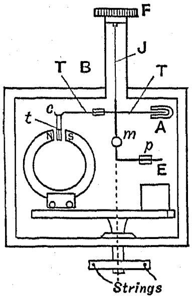


FIG. 101.—Portable Curie Balance with Permanent Magnet.

The permanent magnet NS is mounted in such a manner that it can be turned concentrically below by means of an arm and two strings.

In operating the apparatus the magnet is slowly turned so as to recede from the specimen. This is continued and observations are made when the deflection produced on the suspended system reaches a maximum.

Readings are taken for deflections on both sides of zero.

The most convenient method of calibrating is to use distilled water as a standard substance the susceptibility of which, at ordinary temperatures, is $0{\cdot}79 \times 10^{-6}$.

Observations are first made on the empty glass tube. If the deflection so obtained is θ'', then

$$\frac{k}{k_1} = \frac{m_1}{m}\,\frac{\theta \pm \theta''}{\theta_1 \pm \theta''},$$

where k = susceptibility being measured on substance of mass m and θ is deflection produced.

k_1, m_1, and θ_1 are similar quantities for the standard substance.

The sign + must be used for θ'' if the susceptibility of the glass tube is of opposite sign to that of the body being examined.

A further correction must be made for the susceptibility of air when measuring the susceptibilities of gases.

For a paramagnetic body this correction is $+0{\cdot}041(1/r\Delta+1)$ where r = approximate susceptibility of the material and Δ = its density.

For a diamagnetic body the sign is negative.

§ (70) MUTUAL INDUCTANCE METHOD FOR TESTING FEEBLY MAGNETIC MATERIALS.—This method forms a very rapid and convenient means of measuring approximately the susceptibility of various materials to see if they are suitable for use in apparatus requiring non-magnetic parts. Parts such as brass rod, castings, screws, marble, stone, etc., may be readily measured thus.

The method shown diagrammatically in *Fig.* 102 involves the use of a coil having two windings, and so forming a fixed mutual inductance of large amount (1 or 2 henries). The primary winding is permanently connected to a good battery; the current must be extremely steady and the connections should be few and good.

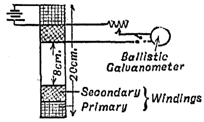


FIG. 102.—Mutual Inductance Method of measuring Feebly Magnetic Substances.

The secondary is connected directly to a ballistic galvanometer of long period.

The material to be tested, of suitable size, is quickly withdrawn from the centre of the mutual inductance, and the resulting throw observed.

The calibration is made by means of a large test-tube filled with a known weight of ferrous sulphate or other suitable material of known susceptibility.

§ (71) METHOD OF O. C. CLIFFORD.[1]—In this method, which was used to determine the susceptibilities of tin and bismuth and their alloys, a special form of torsion balance was used as indicated in *Fig.* 103. The electromagnet was provided with pole pieces NS, which were bored through along the axis shown dotted. The pole pieces were also fitted with small projecting pieces *p*, *p*, thus producing a localised variable field.

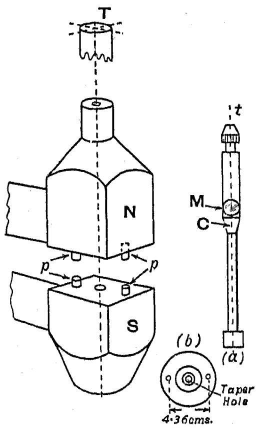


FIG. 103.—Clifford's Method of measuring Small Susceptibilities.

The upper pole piece carries an extension, at the top of which is a turning head, which can be traversed along two axes at right angles to one another. The turning head has a clamping arrangement and a tangent screw. Suspended from the turning head T by a phosphor bronze strip is a spindle, as shown at (*a*). The spindle carries a mirror M and immediately below is turned taper as shown at C. A disc (*b*) with accurately fitting taper hole is provided, and swings, when mounted, freely between the pole tips.

[1] *Phys. Rev.*, 1908, xxvi. 424.

The spindle and disc are of celluloid. The disc is provided also with a collar on which an inertia ring can be slipped centrally. There are two small holes near the edge of the disc 4·36 cm. apart, on a diameter.

Small bismuth cylinders about 6 mm. diameter and length can be attached to the disc by projecting pins fitting the small holes. These are strongly diamagnetic, and so are repelled away from the poles when the field is applied.

The specimens to be tested are attached to the top of the bismuth cylinders.

The method of measuring the susceptibility consists in observing the deflections produced, first when only the bismuth cylinders are in position and then when both bismuth and the material under test are together.

If V = volume of test piece,
$\mathbf{H}$ = magnetic field,
$d\mathbf{H}/dx$ = rate of change of field, x being along an arc,
f = force in dynes on test piece,
k = susceptibility,

then
$$k=\frac{f}{V\mathbf{H}d\mathbf{H}/dx}.$$

The couple produced by the force f is balanced by the torsional control of the suspension, thus $f=u\theta/r$ where

u = torque per unit angle of twist,
r = radius at which f acts.

u is determined by adding an inertia ring and determining times of oscillation before and after.

$$u=\frac{4\pi^2K}{t_2^{\,2}-t_1^{\,2}}.$$

K = moment of inertia of added ring.

The field $\mathbf{H}$ was determined around an arc of about 1 radian at the radius of the pins p for various exciting currents, by the help of a search coil of the same cross-section as the specimens, and mounted so that it could be quickly jerked out from any set position.

The $d\mathbf{H}/dx$ curves were obtained, by plotting, from the $\mathbf{H}$, x curves. Corrections must be made for the celluloid table and spindle owing to want of perfect uniformity in them.

The results obtained for the pure bismuth tested were in very good agreement with those obtained by V. Ettenhausen.[1]

Since the result, when testing other materials, depends on the difference between the deflections produced by bismuth alone and bismuth + specimen, the observations must be made with considerable accuracy.

It is not necessary to use the bismuth when measurements on diamagnetic materials are made.

[1] *Pogg. Ann.*, 1877, clx. 1.

§ (72) Measurements of Prof. E. Wilson.[2] —This instrument is a modification of the Curie balance and was devised in connection with measurements on rock specimens. The outline of the design is given in *Fig.* 104. The electromagnet is cut from a plate of stalloy (3 per cent Si iron) and has a parallel air gap 1·5 cm. wide; the poles are tapered in plan as in *Fig.* 104A, leaving opposing faces 1 cm. wide and 3·75 cm. long. Any magnetic field up to about $\mathbf{H}=2000$ could be obtained in the air gap.

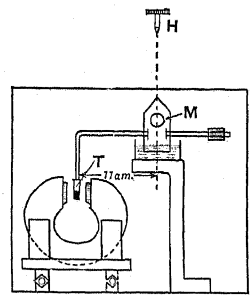


Fig. 104.—Wilson Type of Curie Balance.

The suspended system consists of a light aluminium arm bent at the side over the magnet so as to enable the specimen T to be suspended freely in the magnet gap at a radius of approximately 11 cm.

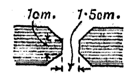


Fig. 104A.—Plan of Poles of Electromagnet of *Fig.* 104.

The horizontal beam passes through an aluminium damping vane, the lower end of which is submerged in oil. A mirror is attached to the vane at M.

The distribution of the magnetic field in the air gap was measured by observing the torque on a fine wire mounted suitably in the gap and carrying a known current. This method is accurate and allows a very localised measurement of the field to be made.

The method of determining the instrumental constant for the space occupied by the specimen when in the position of maximum deflection was determined in various ways, and is related to the susceptibility by the equation $K=C\theta/I^2V$,

where C = constant of instrument,
θ = deflection,
V = volume of specimen,
I = current in amperes.

The effects of the finite size of the specimen were investigated.

The susceptibilities of the following substances have been determined: various iron ores, micas, rocks, sands, aluminium and alloys of aluminium, glasses, tourmaline, and solutions of manganese sulphate and ferrous sulphate in water.

[2] "The Measurement of Magnetic Susceptibility of Low Order," E. Wilson, *Proc. Roy. Soc.* A, 1920, xcvi. 429.

§ (73) MAGNETIC PROPERTIES OF HEMATITE.[1]—This research on crystals of hematite obtained from various sources makes use of the same principle as the Curie balance, but small spheres are used and the non-uniform field is provided by a magnet with cylindrical pole faces. Such pole faces were found to give a nearly constant field gradient over a considerable space.

The formula $F_y = \mathbf{I}_x v(d\mathbf{H}_x/dy)$ was used. The force F was measured by observing the bending produced in a disc of cover glass against the centre of which the sphere was pressed by the force. The measurements were made by means of an interferometer by observing the shift of the fringes.

Measurements were also made on hysteresis. The specimens examined fell into two classes; those showing no signs of hysteresis and possessing intensity proportional to the field in the direction of the axis of symmetry of the crystal. The other class showed marked hysteresis along the axis. In the first group the specimens are paramagnetic along the axis and ferromagnetic in the principal plane of the crystal. In the second group the crystals are ferromagnetic in all directions.

§ (74).—A. Piccard and E. Cherbuliez[2] have described a novel method for the study of very dilute paramagnetic solutions.

A circular tube contains in its lower half the solution to be examined and in its upper half the pure solvent. The part of the tube containing one of the surfaces of separation is placed between the poles of an electromagnet while the part of the tube containing the other surface of separation is attached to a cathetometer so as to be movable vertically. The circulation of the liquid is rendered visible by small particles floating at a position where the tube is capillary. The level of the surface attached to the cathetometer is adjusted until the circulation ceases. This operation is performed with and without the magnet being excited. The difference between the readings represents the magnetic ascension of the solution with relation to the solvent. If the intensity of the field, densities of the liquids, susceptibility of the solvent, and concentration of the solution are known the susceptibility of the dissolved substance is easily calculable.

§ (75) G. MESLIN'S METHOD.[3]—In this method a combination of a torsion balance and an electromagnet is used. The two coils are never excited simultaneously but only one after the other. A study of the magnetic field enables the action at each point in the field to be determined and obviates turning the torsion head to bring the specimen back to its original position.

§ (76) TABLES OF PERMEABILITY AND HYSTERESIS OF SELECTED MATERIALS.—The three tables give typical values of **B** for a number of different materials; they include values of coercivity and remanence. A large number of the columns are from E. Gumlich, *Magnetische Messungen*. Others are representative of measurements made at the N.P.L.

TABLE I

VARIOUS MATERIALS, INCLUDING MAGNET STEEL

H.	B. Nickel.	B. Cobalt.	B. Cobalt-iron.	B. Hensler Alloy.	B. English Magnet Steel. Average.	B. K.S. Magnet Steel. 35 per cent Co.	B. Also K.S. Magnet Steel.*
						Bar, 3 cm.×1 cm.	Rod, 5 mm. diam.
5	720	570	7,650	800	..	..	..
10	3,000	1,700	11,470	2320	400	..	..
20	3,970	3,400	14,480	3250	900	..	..
50	4,660	5,960	18,300	3800	3,800	..	720
100	5,120	7,840	21,100	4110	11,700	1,000	1,600
150	..	9,000	22,450	4250	13,500	..	3,000
300	6,230	..	23,700	..	15,300	8,000	11,800
500	6,550	11,500†	24,100	..	16,550	12,000	14,550
1000	7,110	13,500	24,600	..	..	15,100	17,250 ‡
2000	8,140	17,000	25,600	..	..	18,000	..
3000	9,140	18,500	26,630	..	..	19,500	..
4000	10,150	19,700	27,660	..	..	..	..
Coercivity .	7·5	..	2·72	7·3	63	160	238
Remanence	3,470	..	8,230	2550	10,400	9,200	10,600

* *Electrician*, 1920, lxxxv. 706. † Another sample. ‡ H = 1000.

[1] T. T. Smith, *Phys. Rev.*, Dec. 1916, viii. 721-737.

[2] "New Method of Measurement of Paramagnetic Substances in very Dilute Solution," A. Piccard and E. Cherbuliez, *Arc. des Sci.*, 1915, xl. 342.

[3] G. Meslin, *Comptes Rendus*, June 1905, cxl. 1683-1685, and July 1905, cxli. 102-106.

TABLE II

IRON AND IRON-CARBON MATERIALS

Permeability Curves.	Electrolytic Iron-sheet.		Swedish Charcoal Iron.		Dynamo Steel 0·044 per cent C. Annealed.	Dynamo Steel 0·085 per cent C. Annealed.	Cast-iron Annealed.	Carbon-steels (slowly annealed from 930° C.).			
	Unannealed.	Annealed.	Unannealed.	Annealed.				0·23 per cent C.	0·69 per cent C.	0·99 per cent C.	
H.	B.	B.	B.	B.	B.	B.	B.	B.	B.	B.	H.
0·25	..	2,200	300	310	3,100	450	..	..	..	..	0·25
0·5	180	7,500	900	1,000	7,100	1,550	..	..	..	..	0·50
0·75	350	9,300	2,250	3,400	8,950	3,700	..	..	..	..	0·75
1·0	600	10,240	5,000	6,350	10,200	5,650	..	1,000	280	200	1·0
1·5	1,520	11,400	8,000	8,400	11,730	8,200	..	2,400	..	..	1·5
2·5	4,370	12,800	10,500	10,550	13,400	10,800	900	5,500	850	580	2·5
5·0	8,920	14,470	12,900	12,940	15,000	13,570	2,950	9,200	3,700	1,850	5
10·0	12,750	15,500	14,600	14,630	15,680	14,980	5,150	12,000	8,500	6,550	10
20	15,300	16,200	15,700	16,100	16,130	15,680	6,820	14,120	11,950	10,600	20
50	17,150	17,100	16,900	17,120	17,100	16,700	8,620	15,970	14,600	13,850	50
100	18,380	18,050	17,930	18,130	18,280	17,770	9,950	17,200	16,080	15,500	100
150	19,160	18,870	18,700	18,850	19,100	18,620	11,020	18,050	17,000	16,440	150
200	19,710	19,450	19,400	19,400	19,550	19,380	11,920	18,600	17,460	17,000	200
300	20,650	20,700	20,200	20,180	20,420	20,200	12,800	19,560	18,480	17,900	300
500	21,630	21,670	21,200	21,150	21,460	21,410	14,130	20,700	19,600	19,000	500
1000	22,520	22,570	22,120	22,040	22,320	22,340	16,200	21,750	20,750	20,230	1000
2000	23,620	23,620	23,200	23,140	23,380	23,380	18,120	22,910	22,060	21,650	2000
3000	24,620	24,620	24,210	24,180	24,420	24,420	19,490	23,920	23,200	22,830	3000
4000	25,620	25,620	25,190	25,170	25,420	25,380	20,670	24,910	24,250	23,900	4000
Coercivity .	2·8	0·37	1·1	0·76	0·37	0·88	4·6	2·35	6·3	7·5	
Remanence	11,440	10,850	11,400	9,850	11,050	10,250	5,300	10,600	11,100	9,950	

TABLE III

IRON-SILICON, IRON-ALUMINIUM, AND OTHER IRON ALLOYS

	Iron-silicon.				Iron-aluminium.			Nickel Steel Bars.		
	Rod. 1·03 per cent Si, 0·25 per cent C. Slowly annealed from 800° C.	Sheet, 0·5 mm. thick. 1·03 per cent Si, 0·25 per cent C.	Sheet, 0·5 mm. 2·4 per cent Si, 0·2 per cent C. Slowly annealed.	Sheet, 0·5 mm. 4·45 per cent Si, 0·29 per cent C. Slowly annealed.	Rod. 2·17 per cent Al, 0·13 per cent C. Slowly annealed.	Sheet, 0·5 mm. thick. 2·17 per cent Al, 0·13 per cent C.	Sheet, 0·5 mm. 5·66 per cent Al, 0·15 per cent C.	12·22 per cent Ni, 0·41 per cent C.	Special. 26 per cent Ni, 0·14 per cent C.	Manganese Steel (13 per cent Mn).
H.	B.	B.	B.	B.	B.	B.	B.	μ.	μ.	μ.
0·25	200	420	500	200	400	400	320	(2·0)	..	..
0·5	650	1,500	1,530	620	900	1,250	900	..	..	..
0·75	1,500	5,200	4,500	1,480	1,700	2,700	1,700	..	..	..
1·0	2,700	8,450	7,650	3,000	2,450	5,000	2,860	2·0	1·18	..
1·5	5,100	10,380	10,680	8,500	3,900	8,400	5,370	2·0	..	..
2·5	8,260	13,230	12,700	11,420	6,200	11,040	7,870	2·02	..	..
5	11,500	14,620	14,120	12,840	9,340	13,000	10,470	2·05	1·20	$1{\cdot}02_5$
10	13,880	15,300	14,830	13,520	12,370	13,780	12,700	2·10	1·22	1·04
20	15,200	15,870	15,420	14,200	14,250	14,550	13,830	2·45	1·19	1·07
50	16,520	16,820	16,420	15,220	15,680	15,500	14,980	3·04	1·158	$1{\cdot}13_5$
100	17,550	17,880	17,420	16,300	16,970	16,420	16,000	2·71	1·153	..
150	18,400	18,600	18,200	17,070	17,770	17,220	16,670	2·40	..	..
300	19,850	..	..	..	19,000	..	..	1·97	..	..
500	20,850	..	..	..	19,850	..	..	1·67	..	..
1000	21,700	..	..	..	20,840	..	..	1·41	..	..
2000	22,850	..	..	..	22,000	..	..	1·25	..	..
3000	23,870	..	..	..	23,050	..	..	..	..	..
4000	24,850	..	..	..	24,040	..	..	..	..	..
Coercivity	1·45	0·77	0·83	1·06	0·91	0·96	1·08	..	..	..
Remanence	9,850	14,800	14,080	13,000	4,900	11,700	6,900	..	..	..

D. W. D.

MAGNETISM, MODERN THEORIES OF

AMPÈRE'S theory of molecular currents, formulated on Oersted's discovery of the magnetic effects of an electric current, may be regarded as the first definite step towards an electronic theory of magnetism. The older theories of Poisson and Weber, which aimed at an explanation of the relatively large magnetic effects common to bar magnets, were based on the assumption that the molecules of the material were miniature magnets, but no attempt was made to explain more fundamentally the origin of magnetism possessed by each molecule, and Maxwell's improvement of Weber's theory was directed towards the explanation of hysteresis effects in terms of certain quasi-elastic forces.

The discovery of the electron, followed by the identification of a moving electron with an electric current, and its inseparable magnetic field, have led to the electron theory of matter. On this theory, the magnetic effects of different kinds of matter are referred to the magnet-fields of electrons in orbital motion within atoms or molecules.

§ (1) THE THREE CLASSES OF SUBSTANCES.—As long ago as 1895, P. Curie [1] had carried out an exhaustive examination of the variation of magnetic susceptibility with temperature for a large number of substances. His results led him to divide substances into three classes: (i.) diamagnetic substances, whose susceptibility per unit mass (sometimes called the *specific* susceptibility) was independent of the temperature; (ii.) paramagnetic substances, whose specific susceptibility was approximately inversely proportional to the absolute temperature; (iii.) ferromagnetic substances, whose specific susceptibility varied with the absolute temperature in an irregular and complicated manner. These substances could be permanently magnetised, while paramagnetic and diamagnetic substances lost their magnetic properties as soon as the external field was withdrawn. Du Bois and Honda [2] in 1910 measured the specific susceptibilities of most of the elements at different temperatures and, except in the cases of a few elements, concluded that the rules of Curie given above were invalid. At room temperature, their results indicated a periodic variation of the specific susceptibility with atomic weight. Although, as will be seen later, there are a good number of exceptions to the rules (i.) and (ii.), yet these latter hold to a first approximation for many substances, and Curie's suggestion that the paramagnetic and diamagnetic effects were in a way distinct seemed plausible.

[1] P. Curie, *Annales de Chem. et de Phys.*, 1895, [vii.] v. 298.

[2] Du Bois and Honda, *Ann. der Phys.*, 1910, [iv.] xxxii. 1027; *Versl. Kon. Ak. v. Wetensch.*, Amsterdam, 1910, xii. 596.

(i.) *Diamagnetic Substances.*—The data thus furnished by Curie were utilised by P. Langevin,[1] who in 1905 developed a theory of diamagnetism and paramagnetism in terms of electronic or Ampèrean currents. It is assumed that an electron revolving in an orbit is equivalent to a current flowing round a minute circuit having no resistance. The magnetic moment of such an orbit is $M=\epsilon S/\tau$, where ϵ is the electronic charge, S the area of the orbit described, and τ the period of revolution. Langevin treats primarily of cases where the mutual influences of the molecules are inappreciable. He does not consider ferromagnetism—this extension was made by Weiss and others later—and he confines his study of paramagnetism to gases, particularly oxygen.

On this theory, a diamagnetic molecule is one containing a congeries of electrons in some form of orbital motion such that their aggregate magnetic moment is zero initially. Thus the molecule may contain a system of electrons, some of which are spinning right-handedly and others left-handedly about certain relatively fixed positive charges (the atomic nuclei). When the magnetic field is applied, the balance of the systems is destroyed, and by the simple laws of induction it can be shown that the molecule as a whole acquires a negative magnetic moment[2] given by

$$\Delta M=-\frac{\epsilon^2 H\Sigma r^2}{4m},$$

while the specific susceptibility is

$$\chi=-\frac{N\epsilon^2\Sigma r^2}{4m},$$

where H is the applied field, ϵ the charge on each electron, m the mass of the electron, r the radius of the electron orbit, Σ denotes a summation extending over all the electrons in the molecule, and N is the number of molecules per gram. The above equations will hold for all molecules whether the aggregate moment of the molecule is zero or not, and therefore every substance must possess a diamagnetic moment, though, as we shall see later, it is completely masked in paramagnetic and ferromagnetic media.

It should be mentioned here that the expression ΔM given above for the induced moment is compatible with the magnitude of the Zeeman effect shown by spectral lines quite apart from the magnetic nature of the matter under investigation. Moreover, as long as the molecules exert no mutual influences on one another, the spectral frequencies are independent of temperature in agreement with the Curie rule (i.) for diamagnetic substances. When crystallisation sets in, the mutual forces between the molecules become very great, and the specific susceptibility is in general slightly affected at the fusion point.

[1] Langevin, *Annales de Chem. et de Phys.*, 1905, [8] v. 70.
[2] "Magnetism, Molecular Theories of," § (3), equation (7).

(ii.) *Paramagnetic Substances.*—A paramagnetic molecule on Langevin's theory is one wherein the aggregate magnetic moment of the electronic orbits is not zero. Such a molecule is therefore equivalent to a minute magnet, and, under the influence of an external field, it would tend, like an ordinary magnet, to set its axis parallel to the direction of the field. Such an orientation will obviously depend on the temperature, since molecular collisions will tend to re-distribute the axes of the orientated molecules.

The second rule of Curie (ii.) states that the specific paramagnetic susceptibility varies inversely as the absolute temperature. This may be established thermodynamically if we assume that the induced magnetic moment in the substance is directly proportional to the applied field. Langevin also gives an alternative treatment, based on Boltzmann's theory of distribution of the direction of molecular axes, and shows that the specific susceptibility is given by

$$\chi=\frac{M^2N}{3RT}=\frac{C}{T},$$

where M is the magnetic moment of a molecule, N the number of molecules per gram, R is the gas constant, T the absolute temperature, and C is Curie's constant per gramme.

(iii.) *Ferromagnetic Substances.*—The extension of Langevin's theory of paramagnetism to ferromagnetism is due to Weiss.[3] In ferromagnetic substances complications are introduced by the mutual effects of the molecules on one another, and although such interference really occurs among the molecules of paramagnetic and diamagnetic substances also, there is some peculiarity about the molecules of the ferromagnetic elements which renders this molecular co-operation very pronounced. When a magnetic field is applied to a ferromagnetic substance a relatively large magnetic moment is, in general, induced, and, if the field be reduced to zero, a certain amount of this induced moment remains, constituting the permanent magnetism. This is the residual magnetism, and a reverse magnetic field must be applied in order that this induced moment may be destroyed, this reverse field being a measure of the coercive force of the specimen. All these effects point to a sluggish action on the part of the molecules; to a mutual interference of a very pronounced character. The phenomena involved are admirably illustrated by Ewing's model,[4] in which numbers of small magnets are pivoted so as to be free to move under their mutual fields and also controlled by an external field. Such a model shows the phenomena of hysteresis in a very realistic way. For each ferromagnetic substance there

[3] Weiss, *Journal de Phys.*, 1907, vi. 661.
[4] Ewing, *Magnetic Induction in Iron and other Metals*, 3rd ed., chap. xi.

is a certain temperature above which this mutual co-operation ceases to have these remarkable magnetic effects, the ferromagnetic substance having changed into a paramagnetic substance and following more or less closely the Curie rule for paramagnetic substances. This temperature is known as the critical temperature. The susceptibility-temperature curve for iron is shown in *Fig.* 1. In the γ-range the specific susceptibility is inversely proportional to the absolute temperature, but as the iron is cooled through the critical temperature, T_0, the susceptibility rapidly increases and continues to do so throughout the β-range. Weiss attributed this rapid increase to the operation of a molecular field *below* the critical temperature.

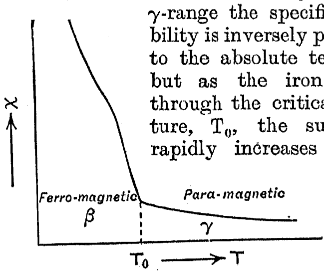


FIG. 1.

According to him [1] if χ_γ is the specific susceptibility of γ iron, χ_β that of β iron, C the Curie constant per gramme, T the absolute temperature, H the external applied magnetic field, I the intensity of magnetisation, A the constant of the molecular field, so that the strength of that field is AI, and ρ the density, we have

$$\chi_\gamma=\frac{C}{T},$$

and producing the γ-curve to just below T_0,

$$\chi_\gamma=\frac{I}{\rho(H+AI)},\quad \chi_\beta=\frac{I}{\rho H}.$$

Thus $$\frac{T}{C}=\frac{1}{\chi_\gamma}=\frac{\rho(H+AI)}{I}=\frac{1}{\chi_\beta}+\rho A.$$

Hence $$\chi_\beta=\frac{C}{T-CA\rho}.$$

The large value of the ratio χ_β/χ_γ depends upon the recognition of the constant A of the molecular field (which is equal to AI). This molecular field is representative of the mutual co-operation of the molecular magnets and has a value much larger than any field which we can produce artificially. The following table gives the values of the molecular field H_c, A, and I, for some ferromagnetic substances :

Substance.	H_c.	A.	I.
Iron	$6{\cdot}53\times10^6$	$0{\cdot}38\times10^4$	1720
Nickel . . .	$6{\cdot}35\times10^6$	$1{\cdot}27\times10^4$	500
Magnetite . .	$14{\cdot}3\times10^6$	$3{\cdot}31\times10^4$	430

Weiss [2] showed further that the energy of the molecular field, viz. $\frac{1}{2}AI^2$, is a measure of the thermal absorption or evolution accompanying the transition at the critical temperature. While, however, postulating the existence of a molecular field, Weiss did not suggest any theory of its origin. He was satisfied to regard it merely as a directive force of the same nature as that which determines the crystalline lattice. His experiments on the molecular susceptibility of magnetite above the Curie point showed that this quantity, when plotted against the reciprocal of the absolute temperature, did not give, as the second Curie rule implies, a single straight line passing through the origin, but a series of straight lines separated by short discontinuous parts, the slopes of the lines being in the ratios 4 : 5 : 6 : 8 : 10.[3] This could imply an increase of molecular complexity with rise of temperature or a change of the magnetic moment of the molecule. Weiss considered the latter as more probable and concluded that the moment of the molecule of magnetite varied, but was always a multiple of a unit, $16{\cdot}4\times10^{-22}$ C.G.S. e.m. units, which he called the magneton. Later, Weiss, in collaboration with K. Onnes,[4] determined the saturation molecular susceptibilities of iron, nickel, and other substances at low temperatures, and showed that these, too, were definite multiples of the same unit. His later results derived from various paramagnetic salts, both crystalline and in solution, do not, however, appear to be quite satisfactory in confirming the reality of this empirical unit. More accurate measurements of susceptibility and a redetermination of Avogadro's number give the value of the magneton as $18{\cdot}54\times10^{-22}$ C.G.S. e.m. units. It is curious that the magnetic moment of the electron orbit on Bohr's theory is an integral multiple of $92{\cdot}7\times10^{-22}$, which is exactly five times the above unit.

The dependence of diamagnetic susceptibility on temperature, the departures from the hyperbolic law of paramagnetism, and the fact that the specific susceptibility is a periodic function of the atomic weight, have led Honda and Okubo [5] to develop a new theory of magnetism based on molecular rotations. There appears to be no doubt that a gyroscopic effect of the type involved in this theory exists, but the evidence available seems to point to the fact that magnetism does not originate from a molecular rotation. It may possibly arise from a gyroscopic property of the electron itself (see below, § (3) (ii.)).

§ (2) EFFECT OF CRYSTALLISATION.—It has been stated above that there are numerous departures from the simple Curie laws regarding

[1] Weiss, *Journal de Phys.*, 1907, vi. 686.
[2] Weiss and Beck, *Journal de Phys.*, 1908, vii. 249.
[3] Weiss, *Comptes Rendus*, 1911, clii. 79 (cf., however, Takagi, *Science Reports*, *Tohoku*, 1913, ii. 117; Honda and Ishiwara, *ibid.*, 1915, iv. 250).
[4] Weiss and Onnes, *Journ. de Phys.*, 1910, ser. 4, ix. 555.
[5] Honda and Okubo, *Science Reports*, *Tohoku*, 1918, vii. 141.

the dependence of paramagnetism and diamagnetism on temperature. Thus most substances show a change of susceptibility on crystallisation, and, in a number of experiments carried out by the writer,[1] it was found that the Curie rule of constancy of diamagnetism held very approximately for many organic substances as long as there was no appreciable change of molecular aggregation, *i.e.* as long as there is no change in the mutual forces between the molecules. When crystallisation takes place, the specific susceptibility changes by a few per cent in many cases. This is typically so with organic aromatic compounds and a number of elements (Honda, Owen, and Ishiwara).[2] This implies a molecular distortion, and, since a magnetic field distorts the electronic orbits, we may provisionally interpret the mutual influences of the molecules during crystallisation in terms of a magnetic molecular forcive. If ΔM is the change of magnetic moment produced in an electron orbit of moment M by applying a magnetic field H, then it can be shown[3] that

$$\frac{\Delta M}{M} = -\frac{H\tau\epsilon}{4\pi m},$$

where τ is the period of rotation of the electron (of the order 10^{-15} sec.), and ϵ/m the ratio of charge to mass ($1{\cdot}77 \times 10^7$ e.m.u.). The largest field which we can produce in the laboratory is of the order 10^5 gauss, and, therefore,

$$\frac{\Delta M}{M} \doteqdot -10^{-9}H = -10^{-4}.$$

Suppose that on crystallisation the intermolecular forces were equivalent to a molecular field of 10^7 gauss, then

$$\frac{\Delta M}{M} \doteqdot -10^{-2},$$

which would correspond to a change of 1 per cent in the specific susceptibility, a change which is of the order actually shown by many substances.

Thus, just as Weiss introduces a magnetic field in ferromagnetics below the critical temperature, and neglects the mutual influences of the molecules, so we may introduce a molecular field of the same order in crystalline diamagnetics and neglect the complications introduced by the forces of crystallisation. But on account of the compensated nature of a diamagnetic molecule (§ (1) (i.)), the molecular field in the latter case is of an alternating character, the distance over which it is unidirectional being comparable with atomic dimensions. All kinds of atoms, whatever their nature, contain electrons revolving in orbits with comparable frequencies, and, as the number of such orbits is of the same order in different types of atoms, it follows that the magnetic moment of each orbit is of the same order, and further, that, in spite of the compensated character of the orbits constituting a diamagnetic system, the local force in between a pair of molecules of the crystal structure will be comparable with the force between a pair of molecules in a ferromagnetic. The aggregate of the local intensity of magnetisation of a diamagnetic crystal per unit volume will be comparable with that of a ferromagnetic. It can further be shown that the energy associated with the crystalline formation, *i.e.* the energy of the local molecular field, will be of the order 25 calories per gram.[4] This is of the order of the latent heat of fusion of many organic substances and elements.

Other evidence confirming a molecular field of this order of magnitude in diamagnetics and paramagnetics is furnished by double refraction, magnetostriction, and magneto-rotation data, and this suggests the idea of an intense molecular field in substances which show an inappreciable change of specific susceptibility on crystallisation. A liquid submitted to a magnetic field becomes slightly double refracting and shows a minute change of volume. If we suppose that on crystallisation the mutual action between the molecules is determined by a molecular field of 10^7 gauss, the double refraction will be of the same order as the natural double refraction of quartz, and the change of volume becomes of the order 0·1 c.c. per c.c., which is comparable with the change of volume of a large number of organic liquids and elements on crystallisation. Measurements of the susceptibilities of diamagnetic liquids and crystals

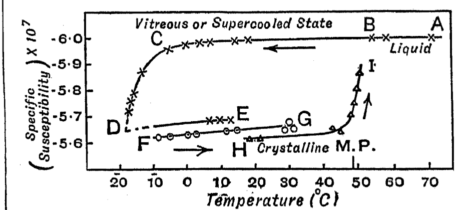


FIG. 2.

have further shown that hysteresis loops with respect to temperature exist, which are very similar to those shown by nickel steels (*Fig.* 2).

Departures from the Curie rule for paramagnetism, particularly at low temperatures, have been observed by K. Onnes[5] and other Dutch physicists, and corrected formulae somewhat of the type used by Weiss have been applied to account for the discrepancies.

In all these cases evidence is available showing that in the crystalline state the mutual interaction of the molecules is very great, in fact the intermolecular forces determine the crystalline symmetry and also the rigidity of the media. So far no evidence has been brought forward which assigns a particular nature to the molecular field. All that has been shown is that a magnetic molecular field of the order 10^7 gauss is capable of accounting for many of the properties of crystalline media. It is, however, at least plausible that the molecular

[1] Oxley, *Phil. Trans. Royal Soc.*, 1914, ccxiv. A, 109; 1915, ccxv. A, 79.

[2] Ishiwara, *Science Reports, Tohoku*, 1914, viii. 303; Owen, *Ann. der Phys.*, 1912, xxxvii. 657.

[3] See "Magnetism, Molecular Theories of," § (3).

[4] Oxley, *Phil. Trans. Royal Soc.*, 1920, ccxx. A, 253.

[5] Onnes and Oosterhuis, *Kon. Ak. v. Wetensch.* Amsterdam, 1912, xv. 322.

field is in part of a true magnetic nature. Further evidence supporting this view is given by a series of classical researches conducted by Tyndall,[1] who examined the deportment of over 100 paramagnetic and diamagnetic crystals when freely suspended in a uniform magnetic field. Whenever the crystal possesses a predominant cleavage, this cleavage invariably sets axially (*i.e.* parallel to the line joining the pole pieces) if the crystal is paramagnetic, and equatorially if the crystal is diamagnetic. This implies that the closer packing of the molecules in a direction parallel to the principal cleavage involves stronger coupling forces in this direction, *i.e.* the mutual interaction of the molecules is a maximum in this direction as we should expect. The fact that a magnetic field is so decisive in isolating the closeness or openness of the packing of the molecules suggests that magnetic forces play a predominant part in determining crystalline symmetry, in other words the cohesive forces are in part at least of a magnetic nature. Further, the existence of minor cleavages suggests that the electrons are distributed round the atomic nuclei in three dimensions and that the symmetry of the electron pattern thus formed in atoms and molecules is the deciding factor determining crystalline symmetry. It is interesting to note that crystals of the simple cubic form (*e.g.* rock salt and sylvine), which possess three equal and mutually perpendicular cleavages, show no appreciable tendency to set in any particular way, except as determined by their outward form, when suspended in a magnetic field. In these latter cases the crystalline lattice, as X-ray analysis shows, has an ionised atomic structure. Each ion is symmetrically surrounded by six ions of opposite sign and the cohesive forces are primarily of an electrostatic nature. But in the case of un-ionised crystals, viz. those of non-conducting compounds and elements (the latter consisting of identical atoms), the cohesive forces can hardly be of this simple electrostatic type, since in either case there is no reason why an electron transfer, such as is involved in ionisation, should take place at all. The type of coupling is of a new kind.

§ (3) NATURE OF ATOMIC STRUCTURE. (i.) *The Cubical Atom.*—On the theory of atomic structure advanced first by G. N. Lewis[2] and afterwards extended by Irving Langmuir,[3] viz. the cubical atom theory, two kinds of chemical combination are recognised, (*a*) the ionised atomic type such as we get in rock salt and sylvine, (*b*) a different type in which no ionisation is involved and in which the valencies, *i.e.* the couplings between the atoms, are determined by pairs of electrons. The atoms held together by a single valency bond are said to hold electrons in common, and the chemical evidence, as pointed out by Lewis, indicates that the electrons of each pair are specially closely related, the pair acting as a single unit. This is precisely what the magnetic evidence discloses.

[1] Tyndall, *On Diamagnetism and Magnecrystallic Action*, 1870, p. 23.
[2] G. N. Lewis, *J. Amer. Chem. Soc.*, 1916, xxxviii. 72.
[3] Langmuir, *J. Amer. Chem. Soc.*, 1919, xli. 868.

The latest available evidence appears to show that the electron is a more complex unit than was hitherto thought, and that it is probably endowed with specific magnetic as well as electrostatic properties. The electron is therefore also a magneton. On the cubical atom theory the most stable grouping of electrons is that of the pair referred to above; the next is that of the octet, or eight electrons arranged at the corners of an imaginary cube. This cube may be very much distorted when atoms are in combination. A good instance of this is the carbon atom which contains six electrons. Two of these form a stable pair close to the nucleus, the other four are arranged at the corners of a tetrahedron. When the four valencies of the atom are satisfied each of these four electrons is closely related to another similar electron, provided by the new atoms, so that a stable but very much distorted octet is formed. Such a structure appears to fit in very well with the requirements of stereo-chemistry, the Baeyer strain theory, and with the existence of the triple bond.

(ii.) *The Magneton Theory, the Anchor Ring Electron.*—Assuming that the electron has the shape of an anchor ring of negative electricity, revolving in its plane about its centre with a high velocity, Parson[4] has suggested a magneton theory of matter which is in some respects closely allied to the cubic atom theory. Parson showed that eight such electrons arranged in cubical formation would set themselves so that the system is internally self-compensated, having practically no external field. Such compensation is required on Langevin's theory to account for diamagnetism. The unique distribution of positive charges which Parson considered necessary for their equilibrium seems to have been the main ground for the non-acceptance of this idea. It appears, however, that the theory contains some fundamental truth, and quite recent work, which is referred to below, has tended to confirm Parson's conception of a combined electron-magneton unit, though much more must be done before this question can be satisfactorily settled. The properties of such magneton groups do, however, provide a means of circumventing the difficulty regarding the periodic variation of susceptibility with atomic weight (Honda and Okubo, end of § (1)).

Compton and Trousdale[5] have made X-ray examinations of magnetite, haematite, and pyrrhotite, and they conclude that the elementary magnet must be the electron or the nucleus. If it were the atom or molecule, or even a group of molecules, we should expect

[4] Parson, *Smithsonian Misc. Collections*, 1915, lxv. 1.
[5] Compton and Trousdale, *Phys. Rev.*, 1915, v. 315.

that the X-ray diffraction pattern would be different when the substance was magnetised from the pattern produced by the unmagnetised sample, which is not the case. Further, Forman[1] has shown that when a beam of X-rays traverses iron in a direction parallel to the magnetising field, there is a definite increase in the absorption of the transmitted beam. These results relegate the elementary magnet to the electron or nucleus. The extraordinary variation of magnetic property with valency finally suggests that the electrons, rather than the nuclei, locate the magnetic elements. Thus it appears that magnetisation is not accompanied by rotations of molecular or atomic systems, but by rotations of minute electron orbits or vortices about their own centres of gravity, each electron, however, not being displaced from its position relative to the nucleus. Such a view may conceivably explain why many *crystalline* paramagnetic salts obey the Curie rule over considerable temperature intervals, their susceptibilities being inversely proportional to the absolute temperature.

One or two additional extensions of the ring electron idea have been made by Allen,[2] who has pointed out its promising capacity to explain optical activity and optical isomerism. He has also made estimates of the ring electron constants. His calculations show that quite near to the ring electron the local magnetic field is of the order 10^8 gauss, a value consistent with that deduced from the experimental determination of the variation of specific susceptibility on crystallisation.

(iii.) *Bohr's Theory.*—In connection with all these phenomena the theory of atomic structure developed by Bohr and Sommerfeld,[3] which is particularly successful in interpreting spectral phenomena, must be considered. The electrons on this theory describe orbits whose radii are comparable with atomic dimensions. These orbits are sometimes very elongated ellipses whose major axes are very large compared with the conventional atomic diameter. With orbits of this character it appears impossible to account for diamagnetic phenomena. It should be pointed out that by far the larger proportion of substances are diamagnetic, and the atoms of each of these would seem to require a uniquely balanced system of electron orbits which the Bohr theory, as developed at present, seems hardly capable of supplying. It should be noted that Bohr's theory is successful mainly in its applications to the neutral hydrogen atom and to the positively charged helium atom, and for these only when the respective atoms are widely separated from each other in the discharge tube. In such cases the mutual magnetic effects of pairs of electrons which inevitably play their part in ordinary matter, particularly in the crystalline state, may undoubtedly be neglected. The Lewis-Langmuir theory has, however, some elements suggestive of the ideas at the base of Bohr's theory, and it may eventually be found that the mathematical interpretation of the phenomena of radiation, as given by Bohr's equations, applies equally well to the three-dimensional distribution of electron orbits required to explain phenomena not connected with radiation. The electrons may be complex units which form a sort of space pattern on a series of spherical or ellipsoidal surfaces surrounding each atomic nucleus. These would correspond to the stationary states of Bohr's theory, and radiation would be caused when an electron passed from one equilibrium surface to another. In some such way as this it may be possible to reconcile the apparently divergent views resulting from studies of matter in a radiating and non-radiating condition.[4]

A. E. O.

1 Forman, *Phys. Rev.*, 1916, vii. 119.
2 H. S. Allen, *Phil. Mag.* xl. 426; xli. 113.
3 Sommerfeld, *Atombau und Spectrallinien*, 1919.

MAGNETISM, MOLECULAR THEORIES OF

§ (1) Magnetic Classification.—If we classify all the substances according to their magnetic properties they are divided into three classes,[5] that is,

(i.) Diamagnetic substances,
(ii.) Paramagnetic substances,
(iii.) Ferromagnetic substances.

The first two classes of substances are very weakly magnetisable, and their intensity of magnetisation is proportional to the magnetising field. The diamagnetic substances polarise in the opposite direction to the magnetising field, but the paramagnetic substances in the same direction as the field. The ferromagnetic substances are easily magnetisable in the direction of the field, that is, polarise strongly by a comparatively small field.

The modern theory of magnetism, to explain these properties of the substances, is based on the electron theory of matter. According to the theory, the atoms constituting matter consist of positive nuclei and electrons revolving round them; these revolving electrons, being equivalent to a circuit carrying an electric current, exert magnetic force in the neighbourhood. Thus the atomic magnetism is explained by the revolving electrons. If a molecule consists of atoms, its molecular magnetism depends on the velocity of the

4 Oxley, *Proc. Royal Soc.*, 1920, xcviii. A, 264.
5 See also "Magnetism, Modern Theories of."

revolving electrons, the radius of the orbits, as well as the plane of the orbits. Since these quantities are different in different substances, the magnetic properties will differ from substance to substance.

According to the electron theory, each element possesses a definite number of revolving electrons equal to its atomic number, this number increasing with the atomic weight. But the magnetic property of the elements does not increase with the atomic weight, but varies periodically with it.[1] This indicates that within the atoms there must be electrons revolving in opposite sense, or revolving in orbits in different planes, and thus partially counteracting the magnetic effects of each other.

In the above theory of magnetism it is necessary to assume that the electronic orbits in an atom have definite orientations relative to the structure of the positive nucleus, that is, the change of the orbital planes is always accompanied by a corresponding change of orientation of the atom as a whole; otherwise, as shown by J. J. Thomson [2] and W. Voigt,[3] the magnetic property of a substance consisting of such atoms cannot be explained.

§ (2) Paramagnetic Substances. (i.) *Gases.*—After Langevin,[4] we shall consider the case of a paramagnetic gas, whose molecules are all small magnets of equal strength. Before the magnetic field is applied, the axes of the molecular magnets are supposed to be uniformly distributed in all directions; but when the field acts on them this uniform distribution is slightly disturbed, becoming a little denser towards the direction of the field. Langevin assumes the law of distribution to be the same as the density of a gas acted on by the force of gravity; that is, if n be the whole number of molecules in one gram atom of the gas, the number of molecules, whose direction of the magnetic axes makes an angle lying between α and $\alpha + d\alpha$ with the field H (*Fig.* 1), is given by

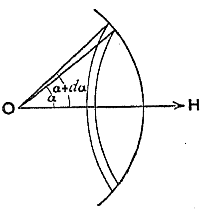


Fig. 1.

$$dn = \mathrm{C}e^{\frac{\mathrm{MH}\cos\alpha}{r\mathrm{T}}}d\omega, \quad d\omega = 2\pi\sin\alpha d\alpha,$$

where M is the magnetic moment of the molecules, r the gas constant referred to one molecule, T the absolute temperature, and C a constant.

If we put $\mathrm{MH}/r\mathrm{T} = a$, and integrate dn with respect to α, we have

$$n = \int dn = 2\pi\mathrm{C}\int_0^\pi e^{a\cos\alpha}\sin\alpha\,d\alpha = \frac{4\pi\mathrm{C}}{a}\sinh a,$$

hence
$$\mathrm{C} = \frac{an}{4\pi\sinh a}$$

$$\therefore\ dn = \frac{an\sin\alpha}{2\sinh a}e^{a\cos\alpha}d\alpha. \quad . \quad . \quad (1)$$

Then the magnetic moment per gram atom is given by

$$\sigma = \int_0^\pi \mathrm{M}\cos\alpha dn = n\mathrm{M}\left(\coth a - \frac{1}{a}\right).$$

Since $n\mathrm{M}$ is the saturation value of the magnetic moment, we denote it by σ_0; then we have

$$\sigma = \sigma_0\left(\coth a - \frac{1}{a}\right). \quad . \quad . \quad . \quad (2)$$

If a be very small and its third power be negligibly small compared with a, we have

$$\frac{\sigma}{\sigma_0} = \frac{a}{3} = \frac{\mathrm{MH}}{3r\mathrm{T}} = \frac{\sigma_0\mathrm{H}}{3\mathrm{RT}}, \quad . \quad . \quad . \quad (3)$$

where R is the gas constant. Thus, for a small value of a, σ is proportional to a. As a becomes very large, σ approaches asymptotically to σ_0. Hence the σa curve has a form as shown in *Fig.* 2.

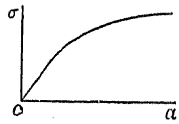


Fig. 2.

Since the susceptibility χ is given by $\chi = \sigma/\mathrm{H}$, we have

$$\chi\mathrm{T} = \frac{\sigma_0^2}{3\mathrm{R}} = \text{const.} \quad (4)$$

This is the law experimentally obtained by Curie in the case of oxygen and some other substances.

In the case of the gas, besides the translational motion, the molecules are continuously rotating, and therefore the actual conditions are not so simple as Langevin considers. For, when a magnetic field is applied to the gyromolecules, by virtue of their magnetic moments, the axes of rotation make a small gyrostatic motion, that is, they undergo precession and nutation round the direction of the field. The amplitude of the nutational motion measured from the direction of the field extends from the initial angle to an angle which is slightly less than the initial. This motion will therefore make a certain contribution to the increase of the magnetic moment of the gas in the direction of the field, that is, the origin of the paramagnetic properties of the gas, while the precessional motion itself does not cause any change in the magnetic moment. Taking the time-mean of the magnetic moment of each molecule in the direction of the field during a complete period of the nutational motion, and integrating it over all molecules, Dr. J. Okubo and the present writer [5] obtained another but a similar expression to that of Langevin.

(ii.) *Solids.*—In the case of a solid, in which

[1] K. Honda, *Ann. der Phys.*, 1910, xxxii. 1027, *Sci. Rep.*, 1911, i. 1; M. Owen, *Ann. der Phys.*, 1912, xxxvii. 657.
[2] J. J. Thomson, *Phil. Mag.*, 1903, [6], vi. 673.
[3] W. Voigt, *Ann. der Phys.*, 1902, ix. 115.
[4] P. Langevin, *Ann. de chem. et phys.*, 1905, (8), v. 70.
[5] *Sci. Rep.*, 1914, vii. 141.

the molecules exert a large mutual action upon each other, Langevin's theory does not apply in its original form. RT in the expression for a is the kinetic energy of the molecules, whose magnitude may be regarded as a measure of the resistivity against the turning of the molecules in the direction of the magnetic field. The mutual action of the solid molecules affects the orientation of the molecules in the same way as their kinetic energy, and therefore the effect of taking this action into account is equivalent to an increase of the kinetic energy by a term Ω. Ω may be called *an equivalent kinetic energy*. Hence in the case of solid we may assume

$$dn = Ce^{\frac{H\sigma_0 \cos a}{RT+\Omega}} d\omega.$$

Putting
$$\frac{H\sigma_0}{RT+\Omega} = a',$$

we obtain as before

$$\frac{\sigma}{\sigma_0} = \frac{a'}{3} = \frac{H\sigma_0}{3(RT+\Omega)},$$

or
$$\chi = \frac{\sigma_0^2}{3(RT+\Omega)}. \quad . \quad . \quad . \quad (5)$$

If we write $\Omega = R\Delta$, where Δ is a positive constant, and consider it to be a constant independent of temperature, the above relation becomes

$$\chi(T+\Delta) = \frac{\sigma_0^2}{3R} = \text{const.} \quad . \quad . \quad (5')$$

This law is given by K. Onnes and A. Perrier, and found to be satisfied by a number of solid substances at low temperatures.

(iii.) *Ferromagnetic Substances above the Critical Point.*—Next, consider the case of ferromagnetic substances at temperatures higher than the critical point. It is well known that the transition from the ferromagnetic substance to the paramagnetic is continuous, there being no definite temperature which distinguishes one state from the other. But for a rough approximation we may conceive such a temperature to exist; above it the substance is paramagnetic, and during cooling it begins to be ferromagnetic at the point. Hence at a critical temperature T_1 the susceptibility becomes very large in comparison with its value at higher temperatures. Thus in this case we may write

$$dn = Ce^{\frac{H\sigma_0 \cos a}{R(T-T_1)}} d\omega,$$

whence we have as before

$$\chi = \frac{\sigma_0^2}{3R(T-T_1)},$$

or
$$\chi(T-T_1) = \frac{\sigma_0^2}{3R} = \text{const.} \quad . \quad . \quad (6)$$

This relation was first obtained by P. Weiss. Except at temperatures near T_1 and transforming ranges, the relation is fairly well satisfied in the case of ferromagnetic substances. The above relation holds good of course for the paramagnetic substances, which become ferromagnetic at a critical point lying below the room temperature.

§ (3) Diamagnetic Substances.—Consider (*Fig.* 3), in an atom, an electron of mass m and charge e moving with a velocity v in an orbit of radius r. The period τ of the revolution is given by

$$\tau = \frac{2\pi r}{v};$$

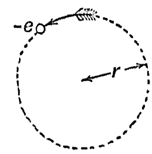


Fig. 3.

and the magnetic moment of the atom is obtained by multiplying the area of the orbit by e/τ the value of the current. Thus

$$M = \frac{\pi r^2 e}{\tau} = \frac{rev}{2};$$

its direction is perpendicular to the plane of orbit. If a magnetic force H acts normally on this plane, the electron undergoes an outward electromagnetic force. Besides, so long as the magnetic field varies, a force tangential to the electronic orbit acts on the electron. Hence on applying the magnetic field both v and r vary; but it can be shown that the variation of r is negligibly small when compared to that of v, we therefore neglect it.

Now $\mathbf{B}$, the flux of the magnetic force through the orbit, is

$$\mathbf{B} = \pi r^2 H;$$

hence the electromotive force E acting on the electronic orbit is

$$E = -\frac{d\mathbf{B}}{dt} = -\pi r^2 \frac{dH}{dt};$$

the work done on the electron during a single revolution is therefore

$$W = eE = -e\pi r^2 \frac{dH}{dt}.$$

On the other hand, if f be the force acting on the electron during a revolution, we may write

$$W = 2\pi r f;$$

hence
$$f = -\frac{er}{2}\frac{dH}{dt},$$

or
$$\frac{dv}{dt} = \frac{f}{m} = -\frac{er}{2m}\frac{dH}{dt}.$$

The change of the magnetic moment due to the magnetic field is then

$$\frac{dM}{dt} = \frac{re}{2}\frac{dv}{dt} = -\frac{r^2e^2}{4m}\frac{dH}{dt}.$$

If we integrate this equation between $t=0$

and t_1, t_1 being the time required for the magnetisation,

$$M' = M - \frac{r^2e^2}{4m}H, \quad . \quad . \quad . \quad (7)$$

if $\quad t=0, \quad H=0, \quad$ and $M=M$,

and if $\quad t=t_1, \quad H=H, \quad$ and $M=M'$.

If the field makes an angle θ with the normal to the plane of orbit, the magnetic moment in the direction of the field is given by

$$M' = M \cos\theta - \frac{r^2e^2}{4m}H\cos^2\theta;$$

hence the magnetic moment per one gram atom of the substance is given by

$$\chi H = \int_0^\pi M\cos\theta dn - \frac{r^2e^2H}{4m}\int_0^\pi \cos^2\theta dn. \quad (8)$$

If in a molecule several electrons are revolving, it may happen in consequence of the orientation of their orbital motions that the resultant magnetic moment of the molecule vanishes; in this case $M=0$, and the distribution of the magnetic axes of the molecules remains, after the magnetic field is applied, uniform in all directions. Hence we have

$$dn = \frac{n}{2}\sin\theta d\theta,$$

and from equation (8) we get

$$\chi = -\frac{nm}{12}\left(\frac{e}{m}\right)^2\Sigma r^2, \quad . \quad . \quad (9)$$

where Σ refers to the revolving electrons in an atom. Thus the magnetisation produces a diamagnetic effect. The above theory of diamagnetism is due to P. Langevin.[1]

If the magnetic moment of the molecules is not zero, the distribution of the magnetic axes of the molecules becomes denser in the direction of the magnetic field, and hence can be given by

$$dn = Ce^{a'\cos\theta}d\omega, \quad a' = \frac{\sigma_0 H}{RT+\Omega};$$

hence in equation (8) the first term is the paramagnetic contribution before referred to; denoting it by χ_p, we have, for a small value of a',

$$\chi_p = \frac{1}{H}\int_0^\pi M\cos\theta dn = \frac{\sigma_0^2}{3(RT+\Omega)}. \quad (10)$$

If we neglect the terms higher than the second power of a', the second term in the expression for χH becomes, as before,

$$\chi_d = -\frac{nm}{12}\left(\frac{e}{m}\right)^2\Sigma r^2;$$

hence the resultant susceptibility is

$$\chi = \chi_p + \chi_d$$
$$= \frac{\sigma_0^2}{3(RT+\Omega)} - \frac{nm}{12}\left(\frac{e}{m}\right)^2\Sigma r^2. \quad . \quad (11)$$

[1] *Ann. de chemie et phys.*, 1905 (8), v. 70.

Thus, if $\chi_p + \chi_d > 0$, the substance is paramagnetic; and if $\chi_p + \chi_d < 0$, it is diamagnetic.

As we see from the above theory, the diamagnetic term χ_d depends only on the orbital motion of the electrons, and this motion cannot change with thermal motion, the change of state, or an allotropic transformation. Hence χ_d must be unaffected by temperature, as well as these changes of states. But χ_p will depend on these factors. Since the observed diamagnetism is the resultant of χ_p and χ_d, provided $\chi_p + \chi_d < 0$, it is to be expected that the observed diamagnetism may change with temperature, melting or the allotropic transformations, etc. Such cases are actually observed.

The fact that the diamagnetic susceptibility of a compound has generally a value, which is peculiar to it and different from the arithmetical sum of the susceptibilities of its components, may be explained as the change in χ_p of the compound molecules. The addition law of diamagnetism, as found by Pascal, is only applicable to these compounds having similar constitution; it can also be understood from the above view. The remarkable change of susceptibility on change of temperature observed by the present writer in tin[2] indicates (*Fig.* 4) that by the allotropic change of grey tin into the ordinary at 32°, χ_p increases so much that the apparent susceptibility here changes sign from negative to positive, and that by melting, χ_p abruptly decreases and the resultant susceptibility becomes again negative. The fact that in some crystals, for example, graphite and antimony, the diamagnetic susceptibility depends considerably on the direction of their crystallographic axis, is also explained in the same way. In a crystal all the molecules are regularly arranged with respect to its crystallographic axes, and, therefore, the mutual action may differ in different directions. The value of χ_p depends therefore on the directions of these axes, and the observed diamagnetic susceptibility χ of the crystal may differ for the different crystallographic axes.

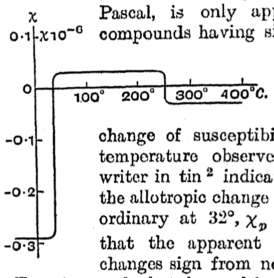


Fig. 4.

§ (4) Relation between Paramagnetic and Ferromagnetic Substances.—It is usual to distinguish the ferromagnetic and paramagnetic substances by the fact that in the first substances the susceptibility is very large and therefore their magnetisation can easily be measured with a magnetometer, while the susceptibility of the latter is so weak that the magnetisation cannot be measured magnetometrically. But the investigation of the binary alloys forming a solid solution with each other shows that an alloy of a ferromagnetic metal with a paramagnetic can be obtained, which has a susceptibility of any value lying between those of the ferromagnetic and paramagnetic metals.

[2] K. Honda, *Ann. der Phys.* xxxii., *loc. cit.*, *Sci. Rep.* i., *loc. cit.*

On the other hand, the paramagnetic susceptibility is independent of the strength of the magnetic field, while the ferromagnetic susceptibility varies considerably with the field. But it is to be remarked that we can also obtain an alloy whose susceptibility changes with the field in such a degree that the variation lies between zero and a large value corresponding to a ferromagnetic substance. Thus, we cannot distinguish the one class of the substance from the other by the above definitions; in fact, the ferromagnetic and paramagnetic substances are two extreme members of a series, whose susceptibility varies continuously from the former to the latter.

We shall therefore compare the magnetic moments of molecules in the ferromagnetic and paramagnetic substances. As a concrete example, let us consider iron and manganese. The intensity of magnetisation of iron increases at first rapidly with the magnetic field, and soon reaches an asymptotic value of 220 C.G.S. per unit mass. In the case of manganese the intensity of magnetisation is extremely small in ordinary fields, but it increases proportionally with the strength of the field, its proportional constant being 10^{-5}. Hence the ratio of these two magnetisations is very large in ordinary fields; but with an increasing field, it becomes always less. For example, at H=100 the ratio is about 180,000; but at H=20,000 it is only 1100. Though the susceptibility will not remain constant in a still higher field, it is highly probable that in a sufficiently strong field, which has not been reached up to the present, the above ratio will be far less than 100. Hence it is to be concluded that the magnetic moments of iron and manganese molecules do not differ so much from each other as is generally believed to be, and that, from the point of view of molecular magnetism, ferromagnetic substances do not take any special position with respect to other substances.

Admitting that the molecular magnetisms of ferromagnetic and paramagnetic substances do not differ much from each other, what causes then the great difference of magnetisability at ordinary fields? This difference is due to the great resistance to magnetisation offered by the paramagnetic substances. The principal cause of this great resistance is very probably due to the kinetic state of molecules, that is, to their thermal condition, as Langevin considers, but not to the mutual actions between the molecules. The present writer holds the view that in the case of ferromagnetic substances thermal resistance is negligibly small in comparison with that of mutual action, while in the case of paramagnetic substance the former effect predominates over the latter; he also considers that the thermal resistance is caused by the rapid revolution of the molecules about their own axes, this motion causing a gyrostatic effect, when acted on by a magnetic field. In other words, at ordinary temperature, the molecules of a ferromagnetic substance are assumed to have a comparatively slow revolution about their magnetic axes, the revolution about the perpendicular axis being of course absent. Hence, in the ferromagnetic molecules, gyrostatic effect is negligibly small, and mutual magnetic action alone predominates.

§ (5) Ferromagnetic Substances.—Three well-known ferromagnetic metals are iron, nickel and cobalt; crystallographically they all belong to the cubic system. Usually these metals consist of an aggregate of an immense number of minute crystals, whose axes are uniformly distributed in all directions. In each of these microcrystals the molecular magnets are arranged in the space-lattice of cubic system. According to Ewing's original theory,[1] all magnets in each microcrystal naturally assume one of three orientations of stable equilibrium which are parallel to the sides of the space-lattice; but even in the same microcrystal a number of such groups may be formed, as shown in *Fig.* 5. As the direction of these groups of molecular magnets is uniformly distributed in all directions, their external action is, as a whole, zero. If an external field acts on the substance, all the elementary magnets in each crystal, or in each group, will tend, as a whole, to turn with their axes in the direction of the field, but they are partially prevented from doing so by action of the mutual magnetic force, tending to draw these magnets back to their original stable orientation. With the increase of field the molecules will more and more turn in the direction of the field, and consequently the intensity of magnetisation becomes greater, tending to an asymptotic value. The above theory of Ewing, which coincides with the view proposed in the last section for the ferromagnetic substance in connection with the paramagnetic, is very simple in its content, but agrees with many observed facts quite satisfactorily. In what follows we shall discuss this theory of magnetism[2] more in detail.

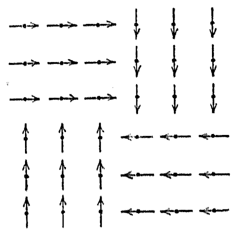


Fig. 5.

§ (6) Magnetisation of a Single Complex.—A group of elementary magnets distributed in a space-lattice, and having their magnetic axes all parallel to the direction of one of

[1] *Roy. Soc. Proceedings*, 1890, xlviii. 342. See, however, for a later view, *Roy. Soc. Proceedings*, 1922, c. 449.

[2] K. Honda and J. Okubo, *Sci. Rep.*, 1916, v. 153, *Phys. Rev.*, 1917, x. 705.

the stable orientations, is called *elementary complex*.

Suppose we have an elementary complex having a space-lattice of squares, in the plane of which an external field H acts, as shown in *Fig.* 6. The action on each of these magnets by its neighbours arises from them all, but we may with a fair approximation suppose that the action of eight only of the surrounding magnets is effective and those of the rest negligible. On this supposition it is easy to calculate the magnetic force acting on one of these magnets.

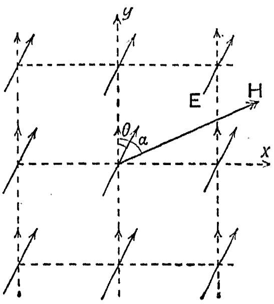


FIG. 6.

If α be the angle between the undisturbed position of the magnetic axes and the direction of H, θ the angle through which the axes are deflected, and we assume the pole of each magnet acted on by sixteen neighbouring poles, we obtain the equation

$$H \sin (\alpha - \theta) = A \sin 4\theta, \quad . \quad (12)$$

where A is a function of a/r the ratio of the half side of the lattice to the half length of the magnets; A is infinite if a/r is unity, but rapidly decreases.

Again if I_0 be the intensity of magnetisation in the direction of the axes of the magnets, I that in the direction of H,

$$I = I_0 \cos (\alpha - \theta). \quad . \quad . \quad (13)$$

Thus if $I/I_0 = i$, $H/A = h$, we have

$$i = \cos (\alpha - \theta), \quad . \quad . \quad (14)$$

$$h \sin (\alpha - \theta) = \sin 4\theta. \quad . \quad . \quad (15)$$

I_0 and A depend on the properties of particular substances. But if we use the reduced i and h instead of the actual intensity of magnetisation and field, relations (14) and (15) apply for all ferromagnetic substances belonging to the regular system. The last two relations may be considered as the laws of magnetisation.

If h and α be given, θ can be found from equation (15), which is of the eighth degree in $\sin \theta$ or $\cos \theta$; hence we cannot solve it analytically. However, as θ is given as the intersections of the two curves

$$y = \sin 4\theta \text{ and } y = h \sin (\alpha - \theta),$$

we can easily find its value by a graphical method. In *Fig.* 7, curve I represents $y = \sin 4\theta$, and curves a, b, c, d, those of $y = h \sin (\alpha - \theta)$ for $\alpha = 30°$, 70°, 120°, and 160° respectively, h

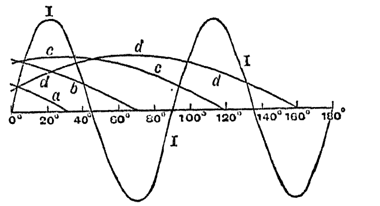


FIG. 7.

being taken as 0·6. By giving different values to h the curve of magnetisation can be obtained.

In *Fig.* 8, four curves representing the relation between i and h are given, in which for the angle α are taken angles of 30°, 70°, 120°, and 170° respectively. They give the intensity of magnetisation in the direction of the respective fields, when the magnitude of the latter is so varied that it is always in equilibrium with the internal resisting force $\sin 4\theta$. In the curve for 30° the initial point a corresponds to the value of $\cos 30°$; as h increases θ becomes greater, but always less than α, and therefore $i = \cos (30° - \theta)$ steadily increases, tending asymptotically to the value of $i = 1$ with $h = \infty$. In the curve for $\alpha = 70°$ the point b corresponds to the value of $\cos 70°$; as h increases from 0, θ and therefore $\sin 4\theta$ also increases. Since, however, the latter quantity reaches a maximum at $\theta = (\pi/8)$, h must be diminished from a certain value of θ upward, if the magnetisation is to be effected statically or reversibly. With $\theta = (\pi/4)$, the resisting force $\sin 4\theta$ vanishes and therefore h must be diminished to zero; with a further increase of θ, $\sin 4\theta$ changes sign, and therefore h must be applied in an opposite direction if the magnetisation is to be made reversibly. If θ approaches to 70°, H becomes $-\infty$ in the limit and the magnetisation tends asymptotically to unity. The curve for $\alpha = 120°$, which begins at the point c on the negative side of i passes through a maximum and a minimum of h, and coincides with the curve for $\alpha = 30°$, as the value of i

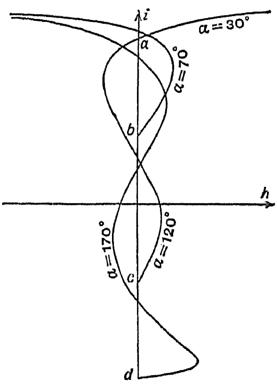


FIG. 8.

increases. The curve for $\alpha=170°$, beginning at a point d on the negative side of i, passes through two maxima and one minimum of h with the increase of i and approaches asymptotically to the line $i=1$.

In the ordinary case of magnetisation the field is continuously increased, and therefore the magnetisation is only partly reversible. But it is easy to see in what way the magnetisation in the direction of the field increases by applying a continuously increasing field.

Case (i.), $0<\alpha<\pi/4$.—The component magnetisation in the direction of the field increases with h and becomes 1 for $h=\infty$. If the field is gradually reduced i takes its original value, and there is no hysteresis.

Case (ii.), $\pi/4<\alpha<\pi/2$.—i increases with h continuously up to the maximum resisting value of the force; here it undergoes an abrupt change and takes a value corresponding to a change of $\pi/2$ in the initial orientation of molecular magnets. With a further increase of the field i continuously increases as though the initial orientation were $\alpha-(\pi/2)$. If the field is reduced, i takes a value quite different from its initial, as shown in *Fig.* 9 (DCE); that is, a hysteresis phenomenon occurs.

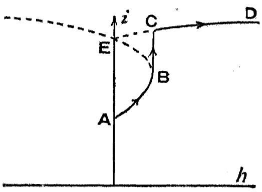


FIG. 9.

Case (iii.), $\pi/2<\alpha<3\pi/4$.—i increases with h, at first continuously and then abruptly, when the resisting force reaches a maximum. After this, the curve of magnetisation follows the course corresponding to the case with the initial orientation of $\alpha-(\pi/2)$ (*Fig.* 10). With the removal of the field a hysteresis phenomenon (*Fig.* 10, DCE) is also observed.

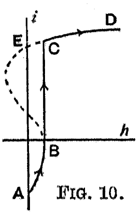


FIG. 10.

Case (iv.), $3\pi/4<\alpha<\pi$.—The curve of initial magnetisation is the same as in the above cases. If the first maximum of the resisting force is less than the second maximum, its next magnetisation is the same as in the case with the initial orientation of $\alpha-(\pi/2)$ (*Fig.* 11 (a)); if the first maximum is greater than the second, the magnetisation is the same as that for the initial orientation of $\alpha-\pi$ (*Fig.* 11 (b)). The subsequent magnetisation takes place continuously. By reducing the field the corresponding hysteresis (*Fig.* 11, FEG) is observed.

The relation between the initial orientation α and the maximum resisting force h_m can be found in the following way:

From the equation

$$h=\frac{\sin 4\theta}{\sin(\alpha-\theta)},$$

we have

$$\frac{dh}{d\theta}=\frac{5\sin(\alpha+3\theta)+3\sin(\alpha-5\theta)}{2\sin^2(\alpha-\theta)}.$$

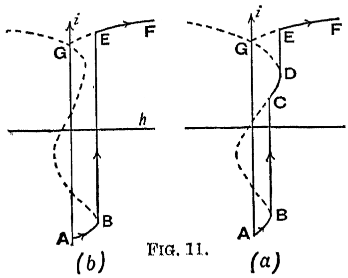


FIG. 11.

If the value of θ corresponding to the maximum force be denoted by θ_0 we have

$$5\sin(\alpha+3\theta_0)=3\sin(5\theta_0-\alpha), \quad . \quad (16)$$

and

$$h_m=\frac{\sin 4\theta_0}{\sin(\alpha-\theta_0)}. \quad . \quad . \quad (17)$$

The existence of such values of θ_0 can be understood from *Fig.* 8. The calculated values of h_m for different values of α are given in the following table and in *Fig.* 12.

α.	h_m.	α.	h_m.
45°	4·000	100°	1·025
50	2·625	120	1·008
60	1·750	140	1·137
70	1·405	160	1·541
80	1·205	170	2·018
90	1·088	180	4·000

Curve a in *Fig.* 12 refers to the first maximum; in the interval between 135° and 180° a second maximum is also possible. However, as h_m corresponding to α for the first maximum is equal to that corresponding to $\alpha+\pi/2$ for the second maximum, curve b for the second maximum has the same form as curve a, being only displaced through $\pi/2$.

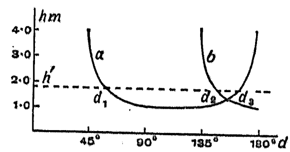


FIG. 12.

§ (7) MAGNETISATION OF FERROMAGNETIC SUBSTANCES.—Hitherto we have considered exclusively the magnetisation of a single complex; but we are now able to study the magnetisation of a mass of ferromagnetic substance, such as iron, which consists of a great number of such elementary complexes with

their magnetic axes uniformly distributed in all directions. Now, the faces of the elementary cubes or the complexes are in actual cases directed in all directions; but for the sake of the simplicity of calculation, it is here assumed that the complexes have one of their faces all parallel to a common plane, other faces being distributed quite arbitrarily, and the magnetic field acts parallel to this plane. The problem is then reduced to the two-dimensional. The magnetisation of this simple case does not obviously differ from that of the actual case in its character.

Let N be the number of elementary complexes; if there is no magnetic force acting on these complexes, the number of complexes whose magnetic axes make, with the direction of the field, an angle lying between α and $\alpha+d\alpha$, is equal to

$$dN=\frac{N}{2\pi}d\alpha.$$

If M be the magnetic moment of a complex of an initial angle α, then the component of magnetisation in the direction of the field is $M\cos(\alpha-\theta)$. Considering M to be the same for all complexes, the total magnetisation due to these complexes is

$$I=\int_{-\pi}^{+\pi}\frac{MN}{2\pi}\cos(\alpha-\theta)d\alpha=\frac{I_0}{\pi}\int_0^{\pi}\cos(\alpha-\theta)d\alpha,$$

where $I_0=MN$ is the saturation value of the magnetisation. Hence we have for i

$$i=\frac{1}{\pi}\int_0^{\pi}\cos(\alpha-\theta)d\alpha. \quad . \quad . \quad (18)$$

The relation connecting α and θ must, however, be different from that for a single complex. In this case, besides $A\sin 4\theta$, which measures the force due to the sixteen neighbouring poles, we must also consider the magnetic force due to surrounding complexes. If no field acts on the substance, the resultant effect of the surrounding complexes is obviously zero; but in its magnetised state this is not the case. To calculate this force exactly is impossible; but it is not difficult to estimate approximately its mean effect. Since the total action of a complex on a magnet within it is the same as the sum of the effects of neighbouring magnets, those of the distant ones being very small, we may consider the form of the complex under consideration to be a sphere, without causing sensible error in the value of $A\sin 4\theta$. The magnetic effect of other complexes on the magnet under consideration may approximately be replaced by that due to a uniform distribution of magnetisation with a mean intensity in the space in which other complexes are found. As the boundary of the said complex is assumed to be a sphere, this force is $(4\pi/3)I$ acting in the direction of the external field, and does not generally coincide in direction with that of the axis of the magnet under consideration; and hence it exerts a couple tending to turn the magnet in the direction of the field. Hence instead of relation (15) we must use the following formula:

$$\left(H+\frac{4\pi}{3}I\right)\sin(\alpha-\theta)=A\sin 4\theta.$$

But for a given value of H, I is a constant, so that for a while we may regard $H+(4\pi/3)I$ as an external field and proceed to calculate I for different assigned values of it. After finding I, the actual field may be found by simply subtracting $(4\pi/3)I$ from the assigned field. Hence the same relation as (15), that is,

$$h=\frac{\sin 4\theta}{\sin(\alpha-\theta)},$$

may also be used in the present case.

If h be given, this equation gives θ in terms of α, and if this value of θ be substituted in equation (18), this gives the intensity of magnetisation i in terms of h, and thus the problem is formally solved. But in actual calculation some complications are involved, and we must separately consider cases corresponding to several graded values of h.

(i.) *External Field Small.*—First let us consider the case when h is very small; then θ is also small, and therefore $\sin 4\theta=4\theta$. From the above equation we get

$$h(\sin\alpha-\theta\cos\alpha)=4\theta,$$

$$\therefore\ \theta=\frac{h\sin\alpha}{4+h\cos\alpha}.$$

Equation (18) gives

$$i=\frac{1}{\pi}\int_0^{\pi}(\cos\alpha+\theta\sin\alpha)d\alpha=\frac{1}{\pi}\int_0^{\pi}\theta\sin\alpha d\alpha$$

$$=\frac{h}{4\pi}\int_0^{\pi}\sin^2\alpha\left(1+\frac{h}{4}\cos\alpha\right)^{-1}d\alpha$$

$$=\frac{h}{4}\left(\frac{1}{2}+\frac{h^2}{8\cdot4^2}+\frac{\pi}{16}\frac{h^4}{4^4}+\ .\ .\ .\right)$$

$$=0\cdot125h+0\cdot00195h^3+0\cdot00007h^5+\ .\ .\ . \quad (19)$$

As it ought to be, i is an odd function of h. If h be sufficiently small, the terms of any order of h as high as, or higher than, the third can be neglected, and i and h are linearly related to each other. This fact was verified by experiments of Bauer,[1] Lord Rayleigh,[2] and others. In this case the magnetisation is perfectly reversible, that is, there is no hysteresis. In actual cases, however, even in very weak fields, the time effect or magnetic effect, which is not considered in the above theory, produces a decided hysteresis.

[1] Bauer, *Inaug. Diss. Zürich*, 1879; *Wied. Ann.*, 1880, ii. 399.

[2] *Phil. Mag.*, March 1887; see also Ewing's *Magnetic Induction*, p. 124.

(ii.) *External Field Large.*—Secondly, we consider the case where h is large. To change the integration variable from α to θ we differentiate equation (15) and obtain

$$\frac{d\alpha}{d\theta}=\frac{4\cos 4\theta}{h\cos(\alpha-\theta)}+1$$

and also

$$\cos(\alpha-\theta)=\pm\frac{1}{h}\sqrt{h^2-\sin^2 4\theta},$$

$$\therefore\ i=\frac{1}{\pi}\int\left\{\frac{4\cos 4\theta}{h}\pm\frac{1}{h}\sqrt{h^2-\sin^2 4\theta}\right\}d\theta. \quad (20)$$

According to the magnitude of h, all the complexes, during magnetisation, do not necessarily change their angle of deflection continuously; in fact, some of these complexes make an abrupt rotation of $\pi/2$ or π. Hence in evaluating the above integral it is necessary to divide the limits of integration into several parts. If h be given, we can find from *Fig.* 12 the value of α for which h is equal to h_m; the values of θ for these values of α may then be found from equation (15). We have generally three values of α and θ, let us call them by α_1, α_2, α_3, and θ_1, θ_2, θ_3. Then we have

$$\int_0^{\pi}=\int_0^{\alpha_1}+\int_{\alpha_1}^{\alpha_2}+\int_{\alpha_2}^{\alpha_3}+\int_{\alpha_3}^{\pi}.$$

In the first and fourth integrals the molecular magnets in the complexes belonging to these integrals remain stable, since the field is less in these cases than the critical value. The magnets in the complexes belonging to the second integral all lie beyond the position of stable equilibrium, and therefore the magnetisation is the same as if the initial orientation of the complexes were $\alpha-\pi/2$. Hence the limit of the second integral must be changed from α_1 and α_2 to $\alpha_1-\pi/2$ and $\alpha_2-\pi/2$. In the third integral the magnets in the complexes lie beyond the first and second positions of stable equilibrium, and therefore the magnetisation is the same as if the initial orientation were $\alpha-\pi$. Hence the limits of the third integral are to be changed from α_2 and α_3 to $\alpha_2-\pi$ and $\alpha_3-\pi$. If the integration variable be then changed from α to θ, we have

$$\int_0^{\pi}=\int_{\theta_1}^{\theta_1'}+\int_{\theta_2}^{\theta_2'}+\int_{\theta_3}^{\theta_3'}+\int_{\theta_4}^{\theta_4'}. \quad (21)$$

Now from equation (20) we have

$$i=\frac{1}{\pi h}(\sin 4\theta'-\sin 4\theta)\pm\frac{1}{\pi}\left\{\int_0^{\theta'}\sqrt{1-k^2\sin^2 4\theta d\theta}\right.$$
$$\left.-\int_0^{\theta}\sqrt{1-k^2\sin^2 4\theta d\theta}\right\},$$

where $k^2=1/h^2$ Hence if E be an elliptic integral of the second kind, we have

$$i=\frac{1}{\pi h}(\sin 4\theta'-\sin 4\theta)\pm\frac{1}{4\pi}\{\mathrm{E}(k_1 4\theta')-\mathrm{E}(k_1 4\theta)\}. \quad (20')$$

The double sign of the second term must be so chosen that upper and lower signs correspond to $\alpha-\theta>\pi/2$ and $\alpha-\theta<\pi/2$ respectively, with the condition that if an abrupt turning of the molecules through $\pi/2$ takes place, α and θ are measured from the new position of equilibrium.

In the following tables and in *Fig.* 13 the result of our calculation according to the above relations is given. Up to $h=0{\cdot}5$, i was calculated by equation (19), while for higher fields it was obtained by means of equation (21), by taking the sum of the four integrals having different limits of integration.

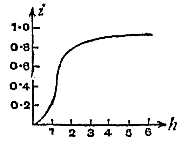


FIG. 13.

h.	i.	h.	i.
0·1	0·0125	2·0	0·816
0·5	0·0627	2·5	0·875
1·0	0·183	3·0	0·909
1·5	0·677	..	...

Thus the form of the curve of magnetisation agrees precisely with that experimentally found. This curve starts from the origin at a definite angle, and increases at first linearly with the field. With a further increase of field, the magnetisation increases more and more rapidly; in a certain field its rate attains a maximum and then gradually decreases. The curve of magnetisation passes therefore through an inflexion point, and gradually approaches to an asymptotic value 1 as the field is increased. This curve is the normal curve of magnetisation with the reduced intensity of magnetisation and field; it is common for all the ferromagnetic substances belonging to the regular system. The curve of magnetisation belonging to a particular substance can be obtained by multiplying I_0 and A, characteristic constants of the substance, by i and h respectively.

If the curve of magnetisation be plotted against the actual field as explained at the beginning of the present paragraph, the characteristic form of the curve will not materially change.

§ (8) RESIDUAL MAGNETISM AND HYSTERESIS PHENOMENON.—If a mass of iron is once magnetised to saturation and then the field reduced to zero, there remains a residual magnetism. The amount of this residual magnetism can easily be found in the following way: The complexes whose magnetic directions lie initially between 0 and $\pi/4$ will return to their original position with $h=0$; the complexes whose magnetic directions were initially $\pi/4>\alpha>\pi/2$, or $\pi/2>\alpha>3\pi/4$, take a new position of equilibrium differing from the initial by $\pi/2$ with $h=0$. Lastly, the complexes whose magnetic directions were initially $3\pi/4>\alpha>\pi$ will come to a new position differing by π from the initial with $h=0$. Hence, if the field be reduced to zero, the

magnetic directions of all the complexes are distributed uniformly within an angle making $\pi/4$ on both sides of the field. The residual magnetism may therefore be found thus :

$$R=2\int_0^{\pi/4} M\cos\theta dn, \quad dN=\frac{2N}{\pi}d\theta,$$
$$=\frac{4I_0}{\pi\sqrt{2}};$$

hence the reduced residual magnetism is

$$r=\frac{R}{I_0}=0{\cdot}8927. \quad . \quad . \quad . \quad (22)$$

Thus there remains a residual magnetism of about 90 per cent. The experiments with very long iron wires confirm the correctness of this conclusion.

According to the above consideration, the process of reducing the field from ∞ to 0 is reversible, that is, the magnetisation during the reduction of the field from ∞ to 0 exactly coincides with the new magnetisation from 0 to ∞, the initial magnetisation being r. This curve of magnetisation can easily be found, because the initial orientation of the complexes is known to be uniformly distributed within an angle subtended by the lines inclined at $\pi/4$ to the field. If h be small,

$$i=\frac{4}{\pi}\int_0^{\pi/4}\cos(\alpha-\theta)d\alpha \quad \text{and } \theta=\frac{h\sin\alpha}{4+h\cos\alpha},$$
$$\therefore i=0{\cdot}8927+0{\cdot}047h-0{\cdot}083h^2+\ldots \quad (23)$$

For a large value of h we find from equation (21) the value of i on simple substitution of the limits of integration.

Starting from the residual magnetism, the magnetisation by a gradually increasing negative field can be calculated in a similar way. This case is equivalent to the magnetisation by a positive field of a group of complexes whose initial magnetic directions are uniform and given by $\pm(3\pi/4)>\alpha>\pi$. For small values of h we have

$$i=-\frac{4}{\pi}\int_{3\pi/4}^{\pi}\cos(\alpha-\theta)d\alpha=0{\cdot}8927-0{\cdot}047h$$
$$-0{\cdot}083h^2-\ldots \quad (24)$$

For large values of h we find i from equation (21), as in the former cases. The results of calculation are included in the following table :

h.	i.	h.	i.
∞	1·000	−1·0	0·815
3·5	0·973	−1·5	0·015
3·0	0·962	−2·0	−0·584
2·5	0·956	−2·5	−0·786
2·0	0·944	−3·0	−0·847
1·5	0·932	−5·0	−0·981
1·0	0·922	$-\infty$	−1·000
C 0	0·893	..	..

In this way we can obtain a well-known hysteresis loop, when the field is varied between $+\infty$ and $-\infty$, as shown in *Fig.* 14. It possesses all the characteristics shown by iron, nickel, and cobalt.[1]

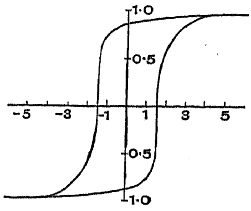


FIG. 14.

The hysteresis loop accompanying a cyclic change of magnetic field between $+h$ and $-h$ can also be calculated in a similar manner. For this purpose the residual magnetism obtained by reducing the field from h to 0 will be at first calculated. Then the curve of magnetisation having this residual magnetism as the initial will be calculated; it must coincide with the curve of demagnetisation obtained by reducing the field from h to 0. Next, the curve of magnetisation from 0 to $-h$ having the state of residual magnetism as the initial, will be calculated, and so on. In this way we have obtained a complete cycle of magnetisation.

The residual magnetism when the field h is reduced to zero is easily known, because, for a given value of h, we can find from *Fig.* 12 the values of α having h as the maximum resisting force, and therefore it can be completely known how many complexes, which had initially a uniform distribution of their axes, will return to their original position on reducing the field to zero, and how many of them will rotate through one or two right angles from their initial positions. Hence the residual magnetism can be calculated by the following expression :

$$r=\frac{1}{\pi}\left\{\int_0^{\alpha_1}\cos\alpha d\alpha+\int_{\alpha_1}^{\alpha_2}\cos\left(\alpha-\frac{\pi}{2}\right)d\alpha\right.$$
$$\left.+\int_{\alpha_2}^{\alpha_3}\cos(\alpha-\pi)d\alpha+\int_{\alpha_3}^{\pi}\cos\alpha d\alpha\right\}.$$

Since the orientation of the magnetic direction of these complexes in the residual state of magnetisation is thus completely known, a further magnetisation with positive and negative field can be calculated in the same way as the case above discussed. In this way we calculated three curves of hysteresis for different values of h, which are shown graphically in *Fig.* 15. The curves are found to agree with the results of experiments.

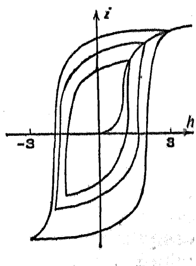


FIG. 15.

[1] K. Honda and J. Okubo, *Sci. Rep.*, 1917, vi. 183.

§ (9) Effect of Temperature on Magnetisation.—The effect of temperature [1] will be lastly considered. In the case of iron, for example, the magnetisation in very weak fields increases with the rise of temperature at first very slowly, but from 600° upwards its rate of increase becomes always greater. At about 750°, the magnetisation reaches a sharp maximum, and then falls abruptly to zero. As the field is increased, the increase at high temperatures becomes always less, and in a field of about ten units the magnetisation remains almost constant up to about 730° and then rapidly falls. With a further increase of the field, the temperature at which magnetisation begins to decrease becomes always lower; and in a strong field of several hundreds, it decreases from a temperature far below the room temperature, the rate of decrease becoming always greater as the temperature rises.

From these facts we may conclude that the temperature affects the magnetisation in two opposite ways, that is, the first effect, which exists in all fields, is to diminish the magnetisation, and the second, which is noticeable only in weak fields, is to increase it. According to the present writer, the first effect is, as was already remarked, due to the rapid revolution of the molecules about their magnetic axes. The angular velocity of this motion is assumed to be comparatively small at room temperature, and the gyrostatic resistance to the turning of the magnetic axes in the direction of the field is very small in comparison with that of the mutual action of the molecules. But as the temperature is increased, the angular velocity becomes always greater, and hence the gyrostatic action of the molecules increases, the substance becoming thereby less magnetisable. Thus the first effect of temperature is explained. Here it is to be assumed that when the field ceases to act, the direction of the magnetic axes of the molecules takes a distribution uniform in all directions by virtue of thermal impacts. From the theory of specific heat of solid, it is concluded that the molecules in the solid do not possess any freedom of rotation. In ferromagnetic substances the axial rotation above referred to is to be considered as dependent on the thermal vibrations of the molecules, its energy being extremely small compared to that corresponding to the energy of free rotations.

The second effect is due to the abrupt turning of the molecules towards the field by virtue of thermal motions. If the thermal agitation be zero, molecular magnets in each complex will take a common direction determined by the external field and internal resisting force. Suppose this direction to make an angle θ with the field. In virtue of the thermal energy, they will in an actual case execute translational and rotational vibrations about their mean positions. The amplitude of their rotational vibrations will actually differ from one magnet to another; but as the first approximation, we may consider their mean value to be β. Since, in each complex, the molecules exert their mutual action on each other, the rotational vibration of molecules with the same phase takes place more easily than in the case of those with arbitrary phases. Hence in a stationary state we may, as the first approximation, suppose that all the magnets in each elementary complex oscillate with a common phase, but that the phase of the oscillation differs from one complex to another. If for a complex (α), $\theta+\beta<(\pi/4)$, then the molecular magnets in the complex will oscillate about its mean orientation θ; on the other hand, if $\theta+\beta>(\pi/4)$, they will undergo an abrupt turning and take a position as if the initial orientation were $\alpha-(\pi/2)$, causing thereby an increase of magnetisation. Hence, even in weak fields, where there is no complex in which molecular magnets abruptly turn in the direction of the field, when there is no thermal motion, the molecular magnets will more and more begin to make an abrupt turning with the rise of temperature, thus causing an increase of magnetisation. The second effect of temperature is then explained.

If the field becomes greater, the increased number of complexes turns abruptly towards the field, even if there is no thermal motion; and consequently the increase of magnetisation due to the thermal vibration becomes always less. In a sufficiently strong field, where all the complexes have finished their possible abrupt turning, the effect of temperature in increasing magnetisation must vanish, and there exists only the effect of diminishing magnetisation due to axial rotation of molecules. Thus the effect of temperature on magnetisation is explained by our theory, at least qualitatively.

Working quantitatively the above idea, Dr. J. Okubo and the present writer obtained a number of the magnetisation-temperature curves for different fields, and found them to agree completely with the observed curves.

K. H.

§ (10) Ewing's Modern Model.—The original theory of Ewing has recently undergone an important modification at the hands of its author. It was supposed that the molecules themselves constituted the Weber elements, to whose orientation under the impressed magnetic force the properties of the iron were to be ascribed. But it is now recognised that magnetism is an attribute of the atom, as explained in § (1), not of the molecule; that each atom of a ferromagnetic metal contains one or more Weber elements which

[1] K. Honda and J. Okubo, *Sci. Rep.*, 1916, v. 325.

possess magnetic moment, probably in consequence of the circulation of electricity in orbits about a central nucleus, and that the element which turns under the influence of an impressed field is not the atom as a whole but something within it. The original theory accounts for hysteresis and also for the fact that for small values of the magnetising force there is no hysteresis, because for small deflections of the Weber elements the phenomenon is reversible. But Ewing has now pointed out that because the limits of reversible deflection are exceedingly narrow, it is necessary to assume that the atomic magnets are spaced very closely. According to him, the ratio a/r in § (6) must not exceed about 1·02, and under these conditions the magnetic field required in the original model to change from the reversible to the irreversible condition is many thousands of times greater than the field which is actually necessary to produce strong magnetisation in iron.

Abandoning then his original model, Ewing has described[1] a new model in which the electrons of each atom are considered to form two groups. Of these groups one constitutes an outer shell in which the electrons have paths that remain more or less fixed relatively to each other; the other group, which occupies an inner part of the atom and may consist of one or more electrons, constitutes the Weber element. It is free to turn under the influence of an impressed magnetic force, and it can take up any one of a number of stable positions relatively to the outer electrons. In each of these stable positions, however, the stability is weak, and it also has a very narrow range of stable deflection before turning over from one position to another. Within small limits of the force the effect is reversible. The path of the electron or electrons which make up the Weber element will revert it to its original position on the removal of the force. If, however, this limit be exceeded, the path will settle down to some other possible position, and on the removal of the force some amount of magnetism will be left. The suggestions thus fit in with Bohr's theory of the atom.[2]

In some of Ewing's models the electron orbits are represented by coils conveying currents, as, for example, in *Fig.* 16, where a central circular coil W represents the Weber element and is pivoted so that it may turn about a diameter as axis, and the two coils A and B represent elliptical orbits whose plane is fixed. In this model the centre of the coil W, which is also the common focus of the two ellipses, is supposed to coincide with the nucleus of the atom. The currents in A and B are directed so that the equilibrium of W depends on the difference of their effects. If everything be symmetrical the equilibrium of W is neutral, but if the distances be slightly

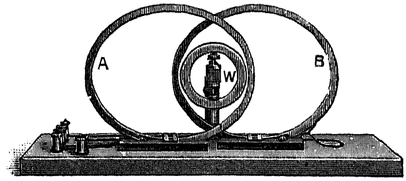


FIG. 16.

different, or if in some other way the forces exerted by A and B on W be slightly different, then W will have a small amount of stability. The external impressed field which is required to upset it can thus be made very small. At the same time the angle through which it turns before becoming upset is also small. The model is to be completed by supposing other similar elliptical orbits to surround W in other planes.

In other models Ewing represents the action of the electrons by means of small magnets. Thus in *Fig.* 17 the Weber element is represented by a short pivoted magnet in the middle,

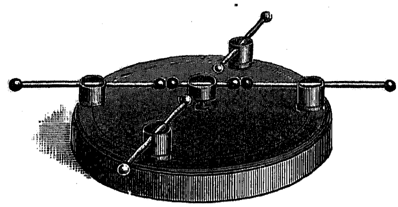


FIG. 17.

and the action of the outer electrons is represented by fixed magnets surrounding it. All of their inner poles are of the same name. Hence the pivoted magnet takes up a position of feeble stability between two of the fixed magnets, and when disturbed by the impressed magnetic force, it undergoes stable deflection through a narrow range before tumbling over into a position of stability with respect to another pair of the fixed magnets. As before, the stability in any one position is due to the difference between two opposing effects.

A more complete model embodying the same ideas in space of three dimensions is illustrated in *Fig.* 18. There the fixed magnets are arranged with cubic symmetry, pointing along the trigonal axes of a cube, so that there are eight inner poles. The Weber element also (in this example) forms an octet of magnetic poles turning as a whole from one to another position of stability in the manner already described. With a model arranged in this way, Ewing is able to reproduce many of the known characteristics of ferromagnetism, including not only the general features of the magnetisa-

[1] *Proc. Roy. Soc.*, Feb. 1922; *Proc. R.S.E.*, Feb. 1922; *Phil. Mag.*, March 1922.
[2] See "Electrons and the Discharge Tube," § (28).

tion curve, but also the magnetic aeolotropy which is produced by strain. He further applies the model to explain the influence on

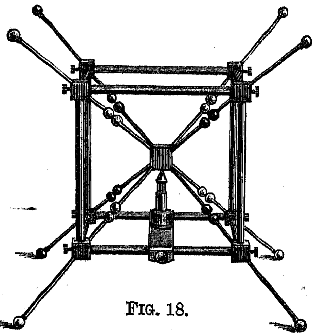


Fig. 18.

the magnetic quality of a ferromagnetic metal of any foreign substance whether present as an impurity or in chemical combination, and he shows in particular that it will account for the extreme aeolotropy which is observed in the crystals of certain ferromagnetic compounds, such as pyrrhotite, which (as Weiss showed) is magnetisable in one plane only.

Ewing's new model preserves all the qualitative advantages of his original model, and is free from the quantitative difficulty already indicated. It may be convenient to recapitulate the chief points of the paper.[1] The author writes:

While retaining the idea of magnetic control of the Weber element as determining the susceptibility and hystereses of ferromagnetic substances, I have been led to replace my older model by a new one in which the Weber element in each atom is controlled by other parts of the same atom instead of only by the Weber elements in the adjoining atoms. The position of the atom in the crystal secures that these points are more or less completely fixed. They probably constitute a shell, or series of concentric shells, within which the Weber element may turn in response to the impressed field, and by which its turning is so controlled that it is capable of small stable deflections or of large irreversible deflections, so that it may swing over with dissipation of energy from one stable position to another when the range of stable deflection is exceeded. This range is in general very narrow. Experiments on the magnetic quality of soft iron show that it is less in that metal than 1°. At the same time they show that the control must be weak, for a small magnetising force suffices to upset the equilibrium of most of the Weber elements. The old model was defective because in it a narrow range of stable deflection could be secured only by placing the magnets so close together that their stability became far too great. Its quantitative failure in this respect was made apparent by considering the magnitude of the field required to break up pairs or rows of pivoted magnets. In the new model a narrow range of stable deflection is secured without excessive stability. This is because the model is so devised that the control of the Weber element in each atom depends on a slight inequality between opposing forces. These forces are exerted separately on different parts of it by other constituents of the atom. Examples of models satisfying this condition are described with the equivalents of both large and small electron orbits which reproduce the known characteristics of ferromagnetic induction.

[1] *Proc. Roy. Soc.* A, Feb. 1922, c. 457.

Magnetism, Residual, and Hysteresis Phenomenon, on Ewing's theory. See "Magnetism, Molecular Theories of," §§ (8)-(10).

MAGNETISM, TERRESTRIAL, ELECTROMAGNETIC METHODS OF MEASURING

§ (1) Introductory.—In 1914 Schuster[2] suggested a new type of magnetometer for the measurement of H, of which the principle is as follows.

(i.) *Principle of the Method.*—In *Fig.* 1 let AB point accurately to the magnetic north, and let Fi represent the direction and magnitude of the horizontal magnetic intensity produced by a current i through a system of coils, the latter being of such size and arrangement that Fi is practically uniform throughout a sphere, having a diameter equal to, or greater than, the length of a small indicator magnet NS. If Fi is greater than H (the earth's horizontal intensity), and if the component of Fi along AB is in opposition to H, the indicator magnet may be made to set at right angles to AB by rotating the coil system and so altering the value of α. H is then determined by

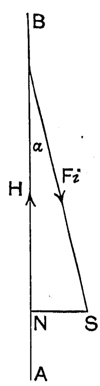


Fig. 1.

$$H = Fi \cos \alpha.$$

If the magnitude of i can be easily adjusted, it may be arranged for α to be very small. Cos α will then be nearly equal to unity, and will vary slowly for comparatively large changes in α.

The direction AB may be most easily determined by reversing the direction of the current, or by turning the coil system through 90°, and then rotating the system until there is no deflection of the indicator magnet when the circuit is either made or broken.

If Fi is less than H, and the indicator magnet is caused to set at right angles to Fi, instead of to H, then

$$H = \frac{Fi}{\cos \alpha};$$

[2] *Terrestrial Magnetism*, March 1914, xix.

that is, the horizontal intensity of the earth's magnetic field in the vertical plane at right angles to the axis of the magnet is equal to Fi, but in the opposite direction. An instrument of this type may therefore be used to measure the component of H in any direction.

(ii.) *Precision of Measurement.*—The accuracy of the method depends on the limits of error within which F, the constant of the coil system, can be calculated and the current intensity measured. In 1914, F. E. Smith, at the National Physical Laboratory, designed, at the request of Sir Arthur Schuster, a suitable coil and magnet system, and this has been used at the National Physical Laboratory for absolute measurements of "H." The value of "F" is believed to be known within 1 part in 100,000.

§ (2) Description of the Magnetometer.—For constructional purposes, as well as to ensure a uniform magnetic field at and near the centre of the instrument, a Helmholtz-Gaugain coil arrangement was adopted. The equivalent coils on each side are of twelve turns of 30 cm. radius, of 1 cm. length, and of bare copper wire; they are wound on a marble cylinder, which, when mounted, can be rotated about a vertical axis. The magnet at the centre is 1 cm. long and about 5 sq. mm. in cross-section; it is supported on a V of aluminium foil by a fine quartz fibre, to which are attached also a reflecting mirror and a damping vane. The damping of the system can be adjusted and can be made aperiodic. The magnet is easily removed from its support, and a copper wire of the same weight and nearly of the same dimensions can be substituted for it. This enables most of the torsion on the fibre to be removed. Reflecting mirrors are attached to the marble cylinder and to the case enclosing the magnet. After the magnet has been deflected, the fibre is turned through an equal angle, and no torsion, or change of torsion, in the fibre is therefore possible. The current is measured in absolute units by means of a previously standardised combination of a standard cell and a standard resistance. The leads to and from the coils are planned so that the current through them shall have no effect on the magnet, and the construction of the apparatus enables a check measurement to be made. A measurement of H, with calculation complete, occupies (after adjustment of the current) about four minutes. The total probable error —including error due to lack of knowledge of the current—is about 4 parts in 100,000.

(i.) *The Coils.*—The coils are wound with hard drawn copper wire in tension, the effective load on the wire being 4 kilogrammes. During winding the cylinder was rotated very slowly, and frequently the motion of the lathe was stopped for measurements to be made of the diameter of the wire.

(ii.) *The Support.*—The support is of gun-metal, and consists of a turn-table carrying two webs which form a cradle on which the marble cylinder rests. The upper portion of the table turns about a central stud, and is supported by phosphor bronze balls placed in a circular race. The upper portion carries also a vernier moving over a circular scale of silver fixed to the lower portion or base of the turn-table.

(iii.) *The Suspended System.*—The magnet, together with a reflecting mirror and a damping vane of aluminium foil, is suspended from a fine quartz fibre 25 cm. long. The suspended portions swing in a square box, of which two opposite sides are of plate glass.

The upper parts of the other sides of the case are also of plate glass, but the lower portions are of brass, and carry adjusting screws for the damping buffers. The buffers have their inner surfaces platinised and are not lacquered; also, to eliminate any electrostatic attraction, a glass tube containing a small quantity of radium bromide is placed near the damping system.

The top of the case is of brass, and, in addition to supporting a tube carrying the torsion head and fibre, it supports 4 reflecting mirrors at right angles. These serve to indicate movements of the case of exactly 90°, 180°, etc. Positions exactly 90° apart are obtained by rotating the case and adjusting the 4 mirrors in azimuth until the angle between the images is independent of the mirrors used, *i.e.* the axes of the mirrors must be 90° apart.

The whole of the suspended system, together with torsion head, fibre, damping buffers, mirrors, etc., can be turned rapidly about a large central stud. A few seconds enable the angle turned through to be adjusted exactly to 90°, 180°, or any other angle previously arranged for.

§ (3) Calculation of the Constants.—For a precise knowledge of H it is necessary to have an accurate knowledge of—

(*a*) The value of the current in absolute measure.

(*b*) The constant F of the coil system.

The uniformity of the magnetic field, the effect of any torsion on the fibre, the possible magnetic effect of the current in the circuit external to the coils, the magnetic permeability of the coil supports, and possible electrostatic effects on the suspended system, are also of considerable importance, but merely demand reasonable care in the design and test of the apparatus and in experimental manipulation.

(i.) *The Current.*—In the experiments made at the National Physical Laboratory the current was measured by a combination of a

standard cell and a resistance, the combination value of which had been previously determined by means of an absolute current balance. The absolute values of the currents used are believed to be known within 3 parts in 100,000.

(ii.) *Determination of the Constant F.*—Each half of the coil system consists of 2 twin coils, each of which is 60 cm. in diameter, of six turns, and of $1\frac{5}{8}$ mm. pitch. The 2 coils form two interwoven helices, the mean planes of which are coincident, but the start and end of one coil are 180° apart from the start and end of the other. This twin coil construction with bare wires enables the diametral and axial measurements of the coils to be made with great accuracy, and also enables the insulation resistance between neighbouring turns to be measured with ease.

The diameters of the coils were measured twice; once along the rake of the helices so that the measurement related to a single coil, and once with the measuring faces of the machine at right angles to the axis of the cylinder. In the latter case the measured distance is really the addition of the lengths of the radii of the 2 twin coils at the point. The diametral measurements were made in 24 axial planes at angular distances of 7·5° apart. The mean diameters at 17° C. of the coils, reckoned to the axis of the wires, are

A_1 . .	60·0029 cm.	one side of system.
A_2 . .	60·0026 cm.	
B_1 . .	60·0015 cm.	other side of system.
B_2 . .	60·0008 cm.	
Mean . .	60·0019 cm.	

The agreement indicates the degree of precision which can be obtained in this kind of work.

The axial lengths of the coils and the distance between the mean planes were determined from measurements along twelve generating lines. The mean axial length of the coils is 0·9987 cm., and is not of great importance; the distance between the mean planes of the coils on opposite sides is 30·0098 cm. at 17° C. The thermal coefficient of linear expansion of the marble was determined to be $7{\cdot}9 \times 10^{-6}$ per 1° C.

For unit current in the circuit the axial intensity at the centre of a Helmholtz-Gaugain system of N turns (the breadth of a coil being comparatively small) is, within less than 1 part in a million, N times that due to unit current in two turns flowing in circles in the mean diametral planes of the coils. The axial intensity F_x (usually written as F, the constant of the coil system) at the centre can be calculated from the equation

$$F_x = \frac{4\pi N a^2}{(a^2+l^2)^{\frac{3}{2}}}, \quad . \quad . \quad (1)$$

where N is the number of turns in half of the complete system, a is the mean radius of the coils, and $2l$ is the distance apart of the mean planes.

In the Schuster magnetometer

$$2a = 60{\cdot}0019 \text{ cm. at } 17° \text{ C.},$$
$$2l = 30{\cdot}0098 \text{ cm. at } 17° \text{ C.},$$
$$N = 12,$$

and the value of F_x is

$$3{\cdot}59595 \text{ cm.}^{-1}.$$

It is believed that the value of F_x is known within 1 part in 100,000.

(iii.) *Uniformity of Field near the Centre of the Coil System.*—In the case of two single turns placed coaxial and with their planes parallel, the intensity of the axial magnetic field at the centre for unit current through the turns is

$$F_x = \frac{4\pi a^2}{(a^2+l^2)^{\frac{3}{2}}}, \quad . \quad . \quad . \quad (2)$$

where a is the radius of a turn, and $2l$ is the distance between the planes of the turns.

In the ideal Helmholtz-Gaugain arrangement $l=a/2$, and in such a case the above expression reduces to

$$F_x = \frac{6{\cdot}4\pi}{a\sqrt{5}} = \frac{8{\cdot}99176}{a} \quad . \quad . \quad . \quad . \quad (3)$$

Nagaoka[1] has shown that the axial intensity at any point near the centre can be calculated from the expression

$$F_x = \frac{6{\cdot}4\pi}{a\sqrt{5}}\left\{1 - \frac{18}{125a^4}(8x^4 - 24x^2y^2 + 3y^4)\right\}, \quad (4)$$

where x and y are the axial and transverse coordinates of the point with respect to the centre.

The transverse magnetic intensity is zero at the centre of the system and at all points in the plane midway between the coils. Its value at a point near the centre can be calculated from the expression

$$F_y = \frac{2304\pi}{625a^5\sqrt{5}}xy(4x^2 - 3y^2). \quad . \quad . \quad (5)$$

It follows from (4) and (5) that at any point near the centre the ratio of the transverse to the axial intensity can be calculated. This ratio can be calculated with considerable accuracy by means of the equation

$$\frac{F_y}{F_x} = 0{\cdot}576xy(4x^2 - 3y^2)/a^4. \quad . \quad . \quad (6)$$

In a system in which a is 30 cm. the above expression reduces to

$$\frac{F_y}{F_x} = (7{\cdot}1 \times 10^{-7})xy(4x^2 - 3y^2). \quad . \quad . \quad (7)$$

In the case of the Schuster magnetometer Smith has calculated that within a sphere of 30 mm. diameter, with its centre coincident with that of the system, the axial intensity does not vary by more than 1 part in 100,000, and the transverse intensity is at no point equal to one hundred thousandth part of the axial intensity.

A valuable feature of such coil systems is the case

[1] *Phil. Mag.*, 1921, xli.

with which two of them can be compared if their centres can be made to nearly coincide. To construct accurately gauged coil systems is very expensive, and such need only be used as standards.

(iv.) *Effect of Torsion of Fibre.*—When there is torsion on the fibre and no current in the coils, let θ (*Fig.* 2) be the angle which the axis of the magnet makes with the meridian. If M is the moment of the magnet and H the horizontal intensity, the couple MH $\sin\theta$ is balanced by the torsional couple of the fibre. If T is the torsional couple for one degree of twist, and ϕ is the actual angle of torsion,

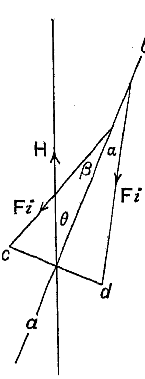


FIG. 2.

$$\mathrm{MH}\sin\theta = \mathrm{T}\phi. \quad . \quad (1)$$

When a current i is passed through the coils so as to produce no deflection of the magnet, an axial magnetic intensity Fi is produced which is at an angle θ with the meridian. In practice, therefore, the direction *ab* must, as a first approximation, be taken as being in the meridian.

When making a measurement, the coil system is next moved through a small angle α, and the direction and intensity of the current is changed in order to deflect the magnet through exactly 90°, *i.e.* into the position *cd*. The condition for equilibrium is

$$\mathrm{MH}\cos\theta - \mathrm{T}\phi = \mathrm{F}i\mathrm{M}\cos\alpha. \quad . \quad . \quad (2)$$

The magnet is now deflected from its new position through exactly 180°. To produce this effect the coil system is rotated through a relatively small angle $(\alpha+\beta)$. The axis of the magnet is still shown by *cd*, but the poles of the magnet are reversed in position.

The condition for equilibrium is

$$\mathrm{MH}\cos\theta + \mathrm{T}\phi = \mathrm{F}i\,\mathrm{M}\cos\beta. \quad . \quad . \quad (3)$$

From equations (2) and (3) we have

$$\mathrm{H} = \frac{\mathrm{F}i\,(\cos\alpha + \cos\beta)}{2\cos\theta}; \quad . \quad . \quad . \quad (4)$$

also
$$2\mathrm{T}\phi = \mathrm{F}i\mathrm{M}\,(\cos\beta - \cos\alpha). \quad . \quad . \quad (5)$$

Combining (5) with (1) there results

$$\sin\theta = \frac{\mathrm{F}i}{2\mathrm{H}}(\cos\beta - \cos\alpha). \quad . \quad . \quad (6)$$

It is apparent that if torsion in the fibre exists α will not be equal to β, conditionally that H is constant over the interval of time (about 1 minute) between the measurement of α and β. It is apparent also that the torsion in the fibre can be removed by adjustment of the torsion head until α is equal to β. However, since in practice Fi can be made equal to H within less than 1 per cent, the value of θ can be calculated within this limit by the equation

$$\sin\theta = \frac{\cos\beta - \cos\alpha}{2}. \quad . \quad . \quad . \quad (7)$$

Knowing θ an adjustment can be made, if necessary, until its value is negligibly small.

Equation (4) shows also that the first adjustment, *i.e.* adjusting the axis of the coil to be parallel with the equivalent axis of the magnet, need only be a very approximate one. In practice such adjustment within less than 1′ of arc is easily made, but if an error of a quarter of a degree is made and θ is taken as zero the error introduced is only 1 part in 100,000.

§ (4) THE CIRCUIT.—In general the voltage applied was about 100, and 200 ohms of manganin were used as ballast resistance. This enabled the current to be maintained constant over considerable periods of time, within less than 1 part in 1,000,000. The main part of the circuit, including the galvanometer, was in a building about 100 yards from the magnetometer. This was convenient, but not essential.

§ (5) DETERMINATION OF H. (i.) *Preliminary Experiments.* — After the magnetometer was mounted in position and the cylinder adjusted until its axis was horizontal the following preliminary experiments were made :

(*a*) Determination of effect of leads. The effect was found to be negligible.

(*b*) Adjustment of torsion on fibre. The torsion was made negligibly small.

(*c*) Adjustment of position of damping buffers until the magnet system was nearly aperiodic in magnetic fields having an intensity of about H/30.

There was no necessity in subsequent determinations to repeat these adjustments, but checks were occasionally made.

(ii.) *The Measurements.* — The auxiliary current circuit having been completed the measurement of H comprises the following operations :

(*a*) Adjustment of position of coil system until its axis coincides with the horizontal magnetic axis of the suspended magnet. In this adjustment the axial field due to the coil system is approximately in the same direction as the earth's magnetic field. Thirty seconds suffice to make a sufficiently accurate adjustment.

(*b*) The current through the coils is reversed and the cylinder turned through a few degrees. The torsion tube, etc., are rotated through exactly 90°. The axis of the magnet now tends to set in a direction nearly at right angles to its first position, and is made to do so exactly by adjusting the position of the coil system. Let the angle between the axis of the coil system and the meridian be α.

(*c*) The coil system is swung through a few degrees with a view to making the resultant field reverse in direction. The torsion tube, etc., are rotated through exactly 180°, and the axis of the magnet now tends to set in a

direction approximately 180°, away from its position in (*b*). The reversal is caused to be exactly 180° by adjustment of the position of the coils. Let the angle between the axis of the cylinder and the meridian be β.

The measurement is now complete, and H is given by

$$H = Fi\,(\cos\alpha + \cos\beta)/2.$$

F is the constant of the coil, and i is calculated from the combination value, in absolute measure, of the standard resistance and standard cell.

§ (6) Results.—In practice the measurements are exceedingly simple and are rapidly made. If the current is steady, a complete measurement involving operations (*a*), (*b*), and (*c*) can be conducted within three minutes. The above table gives the results of a few observations made at the National Physical Laboratory on Friday, July 15, 1921, between 1 and 2 A.M. When making these measurements, adjustment (*a*) was made only at the commencement and end of the observations. It should be noted that the value of the current was intentionally altered so as to vary the values of i and of α and β. In some measurements the direction of the current in the circuit was also altered, and the coil swung through 180°, approximately, to obtain the proper direction of the axial field.

A sensitive magnetograph for recording the changes in H was in operation simultaneously. A change of 1γ is represented[1] on the magnetograph charts by a displacement of about 2·5 mm., and the time scale is such that 7 mm. represents 1 minute. The temperature was very constant. The changes in H, as recorded by the magnetograph, are given in the last column of the table; the value at 1·0 A.M. is taken to be that obtained with the magnetometer.

The dropped figures are for comparison purposes only. The observed and recorded changes agree, with one exception, within the limit ($0{\cdot}5\gamma$) with which the record was read.

A general conclusion is that this type of instrument can be used to measure the horizontal intensity, or a component of it in any direction, with a probable error of not more than a few parts in 100,000, and in an interval of time of not more than a few minutes. The constants being known, the calculation of H consists in multiplying or dividing a product by the cosine of an angle.

F. E. S.

Table

July 15, 1921. $F = 3{\cdot}5958_5$.

Time.	i in C.G.S. Units.	α.	β.	$\frac{\cos\alpha+\cos\beta}{2}$.	$H = Fi\left(\frac{\cos\alpha+\cos\beta}{2}\right)$.	Changes in H from Magnetograph.
A.M.						
1·0	0·051598	5° 18′	5° 12′	0·99580	$0{\cdot}18475_9$	0·18476
1·5	,,	5° 20′	5° 14′	·99575	75_0	75
1·11	,,	5° 26′	5° 22′	·99556	71_5	71_5
1·14	,,	5° 30′	5° 24′	·99548	70_0	70_5
1·20	0·051428	3° 6′	2° 57′	·99860	66_9	67
1·24	,,	3° 2′	2° 54′	·99866	68_0	68
1·27	0·051767	7° 14′	7° 10′	·99211	67_7	67_5
1·29	,,	7° 14′	7° 9′	·99213	68_1	67
1·32	0·051405	2° 32′	2° 24′	·99907	67_3	67_5
1·38	,,	2° 28′	2° 22′	·99911	68_0	68

[1] $1\gamma = 0{\cdot}00001$ C.G.S. unit of magnetic intensity.

MAGNETISM, TERRESTRIAL, OBSERVATIONAL METHODS

The instruments employed in Terrestrial Magnetism are of two distinct classes, one serving for the determination of absolute values, the other recording variations of the elements. Three elements serve to define the magnetic field. The three usually determined absolutely are the declination, or angle between the magnetic and astronomical meridians, the horizontal component of force, and the inclination or dip. The two former elements are measured by the unifilar magnetometer, the last by the dip-circle or dip-inductor. Of the two last-mentioned instruments the dip-inductor is the more exact, but the dip-circle the more portable.

Values of magnetic force are usually given to 1γ ($=0{\cdot}00001$ C.G.S. magnetic unit), but this accuracy can hardly be claimed for measurements made with magnetometers of the Kew or similar patterns. For one thing, there are no instruments generally recognised as standards, and the instruments at the leading observatories appear to differ sensibly from one another. The problem is not of the ordinary laboratory type. The values of magnetic elements at a fixed station are in a constant

state of flux, and in many places a small change in the position of the observing station is not without visible effect on the result. Changes in the values of the magnetic elements at a given station can be measured to a considerably higher degree of accuracy than appertains to the absolute values themselves, but the accuracy ordinarily aimed at in measurements of magnetic curves is only to the nearest 1γ.

§ (1) UNIFILAR MAGNETOMETER.—This instrument serves for determining both declination D and horizontal force H. As shown in *Fig.* 1, it is arranged for taking the vibration experiment in the determination of H. The collimator magnet shown suspended has the stirrup attachment for taking the inertia bar used when finding the moment of inertia of the collimator magnet. A magnet used for observing D only does not require this stirrup, but requires two shanks to enable readings to be taken with the scale "erect" and "inverted," for the purpose of eliminating the error of collimation, due to the non-coincidence of the magnetic and optical axes. *Fig.* 2 shows a form of magnet which serves to determine both D and H. The magnet is a hollow cylinder, about 10 cm. long and 1 cm. in external diameter. In one end, usually the north end, is inserted a cell holding a lens; in the other end is a cell containing a fine scale on glass. Light reflected by the mirror shown in *Fig.* 1 passes through the cylinder to the telescope. The scale, being in the principal forces of the lens, is read by the telescope focussed for infinity.

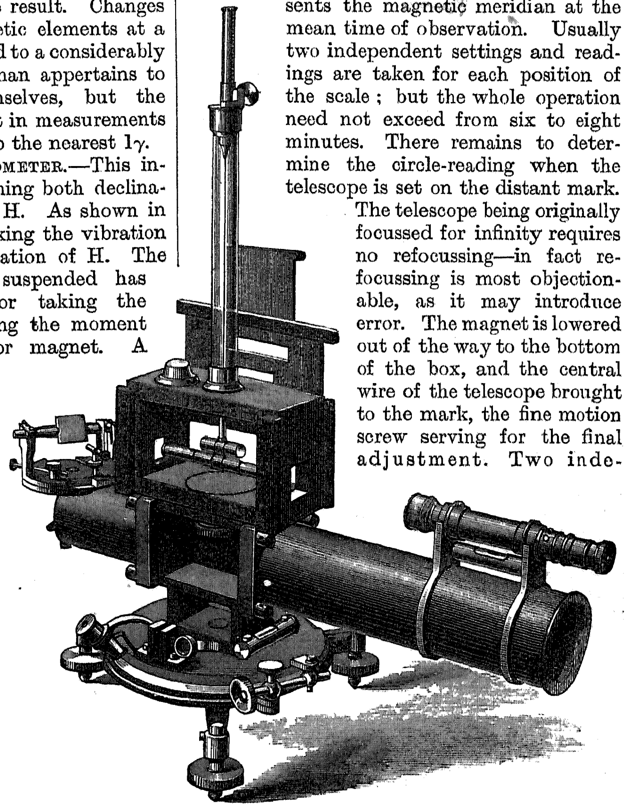


FIG. 1.

At a fixed observatory the observation of D takes only a few minutes. It consists in measuring the angle between the magnetic axis of the collimator magnet and a horizontal line of known geographical azimuth. This line is given by a distant mark, whose position has been determined astronomically. The telescope, shown in *Fig.* 1, has horizontal and vertical cross-wires; the divisions in the magnet scale are vertical. The position of the horizontal circle to which the telescope is rigidly attached is altered, a slow-motion screw being used for the final adjustment, until the central division on the scale coincides with the vertical line in the telescope, or more usually until the apparent angle of swing is bisected by the vertical line. Suspended by the one shank the magnet has its scale apparently erect, suspended by the other its scale is inverted. The mean of the circle readings with scale erect and inverted represents the magnetic meridian at the mean time of observation. Usually two independent settings and readings are taken for each position of the scale; but the whole operation need not exceed from six to eight minutes. There remains to determine the circle-reading when the telescope is set on the distant mark. The telescope being originally focussed for infinity requires no refocussing—in fact refocussing is most objectionable, as it may introduce error. The magnet is lowered out of the way to the bottom of the box, and the central wire of the telescope brought to the mark, the fine motion screw serving for the final adjustment. Two independent settings and readings are desirable. A suitable form of mark is a fine white vertical patch between two black patches.

Supposing no torsion in the suspension, the difference between the mean circle readings

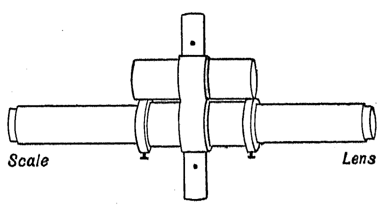


FIG. 2.

which correspond respectively to the magnetic axis and to the distant mark gives the bearing of the magnetic meridian relative to the mark. Adding to this the known bearing of the mark relative to true north, we have the magnetic declination.

Torsion in the suspension is the greatest

obstacle to accuracy. Even in the most skilful hands, a torsion correction cannot always be avoided, and its determination is a difficulty. A plummet approximately equal in weight to the magnet should be hung up for some time before the magnet is suspended. The torsion head at the top of the suspension tube, visible in *Fig.* 1, should be turned until the keel of the plummet coincides with a fixed line in the base of the magnet box. This line is parallel or perpendicular to the axis of the telescope according to the way in which the magnet is suspended. The plummet being non-magnetic comes to rest in a position in which the suspension is free of torsion. Thus with proper care the suspension should be free from torsion when the D observation begins.

But torsion may be introduced during the observation. To test this, the plummet is put on after finally taking off the magnet, and the final position of rest of its keel noted. Suppose, for example, this position is 20° to west of the line above mentioned in the magnet box. Then we finished the observation with 20° of torsion, and commenced it—or at least we hope so—with 0° of torsion. It is accordingly assumed that on the average there was 10° of torsion during the observation. To determine the consequences of this, a torsion experiment is made, the magnet being re-suspended. By means of the slow motion screw the central division of the scale—say 40·0—is brought into coincidence with the vertical line in the telescope, the index on the movable part of the torsion head coinciding with zero on the fixed scale. The torsion head being turned through 180° in one direction, the scale reading becomes, say, 41·4. The torsion head is brought back to its original position and the reading is say 40·2. This gives for the effect of 180° of torsion a twist of the magnet of 41·4 − 40·1, or 1·3 divisions out of the magnetic meridian.

The operation is repeated, but with the torsion head turned in the opposite direction, and another result, say 1·1 divisions, is obtained as the effect of 180° of torsion. Combining the two operations we get 2·4 divisions for 360° of torsion. Supposing 1 scale division to represent 1′ 48″—a fairly average state of matters in Kew pattern unifilars—we find that the observed 10° of torsion called for a correction of − 7″ in the observed westerly declination.

In one type of unifilar the plummet is a flat disc with degree divisions on its perimeter. To enable these to be read, without altering the telescope, a lens is carried by the magnet box in such a way that it can be brought when required immediately in front of the object glass. The readings of the disc at three successive positions of rest, the damping being slow, give a sufficiently accurate value for the position of rest of the plummet, and so the necessary information as to the angle of torsion.

Some observers take the torsion experiment before suspending the plummet; but this increases the risk that the torsion observed has been introduced subsequent to the real D observation. One of the advantages claimed, and doubtless truly, for a metal suspension (usually phosphor bronze, or tungsten) over the more usual silk suspension, is that the torsion being independent of the state of moisture is practically constant, and need not be redetermined except at rare intervals. With silk it is important to keep the uncertainties as small as possible by using a fine suspension. The occasional application of glycerine to the suspension is also useful. A really skilled observer with a fine silk suspension more often than not finds no torsion correction necessary. If the correction found is large, *e.g.* 20″, the observation, if taken at a fixed observatory, had better be scrapped. In the field a lower standard of accuracy is reasonable.

When, as is usually the case in field work, there is no mark of known azimuth, the bearing of the magnetic axis may be observed relative to that of some convenient distant object, *e.g.* a chimney or church steeple. The astronomical bearing of the selected object is then determined at a convenient time with a theodolite or in some other way. If a separate theodolite is used care must be taken, especially if the selected mark be not very distant, that the position of the magnetometer is exactly recovered. The Survey of India magnetometers have a vertical circle, and their telescope reversible on V's, and movable in altitude exactly like a theodolite telescope, so that observations can be made of the pole star or the sun for both altitude and azimuth. The field instruments of the Carnegie Institution of Washington form a combined magnetometer and theodolite, the one superstructure replacing the other on a single divided circle. Rücker and Thorpe slightly modified the Kew pattern instrument shown in *Fig.* 1 so as to take a solar observation.

The plane mirror which reflects the light through the collimator magnet serves as a sun's transit mirror. The horizontality of its axis is tested with a striding level. The perpendicularity of the normal to the mirror to its axis can be tested by reversing the mirror. The telescope was modified so that a reflection of the vertical wire can be seen in the mirror, and the perpendicularity of the mirror when vertical to the collimating axis of the telescope can be secured. The instrument being oriented so that the telescope and sun are in one vertical plane, the mirror is turned until the image of the sun, viewed through suitable coloured glass, appears in the field of view, and the time of transit answering to a given azimuth reading of the circle is determined. With the sun at a convenient height, and the time and the geographical position of the station accurately known, results are obtainable of sufficient accuracy for field-work. Special arrangements, however, are necessary —for which wireless might be useful—to secure the time accuracy essential.

At some places, and at some seasons of the year, one may have to wait a long time for an astronomical observation. Thus declination observations in the field and at a fixed observatory are two very different things.

§ (2) Observation of H.—Two processes are involved: the determination of a time of swing and the measurement of deflection

angles. The former gives the product mH, the latter the ratio m/H, where m is the magnetic moment of the collimator magnet.

(i.) *The Vibration Experiment.*—Prior to the vibration experiment a plummet is suspended, and the torsion head is turned until the keel of the plummet is in its normal direction. The magnet should be suspended for some little time before the observation begins, to ensure its having the temperature of its surroundings, which is measured by a thermometer having its bulb inside the magnet box. By means of the slow motion screw the circle is turned until the central division of the magnet's scale coincides with the vertical wire in the telescope. When the magnet is set swinging, its equation of motion, neglecting air resistance, may be written

$$K\frac{d^2\phi}{dt^2}+mH\sin\phi+\tau\phi=0,$$

where K is the moment of inertia of the magnet and stirrup, m the magnetic moment of the magnet, τ the coefficient of torsion of the suspension, and ϕ the inclination at the instant of the magnet to the magnetic meridian. If the arc of vibration is small, $\sin\phi$ may be replaced by ϕ to a first approximation, and the half period of the complete vibration is given by $T=\pi\{K/(mH+\tau)\}^{\frac{1}{2}}$.

The practice of speaking of T as the time of vibration, meaning thereby the interval between successive transits in opposite directions, is so well established that serious confusion would probably result when dealing with Terrestrial Magnetism from following the usual terminology of physics. The times are usually taken with a chronometer.

From previous knowledge, or a rough preliminary observation, the observer knows the approximate time of 5 swings, say 22 seconds. Suppose he starts the observation about 10 h. 10 m. He keeps his eye on the chronometer for a short time, counting the beats so as to get his eye and ear in. After say 20 seconds he transfers his eye to the telescope counting the beats. The first transit, the scale moving from left to right, occurs say between 25 seconds and 26 seconds. The fraction 0·6 seconds, say, is estimated from the apparent positions of the scale relative to the wire in the telescope at successive full second beats. The result 10 h. 10 m. 25·6 s. is at once entered on the observing sheet. The observer then looks at the chronometer, picking up the beats as before and some 10 seconds before the expected transit, *i.e.* at about 38 seconds, transfers his eye to the telescope and takes the time of transit as before, the scale this time swinging from right to left. It is usual to observe each fifth transit up to transit 65, the times for transits 0 to 60 appearing in one column, those for transits 5 to 65 in another. Subtracting the observed times for transits 20 and 25 from the observed times for transits 60 and 65 respectively, and adding the intervals thus obtained to the observed times of transits 60 and 65, the observer finds the times at which transits 100 and 105 are to be expected. Some 10 seconds before the expected time of the 100th transit he begins to observe again after picking up the beats from the chronometer, and observes each fifth transit as before up to the 165th. The temperature of the thermometer should be read immediately before and after the observation.

The entries on the observing sheet supply fourteen independent estimates of the time of 100 vibrations, seven with scale moving to right, and seven with scale moving to left. Under normal conditions, in England, a skilled observer will find all his times agreeing to within 0·5 second, sometimes to within 0·2 second. Absolute agreement of the fourteen estimates is not unprecedented, but should be very rare. Declination is seldom invariable for ten minutes at a time, and consequently at the end of the observation the elongations on the two sides of the vertical wire are seldom equal. There are usually also progressive changes in H itself and in temperature, thus some little variability in the times is to be expected.

In high magnetic latitudes, where H is low and the time of swing long, disturbance is the rule. It may then be advisable to observe every third transit, and to reduce the number, *e.g.* to observe transits 0, 3, 6 to 39, and then transits 60 to 99, and to take the time of 60 instead of 100 swings. Even greater reduction than this in the number of swings may be expedient.

Immediately after the vibration experiment, a torsion experiment is made, exactly as in the D observation. This gives τ/mH and supplies a correction which reduces the observed time T to what it would have been if H had been the sole controlling force. A small correction is usually also necessary for "rate" in the chronometer. The size of the vibration arc desirable at the commencement of the observation depends on the damping. Times of transit are difficult to take if the scale crosses the central wire either very fast or very slow. A commencing semi-arc of 45′ ordinarily suffices; the correction to reduce to the infinitely small arc of vibration should then be negligible.

(ii.) *The Deflection Experiment.*—For the second process, the deflection experiment, the unifilar is set up as shown in *Fig.* 3. The deflecting magnet shown on the carriage on the deflection bar is the one that has just been swung. The deflected or "mirror" magnet, as it is usually called, is shown separately in *Fig.* 4. The observer views the reflection from the mirror of the small ivory scale supported above the telescope. When the centre of this scale seems to coincide with the vertical wire of the telescope, the deflecting and deflected magnets are at right angles, *i.e.* in the second position of Lamont (not Gauss, as is sometimes said). The

magnet and carriage and the thermometer should be put on the bar some minutes before readings begin. There should be a counterpoise, not shown, on the other arm of the bar. A common arrangement is to have the thermometer as part of the counterpoise, in the opposite arm to the magnet. In some patterns the magnet and thermometer bulb are within a box which serves as carriage, a somewhat heavy counterpoise being then necessary.

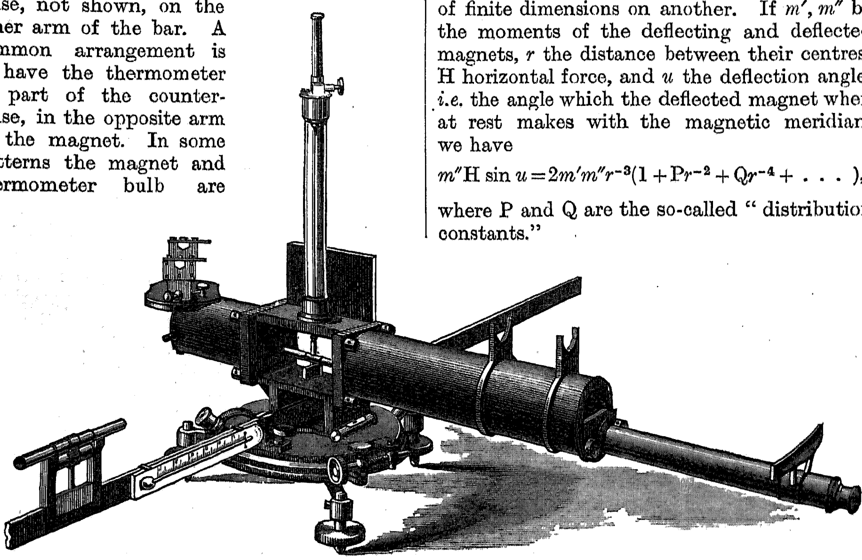


FIG 3.

Deflections are commonly taken only at two distances, usually 30 and 40 cm. Observations are made first with the carriage on one arm of the bar and then on the other, the order of operations being such as to give the same mean time for the deflections at the two distances. If D and H were invariable, and everything perfectly symmetrical, the deflection angles for the deflecting magnet at the same nominal distance on the two arms should be equal. In practice the angles usually differ, however perfect the observational conditions, the differences being sufficiently systematic to afford a useful check on the accuracy of setting of the carriage at the desired division on the scale. The reduction tacitly assumes that if u and u' be corresponding deflection angles, for a common distance on the two arms,

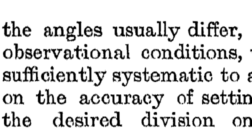


FIG. 4.

$$\sin \tfrac{1}{2}(u+u') = \tfrac{1}{2}(\sin u + \sin u').$$

The consequent limitation of $u-u'$ requires consideration when the deflection angles are large.

The necessity for more than one deflection distance arises from the complexity of the formula for the couple exerted by one magnet of finite dimensions on another. If m', m'' be the moments of the deflecting and deflected magnets, r the distance between their centres, H horizontal force, and u the deflection angle, *i.e.* the angle which the deflected magnet when at rest makes with the magnetic meridian, we have

$$m''\mathrm{H}\sin u = 2m'm''r^{-3}(1+\mathrm{P}r^{-2}+\mathrm{Q}r^{-4}+\ .\ .\ .\ ),$$

where P and Q are the so-called "distribution constants."

Supposing the magnets replaceable by equal positive and negative magnetic charges on their axes, the distance apart of these charges, or "pole-distance," being 2λ for the deflecting and $2\lambda'$ for the deflected magnet, we have

$$\mathrm{P}=2\lambda^2-3\lambda'^2,\quad \mathrm{Q}=(\tfrac{3}{8})(8\lambda^4-40\lambda^2\lambda'^2+15\lambda'^4).$$

More complete expressions involving the cross dimensions have been given by Borgen, but in magnets of ordinary shape his additional terms should be small. Taking the above expressions, we find

$$\mathrm{P}=0 \text{ when } \frac{\lambda'}{\lambda}=\sqrt{(\tfrac{2}{3})}=0{\cdot}8165,$$

$$\mathrm{Q}=0 \text{ when } \frac{\lambda'}{\lambda}=0{\cdot}4667.$$

Some authorities assert at least an approach to constancy in the ratio of the pole distance to the total length of a magnet, Borgen favouring 0·805 and Kohlrausch 0·83. Experiments at Kew Observatory have shown differences as large as 0·05 in the ratios obtained for magnets of very similar construction. If we assume the ratio of pole-distance to length the same for deflecting and deflected magnets, of respective lengths l and l', we can make P vanish by having $l'/l=0{\cdot}8165$, or we can make Q vanish by having $l'/l=0{\cdot}4667$. We cannot, however, make both P and Q vanish, and unfortunately the value of l'/l which makes Q vanish makes P large and conversely.

If we deflect at only two distances we cannot eliminate both P and Q, unless one of the two is zero. The two-distance deflections seem always reduced on the hypothesis

that P is to be eliminated, the Q term being negligible. Supposing we may neglect Q, then writing for shortness

$$\tfrac{1}{2}r_1^3 \sin u_1 = A_1, \quad \tfrac{1}{2}r_2^3 \sin u_2 = A_2,$$

where r_1, r_2 are the two deflection distances, and u_1, u_2 the two corresponding mean deflection angles, we have

$$\frac{(1+Pr_1^{-2})}{A_1} = \frac{(1+Pr_2^{-2})}{A_2} = \frac{H}{m'}.$$

Thence we easily find

$$P = (A_1 - A_2) \div (A_2 r_1^{-2} - A_1 r_2^{-2}).$$

Owing to observational errors and fluctuations in H, it is not practically possible to determine P from a single observation. In practice A_1 and A_2 in the above formula represent the mean results from a large number of observations. If, as is not unusual at a fixed observatory, P is determined from a whole year's observations, and treated as invariable for the year, fictitious discontinuities in the calculated values of H will naturally appear at the year's end. This may be largely avoided by calculating for each month a value for P based on the observations of 3, 5, or 7 months centring in that month.

The bigger r_1 and r_2 the less important are the P and Q terms. For values of H of the order 0·05 and values of m' approaching 1000, deflections may be made at distances for which Q is really negligible and the contribution from P very small. But in Britain, and still more in India, the increase of r above 40 cm. leads in general to unduly small deflection angles. When the deflection distances are 30 and 40 cm., Q is in general not negligible when l'/l is as usual of the order 0·8.

To meet this difficulty one plan has been to give l'/l the low value 0·467 for which Q is theoretically zero. The theoretical value is, however, at best only an approximation, and a very short mirror magnet is less convenient to handle. The most obvious way out of the difficulty is to deflect at three distances, *e.g.* 22·5, 30, and 40 cm., or 30, 35, and 40 cm. If suffixes 1, 2, 3 distinguish the three distances, we have three relations of the type

$$\left(\frac{m'}{H}\right)(1+Pr_1^{-2}+Qr_1^{-4}) = W_1 \equiv \tfrac{1}{2}r_1^3 \sin u_1.$$

These give us

$$P = \{W_1r_1^4(r_3^4-r_2^4)+W_2r_2^4(r_1^4-r_3^4) + W_3r_3^4(r_2^4-r_1^4)\} \div D,$$

$$Q = (r_1r_2r_3)^2\{W_1r_1^2(r_2^2-r_3^2)+W_2r_2^2(r_3^2-r_1^2) + W_3r_3^2(r_1^2-r_2^2)\} \div D,$$

where

$$D \equiv W_1r_1^4(r_2^2-r_3^2)+W_2r_2^4(r_3^2-r_1^2) + W_3r_3^4(r_1^2-r_2^2).$$

With an ordinary English magnetometer, when deflections are made at 30 cm. and 40 cm., the neglect of Q means an error of 3γ to 6γ in H in England, and twice as much in India. On the other hand, having a third distance materially lengthens the observation and the reduction work, and in some instruments the apparent fluctuations in P and Q encourage a suspicion that causes other than real changes in the magnets are at work. Until some method of greater accuracy is evolved, the use of three distances seems a necessary burden at first-class observatories. But for less important stations, where observations are not numerous, or for field work, the best course is to observe at two distances only. If occasional comparisons were made of the instruments in general use in a country with those at some one central station where three distances were used, corrections might be got out applicable to the results given by deflections at two distances only.

§ (3) Constants of the Magnetometer.—A point to be noticed is that the magnetic moments m and m' of the collimator magnet during the vibration and deflection experiments are not identical. In the first place, the magnetic moment diminishes with rise of temperature according to the formula

$$m_t = m_0(1 - qt - q't^2),$$

where the suffix refers to the temperature in the centigrade scale, and q and q' are constants. Usually the mean temperatures during the vibration and deflection experiments differ. One might correct one only of the two results for the usually small difference between the two temperatures; but in practice it is simpler to apply corrections to both results, to reduce them to what they would have been if the observation had been taken at 0° C., or 62° F., or other standard temperature. In the second place, in addition to a permanent moment which for the present purpose we may call m, the magnet has a temporary moment (positive or negative) proportional to the component of the earth's field in the direction of its length. In the vibration experiment the magnet is always practically in the magnetic meridian. In the deflection experiment when the mirror magnet suffers a displacement u out of the magnetic meridian, the collimator magnet being perpendicular to it makes an angle $\pi/2+u$ with the meridian. Thus the magnetic moment is altered from m to $m+\mu H \cos(\pi/2+u)$ or $m-\mu H \sin u$. As a first approximation $2m/r^3 = H \sin u$, thus the second moment may be written $m(1-2\mu r^{-3})$.

It is usual to assume the corrections for torsion, temperature, and induction to be small enough to be additive.

Temperature, besides altering the magnetic moment, alters the moment of inertia and likewise the length of the deflection bar. A less obvious error arises from the bending of the deflection bar due to its own weight and that of the magnet and carriage. Before numerical results can be obtained with a unifilar the temperature and induction coefficients and the moment of inertia of the collimator magnet have to be found. The

scale values applicable to the collimator and mirror magnets are also wanted. The inertia bar has to be weighed and measured. The deflection bar has to have its errors of graduation determined, and its bending observed under the weights it has actually to carry. At present in this country the constants are obtained by the co-operation of the Kew Observatory (Meteorological Office) and the National Physical Laboratory.

The most crucial " constant " is the moment of inertia. It is usually derived from observations of the time of swing of the magnet with and without an inertia bar special to the instrument. The moment of the inertia bar is usually calculated from its mass and dimensions. An alternative plan would be to employ a standard inertia bar for all magnetometers. The weak point in the first alternative is that the density of individual inertia bars may not be uniform. The second alternative has several drawbacks. The number of vibration experiments required for the satisfactory determination of the moment of inertia of a magnet is greater than can well be provided for at a testing station for a reasonable fee. Unless a special inertia bar accompanies the magnet, the observer into whose hands it comes cannot determine the moment of inertia for himself, and as the moment naturally alters with use it is certainly desirable he should be able to do this, if not at first, certainly after a period of years. An error in the accepted value of the moment of inertia means an error in the value of H which is directly proportional to H, and is thus at a fixed station practically invariable.

§ (4) Dip Circle.—The simplest form of this instrument is used for determining only the dip or inclination. Dip needles of 4 feet length were once used; then came needles of 1 foot or 10 inches; the usual length now is about $3\frac{1}{2}$ inches. *Fig.* 5 shows the usual shape of needle; but the needle No. 4 in the foreground and the frame holding it serve for the determination of total force, to be described presently. The axle of the true dipping needle, seen in the background, rolls on agate edges. A " lifter " serves to raise the axle off the knife edges and lower it again. The lifter in the instrument shown is of the " horizontal " type, which gives a greater amount of swing to the needle than the " vertical " type. With the horizontal lifter one or both ends of the needle become invisible when the angle of dip is very low. The vertical lifter on the other hand may be an obstruction when the dip is very high. The reading microscopes serve for sighting the ends of the needle. The graduation in the vertical circle is sometimes continuous from 0° (or 360°) to 350°, but usually, as in the figure, each quadrant is separately graduated from 0° to 90°. There is also a horizontal circle, usually divided from 0° to 90° in four quadrants, and a spirit-level, not shown in the figure.

After levelling the circle, the first thing is to bring the plane of oscillation of the dip

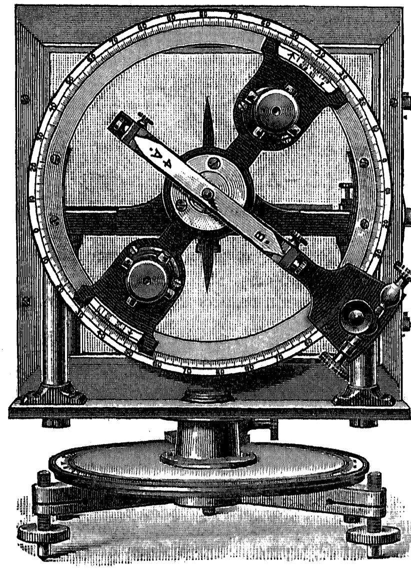


Fig. 5.—Dip Circle.

needle to coincide with the magnetic meridian. Use is made of the fact that when the plane of oscillation is perpendicular to the magnetic meridian, the needle having no horizontal force on it in its plane of motion becomes vertical. The verniers on the vertical circle are set to 90°, and the circle is turned in azimuth until the needle swings equally on the two sides of the wire in the field of view of the microscope. The observation is made independently for the upper and lower microscopes, and with the instrument facing both to north and to south. The mean of the four readings of the azimuth circle is accepted as giving the plane perpendicular to the magnetic meridian, and the direction perpendicular to this is the magnetic meridian required. The horizontal circle is usually fitted with stops, and once these are suitably fixed the meridian can be recovered at once, whether the circle is facing east or west.

If observations be made in a plane inclined to the magnetic meridian at an angle a, the relation between the true dip i and the observed dip i' is

$$\tan i = \cos a \tan i'.$$

If a is small, we obtain as a first approximation

$$i' - i = \left(\frac{a^2}{4}\right) \sin 2i.$$

The error in the dip due to observing out of the magnetic meridian thus varies as the square of the error in the setting, and for a given error of setting is greatest where the true dip is 45°. If we suppose $i=67°$—an approximate value at present for London—for an error of 1° in setting we find $i'-i=11''$. As the probable error in a dip observation is not less than 0·5′, accuracy of 0·25°, or even 0·5°, in the setting suffices. It is thus customary at a fixed observatory to determine the meridian afresh only occasionally.

For an ordinary dip observation two needles are usually employed, say Nos. 1 and 2; one end of each needle is marked A, the other B. Before observing, the axles of the needles and the agate edges are cleaned with pith. A complete scheme of observations is as follows:

Needle No.	Circle Facing.	Side of Needle next Observer.
	End A of Needle Dipping	
1	East	Face
1	West	Face
1	West	Back
1	East	Back
2	East	Face
2	West	Face
2	West	Back
2	East	Back
	End B of Needle Dipping	
2	East	Face
2	West	Face
2	West	Back
2	East	Back
1	East	Face
1	West	Face
1	West	Back
1	East	Back

The needles are freshly magnetised—usually by stroking with bar magnets—before each half of the observation.

In any one position the procedure is as follows: By means of the lifter get the needle to swing through a suitable angle. Using the slow motion screw, bisect the angle by the wire in the eyepiece of the lower microscope and read the lower vernier; then bisect the angle for the upper microscope and read the upper vernier. Raise the needle again and get a fresh swing and repeat the settings and readings, first with the upper microscope then with the lower. A complete observation set out as above entails 64 readings. The whole operation, including the two strokings of the magnets, need not take more than 45 minutes. The time required depends much on the skill of the observer. It tends to increase in damp weather when repeated cleaning of the axles and agates becomes necessary.

The necessity for reversing the needles' magnetism arises from the fact that the centre of gravity does not coincide, except by rare accident, with the centre of the axle. When the centre of gravity is below the centre of the axle, gravity tends to increase the dip, the reverse happening when the centre of gravity is above that of the axle. By taking a mean of the dips with poles A and B dipping, gravity is eliminated if we may treat the needle as a rigid body. Reading both ends of the needle eliminates any error of centring.

If observations of dip be taken in two orthogonal planes inclined to the magnetic meridian at angles α and $90°-\alpha$, and if i_1 and i_2 be the observed dips in the two planes, and i the true dip at the place, then we have

$$\cot i_1=\cos\alpha\cot i,\quad \cot i_2=\sin\alpha\cot i,$$

whence $$\cot^2 i=\cot^2 i_1+\cot^2 i_2.$$

Thus the true dip can be found from extra-meridian observations without determining the meridian. This method might have advantages near the magnetic poles, where the meridian is difficult to determine accurately.

Various forms of dip circle have been devised for use at sea. The most successful of these, the invention of the late Capt. E. W. Creak, F.R.S., and known as the "Lloyd-Creak," closely resembles the ordinary land circle. The chief difference is that there are no agate edges, but pivots in which the ends of the axle rest. To surmount friction a "scratcher" is provided, having a roughened surface, which is drawn to and fro over a projecting knob attached to the framework of the circle.

§ (5) Total Force Observation.—The value of the resultant or total magnetic force at the spot can be obtained with a dip circle after the method devised by Dr. Humphrey Lloyd of Dublin. It applies equally to the ordinary and to the Lloyd-Creak circle. A small contrivance seen in *Fig.* 5 holds a needle—usually called No. 4—with its blade parallel to the vertical plane in which an ordinary needle—usually No. 3—swings. The axle of No. 4 is at the centre of the vertical circle and level with that of No. 3. The line joining the ends of No. 4 is perpendicular to the line joining the centres of the two reading microscopes. Thus when the ends of No. 3 appear central in the microscopes the magnetic axes of the two needles, if coincident with their axes of figure, must be orthogonal.

Fig. 6 shows the two positions A_1B_1, A_2B_2 of the deflecting needle No. 4, and a_1b_1, a_2b_2 the corresponding positions of the deflected needle No. 3. The letters A, a denote north, the letters B, b south poles; OH is horizontal and OV vertical. The dotted line midway between a_1b_1 and a_2b_2 shows the natural dip, *i.e.* the position No. 3 would assume in the absence of No. 4. The deflection angle $\frac{1}{2}a_1Oa_2$ is denoted by u'. If the natural dip is low, or No. 4 very strong, Oa_2 may come

above OH; while if the natural dip is high Oa_1 may fall on the other side of OV.

The second part of the experiment consists in observing the dip of No. 4 when carrying a

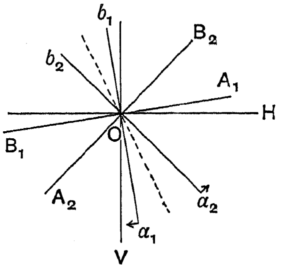


FIG. 6.

small weight, which screws into a small hole in the blade. Let η denote this dip and let

$$i-\eta=u,$$

where i is the true dip as given by ordinary dip needles. To simplify matters we shall suppose that No. 3 and No. 4 when unweighted have their centres of gravity in their axles, and that the magnetic moment m_4 of No. 4 is a constant. Let w be the auxiliary weight, c its distance from the axis of No. 4, m_3 the magnetic moment of No. 3, and R the total force, then we have

$$m_4\mathrm{R}\sin u=wc\cos\eta,$$

$$m_3\mathrm{R}\sin u'=m_3m_4\mathrm{F},$$

where F represents the couple which a needle of unit moment in the position of No. 4 would exert on a needle of unit moment in the position of No. 3.

Multiplying the two equations we get

$$\mathrm{R}^2\sin u\sin u'=wc\mathrm{F}\cos\eta.$$

Distinguishing the corresponding quantities at a base station by the suffix o, we have

$$\mathrm{R}_0{}^2\sin u_0\sin u_0'=wc\mathrm{F}\cos\eta_0.$$

Eliminating $wc\mathrm{F}$, we get

$$\mathrm{R}^2=\frac{\mathrm{A}\cos\eta}{\sin u\sin u'},$$

where the constant

$$\mathrm{A}\equiv\frac{\mathrm{R}_0{}^2\sin u_0\sin u_0'}{\cos\eta_0}$$

is determined from the observations at the base station, where the value of R_0 is known.

An assumption tacitly made is that F is the same for the field and base stations, which implies that the law of distribution of the magnetism in Nos. 3 and 4 should remain unchanged. To increase the likelihood of this, needles 3 and 4, after being magnetised at the base station, do not have their magnetisation reversed.

It is also desirable that the base station should not be in a very different latitude from the area being surveyed, and that base value determinations should immediately precede and follow those made in the field.

Several weights are usually supplied, each with a separate constant.

A recent form of holder employed by the Carnegie Institution of Washington gives a choice of two distances between needles 3 and 4 which ensures a wider range of usefulness. By making provision for a compass needle on the top of the box, and a sighting tube or telescope on the frame, the Carnegie Institution have also adapted the dip circle to serve as a declination instrument. These additions make the dip circle a useful instrument for rough survey work, when it is essential to cut down transport. But it cannot replace the ordinary magnetometer when high accuracy is desired.

The dip circle suffers from a small defect detected by the Hon. Henry Cavendish, but not generally known until rediscovered a century later by Sir A. Schuster.[1]

A needle is not a rigid body, and the elastic bending of the needle due to gravity gives a displacement to the centre of gravity which tends in all cases to reduce the observed dip. For needles of the same shape, the error varies as the square of the linear dimensions, and for a given needle is a maximum where the dip is 45°. For the large needles of Cavendish's time the error might amount to several minutes, but for modern $3\frac{1}{2}$-inch dip needles the error, according to Sir A. Schuster's figures, is only of the order 0·05′.

§ (6) MAGNETOGRAPHS.—The object of a magnetograph is to give a continuous record of magnetic changes. For complete information three instruments are required. The usual elements recorded are declination (D), horizontal force (H), and vertical force (V); but in place of D and H two rectangular components of horizontal force are sometimes substituted. *Fig.* 7 shows the disposition of the Kew pattern magnetograph. The glass covers 1, 2, 3 enclose respectively the D magnet, with a unifilar suspension, the H magnet with a bifilar suspension, and the V magnet with a knife edge resting on an agate plane. *Fig.* 8 shows the suspension of the D magnet. The open frame inside which is the magnet—a rectangular parallelepiped $14\times2\times0{\cdot}3$ cm. approximately—is of copper and serves as a damper. Rigidly attached to the magnet is a semicircular mirror, and immediately below this in the figure is another similar mirror, which is fixed. As *Fig.* 7 shows, there is a separate source of light for each magnet. This illuminates a slit in front of a collimator, and the light focussed by the collimator falls on the two semicircular

[1] *Phil. Mag.*, March 1891, p. 275.

mirrors. The reflected light passes down the long tubes leading to the central box 4, which contains three drums—one for each element—on which is wound the photographic paper. These drums are driven by a single clock. Before reaching the paper the light passes through a hemi-cylindrical lens, which condenses the light reflected from each mirror to a sharp dot. Each sheet when developed shows two lines, one straight answering to the fixed mirror, the other curved answering to the mirror attached to the magnet. The distance between corresponding points in the two lines varies as the angle between the two mirrors, and so measures the changes of direction of the magnet. Every second hour—in some patterns every hour—the clockwork interposes a shutter which cuts off the light either from the fixed or the movable mirror for a short time. The breaks thus caused in the trace enable time to be measured with high accuracy.

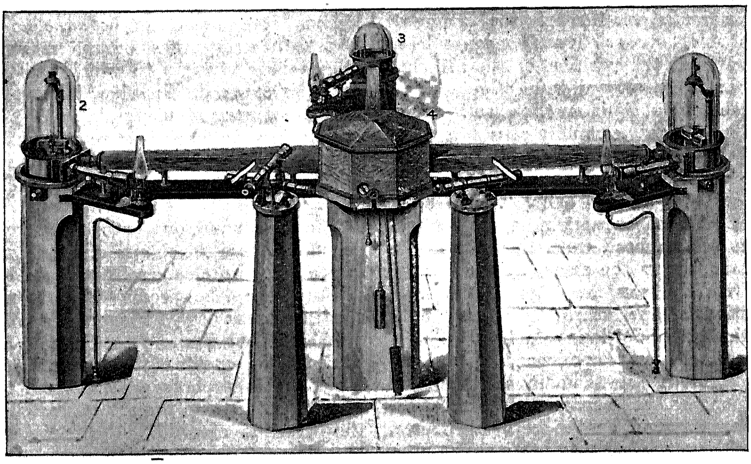


FIG. 7.—Kew Self-recording Magnetometers.

The suspension of the D magnet may be a single fine wire or a bundle of silk fibres. There is a torsion head enabling a torsion coefficient to be found, but in the ordinary Kew instrument this is negligible. Torsion can be removed by means of a plummet. When torsion is negligible—which should never be assumed without trial—the scale value depends only on the distance between the mirror and the photographic paper, the condensing lens being replaced by the equivalent air thickness. Thus the scale value should remain practically invariable when the instrument is regarded as a measurer of angular change. Most Kew pattern instruments have a scale value of about 1 mm. $=1\cdot15'$—though in the instrument used at Kew itself 1 mm. $=0\cdot87'$. A common scale value in other patterns is 1 mm. $=1'$ approximately.

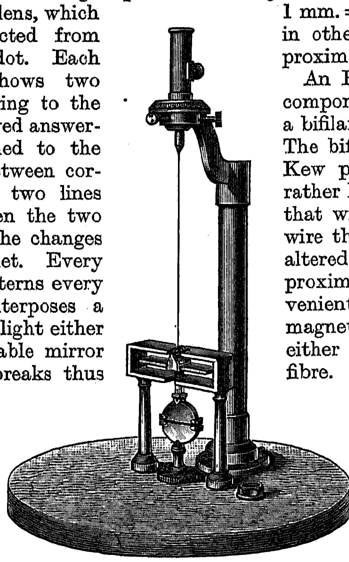


FIG. 8.—Suspended Magnet.

An H (or north (N), or west (W) component) magnetograph may have a bifilar or a stiff unifilar suspension. The bifilar suspension is in use in the Kew pattern where the magnet is rather heavy. It has the advantage that without altering the suspending wire the sensitiveness can be readily altered, and can be kept very approximately at any prearranged convenient value. In patterns with light magnets a unifilar suspension is usual, either a fine wire or a stiffish quartz fibre. In this case a prearranged scale value is difficult to secure, unless one uses an auxiliary magnet to alter the natural field of force, a procedure not without risk.

If changes in N and W are to be recorded, the two magnets should be respectively east-west and north-south when in their normal positions. These directions being fixed, all that is necessary is to alter the torsion head from time to time to neutralise the effect of any secular change either in the earth's field or in the magnetic moment of the magnetograph magnet. No change is necessary in the position of the mirror relative to the

magnet. If the reflected light fell originally on the photographic paper it should continue to do so. The D and H magnets, however, should be respectively in and perpendicular to the normal position of the magnetic meridian at the time. The D magnet looks after itself, but the position of the attached mirror has to be altered from time to time to keep the dot of light on the sheet. When declination changes as rapidly as of late years in England—nearly 10′ per annum—the position of the H magnet requires adjustment every few years. Having calculated the length of ordinate answering, say to 30′, one turns the torsion head until the spot moves this amount across the sheet. If this brings the dot unduly near the edge of the sheet the position of the mirror is altered until the position of the dot is satisfactory.

The vertical force experienced by a horizontal magnet is the same in all azimuths. Thus as long as the V magnet does not tend to get slewed round by the horizontal force, it does not matter what azimuth it is in. For the northern hemisphere the centre of gravity of the magnet and its attachments is a little on the south of the knife edge, the gravitational couple balancing the magnetic couple. The rough adjustment is usually made with a weight sliding on the magnet, and the fine adjustment by a horizontal screw carried by the magnet. A fine vertical screw serves to secure the sensitiveness desired by altering the height of the centre of gravity. Rise of temperature increases the gravitational couple by increasing the horizontal distance between the centre of gravity and the knife edge, and diminishes the magnetic couple by lowering the magnetic moment of the magnet. The two effects conspire, and a temperature compensation is called for. It usually takes the form of a horizontal strip of zinc parallel to the magnet, which projects from the framework to which the knife edge is attached, on the same side as the naturally dipping pole. On the zinc is a sliding weight. The larger expansion of the zinc as compared with steel supplies a compensation which can be adjusted by altering the position of the weight. Another method of securing temperature compensation is to use an auxiliary magnet.

The H magnet has naturally also a temperature coefficient. A rise of temperature diminishes the magnetic couple. The effect on the torsional couple depends on how it is produced. With a bifilar suspension the resultant temperature coefficient is usually small, but not negligible. With a quartz fibre suspension it may be considerable. Temperature coefficients are easily determined at a station where another magnetograph exists. The magnetograph whose coefficients are desired is exposed to large changes of temperature, and their effect can be ascertained by comparing the traces from the two instruments. If the temperature coefficients are known, corrections can be supplied if there is a satisfactory thermograph record of the temperature changes to which the magnetograph is exposed. In view, however, of the various sources of uncertainty, it is a great convenience to have the magnetograph in a chamber whose temperature is constant, or which has at least a very small diurnal variation.

The scale value of a force magnetograph is calculated from the change of ordinate answering to a known change of force. This change of force may be produced by a measured electric current or by a magnet. A convenient method is that due to J. A. Broun. A hollow circle fits the outside of the cylinders containing the magnetograph magnets. From the ends of a diameter there project graduated arms in the direction of radii. A carriage can slide on either arm, supporting the deflecting magnet at the level of the deflected. In the case of the H magnet the arms are in the magnetic meridian, and the deflecting magnet parallel to the arm, and so perpendicular to the H magnet. The centre of the deflecting magnet being at a fixed distance from the centre of the circle, suppose the mean change of ordinate when the magnet is turned end for end, first on the north then on the south arm, to be n mm. Transfer the circle to the cylinder containing the D magnet, so that the arms are perpendicular to the magnetic meridian, and repeat the deflection experiment on the D magnet, the distance of the deflecting magnet being the same as before. Suppose the observed change of ordinate to represent n' minutes. As the D and H magnets were at the same distance from the deflecting magnet, and similarly oriented with respect to it, and as the turning the deflecting magnet end for end eliminates any effect due to temporary induction, the changes of force were the same in the two cases. These changes were:

in the case of D, $H \times \cdot 000291 n'$,
in the case of H, $n S_h$,

where S_h is the scale value desired. Thus

$$S_h = \left(\frac{n'}{n}\right) \times \cdot 000291 H,$$

where H, the local value of the horizontal force, is supposed known. It is assumed that any contribution from the distribution constants P and Q is either negligible or the same in the two cases. As the distance of deflection is large—usually 70 to 90 mm.—and the D and H magnets are of the same size and shape, the assumption is fairly justified.

In the V scale-value experiment the deflecting magnet is vertical when deflecting the V magnet, the deflection bar being parallel to that magnet. In the complementary deflection of the D magnet the deflection bar is in the magnetic meridian, but the deflecting magnet perpendicular to the bar. The calculation of the scale value proceeds on the same lines as above.

The scale value of a force magnetograph may depend on more than one thing. Suppose the normal position of a magnetograph magnet to be inclined to the magnetic meridian at an angle θ; and suppose that the deflection from the natural position in the magnetic meridian has been produced by turning a torsion head through an angle ϕ. The torsion angle is then $\phi-\theta$, and the torsion couple $\tau(\phi-\theta)$, where τ is a constant. This couple must balance the magnetic forces tending to restore the magnet to the magnetic meridian. Thus we have

$$\tau(\phi-\theta)=m\mathrm{H}\sin\theta,$$

where m is the moment of the magnet, H the horizontal force. Suppose we apply a small deflecting force $\Delta\mathrm{F}$ perpendicular to the magnet, and that θ alters to $\theta+\Delta\theta$. Then we must have

$$\tau(\phi-\theta-\Delta\theta)=m\{\mathrm{H}\sin(\theta+\Delta\theta)+\Delta\mathrm{F}\}.$$

Supposing $\Delta\theta^2$ negligible, we thence deduce

$$\Delta\mathrm{F}=-\Delta\theta\left(\mathrm{H}\cos\theta+\frac{\tau}{m}\right)$$
$$=-\Delta\theta\left\{\mathrm{H}\cos\theta+\frac{\mathrm{H}\sin\theta}{(\phi-\theta)}\right\}.$$

If $\theta=0$, then we are using the instrument to measure changes of force perpendicular to the magnetic meridian (and so changes of declination); if $\theta=\pi/2$, we are measuring changes of H. Thus if S_d and S_h represent the scale values of this instrument when used to measure D (as a force) and H, and S is the scale value when the instrument is used to measure changes in that component of the horizontal force which is inclined to the magnetic meridian at the angle $(\pi/2)-\theta$, we have

$$\mathrm{S}=\mathrm{S}_d\cos\theta+\mathrm{S}_h\sin\theta.$$

Also, it should be noticed, $\mathrm{S}_d\propto\mathrm{H}$, and $\mathrm{S}_h\propto\mathrm{H}/(\phi-\theta)$. The smaller the scale value, the more sensitive the instrument. The sensitiveness of a unifilar arrangement measuring D changes as forces is greater the less the value of H. One can thus increase the sensitiveness by means of an auxiliary magnet placed so as to reduce the earth's field. A unifilar arrangement measuring any changes except those in D has its sensitiveness larger the larger the value of $\phi-\theta$, and so the larger ϕ. One can get a very big value of ϕ by using a suspension so fine that the torsion head has to be turned through a large multiple of 2π to bring the magnet to the desired position. This device[1] has been used by Mr. F. E. Smith, F.R.S., for showing small changes of force in a conspicuous way.

The late Professor W. Watson, F.R.S., designed magnetographs in which each magnet system consisted of several parallel magnets, somewhat after the fashion of the Kelvin compass. He also devised a vertical force magnetograph in which the magnet is carried by a horizontal quartz fibre. Changes of vertical force tend to twist or untwist the fibre, and so alter the direction of a beam of light reflected from an attached mirror. Similar devices have been introduced independently by Professors Tanakadate and Krogness.

In some instruments the changes in the inclination to the horizon of the vertical force magnet are converted into motion of the beam of light in a horizontal plane by means of a prism or a second mirror. This is the case, for instance, in the Eschenhagen instrument which has become common of late years. That instrument employs only a single drum, changes of H, D, and V being recorded on a single sheet. It is usual in this type to have two mirrors carried by one of the magnets, usually the H magnet. The inclination of these two mirrors is such that if a large disturbance tends to throw the spot of light from one of the mirrors off the sheet, the light from the second mirror comes on. This, or some analogous arrangement, is almost a necessary device where high sensitiveness is adopted, unless two independent magnetographs are used, one with high, the other with low sensitiveness.

C. C.

MAGNETISM, TERRESTRIAL AND SOLAR, THEORIES OF

I. The Earth's General Magnetic Field, and Solar Magnetism

§ (1) The General Magnetic Field of the Earth.—The earth's general magnetic field is directly measurable only in the immediate neighbourhood of the ground. General inspection of such surface measurements suffices to show that at least the greater part of the field has its origin within the earth. This conclusion is rendered more precise by spherical harmonic analysis of the surface field, as first applied by Gauss.[2] Modern analyses, such as that by Schmidt,[3] limit the portion of the surface field which can be of external origin

[1] See "Magnetism, Terrestrial, Electromagnetic Methods of Measuring."

[2] Gauss, "Allgemeine Theorie des Erdmagnetismus," *Werke*, v. 121.

[3] Schmidt, *Archiv d. Deutschen Seewarte*, 1898, xxi.

to less than 1 per cent, the chief part of this being a uniform field of intensity 0·003Γ or 300γ (1γ = 10^{-5}Γ, Γ denoting the C.G.S. unit); even this small fraction may arise mainly from errors in the magnetic data analysed. A small portion of the field seems on analysis not to have a potential; this may be likewise due to inadequacy of the data, though some part of it may be produced by stationary currents of electricity passing from the earth to the ground or *vice versa*.

Since there is no appreciably magnetic matter above the earth's surface, any part of the field which is of external origin must be due to electric currents, the precise situation of which is unknown; apart from their small contribution, the value of the field at any point above the earth can be calculated from the spherical harmonic analysis of the surface field. This cannot be done for the field inside the earth, because the situation of the internal seat of magnetisation is unknown.

The magnetic field at the earth's surface agrees roughly with the value it would have if the earth were a uniformly magnetised sphere (a powerful small bar magnet at the earth's centre would have the same external field, though as a physical conception of the true cause the idea can be dismissed). In such a sphere the lines of magnetic induction in the interior run parallel to the magnetic axis, completing their circuits along curved paths outside the sphere. The magnetic potential at a point outside the sphere is $H_0a^3 \cos\theta/r^2$, and $H_0 r \cos\theta$ within it, r denoting the distance of the point from the centre, θ the angular north polar distance, a the earth's radius, and H_0 the surface magnetic intensity at the equator. At the equator the magnetic force is horizontal, while at the poles it is vertical, and of intensity $2H_0$. The "dip" of the lines of magnetic force, *i.e.* their inclination ϕ to the horizontal, is given by $\tan\phi = 2\cot\theta$, and is downwards in the northern, and upwards in the southern hemisphere. These relations are approximately fulfilled in the case of the earth's field, relative to the diameter having its poles at 78° N., 68° W., and 78° S., 112° E. as magnetic axis. Since the earth's field is subject to a "secular variation" (§ (2)), slow but effecting great changes in the course of time, it is necessary to specify the date to which these figures refer; they are for the epoch 1885, being the mean of the results derived by Adams [1] and Petersen [2]; the value found for H_0 was 0·3227Γ, which corresponds to an intensity of magnetisation 0·08 C.G.S. for a uniformly magnetised sphere, equal in size to the earth and having the same surface field.

[1] Adams, *Brit. Assoc. Rep.*, Bristol, 1898.
[2] Berghaus' *Physical Atlas*, Gotha, 1892, section by Neumayer.

The complete expression for the earth's magnetic potential contains many other terms besides the one just described, though these are much less important. They represent deviations from the field of uniform magnetisation, which range without any distinct break from regional inequalities affecting many hundred thousands of square miles (as in the Siberian region of abnormal magnetic intensity) down to the most local irregularities. In Northern Siberia the deviation of the magnetic intensity from the value corresponding to the above field $H_0a^3 \cos\theta/r^2$ is of the order 10 per cent.

The more local the deviations, on the other hand, the smaller, in general, is their magnitude, and the more rapidly do they diminish with increasing distance from the earth: for the degree n of the harmonic terms by which they are expressed must be high in proportion to the smallness of the region affected, while the intensity of the corresponding component of the field varies with the distance r according to the law $r^{-(n+1)}$ Thus as r increases, the importance of the higher harmonics steadily diminishes relatively to that of the leading term, for which n is 1; at a distance of a few diameters the earth's field must approximate still more closely to the field of uniform magnetisation—a point which may be of some importance in the theory of aurorae (§ (21)). Except in the neighbourhood of strongly magnetic mineral deposits, however, the variation of the magnetic intensity with height above the earth is too small to be measured by present methods.

Partly because of the inclination of the magnetic to the geographical axis, and partly owing to the small deviations from the field of uniform magnetisation, the horizontal component of magnetic force is at most places not directed along the geographical meridian; its inclination to the latter is termed the magnetic declination. The surface magnetic field at any point is usually determined by measuring the horizontal force (H.F.), the declination, and the dip; these are known as the magnetic elements at the place. For some purposes it is convenient to use the three equivalent elements consisting of the north, west, and vertical components of force, generally denoted by X, Y, Z.

The regional deviations of the field, and particularly the irregularities in the horizontal force, cause the poles of magnetic dip, where the horizontal component vanishes and the magnetic needle becomes vertical, to differ from the poles of the magnetic axis. They occur [3] at 73° N., 102° W., and at 73° S., 156° E., and are thus not even at opposite

[3] Cf. Schmidt, art. "Erdmagnetismus," *Encyk. d. math. Wiss.*, vi. 1. 10. 359, and references there given.

ends of a diameter; the line joining them misses the earth's centre by about 700 miles.

The part of the earth's field which is symmetrical about the axis of rotation consists mainly of the resolved component of the field of uniform magnetisation; Bauer [1] has found that the residual portion can be represented, to within 1 per cent of the whole symmetrical portion, by a single further harmonic term (P_3).

The whole magnetic field rotates with the earth; otherwise, for instance, the poles of magnetic dip would circle round the axis of rotation once each day.

§ (2) THE SECULAR VARIATION OF TERRESTRIAL MAGNETISM.—The known facts regarding the secular variation of the earth's magnetic field are and must long remain inadequate for a proper understanding of this variation. Trustworthy observations of the direction of the magnetic force extend back for about 400 years at London and a few other European stations; measurements of the intensity of the field cover a period of less than a century. Bauer [2] has shown that the direction of the force at any station describes a conical path in the clockwise direction; for London this cone is roughly circular, with an angular radius of about 5°; more than three-quarters of the circuit has been completed, but it is impossible to say whether the curve will close and repeat itself, or what the future course of the variation will be. So far as it is possible to speak of a period, the time suggested by the curve is about 480 years; but it is uncertain whether this is a world-wide or only a local period. At London the total observed range in declination is about 35°, and in dip approximately 10°.

The rate of secular variation changes slowly and somewhat irregularly; at Greenwich, for instance, in declination the annual change was 5′·6 from 1895 to 1900, and increased during the years 1900–1910 to 9′ over the period 1910 to 1915. Over regions as large as England, however, its value at any one time is very approximately a linear function of position, even over districts where there is local magnetic disturbance.[3] The changes in the resolved (horizontal and vertical) components of the total force are much greater than those in this force itself, and are due mainly to the change in direction of the total force. The mean annual change in horizontal force at Greenwich during the years 1865–1895 was 22γ increasing; since 1911 the horizontal force has been decreasing.

The secular variation affects both the main portion, *i.e.* the field of uniform magnetisation, and the other harmonic components, which represent the deviations from this field. The change in the direction of the magnetic axis has been investigated by van Bemmelen,[4] using principally the observations since 1600; during this period the north pole of the axis seems to have described a slightly curved path, neither circular nor centred at the geographical pole, but mainly southwards till about 1850 (descending from latitude 85° to about 77½°), and subsequently slightly westwards and northwards. It has thus never been known to be appreciably more distant from the geographical pole than now.

The secular variation has been discussed by Carlheim-Gyllensköld [5] by the use of spherical harmonic analysis; he concluded that the principal non-zonal harmonics retain approximately constant amplitudes, but precess uniformly from east to west round the geographical pole. This does not necessarily conflict with van Bemmelen's determination of the motion of the magnetic axis. The period of precession for the transverse portion of the field of uniform magnetisation, according to Carlheim-Gyllensköld, is about 3000 years, in so far agreeing with van Bemmelen as indicating that the westward motion of the pole of the magnetic axis is very slow. The periods found for the tesseral harmonics, P_2^1, P_2^2, were 1380 and 450 years, in round numbers; the somewhat rapid precession of the latter term seems to correspond to the secular variation of declination observed in Europe.

§ (3) THE GENERAL MAGNETIC FIELD OF THE SUN.—By the study of the Zeeman effect on the lines of the solar spectrum from various parts of the sun's surface Hale and his collaborators [6] have shown that the sun possesses a general magnetic field, in some respects similar to that of the earth, in others very dissimilar. Its intensity can be observed at different levels within a certain range in the sun's atmosphere, by considering spectral lines known, from other researches, to originate at different ascertainable levels. The lowest layer at which measurements have been made is at a height of 250 kilometres, while above 450 kilometres the intensity of the field sinks to the order of 10Γ, the limit below which the Zeeman effect ceases to be measurable.

The field at any given level seems to correspond roughly with that of a uniformly magnetised sphere, though since the measures relate to the component of magnetic force along the line of sight, the distribution of the direction and intensity of the field in different latitudes has to be partly inferred.

[1] Bauer, *Terr. Mag.*, 1912, xvii. 115.
[2] Bauer, *In.-Diss.*, Berlin, 1895.
[3] Walker and Cox, *Roy. Soc. Phil. Trans.* A, 1919, 218.
[4] van Bemmelen, *Batavia Obs.*, 1899, xxii. App. I.; *Terr. Mag.*, 1907, xii. 27.
[5] Gyllensköld, *Stockholm Obs. Publ.*, 1896, v., or *Met. Zeit.*, 1897, xiv. 39.
[6] Hale, *Astrophysical Journal*, 1913, xxxviii. 31; Hale, Seares, van Maanen, and Ellerman, *ib.*, 1918, xlvii. 1; also *ib.*, 1913, xxxviii.

Attempts to detect deviations of the field from one of uniform magnetisation have been made, and appeared to indicate that at a given level the intensity at the equator is about twice as great as would correspond to the intensity at the poles. The investigators have since suggested, however, that this may not be the real significance of their measures, owing to the possibility of certain systematic errors.

The most striking deviation of the field from that of a uniformly magnetised sphere is found in the rapid radial diminution of its intensity, which falls from a value 50Γ at 250 kilometres to 10Γ at about 450 kilometres height in the solar atmosphere; these values refer to the polar intensity, assuming that the distribution of intensity at any radius agrees with that of a uniform magnetisation. A field of the latter type should diminish in intensity with increasing radial distance r according to the law r^{-3}, and r is of the order $6 \cdot 10^{10}$ cm. at the sun's surface; this corresponds to a reduction of 0·1 per cent in the field intensity between the given levels, instead of a diminution in the ratio 5 : 1. Laplace's equation cannot hold good in this layer, so that magnetising (or demagnetising) agents must exist there.

The direction of the sun's magnetic field is related to its direction of rotation in the same way as in the case of the earth; the magnetic axis of the sun is likewise oblique to the axis of rotation, though by a smaller amount (about 5°). This axis is not stationary relative to the sun, but exhibits a rapid secular variation, consisting of a precession round the axis of rotation, in the westward direction (*i.e.* opposite to the direction of rotation). Viewed from the earth, the magnetic axis rotates once in 31·4 days, while the sun itself rotates in 27·3 days, the period being slightly different at different latitudes on the sun. Allowing for the earth's orbital motion, this represents a precession of the magnetic axis relative to the sun's surface in a period of nearly eight months.

§ (4) THE MAGNETIC FIELDS OF SUNSPOTS.—The magnetic fields first discovered on the sun were local fields associated with sunspots.[1] The sun is unlike the earth in this respect also, that these local variable fields are much more intense than the general field, intensities of 3000Γ being not uncommonly observed in sunspots.

Sunspots, whether large or small, tend to occur in pairs, the members of a pair being of opposite magnetic polarity. The line of centres of sunspot-pairs is usually nearly parallel with the sun's equator. The spots rotate with the sun, and in any pair the member in advance has a polarity which is usually of definite sign, this being dependent on the epoch and the hemisphere in which the spot occurs. The sign is opposite on the two sides of the solar equator, and the entire system of polarities seems to suffer reversal at sunspot minimum epoch. This, at any rate, was observed at the first (hitherto the only) minimum epoch since the discovery of these magnetic fields. No return reversal occurred at the subsequent sunspot maximum. Before the minimum epoch of 1912 the advance members of binary spots in either hemisphere were of the same polarity as the pole of the general magnetic field in the same hemisphere; since then the reverse relation has existed. Unipolar spots, comprising less than 40 per cent of the whole, have the same sign as the advance member of a binary spot in similar circumstances; they usually exhibit some of the characteristics of binary spots.

The strength of the field in a sunspot diminishes with increasing elevation in the solar atmosphere, but much less rapidly than in the case of the general magnetic field of the sun.

II. THEORIES CONCERNING THE EARTH'S GENERAL MAGNETIC FIELD AND SOLAR MAGNETISM.

§ (5) PERMANENT MAGNETISM.—Perhaps the simplest hypothesis that can be proposed to account for the earth's field is that the earth is largely composed of iron or other ferromagnetic material, permanently magnetised, nearly uniformly, along an oblique axis.

The intensity of the surface magnetic field of a uniformly magnetised sphere does not depend on its size, for a given intensity of magnetisation I: for the magnetic moment M is $\frac{4}{3}\pi a^3 \mathrm{I}$ (a being the radius), while the surface magnetic intensity varies as M/a^3 (which is the actual value of H_0 at the equator), and is therefore independent of a. The observed value of H_0 (§ (1)) corresponds to $\mathrm{I}=0 \cdot 08$ C.G.S. Schuster[2] estimates that the maximum value of I which can be imparted to a sphere is about 100, so that a comparatively small fraction of the earth's volume, magnetised strongly, or a large portion, magnetised to one-thousandth of the full intensity, would account for the observed field.

Under ordinary conditions iron loses its capacity for magnetisation at temperatures which are reached at a depth of about 20 kilometres under the land surface of the globe, supposing the rate of increase of temperature with depth to be about 1° C. for every 40 metres; magnetite ceases to be

[1] Hale, *Astrophysical Journal*, 1908, xxviii. 363; Hale, Ellerman, Nicholson, Joy, *ib.*, 1919, xlix. 365.

[2] Schuster, *Phys. Soc. Proc.*, 1912, xxiv. 127.

magnetic at a lower temperature, reached at about 15 kilometres. If there can be no permanent magnetism below such depths, this hypothesis requires the existence of a thin shell highly magnetised and quite near the surface of the earth. The secular variation would then be due to changes in the direction of magnetisation of this shell; the forces producing the changes must be great, since the material must be retentive in order to preserve its magnetisation. Again, the hypothesis of permanent magnetisation has to assume as a coincidence the close approximation of the magnetic to the geographic axis. The required degree of magnetisation of the thin shell is about half the maximum possible. No satisfactory explanation of the supposed magnetisation has been offered. The supposition that it can be due to magnetic induction by outside forces still acting has been considered, but is untenable; less than 1 per cent of the surface magnetic field comes from outside the earth, and at most 1 or 2 per cent of the internal field could be due to the effect of this on the core or shell; moreover, whatever the permeability, a magnetic sphere placed in a uniform field would not exhibit a field of the actually observed type—the primary field at the equator could never be reversed, while at the poles even an infinite permeability would only treble the primary field.

Some of these difficulties would be lessened if the critical temperature at which iron loses its retentivity were raised by increase of pressure. Experiments designed to test this possibility have proved inconclusive, though on the whole tending to show that, on the contrary, increased pressure actually diminishes the critical temperature.[1]

While the hypothesis of permanent magnetisation is, therefore, improbable even as applied to the earth, it is much more so as applied to the sun, because of the far higher temperature, and also on other accounts.

§ (6) Galvanic Currents.—Gilbert's theory of the earth's magnetism [2] was of the form just discussed, other possibilities being then unknown. Halley [3] endeavoured to account for the larger irregularities of the field, and to explain the secular variation by a theory on the same basis, which, while not physically admissible, may be briefly mentioned. He concluded that the earth has two poles, or points of attraction, in each hemisphere, and suggested that it consists of a solid globe surrounded by an outer concentric shell, both uniformly magnetised, in different directions, the two rotating independently, with slightly different speeds. Hansteen [4] revived part of Halley's theory a century later by assuming that there are inside the earth two bar magnets, not small compared with the earth's radius. This view is without either physical probability or analytical advantage.

Instead of being permanently magnetised, the earth, it may be suggested, is an electromagnet, its field being produced by ordinary galvanic currents flowing within it. Schuster [5] has pointed out the objections to various forms of this theory. It cannot be supposed that the currents are a survival from some past order, because their rate of decay would be so great that they could not endure as does the magnetic field. The rate of decay can be deduced with the aid of a mathematical analysis due to Lamb,[6] using the value of the internal electric conductivity determined from the study of the diurnal magnetic variations (cf. §§ (13), (24)). On calculation it appears (as Schuster pointed out in the Halley Lecture for 1917) that the currents would be reduced in the ratio $1/e$ in between two and three days. On the other hand, it is doubtful whether such currents can be permanently maintained; there is no known source of energy inside the earth which could supply the loss due to electrical resistance except the unequal distribution of temperature, and thermo-electric forces would not act in the manner required.

A possible cause of such internal electric currents has, however, recently been proposed by Larmor,[7] though chiefly with reference to the analogous solar problem. If there is any residual internal circulation occurring in meridian planes (and surface phenomena suggest that this may be so, on the sun) this will, in the presence of a magnetic field, induce electromotive forces in the moving matter; if any conducting path round the solar axis be open, an electric current will flow round it, which may increase the inducing magnetic field. The internal circulation may thus act like the motion in a self-exciting dynamo, and maintain a permanent field (produced from insignificant beginnings) at the expense of some of the energy of internal motion.

Owing to the remarkable radial limitation of the sun's general magnetic field, this theory would seem to require currents of one sign within the sun, producing a magnetic field of the observed direction, and currents of the opposite sign in the solar atmosphere, neutralising the deeper field. Any theory of the sun's magnetism which does not at one and

[1] Schuster, *Phys. Soc. Proc.*, 1912, xxiv. 127.
[2] Gilbert, *De magnete*, 1600.
[3] Halley, *Roy. Soc. Phil. Trans.*, 1683, 1692. Cf. Balfour Stewart, Art. "Terr. Mag.," *Encyc. Brit.* (1882, 9th edition).
[4] Hansteen, *Magnetismus der Erde*, Christiania, 1819.
[5] Schuster, *Phys. Soc. Proc.*, 1912, xxiv. 127.
[6] Lamb, *Roy. Soc. Phil. Trans.*, 1883, p. 526; *ibid.*, 1889, p. 513.
[7] Larmor, *Brit. Assoc. Rep.*, Bournemouth, 1919, p. 159.

the same time account for both the presence and narrow limitation of the field (cf. § (9)) suffers from this complicating requirement.

As applied to the earth, this theory demands fluidity and residual circulation in deep-seated regions ; such circulation existing in a celestial body would be extremely permanent, as the large size makes effects of ordinary viscosity nearly negligible. The secular variation of terrestrial magnetism would in this case be explicable without assuming changes in the direction of material circulation, if the internal conducting paths can alter. The almost absolute fixity of length of the astronomical day shows that the material structure of the earth possesses extreme stability.[1]

Larmor also applies the theory to sunspots, regarding them as superficial sources or sinks of vortical radial flow: a strong radial magnetic field, owing to induced electric currents round the axis of the vortex, would be a natural accompaniment; and if the inflow at one level were compensated by outflow at another level, the flatness and vertical restriction of the field would be intelligible.

§ (7) Some General Considerations.—Most theories of the earth's general magnetic field have concerned themselves almost entirely with the leading harmonic component (corresponding to a uniform magnetisation) or, more particularly, to the part of this which is symmetrical about the earth's axis. The close approximation of the direction of magnetisation to that of the axis of rotation has usually been regarded as not merely a coincidence. The discovery of the sun's magnetic field, also only slightly oblique, has strengthened this view, and favours the opinion commonly held, that the magnetisation of these bodies is intimately connected with their rotation. Schuster and Kelvin many years ago advanced the suggestion that any large rotating body may be magnetised in virtue of its rotation—size being referred to, because no comparable magnetisation is observed in rotating bodies of ordinary size (cf. § (10)). However, as has already been noted (cf. § (5)), for a given intensity of magnetisation the increase of magnitude does not augment the surface magnetic intensity.

The comparative magnitude of the observed fields of the earth and sun, and also of the observed effects of rotation on bodies of ordinary size, has been used [2] in an attempt to distinguish between various hypothetical laws of dependence of intensity of magnetisation on size, mass, and angular velocity, assuming the effect to be a fundamental one independent of the nature and state (in other respects) of the rotating body. Some possibilities can be excluded by this means, but it has to be remembered that the sun's magnetic field differs from that of the earth in certain respects so important that it is dangerous to assume that the operative cause is the same in the two cases.

[1] Larmor, *Brit. Assoc. Rep.*, Bournemouth, 1919, p. 159.

[2] Swann, *Phil. Mag.*, 1912, xxiv. 80; *Terr. Mag.*, 1917, xxii. 149.

Theories which are based on some effect of rotation can in any case only account for the part of the magnetic field which is symmetrical about the axis of rotation. Some additional hypothesis is required to account for the obliquity of the magnetic axis. A want of symmetry or homogeneity in the internal constitution of the earth and sun might be supposed accountable, though the variability of direction of the magnetic axis seems almost to exclude this view. Nevertheless, if the symmetrical part could be satisfactorily explained, a great advance would have been made.

One form of rotational theory which has been suggested may be briefly mentioned and dismissed, viz. that the field is due to internal electric currents induced owing to the earth's rotation in a supposed cosmic magnetic field. The existence of such a field, of adequate intensity, would be more difficult to explain than the field of the earth itself.

§ (8) Rotation of Charge.—Perhaps the simplest form of theory which ascribes the magnetisation of the earth and sun to their rotation is that in which these bodies are supposed electrically charged, the field being due to the rotation of the charge, which is equivalent to a current. This hypothesis, however, proves to be untenable in any likely form. Whether the charge is spread uniformly over the surface, or is distributed throughout the volume, if it is sufficient to produce the observed magnetic fields it must be accompanied by an electric potential gradient far exceeding that observed near the earth's surface or than could be maintained in the sun.[3] In the latter case the mutual repulsion of the charge would overcome gravity, and the charge would escape till it sank to a value negligible as regards its magnetic effects. Moreover, though the magnetic field thus produced would be of the right type, as observed with a compass needle sharing the earth's orbital, but not its rotatory motion, Schuster [4] has shown that the type as observed with a needle rotating with the earth would be different; the charge induced on the needle would cause an apparent reversal of the horizontal component of magnetic intensity, while leaving the vertical component unchanged in sign. The theory also offers no explanation of the radial limitation of the sun's field.

[3] Brunt, *Astr. Nach.*, 1913, cxcvi. 169.

[4] Schuster, *Phys. Soc. Proc.*, 1912, xxiv. 127.

§ (9) Rotation of Separated Charges.—A modified form of the last theory has been proposed, which overcomes the difficulty of the external electrostatic field, and avoids the second objection pointed out by Schuster. The rotating body is supposed to be electrostatically neutral, as a whole, but is regarded as having the opposite charges separated by some cause, and hence at unequal mean distances from the centre or axis of the body. J. J. Thomson,[1] Sutherland,[2] and others have suggested possible causes of separation or polarisation, depending on the possibly unequal action of gravity or centrifugal force on the two charges; or on the radial variation of temperature, which by the thermo-electric effect should produce a slight separation of the charges. The latter effect is very small, while the others are at present only conjectural.

In order to explain the earth's field on these lines, the mean distance of the negative charge would have to be greater than that of the positive charge. Swann [3] has calculated the amount of negative surface charge necessary, on the rather favourable assumption that the equivalent positive charge is uniformly distributed throughout the earth's volume. The calculated magnitude of the charge is so great that the internal electric field called into play would at once destroy the separation of the charges. Brunt [4] has made a similar calculation for the sun, and has arrived at the same conclusion; the greatest magnetic field which could be produced by the rotation of electric charges in the sun is about 10^{-15} times the observed field.

If instead of separated charges an electric polarisation be assumed, the internal electric field would likewise be too great for matter to sustain. This field would, as Larmor [5] has pointed out, be neutralised by a distribution of electric charge on the surface and (where the polarisation was not uniform) in the interior; but though the compensation of the electric field might be complete, the two electrical distributions would in general not be electromagnetically equivalent. Their rotation would, indeed, produce a magnetic field which would be comparable with that of either distribution separately. The circumstance that each of the compensating electric fields is separately very great need not be regarded as an objection, for it is recognised that molecular electric fields are, in fact, enormous.

As applied to the sun, this form of theory has the great merit that it can explain a radially limited field, if polarisation of the solar atmosphere is assumed as the cause. The extremely rapid radial diminution of the sun's field suggests that the field is produced in the atmosphere itself, the density of the latter being the only other quantity which suffers a similar diminution and which seems likely to be at all concerned with the field.

On calculation it appears that the polarisation necessary to account for the observed sign of the solar field is such that the negative ends of the electric doublets must be at the greater radial distance from the sun's centre. The speculation may be hazarded as to whether a possible cause of such polarisation may not be found in some directive effect of the intense outflow of radiation from the sun on the electric doublets, such as the un-ionised hydrogen atoms, known to exist there.

Larmor [6] also points out that a crystal possesses intrinsic electric polarisation, since its polar molecules are orientated: if this polarisation were nearly complete, owing to pronounced natural orientation, a crystal of the size of the earth would produce an enormous electric field, compensated electrically, but not magnetically, for the effects of rotation: thus a planet whose materials have crystallised out in some rough relation to the direction of gravity, or of its rotation, would possess a magnetic field; he does not apply this theory to the earth, however, because the material stability of its interior seems to preclude the possibility of explaining the secular variation of a field thus produced.

§ (10) Direct Magnetisation by Rotation.—A further form of rotational theory depends on the fact that in ferromagnetic substances the ultimate magnetic elements are possessed of angular momentum, and therefore act like small gyroscopes on rotation; this was first pointed out by Maxwell. When rotated the elements should tend to set their axes of magnetisation (and rotation) along the axis of rotation of the body of which they form part, and the latter should become magnetised in this direction.

Schuster [7] and Swann [8] have discussed this effect; the former suggested that the main result might be a precession possibly capable of explaining the secular variation, rather than an actual magnetisation. Swann pointed out that the magnetisation, if it existed, should be the same for all volume elements, so that the magnetic intensity at the surface should be the same whatever the size of the body; spheres rotated in the laboratory, however,

[1] Thomson, *Phil. Mag.* (5), 1894, xxxvii. 358.
[2] Sutherland, *Terr. Mag.*, 1900, v. 73; 1903, viii. 49; 1904, ix. 167.
[3] Swann, *Phil. Mag.*, 1912, xxiv. 80; *Terr. Mag.*, 1917, xxii. 140.
[4] Brunt, *Astr. Nach.*, 1913, cxcvi. 169.
[5] Larmor, *Brit. Assoc. Rep.*, Bournemouth, 1919, p. 159.
[6] Larmor, *Brit. Assoc. Rep.*, Bournemouth, 1919, p. 159.
[7] Schuster, *Phys. Soc. Proc.*, 1912, xxiv. 127.
[8] Swann, *Phil. Mag.*, 1912, xxiv. 80; *Terr. Mag.*, 1917, xxii. 149.

at much greater angular speeds than that of the earth, show no comparable surface field.

S. J. Barnett[1] has recently found that rotation does actually produce magnetisation in ferromagnetic bodies. An associated effect, predicted but unsuccessfully sought for by O. W. Richardson[2] in 1907, has since been found by Einstein and Haas,[3] viz. that magnetisation of a body should subject it to a torque round the direction of the field; the measured torque was in close agreement with the theory, and indicated that the magnetic elements had little or no freedom to precess.

The magnetisation observed by Barnett was directly proportional to the angular velocity, and independent of the size of the body; its sign indicates that the rotatory energy and momentum of the magnetic elements is mainly due to the rotation of negative electricity, and thus agrees with the sign of the magnetisation of the earth and sun. Its amount, however, is very small, and corresponds to that which would be produced by a magnetising field of intensity $5.10^{-7}N$, where N is the angular velocity in revolutions per second. In the case of the earth N is the reciprocal of 24.60.60, so that the observed field is far greater than the laboratory experiments will allow, unless some important difference of conditions must be accounted for. It has been suggested[4] that the high temperature of the earth's interior may be such a determining condition; the fact that in Barnett's experiments the magnetisation was found to be proportional to N indicates that the constraints on the gyroscopic elements are elastic: these constraints should diminish with rise of temperature, giving increased freedom for tilting of the elements towards the axis of rotation, though at the same time increasing the tendency for them to assume a random distribution. How far the resultant effect of high temperature would favour increased magnetisation is a question which at present only experiment can decide, and experiments of the kind are being made. The gap between previous laboratory results and the terrestrial field is, however, a large one.

If the magnetic elements are the molecules, the increased freedom to tilt which should accompany rise of temperature would be diminished by increase of pressure. This effect might be less, if the ultimate magnetic elements are the electrons, moving in orbits small compared with the molecular diameter, or (if ring electrons exist) revolving about their own axes. It is possible that the earth's field, and perhaps even the secular variation, may be explained by the gyroscopic alignment and precession of such elements, but the matter is one for conjecture rather than proof until molecular structure is better understood. On the whole, it seems unlikely that the sun's field is due to this cause, in view of the considerations already mentioned relating to its radial diminution. This objection does not apply to the fields in sunspots, where also the rotation may be much more rapid; the continual collisions present the main difficulty in this case, since the fields observed are so strong that a considerable degree of average alignment of the magnetic elements would be necessary.

§ (11) The Irregularities in the Earth's Magnetic Field.—Little more progress has been made towards the explanation of the broader irregularities in the earth's magnetic field than of its main features or of the secular variation. Certain quite local disturbances in the field can be associated with deposits of more or less magnetic minerals near the surface,[5] but the constitution of the earth's crust is known only to a very slight depth comparatively. As regards the larger irregularities, attempts have been made to connect them with other terrestrial features, such as distribution of land and water, and anomalies of mean temperature over the earth. Perhaps the most promising of these attempts is that of Wilde,[6] who constructed a "magnetarium" designed to reproduce the main features of the earth's surface field and its secular variation. It consisted of a globe 18 inches in diameter representing the earth, with plates of sheet iron attached to its under side beneath the ocean areas (and also, apparently, in one or two land areas); within and attached to this was a spherical shell of wire gauze supporting a current-winding with its coils in planes parallel to the equator. Within this, again, was a smaller sphere wound with wire in an independent circuit, and with the coils in a plane inclined at 23·5° to the equator of the first sphere. The axis of the inner sphere could be made to revolve, at the above constant inclination, round the axis of the outer globe. The whole arrangement was constructed so that any part of the globe could be brought under a support on which either a compass or a dip needle could be mounted. Wilde claimed that this model reproduced the main features of the earth's field and its secular variation, though different opinions seem to have been held on this point. Rücker, in a paper[7] critically reviewing that part of Wilde's work which was not concerned

[1] Barnett, *Phys. Rev.*, 1915, vi. 239; 1917, x. 7.
[2] Richardson, *Phys. Rev.*, 1908, xxvi.
[3] Einstein and Haas, *Verh. d. Deutsch. Phys. Ges.*, 1915.
[4] *Nature*, Nov. 25, 1920, p. 407; March 3, 1921, p. 8.

[5] Walker and Cox, *Roy. Soc. Phil. Trans.* A, 1919, ccxviii.
[6] Wilde, *Roy. Soc. Proc.*, 1894, lv. 210, and a privately printed memoir.
[7] Rücker, *Terr. Mag.*, 1899, iv. 113.

with the secular variation, but only with the existing field, took a view which on the whole was favourable to Wilde's claims; he also discussed how far the plates of sheet iron under the ocean, which modified the oblique uniform field produced by the two coils, could correspond to magnetisable matter existing in the earth. The depth under the land surface at which the critical temperature for iron is reached has been estimated at about 20 kilometres (cf. § (5)). The ocean bed, however, is at a temperature only slightly above 0° C., which is therefore less than that of the land surface in most parts of the earth; thus the level at which magnetisability ceases must extend further downwards beneath the oceans than beneath the land by more than the average oceanic depth (about 4 kilometres), assuming that the underlying minerals are the same in the two cases. The theory of isostasy, moreover, asserts that under the oceans the material of the earth's crust must be denser than under the land. If, therefore, the crust is partly composed of rocks of density about 2½ (practically non-magnetic), and partly of heavier material of density ranging up to about 5, the layer of separation or transition must be higher under the ocean than under the land. If the heavier material is iron, on both accounts the sub-oceanic thickness of the magnetisable stratum of this must exceed by more than 4 kilometres the thickness under the land. This additional thickness may be the physical circumstance to which Wilde's sub-oceanic sheets of iron correspond. Rücker proceeded to calculate whether this difference of thickness was adequate, making reasonable assumptions as to the permeability of the layer, to afford a possibility of explaining the observed irregularities in the earth's magnetic field. He concluded that it was adequate if the material of the stratum in question is iron, but not if it is magnetite or basalt.

III. The Magnetic Variations of Short Duration

§ (12) Terrestrial Variations as viewed from the Sun.—In considering the magnetic variations of short duration, periodic or irregular, it is desirable to take a world-wide view of the phenomena, as far as possible, and to picture the earth as seen from the heavenly bodies—the sun and moon—which govern its magnetic and meteorological changes. *Fig.* 1 represents the earth as viewed from the sun, in such way that the north pole is seen uppermost. The earth's orbital motion is then from right to left, while its axial rotation, and the orbital motion of the moon, are in the same sense. The sun also revolves about its own axis in the same direction, so that any radial streamer emitted from it would, if directed towards the earth, sweep past the latter from right to left.

Over the right half of the earth, in *Fig.* 1, the sun has "crossed the meridian," and the local time is after noon; this hemisphere may therefore be referred to as the P.M. (*post meridiem*), and the left-hand the A.M., hemisphere. Similarly we may speak of the day and night (or the sunlit and dark) hemispheres, which are separated by the "twilight circle" of the earth.

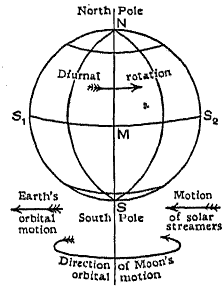


FIG. 1.—Illustrating the motions of the earth, moon, and solar streamers as viewed from the sun (§ (4)).

NMS = Noon meridian.
NS_1S = Sunrise meridian.
NS_2S = Sunset meridian.

The day or sunlit hemisphere is directly visible, the night or dark hemisphere is behind; the P.M. and A.M. hemispheres are respectively to the right and left of the noon meridian.

Various diurnal terrestrial changes are approximately repeated, at the same local time, all round any given circle of latitude. This is a consequence of, and holds good to the extent of, the earth's symmetry (as regards the factors affecting any particular phenomenon) about its axis. For a first approximation, and as regards the middle belt of the earth, between N. and S. latitudes 60°, the diurnal magnetic variations may be considered as depending, at any given latitude, only on the local time. Locally regarded, the changes represent the influence of the moving sun or moon; viewed from the standpoint of the latter, the diurnal variation represents the passage of conditions at each station through stages characteristic of positions fixed relative to the heavenly body, as the diurnal rotation carries the station through all such positions on its own circle of latitude. At a given season of the year, *i.e.* for a given value of the geocentric latitude of the sun (or moon), the co-ordinates determining those fixed relative positions are the latitude and the local time.

The most important periodic magnetic changes are the solar and lunar diurnal variations, which for brevity will be denoted by (S) and (L) respectively. In some respects the latter is the simpler, and it will be considered first.

§ (13) The Lunar Diurnal Magnetic Variation (L).—When deduced from one or more complete lunar months, (L) is purely semidiurnal in all elements (§ (1)) and at all

stations. *Fig.* 2 [1] shows for a number of stations the variations, in force units, along the three directions north (H.F.), west (declination), and upwards (V.F.). The lunar day is reckoned from one "lunar midnight" or lower transit to the next, in twenty-four lunar hours (approximately equal to twenty-five solar hours). The variation is symmetrical in H.F. north and south of the equator, and anti-symmetrical in declination and (outward) vertical force. Owing to the very small amplitude, the variations as determined are subject to more accidental error than in the case of (S) (cf. *Fig.* 6).

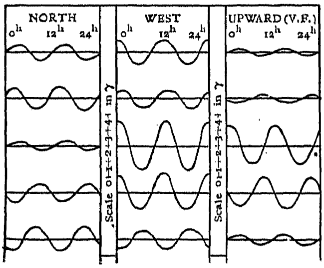


FIG. 2.

Stations (from top downwards): Pavlovsk (60° N.), Pola (45° N.), Zikawei (31° N.), Manila (15° N.), Batavia (6° S.).

As explained in § (12), the lunar diurnal variation (L) can be regarded as due to the rotation of the earth within a certain magnetic field (the L-field) fixed relative to the moon. The semidiurnal character of the curves in *Fig.* 2 denotes that this field is symmetrical with respect to any plane through the earth's axis; it is therefore sufficient to consider one such hemisphere. For simplicity the hemisphere between the meridians of 9^h and 21^h local lunar time will be chosen; at these hours (approximately) the west force variation at the equinoxes vanishes and changes sign. This hemisphere is, of course, only partly visible from the moon. The field over this hemisphere is represented in *Fig.* 3, so far as is possible in a single diagram; the full lines indicate the direction of the horizontal component of the field, *i.e.* the component lying along the spherical surface of the earth. Thus, between the latitudes F, F′, at which the north component of (L) changes sign (*Fig.* 2), the north component in *Fig.* 3 attains its maxima at about 9^h and 21^h, and its minima at about 3^h and 15^h, and *vice versa* beyond these latitudes. The west component is similarly shown as attaining its maxima at 0^h and 12^h, at stations north of the equator, and its minima at the same hours at southern stations. The diagram does not indicate the intensity of the field, but the intensity of the north component, for instance, naturally vanishes at F and F′, where the direction of the horizontal force variation is indeterminate. Thus *Fig.* 3 synthesises many of the features of *Fig.* 2.

The dotted lines in *Fig.* 3 are drawn transversely to the lines indicating the direction of the horizontal component of (L); a system of currents along these lines in a spherical current sheet would give rise to a magnetic field, the direction of which, over a concentric internal sphere, would be that indicated by the full lines. (Over a concentric external sphere the magnetic force would be oppositely directed.) The component over a concentric sphere, internal or external, is, of course, the "horizontal" component on that sphere, as in *Fig.* 3. The magnetic field would also have a vertical component, which would be in the same direction inside and outside the current sheet; it would be upwards over such regions as NZM′ and SZZ, and downwards over the rest of the globe. The corresponding lunar diurnal variation of V.F. would hence be the same as in *Fig.* 2, having maxima at 9^h and 21^h, and minima at 3^h and 15^h, in the northern hemisphere, and *vice versa* in the southern. Thus the variations shown in *Fig.* 2 can be accounted for by supposing that the earth is within a spherical current sheet such as *Fig.* 3 depicts —a conclusion which agrees with that reached by the more precise process of spherical harmonic analysis, though the latter also indicates that a fraction of the L-field is due to a similar but nearly oppositely directed system of electric currents within the earth.[2] The

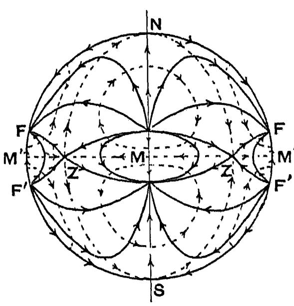


FIG. 3.—Electric current-lines (dotted) and lines of horizontal magnetic force at the earth's surface, resulting by electromagnetic induction from the tidal atmospheric circulation.

The layer in which the electric currents flow is for simplicity assumed to be of uniform conductivity. The obliquity of the magnetic to the geographical axis of the earth is neglected. The above curves give an approximate representation of the semi-diurnal component of the diurnal magnetic variations (except as regards phase), viewed from the sun or moon.

[1] *Roy. Soc. Phil. Trans.* A, 1917, ccxviii. 113; cf. values of C_2, θ_2 for the equinox.

[2] *Roy. Soc. Phil. Trans.* A, 1917, ccxviii. 113.

latter system reinforces the horizontal portion of the external field, while partly neutralising the vertical portion, according to the principles just mentioned; thus the observed vertical component of (L) is smaller in proportion to the horizontal component than it would be if its origin were purely internal or purely external, though its sign shows that it is mainly of external origin. The internal portion is explicable as arising from an internal system of electric currents, induced in the conducting parts of the earth by the external current sheet. The latter, though stationary relative to the moon, is varying relative to the moving earth.

§ (14) THE CAUSE OF THE LUNAR DIURNAL MAGNETIC VARIATION.—The semidiurnal character of (L), and consequently of its associated system of atmospheric electric currents—for the external currents can hardly be outside the atmosphere—suggests that the moon produces its effect by tidal action. This is confirmed by the fact that (L) increases in intensity from apogee to perigee approximately in the ratio of the corresponding values of the lunar tidal force.[1] The barometer itself shows a semidiurnal variation of atmospheric pressure, indicating the existence of an atmospheric lunar tide, and this must involve a circulation of the air, the flow being mainly horizontal. *Fig.* 4 shows the distribution of pressure and motion corresponding to such a tide, as viewed

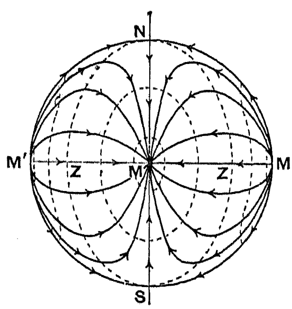


FIG. 4.—Lines of equal pressure (dotted) and of instantaneous direction of motion in a tidal system of circulation in the earth's atmosphere, viewed from the tide-producing body (§ (13)).

In this drawing there is supposed to be no lag of phase, so that maximum pressure occurs at M and minimum at M′. The opposite side of the earth resembles that shown here. The meridians NZS are lines of zero pressure, separating regions over which the pressure is in excess and defect respectively.

from the tide-producing body, assuming there to be no phase-lag.

An atmospheric circulation of this kind will induce electromotive forces in the atmosphere by intersecting the earth's permanent magnetic field. The horizontal component of these forces, produced by the intersection of the vertical magnetic component, is the one which is of importance in impelling a spherical electric current system. Since the E.M.F. will be everywhere normal to the direction of atmospheric flow, its distribution will be as indicated by the dotted lines in *Fig.* 4, allowance being made for the fact that the earth's vertical magnetic component is upwards in the southern and downwards in the northern hemisphere. The intensity cannot be indicated on this figure, but it will clearly vanish at the equator, along with the vertical magnetic force itself.

In any atmospheric layer which is electrically conducting, a system of electric currents will result from this distribution of E.M.F. The form of the current-lines is not readily evident from inspection of *Fig.* 4, owing to the fact that the intensity is not indicated; but it may be seen, and spherical harmonic analysis confirms the conclusion, that if the conductivity of the layer is uniform the current-lines will resemble those in *Fig.* 3. We thus have an explanation of how a lunar diurnal magnetic variation of the type represented by the curves in *Fig.* 2 can arise from the tidal action of the moon upon the earth's atmosphere. There can be little doubt that this explanation is the true one, although when this theory is examined numerically a rather surprisingly large difference of phase is found between (L) and the tide as indicated by the barometer.[2]

§ (15) THE SUN'S INFLUENCE ON THE LUNAR DIURNAL MAGNETIC VARIATION.—Thus far, in considering (L) as deduced from the average of a number of complete lunar months, we have been dealing with it in a specially simple form. On any given lunar day the curves representing the L-variation in the three elements are much more complicated than those of *Fig.* 2, and in fact are not at all semidiurnal in type. The modification consists essentially in a magnification of the variation during the hours of solar daylight. Since these extend over different lunar hours at different epochs in the lunar month, the L-curves vary continually throughout the month. But their monthly mean remains semidiurnal, because every section of the lunar day is equally in turn subject to this magnification.

The effect can be best illustrated by a diagram different in kind from *Fig.* 2. The daily course of (L) at a single station, separated from other magnetic variations, can be indicated by the closed curve (in three dimensions) described by the end P of the vector OP, representing the local direction and magnitude of the magnetic intensity. Such a curve sums up for any one station the changes in the three component directions

[1] *Roy. Soc. Phil. Trans.* A, 1915, ccxv. 101; also *Roy. Met. Soc. Quarterly Journal*, 1919, xlv. 113.

[2] *Roy. Soc. Phil. Trans* A, 1917, ccxviii. 113.

(illustrated by the corresponding set of three curves in *Fig.* 2), provided that the rate of description of the vector curve throughout the day is indicated by marking the stages arrived at at successive hours. It is sufficient for the present purpose, and more convenient for plane representation, to consider the projection of the path of P, on some plane, rather than the actual path. The horizontal projection will be used, summing up the L-variations in H.F. and declination. This curve will be referred to as the vector diagram.

In the case of the monthly average of (L), corresponding to *Fig.* 2, the vector diagram

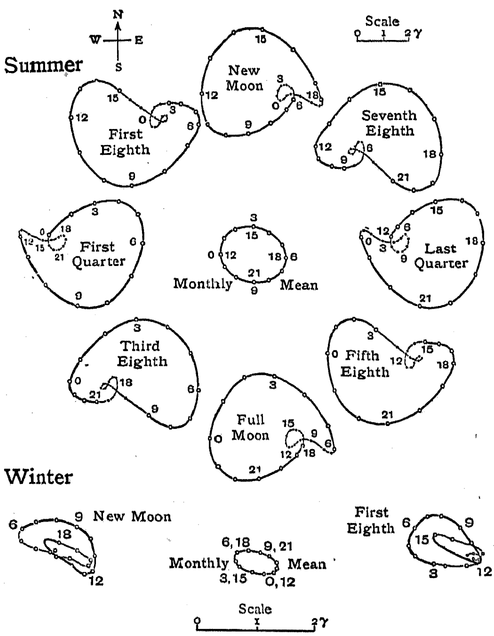


FIG. 5.

consists of an ellipse, which P′ (the projection of P) describes twice daily. *Fig.* 5 relates to Pavlovsk, and shows this monthly mean vector diagram in the centre, together with the corresponding curves at various particular epochs in the lunar month; in the latter the part of the curve described during the solar day-hours is drawn more thickly than the nocturnal part of the curve.[1] It is evident that the moving point P′ still describes two loops daily, but these are no longer identical; the motion of P′—*i.e.* the activity of the L-variation, and the strength of the electric currents producing the L-field—is much greater by night than by day. Thus the sun affects (L) in a very striking way, and destroys its semidiurnal character. The stage at which this influence is exerted in the course of the production of (L) (cf. § (14)) is easily recognised. The sun cannot affect the semidiurnal nature of the lunar atmospheric tide, nor of the resulting system of E.M.F. Hence the increased intensity of the system of *currents* over the sunlit hemisphere indicates that it is the electric conductivity of the atmosphere, in some layer, which the sun affects. This again suggests that the conductivity of the layer concerned is in fact due to some solar ionising agent, which reaches the sunlit but not the dark hemisphere, and is most effective at places where the sun's zenith distance is small. Yet although the conductivity diminishes towards the twilight circle of the earth, it does not wholly die away during the night hours, for the L-field, though small, is still appreciable over the dark hemisphere.

The dependence of the atmospheric electrical conductivity upon the sun's zenith distance is shown also by the seasonal variation of (L). In addition to the vector diagrams for the summer months at Pavlovsk, *Fig.* 5 includes the monthly average diagram, and two of the diagrams for particular phases of the moon, for winter at Pavlovsk. The lunar action is about as powerful in winter as in summer, but the intensity of (L) is much less in winter, a fact which is naturally explained by the diminished ionising action of the sun, owing to its increased zenith distance at that season.

§ (16) Magnetic Activity.—Before considering the other and larger periodic magnetic variation (S) (cf. § (12)) and the irregular variations of short duration, it is necessary to explain the meaning of magnetic activity.

On certain days the three elements of magnetic force everywhere on the earth's surface vary irregularly to an unusual degree. Such a disturbance of the earth's field is called a magnetic storm. On other days the variations are gradual and regular, of a type which has become recognised as characteristic of such magnetically "quiet" days. Ordinarily the case lies between these two extremes, and there is a certain amount of disturbance superposed on the quiet-day type of variation. The intensity of disturbance, or the degree of magnetic activity, as it is termed, may change from hour to hour and day to day. Its larger changes are roughly synchronous all over the earth, however, so that a given interval of time, say a particular hour or day of Greenwich time, can be classified as more or less magnetically disturbed, without reference to any special locality. An international scheme for doing this is actually in operation.

[1] *Camb. Phil. Soc. Trans.*, 1919, xxii. 341.

Magnetic observatories assign character figures 0, 1, or 2 to each successive Greenwich day—0 denoting a quiet day, 2 a disturbed day, and 1 a day of intermediate type. This can be done only very roughly, but the average character figure derived from all the co-operating observatories affords a useful measure of the magnetic activity. The amount of departure from the quiet-day type of variation, corresponding to a given character figure, depends largely on the locality of the station and other factors; but it increases and decreases in a fairly parallel way at all stations.

§ (17) THE SOLAR DIURNAL MAGNETIC VARIATION (S).—Many magnetic observatories publish results for (S) as derived from the five quietest days per month separately. This variation will be denoted by S_q. On other days (S) seems to comprise S_q together with a solar diurnal variation of a different type which varies in amplitude with the degree of magnetic activity, whereas S_q remains constant from day to day (apart from its slow seasonal change). The second part of (S) may be called the disturbance solar diurnal magnetic variation, and denoted by S_d. Thus $(S)=S_q+S_d$. The constancy of S_q is inferred from the fact that if this variation is subtracted from the total variation (S) as derived (i.) from all days, and (ii.) from days of magnetic storm, the difference S_d is of similar type in the two cases, though of much greater amplitude in the latter. *Fig.* 6 shows the S_q-variation in the three elements at a number of stations in the northern hemisphere, together with S_d, derived as in case (ii.) just described. The latter curves relate to the average of about forty moderately intense magnetic storms.[1]

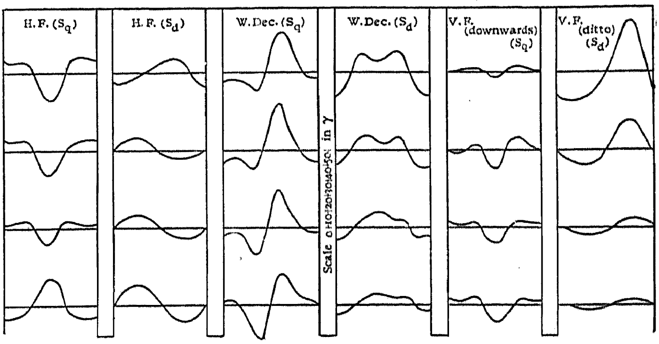


FIG. 6.

Stations (from top downwards): Pavlovsk (60° N.), mean of three stations (average latitude 50° N.), mean of four (37° N.), mean of three (15° N.). The variations are for the mean of one or more years, and are all indicated in force units.

S_q is of about eleven-fold the intensity of (L), with which it has many features in common. *Fig.* 7 shows the vector diagram for the summer and winter seasons at Greenwich, in each case for two sunspot epochs. The variation is clearly greater during the day and in summer than during the night and in winter. It may be seen, moreover, that the curves for S_q in *Fig.* 6 correspond to a magnetic field (the S_q-field), and an atmospheric electric current system similar, except as regards phase, to that illustrated in *Fig.* 3 for (L), but intensified over the sunlit hemisphere.

[1] *Roy. Soc. Proc.* A, 1918, xcv. 61.

Spherical harmonic analysis indicates that the S_q-field has a primary external portion and a secondary induced internal field, mutually related just as in the case of (L). The external field can be approximately accounted for as the product of an atmospheric circulation of tidal type, stationary relative to the sun, in conjunction with a distribution of electrical conductivity over a spherical atmospheric current sheet, similar to the distribution which may be deduced from the study of (L) at any particular lunar epoch.[2] Barometric observations do in fact show that the most important part of the daily atmospheric circulation is approximately of tidal type—in particular,

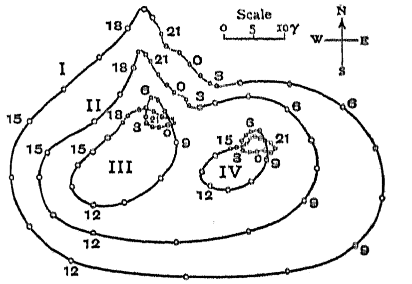


FIG. 7.—Quiet-day vector diagrams of the daily variation of magnetic force in the horizontal plane at Greenwich, 1889-1914.

I. June, sunspot maximum years; II. June, sunspot minimum years; III. December, sunspot maximum years; IV. December, sunspot minimum years.

that it is semidiurnal. Its amplitude is about fifteen times that of the lunar tide, if the two are measured by the corresponding barometric variations. It probably arises from the thermal and not the tidal influence of the sun upon the atmosphere; otherwise, since the tidal force of the moon is more than

[2] *Roy. Soc. Phil. Trans.* A, 1917, ccxviii. 113.

twice that of the sun, the lunar variation of barometric pressure should be this much the greater. As it is, there are grounds for believing that the main part of the solar diurnal atmospheric circulation does not extend throughout the whole height of the atmosphere, but that above some level only the truly tidal portion remains; also that it is above this level that (L) is mainly produced, while S_q is produced at a lower level, in a layer ionised by some solar agent, but not by the same one as that which ionises the layer in which (L) is generated.

These statements rest upon the very different factors governing the changes of amplitude of S_q and of (L). While the former does not vary from day to day with the magnetic activity, its amplitude changes gradually with the sunspot epoch, so that at sunspot maximum its value exceeds that at sunspot minimum by 40 per cent or more. This is illustrated by *Fig.* 6, and can only be attributed to a corresponding increase in the conductivity of the atmospheric layer concerned, for the other factors on which S_q depends do not vary appreciably with the sunspot cycle. It seems probable that the ionising agent is some form of penetrating ethereal radiation, distinct from the general radiation of incandescence of the sun, and that this radiation arises from the sun's surface as a whole, and not from particular sunspots or disturbed areas.

§ (18) The Dependence of (L) on the Magnetic Activity.—The case with (L) is very different; it is still uncertain whether its amplitude is greater at sunspot maximum or not, which at least renders it probable that the difference, if any, is not very great. It is known, however, that the amplitude varies considerably from day to day with the magnetic activity. Like S_d, which can vary by hundreds per cent, nearly vanishing altogether on quiet days, so (L) is several times as intense on, say, the five most disturbed days as it is on the five quietest days of the month. Thus the solar ionising agent that affects the layer in which (L) is mainly produced must vary in parallel with the magnetic activity, and therefore be distinct from the one which governs the amplitude of S_q. There are reasons also for supposing that S_d is produced in yet a third layer of the atmosphere, in spite of the fact that (L) and S_d vary in parallel with the magnetic activity (cf. § (20)).

§ (19) General Remarks on the Diurnal Magnetic Variations. — The above account is a development of the theory originated by Balfour Stewart [1] and Schuster.[2] In spite of its success in many directions, difficulties remain, relating especially to the phase differences between the magnetic variations and the atmospheric motions to which the former are referred. The 24-hour component of the H.F. variation also presents difficulty, as it does not agree with the potential function which fits the other components.[3] Another question which arises is as to the reason for the apparent absence of magnetic effect of the winds which are found to exist in the stratosphere, of much greater intensity than the slow motion associated with the lunar atmospheric tide, viz. about 0·1 km. per hour. It seems necessary to conclude that in the layers of the atmosphere wherein (S) and (L) are produced any other motions which occur must either be relatively small or else extremely variable. The situation of the layers in question is not yet known.

§ (20) Magnetic Disturbance. — Magnetic storm effects can be grouped into three portions, viz. (i.) the rapid irregular fluctuations (F), (ii.) S_d, depending on local solar time, and (iii.) W_d, or the world-wide variation depending on what may be called storm-time, measured from the commencement of the storm. Practically all great magnetic storms commence simultaneously, to within a minute at most, all over the earth. In the case of less extreme disturbance, including even that occurring on the average day, the effects are of the same general type as those observed on days of storm, but the W_d portion cannot then be related to any definite commencement. The time of some standard station must then be used instead, and W_d will vary in rough parallelism with the curve of magnetic activity.[4]

The irregular fluctuations (F) are most intense in and near the auroral zones, one of which encircles each pole of the earth's magnetic axis. They diminish considerably in intensity towards the equator. They are not greater over the sunlit than over the dark hemisphere, the inequality round the earth's axis being rather as between the P.M. and A.M. hemispheres, the former being the more affected.

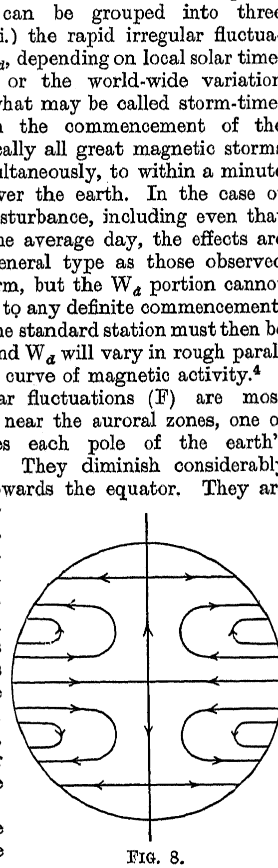


Fig. 8.

In the middle belt of the earth, between the two auroral zones, S_d is almost purely diurnal in type, but near the auroral zones the type changes. Broadly speaking, the variation S_d in the horizontal plane consists of a change of intensity normal to the zone, with but

[1] Balfour-Stewart, *Ency. Brit.*, 1882, 9th ed., article "Terrestrial Magnetism."
[2] Schuster, *Roy. Soc. Phil. Trans.* A, 1889, clxxx. 467; A, 1907, ccviii. 163.

[3] *Roy. Soc. Phil. Trans.* A, 1917, ccxviii. 113.
[4] *Roy. Soc. Proc.* A, 1918, xcv. 61.

little variation parallel to the zone. It is of the same sign on the two sides, while the sign of the vertical force variation is reversed. This indicates that a strong current is flowing along the zone in a somewhat narrow belt. The direction is opposite in the P.M. and A.M. hemispheres, being between the points in the zone on the mid-day meridian (approximately) and on the opposite meridian. *Fig.* 8 gives a diagrammatic sketch of the currents (flowing in the upper atmosphere) corresponding to corresponding currents (in the upper atmosphere, as the sign of the V.F. variation shows) are along the parallels of latitude. Except at the commencement of the storm the direction of flow is from west to east, and the current intensity, after the reversal of the brief initial phase, rises to a maximum (in the cases illustrated this takes from 15 to 20 hours) and then decays. The time taken to attain the maximum is less, the more intense the storm. The subsequent decay is exponential in its

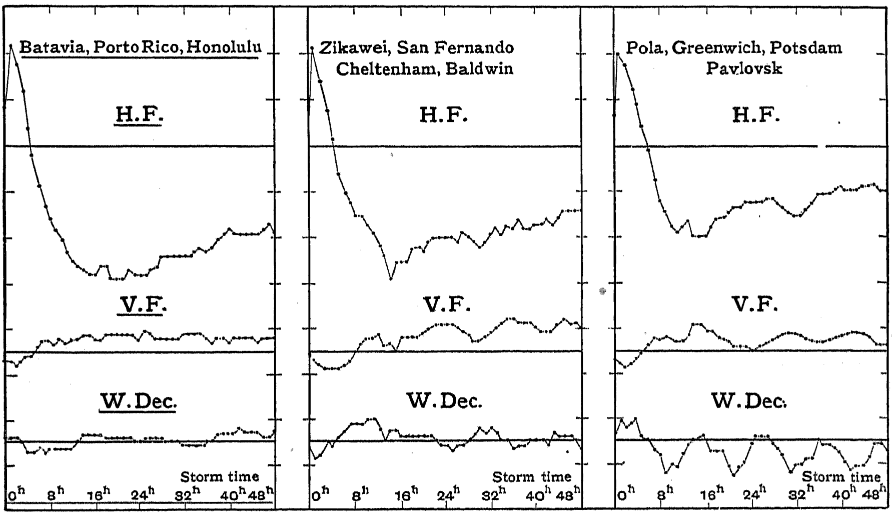


FIG. 9.

S_d, for the region between and including the auroral zones. Further within the zones S_d assumes a different character; the work of Chree [1] has shown that there the vector diagram for S—which is almost exclusively composed of S_d, the S_q part being relatively negligible—is nearly circular. This suggests that the direction of current flow and of H.F. is nearly constant in space, relative to the sun, while the earth's rotation merely alters the geographical direction of the corresponding S_d force throughout the day.

The storm-time variation W_d, being the part independent of local time, is the average effect, in any latitude, at all stations in that latitude. *Fig.* 9 illustrates its character in various latitudes, as determined from about forty moderately intense magnetic storms. The H.F. variation is of the same sign on both sides of the equator, the vertical (radial) force variation changes sign there, while there is little or no change in declination in the middle latitudes to which the diagram refers. Thus the lines of horizontal force in the W_d field are along the magnetic meridians, and the

[1] Chree, *Journal Inst. Elec. Eng.*, 1915, liv. 405.

later stages, though not at first. During the decay the irregular fluctuations F may still be active.

The S_d current system varies in intensity in parallel with W_d, and the two things seem to represent merely parts of one whole, separated for convenience of analysis. S_d represents the inequality of the "regular" storm effects round the parallels of latitude, while W_d is the average effect. *Fig.* 10 shows diagrammatically the combined current system, and indicates that the storm effects in the middle belt of the earth are greater over the P.M. than over the A.M. hemisphere.

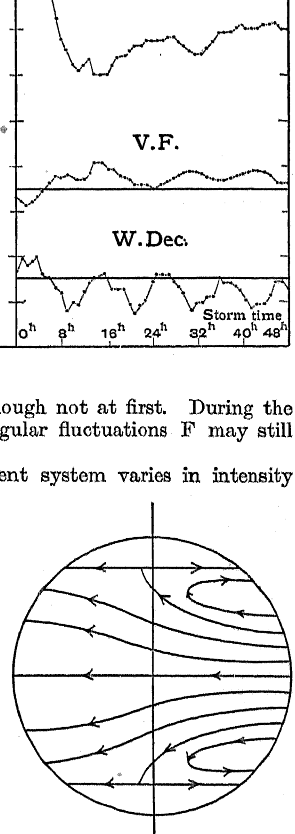


FIG. 10.

§ (21) THE RELATION OF MAGNETIC DISTURBANCE TO THE SUN.—The preceding section has dealt with the form of the disturbance magnetic field and current system, as distributed over the earth relative to the axis of the latter, and to the meridian plane through the sun. It is necessary also to consider the relation of disturbance to the succession in time of certain variable characteristics of the sun, the earth now being regarded as a whole. The sun's physical condition changes intrinsically, and also in its presentation to the earth (on account of the solar rotation); both these changes are clearly reflected in magnetic phenomena on the earth.

The intrinsic cycle of solar activity probably concerns the sun's surface as a whole, spots, faculae, and prominences being indications of local disturbance, but symptomatic of the general condition. These local phenomena are sporadic and irregular in occurrence, and last for a limited period of variable duration ; but their average frequency varies in unison with their distribution in latitude, which undergoes important changes throughout the solar cycle. Particular regions, however, often remain abnormally (though not necessarily continuously) active for several months.

The sun's rotation period (the length of which, relative to the earth, *i.e.* allowing for the earth's orbital motion, is 27·3 days) is of importance to the earth only in so far as there are irregularities in the physical state of the solar surface. The observed relation of magnetic disturbance to the rotation period [1] thus indicates that that disturbance is caused in some way by locally disturbed regions on the sun. The relation is not a periodicity but a recurrence tendency, for the magnetic activity only *tends* to return to its state at any time, after the lapse of about 27·3 days. Great storms show this very clearly, for the number of pairs or series among them, in which the members are separated by one or more periods of 27·3 days, is much greater than can be attributed to chance. More than one such series of recurrences may be in progress at the same time. Sometimes a series has gaps, just as solar outbreaks may occur intermittently in a particular solar area through several rotation periods. The recurrence tendency is shown by ordinary (quiet or disturbed) days as well as by days of storm. Chree,[2] using daily character figures (cf. § (16)), has very clearly proved that the average character figure for the days round about the 27th, 54th, and 82nd day following on a day of specially disturbed or quiet type show a marked but diminishing tendency towards a repetition of this character.

The most convincing interpretation of the recurrence tendency shown by magnetic storms was given by Mr. Maunder,[3] who independently discovered the phenomenon, which had been previously noted by Broun and others. He concluded that magnetic storms must be consequences of the presence near the earth of some agency proceeding from a restricted area of the sun's surface, and travelling outwards in a limited stream in some direction. Such streams as are suitably directed will traverse the space in the earth's neighbourhood, overtaking the earth in its orbit on the P.M. side, as indicated in *Fig.* 1. Should the emitting area remain active over a sufficient period, projecting the stream nearly in the same direction throughout (relative to the solar surface), it may again traverse the space round the earth, after an interval of one or more rotations. In this way there may arise a recurrence tendency with the observed period.

Chree's results may be explained along similar lines, if magnetic disturbance in general is referred to the agency of more or less well-defined streams emitted from particular disturbed localities on the solar surface. When, as the sun rotates, the streams which it emits are projected so as to impinge upon the earth, magnetic disturbance of greater or less intensity results : when such streams happen to be absent from the space round the earth, magnetically quiet conditions prevail.

The recurrence tendency indicates that the solar regions which emit the streams often remain active and approximately stationary on the sun for one or more rotation periods. The constituents of the streams are projected with a speed sufficiently great to prevent any serious dissipation or sideways diffusion within a distance equal to the radius of the earth's orbit. Owing to the continual renewal of the streams from the emitting areas, the whole set of streams (if there are several existent at the same time) will appear to rotate with the sun, like curved spokes of a wheel. There will be a certain lag of the streams, the curvature at any distance depending on the longitudinal and transverse speeds of the constituents ; this angular lag, being probably nearly the same for all streams in the plane of the ecliptic, will not affect the recurrence tendency. The sun's angular velocity is such that the transverse velocity of a stream, relative to the earth, is approximately $4 \cdot 10^7$ cm. per sec., or about one-thousandth of the velocity of light. If the constituents of the stream take 24 hours to travel from sun to

[1] Maunder, *Roy. Ast. Soc., Mon. Not.*, 1905, lxv. 555, and other papers.

[2] Chree, *Roy. Soc. Phil. Trans.* A, 1912, ccxii. 75, and A, 1913, ccxiii. 245.

[3] Maunder, *Roy. Ast. Soc., Mon. Not.*, 1905, lxv. 555, and other papers.

earth, the mean longitudinal velocity would be about four times that transverse velocity.

The earth's angular diameter as viewed from the sun is very small (17·6″), so that any given stream-line would cross its "solid" diameter in 35 seconds; it is therefore not difficult to understand why suddenly commencing magnetic storms seem to start almost simultaneously over the whole earth. Some idea of the breadth of the intense streams concerned may be gained from the duration of storms. A duration of one day (which is not uncommon) would correspond to a breadth, in the ecliptic plane, of about 35 million kilometres, or an angular breadth, viewed from the sun, of about 13°. The sudden commencement which characterises all very intense storms suggests that intense solar streams are somewhat sharply defined, at least on the forward side.

The general distribution of streams in the space round the sun is radial, though with a slight lag or curvature; the differences of intensity of the streams in different directions may be considerable. The radial distribution does not depend upon the streams being projected normally to the sun's surface: at several diameters' distance the parallax due to oblique projection will be small. Owing to the varying situation of disturbed regions on the sun's surface, and to the varying directions of projection, many streams must miss the earth and so fail to produce any changes in the earth's magnetic field. The non-recurrence of particular stream-transits and storms may be due either to change in the direction of projection, to intermission of activity at the source, or perhaps to changes in the earth's heliographic latitude. Ignorance of the direction of projection from particular solar areas precludes the possibility, at present, of identifying the precise solar region which is the source of given magnetic disturbances.[1]

As regards the relation of magnetic disturbance to the intrinsic solar period of about 11 years, Sabine found that the average frequency of magnetic storms shows a marked correspondence with Wolf's sunspot numbers. The latter measure the annual mean frequency of sunspots, and probably well represent the general march of solar activity as a whole throughout the sunspot cycle. But the relation between magnetic disturbance and the sunspot numbers is much less exact than it is for the range in S_q, which for any element and station varies almost linearly with the sunspot number. In the case of magnetic disturbance, years of few sunspots are, on the whole, magnetically quiet, but notably different degrees of magnetic disturbance have been shown by years of equal sunspot development. Again, storms sometimes break out at the very "trough" of the sunspot minimum, though at such times, apparently, only when the sun also shows some sign of special local activity. This corresponds with the irregular occurrence of solar outbreaks, which may appear for a brief period even at minimum epoch.

[1] *Camb. Phil. Soc. Trans.*, 1919, xxii. 341.

§ (22) The Solar Agent responsible for Magnetic Storms and Aurorae.—As regards the nature of the solar agent responsible for the production of magnetic disturbance, it seems possible to assert, negatively, that it cannot be ultra-violet light, since this would affect chiefly the sunlit hemisphere, whereas the disturbance agent favours the P.M. hemisphere and the auroral zones. The latter fact also excludes merely material particles without electric charge, for there is no reason why these should crowd towards the polar regions. Electric particles of some kind offer the only alternative. The experiments of Birkeland[2] and the mathematical calculations of Störmer[3] upon the paths of electric corpuscles projected towards a uniformly magnetised sphere show that the particles would be deflected towards the magnetic poles and fall chiefly upon two zones resembling the auroral zones. The investigation of the paths is a matter of great analytical difficulty, even when the mutual influence of the particles is neglected, but it is clear that the particles can be deflected round the earth and fall on the night hemisphere as well as on the sunlit one. Störmer has also been able to show that conical streams of particles could spread out into thin bands along the auroral zone, like aurora curtains. One of the outstanding difficulties of auroral theory, however, has been to account for the observed angular radius of the auroral zones; the radius should depend on the charge, mass, and speed of projection towards the earth, but none of the hypotheses as to the values of these quantities fits the observed radius. Birkeland originally supposed that the particles were cathode-rays, and afterwards that they were β-particles travelling with a velocity approaching that of light, while Vegard first suggested that they may be α-particles, but has recently reverted to the view that the charge is negative. The calculated radii for negative particles of electronic mass range from 2° to 6°, according to their velocity, while for α-rays the numbers vary from 16° to 19°; the observed radius of the zone of maximum auroral frequency is about 23°, and at times of great magnetic disturbance, which is always accompanied by unusually intense auroral displays, aurorae may be observed at an angular distance from

[2] Birkeland, *The Norwegian Aurora-Polaris Expedition, 1902–1903*, 1913, i. 603.

[3] Störmer; cf. references in *Terr. Mag.*, 1917, xxii. 23; also Vegard, "Nordlichtuntersuchungen," Kristiania, *Vid. Sk. I., Mat. Natur. Kl.*, 1916, and *Geofys. Mem.*, 1920, i.

the magnetic pole as great as 60°. This illustrates a further difficulty of the Birkeland theory, which strictly limits the area over which the particles can enter the earth's atmosphere, whereas auroral observation shows that the assigned limits are transcended, while the evidence of magnetic disturbance suggests that particles enter over a yet wider area still, even round the earth's equator. This may possibly be due to the mutual influence of the charges.

The hypothesis that the solar streams which produce magnetic storms and aurorae are on the whole neutral, but ionised, has been put forward recently by Lindemann [1] in connection with the theory of magnetic storms; it is supposed, on this view, that the light ions are stopped on their first encounter in the atmosphere, while the heavier positive ions penetrate further and produce luminous auroral phenomena where their intensity is sufficiently great. The theory avoids the difficulty which has beset most theories of magnetic storms, in that the stream will not tend to dissipate itself sideways by the mutual repulsion of its members, as it would if the particles were all of the same sign, the stream being of the volume density necessary to produce appreciable magnetic effects. Lindemann has shown, on the basis of reasonable assumptions, that if the particles travel with a speed of about 10^8 cm. per sec., so that they would take roughly two days to reach the earth from the sun, no serious degree of recombination of the ions on the way need be supposed to occur. Calculations (as yet unpublished) show that such a stream might be able to precipitate its particles nearly all over the earth, not excluding the equator, but that there would be very little if any tendency for the particles to favour the polar regions. The facts thus seem to require an appreciable residual charge in the stream.

§ (23) The Mechanism of Magnetic Storms.—Schuster [2] has shown that no form of stream theory of magnetic storms is tenable which attributes the disturbance field to the direct magnetic effect of a stream of electric charges acting like an electric current; the obstacle in the way of all such theories is the one just mentioned, *i.e.* the mutual repulsion of the charges in the stream. The variations of the earth's magnetic field during a storm, and especially the "fluctuations," which vary appreciably over an area no greater than that of England, strongly suggest that the storm is produced in the earth's atmosphere, by electric currents flowing in some layer during the progress of the storm, and afterwards dying away. A recent theory [3] of the production of such currents is based on the fact that the latter, as described in § (20), are of the type which would arise from the radial motion of a conducting layer of the atmosphere, by induction across the horizontal component of the earth's permanent magnetic field. The radial motion must be outwards, in order to explain the diminution of H.F. which is the main regular feature of a magnetic storm. The initial increase of H.F. would similarly be produced by a radial motion inwards.

The radial motion was accounted for in the original form of this theory by attributing it to the mutual repulsion and escape from the atmosphere of the injected corpuscles, supposedly of one sign only of electric charge. As above mentioned, Lindemann [4] has shown that this is not possible, and that the solar stream must be practically neutral. But it seems likely (though the details have not yet been worked out) that the same electromagnetic effects as were deduced in the first form of the theory follow also from this modified form, particularly if, as seems probable, the molecules to which the electrons become attached on the confines of the atmosphere are heavier than the positive ions injected. The latter, which perhaps are hydrogen atoms or molecules, would then rise to neutralise the former. The surface of the earth would be protected from changes of electric potential gradient during the process of injection and neutralisation by the other conducting layers lower down in the atmosphere.

Though the injection seems to be most intense in the auroral zones, as visual observation and also the study of S_d suggest, the electromotive forces at the equator may be considerable, because the H.F. is there at its maximum. The resulting currents would tend to choose the strongly conducting auroral zone of the current sheet as the most favourable return path, as indicated in *Fig.* 10.

§ (24) The Situation of the Conducting Layers in the Atmosphere.—As has already been stated, very little direct evidence is available to enable the height of the conducting layers of the atmosphere to be determined. The layer in which magnetic storms are produced may with fair probability be placed above the lower limit of the auroral layer; this layer extends from about 85 or 90 km. height upwards to heights of 300 km. or more. It seems probable that the diurnal magnetic variations originate at a lower level. Reasons [5] have been assigned in §§ (17), (18) for supposing that S_q and (L) are produced in distinct layers ionised by different solar agents, the second of which varies in

[1] Lindemann, *Phil. Mag.*, Dec. 1919.
[2] Schuster, *Roy. Soc. Proc.* A., 1911, lxxxv. 44.
[3] *Roy. Soc. Proc.* A, 1918, xcv. 61.
[4] Lindemann, *Phil. Mag.*, Dec. 1919.
[5] Chapman and Milne, *Roy. Met. Soc. Quarterly Journal*, 1920, xlvi. 357.

unison with the magnetic activity, and must therefore be associated with the ionised streams from the sun. The S_q layer is probably the lowest, for the corresponding atmospheric circulation is of thermal and not tidal origin, and may be expected to diminish in amplitude with increasing height in the atmosphere. If it were present to any extent in the (L) layer S_q would vary with the magnetic activity, like (L) itself. Since it does not do this, the (L) layer must be the upper of the two. The lunar atmospheric tide, being produced by "body" forces, acting on all parts of the air proportionately to their mass, should have the same velocity of circulation (apart from resonance effects) at all heights. The fact that (L) is greater over the sunlit than over the dark hemisphere, and not greater over the P.M. than over the A.M. hemisphere, at any rate to the same degree, suggests that it arises in a layer different from the auroral layer. This (L) layer must either be more conducting than the auroral layer, or else the lunar tidal circulation must be less intense in the latter, which might possibly be the case if the (L) layer is the lower of the two.

A lower limit for the S_q layer is afforded by considering how far γ-rays, the most penetrating radiation known, would penetrate the atmosphere. The absorption of any kind of ultra-violet radiation is exponential in a uniform layer; also the rate of increase of density of the atmosphere, from the outside inwards, is exponential. Thus the absorption is at first small, gradually increasing to a maximum, and then rapidly diminishing. The height of the level of maximum absorption proves to depend only on the nature of the radiation, and not on its intensity, and the layer is defined more sharply below than above. Thus the height of the S_q, and also of the (L) layer—if the latter is ionised by wave-radiation —will not change with the sunspot period or with magnetic activity. In the case of γ-radiation the height of maximum absorption and ionisation is 26 km., and the intensity falls to 1/10th of this on either side at heights of about 18 and 45 km. These figures refer to direct incidence, *i.e.* at points where the sun is in the zenith. Towards the twilight circle the height is greater, owing to the oblique penetration of the rays. Thus at grazing incidence the maximum intensity is at 50 km. height for γ-rays.

The conductivity of the S_q layer can be estimated from the comparison of the magnetic changes with the corresponding barometric variation. The maximum value in the sunlit hemisphere and in sunspot maximum years must be not less than $25 \cdot 10^{-6}$, a high value which can only be accounted for by some powerful ionising agency.[1] The conductivity of the (L) layer must be at least equally great. The S_q layer is presumably the one chiefly concerned in the deflection of wireless waves round the earth, which makes long-distance wireless telegraphy possible.

[1] *Roy. Soc. Phil. Trans.* A, 1917, ccxviii. 113.

§ (25) Earth Currents.—At times of magnetic disturbance unusually strong earth currents are observed. These bear a close relation to the magnetic fluctuations, and are to be ascribed to induction in the conducting surface layer of the earth by the varying magnetic field of the external current sheet. As is to be expected, the earth currents are greater, for a given amplitude of oscillation in the magnetic field, the more rapid the oscillation. The slow diurnal magnetic variations are associated with relatively small earth currents, although the internal induced field of the diurnal variations is about a third of the whole S_q-field at the earth's surface. The phase of the internal field, however, indicates that it is produced deeper down in the earth; it appears, in fact, that the currents in the shallow moist conducting surface layer are negligible compared with those in the core, and that between the surface and the core there is a relatively non-conducting stratum of about 250 km. thickness. The specific conductivity of the core is estimated at about $3 \cdot 10^{-13}$ C.G.S., which is considerably greater than that of dry rock, but less than a tenth that of sea-water. It is to be expected that the magnetic variations produce much stronger electric currents in the oceans than are observed on land, and that these currents may appreciably affect the diurnal magnetic variations over the ocean.

S. C.

Magnetism, Terrestrial and Solar, Theories concerning:

Direct Magnetisation by Rotation. See "Magnetism, Terrestrial and Solar, Theories of," § (10).

Galvanic Currents. See *ibid.* § (6).

General Considerations. See *ibid.* § (7).

Irregularities in the Earth's Magnetic Field. See *ibid.* § (11).

Permanent Magnetism. See *ibid.* § (5).

Rotation of Charge. See *ibid.* § (8).

Rotation of Separated Charges. See *ibid.* § (9).

MAGNETO, THE HIGH-TENSION

Electric ignition is largely responsible for the enormous progress that has been made, within the last two decades, in the design and construction of high-speed explosion motors. At the present day some system of ignition by high-tension spark is universally adopted on all petrol motors. Although the high-tension magneto is not always used as an ignition

device, its adoption is very general in England and on the Continent for all purposes, whilst in America the battery-coil system is used on the great majority of pleasure cars, and the magneto finds its chief application on motor trucks, tractors, and motor boats, where electric starting is not fitted.

§ (1) WHAT A MAGNETO DOES.—A magneto is a small electrical machine which, when used with a petrol motor, is geared to and driven by the engine (*Fig.* 1). The internal construction is such that, at regular intervals of time, very rapid voltage rises are generated in its high-tension winding, in consequence of which a rapid succession of sparks occur at the electrodes of the sparking plugs fitted to the engine cylinders. At the moment when the spark is initiated, the cylinder contains an explosive mixture of petrol and air which is compressed to about 80 lbs. per sq. in., and owing to the high pressure and the state of turbulence existing in the cylinder, the explosion is rapidly propagated throughout the mixture. The piston receives in consequence of this what is virtually a blow, which in a large aeroplane engine may reach a value of several tons. This blow is communicated to the crank-shaft, and as in a multi-cylinder engine there may be as many as 200 distinct and separate blows per second, it is easy to understand that under these conditions the crank-shaft experiences what is virtually a steady driving torque of considerable magnitude.

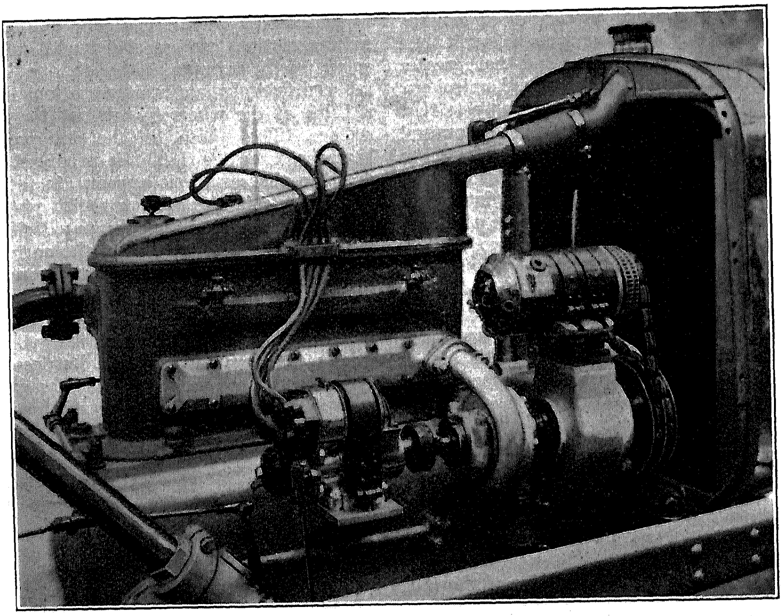


FIG. 1.—Illustration of a Motor Car Engine showing method of mounting Driving Magneto.

The voltage at which the spark occurs is, on the average, in the neighbourhood of 4000, although the magneto is capable of generating a maximum voltage at least three times as great as this. We can, therefore, in very few words, describe a high-tension magneto as an electric generator of very small compass (weighing anything from 5 to 15 lbs.), which is capable of generating at regular time intervals—that may be as small as 0·005 seconds—enormously rapid pressure rises with a peak voltage as great as 12,000 volts.

§ (2) THE PHENOMENON OF A SPARK DISCHARGE.—As the primary function of a magneto is to produce a regular sequence of high-tension sparks in rapid succession, it is desirable at the outset to analyse briefly what happens when a spark occurs between metal electrodes, as in a sparking plug, so that the operation of the magneto in producing these sparks can be more clearly understood. Consider the fundamental case wherein two metal electrodes A and B are linked to the ends of a spark generator. If we assume that the voltage of the generator is slowly rising, an electrostatic field is gradually established in the space between the opposing faces of the electrodes A and B, and the gaseous medium in this space becomes slowly ionised until, corresponding to some critical value of the voltage E, the stress is sufficient to "rupture" the medium. At this moment a flow of electricity occurs

between A and B which manifests itself as an electric spark.

The voltage at which a spark occurs—termed the sparking voltage of the gap—depends on a large number of factors, as follows:

(1) Shape and disposition of the sparking electrodes.

(2) The nature of the gaseous medium.

(3) The pressure and temperature of the gaseous medium.

(4) The state of turbulence in the gaseous medium.

(5) The direction of the current in the gap.

(6) The rate at which the voltage is applied by the spark generator.

Tests which the author has made on large numbers of representative sparking plugs have shown that, even when the gap length is adjusted to some definite value (0·02 in.), the sparking voltage corresponding to normal engine conditions may vary over a 2 : 1 range. This difference is due to variations in the design of the plug electrodes. Corresponding to the normal full-load operating condition, it is doubtful whether the sparking voltage exceeds 5000 in any of the various standard types of plugs now in use, and generally it will be less than this figure. At starting, however, when everything is cold, the sparking voltage will be much higher, and figures of 7000 or 8000 volts are not uncommon. It is obvious that these external conditions determine the "load" that is imposed on the magneto, and the primary aim of a magneto designer should be to produce a machine that is capable of generating a spark at the plug electrodes under the worst conditions likely to obtain in practice.

A great deal of work has been carried out in an attempt to solve the question as to the particular property or characteristic of the spark which determines ignition. This problem will not be touched upon now, and it will suffice to mention that the view is generally held by ignition experts that it is desirable, in designing a high-tension magneto or any other form of spark generator, to ensure that the voltage generated in the machine rises to its maximum value in the shortest possible time. In other words, the rate of voltage rise, or the steepness of the voltage wave front (see *Fig.* 2) must be as large as possible. In the modern high-tension magneto, the rate of voltage rise at the moment of "break" may reach the value of 300 million volts per second, and machines have been constructed which are capable of generating 300 distinct sparks per second. From this it will be realised that the modern high-tension magneto is a very wonderful piece of electrical apparatus.

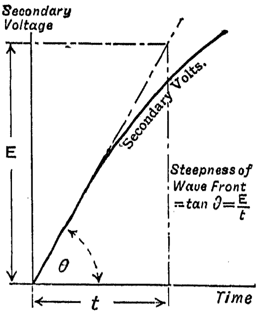


FIG. 2.

§ (3) FUNDAMENTAL PRINCIPLE UNDERLYING THE OPERATION OF THE H.T. MAGNETO.—The basic principle underlying the construction and operation of the high-tension magneto is that of electro-magnetic induction, discovered by Faraday in 1831. Indeed, the modern high-tension magneto has a very intimate connection with the classical ring experiment of Faraday's, shown diagrammatically in *Fig.* 3, whereby he was able to induce an

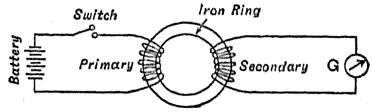


FIG. 3.

E.M.F. in the secondary coil B, first, on closing the circuit of the primary coil A, and secondly, on opening this circuit. The flux and voltage changes corresponding to this sequence of operations are depicted by the curves given in *Fig.* 4.

The evolution of the induction coil was one of the first commercial developments in the electrical field, subsequent to Faraday's epoch-making discovery. To obtain a large

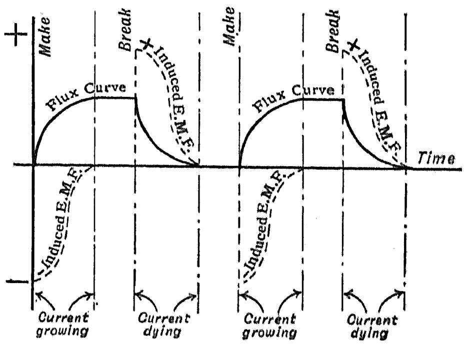


FIG. 4.

induced E.M.F. in the secondary, using the combination of windings represented in *Fig.* 3, it is only necessary to wind the primary with a few turns of thick wire, and the secondary with a relatively large number of turns of thin wire, and at the same time make provision for "making" and "breaking" the primary circuit at a very rapid rate. The French mechanician, Ruhmkorff, did much in his day to perfect the design of the induction coil, and he was the first to use a condenser

in conjunction with the interrupter—a practice which is still adopted not only on the induction coil, but also on the magneto—profiting by the highly important discovery made by Fizeau in 1853.

Now a high-tension magneto embodies many of the essential features of the induction coil. In fact, the main difference between the two forms of spark generator is that with an induction coil, the current, which is first established in the primary winding and then suddenly destroyed, is derived from *some external source* such as a battery, whereas in a magneto this current *is actually induced in the closed primary* circuit by rotation. This sets up an alternating magnetic flux in the windings of both primary and secondary, producing in the former, which are of low resistance, a considerable primary current and in the latter a small voltage. Superposed on this flux in the secondary is the large variable flux due to the mutual induction between it and the primary.

The secondary voltage produced by this varies as the rate of change of the primary current. The interrupter breaks this circuit very suddenly, and the magnetic field associated with the primary current, which for the most part is linked with the secondary winding, is rapidly destroyed, thereby inducing an enormous rise of voltage in the secondary. Following the opening of the primary circuit, the sequence of operations is much the same in the two forms of spark generator, with this important difference, however, that in the magneto, the magnetic field not only dies away to zero, *but actually reverses its direction.* With an induction coil, there is no reversal of flux at " break."

§ (4) Evolution of the H.T. Magneto.—Although the induction coil was perfected nearly seventy years ago, the high-tension magneto has been evolved only within comparatively recent times. Marcus appears to have been the first man to construct a magneto for ignition purposes. His was a low-tension machine, having the now familiar form of H armature; the current generated in the rotating winding was broken at predetermined times in the engine cylinder by a system of cams and levers.

In 1898 Simms and Bosch developed a low-tension magneto, using a fixed H armature in conjunction with rotating iron segments which produced the necessary flux changes in the armature core. Subsequently, by the addition of a secondary winding on the armature core, a high-tension magneto was evolved. This type of machine is known as a sleeve inductor magneto, and it generates four sparks per revolution.

The Bosch Co., of Stuttgart, Germany, must receive the credit of having thoroughly established the fact that a high-tension magneto can be manufactured on a commercial basis to give reliable and efficient ignition in practice. Although this important industry was allowed to develop in Germany, the modern high-tension magneto was first conceived in France by the Frenchman, M. Boudeville, who unfortunately omitted to include a condenser in his scheme for eliminating sparking at the contacts. It is surprising that Boudeville should have overlooked this feature, because the idea of using a condenser for such a purpose is of French origin, the Frenchman Fizeau being the first to suggest, in 1853, connecting a condenser in parallel with the contacts on a Ruhmkorff coil to prevent excessive sparking.

§ (5) Essential Components and Types.—*Fig.* 5 shows in diagrammatic form some of

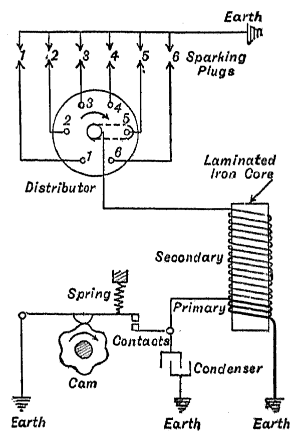


Fig. 5.

the essential components of a modern H.T. magneto. These can be grouped as follows:

(1) A magnet system.

(2) An iron core wound with primary and secondary.

(3) A revolving member for producing cyclic flux reversals in the iron core.

(4) A contact breaker to interrupt the primary circuit at predetermined intervals.

(5) A high-tension distributor.

The earthed contact is actuated by some form of cam, so that the primary circuit is continually being closed and opened for definite angular periods.

The iron core carrying the windings is always laminated, and it may either revolve or be fixed. Depending on the form of construction adopted, we have two distinct types of H.T. magnetos.

(i.) A rotating armature type, in which the familiar H armature, carrying a primary and a secondary winding, revolves between the two poles of a horse-shoe permanent magnet. This type of magneto is only capable of generating either one or two sparks per revolu-

tion of the armature, and it is frequently referred to as a 2-spark machine.

(ii.) A polar inductor type, in which the armature core with its windings is stationary. In this design the rotating member comprises a number of iron inductors (usually four) which co-operate with the fixed poles of the permanent magnet, on the one hand, and with the fixed pole-pieces attached to the armature core on the other hand, so as to produce cyclic reversals of flux in the armature core. This type of magneto is usually designed to generate four sparks per revolution, in which case it is known as a 4-spark machine. Machines can, however, be designed on this principle to give any even number of sparks per revolution.

§ (6) Cycle of Operations in a Magneto.—If we first imagine that the interrupter or contact breaker is inoperative, and that both windings are open circuited, then there will be set up in the armature core, in consequence of rotation, an alternating flux which will follow the flat-topped curve given in *Fig.* 6. If the

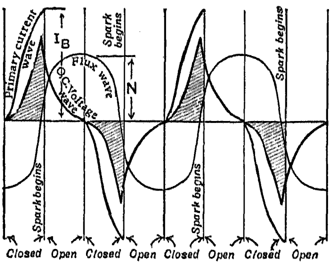


Fig. 6.

exact shape of this curve be known, it is easy to determine the shape of the alternating E.M.F. wave induced in both primary and secondary, by rotation. Concentrating our attention on the primary winding, we have, at any moment,

$$e = -\frac{S_1}{10^8}\frac{d\Phi}{dt} \text{ volts}, \quad . \quad . \quad . \quad (1)$$

where e = instantaneous voltage induced in the primary by rotation,

$d\Phi/dt$ = rate of change of the flux Φ at the moment under consideration,

S_1 = number of primary turns.

The peaky curve, also given in *Fig.* 6, shows how the induced primary voltage e varies with respect to time. It should be particularly noted that the induced E.M.F. e is zero when the flux Φ is a maximum, and the maximum point on the E.M.F. wave is reached when the flux in the armature core is zero; that is, at the moment of its sudden reversal.

Now let us examine what happens when the contact breaker is brought into action. This device is operated mechanically, and it is designed to first short-circuit the primary winding for a given period, and then suddenly open-circuit this winding. The ratio between the period of open circuit and the period of closed circuit is constant at all speeds, and the general practice in magneto design is to make this ratio 0·8. Actually, the primary circuit is closed (with the magneto fully advanced) when the induced voltage, e, is zero, and the contacts do not open this circuit again until the maximum point on the voltage curve has been passed.

We can, therefore, assume that the portion of the voltage curve e, operative during this period of closure (and this part of the wave is shaded in *Fig.* 6) is contributing to the establishment of current in the closed primary circuit, in accordance with the following law:

$$e = ri + \frac{L\,di}{dt}, \quad . \quad . \quad . \quad (2)$$

where i = instantaneous current induced in closed primary winding in amperes,

r = resistance of primary in ohms,

L = instantaneous self-induction of primary in henries,

di/dt = rate of change of the primary current at the instant under consideration.

Knowing the shape of the induced E.M.F. wave on open circuit, it is a relatively easy matter, working from the fundamental equation (2), to apply a step by step method to determine the shape of the primary current curve. A third curve, showing the growth of the current in the primary during the period of closure, is added to *Fig.* 6, whilst *Fig.* 7

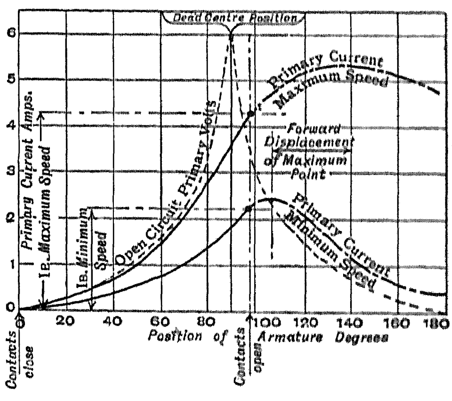
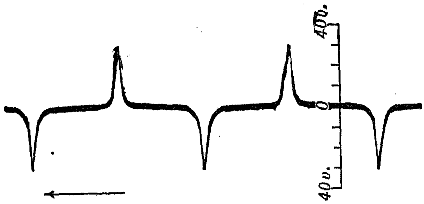


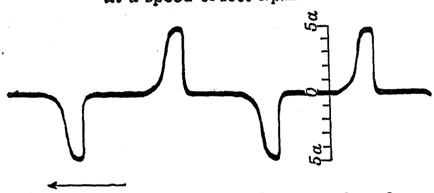
Fig. 7.

gives two typical current curves which have been calculated for the minimum and maximum operating speeds of a magneto. The

oscillograms given in *Fig.* 8, showing actual open circuit primary voltage and primary current waves, should also be of interest.



(a) Open circuit primary voltage wave at a speed of 1000 r.p.m.



(b) Primary current wave showing value of I_B at a speed of 1000 r.p.m.

Fig. 8.

We are mainly concerned with the value of the primary current (I_B) at the moment of "break"; that is, at the instant when the contacts are separated by the cam. The curve given in *Fig.* 9 shows how the value of I_B varies with the speed as measured by the sparks per minute, and it should be noted that the maximum value is closely approached at a low speed. Calling the self-induction of the primary at the moment of "break" L, then we have in the primary winding an amount of electro-magnetic energy equal to $\frac{1}{2}L_A I_B^2$ joules.[1] When the contacts separate, this energy is, as it were, projected with extreme suddenness into the secondary winding at a greatly enhanced pressure, and a portion of it appears as heat liberated by the high-tension spark at the plug electrodes. The transfer of energy from primary to secondary occurs with extraordinary rapidity, by reason of the fact that the collapse and reversal of flux in the armature core (see *Fig.* 10) takes place in a very minute fraction of a second.

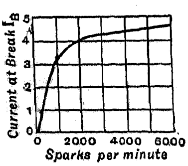


Fig. 9.

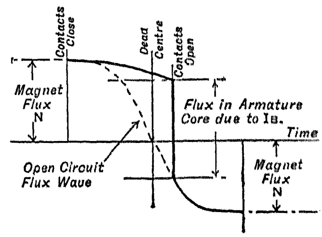


Fig. 10.

The rate of transfer of energy from primary to secondary at "break," as well as the rate of voltage rise in the secondary, is naturally dependent on the rapidity with which the flux collapses and reverses its direction. It has already been pointed out that from a pure ignition standpoint it is desirable to speed up the growth of voltage in the secondary just as much as possible. The practical aspect of this, as reflected in magneto design, is that it is vitally important to laminate carefully the iron core which carries the windings, and eliminate from the magnetic circuit any solid metal parts that may have induced in them eddy currents which, by reacting on the rapidly changing flux, will tend to retard this change.

§ (7) How Flux Changes are produced by Rotation.—We shall now briefly consider the magnetic changes accompanying rotation, which give rise to an alternating voltage in the primary. In what follows it is, of course, assumed that both windings are continually open-circuited, the contact breaker being inoperative.

(i.) *Rotating Armature Type.*—In this type of magneto an H armature, wound with primary and secondary, revolves in the space between two pole-pieces attached to the ends of a U-shaped permanent magnet, the length of air gap being from 0·004″ to 0·006″. *Fig.* 11 shows four positions of the armature, and the distribution of flux in each case.

In position No. 2, each pole-piece completely embraces one end of the armature core, and the flux passing through the core is a maximum for this position. After the armature has rotated to position No. 3, the flux has been but slightly reduced in value, and no marked change occurs until position No. 4 is reached. Here the axis of the armature core is at right angles to the magnetic axis, and the magnet flux in passing from one pole to the other travels, for the most part, through the polar ends of the armature core. That is, the flux passing through the armature core is, at this moment, reduced to zero, and the armature is

[1] Actually, the current which is induced in the closed primary circuit reacts on the magnet flux, in accordance with Lenz's law, so as to prevent any substantial change in the value of the armature core flux during the period of closure (see *Fig.* 10). As the moment of "break" occurs somewhat beyond the position of zero flux, with the primary open-circuited, it is obvious that the magnet flux is considerably distorted just prior to "break," in much the same way that a spring might be stretched by an applied force.

So soon as the constraining influence is destroyed by the opening of the contacts, the magnet flux instantly re-establishes itself in the *opposite* direction in the armature core, thereby inducing a very rapid rise of pressure in the secondary. For dealing with the problem quantitatively it is convenient to consider the current I_B, and the interlinking with the secondary winding of the flux $(LI_B/S_1)\times 10^8$ which this current is capable of generating.

commonly referred to as being in the "dead-centre" position. The slightest angular move-

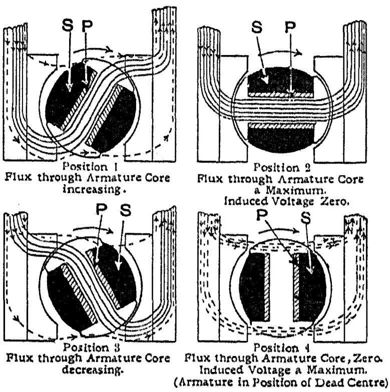


FIG. 11.

ment beyond this critical position of zero flux will cause a sudden reversal of flux in the armature core, for the reason that the polar ends on the armature core overlap the tips of the two fixed poles by a very slight amount—usually only 0·5 mm. on each side—which corresponds to about 1° of angular movement.

It follows from this, therefore, that a very *sudden reversal* of flux in the armature core occurs every time the armature reaches the "dead-centre" position, which happens every half-revolution. This means that there are two flux reversals, and two peaks to the induced primary voltage wave, every revolution. It is thus possible to close and open the primary circuit by means of the contact breaker twice every revolution, and thus generate two sparks. The curves given in *Fig.* 6 actually show four flux reversals and four sparks. They correspond, therefore, to two revolutions of a rotating armature magneto designed to generate two sparks per revolution.

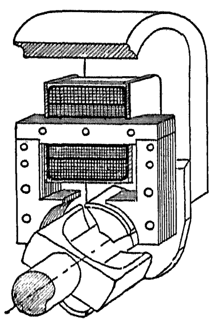


FIG. 12.—4-Spark Polar Inductor Magneto showing Magnetic Circuit.

(ii.) *Polar Inductor Magneto.*—This type of magneto is most commonly built as a 4-spark machine, and we shall therefore deal with this construction. The distinctive feature of this design is that the armature core and windings are fixed, and the requisite flux reversals are produced by a rotor which carries four iron polar inductors (see *Fig.* 12). These are carried by a shaft of non-magnetic material, and spaced 90° apart, the two inductors which are diametrically opposite one another being linked together to form a pair. There are thus two pairs of inductors, and the construction is such that one pair continually receives flux from the north pole of the magnet, whilst the other pair continually delivers flux to the south pole. Taking the inductors in the order in which they are arranged on the shaft, we can look upon them as being alternately north and south poles giving a constant pole sequence of N-S-N-S.

The arrangement of the magnetic circuit is indicated diagrammatically in *Fig.* 13, which shows the rotor in three different positions, and the flux distribution in each case. It should be particularly noted that the laminated armature core carrying the windings is attached to the ends of two laminated members whose polar ends embrace the rotor at points 90° apart. In position No. 1, the flux is flowing from one inductor marked N, through the armature core from left to right, to one inductor marked S. The flux in the armature core is a maximum for

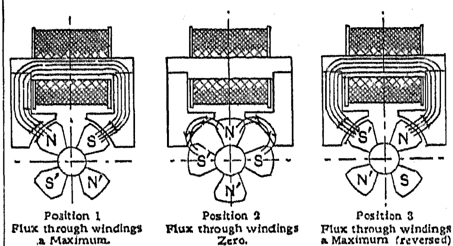


FIG. 13.

this position, and the other two inductors are inoperative.

After the rotor has moved through 45° to position No. 2, a different state of affairs is created. The flux now completes its circuit between the N and S inductors, mainly through the pole faces of the fixed laminated members which support the armature core, and the flux passing through the latter is at this moment zero. We can speak of the rotor as now being in the "dead-centre" position, and a very sudden reversal of flux in the armature core occurs immediately this position is passed. After another 45° movement, the armature core flux is again a maximum, but in the opposite direction, as shown in position No. 3, and it is clear that there must be *four* positions of zero flux and thus four flux reversals in each revolution. It is therefore possible to close and open the primary circuit four times in each revolution, and thus obtain four high-tension sparks. The curves given in *Fig.* 6 correspond to

one complete revolution of a magneto of this type.

§ (8) THE FUNCTION OF THE CONDENSER AND ITS BEARING ON THE PHENOMENON OF CONTACT ARCING.—It has already been mentioned that a condenser is connected in parallel with the contacts which control the primary circuit, its main function being to eliminate the sparking which tends to occur at the moment of "break." Magneto condensers are made up of alternate layers of mica and tinfoil, the thickness of the mica plates ranging between 0·001″ and 0·002″. It is found desirable in practice, owing to the high voltage that is suddenly imposed on the condenser at each "break" and the necessarily thin sheets that have to be used to limit the size of the condenser, to use ruby mica of the very best quality. The capacity of a magneto condenser ranges between 0·075 micro-farad in the case of a small single-cylinder machine to something of the order of 0·25 micro-farad for a large multi-cylinder magneto.

The peculiar property of a condenser in which we are interested arises from the fact that when a rapidly increasing voltage is applied to its terminals, a current will flow into the condenser, the value of this current being, at any moment, proportional to the rate of change of the applied voltage. That is

$$i = C \cdot \frac{de}{dt}, \qquad . \quad . \quad . \quad (3)$$

where i = charging current in amperes,
C = capacity of condenser in farads,
de/dt = rate of change of the voltage between the terminals of the condenser.

So long as the voltage between the ends of the condenser remains constant ($de/dt = 0$) no current will flow through it, and it behaves as a perfect insulator.

Let us then consider what happens at the moment when the contacts begin to move apart. If there were no condenser, the current flowing between the two contact faces would tend to diminish, and corresponding to this change, a large E.M.F. of self-induction would be generated in the coil which would—in conformity with Lenz's law—tend to maintain the current by producing an arc that would be extended in length as the contacts moved apart. The arcing would not only rapidly burn away the contacts, but in consequence of the primary current being prolonged in this way, the rate of change of the flux in the armature core would be very slow, and the secondary voltage correspondingly reduced in value. Without the aid of a condenser, therefore, it is no exaggeration to say that it would be quite impossible to construct a satisfactory H.T. Magneto.

The use of a condenser in parallel with the contacts does two things:

(1) It eliminates arcing at the contact faces.

(2) It speeds up the collapse and reversal of flux in the armature core, thus enabling the induced secondary voltage to reach a very high maximum.

Briefly, the condenser enables these results to be achieved by absorbing the energy which would otherwise dissipate itself in the spark between the contacts, and so retarding the rate of growth of voltage between them for a very brief period of time, subsequent to the moment of break. If we concentrate our attention on the minute air gap between the opposing contact faces—and we are now considering gaps of the order of a few ten-thousandths of an inch only—we can, for the moment, neglect the rest of the circuit, and simply look upon the contacts as two electrodes which are being rapidly separated. That is, the length of air gap is being very quickly increased, which means that the sparking voltage of the gap will also increase in value as the contacts move apart. In *Fig.* 14 we

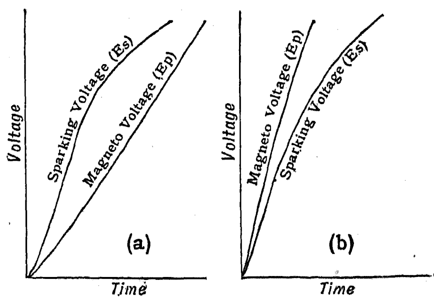


FIG. 14.

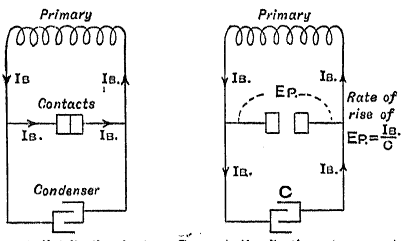
have indicated a hypothetical curve showing how the sparking voltage (E_S) increases with the gap-length for the minute gaps under consideration.

It should be remarked that no very definite information appears to be available concerning the sparking voltage characteristics of very minute gaps, although the author is led to believe, from certain tests which he has made, that in this case the sparking voltage depends greatly on the nature of the electrode material. In the case of large gaps—in excess of 1 mm. length—it is generally agreed that the sparking voltage is not affected by the electrode material.

But as the contacts separate, a rapidly increasing voltage is set up between them by the collapsing magnetic field linked with the primary winding. A second curve, showing the growth of this voltage (E_P), is added to *Fig.* 14. It is purposely shown, in diagram (*a*), to fall below the sparking voltage curve, to indicate the condition of sparkless operation of the contacts. Clearly, with this relationship between the two curves, the actual voltage E_P will always be less than the sparking voltage E_S during the separation of the contacts,

and no sparking will therefore occur.[1] If the E_S curve fell below the E_P curve, a spark would result, which would very quickly degenerate into an arc. This condition is represented in diagram (*b*).

Broadly speaking, the slope of the E_P curve at the origin is controlled by the capacity of the condenser. As this is increased in value, so the rate of voltage rise is reduced, and *vice versa*. The ideal case is depicted diagrammatically in *Fig.* 15, wherein it is



Current distribution just before separation of contacts.

Current distribution at moment of separation, assuming a perfect break.

FIG. 15.

assumed that at the very moment of "break" the whole current I_B flowing through the contacts is, as it were, suddenly deflected from its path and made to flow into the condenser as a charging current. This corresponds to absolutely sparkless operation of the contacts, and gives the rate of voltage rise at the moment of "break," for equation (3) becomes

$$\frac{dE_P}{dt}=\frac{I_B}{C}, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where I_B = primary current at "break,"
dE_P/dt = rate of voltage rise between contacts at "break,"
C = capacity of condenser in farads.

Equation (4) shows at a glance how it is that bad contact arcing can be improved by using a larger condenser. The value of dE_P/dt is thereby reduced, and the induced voltage curve is brought into correct relation with the sparking voltage curve. To obtain the best results with a condenser of given capacity, it is necessary that the self-induction of the condenser circuit should be negligible, so that the current flowing into the condenser at the moment of separation of the contacts can instantly rise to its maximum value.

§ (9) THE INITIATION OF THE HIGH-TENSION SPARK AT "BREAK."—Let us now analyse what happens when the high-tension spark is initiated. We have, in so doing, to concern ourselves with what happens in a very minute fraction of a second—of the order of $\frac{1}{10,000}$ of a second—immediately following the first separation of the contacts. At the beginning of this period the armature—in the case of a rotating armature type of magneto—is in the position shown in diagram I. in *Fig.* 16, which is usually about 5° beyond the "dead-centre" position with the timing of the magneto fully advanced. The current I_B is assumed to be

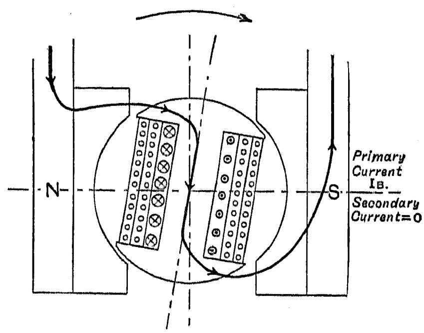


Diagram I. Just before "Break"

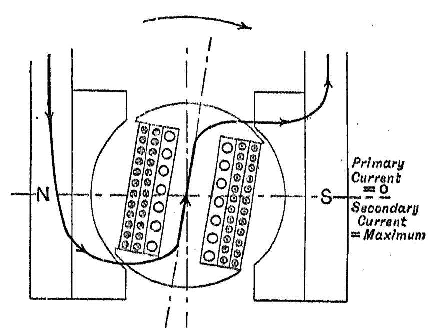


Diagram II. Just after "Break"

FIG. 16.

flowing in the primary winding at this moment, and the path of the flux from N to S pole is distorted in the manner indicated in the diagram.

Then very suddenly the contacts begin to move apart, and, assuming a sparkless break, the primary current will, with extreme rapidity, fall away to zero in accordance with the first portion of the current wave given in

[1] It is rightly contended by several authorities that the problem of securing sparkless operation of the contacts resolves itself into the prevention of the formation of an arc at the moment of separation. This view does not appear to be incompatible with the theory of contact sparking set forth above. It is certain that the conditions obtaining at the contacts just prior to their separation have an important bearing on the matter, and such factors as the current, contact resistance, and the temperature of the contact faces are of paramount importance. At high speeds the contact faces must reach a very high temperature, with the result that a certain amount of volatilisation is bound to occur. Furthermore, the electronic emission may not be unimportant. Both of these factors will tend to reduce the resistance of the gap, and correspondingly, the slope of the E_s curve. If the heating on the contact faces is excessive, the gap resistance may be so small as to cause the formation of an arc as soon as the contacts begin to move apart. It should be noted, however, that E_s would be negligible under such conditions, and therefore, in accordance with the sparking voltage theory, the relationship between E_s and E_P would be such as to give bad contact arcing.

Fig. 17. Correspondingly, the magnetic flux in the armature core instantaneously collapses and *reverses* its direction (see *Fig.* 10), because it should be particularly noted that during this brief period the magnet is tending all the time to send flux through the armature core in the opposite direction to that in which flux is actually flowing prior to "break." The state of affairs

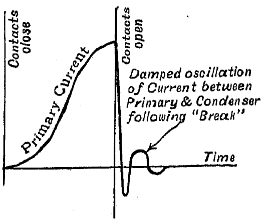


FIG. 17.

indicated in diagram II. in *Fig.* 16 is, therefore, very quickly established, and by that time the H.T. spark will have been initiated, giving a small current in the secondary winding which is so distributed (see diagram II.) as to *retard* the establishment of flux in the reverse direction.

The rapid change of flux in the armature core induces an E.M.F. in the secondary winding which rises to a very large value at a phenomenally rapid rate. Probably there is a period of voltage growth in the secondary of the order of $\frac{1}{100,000}$ of a second before the spark begins. During this minute interval we can look upon the complete secondary circuit comprising the winding, high-tension cable, and sparking plug as equivalent to a condenser of small capacity which is rapidly charged by the rising voltage. In effect, we have a small condenser of the equivalent capacity C_1 linked in parallel with the sparking plug electrodes (see *Fig.* 18), and when the

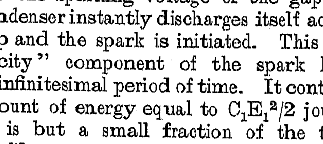


FIG. 18.

voltage between the ends of this condenser reaches the sparking voltage of the gap E_1, the condenser instantly discharges itself across the gap and the spark is initiated. This first "capacity" component of the spark lasts for an infinitesimal period of time. It contains an amount of energy equal to $C_1E_1^2/2$ joules, which is but a small fraction of the total energy liberated in the spark discharge. The capacity component is of an oscillatory nature, as high-frequency oscillations naturally occur in the local circuit formed between the spark gap and the equivalent condenser, when the latter discharges itself across the gap.

If a magneto be connected to a rotary spark gap so that by means of a rotating electrode the spark discharge can be spread over a considerable arc of a circle, the first capacity component will manifest itself as a single bright line discharge. This will be followed by a flamy and coloured discharge which subtends a considerable angle. It is this latter component of the spark which contains the bulk of the heat-energy liberated in the discharge, and it represents—subtracting the losses that occur during the transformation—the electro-magnetic energy stored in the primary winding as given by the formula $LI_B^2/2$, where L is the self-induction of the primary at "break." The total duration of the spark, in certain types of magnetos, may reach 0·003 second, and the general shape of

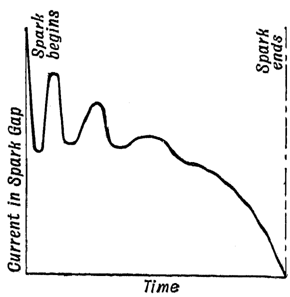


FIG. 19.

the curve representing the spark-gap current is given in *Fig.* 19.

§ (10) THE QUESTION OF SPARK ENERGY.—Recent research has demonstrated that, under *ideal* conditions, it is the first capacity component of the spark that causes ignition. The flamy portion, containing most of the heat-energy, which follows this, is of little value from a pure ignition standpoint. It has already been stated (see *Fig.* 2) that one of the main objects in magneto design should be to secure a secondary voltage wavefront which is as steep as possible, and, other things being equal, this is one of the factors which determine the capabilities of a machine as a spark generator.

It might be hastily assumed from this that there is no advantage in having a flamy portion of the spark which persists for a considerable period of time. This would be a wrong conclusion, because we have, so far, only considered the *ideal* operating conditions. In practice the conditions are frequently far from ideal, and it is these adverse conditions which really determine what the characteristics of a spark should be.

At starting, for example, when everything is cold, the mixture enters the cylinder in the form of a mist, and it is easy to picture petrol globules of quite considerable diameter floating about in the air-stream that passes across the sparking-plug electrodes. Under these condi-

tions it is reasonably certain that the heat-energy liberated by the flamy portion of the spark is efficacious in vaporising some of these globules, and thus producing in the plug gap a localised mixture of petrol and air, approximating to the ideal, which in turn is ignited by the capacity component of the spark.

Considering the normal running condition of an engine, it is probably true that with the engine thoroughly warm and the carburettor functioning in a satisfactory manner, perfect ignition will occur *so long as there is a spark.* We are here faced with a state of affairs where the heat energy of the spark is not, *per se*, of paramount importance, the capacity component being the master of the situation. Admitting this, however, it is important to note that there are factors operating—such as excessive leakage and distributed capacity in the high-tension circuit—which *tend to prevent the occurrence of a spark* by slowing up the rate of voltage rise in the secondary. To guard against imperfect ignition, due to either of these causes, it is therefore necessary to design any form of spark generator to liberate in its spark a considerable amount of energy, and quite apart from the question of ignition, *per se*, the magneto which gives the spark of greatest heat energy will, other things being equal, be best able to meet the adverse conditions imposed in practice. The spark energy curves for different forms of spark generator given in *Fig.* 20 should be of interest in this connection.

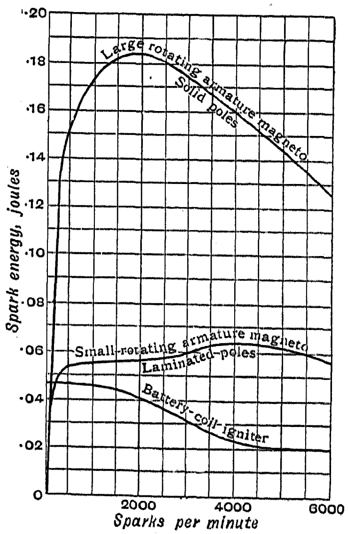


FIG. 20.

§ (11) DETAILED CONSIDERATION OF SOME OF THE MORE IMPORTANT MAGNETO COMPONENTS. (i.) *Magnet.*—The magnet is virtually the source of energy in a magneto. Its function is to maintain, in the armature core, a reasonably constant magnetic flux, despite the excessive vibration and the wide variations in temperature to which the magneto is subjected in service. The criterion of magnetic quality of a magnet steel is given by the shape of that portion of the hysteresis loop shown as a continuous curve in *Fig.* 21. If the product $(B \times H)$ for the range of the demagnetising H, corresponding to the coercive force H_C, be plotted against H, we obtain the chain-dotted curve in *Fig.* 21 which reaches a maximum point. The value of the ordinate of this

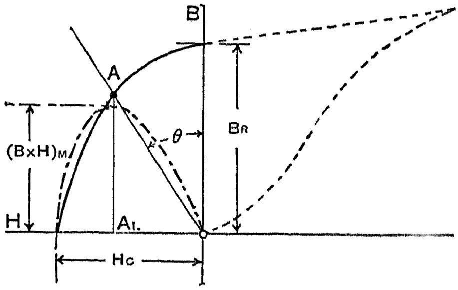


FIG. 21.

second curve corresponding to the maximum point, called $(B \times H)_M$, can be taken as a figure of merit when comparing the characteristics of different magneto magnets.

The active flux density in the magnet under working conditions can be obtained from the BH curve in *Fig.* 21 by drawing the line OA through the origin, such that

$$\tan BOA = \tan \theta$$

$$= \text{Reluctance of magnetic circuit.}$$

The ordinate AA_1 then gives the flux density in the magnet. When calculating the value of the reluctance it is of course necessary to take into account the armature reaction [1] due to the primary current, which produces an equivalent reluctance many times greater than the reluctance of the air gaps and armature core circuit. In a rotating armature type of magneto with tungsten steel magnets the total reluctance is approximately equivalent to a demagnetising force (H) of 35, giving a flux density in the magnet of from 6000 to 7000 lines per sq. cm.

Most magnets used by British manufacturers are made from a steel containing about 5 per cent of tungsten. The remanence (B_R) varies over a range of from 9000 to 11,000, corresponding to which the coercive force (H_C) may lie anywhere between 55 and 70. A magnet of good average quality would have the following characteristics:

$$B_R = 10,000,$$
$$H_C = 60,$$
$$(B \times H)_M = 250,000.$$

It is not considered desirable to use a magnet that has a coercive force less than 54, or for which the value of $(B \times H)_M$ is less than 230,000. The section and length of the magnet depend on the design factors of the magneto. In the case of a small single-cylinder

[1] See "Dynamo Electric Machinery," § (5).

magneto, the cross-section of the steel may be as low as 4 sq. cm., whilst on a large multi-cylinder machine this figure may be increased to 10 sq. cm. The length may vary from 16 cm. to 32 cm.

(ii.) *Pole-pieces and Armature Core Circuit.*—The *rate* at which the flux in the armature core changes at "break" is a factor of paramount importance in determining the efficiency of a magneto as a spark producer. The creation of eddy currents in the pole-pieces or armature core circuit, during the period of rapid flux change, should therefore be avoided as far as possible. Eddy currents, by their reaction, not only retard the flux change, but are detrimental to the extent that they waste a considerable portion of the energy stored electromagnetically in the primary, during the period of transfer to the secondary circuit. The ideal state of affairs, from this standpoint, would be an *air* circuit for conducting the magnetic lines, but this is impracticable because of its magnetic inefficiency.

One of the most important innovations introduced by British magneto designers is the use of laminated magnet pole-pieces in place of the solid cast-iron poles standardised by the Germans. A change of this kind, keeping other factors the same, causes a considerable increase in the spark energy, and greatly improves the low-speed performance, as determined by the fact that the minimum speed at which regular sparking occurs across standard gaps is considerably reduced. The majority of British magnetos are provided with laminated pole-pieces, and it will be found that, in general, these machines give a better performance than the corresponding types provided with solid pole-pieces, despite the fact that it has been possible to use on the laminated pole machines considerably less magnet steel, for reasons that have already been indicated.

Experiments which the author has made show that it is a distinct advantage to use as thin a lamination as practical considerations will permit. A thickness of 0·016 in. is recommended. It is unfortunate that the conventional H armature core cannot be entirely laminated. The end cheeks, between which the laminated centre portion is clamped, are of necessity made of solid material, and the eddy currents generated in them undoubtedly produce a detrimental effect. One of the strong arguments in favour of a polar inductor type of magneto is that the complete armature core circuit can be entirely laminated, and in a recent design of multi-cylinder machine the revolving inductors and pole-pieces are also laminated.

It is interesting to note that—other things being equal—the spark energy at a given speed is approximately proportional to the square of the flux produced in the armature core by the magnet at that speed, when both windings are open-circuited. In a rotating armature type of magneto the active flux in the armature core may vary between 25,000 lines for a small single-cylinder machine to 50,000 or 60,000 lines for a large multi-cylinder magneto. A polar inductor magneto operates with a lower average armature core flux (owing to the greater magnetic leakage in this design), and the figure usually ranges between 20,000 and 25,000 lines for large 4-spark multi-cylinder machines.

(iii.) *Armature Windings.*—In a typical H form of magneto armature enamelled copper wire is used for both primary and secondary, the primary being wound next to the core, and the secondary on top of the primary. Varnished paper, silk, and cambric are used for insulating the successive layers from one another and from the core. Wire of 0·028 in. diameter is frequently used for the primary, whilst the secondary wire may be as fine as 0·0032 in. diameter. The primary turns range between 150 and 200, and the ratio of turns is of the order of 50 : 1. The resistance of the primary is always less than 1 ohm. After winding, the secondary is completely enveloped in insulation, and the whole armature is finally bound tight with special cotton tape. A varnishing and baking process is then applied to ensure that the windings are effectively sealed with a hard coat of varnish, so as to prevent the possibility of moisture penetrating into them.

(iv.) *Contact Breaker.*—Next to the armature, the contact breaker is probably the most vital component part of a magneto. Dealing first with the rotating armature magneto, the contact breaker is always secured to the end of the armature by a single steel screw which serves to make a connection between the insulated end of the primary and the block on the contact breaker base, which carries the insulated and adjustable contact screw. A steel spring-controlled lever is free to oscillate on a bearing, and this carries at one extremity the other contact which is earthed through the control spring. The two contacts are normally brought together by the force of the spring, and they are separated when the fibre heel, secured to the other end of the lever, comes into contact with one of the steel cams fixed inside the housing (called the cam ring) that surrounds the revolving contact breaker.

In *Fig.* 22 we see the details of the contact breaker fitted to the B.T.H. type of magneto. A plain lever arm is used in this design, and the bearing comprises a stout fibre bush, mounted on the contact breaker base, which fits into a highly polished hole in the lever. In another design, very extensively used, the lever has a projecting pin which fits into a fibre bush mounted in a recess in the base of the contact breaker. The cams may either be two separate segments, attached by screws to the cam ring, or they may be ground out of a single ring. This latter construction finds favour amongst British designers, and has a great deal to recommend it from a manufacturing standpoint.

In a polar inductor magneto the contact lever does not revolve, but is actuated by a rotating cam secured to the end of the rotor

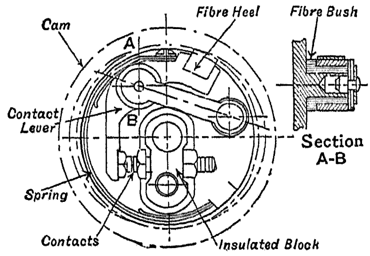


FIG. 22.

spindle, as shown in *Fig.* 23. This construction has many advantages over that shown in *Fig.* 22, and the chief of these is that a lubricated metal bearing can be substituted for the fibre bush bearing adopted in the latter design. The steel lever actually carries a

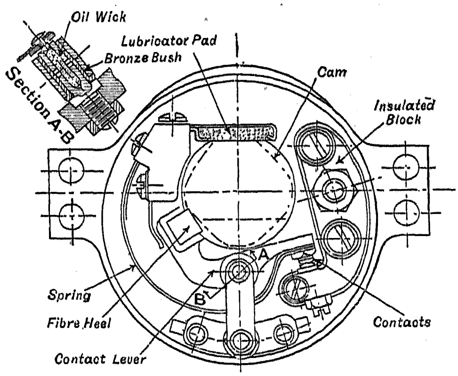


FIG. 23.

bronze bush which works on a steel pin rigidly secured to the contact breaker base. The bearing pin has an axial hole into which a small oiled lubricating wick is pushed. The oil finds its way to the bearing surface through radial holes drilled in the bearing pin.

It is standard practice to use a platinum iridium alloy for the contacts, these being electrically welded to steel screws. The alloy contains from 15 per cent to 25 per cent of iridium. Iridium is added to harden the metal so that it will withstand the severe hammer blows at high speeds. It is fairly common practice to work with a maximum contact gap of 0·012 in., and the control spring is designed to give a pressure between the contacts of from $1\frac{1}{2}$ to 2 lbs. The great majority of manufacturers use a platinum iridium contact tip which is 3·7 mm. diameter and 1 mm. thick. It is worth noting that at high speeds the current flowing through the contacts, prior to break, may reach 5 or 6 amperes.

(v.) *Distributor.*—The distributor is made of a moulded insulating material, containing rubber, to which the name Stabilite was given by the Germans when they first produced this composition many years ago. Until comparatively recent times, it has been standard practice to use a carbon spring-controlled brush which, by revolving inside a circular recess in the distributor, makes contact between the end of the secondary and each distributor segment, in proper sequence. Although this method of distribution is still extensively used, British magneto designers have introduced a design of distributor in which there is no rubbing contact, the high-tension current in each spark discharge leaping across a small air gap (usually about 0·015 in. long) interposed between the tip of a rotating electrode and each metal segment. This design of distributor—called a spark-gap type—enables a protruding form of segment to be used, which eliminates the possibility of arcing taking place on the surface of the insulation. It is, however, vitally important to ventilate a distributor of this type thoroughly, so that the deleterious products of ionisation generated inside the distributor by the sparking may be quickly expelled into the outside air. This is usually done by fixing a fine mesh gauze window either in the front or side of the distributor.

A spark-gap type of distributor enables a safety spark gap of the rotatory form to be readily incorporated in its construction. When a carbon distributor brush is used, the safety spark gap is always fixed inside the magneto, in such a position that the products of ionisation cannot readily escape. The ozone and nitric acid slowly produced in this way have a detrimental effect on adjacent steel and insulating parts. In the case of the rotary safety gap these products are churned up with the air inside the distributor, and expelled through the gauze ventilation window provided. *Fig.* 24 gives details

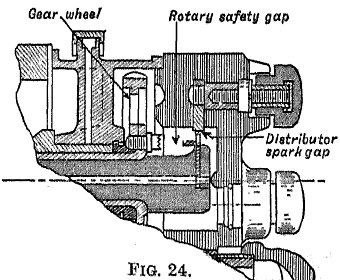


FIG. 24.

of a modern design of spark-gap distributor with rotary safety spark gap, as used in a well-known British magneto.

The use of a spark gap in the distributor is beneficial from an ignition standpoint, as it enables

the magneto to contend more surely with excessive plug leakage. If a carbon brush be used, the end of the secondary winding will be connected directly to the sparking plug during the infinitesimal period of voltage growth in the secondary following "break," with the result that a low resistance plug will cause a rapidly increasing leakage current to flow through the plug during this period, and *before the passage of the spark.* This current, flowing in the secondary winding, will react on the rapidly changing magnetic field and retard its change. Consequently, the rate of voltage rise in the secondary will be retarded, and the voltage may, under certain conditions, not attain to the sparking voltage of the gap.

The mere introduction of a spark gap has the effect of virtually insulating the secondary winding from the rest of the circuit during this period of voltage growth, and the steepness of the voltage wave-front is thus unaffected by leakage in the external circuit.[1] The curve given in *Fig.* 25 is of interest in this connection. It shows the rise of voltage that occurs across a spark gap (l_1) connected to a magneto and having in parallel with it a resistance of 112,000 ohms, when a small auxiliary gap (l)—corresponding to the distributor spark gap—is introduced in series with it. A small gap only 0·02 in. long had the effect, under these conditions, of increasing the voltage across the main gap (l_1) from 2200 to 3800.

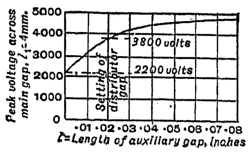


FIG. 25.

§ (12) INSTALLING A MAGNETO IN SERVICE.—A magneto is driven from the crank-shaft of the engine through suitable gearing, which gives a ratio dependent on the number of engine cylinders and the number of sparks generated during each revolution of the magneto shaft. It is desirable to interpose some form of flexible coupling in the drive to protect the revolving part in the magneto from undue stress that may result either from faulty alignment or large acceleration forces.

The gear ratio can be calculated as follows:

$$\text{Gear ratio} = \frac{\text{Magneto speed}}{\text{Crank-shaft speed}} = \frac{X}{2Y},$$

where

X = number of engine cylinders,
Y = number of sparks generated during each revolution of the magneto shaft.

This formula is true only for a 4-stroke or 4-cycle engine. Thus a 4-cylinder 2-spark magneto would be driven at crank-shaft speed, whilst a 12-cylinder 4-spark magneto would run at one and a half times engine speed. In the case of a 2-stroke 2-cycle engine we have

$$\text{Gear ratio} = \frac{X}{Y}.$$

After the magneto has been mounted on the engine platform it is necessary to "time" it with respect to the engine before tightening the driving shaft coupling. This operation consists in bringing the driving shaft and the magneto spindle into proper angular relation with one another, so that the following conditions are fulfilled:

(1) The distributor brush or electrode must be opposite a distributor segment which may be called No. 1, and the high-tension lead connected to the corresponding distributor terminal should be coupled to the sparking plug in No. 1 cylinder.

(2) With the timing lever of the magneto in the fully advanced position the contacts should be just on the point of separating when the piston in No. 1 cylinder is removed from the end of its compression stroke by an amount which corresponds to from 20° to 35° of angular movement of the crank-shaft. The exact "angle of advance" will depend on the design of the engine. If correctly timed, the moment of "break" will occur, with the timing lever fully retarded, when No. 1 piston is at the end of its stroke. Two wiring diagrams for a 4-cylinder magneto, corresponding to clockwise and anti-clockwise rotation, are given in *Fig.* 26. These diagrams indicate the order

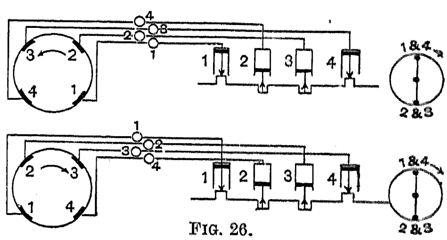


FIG. 26.

in which the distributor terminals should be connected to the four sparking plugs to give a firing sequence of 1-3-4-2, these numbers indicating the order in which the cylinders are arranged side by side, but not the order of the distributor terminals.

§ (13) REPRESENTATIVE BRITISH MAGNETOS DESCRIBED.—One of the great commercial achievements of recent years has been the establishment of a British Magneto Industry. It is therefore proposed to describe in some little detail two well-known types, representative of the two distinct classes of magnetos already discussed, which have, in recent years, been developed to a very high degree of perfection in this country.

(i.) *The M.-L. Light-weight Magneto of the Rotating Armature Class, Type "G."*—This is

[1] It is also probable, more particularly in the case of a magneto giving a period of open circuit, which is a small fraction of the period of closed circuit, that the introduction of a spark gap in the distributor is beneficial in that it quenches the high-tension spark, and thus eliminates the damping effect that will result from current lingering in the secondary circuit.

a light-weight magneto, suitable for medium-sized engines up to 90 mm. bore. It is built for 3-, 4-, and 6-cylinder engines, and is specially designed to give easy starting and efficient high-speed operation. In the case of a 4-cylinder engine it is capable of giving a slow-running speed of 75 r.p.m. The detailed internal design of the 4-cylinder machine (Type G 4) is revealed in *Fig.* 27, which gives a sectional view of the machine.

In this design the laminated pole-pieces are riveted to an aluminium body casting, to hold them in place, and the magnet fits tightly on to the pole-pieces, being secured by two screws which pass into suitable bosses on the main casting. There is a separate end-plate carrying one of the ball-bearings, attached to the main body casting by screws at the driving end of the machine, and the usual form of inspection cover is mounted upon this. At the other end, a brass end-plate, carrying the other ball-bearing, spigots into the bore in the main casting, and is attached to it by means of screws. This end-plate has an annular projection which is carefully machined inside to receive the steel cam ring, to which is attached the timing lever, and on the end of which the contact breaker cover, made of moulded insulating material, is held in place by a stout leaf spring.

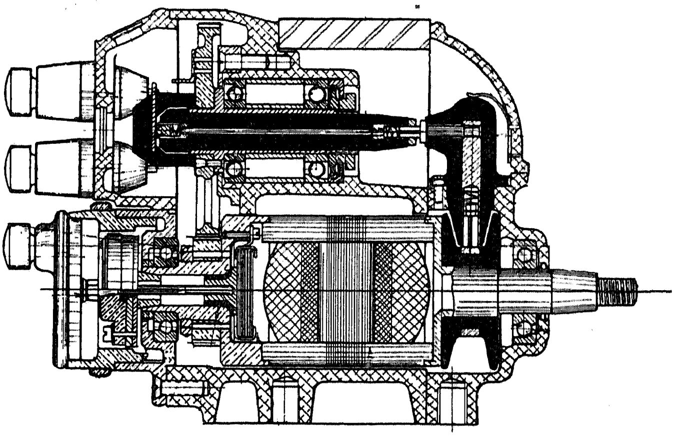


FIG. 27.—Sectional View of M.-L. (Type G 4) Light-weight 4-cylinder Magneto. This design is typical of the modern British rotating armature magneto.

The distributor is of the spark-gap type, but of unique design, in that the main body is made of aluminium. The moulded insulation terminals are mounted separately on the front face of the metal distributor housing. Each terminal has a conical end, which projects into the space formed in the metal housing, and a central nickel rod protrudes from the end of the cone to form the distributor electrode. The high-tension current has to pass, at each discharge, between the tip of the metal electrode carried by the distributor rotor, and the inner face of the circular electrode fitted to each terminal. The safety spark gap is of the rotary type, and is indicated in *Fig.* 27. A fine mesh gauze window is fitted in the centre of the metal distributor housing to provide adequate ventilation.

One special feature of the design, first adopted by the M.-L. Company, is worthy of note. The two cams, instead of being separate units attached to a cam ring, as in all German designs, are formed out of a hardened steel ring which itself forms the cam ring. The active faces of the cams are ground to a high degree of accuracy. This design is a distinct improvement on the use of separate cams, as it enables the two contact gaps, as well as the intervals between successive "breaks," to be made more closely alike.

There are several other designs of type "G" magnetos on the British market, and although these may differ somewhat as regards their detailed design—the Watford magneto, for example, is provided with solid pole-pieces and a carbon brush type of distributor—they are all interchangeable on an engine, as the main overall dimensions standardised by the British Engineering Standards Association [1] for the type "G" magneto are closely worked to by all the British manufacturers. In some of these designs the laminated poles are cast into the aluminium body, being first secured to a gun-metal base plate provided with substantial bosses for taking the fixing screws. One manufacturer in particular uses a single unit aluminium die-cast body, in which design both the driving and distributor end-plates are cast integral with the body. This gives an excellent mechanical construction.

(ii.) *The B.T.H. Polar Inductor Magneto—Type "AV."*—This is a 4-spark machine, working on the polar inductor principle, which is standardised for 8- and 12-cylinder engines. It finds its chief application on large 8- and 12-cylinder aero engines. The two designs differ from one another only in respect of the number of distributor terminals, and the internal gear ratio, which is 2 : 1 in the case of the 8-cylinder machine (Type AV 8 S) and 3 : 1 for the 12-cylinder machine (Type AV 12 S). The

[1] See *British Engineering Standards Committee Report on British Standard Dimensions of Magnetos for Automobile and Aircraft Purposes*, Sept. 1917, No. 80.

12-cylinder model, illustrated in *Fig.* 28, is the only 12-cylinder magneto manufactured in this country.

We have already dealt with the peculiar form of magnetic circuit embodied in this

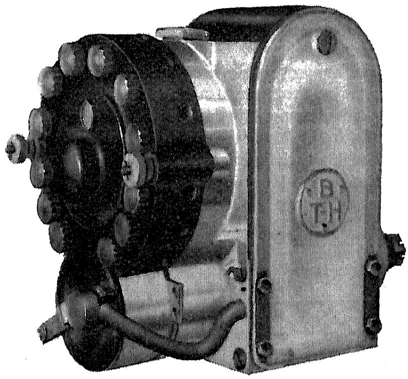


FIG. 28.—B.T.H. 12-cylinder Polar Inductor Magneto (Type AV).

design, by means of which it is possible to secure four reversals of armature core flux during each revolution of the rotor, and thus a corresponding number of high-tension sparks. Any of these magnetos (whether built for 8- or 12-cylinder working) is capable of producing regular sparking across standard three-point test gaps set to discharge at 8500 volts over the following speed range:

	Sparks per Minute.	Rotor Speed.
Minimum . .	400	100 r.p.m.
Maximum . .	16,000	4000 „

The upper limit allows a safe margin, even in the case of a 12-cylinder engine requiring six sparks for each revolution of the crank-shaft. Taking the maximum crank-shaft speed to be 2200 r.p.m., the maximum number of sparks per minute required for efficient ignition would be 13,200—that is, 220 per second.

The detailed construction can be readily understood by referring to *Fig.* 29, which illustrates a magneto in the dismantled condition. The rotor assembly is a thoroughly sound mechanical construction, comprising a "straight-through" shaft made of 25 per cent non-magnetic nickel steel, on to which the two polar inductors are first pressed and then finally riveted. The annular portion of each inductor revolves with very fine clearance inside the corresponding pole-piece which is spigoted and screwed to the main body casting, known as the inductor housing. The latter is usually made of gun metal, and four tapped holes are provided in the base for receiving the fixing screws.

The armature core is provided with two metal flanges to form a spool for receiving the primary and secondary windings. The condenser is mounted on top of this spool, and the combination of core, windings, and condenser forms a definite sub-assembly. This can be readily removed from the magneto by simply withdrawing the two screws which secure the ends of the armature core to the two laminated vertical poles that are riveted to the inductor housing. A novel form of condenser is used, as the alternate plates of mica and tinfoil are of rectangular shape, and no rivets are used in attaching the tinfoil sheets to the contact plates.

One of the distinctive features of this design is the manner in which the various components have been grouped to form distinct sub-assemblies, which greatly facilitates the work of dismantling and assembling a machine. In the case of the gear-wheel which revolves behind the distributor, this has fixed to it a

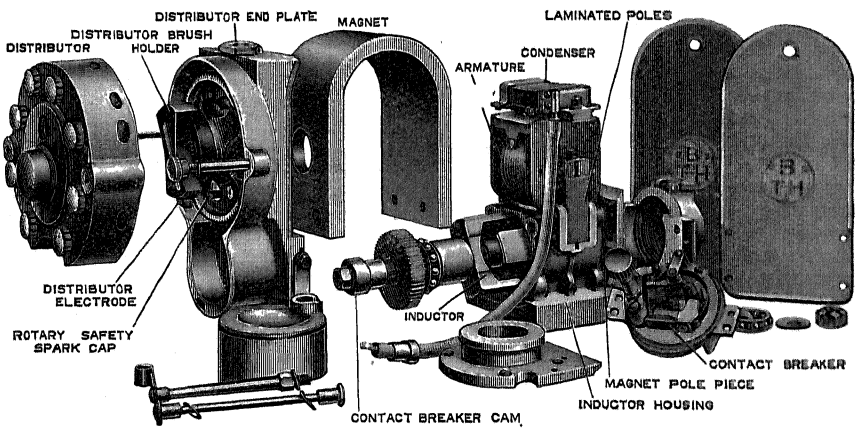


FIG. 29.—Components of B.T.H. Polar Inductor Magneto (Type AV).

tubular steel spindle which revolves in a journal bearing. The stem of the distributor electrode insulator is a sliding fit inside the tubular spindle, and carries at its inner end a small carbon pick-up brush which bears lightly on a contact, moulded in a block of insulation embracing the armature. The end of the high tension winding is connected to this stationary contact, so that current passes from it to the pick-up brush, and thence through the rotor stem to the distributor electrode. The whole combination of gear-wheel, spindle, bearing, and rotor forms a distinct unit, which is attached to the aluminium distributor end-plate by means of screws.

The distributor is of the spark-gap type, and the high tension cables are secured in radial holes by means of steel-pointed screws (with insulated heads) accessible from the front of the distributor. As on all aeroplane magnetos, there is a centre terminal which makes contact with a trailing electrode displaced from the main electrode by about 30° in a backward direction, being insulated from it. The centre terminal is connected to a separate hand-operated magneto when starting. The contact breaker is mounted underneath the distributor, the contact lever being actuated by a four-point cam mounted on the end of the rotor shaft. The details of this particular design of contact-breaker mechanism are clearly shown in *Fig.* 23. A. P. Y.

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MAGNETO-ELECTRIC MACHINE: a generator the magnetic field of which is produced by permanent magnets. See "Dynamo Electric Machinery," § (3).

MAGNETOGRAPH: an instrument devised to give a continuous record of changes in the earth's magnetic field. See "Magnetism, Terrestrial, Observational Methods."

MAGNETOMETER: an instrument for measuring magnetic fields. See "Magnetic Measurements and Properties of Materials," § (2) (ii.).

MAGNETOMETER METHODS: use of, for the determination of the magnetic properties of rods and bars. See "Magnetic Measurements and the Properties of Materials," § (23).

MAGNETOMOTIVE FORCE. The difference between the magnetic potentials at two points is called magnetomotive force. It measures the work done in moving unit positive magnetic pole from the power at lower potential to that at higher on the assumption that the potentials are not altered by the motion.

MAGNETON: a name given by Weiss to his postulated fundamental unit of magnetic moment of the molecule of ferromagnetic substances. See "Magnetism, Modern Theories of," § (1) (iii.).

MAGNETOS: Components and types of. See "Magneto, The High-tension," § (5).

Polar Inductor Type: a magneto in which the armature is fixed, the flux changes being produced by a rotor carrying magneto polar inductors. See *ibid.* §§ (7) (ii.) and (14).

Rotating Armature Type. See *ibid.* §§ (7), (14).

MAGNETS, METHODS OF DESIGN, AND PREDETERMINATION OF PERMANENT

THE following notation is used throughout this article:

The Notation

B = flux density.
Φ = total flux.
H = magnetising force.
V = difference of magnetic potential.
K = magnetic conductance.
q = magnetic conductance per unit zone width (conductance factor).

§ (1) INTRODUCTION. — The function of a permanent magnet is to maintain a magnetic field in some part of space outside its own volume. When a magnet is required which will maintain a field of specified configuration, the problem of its design arises. To test any proposed design it is essential to be able to calculate the field when the dimensions of the magnet and the properties and state of its material are given. Evershed [1] bases the solution of these problems on the demagnetising part of the BH curve of the material of the magnet. *Fig.* 1 (*a*) gives a typical demagnetisation curve for a magnet steel.

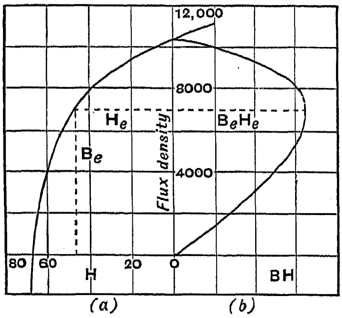

FIG. 1.

§ (2) THE FUNDAMENTAL RELATIONS.—Consider now a ring of the steel in question which has been magnetised to saturation and has its maximum remanent flux density after the removal of the magnetising force.

Let CDPE (*Fig.* 2) represent the demagnetisation part of the BH curve. The remanent flux density of the ring is represented by OD and is a maximum for the material.

Now let an actual magnet be treated in the same way; its remanent flux density will be less than the maximum because of the effect of the ends which exert a demagnetising force H_o (say) on the material of the magnet.

[1] The methods described in this article are those given by S. Evershed in his paper "Permanent Magnets in Theory and Practice," *J. Inst. El. Eng.* lviii. 780-825.

Let PN in the figure represent this remanent flux density B_o, then the flux density has been reduced from OD to PN by a demagnetising force ON. Thus ON represents H_o, the demagnetising force due to the poles.

Now consider the case of a magnet in which both H_o and the flux density B_o are uniform

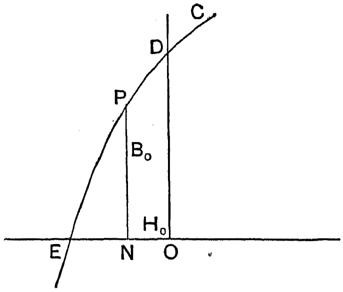

FIG. 2.

throughout. Let A be the area of its cross-section, L the length between the poles, Φ the total flux across the air gap, V the magnetomotive force or magnetic potential difference between the poles, and K the magnetic conductance of the field outside the steel.

Then the energy in the external field is $\Phi V/8\pi$. But

$$\Phi = B_o . A . \qquad (1)$$

and
$$H_o = \frac{V}{L},$$

or
$$V = H_o L. \qquad (2)$$

Hence
$$\frac{1}{8\pi}\Phi V = \frac{1}{8\pi} B_o H_o \times LA. \qquad (3)$$

Thus the energy in the external field is equal to $B_oH_o/8\pi$ multiplied by the volume of the steel. Also

$$\frac{\Phi}{V} = \frac{B_o}{H_o} . \frac{A}{L} = K. \qquad (4)$$

With the help of (3) and the demagnetisation curve, a magnet can be designed to fulfil a specified requirement. Similarly from (4) and the demagnetisation curve the field of a magnet of given dimensions, material, and state can be predetermined.

The above equations can be put into a more general form thus. Consider a tube of induction passing between two points P_1, P_2 (*Fig.* 2A) on the opposite poles of a magnet, outside the magnet by the path P_1QP_2, inside by the path P_1RP_2. Let dS_1 be the section of the tube, σ_1 the surface density of the magnetisation, and V_1 the potential at P_1, with similar symbols for these quantities at P_2.[2] Let B be the flux density, Φ the total flux. Then the energy in the tube $P_1QP_1 = \frac{1}{2}\sigma_1 dS_1(V_1 - V_2)$.

[2] See "Electrostatic Field, Properties of," § (6), and "Magnetic Hysteresis."

Also $\frac{1}{2}\sigma_1 dS_1 = \frac{1}{2}\sigma_2 dS_2 = (1/8\pi)BdS$, where B and dS relate to a section of the tube in the magnet. Hence the energy in the external tube $=(1/8\pi)(V_1 - V_2)BdS$.

Again, let x be the distance from P_1 measured along the tube in the magnet of any point R; H is the

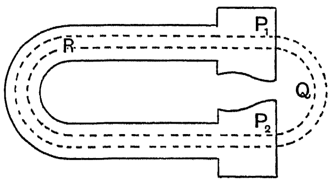

FIG. 2A.

demagnetising force at R due to distributions on the poles at potentials V_1 and V_2. Hence $V_1 - V_2 = \int H dx$, the integration extending round the tube.

Thus the energy in the external tube P_1QP_2 is $(1/8\pi)BdS\int Hdx$, or, since BdS is constant along tube, energy in external tube $=(1/8\pi)\int BHdSdx$.

Hence total external energy $=(1/8\pi)\iint BHdSdx$, the integration extending throughout the magnet, while the total external flux is given by

$$\Phi = \int B \cos \epsilon dS,$$

the integration extending over the surface of the magnet, and ϵ being the angle between the direction of B and the outward drawn normal. We also have

$$\frac{\Phi}{V_1 - V_2} = K.$$

In the case in which the flux density is constant within the magnet these equations reduce to those already given.

§ (3) DATA FOR DESIGN.—In the following, the magnet is assumed to have been fully magnetised. Permanent magnets are, however, seldom used in this condition. A percentage reduction in strength, depending on the use for which the magnet is intended, is almost always advisable. The figures taken for the flux and potential difference in the required field, which are the data for the design, must therefore include an allowance for the difference between the maximum and the working strength of the magnet, so that the magnet as designed and fully magnetised and then reduced by the given percentage will fulfil the required purpose.

§ (4) UNIFORM FLUX DENSITY.—First consider an ideal case—the design of a magnet of uniform cross-sectional area, with no leakage between the limbs, which is to maintain a uniform field of strength h in the air-gap between the parallel faces, each of area S, of two soft iron pole pieces, distance g apart. The reluctivity of soft iron being small, the magnetic potential is assumed to be uniform throughout each pole piece. This is practically justifiable provided the flux density be not excessive.

The external field of the magnet consists of (1) the uniform field h—the useful field—extending throughout the volume Sg, (2) the rest of the field.

Let $\Phi_u(=hS)$ be the flux in (1) and Φ_p the flux in (2) and let $\Phi = \Phi_u + \Phi_p$.

Let $V(=hg)$ be the known potential difference between the pole pieces.

Then
$$\Phi_u = K_u \cdot V,$$
$$\Phi_p = K_p \cdot V,$$

where K_u and K_p are the known conductances of the fields (1) and (2) respectively, deduced from the dimensions and form of the terminal pole pieces. Then $\Phi = (K_u + K_p)V$ is known. Equation (3) gives $LA = \Phi V/BH =$ the volume of the magnet. Taking any flux density B_1, less than the remanent value, make $A = \Phi/B_1$. If H_1 be the value of H corresponding to B_1, given by the demagnetisation curve, make $L = V/H_1$. Then A and L are the cross-section and mean length of the required magnet.

§ (5) THE ECONOMIC MAGNET.—The energy of the magnetic field of the ideal magnet is $\Phi V/8\pi$ ergs and the volume of the magnet is $\Phi V/BH$. If B, and consequently H, be fixed the volume is proportional to the energy in the field to be maintained. The product BH is a measure of the energy in the field which unit volume of the magnet is capable of maintaining. This varies with B in the manner shown by the curve in *Fig.* 1 (*b*), derived from the demagnetisation curve. The maximum value of BH is $B_e H_e$. In many cases economy of space and steel is of importance. In these cases B should be taken equal to B_e. Then the dimensions of the magnet of least volume—the economic magnet—which meets the requirement defined by Φ and V will be $A_e = \Phi/B_e$ and $L_e = V/H_e$.

A more complex field than that defined in § (4) involves merely more labour in evaluating Φ and V and nothing else.

§ (6) PREDETERMINATION OF FIELD.—The problem of predetermining the field of the ideal magnet of given dimensions is treated as follows. Equation (4) gives

$$\frac{B}{H} = \frac{L}{A} \cdot \frac{\Phi}{V} = \frac{L}{A}(K_u + K_p),$$

where $K_u + K_p$ is the terminal magnetic conductance of the magnet and may be estimated from the dimensions of the terminal apparatus. Since L and A are known, the ratio B/H can be evaluated. Find the point on the demagnetisation curve for which B/H has this value and let the co-ordinates of this point represent B_1 and H_1. Then $\Phi = B_1 A$ and $V = H_1 L$ and the field is determined.

By drawing a curve connecting B/H with B, the data for which may be derived from the demagnetisation curve, the determination of the particular flux density for which B/H has a given value is made easier. A curve connecting B/H with H is also helpful.

§ (7) LEAKAGE.—In the above ideal case it has been assumed that there is no leakage between the limbs of the magnet, and that the flux density in the core is uniform. Leakage is unavoidable in practice, and as magnet steel is supplied in bars of uniform cross-section, the flux density in a magnet must vary. This fact must therefore be taken into consideration both in design and predetermination.

In an actual magnet the flux density varies from a maximum at the neutral section to a minimum at the ends. The value of BH varies correspondingly. The average value of BH in each limb must be less than its maximum value. Hence the dimensions of an actual magnet can never be so favourable to economy of steel as those of the ideal magnet of § (5). Design must aim at the highest average BH in the whole magnet consistent with the imposed condition of uniform sectional area. If B_eH_e be the maximum value of BH, it is found in practice that when a magnet is so designed that the average value of H in the steel is not very different from H_e the range of variation in flux density is most favourable for economy of steel. Examination of the curve in *Fig.* 1 (*b*) supports this; for BH is a maximum when $H=H_e$, so that the actual and percentage change in BH for a finite small change in H is smaller when the average of H over this range is H_e than when it has any other value.

§ (8) VARYING FLUX DENSITY. — Let us estimate the dimensions of an economic magnet which will fulfil some energy requirement, ΦV, and with the data thus obtained find out what the magnet will do. Guided by these results, make any corrections in the size of the magnet as may be necessary. The dimensions of the magnet can only be arrived at after one or more approximations. Making $L=V/H_e$, experience shows that it is seldom that any alteration in length is necessary. Let K_c be the sum of the conductances of the leakage paths between the limbs of the magnet and let ϕ be the leakage flux. Then ϕ is approximately equal to $\frac{1}{2}VK_c$. The total flux is $\Phi+\phi$, and if the average flux density be B_e we must make $A=(\Phi+\phi)/B_e$ One method of approximation is the following. First make $A_0=\Phi/B_e$, and from the leakage conductance based on A_0 and L calculate ϕ_1 from $\phi=\frac{1}{2}VK_c$. The total flux is now $\Phi+\phi_1$, which gives for A the value $A_1=(\Phi+\phi_1)/B_e$. Using A_1, the leakage conductance is determined anew and the new leakage ϕ_2 from which is obtained $A_2=(\Phi+\phi_2)/B_e$. Proceeding in this way final values for A and ϕ are obtained after a few approximations, making $A=(\Phi+\phi)/B_e$.

§ (9) USE OF RATIO EQUATION.—Having thus got L and A for the magnet and the conductance K_c of the leakage paths from limb to limb, assume that the leakage conductance is equivalent to a fictitious conductance αK_c connected across the ends of the magnet, α being a fraction. The ratio equation (4) becomes $B/H=(L/A)(K_u+K_p+\alpha K_c)$. Referring to the demagnetisation curve, let H_a be the value of H which gives B/H the above ratio. Then $V=H_a . L$ and $\Phi_u=V . K_u$, which determines the useful field while the terminal flux is $\Phi=V(K_u+K_p)$. If H_a be not sufficiently close to H_e it may be necessary to modify the length and also the sectional area in the design in order to obtain values for Φ and V sufficiently close to those postulated by the field. The value of α lies between $\frac{1}{3}$ and $\frac{1}{2}$; the safest course in making a forecast is to make it $\frac{1}{2}$. Great precision in estimating αK_c is not called for in a well-designed magnet.

The above method is approximate; it gives no information about the distribution of the flux in the steel. An accurate and detailed forecast of the performance of the magnet may be made by the integration method given in the following section.

§ (10) INTEGRATION METHODS.—Suppose the magnet to be divided into a number of lengths or regions within each of which B and H are treated as constant quantities. Their values are supposed to change suddenly in passing from one region to the next. Thus leakage will take place solely at the junction between two adjacent regions. A method of subdivision is indicated in *Fig.* 3, where the magnet is divided into five pairs of regions symmetrically disposed on either side of the central region. The magnet, whose function is to create the field between the pole pieces P_1, P_2, is shown spread out in the figure, its axis appearing as a straight line. The number of subdivisions necessary depends to some extent on the leakage field. Magnets

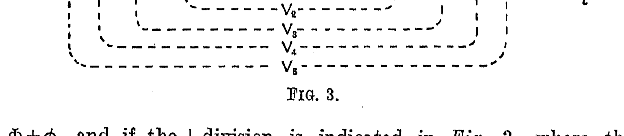

FIG. 3.

without pole pieces generally require finer subdivision than magnets with pole pieces, in order to obtain reliable values for Φ, V, and flux density distribution.

The junctions between adjacent regions being the centres of leakage zones, there will be five pairs of leakage zones on either side of the neutral section. The conductance of the path between any pair of zones will be equal to q multiplied by the width of the zone, q being the conductance per unit length at the centre of the zone.

Assume the flux density in the central region to be B_1. Reference to the demagnetisation curve will give the corresponding H_1. This H_1 multiplied by $\frac{1}{2}s_1$ on each side of the neutral section gives $V_1 = H_1 s_1$, the difference of potential between the centres of the first pair of leakage zones. The conductance of this first leakage path is $q_1 s_1$, and the decrement in flux density at the junctions between the central region and the adjacent regions will be $V_1 q_1 s_1/A = \delta_1 B$. Subtracting $\delta_1 B$ from B_1 gives B_2, the flux density at the bottom of the first step in each limb. The demagnetisation curve gives H_2 corresponding to B_2 and the next increment in potential is $H_2(s_1+s_2)$. The equation $V_1 + H_2(s_1+s_2) = V_2$ gives V_2, the potential difference between the centres of the second pair of leakage zones. The decrement of the flux density at the second step will be $V_2 q_2 s_2/A = \delta_2 B$. Hence $B_2 - \delta_2 B = B_3$ gives B_3, the flux density at the bottom of the second step in each limb. Deriving H_3 from the curve and proceeding step by step to the ends of the magnet, a terminal density B_6 is found, giving the terminal flux Φ, where $\Phi = B_6 . A$. The successive additions of potential will at the same time give the terminal potential difference V.

Let $K_t (= K_u + K_p)$ be the conductance between the pole pieces. If the initial density B_1 has been correctly chosen, then Φ/V will be equal to K_t, and the field of and distribution of flux in the magnet will have been determined. Generally, however, Φ/V will not be equal to K_t and it will be necessary to make a second and also a third calculation starting with different initial densities. Assuming these three integrations made with initial densities B', B'', B''', resulting in terminal ratios Φ'/V', Φ''/V'', Φ'''/V''', it is essential that K_t should be somewhere between the greatest and least of these terminal ratios.

Treating the connected quantities B and Φ/V as the co-ordinates of a point, plot the three points $(B', \Phi'/V')$, etc., and draw a smooth curve through them. The particular value B_1 of the initial flux density, which after integration will give a terminal ratio Φ_t/V_t equal to K_t, is found by reading from the diagram thus constructed the abscissa of the point on the curve whose ordinate is K_t. The value thus obtained is tested by carrying out a fourth calculation starting with this initial density. It should result in a ratio Φ_t/V_t equal to K_t. The field of the magnet and the distribution of flux in it is thus determined.

§ (11) Tests of Design.—The calculations just detailed will give the following quantities :

Φ_t = the terminal flux of the magnet,
V_t = the terminal potential difference,
B_1 = the flux density at the neutral section,
B_t = the flux density at the ends of the magnet.

Comparing Φ_t and V_t with Φ and V which specify the field, it will be seen if any modification in length or sectional area be necessary. If economy of steel be important the mean flux density $\frac{1}{2}(B_1 + B_6)$ ought not to differ widely from B_e; nor should V_t/L differ much from the economic value H_e. If these quantities agree reasonably well there will be some approach to an economical use of steel in the magnet.

§ (12) Efficiency of Design.—The energy of the leakage field between the limbs can be determined from the data obtained in the fourth calculation. Adding this to $\Phi_t V_t/8\pi$ a measure of the total energy of the external field of the magnet is arrived at. Comparing this with the maximum external energy of the field $B_e H_e \times LA/8\pi$ which the steel in the magnet is capable of maintaining in the ideal case, it will be seen how far the proposed magnet falls short of the economy obtained in the ideal magnet. This affords a measure of the efficiency of the design.

Table I

Maximum External Magnetic Energy per c.c. of Various Kinds of Iron and Steel

Material.	Condition.	Ergs per c.c.	Coercive Force.
Lowmoor iron .	Annealed	183	2
Mild steel . .	Hardened	970	7
Pianoforte steel .	Hardened	5,600	41
Silver steel .	Rolled	2,500	18
	Hardened	7,900	68
Tungsten steel	Annealed	3,900	18
	Rolled	5,300	28
	Hardened	13,100	67

§ (13) Comparison of Actual and Designed Output.—The methods of design and predetermination given above are found to be very reliable. The values of B_1 and Φ_t, obtained by the method of § (10), differ by less than 1 per cent from the values obtained

experimentally in tests on magnets constructed to the design when the differences in the qualities of the steel in the magnet and in the test piece—which gives the BH curve—are eliminated. These differences are found to be the main and most serious cause of the discrepancies observed between the designed and actual output of magnets.

§ (14) Evaluation of Conductances.—(i.) The magnetic conductance of an air-gap of width g between the flat ends of two coaxial cylindrical pole pieces of radius a is S/g, where $S=\pi a^2$, provided g does not exceed about $\frac{1}{6}a$. When g is greater than this the formula underestimates the conductance.

The conductance of the paths originating on the curved surface of the cylinders at a distance x from the gap per unit length is given by

$$q=\frac{2\pi}{\rho+z},$$

where $$\rho=\frac{g}{a+x}.$$

Writing $x=na$, the value of z for different values of n is given in the following table:

Table II

n.	z.	n.	z.
0·0	0·000	1·3	2·36
0·1	0·296	1·5	2·59
0·2	0·561	1·7	2·79
0·4	1·013	1·8	2·88
0·6	1·391	2·0	3·05
0·8	1·722	4·0	4·23
0·9	1·859	8·0	5·56
1·0	2·000	16·0	6·92
1·1	2·13	20·0.	7·45
1·2	2·25	24·0	7·75

To find the total conductance of the paths extending from the edge of the air-gap to a distance x, the integral $\int q dx$ has to be evaluated. This can be done by drawing a curve connecting q with x and measuring its area. The upper limit in the integral $\int q dx$ is about half the length of the pole piece. The conductance of the paths originating over the rest of the surface of the pole piece is $\frac{1}{2}(1\cdot77\sqrt{S'})$, where S' is the entire surface of the pole piece.

These results apply also to rectangular pole pieces. If s and w be the sides of the opposing faces, sw must be substituted for S and $(s+w)/\pi$ for a in the above.

(ii.) The conductance q per unit axial length for two parallel cylinders of diameter d with axes distant l apart is given by

$$q=\frac{\pi}{\log_e\{n+\sqrt{n^2-1}\}},$$

where $$n=\frac{l}{d}.$$

The value of q for different values of n is given in the following table:

Table III

n.	q.	n.	q.
1·0	Inf.	5	1·37
1·1	7·10	6	1·27
1·2	5·04	7	1·19
1·3	4·15	8	1·13
1·5	3·26	9	1·09
2·0	2·38	10	1·05
3·0	1·78	15	0·93
4	1·52	20	0·85

In applying these results to limbs of any other section it is necessary to make $d=$(perimeter of section)/π.

(iii.) The conductance q of the paths originating on the surface of a uniformly magnetised ellipsoid of revolution of major axial diameter $2c$ and equatorial diameter $2a$ per unit distance measured parallel to the major axis, the direction of magnetisation, is given for different values of c/a in the following table:

Table IV

$\frac{c}{a}$.	q.	$\frac{c}{a}$.	q.
1	6·28	14	1·32
2	3·77	17	1·23
3	2·87	20	1·15
4	2·41	25	1·07
6	1·94	30	1·01
8	1·69	40	0·93
9	1·59	60	0·83
10	1·52	80	0·76
12	1·40	100	0·73

One example of the use of Tables III. and IV. is to find the conductances of the leakage paths between the limbs of an ordinary bent magnet. For the region on either side of the neutral section to the end of the bend q may be taken from Table IV., where c/a is taken equal to the length divided by the virtual diameter. For the parallel parts of the limbs q may be taken from Table III. Magnets of this type form a very numerous class; hence the usefulness of Tables III. and IV.

R. LL. J.

Magnets for Ammeters and Voltmeters. See "Direct Current Indicating Instruments," § (2).

Manganin: a material of which standard resistances are made. Dependence of electrical properties on composition. Work of Hunter and Bacon, and the N.P.L. See "Electrical Resistance, Standards and Measurement of," § (4).

Mansbridge Condensers, Construction of. See "Capacity and its Measurement," § (28).

Manual and Automatic Telephone Switching, Analogy between. See "Telephony," § (8).

Matsumoto's Theory: a method of considering the problem of the determination of the heating of current-carrying electric cables. See "Cables, Insulated Electric," § (5).

Maximum Demand Indicator. See "Watt-hour and other Meters for Direct Current. II. Watt-hour Meters," § (33).
Atkinson-Schattner Type. See *ibid.* § (34).
Merz-Price Type. See *ibid.* § (35).
Wright Type. See *ibid.* § (33).

Maximum Overload Device: a device which will cause a switch to open circuit if the current flowing exceeds a certain predetermined value. See "Switchgear," § (8).

Maxwell. The name given to the unit of magnetic induction on the practical system of electrical measurements.

1 Maxwell = 1 C.G.S. unit or line of induction.

See "Units of Electrical Measurement," § (28).

Coefficients of Capacity, etc.: coefficients in the expressions for a system of conductors. See "Capacity and its Measurement," § (4).

Distribution Law applied to electron atmospheres: experimental investigation of. See "Thermionics," § (5) (ii.).

Maxwell Bridge, The: for the determination of capacity in electromagnetic units. Theory of, and application to the measurement of "v." See "v," § (5).

Mean Effective Value or Root Mean Square of a Variable Quantity. The value found by forming the sum of the squares of the variable quantity, dividing by the number of individual values taken to form the sum, and extracting the square root.

If the quantity varies continuously, being given by an equation $x = Xf(t)$, then for the sum we substitute an integral.

The energy dissipated at any moment as heat in a conductor carrying a current varies as the square of the current or of the E.M.F. at that moment; or, if there be no lag between the current and the E.M.F., the power necessary to generate the current varies as its square. Thus if the current be variable its mean effective value or root mean square is the value of a constant current which dissipates the same energy, or, assuming the absence of lag, requires the same power for its production as the variable current.

If $\overline{X}$ be the mean effective value or root mean square of a quantity $x = Xf(t)$ during a time T, then

$$\overline{X} = \left\{\frac{1}{T}\int_0^T x^2 dt\right\}^{\frac{1}{2}}$$
$$= X\left[\frac{1}{T}\int_0^T \{f(t)\}^2 dt\right]^{\frac{1}{2}}.$$

If $\qquad f(t) = \sin nt,$

then $\quad \{f(t)\}^2 = \sin^2 nt = \frac{1}{2}(1 - \cos 2nt).$

Thus

$$\overline{X} = X\left\{\frac{1}{2T}\int_0^T (1 - \cos 2nt)dt\right\}^{\frac{1}{2}}$$
$$= X\left\{\frac{1}{2T}\left(T - \frac{1}{2n}\sin 2nT\right)\right\}^{\frac{1}{2}},$$

and if T be any number of complete periods $\sin 2nT = 0$. Hence

$$\overline{X} = \frac{X}{\sqrt{2}} = \frac{X}{1{\cdot}414} = X \times {\cdot}7072.$$

Thus the mean effective value taken over any number of complete periods of a quantity which varies sinusoidally is found by dividing the amplitude by 1·414, or multiplying it by ·7072, while if the mean effective value be known the amplitude is given by multiplying it by 1·414.

Mean Value of a Variable Quantity. The value found by forming the algebraical sum of the values of the variable and dividing by the number of values taken to form the sum.

If the quantity varies continuously being given by an equation $x = Xf(t)$, then for the sum we substitute an integral.

Thus $\bar{x}$, the mean value of x over any period T,

$$= \frac{X}{T}\int_0^T f(t)dt.$$

If the function be sinusoidal so that

$$f(t) = \sin nt,$$

$$\bar{x} = \frac{X}{T}\int_0^T \sin nt dt = \frac{X}{nT}(1 - \cos nT).$$

If T be a complete period, we have

$$nT = 2\pi, \quad \cos nT = 1, \quad \bar{x} = 0.$$

If T be a half period, then

$$nT = \pi, \quad \cos nT = -1, \quad \bar{x} = \frac{2X}{\pi}.$$

Thus the mean value in each half period of a current or an E.M.F. which varies sinusoidally is found by multiplying the amplitude by $2/\pi$, or conversely the amplitude of such a current is given by multiplying its mean value taken over a half period by $\pi/2$.

MICROPHONE, THE HOT WIRE

THIS instrument was designed in the first instance for the detection of enemy guns during the war, but latterly it has been modified so that it can be employed for the detection and measurement of continuous sounds.

As originally constructed, the microphone consisted of a closed box or metal cylinder of capacity from 10 to 20 litres. In one of the walls of this box a small opening is made and the opening is fitted with a short tube about 1 cm. in diameter and 2 cm. in length. At the point where the tube enters the box a grid of very fine platinum wire prepared by the Wollaston process can be inserted, so that the grid intercepts the blast of air which enters the box on the arrival of the gun sound.

In order to show the existence of the blast the platinum wire is heated by a small electric current, and its resistance is balanced in a bridge circuit in which an Einthoven galvanometer is employed. The arrival of the sound produces a change of temperature of the grid which is clearly shown by a sharp fall in the resistance of the grid and consequent deflection of the galvanometer. This simple device will give detection of very faint gun sounds within wide limits of variation of dimensions, either of the box or of its orifice, nor is it necessary to employ the finest platinum wire or the most symmetrical grid.

Definite research, however, has led to the adoption of a certain form of containing vessel or box with an orifice of suitable dimensions, and, moreover, the flatness and symmetry of the grid, coupled with certain dimensions of wire, have proved to be of advantage. It has also been shown that the box and its orifice should be deprived, as far as possible, of all resonating properties so that the record of the galvanometer, as produced on a moving sensitised strip, should not be too much disturbed by vibrations set up in the box through shock excitation. This is the more readily secured by the drilling of small holes or the opening of fine cracks in the walls of the box.

The above description refers to a microphone which is largely non-resonant and designed to deal with impulsive sounds. For the detection of continuous faint sounds,

especially those associated with a definite musical note, everything is to be gained by encouraging resonance, and for this purpose the microphone has been modified so that the containing vessel and orifice constitute an ordinary Helmholtz resonator. Within the orifice the hot wire grid is now inserted, and special attention has to be devoted to dimensions and form, both of the containing vessel and its orifice.

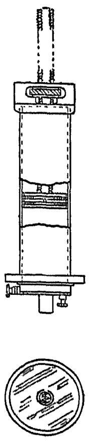

FIG. 1.

The accompanying figure (*Fig.* 1) shows one form of tunable microphone and the grid with its mounting. The latter is a circular sheet of mica perforated at its centre by a circular hole. A porcelain bridge placed diametrically across the hole supports three loops of the platinum wire whose diameter is approximately 0·004 mm., and whose resistance varies from 150 ohms at air temperature to about 300 ohms when heated by the steady working current. The electrical circuit is completed by contact with circular silver electrodes fixed to opposite sides of the mica plate. In order to tune the microphone the volume may be varied by the use of a screw plunger, as in the figure, or some change may be made in the dimensions of the orifice. It is, however, generally desirable to use the former method of tuning as change in orifice generally results in change of sensitivity.

A full description of the instrument, together with the theory and mode of working, is given in a paper by Tucker and Paris,[1] but the following summary may be given:

The effect of sound on the microphone may be observed by a steady drop in electrical resistance of the grid as measured by a Bridge or by the use of a Valve Amplifier which magnifies the periodic resistance changes. The amplifier is used with a telephone or vibration galvanometer.

By the first method the resonance curves of various types of microphone have been obtained.

Control of sensitivity is satisfactorily secured by variation of the heating current. By application of the Bridge method it is found that for a sound of given intensity the steady resistance change is a linear function of the excess of temperature of the grid above that of its surroundings as measured on the Platinum scale.

A controlling factor in the sensitivity of the microphone is the free convection current arising from the heated grid, and when in use it is desirable to have the plane of the grid horizontal.

[1] "A Selective Hot Wire Microphone," *Phil. Trans. Roy. Soc.* A, 1921, ccxxi. 389.

The paper just quoted shows how oscillatory air currents produce in the grid:

(1) A steady drop of resistance due to average cooling.

(2) A periodic resistance change of the same frequency as that of the sound.

(3) A periodic resistance change of twice this frequency.

W. S. T.

MICROPHONE HUMMER: an instrument used for supplying alternating current (approximately sine wave) of audio frequencies to bridges, etc. See "Inductance, The Measurement of," § (15).

MICROPHONE-TELEPHONE: a detecting instrument for use at low frequencies, at which the ordinary telephone is very insensitive. See "Inductance, The Measurement of," § (34).

MICROPHONES OR TELEPHONE TRANSMITTERS

§ (1) INTRODUCTORY.—The action of a microphone is based on Hughes' discovery in 1878 that a loose contact, in a circuit containing a battery and a telephone, may give rise to loud sounds in the telephone. The variations of resistance at the contact produce large changes in the current in the circuit and hence loud sounds in the telephone.

In one of the original forms of the apparatus a pointed rod of carbon rests in horizontal grooves in two carbon blocks attached to a sounding-board. A battery and telephone are connected to the carbon blocks; small vibrations produced by sound-waves falling on the apparatus cause the loose rod to move on its supports, and the varying resistance at these points produces changes in the current, and hence sound in the telephone. Hughes' original apparatus has recently been discovered stored with some miscellaneous goods, and is now on exhibition in the Science Museum.

§ (2) TELEPHONE TRANSMITTERS.—A telephone transmitter consists in general of a diaphragm which vibrates in response to the pressure variations of sound-waves, together with means whereby the vibrations of the diaphragm are caused to produce either a real E.M.F., as when a magnetic receiver is used as a transmitter, or a variation of the resistance of a part of the circuit through which direct current is flowing. A corresponding variation is thus produced in the current, and shows itself in the receiving apparatus at the other end of the line. The latter arrangement is the only one which has hitherto been commercially used for telephony, and it has been found that particles of carbon in light contact produce the greatest change in resistance when

subjected to small variations of pressure. A commercial transmitter therefore consists of a diaphragm which transmits its motion to carbon granules or pellets confined between electrodes connected to a battery.

The pressure variations in sound-waves are extremely small, but the sensitiveness to pressure of the resistance at the points of contact of particles of carbon is so great that an easily measurable alternating current can be produced by the action of the waves.

Since the electrical power produced by the transmitter is drawn from a battery it may greatly exceed the acoustic power absorbed by the transmitter, but there are limitations to the transmitter output which are imposed by the physical properties of carbon. By increasing the voltage applied to the transmitter the current would be increased, and hence also the apparent alternating E.M.F. generated, which is the product of the variable component of resistance and the instantaneous current.

When, however, the voltage per carbon contact in series exceeds a certain amount, heating and arcing occur between the contacts, causing loud hissing and crackling sounds to be heard in the telephone; this is known as "burning." It has been found that burning once started tends to grow worse, eventually rendering the transmitter useless.

The type of common battery transmitter in general use has been described elsewhere; [1] other types in more or less general use are described below.

§ (3) Centrally damped Transmitters.—A type of transmitter which is in wide use in the United States is a variation of the ordinary pattern described under "Telephony"; the carbon button or cell is the same, but instead of the front electrode being locked to the aluminium diaphragm by a nut, it terminates in an insulated stud, which is pressed against the diaphragm by a spring. The two damping springs pressing on the diaphragm are absent, and the diaphragm rests upon its edge, which is turned up all round, instead of upon the flexible support of a rubber ring. This type of transmitter is known as the "centrally damped transmitter" on account of the central position of the controlling spring.

The advantages gained by the "centrally damped" construction are, in addition to those of a manufacturing character, higher efficiency and slightly better quality of speech reproduction. The higher efficiency is due to the reduction in the amount of lost energy absorbed by the diaphragm support; the better quality is due to the simplification of the vibration characteristics and the slightly higher natural frequency of the vibrating system which is found in this transmitter.

[1] See "Telephony," §§ (12)-(14).

In local battery sets it is the usual practice to use common battery transmitters such as have been described above, and to use three primary cells. There are, however, still in use a number of specially designed local battery transmitters used with two primary cells. These transmitters are similar to the common type of transmitter in all respects, except that the carbon button is larger in diameter, the electrodes nearer together, and the carbon granules used are coarser. The object of these changes is to reduce the resistance to a value suitable for the local battery circuit.

§ (4) Microtelephones.—All of the transmitters so far described are of substantial proportions, and are generally used on desk sets or wall sets, so that they do not require to be supported by the person using them. There is, however, a strong tendency in European countries to favour a light combination of transmitter and receiver, so arranged on a common handle that the set can be held in one hand with the receiver at the ear and the transmitter in front of the mouth; this apparatus is known variously as a hand-set, a hand microtelephone, or as a microtelephone. The advantages of this type of instrument are the comfortable and unstrained position which can be adopted when using it, and the possibility of changing one's position naturally and freely during a long telephone conversation.

Among the disadvantages are the loss of efficiency due to the distance separating the mouthpiece from the mouth of the operator and the greater tendency to burning troubles, arising from the fact that the instrument must operate in any position.

The design of a transmitter for a microtelephone differs considerably from the design of transmitters used in fixed sets; the microtelephone transmitter must be lighter and easily replaceable in the hand-set in case of damage. It must be so designed that in all positions it will give satisfactory service, and will not cause the exchange supervisory lamp to glow by opening the exchange circuit.

The gap between the transmitter and receiver must be great enough to suit persons having an abnormally large separation between ear and lips, and consequently the majority of users do not find their lips close to the mouthpiece. It is therefore, above all, necessary that the transmitter should provide as great a modulated current when the speaker's lips are at a distance as the fixed transmitter produces when the lips are close. At the same time forcing or speaking close up must not render the speech unintelligible through increased transmitter distortion, or by supplying more power to the distant receiver than it can faithfully translate into sound-waves.

The transmitters described below are preferably used in microtelephones, but they are occasionally met with in fixed telephones. They are all unit types which can be easily fitted into a cup formed on the microtelephone handle, and are known as " inset transmitters " or " capsules." Sometimes the transmitter is connected to terminal posts by flexible conductors, but often it is merely slipped into place and makes contact with suitable springs.

The capsule transmitter most commonly used in this country is shown in *Fig.* 1, and

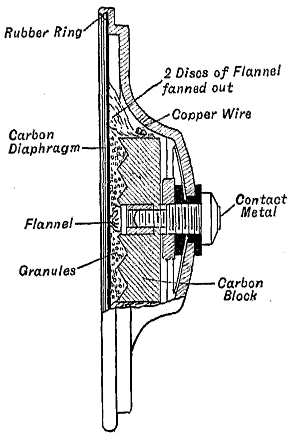

Fig. 1.

consists of a shallow brass cup, to the inside of which is fixed a circular carbon block insulated from the outer case. The mouth of the cup is closed by a carbon diaphragm resting on a turned seating, and the edge of the cup is spun over the edge of the diaphragm, a thin rubber ring being first inserted. The space between the block of carbon and the diaphragm is filled with carbon granules, which are confined by large flannel washers bound round the carbon block and fanned out at the edges. The surface of the carbon block is formed with concentric grooves to increase its effective surface and to prevent the carbon piling together in the lower part of the transmitter. This piling leads to low-efficiency, and is technically known as " packing." Connection is made to the carbon diaphragm, which forms one electrode, by means of the metal cup; the insulated screw used for fixing the carbon block forms the other terminal.

Another type of inset transmitter also using a carbon diaphragm fastened to the front of a brass cup is shown in *Fig.* 2. This has a large back electrode covered by a thick felt disc cut like a six-spoked wheel, and the carbon granules are distributed among the six spaces which are closed in front by the diaphragm and at the back by the carbon block. The spokes of the wheel-shaped piece of felt fit into diametral grooves in the carbon block, and the rim fits closely round it. The felt which is pressed against the diaphragm by a star-shaped spring resting in the grooves in the back electrode increases the damping of the diaphragm and keeps the granules in their proper places. This transmitter is designed to utilise a relatively large contact area on the electrodes. This keeps it cool, and so assists in securing a relatively high resistance, a feature which favours efficient operation with a small direct current. The high resistance is also partly due to the separation of the carbon into separate pockets, so that it is nowhere subjected to much pressure from superimposed granules.

Carbon diaphragms are not impermeable to moisture, so that it is usual either to enamel the outer side of the diaphragm or to place a thin diaphragm of mica, celluloid, or tinfoil in front of it; a mechanical protection consisting of a stiff metal grid is often added.

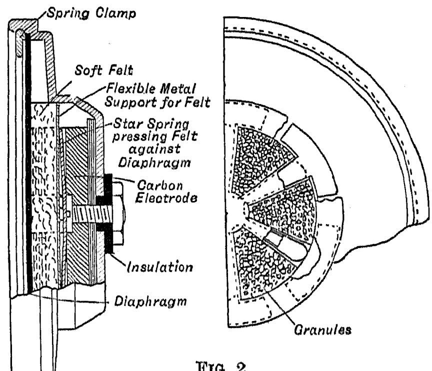

Fig. 2.

With regard to the special conditions to be met when these transmitters are used in microtelephones, it may be said that the lightness of the diaphragm resulting from the absence of any attached heavy electrode makes them very sensitive to feeble sounds and faithful in reproduction. The weakness of the forces acting on the diaphragm when the mouth is not too near also favours good articulation. The necessity of filling the carbon chamber fairly well in order to avoid opening circuit in the horizontal position prevents the carbon becoming too loose, owing to excessive vibration when the speaker's mouth is close to the diaphragm, so that the articulation, although less good under these conditions, is not so bad as to lead to unintelligibility. The speech efficiency with the mouth at an average distance is, however, less

than that of a good desk instrument spoken into with the lips close.

A capsule transmitter which is more robust than the carbon diaphragm type is shown in *Fig.* 3. It differs from those described above in having an aluminium diaphragm supporting a small circular polished carbon electrode. This front electrode projects through a circular hole in a mica washer, which forms the front

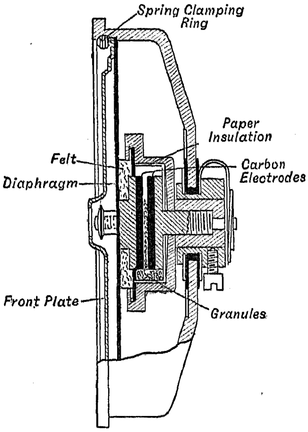

FIG. 3.

of the brass carbon chamber, at the bottom of which is a second circular polished electrode. The chamber is lined with paper for insulation purposes, and contains carbon granules. The back electrode is not as large as the internal diameter of the carbon chamber, so that there is an annular space which enables a certain excess volume of granules to be stored, in order that when the transmitter is held face downwards as in the act of replacing it on the horizontal cradle often provided, the back electrode may not cease to make contact with the granules. In this way continual arcing at the surface of the back electrodes is avoided, and the development of "burning" is prevented. A projecting shank at the back of the carbon chamber passes through an insulated bush in the outer cup and forms a spring terminal.

This capsule is capable of being adjusted to have the best value of diaphragm tension by means of a set screw which locks the shank in the insulated bush, the carbon chamber being pressed against a thin felt washer on the diaphragm with the force necessary to produce the desired tension. A particular feature is the shallow-dished plate, with a small hole in the centre, mounted in front of the diaphragm; the confinement of air between the diaphragm and the front plate increases the apparent stiffness of the diaphragm to rapid vibrations, thus effectively raising its natural frequency. The size of the hole through which an alternating flow of air must take place when the diaphragm vibrates controls the damping. Variations in the size of the hole or the depth to which the front plate is dished profoundly alter the characteristics of the capsule.

§ (5) INERTIA TRANSMITTERS.—All the transmitters described above have one feature in common, namely, that of the two carbon electrodes one is flexibly mounted and the other is fairly rigidly mounted. There are, however, instruments known as inertia type transmitters, in which the relative motion of the two electrodes depends upon the inertia of the back part of the button, or electrode assembly, rather than on its stiffness or rigidity of mounting. The kind of transmitter referred to will be immediately understood if the bridge of the ordinary common battery transmitter is supposed to be removed and replaced by a perfectly flexible electrical connection to the back electrode. This type of transmitter has not a simple vibration characteristic; the vibration of the back electrode, not entirely suppressed in any transmitter, here being appreciable since the weakness of the mica annulus limits the weight of the back part of the button. The possible loading of the back electrode is also limited by the fact that it is coupled to the diaphragm, so that the natural frequency of the diaphragm system will be reduced as the mass of the button is increased. The natural periods of the two electrodes are, in general, different, so that there is a distortion, since the two electrodes will vibrate in phase at some frequencies and in opposition at others, and the effect on the alternating electrical output will be the algebraic sum of the motions.

Recently transmitters have been built for special purposes to reduce the vibrational distortion to a minimum. In order to secure the very high natural frequency necessary for this purpose the diaphragm is made of very thin material and mounted in such a way as to be stretched. The aim of this is to avoid resonance at telephonic frequencies, and the system being very stiff the electrical output is small, so that these instruments are not at present suitable for ordinary telephony.

F. B. J.

MÖLLINGER APPARATUS, use of, for the measurement of power losses in iron. See "Magnetic Measurements and Properties of Materials," § (59).

MÖLLINGER'S WINDING APPARATUS: an apparatus for quickly winding ring specimens for magnetic tests. See "Magnetic Measurements and Properties of Materials," § (22) (iv.).

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Ayrton and Perry. See "Measurement of Insulation Resistance," § (3) (i.).

OHM'S LAW states that in any circuit under constant physical conditions the ratio of the electromotive force to the current is a constant. This constant is known as the resistance of the circuit; its reciprocal is the conductivity. See "Units of Electrical Measurement," §§ (8), (9); "Electrical Measurements, Systems of," § (9).

The most satisfactory direct proof of Ohm's law which we possess is given by the experiments[1] of Chrystal and Saunder, carried out at Maxwell's suggestion at the Cavendish Laboratory. Since the resistance of a wire is found to be independent of the direction of the current, it must, if not constant, depend on the square and higher even powers of the current. Thus we may express it as

$$R(1 - h\frac{i^2}{A^2} + \ldots),$$

where h is a constant, i the current, and A the area of the cross-section of the wire; since the effect sought is small we neglect powers of i above the second. Then it can easily be shown that if two adjacent arms of a Wheatstone's bridge through which the same current i passes be formed of a thin wire and a thick wire respectively the position of balance will depend on the current and hence on the E.M.F. of the battery. It is not possible, however, to carry out the experiment merely by altering the E.M.F. of the battery, for the wires are heated by the current and the rise of temperature of the small wire is much greater than that of the large; the ratio of the resistances is thereby altered and the balance point shifted.

Now, however, let the E.M.F. be altered rapidly by means of a commutator—a tuning-fork vibrating 60 times a second was used—from E to E′, so far as the heating effect is concerned the wires will reach a steady state and a balance point can be found; thus as a second experiment change the E.M.F.'s, keeping other conditions the same from E to $-E'$; the heating effect will be the same as in the first part of the experiment, for it is independent of the direction of the current, and so far as it is concerned the balance point on the bridge will not be disturbed, but calculation shows that if there is a term in the resistance depending on the square of the current there will be a displacement of the balance point depending on h, and on the change of the E.M.F., the resistances in the bridge and the diameter of the fine wire. By observing the shift of the balance, if it exists, and knowing the other quantities, the value of h can be calculated.

[1] *Brit. Ass. Rep.*, 1876, p. 49.

Various thin wires were used; the final experiments, three in number, were made with wires of platinum, German silver, and iron respectively. A battery of four Daniels was alternated with one of two—it had been shown by calculation that the effect looked for would be a maximum of $E' = \frac{1}{2}E$ —the latter, in the one experiment, being thrown in in the same direction as the former, and, in the second experiment, in the opposite direction. It appeared as the result of the experiments that the value of h was certainly less than 10^{-12}; or, in other words, the resistance of a conductor one square centimetre in section is not diminished by as much as $1/10^{12}$ of its value—assuming the temperature constant —when a current of 1 ampere traverses it. We may consider, therefore, that Ohm's law is verified and that the ratio of the E.M.F. to the current is constant.

It was necessary to check the values of E.M.F. of the batteries from time to time; this duty was assigned to the present Editor, then a student in the Laboratory. The apparatus he was given to do it with was a Kelvin attracted disc absolute electrometer.

OIL, TRANSFORMER: oil employed for insulating and cooling transformers. See "Transformers, Static," § (22).

OIL-IMMERSED CIRCUIT BREAKERS: automatic switches arranged to break a circuit under oil. See "Switchgear," § (6).

OMEGA: a portable instrument for the measurement of resistance. See "Measurement of Insulation Resistance," § (3) (iii.).

ONDOGRAPH, HOSPITALIER'S. See "A.C. Wave Forms," § (2).

OPTICAL TELEPHONE: an early form of vibration galvanometer. See "Vibration Galvanometers," §§ (5), (11).

OSCILLATING METER. See "Watt-hour and other Meters for Direct Current. II. Watt-hour Meters," § (12).

OSCILLATION GENERATORS, use of thermionic valves as generators of radio- and audio-frequency currents. See "Thermionic Valve, its Use in Radio Measurements," § (5) (i.) and (ii.).

OSCILLOGRAPH: an instrument for recording rapid fluctuations of current and voltage —a strongly damped high-frequency galvanometer. See "Vibration Galvanometers," § (5).

Blondel's photographic recording for high frequencies. See "A.C. Wave Forms," § (4) (iii.).

Broun's cathode ray. See *ibid.* § (3).

Cathode ray for high-frequency delineations and power curves. See *ibid.* § (4) (vi.).

Duddell's. See *ibid.* § (4) (i.).

Early history of. See *ibid.* § (3).

PHOTOELECTRICITY

§ (1) Historical.—In its widest sense the term photoelectricity includes any electrical change brought about by the action of light, but it is usually employed with a more restricted meaning to connote a change in the state of electrification of a body exposed to light. According to Clerk Maxwell's electromagnetic theory of radiation, light consists of an electromagnetic disturbance propagated with finite velocity. In the advancing wave-front there are periodic changes, both of electric and magnetic force, which take place in directions at right angles to each other. The electric theory of matter assumes that material bodies are built up of discrete electric charges—a positive nucleus surrounded by electrons, each of which carries a definite negative charge—a change in electrification then is equivalent to the addition or removal of some of the electrons. When an electromagnetic wave falls on an assemblage of electrified particles such as is supposed to constitute an atom of a metal, the electric force disturbs the equilibrium of the system, and it is not unreasonable to anticipate that under suitable conditions electrons may be separated from the metallic atoms. Such liberation of electrons by light is observed under suitable conditions, and constitutes a photoelectric discharge. It should be noted that this mode of presentation is an inversion of the historical order, for actually the study of photoelectric emission played an important part in the development of the electron theory.

In connection with his experiments on electromagnetic waves in 1887, Hertz [1] observed "a noteworthy reciprocal action between simultaneous electrical sparks." It was found that the passage of an electric spark took place more readily when a second spark occurred in view of the spark-gap. This effect was traced to the action of ultra-violet light from the second spark, for glass plates cut it off whilst plates of rock-crystal made little difference. Ultra-violet light from any other source produced a similar effect. Shortly afterwards Wiedemann and Ebert [2] showed that the seat of the action was the cathode or negative terminal of the spark-gap.

Hallwachs [3] in 1888 made the important discovery that a negatively charged body readily loses its charge when exposed to ultra-violet light, whilst the body under the same conditions, but carrying a positive charge, is not affected. This fundamental experiment in photoelectricity is easily shown to an audience by setting up a gold-leaf electroscope in front of a lantern fitted with an electric arc lamp with solid carbons. A freshly polished zinc plate attached to the electroscope is charged, and then illuminated with light from the arc by opening the side door of the lantern. A large concave mirror may be used to reflect the light upon the zinc plate so as to avoid possible complications due to direct exposure to the arc. As the carbons are moved further apart the rate of discharge is considerably increased, because under such conditions a large amount of ultra-violet light is obtained from the vapours between them. The experiments of Hallwachs were made with polished metal plates which were exposed to ultra-violet radiation. Elster and Geitel [4] found that the electropositive metals, sodium, potassium, and particularly rubidium, give a photoelectric effect when exposed to the light of the visible part of the spectrum. Less electropositive metals show smaller activity, yet even metals such as zinc and aluminium exhibit the photoelectric effect when exposed to sunlight.

When the photoelectric discharge occurs in gases at ordinary pressures the electrons set free at the surface of the illuminated plate form ions by becoming attached to one or more gaseous molecules, and these electrified carriers move slowly through the gas under the influence of an applied electric field. It is only when experiments are carried out in a very high vacuum that the conditions of the discharge are simplified. Elster and Geitel [5] found that at low pressures a transverse magnetic field diminished the photoelectric current, thus providing the method of experimenting by which J. J. Thomson,[6] Lenard,[7] and Merritt and Stewart [8] were able to identify the carriers of negative electricity with the cathode particles of a Crookes' tube. The ratio of the charge e to the mass m of such a carrier, now known as an electron, was determined, and the elementary charge e was found to be the same as in electrolysis.

§ (2) Experimental Methods and Results.—Two methods of experimenting have been of great use in investigating the pure electron discharge in a high vacuum. In the first method the potential of the illuminated plate is measured by connecting it to a gold-leaf electroscope or a quadrant electrometer, whilst all bodies in the neighbourhood are kept at the potential of the earth. In this

[1] Hertz, *Ann. d. Physik*, 1887, xxxi. 383.
[2] Wiedemann and Ebert, *Ann. d. Physik*, 1888, xxxiii. 241.
[3] Hallwachs, *Ann. d. Physik*, 1888, xxxiii. 301.
[4] Elster and Geitel, *Ann. d. Physik*, 1889, xxxviii. 40.
[5] *Ibid.*, 1890, xli. 161.
[6] J. J. Thomson, *Phil. Mag.*, 1899, xlviii. 547.
[7] Lenard, *Ann. d. Physik*, 1900, ii. 359.
[8] Merritt and Stewart, *Phys. Rev.*, 1900, xi. 230.

case the plate acquires a positive charge in consequence of the emission of electrons under the influence of the radiation falling upon it, and the positive potential, V, reached is observed. This gives a measure of the *maximum velocity*, v, of the electrons leaving the plate. For the kinetic energy, $\frac{1}{2}mv^2$, may be equated to Ve, which is a measure of the work done in leaving the plate, giving a simple formula for finding v,

$$\tfrac{1}{2}mv^2 = Ve.$$

In the second method of experimenting, the number of electrons leaving the illuminated surface is determined. The polished metal forms one plate of a condenser, the second plate being a piece of metal gauze, so that light from the source can pass through the meshes of this grid and fall on the metal under test. The grid is connected to the positive, and the plate to the negative terminal of a battery, thus setting up an electric field tending to drive electrons from the plate to the grid. The current flowing round the circuit may be measured in suitable cases by a sensitive galvanometer, or in other cases by a quadrant electrometer. Since the current depends on the number of electrons leaving the illuminated surface per second, it is possible to find how this number depends on the strength of the electric field. As the potential difference is increased the number rises to a maximum value, corresponding to the "saturation current." If the direction of the electric field be reversed, the motion of the electrons away from the plate is retarded, and only those possessing sufficient velocity will travel to the grid. The curve obtained by plotting the current against the potential is called the "velocity distribution" curve, because from it may be deduced the proportion of electrons leaving the plate with any assigned velocity.

From such experiments made in high vacua important conclusions have been reached. Both the number of electrons emitted and the velocity with which they leave the surface are found to be independent of temperature throughout a very wide range. But if the temperature is raised sufficiently (say to 800° C.) thermionic emission begins to take place (see article, "Thermionic Emission"), even in the absence of external illumination. Results of even greater significance in connection with the theory of photoelectric activity have been obtained by varying the character of the light falling upon the plate. The velocity of emission is independent of the intensity of the illumination, whilst the number of electrons is directly proportional to the intensity of the light. The latter result has been found to hold good, at least approximately, over an extremely wide range of light intensities. Elster and Geitel [1] discovered the interesting fact that, in certain cases, the strength of the photoelectric current depends upon the orientation of the plane of polarisation of the incident light. This fact must be taken into account in considering the influence of the wave-length or the frequency of the exciting light upon photoelectric emission. Pohl and Pringsheim [2] found that when the electric vector in the wave-front is parallel to the plane of incidence, the curve for the specific photoelectric activity of the alkali metals rises to a maximum in the visible part of the spectrum. Compton and Richardson [3] came to the conclusion that a similar maximum exists when the electric vector is perpendicular to the plane of incidence, but in this case the maximum is in the ultra-violet. Results of great theoretical interest have been obtained from experiments on the relation between the velocity of the electrons and the frequency of the light. The energy of the escaping electrons is given by the equation

$$Ve = \tfrac{1}{2}mv^2 = h\nu - w_0,$$

where h is Planck's universal constant [4] of radiation ($6{\cdot}55 \times 10^{-27}$ erg. sec.), ν is the frequency of the incident light, and w_0 measures the amount of work done when an electron escapes from the atom to which it is attached.

§ (3) Photoelectric Activity.—It is found that the "electron affinity" w_0 can be expressed in the form $h\nu_0$, where ν_0 is a definite frequency characteristic of the metal on which the radiation falls. The maximum kinetic energy can, therefore, be expressed in the form

$$Ve = \tfrac{1}{2}mv^2 = h(\nu - \nu_0).$$

This relation, which has been called "the fundamental law of photoelectric activity," was first suggested by Einstein,[5] on the basis of the hypothesis now discarded, of the existence of "light quanta." O. W. Richardson,[6] in collaboration with K. T. Compton, and Hughes [7] were the first to verify the result experimentally, and very accurate experiments have since been carried out by Millikan,[8] who has shown that this furnishes one of the best methods of determining the value of Planck's constant. The following table gives some of the values found by this method.

[1] Elster and Geitel, *Ann. d. Physik*, 1894, lii. 433; 1895, lv. 684; 1897, lxi. 445.
[2] Pohl and Pringsheim, *Deutsch. Phys. Gesell. Verh.*, 1910, xii. 215, 349; 1911, xiii. 474.
[3] Compton and Richardson, *Phil. Mag.*, 1913, xxvi. 549.
[4] See "Quantum Theory," Vol. IV.
[5] Einstein, *Ann. d. Physik*, 1905, xvii. 132.
[6] Richardson, *Science*, 1912, xxxv. 783; *Phil. Mag.*, 1912, xxiv. 575.
[7] Hughes, *Phil. Trans.* A., 1912, ccxii. 205.
[8] Millikan, *Phys. Rev.*, 1916, vii. 18, 355.

Photoelectric Determinations of Planck's Constant

Metal.	$h \times 10^{27}$ erg. sec.		
	Richardson and Compton.	Hughes.	Millikan.
Li	..	..	6·584
Na	5·2	..	6·569
Al	4·3	..	6·41 *
Mg	5·2	5·32	6·45 *
Zn	5·1	5·95	..
Sn	4·9	..	..
Bi	3·55	5·70	..
Cu	3·8	..	..
Pt	5·85	..	..

* Observations by Kadesch and Hennings in Millikan's laboratory.

The equation possesses a very high degree of generality, for it applies not only to ordinary light but also to X-rays, and probably is valid not only in the case of emission of electrons under the influence of light, but also when emission of radiation is brought about in consequence of the impact of electrons. The significance of the relation may best be appreciated by considering a particular case, for example, the metal sodium. The characteristic frequency, ν_0, for this metal has the value $5{\cdot}15 \times 10^{14}$ sec^{-1}, corresponding to green light in the spectrum. The equation implies that if the light falling on the sodium has a smaller frequency, or is nearer the red end of the spectrum, no emission of electrons takes place at all. But if the light is rather more bluish than this green light, electrons will be emitted, and the maximum energy of emission will increase in proportion as the difference between the frequency of the exciting light and the characteristic frequency increases. Richardson [1] has shown that Einstein's equation may be derived by thermodynamic and statistical methods on the assumptions that Planck's radiation formula is true, and that the number of electrons emitted is proportional to the intensity of monochromatic radiation.

In accordance with the quantum theory, interchange of energy between matter and aether can occur only by indivisible quanta, the amount of such a quantum being $h\nu$ Thus photoelectric emission falls into place naturally as one instance of this general theory.

§ (4) Photoelectric Fatigue.—The photoelectric activity of a metal surface which has been freshly polished diminishes as the time that has elapsed since the metal was polished is increased. This is known as the "fatigue" of the Hallwachs effect. The phenomenon is a complicated one, and the cause is not necessarily the same in all cases. Thus in the case of an alkali metal exposed to air, oxidation would bring about a rapid diminution in activity; but in the case of metals such as copper and zinc it has been shown that oxidation is not the primary cause of fatigue. The subject has been investigated by Kreusler,[2] E. v. Schweidler,[3] Hallwachs,[4] Allen,[5] Ullmann,[6] and others, and it has been shown that the condition of the gaseous layer at the surface of the plate is the most important factor in determining the photoelectric fatigue of metal plates in gases. It seems probable that in the case of a perfectly clean metal surface in a very high vacuum no fatigue would take place, but the difficulties in even approaching the realisation of such an ideal are so great that it is not surprising that experimentally contradictory results have been obtained. Photoelectric fatigue in a vacuum has been the subject of experiments by Lenard,[7] Ladenburg,[8] Bergwitz,[9] Millikan and Winchester,[10] Robinson,[11] Compton,[12] and others.

§ (5) Photoelectric Activity and Contact Potential.—The phenomena of photoelectric and thermionic emission and contact potential are intimately related to one another. At one period there was a widespread feeling that all these were to be explained as the result of chemical action. Thus with regard to the photoelectric effect Pohl and Pringsheim [13] suggested that as it was much decreased by improving the vacuum, perhaps the whole effect is to be attributed to interaction between gas and metal. Wiedmann and Hallwachs [14] went so far as to say that "the presence of gas is a necessary condition for appreciable photoelectric electron emission" from potassium. Fredenhagen and Küstner [15] came to a similar conclusion with regard to the photoelectric current from zinc, and later Fredenhagen [16] stated that both the photoelectric and the thermionic emission of electrons from potassium depend entirely on the presence of gas. If such views were correct, it would be impossible to get any electric discharge through

[1] Richardson, *Phil. Mag.*, 1912, xxiii. 615; 1912, xxiv. 570. See also "Thermionics," § (5) (ii.).

[2] Kreusler, *Ann. d. Physik*, 1901, vi. 398.
[3] E. v. Schweidler, *Akad. Wiss. Wien, Ber.*, 1903, cxii. 974.
[4] Hallwachs, *Phys. Zeits.*, 1904, v. 489; *Ann. d. Physik*, 1907, xxiii. 459.
[5] Allen, *Proc. Roy. Soc.*, 1907, lxxviii. 483; 1909, lxxxii. 161; *Phil. Mag.*, 1910, xx. 565.
[6] Ullmann, *Ann. d. Physik*, 1910, xxxii. 15.
[7] Lenard, *Ann. d. Physik*, 1902, viii. 149.
[8] Ladenburg, *Ann. d. Physik*, 1903, xii. 568.
[9] Bergwitz, *Phys. Zeits.*, 1907, viii. 373.
[10] Millikan and Winchester, *Phil. Mag.*, 1907, xiv. 188; *Phys. Rev.*, 1910, xxx. 85, 287.
[11] Robinson, *Phil. Mag.*, 1912, xxiii. 255.
[12] Compton, *Phil. Mag.*, 1912, xxiii. 579.
[13] Pohl and Pringsheim, *Phys. Zeits.*, 1913, xiv. 1112.
[14] Wiedmann and Hallwachs, *Deutsch. Phys. Gesell. Verh.*, 1914, xvi. 107.
[15] Fredenhagen and Küstner, *Phys. Zeits.*, 1914, xv. 65, 68.
[16] Fredenhagen, *Deutsch. Phys. Gesell. Verh.*, 1914, xvi. 201.

a perfect vacuum owing to the impossibility of extracting any electrons from the electrodes. But with the great improvements which have been made, particularly in America, in the production of high vacua it has been proved conclusively that in the highest attainable vacuum there is a pure electron discharge, which has been utilised in the Coolidge tube and in thermionic valves and amplifiers with results of increasing importance.[1] Dushman [2] has repeated the experiments of Hallwachs and Fredenhagen in a better vacuum than they employed, and found no difficulty in obtaining both thermionic and photoelectric emission. Most of the failures to obtain electron emission in a high vacuum may be attributed to neglect of the effect of the "space charge," due to the accumulation of negative electrons in the vacuum.

The relation between the velocity of photo-electrons and the contact potentials between metals required by Richardson's theory, and the value of the "electron affinity," w_0, of the metals, have been investigated by Richardson and Compton,[3] and by Millikan [4] and Hennings,[5] and the results of elaborate experiments have proved conclusively that the theoretical results hold with great accuracy. In this work, clean fresh metallic surfaces have been secured by cutting the metal in a very high vacuum. Experiments by Langmuir [6] have shown that films of metal distilled in apparatus which has not been baked out show signs of very marked contamination owing to the gases carried down by them.

§ (6) The Becquerel Effect.—Becquerel [7] in 1865 observed that certain substances used as electrodes in an electrolyte show a difference in potential when one plate is in darkness and the other is illuminated. This effect, which has been studied by Minchin,[8] Wilderman,[9] and others, may be attributed to a thin solid layer sensitive to light, or to the presence of a light-sensitive substance in the electrolyte. Goldmann and Brodsky [10] have shown that there is great similarity between the Becquerel and Hallwachs effects, and the former may be explained by assuming that electrons are liberated from the electrode by light, but these have to pass through a relatively strong field of a double layer at the surface of the electrode. On emerging into the conducting medium they are converted into slow-moving ions. Interesting results have been obtained by T. W. Case,[11] who has constructed photo-chemical cells in which the sensitive substance is an oxide of copper. A copper plate (3 in. by 4 in.) coated with cupric oxide will give in strong sunlight 0·1 volt and a steady current of about ½ milliamp., when a clean copper plate is used as the other electrode and the electrolyte is a dilute solution of common salt. Two copper plates coated with red cuprous oxide and immersed in copper formate will give 0·11 volt and a current density of about 50 microamps./cm.^{2}.

§ (7) Changes in Electrical Conductivity due to Light.—Arrhenius [12] showed that by exposing the silver halides to light an increase in electrical conductivity was produced. This phenomenon has been investigated by Scholl [13] and by W. Wilson,[14] who attribute the increased conductivity to electrons set free by the action of the light. Various other substances have been examined by Coblentz and Emerson.[15] In the case of ordinary metals the number of free electrons is already so large that the additional electrons liberated by illuminating the surface make no appreciable difference in the electrical conductivity. The well-known sensitiveness of selenium to light, shown by the diminution in electrical resistance when thin films are illuminated, may be explained by supposing that the incident light liberates slowly moving electrons, which remain within the substance and thus increase its conducting power.

§ (8) Fluorescence and Phosphorescence.—The separation of electrons under the influence of light has an important bearing on our conception of fluorescent and phosphorescent phenomena. According to the theory of Stark [16] there exists at the surface of the atom a limited number of separable electrons which play the part of valency electrons and serve to bind together the chemical atoms in a molecule. When a valency electron is separated from its atom and becomes attached to a second atom we obtain a positive and a negative atomion. Stark supposes that the carrier of a band spectrum is a single atom or a molecule composed of several atoms, whilst the carriers of the series spectra are positive atomions. Emission in band spectra is supposed to occur when valency electrons which have been partially or totally separated are restored to the atom. One process by which the separation may be effected is photo-

[1] Langmuir, "The Pure Electron Discharge and its Applications in Radio Telegraphy and Telephony," *General Electric Review*, May 1915.
[2] *Am. Electrochem. Soc. Trans.*, 1916, xxix. 125.
[3] Richardson and Compton, *Phil. Mag.*, 1912, xxiv. 570; 1913, xxvi. 549.
[4] Millikan, *Phys. Rev.*, 1914, iv. 73; 1916, vii. 355.
[5] Hennings, *Phys. Rev.*, 1914, iv. 228.
[6] Langmuir, *Am. Electrochem. Soc. Trans.*, 1916, xxix. 155.
[7] Becquerel, *La Lumière*, Paris, 1865, ii. 121.
[8] Minchin, *Phil. Mag.*, 1891, xxxi. 207.
[9] Wilderman, *Proc. Roy. Soc.*, 1904, lxxiv. 369.
[10] Goldmann and Brodsky, *Ann. d. Physik*, 1914, xliv. 849, 901.
[11] T. W. Case, *New York Electr. Soc.*, June 14, 1916; *Am. Electrochem. Soc. Trans.*, 1917, xxxi. 351.
[12] Arrhenius, *Wien. Ber.*, 1887, xcvi. 831.
[13] Scholl, *Ann. d. Physik*, 1905, xvi. 193, 417.
[14] W. Wilson, *Ann. d. Physik*, 1907, xxiii. 107.
[15] Coblentz and Emerson, *Washington Acad. Sc. J.*, 1917, vii. 525.
[16] Stark, *Prinzipien der Atomdynamik*, 1911, ii. 213-27.

electric action. In the case of fluorescence, restoration of the electron follows immediately upon separation; in the case of phosphorescence a finite time elapses between the separation of a valency electron and its return. It has been proved experimentally that fluorescence is generally accompanied by the photoelectric effect, but this is not necessarily the case, as the separation of the electron may be only partial or the electron may be caught by surrounding molecules, especially when the substances are in solution. Phosphorescence, however, implies complete separation—that is, photoelectric emission. The theory of Lenard [1] in many respects resembles that of Stark, but Lenard distinguishes between the photoelectric electrons and the radiating electrons. When the former return to the atom, the latter are set in vibration and consequently emit radiation. Phosphorescent substances are usually good insulators, and the separated electrons may remain attached to other atoms for considerable periods before returning to the atoms from which they originated. The variation of phosphorescence with temperature can be accounted for on this theory, which serves also to give an explanation of the law of Stokes that the wave-length of the exciting light, which brings about the release of the photo-electron, must be less than the wave-length of the phosphorescent light. Lenard found that there were three separate bands which could produce excitation of an emission band of a phosphor of the alkali earths. Pohl [2] pointed out that the wave-lengths of these bands were inversely proportional to the square roots of the natural numbers 2, 3, and 4. This appears to be in agreement with the formula of Lindemann [3] for the wave-length corresponding to the maximum selective photoelectric effect.

§ (9) Photochemical and Photoelectric Changes.—Many instances are known in which chemical changes are brought about by the action of light. According to the electron theory of matter, it is the valency electron, or more probably a pair of electrons, which plays the part of a chemical bond. Any chemical change must be preceded by a physical change, that is to say, there must be some displacement or some commotion taking place within the molecule before actual decomposition or recombination occurs. It may be regarded as practically certain that the first stage in any photochemical reaction consists in the separation, either partial or complete, of negative electrons from positive atoms under the influence of light. This theory at once makes it obvious that absorption of light is necessary before any change can take place at all. It is not at present possible to trace out all the stages of the process of a given photochemical change, but the first step in such a change is the production of ionised molecules, which are then in a condition suitable for recombination in a different fashion. In some cases such a condition of ionisation may persist for quite long periods, as is shown in the phenomenon of thermo-luminescence. Electrons which have been separated from their parent atoms may become attached to other atoms or molecules and, when the conditions are suitable, may remain attached for a considerable time. This conception has been employed by Joly [4] in connection with the formation of the latent image in photography. He regards the latent image as built up of ionised atoms or molecules, upon which the chemical effects of the developer are subsequently directed. Whilst such a physical theory of the latent image is capable of explaining many of the observed phenomena,[5] there are many scientific photographers who hold that it is necessary to assume that the ionisation brings about a chemical change. Joly has drawn attention to the importance of photoelectric action in radiotherapy, and has pointed out that the effects of light on the growth of the cell, which is highly sensitive to ionic concentration, resemble, superficially at least, the effects of radiation in the formation of the latent image. A photoelectric theory of vision has been suggested by H. S. Allen,[6] Sir Oliver Lodge,[7] and Joly according to which photoelectric action takes place in the chemical substance, or, it may be, substances, contained in the rods or cones of the retina when irradiated, so that a separation of electrons occurs resulting in electrification of the nerve cells which set up the nervous impulse to the sensorium.

§ (10) Photoelectric Photometry.—It is instructive to find that the scientific study of an obscure phenomenon has resulted, as in so many other cases, in the development of a method of measurement of increasing technical importance. Hallwachs [8] based a method of photometry on the fact that a plate coated with copper oxide showed constant photoelectric activity over long periods of time, when preserved in a small air-tight vessel. The intensity of the radiation falling upon the prepared surface was determined by measuring the saturation current passing through the photoelectric cell. More sensitive cells, which may be used not only for ultra-violet light

[1] Lenard, *Ann. d. Physik*, 1909, xxviii. 476.
[2] Pohl, *Deutsch. Phys. Gesell. Verh.*, 1911, xiii. 961.
[3] Lindemann, *Deutsch. Phys. Gesell. Verh.*, 1911, xiii. 482.
[4] Joly, *Nature*, 1905, lxxii. 308.
[5] See "Photoelectricity" (H. S. Allen), 1913, pp. 206-216.
[6] H. S. Allen, *Journal of the Röntgen Society*, 1919, xv. 40.
[7] Lodge, British Association, 1919.
[8] Hallwachs, *Phys. Zeits.*, 1904, v. 489.

but also for the visible part of the spectrum, originated from the work of Elster and Geitel [1] on the alkali metals. In the first apparatus the sensitive surface was of potassium, placed in an atmosphere of argon or helium to secure permanence. This was used by Dember [2] in a determination of Avogadro's constant by observations on the absorption of sunlight in the atmosphere. Experiments by Richtmyer [3] and others have extended the law of proportionality between photoelectric current and light intensity, first given by Elster and Geitel, up to high values of the intensity of illumination (about 600 candle-feet). Some idea of the magnitude of the currents obtained is given by the fact that from a sodium surface in a glass cell, a current of about $2 \cdot 6 \times 10^{-8}$ amperes was produced by an arc giving an illumination of 100 candle-feet. By taking suitable precautions to avoid the dark discharge and the dark after-effect, Elster and Geitel [4] found approximate proportionality between the photoelectric discharge from a potassium or rubidium cell and the light intensity for illuminations varying from one third of bright sunlight (about 30,000 lux) down to about 6×10^{-4} lux.

In making photometric determinations of a high order of accuracy, the majority of investigators in this field have concluded that it is not safe to assume for any photoelectric cell a direct proportionality between photoelectric current and exciting radiant power. The subject has been investigated at the United States Bureau of Standards,[5] and a null method of photoelectric spectrophotometry has been developed by K. S. Gibson.[6] This method eliminates the necessity of corrections in connection with the current-irradiation law or the so-called "dark currents" in the photoelectric cells. "The potassium hydride cell now on the market, when used with an incandescent lamp and a glass dispersing prism, gives a maximum response usually near 460 $\mu\mu$; and the photoelectric method, under these conditions, admirably supplements the visual and photographic methods, being best where they become poor, and becoming poor only after they have become reliable."

It should be noted that the small current obtainable from a photoelectric cell under ordinary conditions of illumination may be amplified by means of one or more thermionic triode valves. Used in this way the photoelectric cell may well prove its usefulness not only in connection with photometry, but in signalling without wires and as a means of scientific investigation.

H. S. A.

[1] Elster and Geitel, *Ann. d. Physik*, 1893, xlviii. 625; *Phys. Zeits.*, 1911, xii. 609; 1912, xiii. 739.
[2] Dember, *Gesell. Wiss. Leipzig, Ber.*, 1912, p. 259.
[3] Richtmyer, *Phys. Rev.*, 1909, xxix. 71, 404.
[4] Elster and Geitel, *Phys. Zeits.*, 1913, xiv. 741.
[5] Coblentz, *Bureau of Standards Scientific Paper*, No. 319, 1918.
[6] K. S. Gibson, *id.*, No. 349, 1919.

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Bibliographies of the original papers on Photo-electricity have been prepared by E. v. Schweidler, *Akad. Wiss. Wien, Ber.*, 1898, cvii. II.a, 881; *Jahrbuch der Radio-aktivität*, 1904, i. 358-366, and by R. Ladenburg, *Jahrbuch der Radio-aktivität*, 1909, vi. 428-433. A bibliography of papers on Photoelectric Fatigue has been published by H. S. Allen in the *British Association Reports* (Sheffield), pp. 538-541, 1910. References to later papers may be found from the annual volumes of *Science Abstracts*, mostly under the heading "Discharge in Gases and in Vacuo."

PHOTO-ELECTRONS AND THERMIONS, determination of e/m of. See "Electrons and the Discharge Tube," § (16).

PHOTOMETRY, PHOTOELECTRIC. See "Photo-electricity," § (8).

PHYS. TECH. REICH. METHOD, for high magnetisation tests. See "Magnetic Measurements and Properties of Materials," § (41).

PICOU PERMEAMETER. See "Magnetic Measurements and Properties of Materials," § (36).

PIERCE'S HEAVY SOLENOID, for high magnetisation tests. See "Magnetic Measurements and Properties of Materials," § (43).

PIERCE'S YOKE METHOD, for high magnetisation tests. See "Magnetic Measurements and Properties of Materials," § (45).

PIEZO-ELECTRIC EFFECT, relation of, to unsymmetrical crystal structure. See "Piezo-electricity," § (3).

PIEZO-ELECTRICITY

§ (1) PIEZO- AND PYRO-ELECTRICITY.—Certain crystals exhibit electrical charges at particular regions when heated or cooled, or when subjected to stresses; in the former case the effect is called pyro-electric, and in the latter piezo-electric. The effect of pressure was discovered by F. and P. Curie [7] in 1880. All piezo-electric substances are pyro-electric, and it is doubtful if any pyro-electric effect would be obtained if stresses were eliminated.

§ (2) TRUE PYRO-ELECTRIC EFFECT DOUBTFUL.—The existence of a true pyro-electric

[7] *Comptes Rendus*, 1880, xci.

effect has been questioned by several investigators. A research, carried out jointly by Riecke and Voigt,[1] in which Pfaff's[2] values of the expansion coefficient of tourmaline were used, resulted in such a slight difference to be accounted for by any pyro-electric effect — this difference was about 10 per cent of the total—that these investigators, in view of the uncertainty of the many different measurements on which they based their calculations, came to the conclusion that true pyro-electricity does not exist. This opinion had been previously expressed by Röntgen, and by F. and P. Curie. In a later paper, however, Voigt[3] arrived at the opposite result. The tourmaline which had been investigated by him and Riecke was tested for its thermal expansion by Pulfrich and Kellner, at Jena. Using the Pulfrich-Kellner values of the expansion coefficients of tourmaline, Voigt now found that 18 per cent of the pyro-electric excitation which took place on heating the tourmaline should be regarded as a direct effect of the temperature rise, and would consequently remain were it possible to remove the thermal deformation by suitable pressures.

In a paper published in 1914, in which he summarised his later investigations on pyro- and piezo-electricity, Röntgen[4] also discussed the question of the existence of a true pyro-electricity. His experience shows that the determination, especially of the electric constants of tourmaline, is coupled with a great deal of uncertainty. It is true that all Voigt's constants were determined from samples taken from the same crystal, but it does not seem to be a necessary consequence that all the preparations were the same in their properties, particularly their electrical properties, in view of Röntgen and Riecke's experiments, according to which an apparently homogeneous crystal can be shown to be subject to different degrees of pyro-electric excitation at two different positions. The results of Röntgen's own experiments and of others suggested by him give no indication of the existence of true pyro-electricity in tourmaline. Although Röntgen does not regard this result as certain proof of the non-existence of pyro-electricity, he does find that Voigt's experiments are not sufficient to establish the existence of this effect beyond all doubt. Lindman[5] applies the following formula, derived from Voigt's theory, in which λ/v represents the fraction of the total pyro-electric excitation which is to be regarded as true pyro-electricity:

$$\lambda = v - \mu\frac{(s_{11}+s_{12})\alpha_3 - 2s_{13}\alpha_1 + (s_{33}\alpha_1 - s_{13}\alpha_3)2\delta_{31}/\delta_{33}}{s_{33}(s_{11}+s_{12}) - 2s_{13}^2}.$$

[1] *Wied. Ann.*, 1892, xlv. [2] *Pogg. Ann.*, 1858, civ.
[3] *Wied. Ann.*, 1898, lxvi. [4] *Ann. d. Phys.*, 1914, xlv.
[5] *Ann. d. Phys.*, 1920, lxii.

The quantities s_{hk} ($h=1, 2, 3$; $k=1, 2, 3$) are moduli of elasticity, δ_{31} and δ_{33} the coefficients of thermal dilatation perpendicular and parallel to the principal axis, μ a quantity proportional to δ_{13}, and v a quantity proportional to the total pyro-electric effect. For tourmaline, Lindman found λ/v to be 0·119; *i.e.* of the total effect due to increase of temperature, the true pyro-electricity may be 11·9 per cent. However, this result was not determined directly, but by combining a number of determinations of constants. Hence Lindman considers the existence of a true pyro-electric effect to be doubtful.

§ (3) Piezo Effect and Crystalline Form.—All crystals which exhibit the piezo-electric effect lack symmetry. In crystallography this asymmetry was supposed to be due to a suppression of certain faces on formation of the crystal; it appears to be due to an asymmetric arrangement of molecules of regular form, but asymmetry of the molecules is also possible. Irregularity of structure is also shown by the difference in effect of solvents on the crystal faces and by the rotation of the plane of polarisation of polarised light. Piezo-electric and optical activity in crystals always exist simultaneously, but optical activity also arises from unsymmetrical molecules, as in solutions which are optically active.

Tourmaline is an example of a piezo-electric crystal, and its crystalline form is shown in *Fig.* 1; the two ends of the crystal are markedly different. When the temperature of the crystal is raised, or the crystal is extended, a becomes positively charged, and b is charged negatively; on cooling, or if the crystal is compressed, the signs of the charges are reversed. If the crystal is powdered and the particles heated, or cooled, the polar charges cause them to gather in the form of chains.

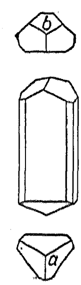


Fig. 1.

The existence of the so-called pyro effect is most easily demonstrated by heating tourmaline crystals in an oven, and dusting them, after removal, with a mixture of red lead and sulphur. The mixture should be shaken through muslin, when, by friction, the sulphur becomes negatively electrified and adheres to one end of the crystal, and the red lead, which is positively electrified, adheres to the other end. If, after being heated, the crystal is allowed to cool, the polarity is reversed; if the temperature is maintained steady no electrical effects are observed at high or low temperatures. A heated crystal, if suspended by a fine fibre, may be attracted and repelled by electrified bodies or by other heated crystals which exhibit the pyro-electric

effect; such crystals are those of Rochelle salt, silicate of zinc, cane-sugar, and quartz. With quartz there is considerable difficulty in finding good specimens, as it is prone to twinning, and two twin crystals have opposite effects, so that over a relatively small area the total effect is often small.

To demonstrate the piezo effect it is convenient to place a sheet of tinfoil on a slab of tourmaline and connect the foil to an electrometer. When pressure is applied the electrometer is deflected, and when the slab is turned over—the tinfoil being still uppermost—the direction of the deflection is reversed. The relation between pressure and the electric charge was found by F. and P. Curie to be a linear one, the phenomenon being represented by the equation $Q=KP$, where Q is the charge in electrostatic units, P the total force supplied, and K a constant, known as the piezo-electric constant. Rochelle salt in the crystalline form was found by the Curies to have the largest piezo-electric constant of any substance so far examined.

Boracite [1] is another example of a piezo-electric crystal; its form is that of a cube with four alternate corners truncated. *Figs.* 1 and 2 show that there are "blunt" and "sharp" ends, or edges, to the crystals of tourmaline and boracite; if such a crystal is pressed between opposite sharp and blunt ends, opposite electrification of these ends results. A similar effect is produced by cooling the crystal. When a natural hexagonal prism of quartz is heated or cooled its six edges are electrified positively and negatively in alternate order.

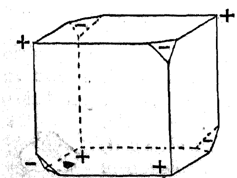


FIG. 2.

§ (4) PREPARATION OF ROCHELLE SALT CRYSTALS.—The Rochelle salt crystal not only has the largest piezo-electric constant, but is also the most easily prepared. Nicholson [2] has grown highly efficient crystals varying in weight from a few grams to 500 grams according to the volume and density of liquor used. In all cases crystals were grown from perfect nuclei possessing definite form, such nuclei being produced in large numbers by an ordinary crystallising process.

The nuclei are immersed in a clear saturated solution of the salt (Rochelle salt is sodium potassium tartrate $NaKC_4H_4O_6 \cdot 4H_2O$), the nuclei and the solution being at the same temperature before immersion of the former. The crystal may be suspended from a thread, or floated on mercury, or supported on a glass plate. Growth of the crystals is brought about by normal evaporation of the solution or by the application of temperature gradients to saturated solutions. Nicholson states that the latter method gives the better crystals, and that improvement in the piezo-electric property is brought about by desiccating the crystals; this is effected by soaking in alcohol and heating, the subsequent loss in weight being 3 per cent. However, Valasek is strongly opposed to desiccation and believes that complete desiccation renders a crystal entirely inactive.

The crystal surfaces of a Rochelle salt crystal may be regarded as consisting of two systems which are normal to each other. One system is parallel with the principal axis and therefore engirdles the crystal; the other system comprises the two basal planes parallel to the other two axes of the crystal. Nicholson advises that the crystals be grown with the principal or crystallographic axis, and one of the other axes, in a horizontal plane; the growth is then dominant along the axes in the horizontal plane, development along the vertical axis being partially suppressed. On cooling rapidly the nuclei increase in size and the crystal acquires a "composite" structure; at each end of the nucleus and along its principal axis there appears a pyramid which forms a polar terminal, and the electrical properties of the crystal has led Nicholson to suggest that during growth the molecules throughout the pyramidal regions are subject to forces which turn them, in planes containing the principal axis, through a right angle. The poles are accordingly at right angles to each other, the pyramids being electrically positive when the rest of the crystal structure is negative, and *vice versa.*

For piezo-electric use the basal planes of the crystal are made slightly concave in the polar regions and waxed tinfoil electrodes are pressed on the crystal.

§ (5) APPLICATIONS. (i.) *Effect of Axial Compression.*—With a crystal of Rochelle salt of composite structure Nicholson has found that the relation between electric potential and various loads applied on the diagonals is not linear, and that it is advantageous to apply permanent static compression. Over a large range of size of crystal, the value of the applied static pressure for maximum rate of change of charge corresponds to an absolute force of approximately 15 kilograms. The static pressure may be applied permanently without fatigue.

For experimental purposes a crystal is compressed between two discs held together by springs; the discs form one electrode and a girdle connection forms the other. If a suitable receiver is connected to the electrodes and the crystal is placed on a table, the ticking of a watch, near the crystal, can be heard.

(ii.) *Effect of Torsion.*—With the composite

[1] S. P. Thompson, *Elec. and Mag.* § (74).
[2] *Amer. Inst. Elect. Eng. Proc.*, 1919.

structure Rochelle salt crystal a twisting couple about the principal axis produces the greatest piezo-electric effect. The relation between the piezo-electric charge and the torsion is linear and is given by $Q = KFL$, where Q is the charge in electrostatic units, F is the force in kilograms, and L is the length of the lever arm in centimetres. For several crystals experimented with, K is as great as 100. A clockwise torque has the same effect, with respect to sign of charge, as is produced by tension or increase of temperature.

(iii.) *Effect of Alternating Stresses.*—If a crystal, mounted as already described, is subjected to fluctuating impulses, it generates a corresponding set of electrical oscillations, and if it is subjected to rapidly changing electric potentials there is produced a corresponding train of impulses.

(iv.) *Application to Phonograph.* — When, through a phonographic record and by means of a special needle attached to a disc of a mounted crystal, the latter is subjected to torsional stresses, the potentials of the electrodes change, and these changes may be amplified. Sufficient power may thus be produced to operate simultaneously hundreds of telephone receivers.

(v.) *Application as Sound Transmitter and Receiver.*—To make the crystal system efficient as a receiver of sound waves Nicholson surrounds the crystal with a twisted cylindrical diaphragm which is attached by means of rings to the two plates compressing the crystal. When sound waves strike the diaphragm, torsional stresses are applied to the crystal, and corresponding electrical oscillations are generated. Singing against the diaphragm of a good receiver, especially when the frequency is near the resonance frequency of the crystal (between 200 and 600), often produces a difference of potential at the electrodes of 15 volts on open circuit. With the use of a triode valve amplifier, good transmission of speech may be obtained, with a piezo-electric crystal as a receiver at one end and a second one as a transmitter at the other.

(vi.) *Application of Piezo Effect to Measurement of Pressure.*—Sir J. J. Thomson [1] suggested in 1919 that since the quantity of electricity liberated is directly proportional to the pressure, pressure changes in explosions might be measured by direct measurements of the quantities of electricity produced. In the case of an explosion in a gun the rise of pressure is complete in less than 0·001 second and the interpretation of mechanical pressure determinations is very doubtful. The principle of the application described by Thomson is explained by *Fig.* 3. Appropriate parts of a crystal are exposed to the explosive pressure to be measured; as the pressure rises there is a corresponding increase in the quantity

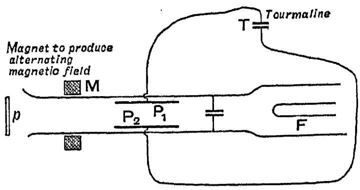


FIG. 3.

of electricity liberated and a proportional increase in the potential of the plates P_1 P_2 in a cathode ray tube. A stream of electrons from a hot tungsten wire is propelled through a fine tube and through the space between the plates; as long as the plates are of the same neutral potential the electrons move in a straight line and hit a screen or photographic plate at the end of the apparatus in a central spot. When, however, the potentials of the plates change, due to the liberation of charges from the crystal, the stream of electrons is deflected, and from the deflection, together with calibration data, the pressure may be calculated. An exposure of less than 0·00001 second is sufficient to affect the photographic plate, and a pressure-time curve may be obtained by moving the plate or by dropping it. In practice the plate is not moved, but an independent rapidly alternating magnetic field is placed between the plates P_1 P_2 and the photographic apparatus. The alternating magnetic field causes the spot formed by the cathode rays to move at right angles to the direction of deflection produced by pressure on the crystal, with the result that, during an explosion, the spot is displaced horizontally by an amount proportional to the pressure, and vertically by an amount proportional to time. The vertical speed of the spot varies harmonically and a time-pressure curve can readily be obtained from such a photograph. In this way the rate of rise of pressure has been determined when a mixture of hydrogen and oxygen is exploded; results have also been obtained with the explosion of a charge of gun-cotton in sea-water. Further, it appears possible to use the method to determine the actual rate of propagation of an explosive wave through tubes by putting one crystal in one part of the tube and a second crystal in another part. In the same way the rate of propagation of an explosive wave through a thickness of a solid may be determined by attaching crystals to the upper and lower surfaces.

F. E. S.

[1] *Engineering*, 1919, cvii. 543. See also "Cathode Ray Manometer," Vol. I.; Keys, "A Piezo-electric Method of measuring Explosive Pressures," *Phil. Mag.*, 1921, xlii. 473.

POSITIVE RAYS

POSITIVE rays were discovered by Goldstein in 1886 in electrical discharge at low pressure. In some experiments with a perforated cathode he noticed streamers of light behind the perforations. This luminosity he assumed was due to rays of some sort which travelled in the opposite direction to the cathode rays and so passed through the apertures in the cathode, these he called "canalstrahlen." [1] Subsequently Wien showed that they could be deflected by a magnetic field.[2] They have been very fully investigated in this country by Sir J. J. Thomson,[3] who called them Positive Rays on account of the fact that they normally carry a charge of positive electricity.

§ (1) NATURE OF POSITIVE RAYS.—The conditions for the development of the rays are, briefly, the ionisation of a gas at low pressure in a strong electric field. Ionisation, which may be due to collision or radiation, means in its simplest case the detachment of one electron from a neutral atom. The two resulting fragments carry charges of electricity of equal quantity but of opposite sign. The negatively charged one is the atomic unit of negative electricity itself,[4] and is the same whatever the atom ionised. It is extremely light and therefore in the strong electric field rapidly attains a high velocity and becomes a cathode ray. The remaining fragment is clearly dependent on the nature of the atom ionised, and therefore of the gas in the discharge tube. It is immensely more massive than the electron, for the mass of the lightest atom, that of hydrogen, is about 1850 times that of the electron, and so will attain a much lower velocity under the action of the electric field. However, if the field is strong and the pressure so low that it does not collide with other atoms too frequently it will ultimately attain a high speed in a direction opposite to that of the detached electron, and become a "positive ray." The simplest form of positive ray is therefore an atom of matter carrying a positive charge and endowed, as a result of falling through a high potential, with sufficient energy to make its presence detectable. Positive rays can be formed from molecules as well as atoms, so that it will at once be seen that any measurement of their mass will give us direct information as to the masses of atoms of elements and molecules of compounds, and that this information will refer to the atoms or molecules *individually*, not, as in chemistry, to the mean of an immense aggregate. It is on this account that the accurate analysis of positive rays is of such importance.

§ (2) METHODS OF DETECTION.—For visual effects the rays are best detected by a screen made of powdered willemite, which glows a faint green when bombarded by them. When permanent effects are required this screen is replaced by a photographic plate. The sensitivity

[1] *Berl. Ber.*, 1886, xxxix. 691.
[2] *Verh. d. Phys. Gesell.*, 1898, p. 17.
[3] *Rays of Positive Electricity and their Application to Chemical Analyses*, Longmans, Green, 1913.
[4] R. A. Millikan, *The Electron*, University Chicago Press, 1918.

of the plate to positive rays bears no particular relation to its sensitivity to light, so far the best results have been obtained from comparatively slow "process" plates of the type known as "Half-tone." The real relative intensities of rays of different mass cannot be compared by screens or photographic plates, except in the possible case of isotopes of the same element; they can only be determined reliably by collecting the rays in a Faraday cylinder and measuring their total electric charge.

§ (3) THOMSON'S METHOD OF ANALYSIS.—The method by which Sir J. J. Thomson made such a complete investigation into the properties of positive rays, and which still remains pre-eminent in respect to the variety of information it supplies, consists essentially in allowing the rays to pass through a very narrow tube and then analysing the fine beam so produced by electric and magnetic fields.

The construction of one of the types of apparatus used is indicated in *Fig.* 1. The

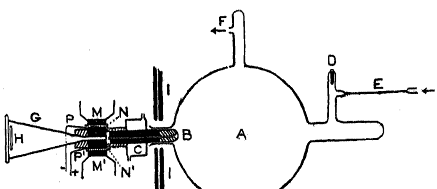


FIG. 1.

discharge by which the rays are made takes place in a large flask A similar to an ordinary X-ray bulb of about $1\frac{1}{2}$ litres capacity. The cathode B is placed in the neck of the bulb. Its face is made of aluminium, and so shaped that it presents to the bulb a hemispherical front provided in the centre with a funnel-shaped depression. This hole through which the rays pass is continued as an extremely fine-bore tube, usually of brass, about 7 cm. long, mounted in a thick iron tube forming the continuation of the cathode as indicated. The finer the bore of this tube the more accurate are the results obtained, and tubes have been made with success as narrow as one-tenth of a millimetre, but as the intensity of the beam of rays falls off with the inverse fourth power of the diameter a practical limit is soon reached. The cathode is kept cool during the discharge by means of the water-jacket C.

The anode is an aluminium rod D, which is generally placed for convenience in a side tube. In order to ensure a supply of the gas under examination a steady stream is allowed to leak in through an exceedingly fine glass capillary tube E, and after circulating through the apparatus is pumped off at F by a Gaede rotating mercury pump. By varying the speed of the pump and the gas-holder communicating with E, the pressure in the discharge tube may be varied at will and maintained at any desired value for considerable lengths of time. The pressure is usually adjusted so that the discharge potential is 30,000 to 50,000 volts. During the discharge all the conditions necessary for the production of positive rays are abundantly present in A. Under the influence of the enormous potentials they attain high speeds as they fly towards the cathode, and those falling axially pass right through the fine tube, emerging as a narrow beam.

This beam is subjected to analysis by causing it to pass between the pieces of soft iron P, P′ which are placed between the poles M, M′ of a powerful electromagnet. P and P′ constitute the pole pieces of the magnet, but are electrically insulated from it by thin sheets of mica N, N′, and so can be raised to any desired potential difference by means of the leads shown in the *figure*. The rays then enter the highly exhausted "camera" G, and finally impinge upon the fluorescent screen or photographic plate H. In order that the stray magnetic field may not interfere with the main discharge in A, shields of soft iron I, I′ are interposed between the magnet and the bulb.

If there is no field between the plates P, P′ the beam of rays will strike the screen at a point in line with the fine tube called the undeflected spot. If an electric field of strength X is now applied between the plates a particle of mass m, charge e, moving with velocity v, will be deflected in the plane of the paper and will no longer strike the screen at the undeflected spot, but at a distance x from it. Simple dynamics show that if the angle of deflection is small $x=k(\mathrm{X}e/mv^2)$. In the same way, if the electric field is removed and a magnetic field of strength H applied between P and P′ the particle will be deflected at right angles to the plane of the paper and strike the screen at a distance y from the undeflected spot where $y=k'(\mathrm{H}e/mv)$, k and k' being constants depending solely on the dimensions and form of the apparatus used.[1] If now, with the undeflected spot as origin, we take axes of co-ordinates OX, OY along the lines of electric and magnetic deflection, when both fields are applied simultaneously the particle will strike the screen at the point (x, y) where y/x is a measure of its velocity and y^2/x is a measure of m/e, its ratio of mass to charge.

Now e can only exist as the electronic charge $4{\cdot}77 \times 10^{-10}$ C.G.S. or a simple multiple of it. Thus if we have a beam of positive rays of constant mass, but moving with

[1] "Electrons and the Discharge Tube," § (9).

velocities varying over a considerable range, y^2/x will be constant and the locus of their impact with the screen will be a parabola pp' (*Fig.* 2). When other rays having a larger mass m' but the same charge are introduced into the beam, they will appear as another parabola qq' having a smaller magnetic displacement. If any straight line p, q, n be drawn parallel to the magnetic axis OY cutting the two parabolas and the electric axis OX in p, q, n it will be seen at once that $m'/m = pn^2/qn^2$. That is to say, the masses of two or more particles can be compared directly by merely measuring lengths the ratio of which is entirely independent of the form of the apparatus and the experimental conditions.

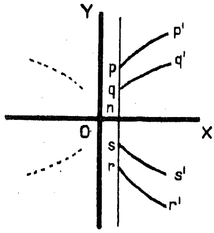


Fig. 2.

This is really the fundamental principle upon which the method is based. A photographic record is obtained on which we can identify at least one parabola as being associated with atoms or molecules of known mass; all the other parabolas can then be measured and compared with this one and their masses deduced. With electric and magnetic fields roughly known there is little difficulty in such an identification, and to make quite sure the absolute value of m/e for the hydrogen atom was determined and found to agree with the values obtained by other methods. In actual practice, since OX is an imaginary line and has no existence on the photograph, in order that the measurements may be made with greater convenience and accuracy the magnetic field is reversed during the second half of the exposure, when—in the case we are considering—two new parabolas will appear at rr', ss', due to m and m' respectively; the masses can now be compared by the equation $m'/m = pr^2/qs^2$: p, q, r, s being any straight line cutting the curves approximately parallel to the magnetic axis. The measurement of these lengths is independent of zero determination, and if the curves are sharp can be carried out with considerable accuracy.

It has been shown that the electrical displacement is in inverse proportion to the energy of the particle. Since this energy is simply dependent on and proportional to the electrical potential through which the charged particle fell before it reached the cathode and not upon its mass, the distribution of intensity along the parabolas will be somewhat similar. There will also be a definite maximum energy corresponding to the whole drop of potential across the discharge tube, with a corresponding minimum displacement on the plate; so that all normal parabolas will end fairly sharply at points p, q, etc., equidistant from the magnetic axis OY. The extension of the curves in the other direction indicates the formation of ions at points in the discharge nearer the cathode which will so have fallen through a smaller potential.

§ (4) Negatively Charged Rays.—As there is intense ionisation in the fine tube the charged particles may easily collide with and capture electrons in passing through it. A singly charged particle capturing a single electron will of course proceed as a neutral ray, and being unaffected by the fields will strike the screen at the central spot. If, however, it makes a second collision and capture it will become a negatively charged ray. Rays of this kind will suffer deflection in both fields in the opposite direction to the normal ones, and will therefore give rise to parabolas of a similar nature but situated in the opposite quadrants, as indicated by the dotted lines in the figure. Such negative parabolas are always less intense than the corresponding normal ones, and are usually associated with the atoms of electronegative elements such as carbon, oxygen, chlorine, etc.

§ (5) Rays with Multiple Charges.—If during ionisation more than one electron is split off, the resulting positive ray will have a double or multiple charge. Taking the case of a doubly charged particle it may give rise to two distinct effects. In the first place, if it retains its double charge while passing through the analysing fields its behaviour will be quite indistinguishable from that of a normal ray of half its mass. In the second place, the particle may retain its double charge through the whole potential fall of the discharge but capture an electron in the fine tube. It will then constitute a ray of normal ratio of mass to charge but with double the normal energy, so that the normal end of the parabolas will be extended towards the axis OY (*Fig.* 2) to a point half-way between that axis and the line pq. Such an extension will be seen on the bright parabola due to carbon in the photograph reproduced in Plate I.

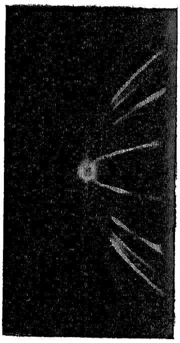


Plate I.—Typical Positive Ray Photograph taken with CO present in Discharge Tube.

Most elements are capable of losing two

electrons, some, such as krypton, three or more, while mercury can lose no less than eight at a time. The results of the multiple charge on atoms of mercury is beautifully illustrated in Plate II. The parabola α

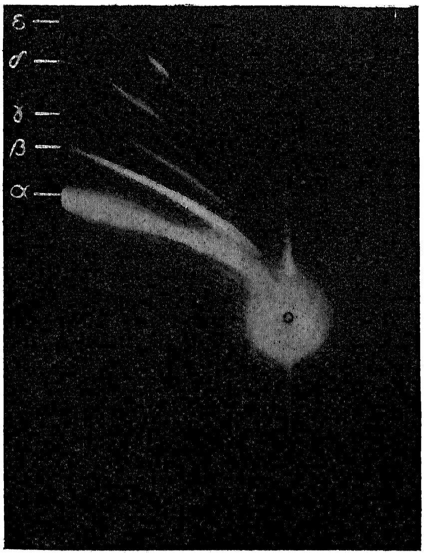


PLATE II.—The Multiple-charged Parabolas of Mercury.

corresponding to normal single charge will be seen extended almost to the origin itself, while above a series of parabolas of diminishing intensity β, γ, etc., indicate the atoms which have retained two, three, or more charges.

§ (6) DEMPSTER'S METHOD OF ANALYSIS.—In Dempster's method [1] positively charged ions are generated from salts fused on a metal strip either by heating or bombardment with electrons. These ions are then accelerated by a measured potential of 600 to 1700 volts in the vessel G (*Fig.* 3), and then strike a fine slit s_1. The beam so produced is bent into a half-circle by a strong magnetic field so that it falls on to a second slit s_2, and finally on to a metal plate connected to an electrometer. In making measurements the magnetic field is kept constant, and the potential plotted against the rate of charge of the plate. As soon as the potential P corresponding to maximum rate of charge has been obtained in this manner the absolute value of e/m can be obtained directly from the formula $e/m = 2P/H^2r^2$, r being the radius of curvature. It is therefore necessary to measure both the electric and magnetic fields. The resolving power claimed is about 1 in 100, and measurements are given making the masses of sodium and potassium ions 23 and 39 respectively. The number of elements to which it can be applied seems somewhat limited, but in respect

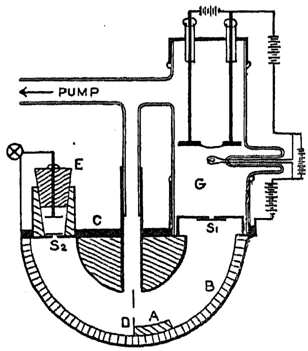


FIG. 3.

of its power of measuring the masses of the positive rays of metals it is of great importance, since, with the exception of mercury, it has not been found possible to obtain these satisfactorily by the ordinary discharge-tube method.

§ (7) ASTON'S METHOD OF ANALYSIS. THE MASS-SPECTROGRAPH. — This apparatus was primarily designed to determine the constitution of neon, for which purpose a greater accuracy and resolving power than that afforded by the previous methods was required. In it the rays are generated in a large discharge tube similar to that used by Thomson, but the electric and magnetic fields are applied so that the deflections are at 180° instead of at 90° to each other.[2] The principle of the method is indicated in *Fig.* 4. The rays are sorted into an extremely thin ribbon by

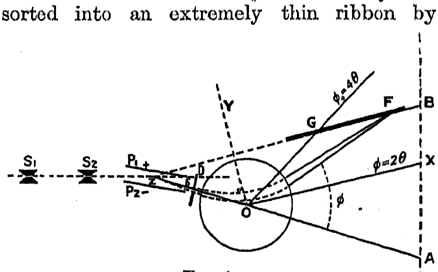


FIG. 4.

passing them through the two parallel narrow slits s_1, s_2. They are then deflected by the electric field between the plates P_1, P_2. This spreads them out into an " electric spectrum " in which the deflection of any particular particle is proportional to e/mv^2 as has already been shown above. After emerging from the electric field the rays may be taken, to a first degree of approximation, as radiating from a

[1] *Phys. Review*, 1918, xi. 316.

[2] *Phil. Mag.*, 1919, xxxviii. 707.

virtual source Z half-way through the field on the line s_1s_2. A group of these rays is now selected by means of the stop or diaphragm D, and allowed to pass between the poles of a magnet. For simplicity the poles are taken as circular, the field between them uniform and of such a sign as to bend the rays in the opposite direction to the foregoing electric field.

If θ and ϕ be the angles (taken algebraically) through which the selected beam of rays is bent by passing through fields of strength X and H, then

$$\theta v^2 = lX\frac{e}{m}, \quad (1) \qquad \text{and} \qquad \phi v = LH\frac{e}{m},$$

where l, L are the lengths of the paths of the rays in the fields. Equation (1) is only true for small angles, but exact enough for practice. It follows that over the small range of θ selected by the diaphragm θv^2 and ϕv are constant for all rays of given e/m, therefore

$$\frac{\delta\theta}{\theta}+\frac{2\delta v}{v}=0, \quad \text{and} \quad \frac{\delta\phi}{\phi}+\frac{\delta v}{v}=0,$$

so that

$$\frac{\delta\theta}{\theta}=\frac{2\delta\phi}{\phi},$$

when the velocity varies in a group of rays or given e/m.

From this it follows (*l.c.* p. 711) that all rays of constant mass, or more precisely of constant m/e, will come to a real focus F, and that the locus of the foci so generated will be along the line GF passing through Z and parallel to the line $\phi=2\theta$. If a photographic plate is placed at GF a spectrum depending on mass alone will be obtained. On account of its analogy to optical apparatus the instrument has been called a mass-spectrograph and the spectrum produced a mass-spectrum.

For the details of construction of the apparatus and its technique the reader is referred to the original papers (*Phil. Mag.*, April 1920, xxxix. 449, and May 1920, p. 611). The relation between the positions of the lines on the plate and the masses they indicate is fortunately nearly linear, but it is not necessary to know it accurately, as all lines are measured with respect to reference lines due to atomic and molecular rays of known mass. Thus carbon (12·00) and oxygen (16·00) give with their compounds a perfectly reliable scale up to CO_2(44). For lines well placed, an accuracy of about one part in a thousand is obtainable.

§ (8) Isotopes.—Owing to the "focus" effect and the use of slits instead of a circular tube, very much greater resolution is possible with this form of apparatus than with the original method, and by its use it was soon made evident that many elements hitherto supposed to be homogeneous were actually mixtures of so-called "isotopes." Of still greater theoretical importance was the result that every mass measured proved to be a whole number on the "oxygen scale," hydrogen being the only element of which the departure from this rule was measurable. The following plate (Plate III.) gives a reproduction of some of the mass spectra obtained.

Compounds of carbon are nearly always present in the discharge tube. These give two characteristic groups of lines, the C_1 group: C(12), CH(13), CH_2(14), CH_3(15), and CH_4 or O(16); and the C_2 group beginning with C_2(24) and containing the very strong line CO or C_2H_4(28). The latter group and part of the former are well shown in Spectrum I. Between these groups may be seen the two lines due to the isotopes of neon 20 and 22.

Spectra II., III., IV. taken with phosgene gas show the very important group of lines corresponding to masses 35, 36, 37, and 38. Lines 35 and 37 are undoubtedly due to the isotopes of the element chlorine. Lines 36 and 38 are almost certainly due to the two hydrochloric acids HCl^{35} and HCl^{37}. The remarks already made about parabolas due to multiply charged rays apply to the lines obtained by this form of analysis. Lines due to particles carrying one, two, three, or more charges are called lines of the first, second, third, or higher order, thus in Spectrum II. the faint lines at 17·5 and 18·5 are chlorine lines of the second order. In Spectrum V. taken with argon the third order line 13·33 is clearly shown, and from this the atomic weight of the principal constituent of this element was deduced to be 40 with great accuracy.

The remarkable property mentioned above of the atoms of the element mercury for carrying numerous charges is well exhibited in mass spectra, for it is difficult to eliminate it from the tube, and its presence is beneficial to the smoothness of the discharge. It is a "complex" element and the characteristic closely packed group of lines due to its isotopes can be seen, progressively weaker in intensity, as high as the sixth order; some of these may be recognised on the Plate. Spectra VIII. and IX. obtained with krypton and xenon show that the former consists of no less than six isotopes and the latter of five. The second-order lines of krypton are clearly shown to the left of Spectrum VIII., closely associated with the argon (40) line, so that the masses of the constituents of krypton can be measured with great accuracy.

The method of determining masses by the position of lines with regard to known reference lines cannot be conveniently applied to the elements hydrogen and helium as these are

too remote from the scale of reference. For these another method is available the procedure for which is as follows:

§ (9) Method of comparing Masses by "Bracketing."—It is not practicable to determine the values of the magnetic field, but it can be kept constant without much difficulty. On the other hand, it is easy to apply electric fields whose ratios are known with certainty. For a given fixed position on the spectrum $mv^2 \propto X$ and $mv \propto H$. Therefore if H is constant $m \propto X^{-1}$. If, therefore, after taking a spectrum we take another with the same magnetic field and, say, exactly double the electric field, the position due to a mass m on the first will be occupied by a line due to a mass $\frac{1}{2}m$ on the second. Hence if V is the original potential on the plates and v a suitable small voltage, and we take three spectra on the top of each other, one with a potential V, one with $2V+v$, and a third with $2V-v$, the magnetic field being the same for all, a line due to a mass m will appear bracketed on each side by lines due to $\frac{1}{2}m$. If the two to one relation is an exact one, the bracket will be symmetrical, as in the case of the hydrogen atom and molecule Spectrum VII. *c*. If the bracket is not symmetrical the ratio of the masses is not 2, as in the case of the hydrogen molecule and helium atom Spectrum VII. *b* and *d*. In this way it was found that the helium atom had a mass 4·00 on the oxygen scale, whereas the mass of a hydrogen atom was not unity but approximately 1·008, a value agreeing well with that obtained by chemical methods.

F. W. A.

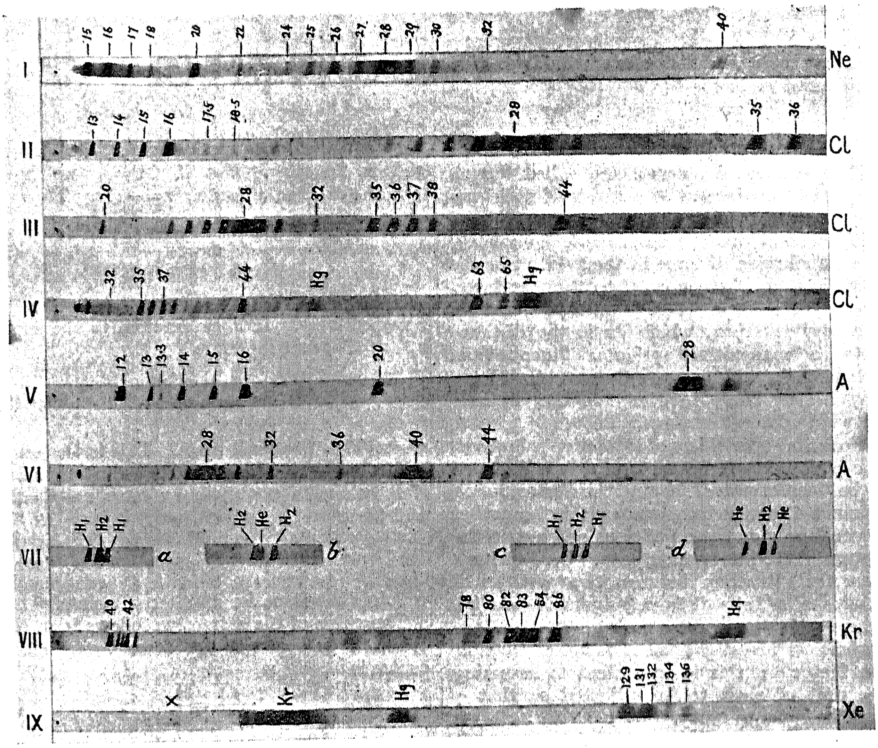


Plate III.—Typical Mass Spectra.

Positive Rays, production and influence in maintaining discharge through gases. See "Electrons and the Discharge Tube," § (7).

Potassium Permanganate, prepared by electrolysis. See "Electrolysis, Technical Applications of," § (30).

POTENTIAL

The *gravitational* potential at a point P in free space, due to a distribution of attracting matter, is the amount of work required to move unit mass from P to infinity, against the attraction of the field. It is therefore a function of the position of P; we denote it by V_P, or simply by V. If PP′ $(=\delta s)$ be a

line-element drawn from P in any direction, $V_{P'}$ will exceed V_P by the amount of work required to bring unit mass from P′ to P, *i.e.* by $F\delta s$, where F is the component attraction in the direction PP′. Hence $\delta V = F\delta s$, or

$$F = \frac{\partial V}{\partial s}. \quad . \quad . \quad . \quad . \quad (1)$$

Thus, if X, Y, Z be the component attractions, per unit mass, parallel to rectangular co-ordinate axes, we have

$$X = \frac{\partial V}{\partial x}, \quad Y = \frac{\partial V}{\partial y}, \quad Z = \frac{\partial V}{\partial z}. \quad . \quad (2)$$

The surfaces V = const. are called "equipotential" surfaces. If a series of such surfaces be drawn for equal small intervals of the constant, they will indicate completely the distribution of force in the field. For the force is everywhere normal to these surfaces, in the direction in which V increases, and its intensity is $\delta V/\delta n$, where δn is the distance between consecutive surfaces. Since δV is constant in this representation, the force varies everywhere inversely as the distance between adjacent surfaces.

So far, no special law of attraction is implied, except that the forces of the field are assumed to be "conservative," so that the work required to move a particle from one position to another depends only on these positions and not on the nature of the intervening path. On Newton's Law the attraction between two particles of masses m, m', at a distance r apart, is $\gamma mm'/r^2$, where γ is the constant of gravitation. In theoretical investigations it is usual to omit the factor γ; this is equivalent to assuming the unit of mass to be adjusted so that γ shall be equal to unity.

Writing then m/r^2 for the force on unit mass at a distance r from a particle m, the work required to produce a displacement δs in a direction making an angle ϕ with that of r is

$$\frac{m}{r^2}\delta s \cos\phi = \frac{m}{r^2}\delta r = -\delta\left(\frac{m}{r}\right).$$

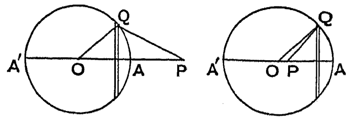
Integrating this from r to ∞, we find that the potential due to m is

$$V = \frac{m}{r}. \quad . \quad . \quad . \quad . \quad (3)$$

Hence the potential at a point P due to any system of particles $m_1, m_2, m_3, \ldots$, whose distances from P are $r_1, r_2, r_3, \ldots$, respectively, is

$$V = \frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3} + \ . \ . \ . = \Sigma\left(\frac{m}{r}\right). \quad (4)$$

The calculation of the potential due to *continuous* distributions is a matter of integration. Thus, to find the potential, at a point P, of a uniform thin spherical shell, we divide the surface into narrow zones by planes perpendicular to the line joining P to the centre O. If x be the distance of one of these planes from O, the mass of a zone will be $\sigma \times 2\pi a\delta x$, where a is the radius, and σ the surface-density (mass per unit area). Hence, if r be the distance (PQ in the figures) of



the edge of the zone from P, its potential is $2\pi\sigma a\delta x/r$. But if $OP = c$, we have $r^2 = a^2 + c^2 - 2cx$, whence $r\delta r = -c\delta x$. Thus

$$V = -\frac{2\pi\sigma a}{c}\int_{r_2}^{r_1} dr = \frac{2\pi\sigma a}{c}(r_2 - r_1), \quad (5)$$

where r_1, r_2 are the least and greatest values of r respectively. If P be external to the shell, $r_2 - r_1 = 2a$, and therefore

$$V = \frac{4\pi\sigma a^2}{c} = \frac{M}{c}, \quad . \quad . \quad . \quad (6)$$

where M is the total mass. This is the same as if the mass were concentrated at O. But if P be inside the shell, we have $r_2 - r_1 = 2c$, and therefore

$$V = \frac{M}{a}, \quad . \quad . \quad . \quad . \quad (7)$$

which is the same for all internal points.

The value of the attraction follows from (1). When P is external the force towards the centre is

$$-\frac{dV}{dc} = \frac{M}{c^2}, \quad . \quad . \quad . \quad (8)$$

the same as if the whole mass were concentrated at the centre. On the other hand, the potential is constant throughout the interior, by (7), and the attraction therefore *nil*. It may be shown that Newton's is the only law consistent with this result. Since a solid sphere or spherical shell may be regarded as made up of concentric shells of infinitesimal thickness, it is evident that, provided the density is a function only of distance from the centre, the preceding statements can be generalised.

If in (3) we denote the co-ordinates of m by a, b, c and those of P by (x, y, z), we have

$$r = \{(x-a)^2 + (y-b)^2 + (z-c)^2\}^{\frac{1}{2}}. \quad . \quad (9)$$

The components of the force due to m are therefore, by (2),

$$X = -\frac{m(x-a)}{r^3}, \quad Y = -\frac{m(y-b)}{r^3},$$

$$Z = -\frac{m(z-c)}{r^3}. \quad (10)$$

From these we deduce by differentiation

$$\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}=0, \quad . \quad . \quad . \quad (11)$$

so long as r is not zero. This relation can obviously be extended, by (4), to any system of particles; it holds throughout any region which is free from attracting matter. It follows from (2) that

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0 \quad . \quad . \quad . \quad (12)$$

in such a region. This is known as Laplace's equation. It may be interpreted as expressing that the "concentration" of the potential is zero at any point of free space. (See "Heat, Conduction of," Vol. I.)

A closely related theorem is that the mean value of the potential over a spherical surface not enclosing any of the attracting matter is equal to the potential at the centre. It is sufficient to prove this for the case of a single particle m situate at an external point P. The mean value in question is then

$$\frac{1}{4\pi a^2}\Sigma\frac{m}{r}\delta S,$$

where a is the radius and r denotes the distance of the surface-element δS of the sphere from P. Now $\Sigma(\delta S/r)$ is the potential of a spherical film of unit surface density at an external point, and is therefore equal to $4\pi a^2/c$, where c is the distance of P from the centre. The mean value is accordingly equal to m/c. From this theorem it follows at once that V cannot be a maximum or minimum at any point of free space. Hence, although there may be points in the field at which a particle would be in equilibrium, such equilibrium would be unstable.

A "line of force" is a line drawn from point to point always in the direction of the resultant force. It is obviously orthogonal to the equipotential surfaces which it meets. The equation (11) is identical in form with the "equation of continuity" of an incompressible fluid, the force (X, Y, Z) taking the place of the velocity, usually denoted by (u, v, w). Hence the lines of force have the same configuration as the lines of motion in a possible mode of motion of such a fluid. Many important theorems follow at once from this analogy. For instance, the lines of force which pass through any small area define a tube called a "tube of force." From the analogy with a tube of flow we infer that (in free space) the product the force into the cross-section is constant along such a tube. We may imagine the field external to the attracting masses to be filled with tubes so arranged that the above product is the same for each. On this understanding we may say that the number of tubes which enter any region is equal to the number which leave it, since in the hydrodynamical analogue the quantity of fluid contained in the region is constant.

The analogy may be extended. Any particle m of the attracting matter may be compared to a fictitious "negative source," or "sink," which absorbs fluid at a constant rate. The total flux inwards across a small spherical surface surrounding the point is $m/r^2 \times 4\pi r^2$ or $4\pi m$. Hence the total flux into any region is equal to 4π times the sum of the included masses.

When we proceed to consider the interior of an attracting body the notion of a finite mass concentrated at a point becomes inapplicable, since the potential and the force there would be infinite. The following conventions are then adopted. We describe about any point P a small closed surface S, and consider the values at P of the potential V, and the force (X, Y, Z) due to the matter *outside* S. It may be proved that, provided the volume density (ρ) at P is finite, these quantities tend to definite limits as the dimensions of S are contracted. These limits are adopted as the *definitions* of the respective functions at P; and it may be shown that the relations (2) still hold. The equation (11), however, requires modification. It may be shown that provided ρ is not merely finite, but is also a continuous function of the position of P, the required relation is

$$\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}=-4\pi\rho, \quad . \quad . \quad (13)$$

whence $\quad \nabla^2 V=-4\pi\rho\,; \quad . \quad . \quad . \quad (14)$

this is Poisson's extension of Laplace's equation, which was proved on the supposition that $\rho=0$. The equation (13) is equivalent to the statement that the flux of force into an elementary region $\delta x\delta y\delta z$ is equal to 4π times the included mass $\rho\delta x\delta y\delta z$. The formula (14) shows that the "concentration" of the potential about any point is proportional to the density there.

The *electrostatic* potential at any point P of an electric field is defined as the potential energy of a unit positive charge at P, *i.e.* it is the work required to bring a small body having this charge from an infinite distance to P, on the supposition that the field is unaltered during the process. There is therefore an opposition of sign as compared with the gravitational potential. The electric force, *i.e.* the force on unit charge, in the direction of a line-element δs is now $-\partial V/\partial s$, and the components parallel to co-ordinate axes are

$$X=-\frac{\partial V}{\partial x}, \quad Y=-\frac{\partial V}{\partial y}, \quad Z=-\frac{\partial V}{\partial z}, \quad (15)$$

in place of (2). On the other hand, since the force between charges of the same sign is repulsive the potential due to any distribution of charges e is given by the formula

$$V=\Sigma\left(\frac{e}{r}\right), \quad . \quad . \quad . \quad . \quad (16)$$

in strict analogy with (4). The formula (13) is replaced by

$$\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}=4\pi\rho, \quad . \quad . \quad (17)$$

whilst (14) is unaltered.

The conceptions and properties of equipotential surfaces, and lines and tubes of force, will hold as in the former subject, with obvious slight modifications of statement. Thus the number of tubes of force which issue from a closed surface will be proportional to the excess of positive over negative electricity in the included space. More precisely, the surface integral of the outward force normal to the surface will be 4π times the above excess.

In the interior of a conductor in electrical equilibrium the electric force is everywhere zero; this is implied by the absence of currents. The potential throughout, and over the surface, is therefore uniform. It follows that the electric force (R) just outside is in the direction of the normal. A tube of force which meets the surface is therefore orthogonal to it, and ends there. Let δS be an element of the surface, and imagine two parallel surfaces to be drawn on the two sides, at distances which are not only small, but small compared with the lateral dimensions of δS. Drawing the normals round the contour of δS we complete the boundary of a small disc-shaped region which encloses a mass $\sigma\delta S$, where σ denotes the surface density, or charge per unit area in that neighbourhood. Since the electric force inside the conductor vanishes, we have $R\delta S=4\pi\sigma\delta S$, by the theorem, or

$$R=4\pi\sigma. \quad . \quad . \quad . \quad (18)$$

If δn be an element of the normal drawn outwards from the conductor this is equivalent to

$$-\frac{\partial V}{\partial n}=4\pi\sigma. \quad . \quad . \quad . \quad (19)$$

This formula may be applied to calculate the "capacity" of some simple forms of condenser. Take first the case of two parallel metal plates whose distance d apart is small compared with the lateral dimensions of the area (S) of either. If the respective potentials are V and 0, the lines of force between will be approximately straight, and normal to them. The electric force is accordingly $R=V/d$, and the densities $\pm\sigma$ of the electrifications on the inner surfaces of the two plates are therefore given by

$$\sigma=\frac{V}{4\pi d}. \quad . \quad . \quad . \quad (20)$$

The total charge on the former plate is σS, and the capacity therefore $S/4\pi d$.

The case of a condenser whose opposed surfaces are spherical is specially simple and the result exact. Let a and b be the radii of the inner and outer surfaces respectively. If $\pm E$ be the total charges on these, the potential just outside the inner surface will be

$$V=\frac{E}{a}-\frac{E}{b}, \quad . \quad . \quad . \quad (21)$$

by (6) and (7). The capacity is therefore

$$\frac{E}{V}=\frac{ab}{b-a}. \quad . \quad . \quad . \quad (22)$$

Putting $b=\infty$ it appears that the capacity of an isolated spherical conductor is equal to the radius, as is otherwise evident from (7).

To find the capacity per unit length of a condenser consisting of two coaxial cylindrical surfaces, we note that the tubes of force in the intervening space are wedge-shaped. Expressing that the product of the force into the cross-section is constant we have

$$r\frac{dV}{dr}=A, \quad . \quad . \quad . \quad (23)$$

where r denotes distance from the axis. Hence

$$V=A\log r+B. \quad . \quad . \quad (24)$$

Determining the constants so that $V=V_1$ for $r=a$, and $=0$ for $r=b$, we find

$$V=V_1\frac{\log(b/r)}{\log(b/a)}. \quad . \quad . \quad (25)$$

The surface density on the inner face is

$$\sigma=-\frac{1}{4\pi}\frac{dV}{dr}=\frac{V_1}{4\pi a\log(b/a)}; \quad . \quad (26)$$

and the capacity per unit length therefore

$$\frac{2\pi a\sigma}{V_1}=\frac{1}{2\log(b/a)}. \quad . \quad . \quad (27)$$

H. L.

POTENTIAL:

To produce discharge through gases. See "Electrons and the Discharge Tube," § (2).

Due to a small magnet. See "Electromagnetic Theory," § (1).

Due to a magnetic shell. See *ibid.* § (2).

Electric: the work done in bringing unit charge of positive electricity from beyond the boundaries of the field to the point in question without disturbing the distribution of the field. See "Units of Electrical Measurement," § (11).

Magnetic: the work done in bringing a unit magnetic pole from an infinite distance to the point considered. See "Magnetic Measurements and Properties of Materials," § (1); "Units of Electrical Measurement," §§ (11), (16).

POTENTIAL ENERGY of a shell in a magnetic field. See "Electromagnetic Theory," § (3).

POTENTIAL GRADIENT, as determining the corrosion of iron in ferro-concrete. See "Stray Current Electrolysis," § (16).

POTENTIAL TRANSFORMERS: transformers employed to facilitate the measurement of

alternating voltages. See "Transformers, Instrument," § (8).

Characteristics of. See *ibid.* § (12).

POTENTIOMETER: use of, for the comparison of standards of electrical resistance. See "Electrical Resistance, Standards and Measurement of," § (13); "Potentiometer System of Electrical Measurement," § (7).

Alternating Current. See "Alternating Current Instruments," § (59).

POTENTIOMETER SYSTEM OF ELECTRICAL MEASUREMENT

§ (1) GENERAL PRINCIPLES.—When a steady current is passed through a wire or series of resistances, the potential difference between any two points is proportional to the resistance between them. If an external source of E.M.F. e is connected through a galvanometer, as shown in *Fig.* 1, to two points a, b,

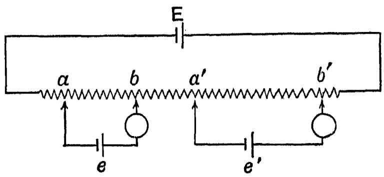


FIG. 1.

then, provided e is less than E, two points can be found at which there is no flow of current through the galvanometer. If another external source of E.M.F. e' be introduced, for which the balancing positions are a', b', the ratio of e to e' will be equal to the ratio of the resistance between a, b, to that between a', b'.

This affords a method of comparing different sources of E.M.F. and of potential drop produced by the passage of currents through standard resistances, which has many advantages.

In practice, the method generally resolves itself into comparing an E.M.F. to be measured with that of a standard cell. Being a null method, the galvanometer can be made as sensitive as is required, since the range is not restricted by the deflection. The method is not affected by external magnetic fields, and its accuracy depends entirely upon the relative values of the resistances and slide wire, which can be readily checked without reference to external standards. It thus constitutes a method which can be used for a large variety of measurements where a high degree of accuracy is required, combined with ease and simplicity of operation.

The earliest form of potentiometer consisted of a uniform wire stretched over a divided scale, but it was modified by R. E. Crompton to the form shown in *Fig.* 2, which is still in general use. A description of this instrument will illustrate the general principles underlying all types of potentiometer.

It consists essentially of fourteen equal coils AB, usually of 10 ohms each, connected in series with a slide wire BC, equal in resistance to one coil. This is connected to an accumulator E through two rheostats R_1 and R_2 for coarse and fine adjustment. The current through the instrument is adjusted by means of the rheostats until the standard cell is balanced across a resistance, the value of which corresponds to a multiple of its E.M.F.

Thus, a Weston cell having an E.M.F. of 1·0183 volts would be balanced across 10 coils and 0·183 of the total length of the slide wire, the pressure drop across each coil of the potentiometer being then 0·1 volt. The double pole switch K enables either the standard cell or the unknown E.M.F. to be connected through to the points P_1, P_2, as required. When the unknown E.M.F. is connected, balance is effected by means of adjustment of the position of the points P_1 and P_2. Under these conditions, the extreme range of the instrument would be 1·5 volts, which, however, as will

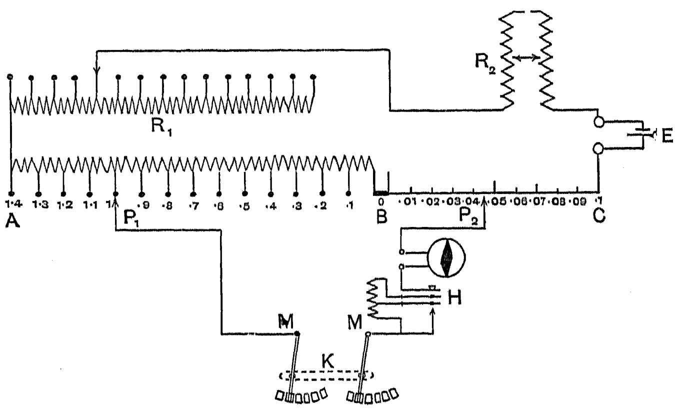


FIG. 2.

be shown later, can be extended in either direction.

Many modifications have been made to the original slide wire form of instrument, more particularly in the adoption of the principle

of the Varley slide and a substitution method by means of which the slide wire can be eliminated. These and other modifications are dealt with in a more detailed description of various instruments in ordinary use.

In the design and construction of all potentiometers, certain requirements have to be satisfied :

(i.) *Internal E.M.F.*—The internal thermo-E.M.F. must be kept so low that it does not appreciably affect the accuracy of the readings. This can be effected either by the selection of a material such as manganin, which has a very low thermo-E.M.F. against copper, or by placing all the joints and contacts inside the instrument where the temperature conditions are not affected by the hand of the operator. In general, where a manganin slide wire is used, since the material is easily worn, the contact should be made by means of a spring operated by a tapping key. When the contact is required to be continuous, a harder material, such as nickelin, is preferable. In this case, however, the high thermo-E.M.F. of the wire and the temperature coefficient requires that it shall be placed inside the instrument.

(ii.) *The Rheostat.*—The adjustable rheostats are a most important portion of the potentiometer, and in the earlier forms were a fruitful source of trouble, since the irregularity of the contact gave rise to fluctuations in the current. Typical rheostats in present use are illustrated in *Fig.* 3 (*a*) and (*b*).

In *Fig.* 3 (*a*), the Crompton type, the wire is wound in a double helix on a grooved ebonite cylinder. The cylinder moves up and down

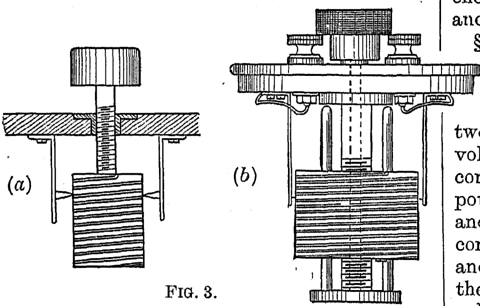


FIG. 3.

on the screw thread, so varying the length of wire included between the spring contacts. The advantage in this type is that the spring contacts are fixed. Some of the instruments made by the Cambridge & Paul Instrument Co. have a double tube; other types have either modifications of this pattern or a slider short-circuiting a portion of a pair of wires, either straight or circular, over which it moves. S. W. Melsom designed a form of cylindrical rheostat in which the cylinder moves up and down, but the spindle and the operating handle have no longitudinal motion (*Fig.* 3 (*b*)), and in which any horizontal motion of the handle is not transmitted to the cylinder, and, since the screw is supported at both ends, uniformity of contact is ensured.

(iii.) *Ease of Operation.* — The method depends for its accuracy on the steadiness of the current flowing through the potentiometer, and the current or pressure being measured; while with modern laboratory and test-room conditions it is comparatively easy to maintain the currents extremely steady.

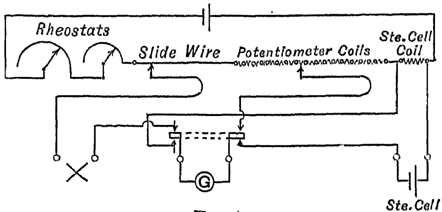


FIG. 4.

It is desirable that the general design and lay-out of the potentiometer should permit of rapid and easy operation. One device in particular which greatly facilitates rapid operation, and increases the accuracy of the readings, is the provision of a separate arrangement whereby the balancing of the standard cell is independent of the position of the ordinary measuring dials. This is generally effected by the provision of a separate key connected as in *Fig.* 4. It will be seen from the diagram that the standard cell can be checked at any time by depressing the key, and without changing the position of the dials.

§ (2) INSULATION.—Although, by suitable arrangement of external circuits, it is possible in general by the aid of a volt box to reduce the total E.M.F. between any two points to an amount not exceeding 1·5 volts, it is still necessary that all internal connections should be well insulated. In most potentiometers it is usual to mount all studs and switches on ebonite, and to run the internal connections in such a way that they are rigid and separate from each other. Sometimes the internal connections are covered with rubber tube or varnished silk, but insulation of this type frequently becomes unsatisfactory after a time, and the use of such materials is not to be recommended. In considering the precautions to be taken to ensure good insulation during the measurement of energy, particularly where ordinary supply pressures are being dealt with, a large measure of safety, as regards leakage, is to be obtained by connecting the instrument as in the diagram (*Fig.* 5). Here the connections from the standard resistance R and the volt dividing box are made in such a way that there is no

great difference of potential between the two pairs of potential wires, and the pressure between the contacts on the potentiometer switch and between any portion of the potentiometer and earth is small. It will be seen that the standard resistance is connected on the "earth" side of this typical circuit, and that the portion of the volt dividing box which is connected to the potentiometer is on the side nearest the standard resistance.

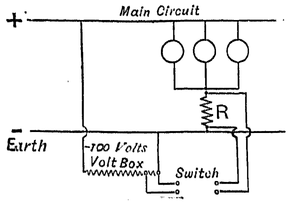


FIG. 5.

Clark Fisher[1] gives a somewhat similar diagram, but with the standard resistance in the earthed main. This is shown in *Fig.* 6. In view, however, of the complexity of a modern earthed system, in which the sheath of the cable may be, and frequently is, in parallel with the copper conductor, it is possible that the connection in the form shown in this diagram will give a value of the current

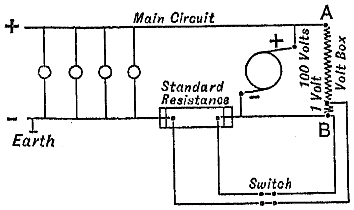


FIG. 6.

which, since it does not take account of any current that may be passing through the cable sheath or the earth, would be lower than the actual current passing through the lamps or other apparatus. Where the connections of a circuit of this kind cannot be arranged, as in *Fig.* 5, and where the earth currents may be, as they frequently are, an appreciable proportion of the total current, it is advisable to connect both the low-pressure coil of the volt dividing box and the standard resistance to the other pole of the supply. This is notably the case where the energy in a three-wire circuit is being measured. In the usual case of a three-wire circuit with the middle wire earthed, if the measuring resistances R and R_1 cannot be connected in the position shown in *Fig.* 7, they must be inserted as in *Fig.* 8, with the result that at the potentiometer there is a high pressure not only to earth, but between various sections of the potentiometer switch. Each of these methods requires a small correction; in the methods illustrated in *Figs.* 5, 6, and 7 for the pressure drop across the standard resistance, and in that of *Fig.* 8 for the small current taken by the volt box.

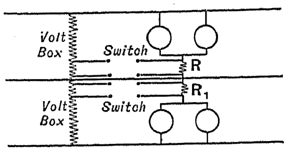


FIG. 7.

The potentiometer is perhaps not frequently used for the actual measurement of energy in a large power circuit, but the same problem presents itself in ordinary testing work, as, for instance, in the case of the calibration of a large watt-hour meter. Here, the pressure coils would be energised by one battery giving the maximum pressure of the watt-hour meter

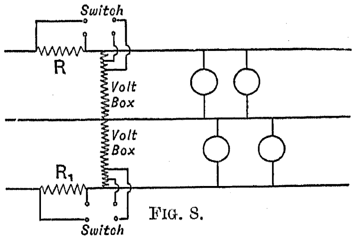


FIG. 8.

circuit, and the current coils from a separate heavy current battery. Either or both the batteries may, and normally will, go to earth at some point, and there is a possibility of having a large pressure on the potentiometer, whatever the method of connecting. E. H. Rayner[2] proposed a plan whereby in such a case both batteries might be earthed at a

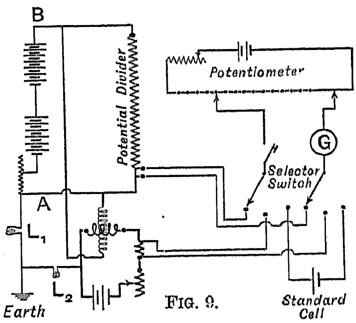


FIG. 9.

common point by means of lamps or some other convenient high resistance (*Fig.* 9), and thus the difference of potential at the potentiometer be restricted to a few volts. Even with this arrangement, however, it is still possible, in the event of the higher

[1] In his book *The Potentiometer and its Adjuncts.* Electrician Printing and Publishing Co.

[2] *Proceedings I.E.E.* xlvii. 7.

pressure going to earth at some other point, that there may be a considerable pressure on the potentiometer, and it is desirable that in every circuit of this kind the insulation of the instrument should be very high, and that the hand of the operator should be protected from the contacts and brushes. This can be effected by, among other methods, the form of construction shown in *Fig.* 10, which was evolved more particularly for use in a strong light where in normal circumstances the ebonite would quickly deteriorate. The ebonite top and the whole of the contacts are covered by means of an aluminium plate, the numerals being engraved on the aluminium, the only projecting pieces being the ebonite handles for turning the switches and the index pointers. Crompton & Co., and other makers, cover the contacts with a glass plate, which serves to protect the contacts from the hand, and also to exclude dust.

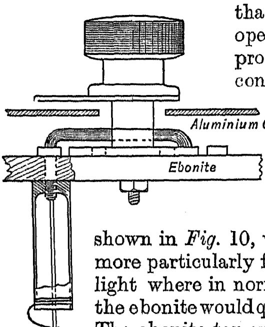


FIG. 10.

In addition, it is essential that the potentiometer itself, and the accessory apparatus, such as the galvanometer, the secondary battery, and all connecting leads, should be thoroughly well insulated from earth. Paraffin blocks may be used, but in view of their tendency to collect dust and to retain a thin film of moisture over the surface, these should be frequently scraped. A more satisfactory method for the galvanometer is to support it on small amberoid pillars, placed under the levelling screws. For the potentiometer, the insulation of the ebonite top, if properly made, should be amply sufficient, and for the secondary battery, paraffin blocks, which are frequently cleaned by scraping. The connecting leads are a source of danger, for it has been found that in the ordinary rubber-insulated wire the compounded covering may take up and retain sufficient moisture to allow of appreciable leakage. These, therefore, should be supported with some sort of insulator, such as silica, that will ensure a high insulation under all normal atmospheric conditions.

§ (3) VARIOUS TYPES OF POTENTIOMETERS. (i.) *Nalder.*—One of the earliest modifications of the ordinary slide wire type of potentiometer was that introduced by Messrs. Nalder & Co. Here the potentiometer sections were mounted on two dials, one having 150 steps, each equivalent to 0·01 volt, and the other 100 steps, each equivalent to 0·0001 volt, so that the whole of the second dial was equal to one section of the first. In this way, although the slide wire was eliminated, it was possible to read to 0·0001 volt on the lower dial.

(ii.) *Feussner.*—The essential feature of this potentiometer is the device which enables the number of direct-reading dials included between the potential points to be increased, and thereby allows the slide wire to be dispensed with. The main part of the E.M.F. is balanced against the two dials A and B (*Fig.* 11), and more exact adjustment is obtained by means of intermediate double dials, here three in number, C, D, and E. Since a modification of the resistance between A and B would alter the current through the potentiometer if it were not provided for, the current through the potentiometer is also carried through the lower series of the double dials in such a way that modification of the resistance in the upper series is compensated for by a modification in the opposite sense of the resist-

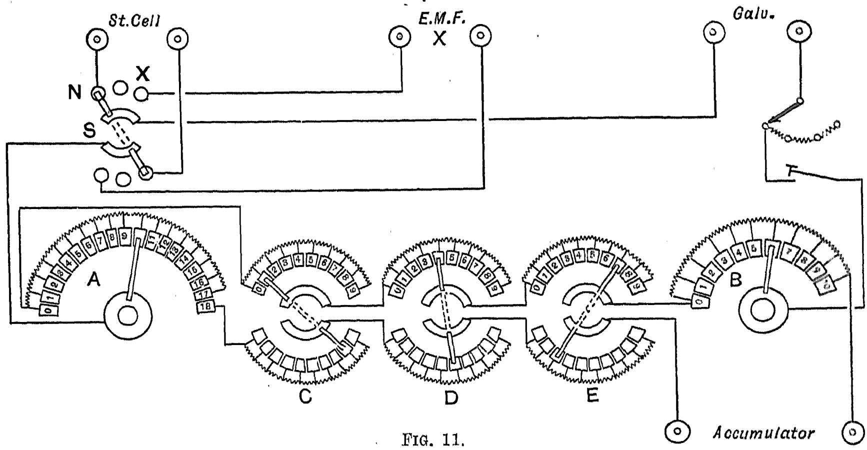


FIG. 11.

ance included in the lower. Thus the total resistance of the potentiometer circuit remains unchanged whatever the setting of the dials, and the current through the instrument is kept constant. Reference to *Fig.* 11, which shows diagrammatically the essential principle of the instrument, will make this clear.

The normal resistance of each coil on the dial A is 1000 ohms; B, 100; C, 10; D, 1; E, 0·1. Thus, for a pressure drop of 2 volts, the instrument has a total resistance of 20,000 ohms, the lowest reading of each section being equal to 0·00001 volt. For general measurements of current and pressure, the total resistance is probably too high, and it is usual to short-circuit seventeen of the coils in dial A, thus leaving the total resistance of the potentiometer equal to 2000 ohms with the lowest reading dial equalling 0·0001 volt per section.

In the more modern form of instrument an additional dial is provided whereby the balancing of the standard cell may be effected without reference to the main reading dials. In this case the additional dial, consisting of ten 1-ohm coils, for the standard cell is connected at the upper end of the dial A, one connection for the standard being taken off at the end of the eighth coil in dial A, an additional resistance of 180 ohms being provided so that the resistance between that point and the stud O on the additional dial is 10,180 ohms. Thus, the additional dial permits of the setting of any standard cell E.M.F. between 1·0180 to 1·0190, it being only necessary to put the selector switch S on to the contacts marked N.

(iii.) *Kelvin-Varley Slide.* — Instruments made by Messrs. H. Tinsley & Co. embody the use of a system known as the Kelvin-Varley slide method. As will be seen from the diagram (*Fig.* 12), the first dial consists of 18 coils, of which two consecutive coils are shunted by a second dial divided into ten sections, the total resistance of which is equal to that of the two coils shunted. Thus the resistance between the double brush is equivalent to one of the coils of the first dial and is divided equally into ten parts on the ten section dial. Each section of the first dial therefore equals 0·1 volt, the second dial 0·01 volt, the whole of the slide wire being equivalent to one coil on the second dial. The instrument is generally fitted with two ranges which are changed by means of a plug. Each of the coils in the first dial has a resistance of ten ohms.

This method was also used by Harker,[1] for a potentiometer designed for thermo-electric work.

[1] *Physical Society Proceedings*, xxviii. 473.

(iv.) *Tinsley.*—In a later form introduced by Mr. Tinsley, the Kelvin-Varley slide method is again used, but the slide wire is replaced

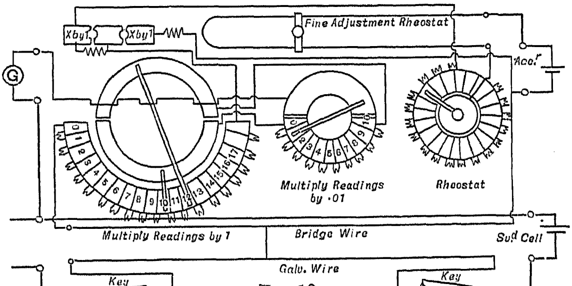


FIG. 12.

by an additional dial. Three dials are employed (see *Fig.* 13), the first ×0·1 having 18 steps, each of 0·1 volt, the second dial ×0·001, shunting two coils of the first, has 100 sections, and divides any one coil on the first dial into 100 parts. The third dial ×0·00001 is equal to one coil of the second dial, and thus each section of it is equal to 0·00001 volt. The total resistance of this instrument is approximately 200 ohms.

The Kelvin-Varley slide method necessitates the use of good contacts on the double brushes, but when this addition is satisfied, it is a very convenient method for extending the scale of

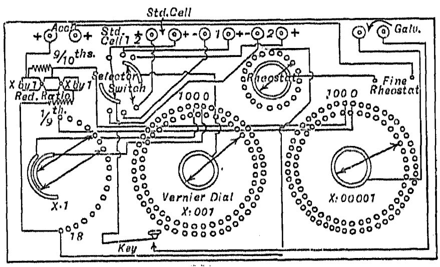


FIG. 13.

a potentiometer. Owing, however, to the fact that the resistance of the connections and contacts to the shunting dial cannot be entirely eliminated, there is a small zero error in instruments of this type.

(v.) *Vernier Instrument.* — R. W. Paul introduced a method of extending the Kelvin-Varley slide principle, which he designated a "Vernier" potentiometer. Six dials are so arranged that each of them, with the exception of the first, is in shunt with two coils of the preceding dial. This system involves the use of somewhat high resistances, the total of the potentiometer being 2000 ohms. It is shown

in detail in *Fig.* 14. Mr. Paul's potentiometers, made by the Cambridge & Paul Instrument Co., Ltd., are generally provided with the device

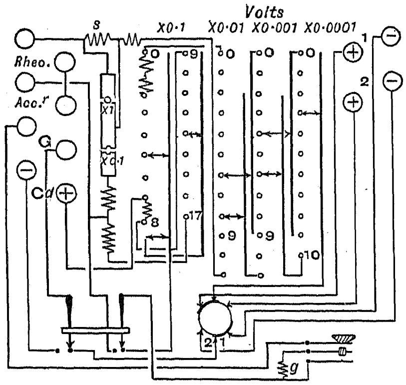


FIG. 14.

for the independent checking of the standard cell, shown in *Fig.* 4.

(vi.) *National Physical Laboratory Instruments.* — In the instrument used for general power measurements at the National Physical Laboratory, the method of subdividing the coils into separate dials has been adopted. This enables the resistance of the instrument to be kept low and eliminates any effects due to contact resistance. Details of a typical instrument of this construction are shown in *Fig.* 15. It will be seen that the potential contact is carried through from one dial to the next and the reading of the instrument is that of the first dial on which the pointer stands against a significant figure. The advantage claimed for this instrument is that the length of scale is equal to that obtained with the Kelvin-Varley slide method without the necessity of using comparatively high resistances, and that it is quick in operation and easy to read. Each of the 100 coils has a resistance 0·1 ohm, the total drop across the whole being 0·1 volt, or when the instrument is used for the measurement of thermo E.M.F., 0·01 volt.

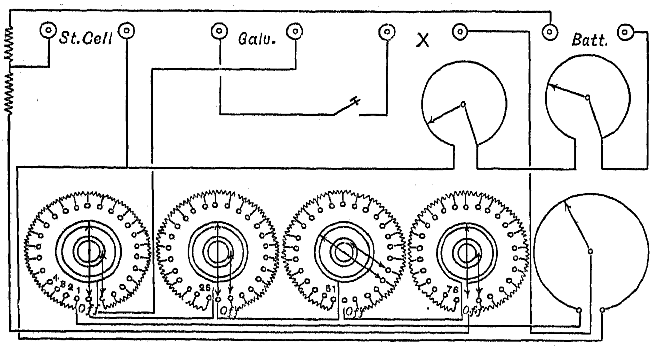


FIG. 15.

§ (4) DEFLECTION TYPE.—A form of potentiometer in which the resistance of the galvanometer circuit is kept constant whatever the position of the dials, was first introduced by Stansfield [1] (see *Fig.* 16). The deflection of the galvanometer will therefore always be proportional to the unbalanced part of the potential difference. This type of instrument was designed for use in the thermo-electric measurement of high and rapidly changing temperatures, where there was not sufficient time to obtain an exact balance for each reading. As used by Stansfield, the plugs were set to the nearest 2000 and 200 microvolts respectively, the further subdivisions being given by means of the deflection of the galvanometer.

(i.) *Brooks's Instrument.*—This instrument has been modified by substituting dials for the plugs used in the earlier form, and is largely used in metallurgical work. Similar types of potentiometer were developed independently by H. B. Brooks of the Bureau of Standards at Washington, for the testing of voltmeters and ammeters. Diagrams of this instrument are shown in *Figs.* 17 and 18, the galvanometer in this case being a sensitive pivoted moving-coil instrument. Brooks investigated the theory of an instrument of this type and finally evolved a type suitable for the testing of ammeters and voltmeters. The theory of the instrument is given in the following paragraphs: [2]

Assuming that the instrument is to be used for measuring electromotive forces higher than those of a single cell, a volt-box will be required, as shown in *Fig.* 17, in which E is the unknown E.M.F. to be measured, e_1 that of the auxiliary storage cell, R the total re-

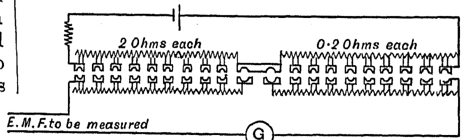


FIG. 16.

sistance of the volt-box, the fall of potential around a fraction of this, R/p, being opposed to

[1] *Phil. Mag.* xlvi. 59.
[2] *Bureau of Standards*, ii. 230 *et seq.*

the fall of potential around a portion, r_1, of the potentiometer wire AB. It was seen that to obtain the condition of constant sensibility a rheostat would be required in the galvanometer circuit, this rheostat being controlled by the motion of the main dial, represented by the

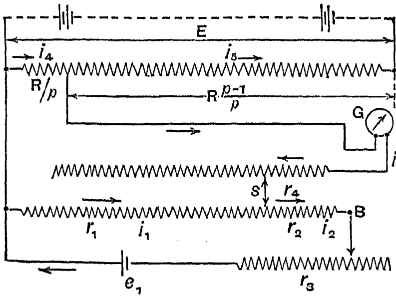


FIG. 17.

slider S. For simplicity the standard cell and connections for using it to check the working current in AB are not shown, as they do not affect the problem. Assuming that the condition of balance does not exist, and denoting

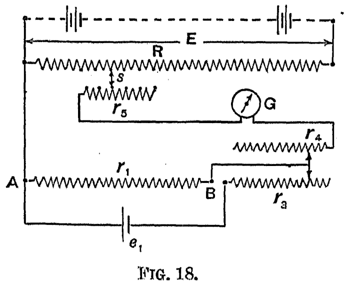


FIG. 18.

the currents and resistances in the various branches by $i_1, i_2, \ldots . . r_1, r_2, \ldots . .$ and applying Kirchhoff's laws, we have the following equations:

$$i_1 - i_2 + i_g = 0, \quad . \quad . \quad . \quad . \quad (1)$$

$$i_4 - i_5 - i_g = 0, \quad . \quad . \quad . \quad . \quad (2)$$

$$i_1 r_1 + i_2(r_2 + r_3) = e_1, \quad . \quad . \quad . \quad (3)$$

$$i_1 r_1 - i_g(r_4 + r_g) - i_4\frac{R}{p} = 0, \quad . \quad . \quad (4)$$

$$i_5 R\frac{p-1}{p} + i_4\frac{R}{p} = E. \quad . \quad . \quad . \quad (5)$$

Solving these equations, we get

$$i_g = \frac{e_1(r_1/r_1 + r_2 + r_3) - E/p}{r_4 + r_g + r_1(r_2 + r_3)/\{r_1 + (r_2 + r_3)\} + R(p - 1/p^2)}. \quad (6)$$

The first term in the numerator of this expression is the fall of potential in the portion r_1 of the potentiometer wire, when the galvanometer circuit is open; it is therefore numerically equal to the setting of the potentiometer. The second term in the numerator is the fall of potential which would exist around the portion R/p of the volt-box if the galvanometer circuit were open. The denominator is the total resistance in the galvanometer circuit, the third term being the resultant resistance of the portion r_1 of the potentiometer wire shunted by the remainder of the battery circuit, and the fourth term is the resistance of the portion R/p in parallel with the remainder of the volt-box, $R(p - 1/p)$. Equation (6), therefore, shows that the current through the galvanometer is equal to the unbalanced portion of the electromotive force divided by the total resistance of the galvanometer circuit; or

$$i_g = \frac{\Delta e}{\Sigma(r)}. \quad . \quad . \quad . \quad . \quad (7)$$

Referring to equation (6), we may denote the first term in the numerator, which may be called the setting, by s. Since the volt-box has a multiplying power of p, the equation may be written in the form

$$i_g = \frac{ps - E}{p\Sigma(r)}, \quad . \quad . \quad . \quad (8)$$

which shows that if $\Sigma(r)$ can be kept constant the galvanometer current will be directly proportional to $(ps - E)$, the difference between the E.M.F. corresponding to the setting and the E.M.F. to be measured. If $\Sigma(r)$ can be kept constant for all settings, it is only necessary to calibrate the scale of the galvanometer properly to make it read directly the unbalanced part of the E.M.F. under measurement. Referring to *Fig.* 17, and equation (6), it will be seen that the resistance r_4 must have such values for different positions of the slider S that the sum of $r_1(r_2 + r_3)/\{r_1 + (r_2 + r_3)\}$ and r_1 will be a constant. The latter has a maximum value at some point between A and B, and minimum values at A and B; so that r_4 must vary accordingly.

A difficulty in the way of using this particular arrangement of circuits lies in the fact that the value of e_1 depends on the condition of the storage cell, and as e_1 varies r_3 must be varied to keep the proper current through AB. This double variation of the setting and of r_3 is difficult to compensate for accurately, since for settings near A, changes of r_3 make very little change in the resultant resistance, while for settings near B the changes of r_3 enter almost undiminished into the resistance of the galvanometer circuit. We may use a number of cells in place of one, giving a larger current through AB, and thus limiting the part of AB used to a relatively small portion near A. This would give a sufficiently accurate compensation for most purposes; but another arrangement of circuits may be used which does not have the objection of requiring a number of cells, one being sufficient, while the compensation for changes in the setting and in the battery rheostat may be made as perfect as desired.

This plan of circuits is shown in *Fig.* 18.

In this arrangement the setting is made on the volt-box instead of on the potentiometer wire. The rheostat r_3 is set so as to give, by reference to a standard cell (not shown), a certain standard current through AB, and therefore a constant fall of potential around it. This fall of potential is balanced as nearly as possible by setting the slider S. The variable resistance r_4 is arranged so that r_4 plus the resultant resistance of the potentiometer circuit is a constant: r_4 thus takes care of changes in resistance required by variations in the E.M.F. of the auxiliary cell, but is independent of the setting. The fraction I/p being in this case a variable, the resistance in the galvanometer circuit must vary with it, as may be seen from equation (8). Since in this second arrangement the E.M.F. term $e_1(r_1/r_1+r_3)$ is a constant, we may denote it by e. We have then

$$i_g=\frac{1}{p}\cdot\frac{pe-\mathrm{E}}{\Sigma(r)}. \quad . \quad . \quad . \quad . \quad (9)$$

For a difference of one volt between the setting pe and the E.M.F. E under measurement, the galvanometer is to give a deflection of m scale divisions.

If I denotes the current required to give a deflection of one scale division, the current $m\mathrm{I}$ must always flow when $pe-\mathrm{E}=1$. Substituting these values, we have

$$m\mathrm{I}=\frac{1}{p}\cdot\frac{1}{\Sigma r}, \quad . \quad . \quad . \quad . \quad (10)$$

$$\Sigma(r)=\frac{1}{pm\mathrm{I}}. \quad . \quad . \quad . \quad . \quad (11)$$

Equation (10) shows that the compensating resistance r_5 in the galvanometer circuit, which is controlled by the movement of the slider S, must vary so as to keep the product $p\Sigma(r)$ a constant. Equation (11) holds for the first arrangement also, shown in *Fig.* 17; in this case, p being a constant, $\Sigma(r)$ must be constant and equal to $1/pm\mathrm{I}$.

The galvanometer used may be either pivoted or, for laboratory use, a reflecting instrument of short period, the first being used where the apparatus is required to be portable and always ready for use. The sensitiveness required for voltage measurement is a deflection of one millimetre for from two to four microamperes. The instrument in any case must have a unit scale.

A point to be considered is the value of the total resistance at which the galvanometer is a periodic. For good results the resistance in the galvanometer circuit should be equal to or slightly greater than this resistance. In the first arrangement of circuits (*Fig.* 17) in which the total resistance of the galvanometer circuit is a constant, the damping may be external to any desired extent; but for the second case, with a large variation in the resistance of the galvanometer circuit, this resistance should be relatively high and the damping largely internal.

In a later paper [1] Brooks describes further modifications of the circuits, and the type of instrument recently constructed for voltmeter testing, and other precision measurements of pressure in the laboratory and test-room.

The modifications made were for the purpose of extending the range and dealing with the difficulties of introducing the proper compensation for the whole range, and also for keeping the damping of the galvanometer constant. The instrument (*Fig.* 19) is described by Brooks [2] as follows:

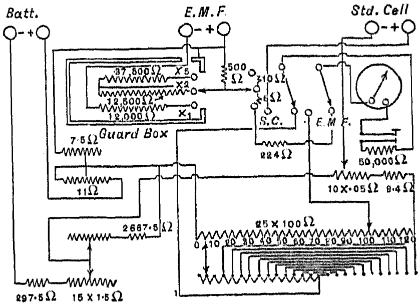


FIG. 19.

The main dial has 25 steps of 100 ohms each. This is in series with a coil of 9·4 ohms and a dial of 10 steps of 0·05 ohm each. The Weston standard cell is balanced around 509·4 ohms plus the amount on the dial, and as the standard current is 0·002 ampere, cells of 1·0188 to 1·0198 volts may be used, and if need be this range may be varied by changing the 9·4 ohm coil. In the storage cell circuit is a series rheostat whose minimum resistance is 297·5 ohms, increasing from this value in 15 steps of 1·5 ohms each. This is r_3 of the preceding discussion. At the same time that r_3 is increased, the resistance in shunt to the potentiometer wire r_6 decreases from a maximum of 6814 ohms to a minimum of 2667·5 ohms. A fine rheostat of 11 ohms in the battery circuit covers any step of the coarse rheostat, and has a compensating resistance of 7·5 ohms maximum in the galvanometer circuit.

The drop is taken from the ends of a 500 ohm coil, which is in series with 12,000 ohms when the range switch is set on ×1. When this switch is at ×2 and ×5 the total resistance between the E.M.F. posts is 25,000 and 62,500 ohms respectively. All of these coils, except the first 500 ohms, are mounted within and well insulated from a brass box which entirely encloses them. The negative E.M.F. terminal post is inside of, and well insulated from, a brass sleeve which projects into the box and is soldered to it. This box is connected by a wire to the positive E.M.F. post, and acts as a "guard wire" to prevent leakage currents from flowing through the circuits of the potentiometer proper. This is a very important precaution, which will be appreciated when we consider the high pressure (625 volts) available for producing leakage, and the fact that the full deflection of the galvanometer is produced by 0·00006 ampere, a current which 625 volts would send through a resistance of over ten megohms; while a current sufficient to give a readable deflection would flow, under this pressure, through 3000 megohms. With the arrangement shown, the maximum pressure which may produce leakage through the galvanometer is 6 volts.

The main dial has a set of compensating coils r_4 for keeping constant the resistance between the sliding contact and the 0 end of this dial. By arranging the values of r_3, r_6, and the standard cell

[1] *Bureau of Standards*, iv. 275.

[2] *Loc. cit.* 287.

coils so that the resultant resistance beyond the 125 end of the dial is 300 ohms, the total resistance in the storage battery circuit being 2800 ohms, the point of maximum resultant resistance to the galvanometer current will come at the setting 70, when r_1 is 1400 ohms; the resistance at 75 will be the same as at 65, and so on. Hence compensating coils are used up to the point 70, and by the use of cross connections no additional coils are needed beyond this point.

While the lowest range provided is nominally 0 to 125 volts, the range 0 to 5 volts may be had by using as the E.M.F. terminal the lever of the range switch, and adding sufficient resistance to the galvanometer circuit to make the proper total. In other words, the normal range of the potentiometer is from 0 to 5 volts, readable to 0·0004 volt (one-tenth of a scale division of the galvanometer). This range may be increased as desired by adding resistance in the E.M.F. circuit at the rate of 100 ohms per volt.

An instrument on a similar principle has been designed for ammeter testing.[1]

It is interesting to note that while the plan of circuits shown in *Fig.* 17 is not convenient for use in measuring voltages higher than those of a single cell, it is the most suitable one to use in a deflection potentiometer for measuring currents. Here the fall of potential at the terminals of a current shunt is to be measured, and this fall of potential being small, it is desirable to use all of it.

The circuits for an instrument of this kind are shown in *Fig.* 20. A current shunt W replaces the

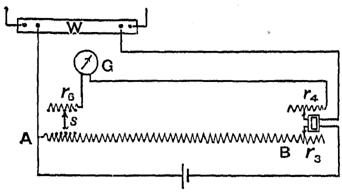


FIG. 20.

portion Rp of the volt-box in *Fig.* 17, and only a limited portion of the potentiometer wire AB is used, corresponding to the drop in the shunt at full load, say 150 millivolts. Since only a small portion of the wire AB is used, the compensating resistance r_4 may correct for the small changes in the resultant resistance as the battery rheostat r_3 is altered, for the position of S corresponding to full load through the shunt. The compensation for smaller loads will not be theoretically exact, but the error may be made negligibly small. In an instrument for this purpose it is an advantage to use a smaller resistance in AB, in order to reduce the necessary sensibility of the galvanometer and to keep within limits the source of error just referred to. By proper design a high grade of portable galvanometer like the one before mentioned, if suitably wound, will be sensitive enough.

§ (5) RANGE OF POTENTIOMETERS.—The range of the early instruments was of the order of 1·5 volts, but by comparatively simple devices it was soon extended, more particularly in regard to the accurate measurement of much lower pressures. This is especially necessary in the measurement of the large currents used in modern practice, for which the construction of standard resistances to give a pressure drop of anything like one volt is impracticable. In the case of a single-range low-reading instrument this is secured by the addition of an invariable resistance, but usually potentiometers are now constructed to have two or more ranges which are available as required.

A method in general use, where the standard cell is always balanced on the higher range of the instrument and a plug or a switch afterwards moved over when it is required to use the instrument on a lower range, is shown in *Fig.* 21. It has a series shunting device whereby the working portion of the potentiometer is shunted by a known resistance, so that the value of each of the sections is reduced in a given ratio, and at the same time a series resistance is put into circuit which keeps the total resistance of the instrument unaltered. As will be seen from *Fig.* 21, with the plug

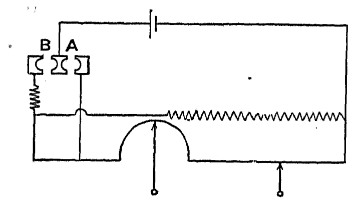


FIG. 21.

in position A, the current flows through the potentiometer coils and rheostats only and the standard cell would be balanced against the appropriate value; when the plug is changed to B the series-paralleling device is in operation and the pressure on the coils is 1/10th or 1/100th according to the range of the normal pressure.

A modification of this method, suggested by F. H. Schofield, allows the balancing of the standard cell to be carried out independently of the position of the range switch, as in *Fig.* 22, and gives greater certainty of measurement when it is required to pass rapidly from one range to another.

§ (6) VOLT DIVIDING BOXES.—For the measurement of pressures higher than one volt, a resistance box is used, by means of which a given proportion of the total pressure can be taken to the potentiometer. The usual allowance is 100 ohms to the volt, but some makers prefer a higher and some a lower value of resistance. The coils are generally wound on grids or flat plates in order thereby to facilitate the dissipation of the heat generated. A frequent source of error arises from the unequal heating of the various sections of the resistance, in consequence of

[1] *Bureau of Standards*, ii. 237.

which their relative value changes. The temperature distribution in a box made up of a large number of sections is usually not quite uniform, and where the sections con-

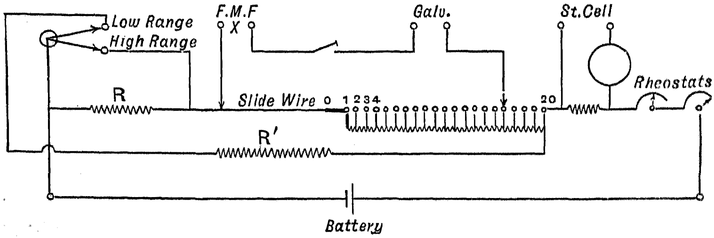


FIG. 22.

nected to the potentiometer are placed on the outside grid, there is often an appreciable change in the volt ratio, due to unequal heating of the coils. This tendency can be greatly reduced, if not entirely eliminated, by so arranging the grids that the coils for connection to the potentiometer come somewhere near the centre of the box, so that their temperature is equal to that of the average of the coils. The difficulty can also be met by taking care that the dimensions of the box and the ventilation of the coils is sufficient to ensure that the rise of temperature should be small, but in practice a box of this kind is somewhat too cumbersome, and a reasonable rise of temperature (20° C.) should be permitted and provided for.

Taking as a typical case a volt-box made by Messrs. Crompton & Co., the coils are wound on grids, each having a surface area of approximately 160 cm.^{2}. The resistance of each is 5000 ohms, designed for a maximum pressure of 75 volts. Thus, for a volt-box for 600 volts, eight grids would be required, having a total area of 1280 cm.^{2}, the surface area being approximately 140 cm.^{2} per watt dissipated. Practice in this direction varies very considerably, especially where the boxes are fitted with ventilating holes. For the most accurate work, the volt-box can be immersed in oil, usually moisture-free paraffin, a method which ensures uniformity of temperature and greatly improves the dissipation of the heat generated.

§ (7) STANDARD RESISTANCES. (i.) *Air Cooled.* (*a*) *Strip Form.*—The resistances ordinarily used for the measurement of current by potentiometer method vary in design with individual makers, but in most cases take the form of a strip or strips of resistance material, usually manganin, supported vertically, except in the smaller sizes, where wire spirals are used. A typical resistance of this type is shown in *Fig.* 23, a resistance made by Messrs. Crompton & Co. to carry a current of 3000 amperes. Here the manganin strips are silver-soldered into copper strips, which are soft-soldered into the end blocks.

(*b*) *Wire Form.*—Another method[1] of constructing an air-cooled resistance is to have a number of manganin wires in parallel, each of them hard-soldered into a copper strip which is then soft-soldered into the cast end block (see *Fig.* 24). The advantage of this form is that if the spacing of the wires is sufficient (about four times the diameter of the wire), a larger

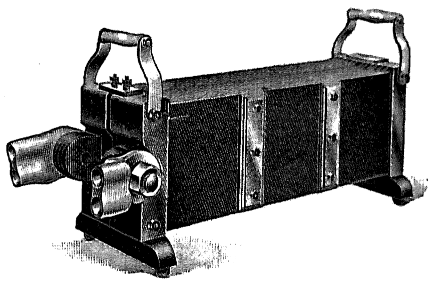


FIG. 23.

heat-radiating surface is secured than with a strip of metal of equal cross-section, and the convection currents set up are usually much more efficient in dissipating the heat.

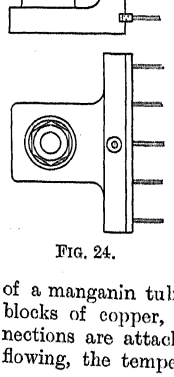


FIG. 24.

A standard resistance of this type, built for a current of 3000 amperes and a pressure drop of 0·15 volt, had 190 wires, each 4 millimetres diameter and 27 centimetres long, the spacing between the centres of the wires being 15 millimetres.

(ii.) *Water Cooled.*—In another form of resistance introduced by Messrs. Crompton & Co., the resistance element consists of a manganin tube hard-soldered into heavy blocks of copper, to which the current connections are attached. When the current is flowing, the temperature of the resistance is

[1] *Electrician*, lxx. 963.

reduced by passing a stream of water through the tube. In this way, a very high current density—in the case of manganin, up to 20,000 amperes per sq. in.—becomes possible and the size of the resistance element, having regard to the amount of heat dissipated, can be very considerably reduced. Tubes, 18 in. in length, are generally used for resistances to carry 1500 amperes with a pressure drop of 1·5 volts. Details of this form of construction are given by Clark Fisher,[1] and with the tubes so designed as to reduce the self-induction to a minimum for use in alternating-current measurements, in a paper by Paterson, Rayner, and Kinnes.[2]

§ (8) Rise of Temperature and Accuracy Characteristics.—The rise of temperature, and consequently the size of a resistance, is based on the temperature curve of the resistance alloy used. With manganin of good quality, the resistance will probably increase up to a temperature of 25° C., and usually after 30° C. to 40° C. will fall. In general, a resistance is designed so that the maximum change in resistance, due to heating by the current, will not exceed ±0·02 per cent, a value which with manganin permits of a temperature rise of about 50° C.

§ (9) Cooling Surface.—The area of a strip resistance is usually taken as being that of both sides of the strip or strips used, but the temperature rise will obviously be affected by the number of strips and the distance between them. In the following table, therefore, a comparison of the cooling surface is given: Column (1), for the total area of resistance metal used, and column (2), in the strip and wire forms, a value based on the overall size of the resistance.

Table

Cooling Surface for Various Types of Resistances

	Column (1).	Column (2).
	sq. cm. per watt.	sq. cm. per watt.
Strip	50	8
Wire	15-20	5
Water-cooled tube	0·2	. .

All of these values are based on similar requirements as regards accuracy.

§ (10) Pressure Drop.—With the older type of potentiometer, which was constructed to measure a pressure drop up to 1·5 volts, it was usual to construct these standard resistances to give a pressure drop of 1·5 volts at full-load current. With the advent of much larger currents, however, this pressure represents an excessive expenditure of energy and material, and is, moreover, unnecessary, since a pressure drop of 0·15 volt is amply sufficient to give the requisite degree of accuracy, assuming that the potentiometers, etc., are selected for the purpose. Therefore, for the measurement of large currents, resistances are usually designed to have a pressure drop of 0·15 volt, and it is generally convenient to have a series of resistance units for various currents, each arranged to have this pressure drop at its maximum current.

§ (11) Design of Main Current Ends.—Special attention has to be paid to the design of the lugs in the case of resistances for large currents, in order that the proper distribution of current through the resistance may not be affected by slight changes in the exact position of the current connections. This source of error is particularly apt to arise where two or more bolts have to be used to connect the main current leads to the resistance, and where they may be placed on one or both of the faces of the lug.

Searle[3] has shown that however a current may be led into a rod, the distribution at a point whose distance from the end is greater than three or four times the greatest diameter of the section is practically independent of the manner of distribution at the end where the current enters. In the case of strips, he also shows that any want of uniformity of current distribution over a section is diminished to 1/20th of its amount if we advance along the strip by a distance equal to the width of the strip.

If, as is more usual in practice, the connecting ends are in the form of sheets to which one or more bolts may be attached, and supposing as an extreme case that the current is led in at one corner, from this it follows that at a section whose distance from the end is three times the width of the strip, the current distribution is sufficiently uniform to ensure that there will be no appreciable inaccuracy due to distortion of the stream lines.

In practice, however, it is not desirable to have the end lug as long as this. Various methods are employed in order to avoid the use of an excessive length of metal between the part to which the current connections are attached and the actual resistance elements, the most common being the interposition of a constriction between the part to which the main current connections are attached and the resistance elements. The conditions, as regards size and method of connection, vary so much in practice that it is scarcely practicable to attempt to state any rule for the dimensions to be employed which will suit all cases. As an example of a resistance standard having

[1] *The Potentiometer and its Adjuncts*, p. 75.

[2] *Journal I.E.E.* xlii. 455. See also "Inductance, The Measurement of," § (47).

[3] *Electrician*, lxvi. 999, 1029; lxvii. 12, 54.

four bolt-holes with end-pieces of the dimensions shown in *Fig.* 25, it was found necessary to make the constriction one inch deep in order to ensure that there should be no change in the value of the resistance, according as one or more bolts were used, whether they were

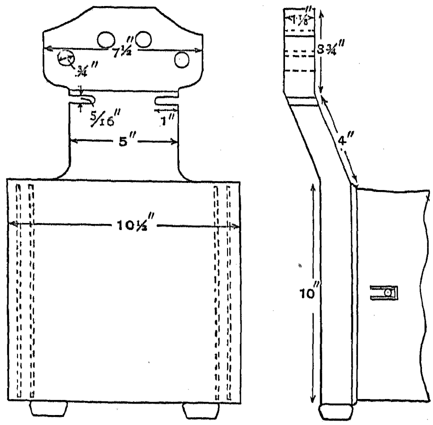


FIG. 25.

connected to one or both faces, and whatever the position of them.

§ (12) POTENTIAL TERMINALS.—A common and very convenient method of taking off the potential terminals, which at the same time admits of easy adjustment of the resistance in either direction, is shown in *Fig.* 26.

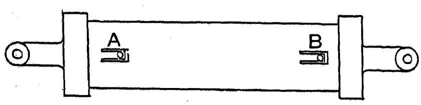


FIG. 26.

The tongues of metal A or B, to which the potential terminals are attached, can be lengthened by cutting the metal with a fret-saw. Lengthening of the tongue A increases, and lengthening of the tongue B diminishes the value of the resistance included between the points. The length of the tongue should be not less than three times its width in order to obviate any difference of potential arising from connections being made at different sides of the terminals.

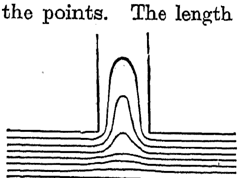


FIG. 27.

Where the potential points of the resistance are taken off at the end blocks, it is necessary to arrange the shape and size of the terminals so that the flow of current through the terminals will not affect the value at the point of connection. *Fig.* 27 shows the way in which the stream lines pass along the base of the terminal, and Searle (*loc. cit.*) investigated the question, and has shown that if the length of the pillar is four times its diameter, the difference of potential at the upper part is less than 1/100,000th of its difference at the base. The usual practice is to make the terminal three times its diameter.

§ (13) PRECISION TYPE LOW RESISTANCES. REICHSANSTALT PATTERN.—For more accurate measurement of current and for use as sub-standards, current-measuring resistances are usually constructed so that they can be immersed in oil. The most common form of resistance of this kind is the Reichsanstalt pattern (see *Fig.* 28), in which the resistance elements are thin strips of manganin mounted in a metal box immersed in oil and provided with a stirring device for keeping the oil in circulation over the strip, and a long spiral

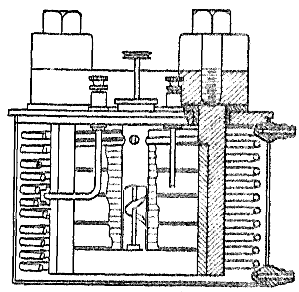


FIG. 28.

tube through which cold water can be passed. This method of oil immersion and supplementary water cooling permits a much higher current density than the air-cooled type, and ensures greater accuracy, since the temperature rise is not only restricted in amount, but can be definitely measured and allowed for by means of a thermometer immersed directly in the oil. A short-circuiting piece is usually provided, which saves the coil from heating when observations are not being taken. Resistances of this type are made in sizes ranging from 0·1 ohm to 0·0001 ohm, the latter size to carry up to 3000 amperes. The maximum temperature rise when running at maximum current with a full flow of water is approximately 20° C., and with a knowledge of the temperature characteristics of the material an accuracy of one part in ten thousand can be obtained. But it must be noted that even with good oil circulation the temperature of the strip when the current is flowing is usually considerably higher than the oil in which it is immersed, and small errors may be introduced in this way. Some actual observations taken on a resistance of this kind, showing the extent to

which the temperature of the strip exceeds that of the oil, are as follows:

Current.	Temperature of Oil.	Resistance.
Amperes.	°C.	International ohms.
200	10·6	0·000100092
	11·8	0·000100094
	14·0	0·000100097
	16·7	0·000100099
	18·2	0·000100100
	22·8	0·000100101
	26·9	0·000100101
	30·0	0·000100101
1000	12·2	0·000100096
	14·4	0·000100098
2000	12·6	0·000100101
	18·4	0·000100100
3000	21·0	0·000100090
	27·0	0·000100087

With a current of 3000 amperes the strip has attained a temperature at which the temperature resistance coefficient is negative, and the resistance has fallen fourteen parts in a hundred thousand below the value which would be indicated by measurement of oil temperature alone.

A type of resistance for use as sub-standard in standardisation work, designed at the National Physical Laboratory by Melsom and Booth, followed somewhat on the lines of the Reichsanstalt pattern, but special attention has been given to the cooling of the element, elimination of soft-soldered joints, and the design of both current and pressure terminals. As will be seen from *Fig.* 29, the resistance element is composed of a number of wires hard-soldered into a copper strip, which is connected at its centre to the current terminals, thus providing better cooling paths for the oil than with strip. The potential terminals are taken direct from the copper strip, and consequently no soft-soldered joints are included in the resistance. The method of connection to the copper end also ensures a more even distribution of current through the resistance elements than occurs in the more usual strip resistance.

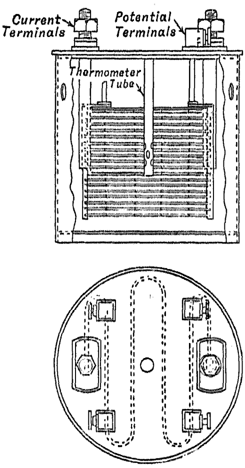


FIG. 29.

§ (14) STANDARD METHODS OF TESTING. (i.) *Potentiometers.*—One of the chief claims of the potentiometer is that its readings do not depend on the absolute resistance of the coils of which it is composed, but on their relative values, and in this way a higher order of accuracy can be obtained, provided that the instrument is so arranged that intercomparison of its various sections can be readily effected. The method of test will necessarily vary with the type of potentiometer. In the case of the Crompton instrument (see *Fig.* 2) provision is made for testing by means of an additional brush and pair of terminals, whereby an intercomparison of each of the coils can be made without the use of elaborate accessory apparatus (see *Fig.* 30). This represents the most

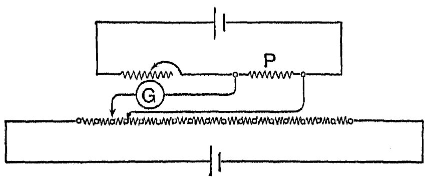


FIG. 30.

simple and accurate means of testing a potentiometer, since all the sections can be compared in turn with a single coil, the actual value of which is immaterial, provided it remains constant during the period of test. Each individual coil can be readily tested without undue trouble to one or two parts in a hundred thousand, and the resultant accuracy, when these values are added together, as they are in the normal use of the instrument, is far beyond that to which the instrument can be read. The initial resistance of the coil P is ten ohms, and for convenience the galvanometer is adjusted so that a deflection of ten divisions on the scale corresponds to one part in ten thousand, the difference between the coils being read directly by deflection of the galvanometer. By using small portable accumulators of, say, 15 ampere hour capacity, the current after a few hours will remain constant for some minutes together to give one part in a hundred thousand, and the whole of the potentiometer sections can be checked by this means as quickly as and more accurately than by any other method.

Where, however, the potentiometer is not provided with testing potentials of this kind, it is usual to build up a separate potentiometer circuit out of, say, ten equal coils, and to compare these with the various combinations of the potentiometer sections. This latter method can also be used for the more complicated Vernier type of potentiometer, although for both types it is frequently more convenient to make temporary connections by means of

sliding contacts to the studs of the individual coils, and to make the test as in *Fig.* 30.

The method used at the Bureau of Standards[1] is a modification of the Matthiessen and Hocking method, and consists of a ratio set made up of 100 resistance coils connected as shown in *Fig.* 31, having a total resistance of 2111·1

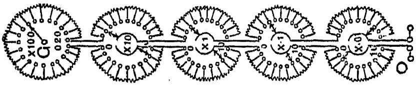


FIG. 31.

ohms, equivalent to a long slide wire of the same resistance, and upon which contact can be made at intervals of 0·01 ohm. The total resistance of 2111·1 ohms is connected across two binding posts and is independent of the setting of the dials, while the resistance between the zero terminal and the galvanometer terminal G is variable in steps of 0·01 ohm from 0 to the maximum. For use in calibrating a potentiometer, the ratio set is connected as shown in the simple diagram in *Fig.* 32, and

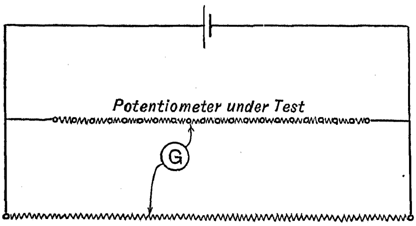


FIG. 32.

is adjusted for various settings of the potentiometer until a balance is obtained at each point. The ratio of any two sections of the potentiometer under calibration is then equal to the ratio of the different corresponding readings on the ratio set.

This method has the advantage of using the bridge system of measurement, and thus avoids the necessity of keeping the current steady during the period of calibration. On the other hand, the accuracy of the method is not so high as that of the simple potential method, and it has the further disadvantage that reliance has to be placed on the large number of coils in the ratio set remaining constant.

(ii.) *Dividing Boxes.*—For the satisfactory testing of pressure dividing boxes, it is essential that the test should be made at a series of pressures covering the normal working range of each section. This requires that standard boxes having the requisite ratios and designed for the same range of pressure as the boxes under test should be available. Such a standard box is usually immersed in oil in order to ensure uniformity of temperature among the sections. The boxes are connected in parallel across the pressure, and the comparison of the resistance is made either by means of a bridge, as shown in *Fig.* 33, where the balance is

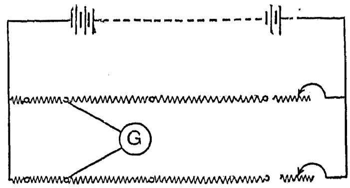


FIG. 33.

effected by means of a small resistance added to one or other of the boxes as required, or by means of a potentiometer. Although the bridge method does not require that the current through the dividing box shall be constant, yet it has been found that in practice a number of observers working over a period of years have preferred the potentiometer method.

(iii.) *Low Resistances.*—The various methods of testing low resistances have been fully dealt with in another article.[2] For air and water cooled resistances up to large current sizes, the conditions, due chiefly to the large size of the resistance and the impossibility of fitting it in to the actual bridges, etc., described earlier, require some modification of the exact arrangement, but, as a rule, the methods used are the same. Generally, owing to the large currents, the Kelvin double bridge method is used wherever possible, but in cases where the resistance of the connecting leads is too high or the standard resistance is unsuitable for this type of measurement, a potentiometer method is used.

For similar work in which it is not possible or convenient to balance the galvanometer exactly, and particularly in the measurement of temperature of cooling metals, Rosenhain suggested the use of a moving scale which would combine the readings of the potentiometer dials and the galvanometer deflections. An instrument designed on this principle consists of a number of turns of manganin wire wound in a double helix on a drum. The drum as it revolved varied the length of wire between the contacts, and to it was fixed gearing which moved a long strip celluloid scale the actual reading of the instrument being indicated by the position of the light spot on the scale.

A portable instrument on the same principle is now made by the Cambridge & Paul Instru-

[1] *Bull. of Bureau of Standards,* xi. 1.

[2] See "Resistance, Standards and Measurement of," §§ (8), etc.

ment Co., Ltd., the galvanometer in this case being a pointer instrument, and the scale an engraved ring moving with the drum.

S. W. M.

POTENTIOMETERS, standard method of checking and testing. See "Potentiometer System of Electrical Measurement," § (14) (i.).

POULSEN ARC, THE: an arrangement for producing undamped electrical oscillations for radio-telegraphy. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (4).

A device for producing high-frequency electrical oscillations of high power. Use of, in wireless telegraphy. See "Wireless Telegraphy," § (17) (iii.).

POWER:

Measurement of, in Alternating Current Circuits. See "Alternating Current Instruments," § (26).

Single-phase, Measurement of. See *ibid.* § (27).

Three-phase, Measurement of. See *ibid.* § (29).

Two-phase, Measurement of. See *ibid.* § (28).

POWER FACTOR, values of, for condensers with various dielectrics. See "Capacity and its Measurement," § (71).

POWER FACTOR OF CONDENSERS: a factor giving the energy loss in the condenser when using alternating current. See "Capacity and its Measurement," § (13).

POWER FACTOR METER: an instrument for indicating the phase angle between potential and current in an alternating current circuit. See "Alternating Current Instruments," § (46).

Gifford Type: a meter in which the current for the moving part is generated in it by induction. See *ibid.* § (47).

POWER LOSS IN CONDENSERS: dielectric hysteresis loss on charge and discharge. See "Capacity and its Measurement," § (12).

POYNTING'S THEOREM. This theorem—see *Phil. Trans. R.S.*, 1884, clxxv. 343—states that there is a general law for the transfer of energy in the electromagnetic field according to which it moves at any point at right angles to the plane containing the lines of electric and magnetic force, and that the amount crossing per second unit area of this plane is equal to the product of the intensities of the two forces multiplied by the sine of the angle between them and divided by 4π, while the direction of the flow of energy is that in which a right-handed screw would move if turned from the positive direction of the electrical to the positive direction of the magnetic force.

The electrical energy within any closed surface in the field is given by the expression $(K/4\pi)\iiint \mathbf{E}^2 dxdydz$, while the magnetic energy is $(\mu/4\pi)\iiint \mathbf{H}^2 dxdydz$. If the conditions within the surface are changing, work may be done by the electromagnetic forces and heat generated owing to the resistance of the conductors. Poynting obtained expressions for the rate of change of the electric and magnetic energy within the surface, the rate at which heat is produced, and the rate at which the electromagnetic forces do work.

He showed that the sum of these four quantities is given by an expression, to which each element of the surface contributes a share, depending on the values of the electric and magnetic intensities at that element. That is, the total change in the energy within the surface is accounted for by supposing that energy passes in through the surface according to the law given by this expression.

If dS be an element of the surface at the point considered, l, m, n the direction cosines of the outward-drawn normal, P, Q, R the components of the electrostatic force at the point, and α, β, γ the components of the magnetic force, then if the surface be taken where the matter is at rest the value of the share contributed by the element dS is given by

$$\frac{1}{4\pi}\{l(R\beta - Q\gamma) + m(P\gamma - R\alpha) + n(Q\alpha - P\beta)\}dS,$$

and the total change of energy per second within the surface is found by integrating this over the surface.

Again if λ, μ, ν be the direction cosines of the lines of intersection of the electrostatic and magnetic equipotential surfaces, it can be readily shown that

$$\lambda = \frac{R\beta - Q\gamma}{\mathbf{EH}\sin\theta}, \quad \mu = \frac{P\gamma - R\alpha}{\mathbf{EH}\sin\theta}, \quad \nu = \frac{Q\alpha - P\beta}{\mathbf{EH}\sin\theta}.$$

Thus the share of an element dS is

$$\frac{1}{4\pi}\mathbf{EH}\sin\theta(l\lambda + m\mu + n\nu)dS.$$

If at the point in question dS be taken so as to coincide with the plane containing **E** and **H**, then $l\lambda + m\mu + n\nu$ is equal to unity and the element contributes the greatest amount of energy to the space, or, in other words, the energy flow is perpendicular to this plane and is equal in amount to $\mathbf{EH}\sin\theta/4\pi$ per unit of area.

Since the energy flows perpendicularly to the plane containing the electric and the magnetic forces it must flow along the equipotential surfaces where these exist, that is, the lines of flow are the intersections of these two surfaces.

— Q —

— R —

RADIATION FROM A MOVING CHARGE OF ELECTRICITY. The space about a moving charge of electricity is a field of both the electric and magnetic force. Through any surface in such a field there is a flow of energy, the flow per unit area at any point being given—see Poynting's Theorem—by the product of the magnetic and electric intensities multiplied by the sine of the angle between them and divided by 4π. The flow takes place in a direction at right angles to the plane through the directions of the two intensities. This energy is radiated from the moving charge.

In the case of a charge Q moving with velocity v and acceleration f it is shown—see "X-Rays," § (2)—that, at a point at a distance r from the particle in a direction making an angle θ with the direction of motion, the resultant electrical and magnetic intensities are, if V be the velocity of propagation of electric waves, given by

$$\mathbf{E} = -\frac{Qf\sin\theta}{4\pi V^2 r}, \quad \mathbf{H} = -\frac{Qf\sin\theta}{Vr}.$$

These two intensities are at right angles to each other, and the directions of both lie on a sphere with the moving charge as centre. Hence the energy is moving outwards at right angles to this sphere, and the rate at which it crosses the area dS is, by Poynting's Theorem, equal to

$$V^2\mathbf{EH}dS.$$

Substituting the values of **E** and **H**, we have

$$\text{Rate of transfer through } dS = \frac{Q^2 f^2 \sin^2\theta}{4\pi V r^2} dS.$$

Now $\qquad dS = 2\pi r^2 \sin\theta d\theta,$

thus

Total rate of transfer of energy

$$= \frac{Q^2 f^2}{2V}\int_0^\pi \sin^3\theta d\theta = \frac{2}{3}\frac{Q^2 f^2}{V},$$

and this is the rate at which the moving source radiates its energy.

Radiators, Electromagnetic: devices for the production of electric waves. Effect of added inductance on. See "Wireless Telegraphy," § (8).

Radio-frequency Alternators: electromagnetic machines for the production of high-frequency current for radio-telegraphy. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (5).

Radio-frequency Currents, calibration of sensitive instruments for the measurement of. See "Radio-frequency Measurements," § (18) (vii.).

RADIO-FREQUENCY MEASUREMENTS

(The Arabic numbers in brackets refer to the Bibliography at the end of the article.)

The following article deals essentially with the various quantities which have to be measured at radio-frequencies on circuits, apparatus, and materials used in radio-telegraphy and telephony. It excludes any treatment of the arrangements and operations connected with radio-telegraphy and telephony as such.

The article is divided into the following parts:

I. Introductory.
II. Measurements of wave-length and frequency.
III. Measurement of current.
IV. Measurement of capacity and design of condensers for measuring purposes.
V. Measurement of inductance and design of inductances for measuring purposes.
VI. Measurement of effective resistance and decrement.

I. Introductory

Radio-frequency measurements stand in a class by themselves on account of peculiar difficulties arising from the extremely rapid rate of change of the currents involved, and from the fact that the quantities being measured are not always definitely located in the circuits. The total inductance of the circuit is not large in many cases, and the small inductances of leads, condensers, etc., are not negligible; similarly, in regard to capacity, there are considerable distributed capacities in leads and inductances; the condensers and inductances also have distributed capacity to each other and to adjacent circuits, earth, walls, the observer, etc. When dealing with measurements of current, voltage, power, effective resistance, etc., the introduction of instruments and apparatus for the purpose of measurements causes variation in factors which it is often desired to keep invariable.

The greatest difficulties usually arise in connection with measurement of effective resistance, including those resistances representing losses in dielectrics, radiation, and in the measuring devices. In these measurements, currents or the ratio of currents have to be measured. The constants of the instruments used may change largely with frequency owing to alteration in distribution of current traversing a vital part of the circuit. Reaction of currents in the applied measuring circuit, on the circuit being measured, must be carefully watched for and eliminated if possible.

In general, the inaccuracies of measurement increase as the frequency is increased.

In many cases the currents are quite different in different parts of a circuit. When a measurement of current is desired judgement must be used as to where to insert the current measuring device.

In dealing with highly resonant circuits profound changes may be produced in the currents and frequency if the coupling of two circuits exceeds a definite and small amount. In general the coupling between a circuit and measuring device should be very small, particularly when the measuring circuit is highly resonant.

There is a vast amount of literature accumulated regarding all kinds of radio measurements. In judging the various results obtained, however, it must be borne in mind that the development of the three-electrode valve, when used as a source of radio-frequency currents, has caused a revolution in measurement owing to the extreme steadiness obtainable, both of frequency and amplitude of the oscillations. The use of the valve as a local source of feeble oscillations of great steadiness has also opened a new field, the development of which is only just commencing. There seems little doubt that in the near future radio measurements will rank amongst the most refined of all commercial electrical measurements, particularly in regard to frequency, since the demands made on accuracy of frequency in transmission and reception will certainly become more and more severe.

II. Measurement of Wave-length and Frequency

§ (1).—The measurement of wave-length or frequency is one of the most fundamental of all radio measurements. Although commonly thought of as synonymous, these two quantities are quite independent from the point of view of measurement.

The measurement of both quantities simultaneously and independently in the same

circuit constitutes a determination of v, the velocity of propagation of the waves whose length has been measured.

Those methods in which wave-length is measured directly as the length of a stationary wave, and those measuring frequency directly as $(\text{time})^{-1}$, are of course absolute. These will be considered in detail first.

Absolute Measurements. — In the present article any methods in which both frequency and wave-length have been simultaneously and independently measured will be considered as independent absolute measurements.

The methods may be classified as follows according to the various essential underlying principles:

Direct measurement from high-frequency alternators.

Stationary waves on Lecher wires—wave-length determinations.

Photography by reflected light from sparks or other luminous discharges in an oscillatory circuit.

Methods in which the ratio of two frequencies is known to be an exact integral number. If the numerical values of this ratio and of one of the two frequencies are known the other frequency is thereby determined. The electrical oscillations of lower frequency are used to produce forced vibrations or waves of a material kind. The frequency or wave-lengths of these latter is then measured absolutely.

§ (2) Absolute Frequency Measurement by High-frequency Alternators. — The measurements of frequency falling under this heading are of so simple a kind as not to require any very special treatment.

Duddell was one of the first to construct a small alternator capable of giving feeble alternating currents of frequencies up to $1{\cdot}2 \times 10^5 \sim$ per sec. The machine consisted of an inductor alternator driven by geared-up belt pulleys. The armature was of toothed disc stampings bolted together. In the machine, capable of giving a frequency of 120,000 $\sim$ per sec., there were 200 teeth, thus necessitating a speed of revolution of 600 turns per sec. The frequency is of course simply the product of the number of teeth and the number of revolutions per second.

H. Boucherot (27) has made absolute measurement of radio-frequencies using a specially constructed high-frequency alternator capable of giving 60,000 $\sim$ per sec. L. Brillouin (27) has stated that it is difficult to obtain an accuracy better than 1 per cent. In order to obtain accuracy by any direct method using an alternator very special arrangements would have to be made to hold the speed steady if an accuracy of 1 in 1000 was required.

§ (3) Stationary Waves on Wires.—The propagation of waves along wires has been the subject of much experimenting, notably by Bezold, Lodge and Chattock, Sarasin and de la Rive, Blondlot, and others before the experiments of Lecher, whose name is attached to the system of wires on which such waves are usually demonstrated. In 1887–88 Lodge and Chattock (1) clearly demonstrated the presence of stationary waves on long parallel wires running round and across a room, and gave means of measuring the length of such waves. Lecher's (2) experiments were carried out in 1890. Hertz (3) in 1891 gave the theory of the propagation of waves along such wires. Since then many workers, Paalzow and Rubens (4), Zenneck (5), Arons (6), Righi (7), Gehrke (8), to mention a few, have studied and made measurements on such waves.

(i.) *Measurements of Trowbridge and Duane* (9).—These measurements were made in 1895 and were the first very complete and careful determinations of wave-length and frequency for the purpose of measuring "v" absolutely.

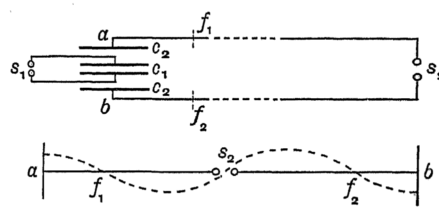


Fig. 1.—Parallel Wire System (Trowbridge and Duane).

The parallel wire system used is shown in *Fig.* 1.

Oscillations are set up in a primary system consisting of spark-gap S_1, leads, and condenser C_1. The parallel-wire measuring system is coupled electrostatically through condensers C_2, C_2; at the far ends of the parallel wires is a small spark-gap S_2.

The resonant condition of the parallel wire circuit is brought about by adjustment of the frequency of the primary exciting circuit containing condenser C_1. Resonance is made evident by sparks at S_2. When the system is vibrating in the mode best suited to measurement there will be potential nodes at f_1, f_2, and S_2 as shown in the developed circuit in which the dotted line represents roughly the potential distribution. The total distance f_1, f_2 measured along the parallel wires and end pieces, including S_2, is the measure of the wave-length of the stationary wave produced.

The points f_1, f_2 are located by means of a bolometer, connections to which can be slid along the wires until a zero deflection is obtained corresponding to zero difference of potential between the points.

Simple Theory of Parallel Wires. — The inductance at high frequency of parallel wires is given in

electromagnetic units by $L=4l\,(\log_e(2D/d))$. Similarly the capacity in electromagnetic units is given by

$$C=\frac{l}{v^2 4\log_e(2D/d)}.$$

Hence $$LC=\frac{l^2}{v^2},$$

where l=length of each wire in cm.,
D=distance apart in cm.;
d=diameter,
v=velocity of waves—cm. per second,

from which it is seen that the wave-length is independent of the diameter and distance apart of the wires.

This is only true so long as other conductors are at a distance from the wires, which is large compared to their diameter and distance apart.

There is a slight retardation of the waves due to resistance of the wires; the effect is usually negligible.

The expression for the velocity of waves along wires when the resistance is not negligible is

$$v=\frac{l}{\sqrt{LC}}\left(1-\frac{R^2}{8\omega^2L^2}\right),$$

i.e. $$v_L=v_0\left(1-\frac{R^2}{8\omega^2L^2}\right),$$

where R and L are effective resistance and inductance in consistent units, per unit length of the wires.

(ii.)—Zenneck's method (10) is shown in *Fig.* 2.

A is a source of high-frequency oscillations, consisting of loop of inductance L, two condensers CC, and spark-gap S, all in series,

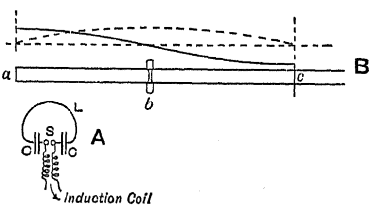


FIG. 2.—Zenneck Parallel Wire System.

the last named being connected to the secondary of an induction coil. This circuit is loosely coupled to a system of parallel wires B, joined at one end a; the wires are bridged at their far ends by a short-circuiting link c, which is adjustable along the wires. A sensitive detector of voltage (vacuum tube) b also bridges the wires at the middle and is adjustable.

In carrying out the tests circuit A is energised; the bridge c and tube b are then adjusted (always keeping b midway between a and c) until maximum brightness of the glow discharge is obtained. When this occurs there will be a distribution of current along the wires as shown by the full line, and a loop of voltage between them as shown by the dotted curve. When the resonant position has been found the length ac gives a direct measure of the half wave-length of the oscillations. This will be exact if the resistance of the wires is negligible compared to their reactance.

(iii.) *Experiments of Diesselhorst* (11).—These experiments, carried out at the Physicalische Technische Reichsanstalt in 1907, included absolute measurements of frequency by a spark method, which are described briefly in § (4). The Lecher wires employed were very long and were zigzag in the open air, as in diagram *Fig.* 3.

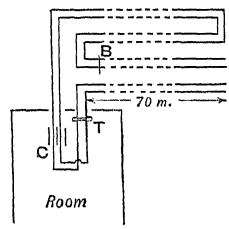


FIG. 3.—Diesselhorst Parallel Wire System.

It had been shown previously that bending or doubling back the wires had no influence on the wave-length (12).

The total length of the loop was about 300 metres. The wires were 1 mm. diameter and 2 cm. apart, located by numerous small ebonite pieces.

The measurements were made with the help of an assistant outside operating the link B. In making the observations the coupling was loosened at C until just enough energy was induced into the system to cause vacuum tube T to light up at resonance. By a system of signals link B was slid along, first in one direction and then in the other, by the assistant whilst the observer watched the vacuum tube. By this means the position of B for maximum resonance was found and observed. The observations were repeated many times and the mean taken. For the shorter waves the positions of B corresponding to $\lambda/4$, $\lambda/2$, $3\lambda/4$, λ, and $5\lambda/4$ were thus determined. The various values of λ so determined were within a few parts in 1000 of the mean value. Wave-lengths up to 1200 meters were measured and compared against a wave-meter and also against an absolute determination of the spark frequency.

A matter to guard against in making measurements with Lecher wires is to discriminate between resonances of the fundamental and of harmonics. In the half-wave oscillator of Zenneck the tube should not light up except at the middle of the loop. If resonance (glowing) is obtained at, say, $\frac{1}{4}$ and $\frac{3}{4}$ along the loop these must be due to a frequency component of twice the fundamental n. If only the frequency $2n$ were present no glowing would occur at the middle, since this point would be a node of voltage.

§ (4) SPARK AND VACUUM-TUBE PHOTOGRAPHY.—The principle of this method of absolute measurement of frequency of oscilla-

tions is to photograph images of sparks or other luminous discharges forming part of the oscillatory circuit. The images of the discharge are formed by reflection from a rapidly revolving mirror on to a photographic plate. By this means the successive discharges of one train of oscillations are drawn out on a time base on the photographic plate.

(i.)—The earliest experiments of this nature were made by Feddersen (13) in 1861, who first showed, by reflection from a rotating mirror, the oscillatory sparks from a Leyden jar, explaining their nature and how to measure their frequency. A great number of experimenters have made measurements by the method; for example, J. Trowbridge and W. Duane (9), 1895, in their measurement of "v" used a Lecher wire system loosely coupled to a Hertzian oscillator. A bolometer similar to that of Paalzow and Rubens (4) was used to indicate the nodes on the wires. The spark used was in the Lecher circuit and not in the primary oscillator, so as to ensure measuring the same waves in the determination of the two quantities. The rotating mirror was driven from a small electric motor at a rate of 70 revolutions per sec. The rate of revolution was measured by a chronometer. In their later experiments cadmium electrodes were used in the spark-gap.

(ii.)—In the measurements by L. Decombe (14) the mirror was revolved at the very high speed of 400 to 500 revolutions per sec. The speed was measured by comparing with a tuning-fork the note emitted by the revolving mirror. In the measurements by C. A. Saunders (15) both wave-length and frequency were determined, but the frequency measurements were made on the sparks in the primary circuit with the help of the rotating mirror. The value found for v was 2·997, so that the experiments were probably made with good accuracy.

(iii.)—The measurements by Hemsalech (16), 1901, were made using a slit illuminated by the spark, thus giving particularly suitable photographic records.

(iv.)—Gehrke (17) in 1904 introduced the glow-light oscillograph, consisting of two strips of aluminium placed end on in the same diametral plane in an exhausted glass tube; a small gap is left between the ends. The size of the strips is about 1·5 cm. wide and 12 cm. long. When a sufficient voltage is applied at the terminals of the tube a glow appears, extending over the negative electrode along a length roughly proportional to the voltage. Viewed edgewise this glow appears as a bright and fairly well-defined line of light. Such a tube has been used by H. Diesselhorst (18) to delineate discharges in oscillatory circuits by photography of the glow after reflection from a revolving mirror.

In these experiments the tube was connected in series with a high resistance (a U-tube containing water), the combination being shunted across the condenser in an oscillatory circuit with spark-gap fed from an induction coil. The glow-light tube was in a dark box at a distance of 25 cm. from a revolving concave mirror. The image of the line of light in the tube when discharging fell on a photographic plate at about the same distance from the mirror as the tube. The rate of revolution of the mirror was 200 per sec. The images on the plate showed black lines alternately on each side of the zero corresponding to the discharges in opposite directions each half alternation. At a frequency of 100,000 ∿ per sec. the alternate images are separated by about 6 mm. on the plate. A direct-reading revolution indicator was mounted on the other end of the motor spindle.

(v.)—In the more accurate measurements on absolute frequency made at the Reichsanstalt in 1907, a spark-gap of small round cadmium rods was used and the distance from the plane rotating mirror was from 3 to 5 metres. The calibration of the apparatus was made with the help of an accurate scale ruled in cms. and placed at the position of the spark-gap. Its image was then photographed after reflection from the mirror. The speed was measured on a chronograph by the help of electrical contacts.

The photographic plates were measured by projection with twenty-fold magnification. In general the first twenty sparks could be measured.

(vi.) *Absolute Measurements of Oscillation Frequency at the National Physical Laboratory.*—These experiments (19) were similar in general arrangement to those of Diesselhorst. The apparatus is shown in the accompanying photograph, *Fig.* 4, and diagrammatically in *Fig.* 5.

The oscillations were generated in the circuit L_1 C_1 S, fed from step-up transformer T with alternating current of 50 ∿ per sec. The spark-gap S consisted of short cadmium electrodes in the form of bluntly-pointed rods about 6 mm. diameter. These were mounted at one end of a long camera box having a side extension carrying a plate-holder and slide at P. The mirror at M was mounted on ball-bearings and was about 4 cm. diameter and 2 metres radius of curvature. The distances SM, MP were each 2 metres, and the angle SMP was fairly small. The plate P was mounted in a slide fitting friction-tight in vertical guides in such a way that it could be slowly moved downwards across the opening in the side arm (the images of the trains of sparks flashing across the plate meanwhile). Under these conditions, with the plate occupy-

ing 5 seconds in moving down a distance equal to its own width, the chances were that 12 trains of sparks would fall on it. In general there were about this number of trains of sparks available for measurement.

The spindle on which the mirror was mounted carried a small commutator above which formed the essential charging and discharging part of a Maxwell's commutator bridge as used in measuring capacity.[1] This bridge forms a very convenient way of holding a speed constant to 1 in 10,000 if desired. The bridge was set to be in balance at a particular chosen frequency (say 50 to 100 ∿ per sec.). In carrying out the measurements everything was set going and allowed to become steady in the oscillatory circuit and in the motor driving the mirror; this latter was so set that it tended to drive the mirror slightly faster than to give the Maxwell balance; by means of a cord passing loosely round another pulley on the spindle a small brake could be applied to it and the balance held by hand. Under these conditions the photographic plate was slowly slid past the images of the spark trains. Readings were also taken, during the exposure, on the loosely coupled wavemeter W. The speed of the rotation of the mirror was accurately measured with the help of a worm-driven contact operating on the chronograph every 100 revolutions of the mirror.

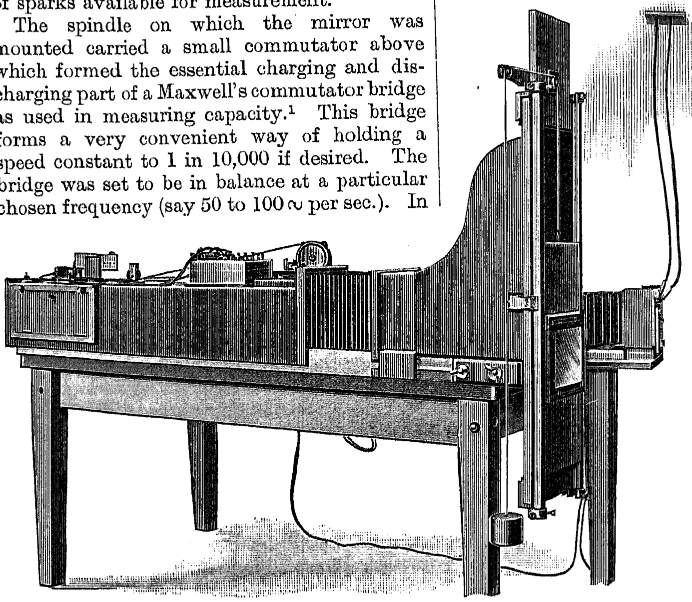


FIG. 4.—Spark Photography Camera (N.P.L.).

A great number of plates were exposed and the measurements on each train made by the help of a scale engraved on glass. Frequencies over the range 150 to 1500 metres wave-length were observed and compared against the standard wavemeter. About 20 sparks were obtained in a train; in general two sparks correspond to one complete oscillation. The alternate sparks could usually be clearly distinguished: the trains being thus -_¯_¯_¯_¯_ in a good case, showing fine lines corresponding to the fine point of light at the surface of one of the electrodes.

The general agreement between the absolute determinations of frequency and the deduced values as given by the wavemeter was very good; the average error over the whole of the observations of each class was about 0·2 per cent.

The method of absolute measurement by spark photography may be said to be very direct and involves very simple procedure, and the results when obtained are unquestionable within certain limits of accuracy. It is this question of accuracy, however, that forms the main point of criticism of the method. The trains of waves on a photographic plate are relatively short (10 to 15 cm. length), and this length measurement is of fundamental importance. The attached wavemeter is averaging the frequency of an enormous number of spark trains of oscillations, whereas the plate averages comparatively few. Individual trains are found to differ by several per cent from the mean. In spite of this, good consistency is found between the means of different plates exposed on sparks of the same frequency. The average accuracy, when a great many trains (say over 100) of oscillations are measured at each of a number of frequencies, is probably of the order of 1 or 2 in 1000. For higher accuracy it would seem that the spark method must give place to the more accurate and uniform harmonic methods. The range of frequencies most suitable for the spark method is from the lowest

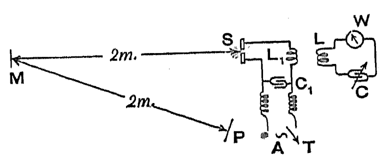


FIG. 5.—Arrangement of Apparatus for Spark Photography.

[1] See "Capacity and its Measurement," § (41).

up to about 10^6 ∿ per sec. At this upper frequency a good link on to the Lecher wire system is obtained.

§ (5) FREQUENCY DETERMINATIONS BY HARMONIC RATIOS.—The methods involving this principle are many and various. Virtually they consist in measuring the unknown frequency in terms of another known frequency. The ratio between the two frequencies is either an integer or a very simple arithmetical ratio. In any case the essence of such methods is that the ratio is known with absolute certainty. Simon (20) and Barkhausen (21) in working with the Poulsen arc for the generation of undamped oscillations at the P.T.R. had developed out the theory of such oscillations; from this it was seen that the wave form depended on the ratio of the time the arc was alight to the time it was extinguished during each oscillation. The departure from sine wave, and hence the increase in the proportion of harmonics, is enhanced by making the time of extinction as large as possible, and also by making capacity large and self-induction small in the oscillatory circuit. By diminishing the direct current feeding the arc the proportion of harmonics is further enhanced. Unfortunately these conditions are antagonistic to constancy of frequency of the fundamental.

(i.) *The Poulsen Arc Method.*—In the experiments of R. Lindemann (22) on measurements of harmonics of the Poulsen arc, a circuit similar to that of Schapira (4) was used as in *Fig.* 6. A second oscillatory circuit B is shunted across the arc in parallel with the main oscillatory circuit A; it consists of a condenser and inductance tuned to the desired harmonic. When exact setting corresponding to any harmonic is obtained, a large building up of current in the side circuit results. H. Diesselhorst and R. Schmidt (23) have used this circuit with a Braun cathode ray tube and have obtained the Lissajous figures corresponding to the compounding of the two oscillations in A and B. From the pattern of the figure obtained the order of harmonic can be seen by inspection so long as it is not of too high order (say 12).

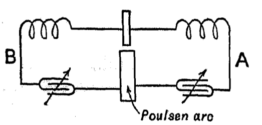


FIG. 6.—Poulsen Arc Circuit (Schapira) for giving Harmonics.

The observations of R. Lindemann were made with the aid of a standard wavemeter. This was first tuned to the fundamental and then to each of the picked-out harmonics 2, 3, 4, etc., in circuit B. The values of the wave-lengths of the harmonics were calculated from the wavemeter readings, and each wave-length so obtained was multiplied by the number representing the order of harmonic which it was. Theoretically the values of calculated fundamental so obtained should come out identical. In practically every case the values were within 1 in 1000 of the observed value and of the mean of all the deduced values of the fundamental.

In some of the observations a fundamental frequency corresponding to wave-length of 6000 metres was used.

Harmonics as high as the 43rd were successfully obtained and measured, thus reaching the region of wave-length equal to 140 metres and giving good overlapping with the Lecher wire measurements of H. Diesselhorst, with which very good agreement was obtained.

(ii.) *Absolute Measurements by Harmonics from Buzzer.*—This method is based on the discovery by Mandelstam when working with oscillations excited by a buzzer. It was observed that the frequency of oscillations produced does not depend only on the natural free period of the oscillatory circuit, but also on the frequency and nature of the contact made by the buzzer. Under suitable conditions those oscillatory frequencies which are exact multiples of the buzzer interrupter frequency have a larger amplitude than others. This property has been utilised by J. Tykocinski-Tykociner (24) to produce a

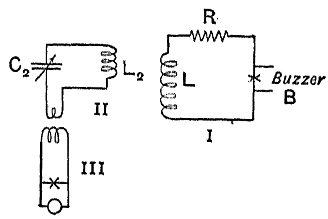


FIG. 7.—Buzzer Circuit giving Harmonics in Tuned Circuit.

source consisting of a series of high-frequency oscillations, as shown in *Fig.* 7.

The buzzer B giving regular interruptions excites an aperiodic circuit I, consisting of inductance L and resistance R. This circuit is loosely coupled to a second circuit II, capable of freely oscillating, and consisting of inductance L_2 and variable capacity C_2. A detecting or measuring circuit III is coupled to II, and contains a thermal element and galvanometer or a detector and telephone; it indicates the maxima which occur as the free frequency of circuit II passes through those values which are exact multiples of the buzzer interruptions. In order to determine the absolute frequency of a particular reinforced harmonic it remains to measure the order of the harmonic, say h where $uh=n$, u being the frequency of the

buzzer and n the frequency of the reinforced oscillations.

A number of ways of determining h exist. (*a*) To count every reinforcement from the beginning or from such a low order of harmonic, say 10th, that there is no doubt whether it is No. 9, 10, or 11. The calculations from a knowledge of L_2 and C_2 in the wavemeter circuit are more than sufficient for this accuracy; by then carefully counting onwards up to any desired frequency of the series the value of h is determined.

This method demands a special circuit II covering the long range, and will necessitate additional condensers or inductance; a stand-by oscillatory circuit (heterodyne) will also be necessary in this case to locate an overlapping harmonic when passing from one condition of circuit II to another at the same frequency.

(*b*) If n_1 and n_2 be two harmonic frequencies picked out on circuit II at condenser readings C_2' and C_2'', also if r be the number of maxima observed in passing from n_1 to n_2, then $r=h_1-h_2$ and

$$n_1=\frac{r\,.\,u}{\{(n_2/n_1)-1\}}=\frac{r\,.\,u}{\{\sqrt{(C_2'+c)/(C_2''+c)}-1\}} \quad (1)$$

The small capacity c is an additional factor which has to be introduced. It represents the self-capacity of the inductance. c can be determined by other methods to an accuracy of perhaps 1 $\mu\mu$F. If r is of higher order (100, for instance), then $C_2'-C_2''$ will need to be at least 400 or 500 $\mu\mu$F in order to be quite certain that h is located accurately.

(*c*) The third method of locating h is quite accurate and free from any uncertainty. It requires the help of a small heterodyne set producing feeble oscillations of fundamental frequency approximately equal to n.

The heterodyne frequency is adjusted to a frequency f until a beat note of unmistakable character and pitch is heard by interference with the reinforced frequency n_1 in circuit II. Condenser C_2 is now gradually reduced whilst the successive reinforced harmonics are carefully counted. When the frequency $2n_1$ is reached it will be located by a beat tone with the first harmonic of the heterodyne of frequency $2f$, and this beat tone will be exactly the octave of the beat tone heard between n_1 and f. The number of resonances counted between n_1 and n_2 is the order h of harmonic of the buzzer giving rise to the frequency n_1 in circuit II.

The measurement of the acoustic frequency of the buzzer presents no special difficulty.

(iii.) *Multivibrator of H. Abraham and E. Bloch* (25).—This ingenious apparatus, developed in 1917, makes use of the three-electrode valve for producing a fundamental note very rich in harmonics. The frequency can be adjusted to almost any value from a few per minute up to 50,000 ∿ per sec. The special feature of the oscillations so produced is the richness of harmonics. The circuit is

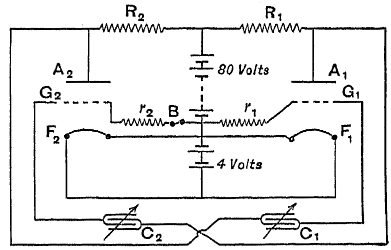


FIG. 8.—Abraham-Bloch Multivibrator.

given in *Fig.* 8, and is reproduced from *Comptes Rendus*, 1919, clxviii. 1105.

In a particular type of the apparatus giving a fundamental of 1000 ∿ per sec. the constants of the circuits are as follows: $R_1=R_2=50{,}000$ ohms wound in the form of inductive disc coils of constantan wire. $r_1=r_2=75{,}000$ ohms carbon compound rods. C_1 and C_2 are subdivided and adjustable condensers allowing any capacity up to 0·008 μF to be obtained. The principle of operation is as follows. Anode A_1 begins to discharge through the right-hand valve. The current thus started in flowing through R_1 causes the potential A_1 to fall; this fall of potential is communicated *via* condenser C_2 to the grid G_2 of the left-hand valve. A diminution of anode current through resistance R_2 thereby ensues; the potential of A_2 therefore rises, and this rise of potential is communicated *via* condenser C_1 to the grid G_1, which causes the current through A_1 to still further increase. The system is thus unstable; current rapidly rises in A_1 and becomes zero in A_2. The grid G_1 loses its positive potential by leakage through C_1. Meanwhile condenser C_2 and grid G_2 are becoming charged through the leak resistance r_2. The negative potential of G_2 soon becomes sufficiently small to allow current to flow through A_2 again, and the cycle described above repeats itself in the other half of the symmetrical circuits. The frequency of the reversals is determined by the values of C_1, r_1, and C_2, r_2. It is approximately of the order

$$n=\frac{1}{C_1r_1+C_2r_2}. \quad . \quad . \quad . \quad (2)$$

When the apparatus has settled down the fundamental frequency is extremely steady under suitable conditions, but is disturbed by variations in anode battery voltage and filament current. If it is desired to keep to a fixed fundamental of say, 1000 ∿ per sec., it is possible to control the frequency very

accurately and hold it constant by introducing into the anode battery circuit a small voltage obtained from an electrically-maintained tuning-fork.

The method of using the multivibrator is shown in *Fig.* 9. M is the apparatus controlled

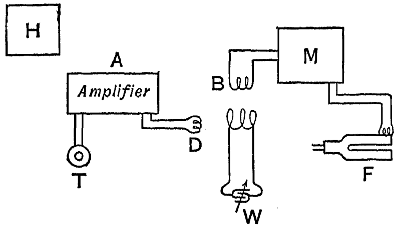


FIG. 9.—Arrangement of Apparatus using Multivibrator.

by tuning-fork F. A coil inserted in series with one of the grid leak resistances *r* (shown at B) conveys the electrical impulses or "blows" to the standard wavemeter W. The coupling between B and W should be very loose. A valve amplifying set A is very loosely coupled to W and serves to make evident the resonances in W.

The coil D of the amplifier can be so adjusted in relation to B that no sound is heard at T when the wavemeter is not resonating.

If now the condenser of the wavemeter is gradually turned, a succession of reinforcements of the fundamental of 1000 ∿ per sec. will be heard in the telephone T. These correspond to the successive building up of resonant feebly damped trains of oscillations in the wavemeter W as its forced resonance under the successive blows of the multivibrator become n, $n+1$, $n+2$, etc., times the fundamental where n is an integer. The position of maximum resonance can be located to an accuracy of about 1 in 1000 by this method of observing the resonance. By substituting for T a vibration galvanometer tuned to the fundamental, a considerable increase in accuracy can be obtained. When observing harmonics of high order, above 100 for example, there is considerable blurring of the resonances owing to the closeness of the successive harmonics. In this region it is desirable to observe resonance with the help of a heterodyne set H, loosely coupled to the amplifier (about 1 metre away). The heterodyne oscillations are carefully adjusted to give a beat note by interference with the particular harmonic being observed. This method gives character to that harmonic and enables the resonance to be much more accurately estimated.

In a series of experiments made at the National Physical Laboratory, the comparison of frequency given by the multivibrator with that given by the standard wavemeter was very satisfactory, the accuracy of agreement of individual observations was somewhat better than could be read on the condenser scale, and the average agreement over a range from 40,000 to 150,000 ∿ per sec. was to about 1 or 2 in 10,000.

For exploring higher frequencies (120,000 to 2×10^6) it is convenient to use a high-frequency multivibrator of fundamental of about 40,000 ∿ per sec. and observe harmonics of this, up to the 50th if desired. The fundamental of the high-frequency multivibrator can be determined by direct comparison with the picked out 40th harmonic of the 1000 ∿ per sec. multivibrator.

The frequency of forced resonance of the wavemeter lies between the free period and the forced period using sine wave excitation. This arises because an impulse is given and the wavemeter then performs free damped oscillations until the next impulse.

If R, L, and C are the effective resistance, inductance, and capacity of the wavemeter, then assuming the resonance maintained by instantaneous blows

$$n = n_0\left(1-\frac{5}{8}\frac{R^2C}{L}\right). \qquad (3)$$

With well-designed wavemeter coils the correction factor $\frac{5}{8}(R^2C/L)$ is absolutely negligible. In the wavemeter tested at the National Physical Laboratory this correction, in the worst case, was less than 2×10^{-5}.

The chief drawback to the multivibrator method of absolute frequency measurements is the disturbance caused by other sounds and by neighbouring circuits. Considerable skill is required in interpreting the sounds heard, particularly when working with the heterodyne. A great many combination and difference tones can be heard together with the note proper of the multivibrator and the beat tone of the heterodyne. With practice the observations become quite certain, however, and the accuracy is all that could be desired. It is a slight disadvantage that the oscillations actually being observed are not undamped. By operating through the intermediary of a valve generator, which would be set by the multivibrator, and then inducing from the undamped source into the standard wavemeter, the system has many possibilities.

§ (6) FREQUENCIES OF LONG WAVES.—With the extension of radio-telegraphy more and more into the region of longer waves, the difficulties in the measurement of frequency diminish. It seems probable that absolute measurements could be made by direct recording on an oscillograph or Einthoven galvanometer up to frequencies of 20,000 or 30,000 per sec. High-frequency oscillographs are made having a free undamped period of 10,000

per sec., and giving a large deflection for 0·1 amp. at low frequencies. The sensitivity would slightly increase up to 10,000 ∿ per sec., but would rapidly diminish above this value.

Quite powerful longitudinal vibrations in steel bars can be produced by acting on them with currents of the resonating frequency. The resonance is extraordinarily sharp, since vibration of this kind has probably less damping than any other kind of mechanical vibration. A bar set to have a frequency of 15,000 ∿ per sec. forms a means of setting a radio-frequency current to an accuracy of about 1 in 10,000.

The method, however, is limited by the limit of audible frequency of the operator.

The cathode ray tube with heated cathode will probably form the most accurate method of absolute measuring and setting of frequency in the future. Further work is desirable in this field. Up to the present the cathode ray tube has only been used to measure the ratio of radio-frequencies; by extending its use to the comparison of acoustic with radio frequency a definite advance would appear probable.

§ (7) Wavemeters.—The wavemeter is the most fundamental and generally used instrument in radio measurements. It is used chiefly for measuring wave-length or frequency, but can also be adapted to measure capacity, inductance, and effective resistance or decrement. With the addition of means of making it self-exciting, it can be used to produce oscillations of known frequency and to make measurements on circuits which have no radio-frequency current traversing them. When equipped with an accurate current-measuring instrument the apparatus can perform nearly all the measurements on circuits which have to be made in radiotelegraphy. Essentially, a wavemeter consists of a circuit in which capacity and inductance separately concentrated are the principal features. In addition, means of determining when certain conditions such as resonance have been realised must be provided.

Wavemeters may be divided into two classes—standard and commercial. The desirable qualities in a standard wavemeter are accuracy, permanence, and simplicity in design; the first and third qualities usually go together. A standard wavemeter should have its parts so designed that their electrical constants may be determined independently at low frequency, and these values used to calibrate it as a wavemeter.

Commercial wavemeters are those in which portability and direct reading are two important qualities; simplicity in reading and easy deduction of the quantity being measured being important, it usually results that the apparatus becomes more complicated and less accurate than the standard type.

Dealing first with the standard wavemeter; this may have both capacity and inductance independently or connectedly variable, or one only may be variable. It is most usual to provide for varying capacity and to use a set of fixed inductance coils specially constructed for the purpose.

Two or three variable air condensers may be conveniently used, as desired, to cover a long range of capacity. These in conjunction with ten or twelve inductance coils will cover the whole range required in radio measurements, *i.e.* from 50 to 25,000 metres.

The important points of construction of the condensers and inductances are dealt with in §§ (22) and (23). It will be sufficient here to mention that the condenser should be shielded and that the shield should be connected either to one of the condenser terminals or to earth.

An essential part of the wavemeter is the device for indicating resonance. In the standard instrument this device should also be capable of giving reproducible readings from which the ratio of currents in the wavemeter may be accurately determined, if desired.

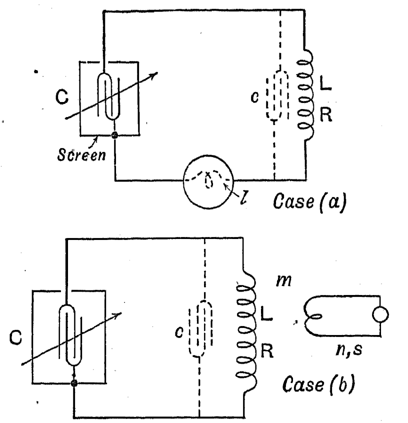
There are two indicating systems which are commonly used and which are suitable for a standard wavemeter. In one a hot-wire current-indicating device of low and constant resistance and inductance is inserted directly in series in the oscillatory circuit. This may very conveniently be a separate heater and thermojunction (see Part III. § (9)) and galvanometer or a self-contained unit such as a Duddell Thermoammeter or similar pointer-indicating instrument. The galvanometer and heater thermojunction system is probably slightly more accurate and considerably more sensitive, but has not the portability and freedom from disturbance of the thermoammeter. Another arrangement consists in providing a single turn or loop connected directly to the current-indicating device and coupled loosely to the wavemeter coil. In this case, so long as certain conditions as to coupling are satisfied, the electrical constants of the loop circuit are unimportant.

This arrangement has the advantage that the wavemeter is left as simple as possible and with the least extension of parts. The wave-length calibration and the damping decrement are almost entirely independent of the resistance and inductance of the measuring circuit. These qualities are, however, gained at the expense of sensitiveness.

The separate loop system may have currents induced in it from other sources than the resonant current in the wavemeter, for instance from any oscillating source of current in the vicinity. Such currents cannot always

be perceived by merely detuning the wavemeter owing to the current-indicating instrument following a square law. An extraneous current of 5 per cent of the wavemeter maximum current would, when acting alone, be scarcely perceptible on the ammeter. Such an amount would cause considerable errors in measurements of effective resistance.

The two cases are shown diagrammatically in *Figs.* 10 and 11.



FIGS. 10 and 11.—Resonance in Wavemeter Circuit.

If C=capacity of condenser,
c=self-capacity of coil,
and $C_0 = C + c$.
Also L=self-inductance of wavemeter coil,
l=self-inductance of current indicator (Case *a*),
and L_0=effective inductance of wavemeter coil.
R=total effective resistance of wavemeter circuit in ohms.
In addition, for Case (*b*)—
Let n=total effective self-inductance of loop circuit,
m=mutual inductance between coil and loop,
s=total effective resistance of loop circuit in ohms.
Then for Case (*a*)—

Free natural resonance is given by

$$(L+l)C_0\omega_1^2 = 1 - \frac{R^2C_0}{4(L+l)}. \qquad (4)$$

Forced resonant frequency under the excitation of sine wave E.M.F. of the form $e = E_0 \sin \omega t$ is given by

$$(L+l)C_0\omega^2 = 1. \qquad (5)$$

Case (*b*)—Assuming coupling loose, *i.e.* m^2 small compared to Ln.

Free natural resonance is given by

$$L_0C_0\omega^2 = 1 - \frac{R_1^2C_0}{4L_0}, \qquad (6)$$

where $$L_0 = L - n \cdot \frac{m^2\omega^2}{S^2+n^2\omega^2} \qquad (7)$$

and $$R_1 = R + S\frac{m^2\omega^2}{S^2+n^2\omega^2}. \qquad (8)$$

Forced resonant frequency is given by

$$L_0C_0\omega^2 = 1 + \frac{n}{L_0} \cdot \frac{m^2\omega^2}{S^2+n^2\omega^2} \qquad (9)$$

with the high-class condensers and self-inductance coils which would be used for a standard wavemeter the correction terms on frequency due to resistance or the additional circuit will be entirely negligible.

If system (*b*) is used then the quantity representing R_1 in equation (8) must be used when making measurements of effective resistance.

When using a standard wavemeter for resistance measurements, a set of specially constructed resistances for insertion into the circuit is very valuable. Suitable forms of resistance are described in the section dealing with resistance.

The setting up and disposition of a standard wavemeter is of considerable importance. The arrangement shown in *Fig.* 12 has been found satisfactory. The screen-connected terminal of the condenser is connected to one terminal of the thermoammeter or other thermal-measuring instrument A. The other terminal of the thermal instrument is con-

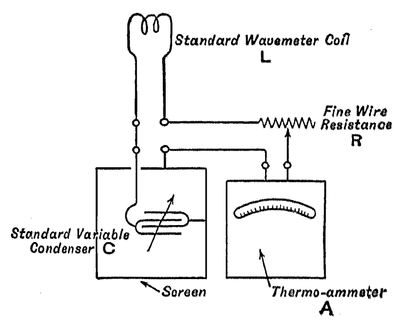


FIG. 12.—Set-up of Standard Wavemeter.

nected to one end of a fine-wire adjustable resistance R, when effective resistance measurements are being made; for simple wavemetering the resistance R is omitted altogether. The standard wavemeter coils are mounted on long ebonite strips so that they may be removed about 20 cm. away from the condenser to avoid eddy currents, which would alter the effective resistance and inductance of the coils. The thermal instrument is mounted on paraffin blocks about 10 cm. from the surface of the table. A long lever attached to the condenser turning head enables fine adjustments to be made and removes the operator's hand to a considerable distance from the wavemeter.

For the most accurate observations, a separate heater and thermojunction is used on a reflecting galvanometer with scale at a distance. After making any adjustments to

the wavemeter, the observer walks to the scale and thus removes himself from the vicinity of the wavemeter. This procedure is, of course, very tedious and slow. For a more permanent and elaborate equipment distant control could be employed to advantage.

§ (8) METHODS OF MAKING OBSERVATIONS WITH WAVEMETERS.—There are various ways of determining the setting of the wavemeter condenser corresponding to resonance when measuring wave-length.

(i.) When operating with steady undamped oscillations, the point of resonance is usually so well defined that the setting is more accurate than can be located on the condenser scale and nothing further is needed.

(ii.) Frequently, however, the wavemeter is used, in conjunction with a source which consists of trains of damped waves, such as the various spark sources, also when calibrating buzzer-excited wavemeters. In these cases the resonance will not be so sharp owing to damping in the source circuit. It is convenient and more accurate to take two readings on the condenser. One is taken with the capacity somewhat smaller than the resonance reading, so that the thermal ammeter reads, say, 70 per cent of the maximum. This value of capacity (C_1) is noted. The condenser is then quickly moved through the resonant capacity to such a value (C_2) on the side higher than for resonance as to give the same reading on the ammeter. Then $C_{res.}=(C_1+C_2)/2$ to a very close approximation. The increased accuracy is due to the steepness of the resonance curve for changes of capacity at the region where $I=0{\cdot}7\ I_{res.}$. If the damping in the source or wavemeter is very large the resonance curve is not symmetrical on each side of the maximum point.

(a) *Equations to Resonance Curve.*—The equations to the resonance curve, when the capacity of the wavemeter condenser is varied and the induced electromotive force is a sine wave of the form $\omega=E_0 \sin \omega t$, are as follows:

If C is the condenser reading and c is $C-C_{res.}$, where $C_{res.}$ = condenser reading corresponding to resonance and $I_{res.}$ the corresponding current,

$$I^2=I_{res.}^2\left(\frac{1}{1+(L^2c^2\omega^2/R^2C^2)}\right). \quad (10)$$

For the case where the source has small damping decrement giving an induced electromotive force of the form $e=E_0 . \epsilon^{-\alpha t} \sin \omega t$, it is more convenient to give the expressions for ratio of I to $I_{max.}$ in terms of decrements of source and wavemeter.

We then have

$$\frac{I}{I_{res.}}=\frac{1}{1+(4\pi^2/(\delta_1+\delta_2)^2)\{1-(\omega_2/\omega_1)\}^2}, \quad (11)$$

where $\delta_1=(\pi R_p/L_p\omega_1)$ = damping decrement per complete period of source,

$\delta_2=(\pi R_2/L_2\omega_1)$ = damping decrement of wavemeter,

ω_1 corresponds to the setting of wavemeter for $I_{max.}$,

ω_2 corresponds to the setting of wavemeter for I.

For the case of a wavemeter with separate loop to measure current the above formulas will be approximately correct if the values of R′, L′, and δ_2', corresponding to the equivalent simple circuit replacing the double circuit wavemeter, are used.

The values of R′, L′, and δ_2' can be obtained from equations (7) and (8).

An examination of equation (10) shows that the curve connecting I and condenser displacement from resonance is approximately parabolic: it is not, however, quite so. If the damping of either source or wavemeter entails changing the condenser more than, say, 5 per cent of its value at resonance in order to reduce I to $0{\cdot}7\ I_{res.}$, the true value of $C_{res.}$ will not be exactly $(C_1+C_2)/2$, but will be about 1 in 1000 lower.

For wavemeters having scales which read directly in wave-length, equations (10) and (11) are more conveniently represented by the expression

$$\frac{I^2}{I_{res.}^2}=\frac{1}{1+(\pi^2/\delta_0^2)\{1-(\lambda_1/\Lambda)^2\}^2}, \quad (12)$$

where $\delta_0=\pi R/L\omega_1$ = log dec. per complete undamped period at resonance,

λ_1 = invariable wave-length of impressed sinusoidal electromotive force,

Λ = variable undamped wave-length corresponding to any setting of the wavemeter.

If the sine wave source is varied in frequency and the wavemeter circuit fixed in inductance, capacity, and resistance, the equation of the resonance curve becomes

$$\frac{I^2}{I_{res.}^2}=\frac{1}{1+\{(\pi^2/\delta_0^2)(\lambda_1^2/\Lambda^2)\}\{(\Lambda^2/\lambda_1^2)-1)\}^2}, \quad (13)$$

where Λ is now fixed and λ_1 variable.

For the case corresponding to equation (11), but with λ in place of ω, we have

$$\frac{I}{I_{res.}}=\frac{1}{1+\{4\pi^2/(\delta_1+\delta_2)^2\}\{1-(\lambda_r/\lambda)^2\}}, \quad (14)$$

where λ_r is the wavemeter reading at resonance, and λ is any other reading.

These equations are rearranged in a form suitable for determining δ_1 and δ_2 in the section on resistance measurement where measurement of decrements is discussed.

(iii.) (a). A null method of setting a wavemeter to resonance with a source for the purpose of measuring frequency has been given as a development of the "dynamometer effect" first shown by Mandelstam and Paplexi (26). At resonance the current in the wavemeter is in phase with the voltage induced into the wavemeter circuit. If this voltage is induced by mutual inductance coupling between source and wavemeter,

the voltage will be in quadrature with the primary current. If, therefore, currents from source and wavemeter pass through fixed and suspended coils of a dynamometer, no deflection will be produced at resonance, but a large deflection will result either to right or left according to the direction of detuning. The shape of the deflection curve will depend on the constants of the wavemeter and on the decrement in the primary circuit. It will be somewhat as in *Fig.* 13, where $(\lambda - \lambda_r)/\lambda$ is

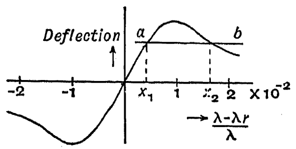


FIG. 13.—Dynamometer Effect (Mandelstam and Paplexi).

plotted as abscissa and deflection as ordinate. The curve is very steep through the zero portion, giving great sensitiveness in the location of the resonance point.

The source and wavemeter are not directly coupled through the dynamometer, but each is coupled to one coil of the dynamometer by a small mutual inductance as in *Fig.* 14, in

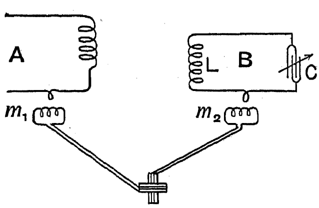


FIG. 14.—Resonance by Null Method using Dynamometer.

which the circuit A represents the source and circuit B the wavemeter, each acting through small mutuals m_1 and m_2 on to the dynamometer.

(*b*) The system can also be used to measure decrements. To carry this out the condenser C is adjusted to resonance and its value noted $=C_r$. Settings are then made on each side of C_r until the deflection is a maximum; calling these C_1 and C_2 we have

$$\delta_1 + \delta_2 = \frac{\pi}{2} \cdot \frac{C_1 - C_2}{C_r}.$$

For greater accuracy the curve shown in *Fig.* 13 should be obtained and a horizontal line *ab* drawn at a part where the curve is fairly steep, cutting it at points x_1, x_2 where $x = (\lambda - \lambda_r)/\lambda$. The sum of the decrements of primary and wavemeter is given by

$$\delta_1 + \delta_2 = 2\pi\sqrt{x_1 x_2}$$

in terms of condenser readings, this becomes to a close approximation

$$\delta_1 + \delta_2 = \pi\sqrt{\left(\frac{C_1 - C_r}{C_1}\right)\left(\frac{C_2 - C_r}{C_2}\right)}.$$

(iv.) Another null method of comparing two wavemeters or oscillatory circuits has been devised by Armagnat (27). The two wavemeters have oscillations excited in them from another source, either buzzer or valve. The two wavemeters act inductively upon another tuned detecting circuit containing detector and telephone. If the wavemeters have equal decrement, adjustments can be made to perfect silence in the telephone. Under these conditions with steady maintained oscillations, sensitivity to 1 in 10,000 in the comparison can be obtained. In this case a tikker is required in the detector circuit.

§ (9) TYPICAL WAVEMETERS. — Much ingenuity has been shown in the development of wavemeters, from the parallel-wire systems to the elaborate modern wavemeters capable of performing a variety of measurements.

The simplest type of wavemeter is the Lecher parallel wires with vacuum tube or bolometer resonance indicating device. For such a system the wave-length is given directly as

$$\lambda = 2a + 2b + c,$$

where a = length from node to node,
b = distance apart of wires,
c = small capacity of vacuum tube (2 or 3 cm.).

If these quantities are all in cm. λ will be in cm. Various additions have been made to the system with a view to shortening its length. Drude (28) added a variable condenser at one closed end, thus concentrating capacity at one part of it and turning the system virtually into a closed system. If the capacity is large the wave-length is given by the ordinary formula $\lambda = 2\pi\sqrt{CL}$, where C includes the equivalent capacity of the wires and L is the total self-inductance of the loop.

More accurately

$$\lambda = 2\pi\sqrt{CL} + \frac{\pi}{3}\frac{a^2}{\sqrt{CL}},$$

where a = half-length of the loop. C and L are in cm.

An improved wavemeter of this type was evolved by Nesper (29), and is indicated diagrammatically in *Fig.* 15. The parallel wires (3 mm. diam.) are connected at one end through a condenser C and loop n in series. This loop is loosely coupled to the current-measuring circuit consisting of loop m, self-inductance l, and thermo-element t. The object of l is to render the current sensitiveness independent of frequency of the main circuit. The short-circuiting bridge is specially con-

structed with good rubbing contacts and can indicate on a scale the self-inductance of the loop. The total length of the wires is 2·5 metres.

The condenser C may be fixed or variable

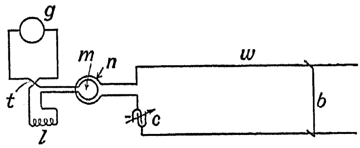


FIG. 15.—Nesper Wavemeter for Short Waves.

to suit different conditions. For short wave-length measurement, small fixed condensers are used of 100 cm. and 1000 cm. capacity for two ranges, 7 to 44 metres and 24 to 140 metres respectively.

§ (10) CLOSED CIRCUIT WAVEMETERS.—One of the earlier types of wavemeter possessing concentrated inductance and concentrated capacity is the Dönitz Wavemeter (30). This wavemeter has some interesting features, and in principle embodies all the properties of the more modern simple wavemeters. The chief point of interest is the method of observing the resonance. This consists of a small current transformer having its secondary winding connected to a fine wire enclosed in the bulb of an air thermometer, as in *Fig.* 16.

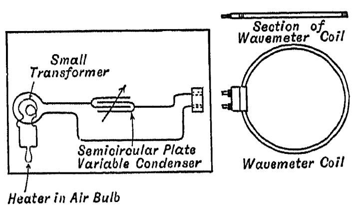


FIG. 16.—Dönitz Wavemeter.

The primary is a small loop in the main circuit. The coupling between the primary and secondary can be varied. The system allows any other thermal instrument such as a hot-wire milliammeter to be inserted in the secondary instead of the hot-wire air thermometer.

The condenser is similar to the modern variable air condenser of the semicircular fixed and moving plate type, and the inductances are five in number. The ratio between the inductance of successive coils is 4 to 1. The condenser is arranged to give a ratio of capacity of 1 to 4 for readings of 20 and 160 on its scale.

The inductances are wound with no supporting frames, but have their windings tied together with silk at intervals; they have a two-pin plug which attaches and connects them on to the terminals of the condenser through the small loop inducing into the air thermometer.

The range with air as dielectric in the condenser is from 100 m. to 3000 m.

A diagrammatic view of the wavemeter is given in *Fig.* 17.

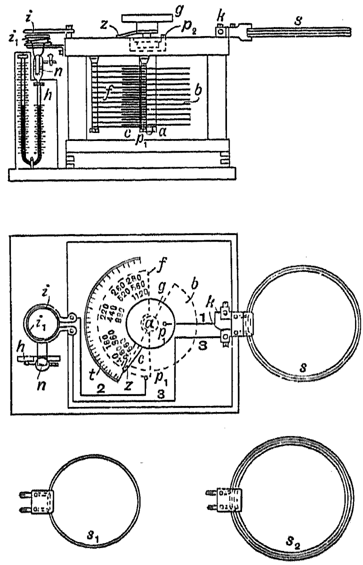


FIG. 17.—Dönitz Wavemeter.

§ (11) FLEMING CYMOMETER (31).—This wavemeter described by Professor J. A. Fleming in 1904 belongs to a class in which both self-inductance and capacity are varied simultaneously by the sliding of one lever. The virtue of such a system is that an approxi-

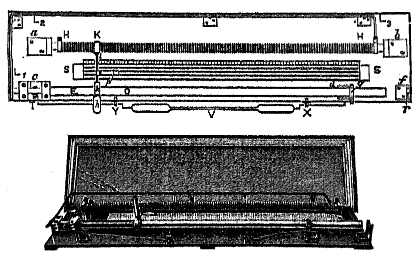


FIG. 18.—Fleming Cymometer.

mately uniform wave-length scale is obtained, and hence a longer range over which useful readings may be taken. A modern form of the instrument is shown in *Fig.* 18. The diagram, *Fig.* 19, illustrates the working principle. F is a helix of thick bare copper wire wound on a screw-cut groove on an ebonite tube; the condenser is of tubular form consisting of one or more inner tubes

covered with a thin sheathing of ebonite; the outer member of the condenser consists of brass tube or tubes which can be slid smoothly over the ebonite by means of a handle H, or slow-motion screw, as in the figure.

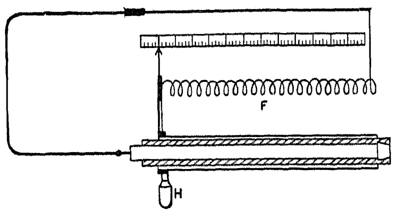


FIG. 19.—Diagram of Connections (Fleming Cymometer).

A sliding contact on the helix is rigidly connected mechanically and electrically to the outer brass tube or tubes. The circuit is completed by a loop in which an indicating hot wire or other element is included (in the earlier forms a neon tube was connected across the points of maximum potential). Except near the end where the helix is short, the inductance and the capacity will each be proportional to the displacement of the handle, and hence the quantity $\sqrt{CL}$ will be approximately proportional to displacement. The instrument cannot be said to possess high accuracy, firstly because the partial short-circuiting of one or more turns of the helix by the slider causes variability in the inductance and effective resistance; secondly, the use of ebonite as dielectric in a condenser is very undesirable owing to large dielectric hysteresis and change with exposure. Arrangements do not appear to have been made to provide various ranges, so that the upper part of the scale where the best accuracy would be obtainable corresponds only to any particular chosen range of frequency covering a ratio of about 2 to 1.

§ (12) TELEFUNKEN WAVEMETER (32).—This is a modern instrument of very good design capable of performing a number of measurements.

As will be seen from the photograph, *Fig.* 20, the variable condenser (right hand) has a scale marked in degrees for accurate measurements and three other scales marked directly in wave-lengths. A special index allows any of the three scales to be read accurately. The dielectric in one pattern of the wavemeter is paraffin oil. Three inductances are supplied to correspond with the scales, additional coils are also obtainable to extend the range in either direction. The coils are disc-shaped and have three contact sockets fitting a three-pin plug on the end of a leather-covered three-ply cable. The plug can be clamped in a universal holder on a solid base, so that the coil may be presented in any manner to the circuit being measured.

Three indicating devices are available: (i.) for accurate measurements—a very sensitive

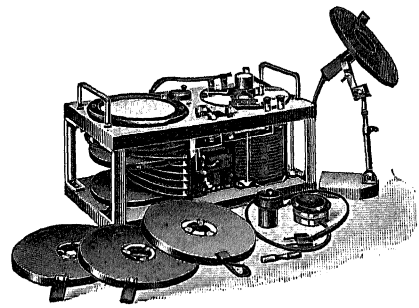


FIG. 20.—Telefunken Wavemeter.

hot-wire milliammeter of about 9 ohms resistance giving full-scale deflection for 40 milliamperes; (ii.) a small neon vacuum tube; (iii.) a crystal detector.

The circuits are given in *Fig.* 21, from which it will be seen that the hot-wire instrument

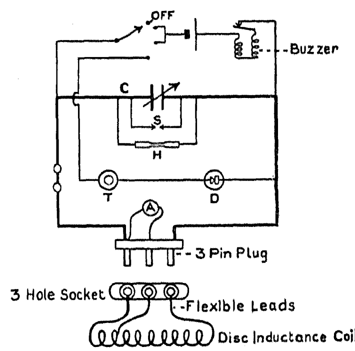


FIG. 21.—Diagram of Circuits (Telefunken Wavemeter).

is connected directly to a turn or two on the coil by a kind of auto-transformer coupling. The mutual inductance between the pair of leads going to the ammeter and the pair of leads going to the condenser of the wavemeter forms a considerable part of the mutual inductance coupling of the ammeter.

A buzzer is also provided for generating feebly damped trains of oscillations in the wavemeter by impact excitation.

§ (13) HAHNEMANN WAVEMETER (33).—This wavemeter is somewhat similar in design to the Telefunken instrument in that it is capable of measuring wave-length, decrement, capacities, inductances, and of acting as a receiver for both damped and undamped waves; a buzzer also permits of its use as a generator of weak, slightly damped oscillations

of known frequency. The inductances are short cylindrical coils specially designed to give approximately constant decrement over their range of frequencies.

The current-measuring instrument is a sensitive hot-wire milliammeter giving full-scale deflection for 100 milliamperes; it has a resistance of approximately 10 ohms. In order to render the calibration and decrement of the wavemeter nearly independent of the constants of the indicating instrument a condenser of suitable value (about 0·04 mfd.) is shunted across the milliammeter.

The proportion of capacity and inductance in the wavemeter is so arranged that the voltage across the condenser suits the helium tube (when this is used as indicator of resonance) at the same coupling as will suit the hot-wire milliammeter.

When used for the measurement of weak signals of undamped oscillations—*i.e.* measuring frequency of waves from a distant station—these are rendered evident to the ear by means of a telephone and interrupted contact driven by the buzzer, *i.e.* tikker.

A spark-gap with glass cover is also provided whereby, with the help of a suitable source of high voltage (induction coil or transformer), damped oscillations of more energy than are given by the buzzer may be generated; on this account the condenser has oil dielectric

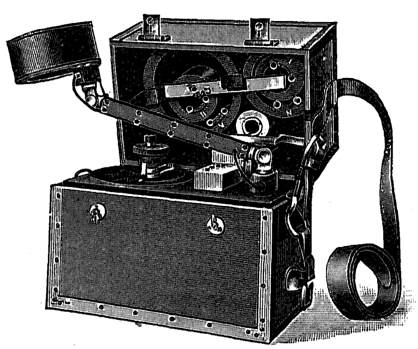


FIG. 22.—Hahnemann Wavemeter.

so as to withstand the high voltage thus impressed upon it.

A photograph of this wavemeter is given in *Fig.* 22.

Other types of wavemeter have both capacity and inductance continuously and simultaneously variable, thus giving a more extended wave-length scale and one approximately linear.[1]

§ (14) DIRECT-READING WAVEMETERS.—There are two types of wavemeter which are direct-reading. They cannot be said to possess the accuracy of the more usual type, but there seems room for development in such instruments if they could be brought to sufficient perfection of construction.

One such wavemeter, constructed by the firm of E. Huth (invented by R. Hirsch (34), Berlin), incorporates the ingenious idea of making the moving system of the variable condenser continuously rotable at a speed of several revolutions per second. On an arm, on the same spindle as the rotating plates, is carried a small capillary helium or neon tube which revolves over a scale calibrated in wave-lengths. As the condenser plates revolve resonance is obtained at a particular position.

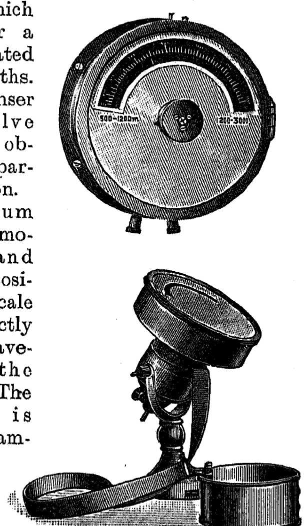


FIG. 23.—Huth Direct-reading Wavemeter.

The vacuum tube glows momentarily and indicates its position on the scale calibrated directly to read the wave-length of the resonance. The apparatus is shown diagrammatically in *Fig.* 23, in which the moving plate system is revolved by a small motor; current from a fixed inductance is led in through slip-rings.

With the sharpness of resonance obtainable from the modern valve-maintained oscillations the position of maximum brightness of the tube on the scale would be indicated with great exactness, and by use of a number of scales and corresponding coils a long range could be covered.

§ (15) FREQUENCY METER OF FERRIÉ (35).—This direct-reading wavemeter operates on the principle of the dependence upon frequency of the currents in two branch circuits, one of which is highly inductive whilst the other is nearly non-inductive.

Thus, in *Fig.* 24, if two currents I_A and I_B are flowing in branch circuits A and B, having inductances and resistances of L_A, R_A and L_B, R_B respectively, the ratio of $I_A{}^2$ to $I_B{}^2$ is

$$\frac{I_A^2}{I_B^2}=\frac{R_B^2+L_B^2\omega^2}{R_A^2+L_A^2\omega^2}.$$

If in circuit B ωL_B is large compared to R_B,

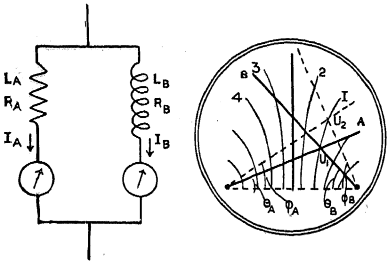
[1] J. de Beaupré, "Ondmetre système Péri à capacité et self-induction variables," *La Lum. Élec.*, 1910, xxxii. (2), 391.

whilst in circuit A R_A is large compared to ωL_A, the ratio of squares of currents becomes approximately

$$\frac{I_A^2}{I_B^2}=\frac{L_B^2\omega^2}{R_A^2}.$$

The application of this principle is carried out as follows: *Fig.* 25—A and B are the pointers of two independent hot-wire ammeters carrying the currents A, B respectively.

For a particular current and a particular frequency the deflections will be θ_A and θ_B respectively, and the pointers will intersect at



FIGS. 24 and 25.—Ferrié Indicating Wavemeter.

some definite point U_1. For larger currents through A and B other deflections, ϕ_A and ϕ_B, will be produced, giving a second point of intersection U_2. Hence for all currents at a particular constant frequency the locus of the points of intersection of A and B will be the curve 1; for other frequencies curves 2, 3, 4, etc., will be obtained.

In the actual instrument the dial consists of a whole sheaf of curves corresponding to the scale of wave-lengths for the range covered.

One of the serious drawbacks of such an instrument is that it cannot give true readings of frequency unless the source acting on the wavemeter is pure, *i.e.* free from harmonics. To secure this it is necessary to interpose a tuneable circuit in between the unknown circuit and the wavemeter. The necessity of adjusting this circuit to resonance partly nullifies the object of the whole device. With some of the modern valve generating systems, however, the wave is very pure. It is doubtful, however, if, in any case, trustworthy readings to an accuracy of 1 per cent could be obtained.

§ (16) HETERODYNE WAVEMETERS (36).—A number of self-generating wavemeters have been developed since the introduction of the small three-electrode valves which can be run from low-voltage anode sources of 6 to 20 volts in the form of dry cells. A typical wavemeter is shown diagrammatically in *Fig.* 26. The oscillatory circuit consists of a variable condenser C and either of a number of coils I., II., III., etc., mounted separate from one another in pairs. Switches A and G work together; switch A connects one coil of any pair to the variable condenser and anode of the valve, whilst B connects the other coil of the selected pair to the grid circuit of the valve. The coils in a pair are suited for the production of oscillations over the range given by condenser C. A telephone headgear set is inserted in series in the anode circuit of the valve.

To use the wavemeter, it is brought up within a few feet of the circuit to be measured. If the frequency of the oscillations is approximately known the wavemeter is set near the expected value; if nearly correct, this will be made evident by a beat tone in the telephone due to interference between the wavemeter and the unknown oscillations. Adjustment is now carefully made until the pitch of the note falls below audibility. Continuing the movement of the condenser in the same direction will result in the beat tone being again heard as a note of rising pitch.

If the frequency is fairly high (above 100,000 ∿ per sec.), the region of silence is sufficiently sharply located and gives the frequency directly.

For low frequencies (below 30,000 ∿ per sec.) there is a rather indefinite region between the

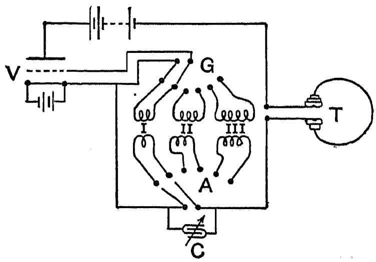


FIG. 26.—Heterodyne Wavemeter.

beat tones of ± 150 ∿ per sec., and if more accuracy is desired, readings on each side, corresponding to a beat tone of say 500 or 600 ∿ per sec., may be taken. The ear can judge easily the equality of pitch to an accuracy of 5 per cent, which represents less than 1 in 1000 of the high frequency. The mean of the two values will be very accurately the measure of the unknown frequency.

When the frequency of the source being tested is quite unknown, it is safest to set the wavemeter to its slowest frequency (longest wave-length) and gradually work through the scales until the beat tone is heard. The reason for this procedure is that the wavemeters belonging to this class usually possess numerous harmonics in the wave given by them. If the source also has harmonics the wavemeter will resonate to these if it is varied gradually from the upper end of its frequency range. A whole series of beat tones is fre-

quently heard in quick succession as various harmonics of source and wavemeter come within audible differences of each other. This is the most serious defect of the heterodyne wavemeter; but, generally speaking, the longest wave at which the wavemeter indicates resonance, if it also gives the loudest beat tone, represents the fundamental of the unknown frequency being determined.

The natural free period of a circuit can be determined by a heterodyne wavemeter by coupling them carefully together. Starting with the coupling rather tight and varying the condenser whilst carefully listening in the telephone, two clicks will be heard, separated by a considerable interval on the condenser scale. These occur as a result of the oscillations in the wavemeter suddenly changing in frequency when a certain difference in the free periods of the two circuits has been established. By loosening the coupling and repeating the observations the clicks will be heard closer together on the condenser scale. A point can be reached at which the two clicks are separated by an interval of 1 per cent on the scale. The mean of the two readings gives the free period of the circuit.

This method of determining the condition of resonance between two circuits has been published by L. W. Austin (37), who gives a number of applications of it to radio measurements.

§ (17) Calculation Devices.—To facilitate calculation of wave-length from inductance and capacity various graphical devices have been used.

One of the most convenient of these is an abac (38) by W. Eccles, in which a scale of inductance is set out around the lower half of an ellipse and capacity scales are set out around the upper half. A thread stretched across between any values of inductance and capacity, intersects a wave-length scale set out on the diameter at the corresponding value of wave-length.

Logarithmic charts have also been devised by Luckey (39) and Sörensen (40) depending on the relation

$$\log \lambda - k = \tfrac{1}{2} (\log L + \log C).$$

By plotting L as ordinate and C as abscissa on logarithmic paper, a family of sloping straight lines is obtained, each corresponding to a definite value of λ (41).

A very exhaustive treatise up to the time of its publication, dealing with wavemeters and measurements therewith, is E. Nesper's book, *Frequenzmesser und Dampfungmesser*.

III. Current Measurements

Current measurement at radio-frequencies presents difficulties peculiar to the nature of the currents themselves and to the smallness of the constants of the circuits in which the measurements are made.

The measurement of current is, moreover, of more importance in radio investigations than in other branches of electrical research, because direct methods of measuring resistance, power, dielectric losses, etc., which are available at commercial frequencies or with direct current, cannot be used, or have not yet been developed for use, at radio-frequency.

Currents of all values, from the very few microamperes received in an aerial up to a few hundred amperes transmitted from a large sending station, have to be measured. There are drawbacks to most of the methods, particularly at each end of the range. There is, however, an intermediate region comprising the range from a few milliamperes up to a few amperes, over which the uncertainties are least and the consequent accuracy the best. This intermediate range is the most important from the standpoint of measurements at present, because most of the wavemeters, decremeters, heterodyne sets, etc., operate with currents of this order.

The requirements which any current-measuring instrument or device should satisfy are—(*a*) the reactance should be negligible or as small as possible, (*b*) the calibration should not change with frequency, (*c*) the power consumed should be as small as possible consistent with accuracy and (in the case of portable instruments) robustness.

It is also desirable that the reading of the instrument should not be affected by the presence of external circuits carrying the same current or currents from the same source.

The limitations imposed by conditions (*a*) and (*b*) have guided the lines of development of current measuring devices almost entirely in the direction of thermally operated instruments.

There are, however, a few methods of measuring current at radio-frequency which do not depend upon thermal effects, more particularly applicable to small current measurements.

A variety of the thermal effects of the current have been made use of for operating the indicating instrument; amongst these may be mentioned:

The temperature of a wire or strip carrying the current, and hereafter called the "element," is measured by a thermojunction in contact therewith.

The element heats the fluid (gas or liquid) in contact with it, and this heated fluid heats one or series of thermojunctions.

The thermo-electric voltage at a junction between the element and a dissimilar metal.

The change of electrical resistance of the element is measured or produces a deflection in a Wheatstone bridge.

Change in length or tension or (usually) a

combination of both as a result of the temperature rise of the element.

Expansion or change in pressure of a gas in a closed space surrounding the element.

Of other methods, the following have been used:

Dynamometer by Mandelstam and Paplexi, both simple type and type in which the currents in the moving part are wholly induced.

Rectifying contact and direct-current galvanometer.

Electrometer.

In addition, current transformers can be very satisfactorily used as intermediary and to provide a constant ratio between the unknown current and the actual and much smaller current which is measured.

The various methods may be divided into three classes: small, medium, and large currents; they are described in this order below. There is, of course, considerable overlapping of the ranges of various devices, and the limits given in brackets are only intended to serve as a rough guide.

§ (18) SMALL CURRENTS (0-10 milliamperes). (i.) *Barretter, Bolometer.*—A simple form of barretter[1] consists of a single fine wire forming one arm of a Wheatstone bridge; this form was used by Paalzow and Rubens (1) in their experiments on stationary waves on Lecher wires. The principle is as in *Fig.* 27 in which *ab* is the fine wire. The Wheatstone bridge is first accurately balanced when no high-frequency current is flowing. The radio-frequency current is led in at points *a*, *b*, and thus by further heating the wire alters its resistance, producing a deflection on the galvanometer.

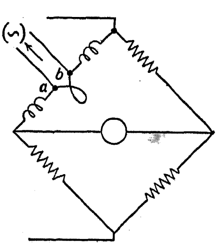


FIG. 27.—Paalzow and Rubens' Barretter.

(ii.).—A great improvement on this system (Rubens and Ritter (2)) consisted in duplicating the bolometer unit and constructing each of four equal arms arranged as a Wheatstone's bridge. The high-frequency terminals are connected to two angles of this unit and the main bridge circuit to the other two, which are always at the same potential. By this arrangement it is secured that no part of the H.F. current passes into the main bridge circuit. The scheme of connections is given in *Fig.* 28, in which A and B are the two bolometer units, R and S are the other two equal arms of the main Wheatstone bridge.

[1] W. Kempe, "The Barretter and its Uses as Indicator in Electrical Oscillating Systems," *Phys. Zeit.*, 1910, xi. 331.

By this arrangement the initial balance is independent of temperature or direct current through the bridge. By the bridge

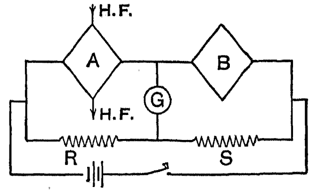


FIG 28.—Barretter Bridge (Rubens and Ritter).

arrangement of the unit itself the radio-frequency and direct current are each excluded from the circuit of the other.

(iii.).—The sensitivity is greatly increased by mounting the fine wires in a vacuum. Paalzow and Rubens used iron wires 0·07 mm. diameter. C. Tissot (3) used wires of platinum 0·01 mm. diameter in vacuum. With this arrangement it was found possible to measure currents down to a few microamperes, *e.g.* those in an aerial 50 km. from a sending station.

(iv.).—Béla Gati (4) used the modified form of bolometer shown in *Fig.* 29, in which WW are the fine wires in air or oil.

L, L are inductances to prevent the radio currents flowing round the rest of the circuit. B, B are single accumulator cells and R, R are resistances which can be inserted to depress the zero point when measurements on a larger range of currents are desired than can be comprised on a single scale of the galvanometer. Gold and platinum wires are used. With a platinum wire 0·5 μ diameter a deflection of ten divisions was obtained for a current of 34 μ amp. Béla Gati also used platinum wires in oil, and found that with a wire 2 μ diameter in oil using a direct current of 100 milliamperes, 4 or 5 times more sensitivity was obtained for the superposed high-frequency current than when the same wire was used in air with 20 milliamperes direct current.

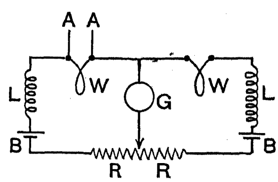


FIG. 29.—Bolometer Bridge (Bela Gati).

(v.) *Duddell Thermo-galvanometer* (5).—This instrument, designed by Duddell in 1904, consists of a Boys micro-radiometer in which the heat received by the thermojunction comes from a fine wire or grid mounted close below it instead of from a distant source. The instrument is shown diagrammatically in *Fig.* 30. T is a small torsion head, Q a fine quartz fibre, M the mirror, G a slender

glass stem, L a silver wire loop (wire about 0·1 mm. diam.), Sb and Bi are antimony and bismuth short rods connected below by a small copper bar. The loop swings freely between the poles of the permanent magnet NS. The heaters w are mounted in separate small holders so as to be interchangeable. The heater can be adjusted so that the thermojunction is from 0·5 to 1 mm. above it. Various types of heater may be used; for larger currents a low-resistance straight wire of gold and about 1 ohm resistance, a finer grid of 10-30 ohms resistance for intermediate small currents, and for the smallest currents platinised glass or quartz threads. The range of currents measurable is from about 1 milliampere down to 10 microamperes. For the latter measurement, however, the resistance of the heater would be some thousands of ohms. This value of resistance is not very suitable for most radio-measurements. The instrument, being very susceptible to temperature changes, is provided with heavy copper blocks surrounding the suspended system. A very slow drifting of zero usually occurs, which necessitates an allowance being made or a re-setting of the suspended system or scale from time to time during use. As a complete self-contained instrument it is probably the best yet devised from the point of view of sensitivity and freedom from inductance or capacity.

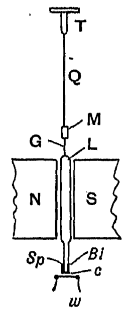


FIG. 30.—Duddell Thermo-galvanometer.

(vi.) *Electrometer.*—Small currents may be measured by means of an electrometer shunted across a condenser. Since the voltage across a condenser varies inversely as the capacity for a given frequency, current sensitivity would be gained by making the condenser small, but, since the electrometer has a by no means negligible capacity and one which necessarily varies with the reading, it is desirable to have considerable external capacity. In order to keep the circuit of low reactance the condenser should also be large. A sensitive electrometer can be used as a shunt to a moderately high resistance (about 1000 ohms); by this means it would be possible to measure currents of 1 milliampere. As a practical case, take $R=1000$, $C=50$ $\mu\mu F$ (capacity of electrometer), and $\omega=2\pi n=10^6$. Then

$$I \doteq \frac{E}{R}(1 + \tfrac{1}{2}R^2C^2\omega^2);$$

this is equal to $E/R(1+0{\cdot}001)$ in the above case, *i.e.* the term containing ω is only one-thousandth of the main term. The insertion of 1000 ohms into a circuit could, of course, only be done in certain special cases.

Using an electrometer in the anode circuit of a three-electrode valve, however, measurements have been made by E. O. Hulbert and G. Breit (6) on the received currents in an aerial.

The arrangement is shown in *Fig.* 31. The small voltage to be measured is applied to the grid filament circuit of a valve or amplifier set containing a number of valves. A high resistance R of the order of 50,000 ohms is in series with the anode circuit of the valve. When the high-frequency voltage acts on the valve there is an alteration in the *mean* anode current due to the curvature of the anode-current grid-voltage characteristic of the valve. It is the change of potential drop across the resistance R due to this change in mean anode current which is measured. The electrometer quadrants EE are connected across the ends of R; a potentiometer P and battery B

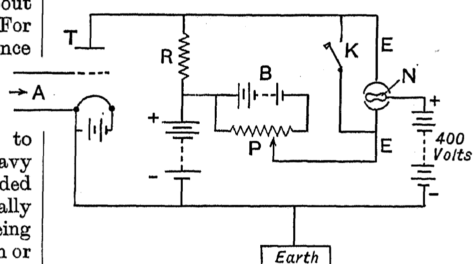


FIG. 31.—Electrometer Method of measuring Small Currents using Three-electrode Valve (Hulbert Breit).

are interposed in one electrometer lead so as to compensate the large potential drop along R in the quiescent state. The needle is charged to a high potential (400 volts), under which condition the deflection was found to be, for a particular Dolezalek electrometer, 2500 mm. on a scale at 4 metres distance when a voltage of 1 was applied to the quadrants.

K is a key for short-circuiting the electrometer quadrants when reading the zero.

Experiments made on a three-stage amplifier showed that the electrometer deflection was nearly proportional to V^2, V being the applied voltage. For this amplifier, applying sine wave voltage at A of wave-length 850 metres, the Deflection/V^2 was equal to 8000.

To calibrate such an apparatus, the scheme given in *Fig.* 32 was adopted. This arrangement would also appear to be suitable for other small voltage-measuring devices, and is

therefore described under separate heading immediately following.

(vi. *a*) *Calibration of Sensitive Current and Voltage Measuring Devices.*—The arrangement shown in *Fig.* 32 consists of a special oscillating circuit containing inductance L and condenser C_1 with thermo-galvanometer G in series. Shunting C_1 are two condensers C_2 and C_3 in series, C_2 being large and C_3 small, so that the value of I (the primary current) may be conveniently measurable when the voltage across C_2 is of the small order being measured.

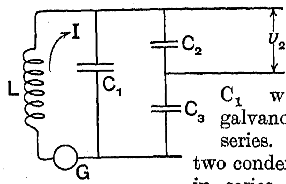


FIG. 32.—Calibrating Circuit for Small Current Measuring Devices.

If I is the total current in the circuit at point G, and V_2 is the voltage across C_2, we have

$$V_2 = \frac{C_3 I}{\omega(C_1C_2 + C_2C_3 + C_1C_3)}, \text{ where } \omega = 2\pi n;$$

a voltage of the same character as that being investigated is induced into L; strictly speaking, the voltage applied should have the same frequency and wave form as the unknown voltage, whether undamped, modulated, or spark oscillations.

(vii.) *Thermo-element.*—The original form (7) of thermo-element consisted of two crossed wires as in *Fig.* 33; one wire is, say, constantan, and the other, iron or steel. The high-frequency current passes through, from A to B, and the galvanometer is connected to *a*, *b*. The heat produced raises the temperature of the junction and so produces a thermo-electromotive force, giving a deflection on the galvanometer. There is some uncertainty of resistance at the junction where the current has to pass from one wire to the other. An improvement consists in mounting the wires as in *Fig.* 34. The thermojunction is quite independent of the heater wire, and is soldered to it at a point near the middle by the smallest possible ball of solder. This arrangement gives more stability to the heater and enables more choice of the thermojunction elements to be employed. By careful adjustment also, the device can be calibrated with direct current, taking care to obtain readings with the current in both directions. With the crossed junction of *Fig.* 33 it is almost essential to calibrate with alternating current because the direct-current drop of potential at the junction causes a large deflection of the galvanometer. For the smaller currents this deflection may be ten times the deflection due to the true thermal voltage generated at the junction.

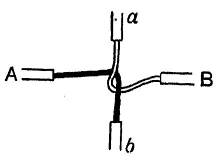


FIG. 33.—Crossed-wire Thermojunction.

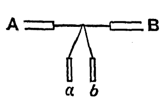


FIG. 34.—Straight-through Heater and Thermo-junction.

The sensitiveness of these directly connected thermo-elements may be very greatly increased by enclosing them in a high vacuum (0·01 to 0·001 mm. Hg) (P. Lebedew (8), J. A. Fleming (9), and H. Brandes (10)). A very sensitive combination is formed by a junction of thin platinum wire to the end of which is fused a small ball of tellurium (L. W. Austin (11), 1911). J. A. Fleming has used bismuth tellurium vacuum thermojunctions attached to constantan wire (9).

With a vacuum thermo-element of good design, currents of the order 0·5 milliampere may be measured, using a heater of 10 ohms resistance and a reflecting galvanometer.

With heaters and attached thermojunctions in air or vacuum, the deflection is almost exactly proportional to I^2. The calibration is independent of frequency; it is best carried out by means of an electrostatic voltmeter at a low frequency, using a high resistance to obtain the necessary voltage drop with the small currents employed.

The foregoing devices are semi-standard devices, *i.e.* those in which the absolute calibration at radio-frequencies can be obtained from calibration either with direct current or with low-frequency alternating current whose value is accurately known.

There are a few devices for measuring small radio-frequency currents which, while valuable for obtaining relative values for the currents, are not so suitable for measuring the actual values in milliamperes.

(viii.) *Repelled Disc Dynamometer.*—This instrument, devised by J. A. Fleming in 1887, has been successfully used in a sensitive form for small currents at radio-frequency in 1899 by Fessenden and G. W. Pierce (12), who has also developed the theory of it. The elements of the instrument are shown in *Fig.* 35. C is a coil of a few turns wound near the open end of an ebonite tube E. The other end of the tube is fixed in a base B carrying the terminals connected to the ends of the coil.

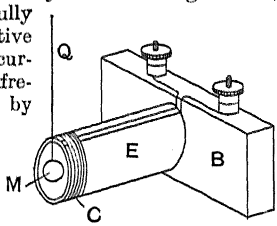


FIG. 35.—Repelled Disc Dynamometer.

Suspended by a quartz fibre Q is a thin silver disc M whose plane makes an angle of 45° with the plane of the coil. A small mirror is attached centrally on the front of the disc. When alternating current is sent through the coil, currents are induced in the disc having such phase relations to the primary current as to cause a torque to exist between disc and coil. The torque is a maximum when the disc is at 45° to the plane of the coil.

If the disc is considered as mathematically equivalent to a ring having effective resistance R, effective self-inductance L, and effective mutual inductance (to the primary) of M, we have for the 45° position, and when the centres of ring and coil coincide,

$$\text{Torque} = I^2N^2\left(\frac{\pi^2 r_1^{\,2}}{r_2}\right)^2 \frac{L\omega^2}{R^2+L^2\omega^2},$$

where r_1 is the radius of the ring, r_2 is the radius of the coil. At high frequencies R tends to a constant value, whilst $L^2\omega^2$ continually increases; the deflection, therefore, tends to become independent of frequency if this is high.

The deflection, if small, is proportional to I^2. The effective resistance and inductance of the instrument will both change with frequency. If only a few turns (say 5) are used of a few cm. diameter, both these quantities will, however, be small (of the order 0·1 ohm and 1 microhenry).

(ix.) *Crystal Rectifier.*—A device more sensitive than any of the foregoing is the crystal detector commonly used on wavemeters and in receiving sets before the advent of the rectifying and amplifying valve.

A very great deal of research has been expended upon the rectifying action at the contact between a metal and crystal or between one crystal and another (Refs. 13 to 21).

Properly mounted, certain combinations are fairly reliable over considerable periods of time.

The exact nature of the action is still somewhat obscure, but from the point of view of measuring devices crystal rectifiers may be considered as unilateral conductors; so that when an alternating voltage is applied to them the resulting mean direct current is a function of the "average" effective value of the alternating voltage. A typical characteristic curve is shown in *Fig.* 36 for a carborundum crystal against a metal. The conductivity is of the order 100 times as great for current in one direction as for that in the opposite direction for voltages below 10 volts.

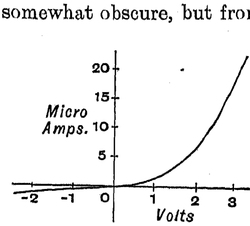


FIG. 36.—Characteristic of Carborundum Rectifier.

The kind of curve obtained, connecting alternating voltage with rectified current, is shown in *Fig.* 37 for carborundum. Similar curves may be obtained for molybdenite against a metal. L. Austin (15), in experimenting with various contacts, found silicon steel to be very suitable for low voltages (below 0·2 volt) and to give rectified currents proportional to the square of the applied alternating voltage. In a particular case with silicon steel contact the law was approximately $I = 2000E^2$, if the resistance in series with the rectifier is negligible, where I = galvanometer current in micro-amperes, E = applied r.m.s. volts (alternating).

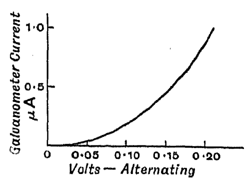


FIG. 37.—Alternating Voltage Characteristic of Carborundum Rectifier.

Amongst the methods of using a crystal rectifier for measuring purposes, two may be mentioned—(*a*) with a direct current galvanometer, (*b*) by the shunted telephone method.

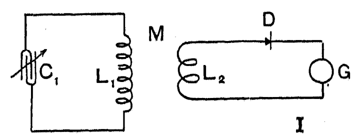


FIG. 38.—Separate Aperiodic Detector Circuit.

(*a*) *Figs.* 38 and 40 show two circuits which are valuable. In *Fig.* 38 the detector and galvanometer are in series in an aperiodic circuit loosely coupled to an exploring oscillatory circuit. This is one of the best methods to use, because the oscillatory circuit is left free and its damping is not much affected by the crystal circuit. The circuit shown in *Fig.* 39 is commonly used on some types of wavemeters, and is more sensitive than any of the other circuits. Generally, a very serious increase in the decrement of the measuring circuit results from this method of connection. The galvanometer or telephone is frequently shunted by a small fixed condenser of a few thousandths of a microfarad to increase the permittance for the pulsating radio-frequency current.

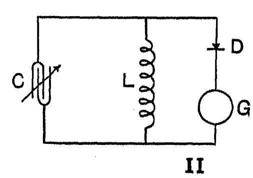


FIG. 39.—Shunt Detector Circuit.

Where it is desired to insert the detector in an oscillatory circuit, less disturbance to

the circuit results if it is connected as in *Fig.* 40, where C_1 is a condenser of many times the capacity of C. The sensitivity is, of course, not so great as in *Fig.* 39, but it is by no means reduced in the proportion of C to C_1 because of the great improvement in the resonance over that of *Fig.* 39. If very great trouble is taken in sorting the crystals, in the case of silicon or carborundum, a contact having an effective resistance of hundreds of thousands of ohms can be found. Such contacts can then quite satisfactorily be used on wavemeters, etc., in the circuit shown in *Fig.* 39.

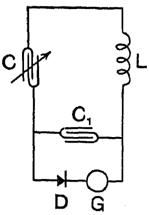


FIG. 40.—Series Detector Circuit.

(*b*) Another method[1] commonly used when comparing two currents with the aid of a crystal rectifier is that known as the shunted telephone method,[2] when the currents being investigated are damped, or undamped periodically interrupted trains of oscillations. The telephone cannot be used for steady maintained oscillations unless some interrupting device is inserted in series with it.

The principle of the method consists in reducing the intensity of sound heard in the telephone to the limit of audibility for each observation of the quantities being compared.

This may be done by shunting the telephone with a variable resistance and adjusting until the limiting point has been reached.

If Z_t is the effective impedance of the telephone for the frequency and wave form corresponding to the sound produced, and R_1 and R_2 are shunt resistances necessary to reduce the telephone current I_t to the value giving just inaudible sound, we have

$$\frac{I_1}{I_t}=\frac{R_1+Z_t}{R_1}.$$

R and Z are, of course, added vectorially and

$$\frac{I_1}{I_2}=\frac{R_2}{R_1}\cdot\frac{R_1+Z_t}{R_2+Z_t}.$$

A convenient arrangement given in *Bureau of Standards Circular*, No. 74, p. 167, is as shown in *Fig.* 41. It consists in using a fixed resistance R in series with the rectifier; the telephone is shunted across a part S of this resistance.

Another method of using the telephone and crystal rectifier is to alter the coupling between the detecting circuit and the circuit being observed, until the critical point is reached. This can be conveniently carried out by means of a low-range calibrated mutual inductance, in which case direct ratios of current may be obtained. When the low-frequency source is available (as in some measurements on magnification of valves), it is convenient to arrange a circuit carrying a quite independent current of the low frequency and of known value. By a change-over switch it is possible to compare directly the sound produced by the rectified radio-frequency current with the telephonic frequency current induced in the telephone, using a calibrated mutual inductance. From the known value of m and of the primary telephonic current, the alternating voltage in the telephone, corresponding to the rectified radio-frequency current, may be determined.

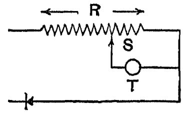


FIG. 41.—Shunted Telephone Method of comparing Currents.

The calibration of detectors when used for measuring purposes should be made immediately before and after use. This can best be done by the use of a sensitive vacuum thermojunction heater in the L_1C_1 circuit of *Fig.* 38, using as large a current as can conveniently be read on the galvanometer in the crystal rectifier circuit. From two or three such readings the law of the rectifier may be obtained and extrapolation downwards made use of.

The most sensitive devices of all for the measurement of minute radio-frequency currents are three electrode valve arrangements in the form of "amplifiers," but specially adapted to the purpose of measurement. This really consists in measuring the "amplification factor" of the complete assemblage of valve apparatus; the measurements must, of course, be made under conditions representing those which will afterwards be used. The amplification factor may be calculated by determining the various static and dynamic characteristics of the valves by methods given in Part VI. on valves, but it is desirable to carry out complete measurements by one of the methods indicated in the references—in particular, the methods given by L. W. Austin (23) on "Current Measurement with the Audion," *Jour. Wash. Acad. Sciences*, 1916, vi. 81, and *Proc. I. Radio E.*, 1916, iv. 251, also *Electrician*, 1917, lxxviii. 465, and *Proc. J. Radio E.*, 1917, v. 239, and by H. Abraham and E. Bloch, "Meas. of Small Alternating Currents," *C.R.*, 1919, clxix. 59. See also Ref. (22).

§ (19) INTERMEDIATE CURRENTS (MEASUREMENT OF)—(10 milliamperes to 3 amperes).—This range of currents lies within a region presenting comparatively little difficulty of

[1] É. Roux, *Jahr. der D. Tel.*, 1917–18, xii. 462.
[2] B. van der Pol, Jr., *Phil. Mag.* (6), 1917, xxxiv. 1841; and G. W. O. Howe, *Phil. Mag.* (6), 1918, xxxv. 131.

measurement, more power is available, and the element carrying the current can be single and is sufficiently small in cross-sectional area for its electrical constants to be invariable with frequency.

(i.) *Duddell Thermo-ammeter.*—One of the most convenient instruments of all for general use is the Duddell Thermo-ammeter; this is a portable instrument operating on exactly the same principle as the Thermo-galvanometer, but of course constructed on more robust lines. It consists, as shown in *Fig.* 42, of a light D'Arsonval moving coil

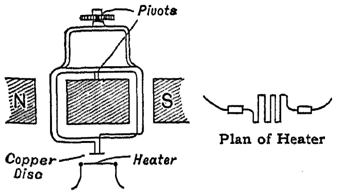


FIG. 42.—Duddell Thermo-ammeter.

system similar to that in an ordinary direct-current milliammeter, except that the ends of the coil terminate in a bismuth and antimony alloy thermojunction below in the axis. The rods of bismuth and antimony are soldered to a small copper disc about 3 or 4 mm. diameter.

Immediately below the disc is the heater, adjustably mounted; the normal distance separating heater from the disc is about 1 mm.

Two ranges of the instrument are standard; one range reads full scale for 100 milliamperes, and has a heater in the form of a small grid of specially high resistance wire, thus (about full size); the resistance is about 1·4 ohms.

Another range is one having a maximum reading of 10 milliamperes with a heater resistance of 150 ohms. The heater in this case consists of a platinised film on mica.

Intermediate ranges can, of course, also be made. An instrument reading full scale for 1 ampere has also been quite successfully constructed. In these instruments the movement of the pointer is rather sluggish, requiring about twenty seconds to reach within 0·1 division of its final reading when a steady current is suddenly switched on. With spark or arc generated oscillations this sluggishness is an advantage, as it smooths out sudden small irregularities. With the valve generators, however, such slowness is of no value. The instrument is almost free from zero creep and has a negligible temperature coefficient and negligible self-inductance (when the leads to the terminals are properly arranged) except for the case of circuits having exceptionally small self-inductance. The inductance is of the order 0·25 microhenry when the terminals are close together (desirable) and the internal leads are short and lightly twisted together.

(ii.) *Vacuum Heaters with Attached Thermo-junctions.*—These when used with portable or low-sensitivity galvanometers are very convenient, and more flexible than the Duddell instrument in that the sensitivity can be changed and the range also altered by choice of vacuum bulbs with various resistance heaters. A heater of 1 ohm resistance will, with 100 milliamperes, give a good deflection on a pivoted pointer galvanometer. With a robust reflecting instrument having good stability of zero the same equivalent deflection is obtained with 20 milliamperes.

A range of such vacuum heaters having resistances from 1 to 30 ohms is made by the Cambridge and Paul Instrument Co. They are also made by Messrs. Hans Boas in Germany. The thermojunction in those made by the C. and Paul Inst. Co. is specially constructed so that calibration may be accurately carried out with direct current. The deflections are nearly equal for reversed and direct current. *Fig.* 43 shows such a bulb. The two disadvantages attending these vacuum heaters are: (*a*) the whole galvanometer circuit is attached directly to the oscillatory circuit, so that parasitic currents may flow *via* the junction and the earth capacity of the galvanometer, thus giving fictitious deflections; (*b*) if the heater is burned out the whole bulb and thermojunction are rendered useless.

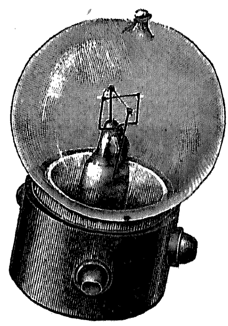


FIG. 43.—Vacuum Bulb Thermojunction Heater.

(iii.) *Separate Heater and Thermojunction—(a) in Air.*—In these the heater consists of a short straight or crinkled wire; above this is mounted a set of thermojunctions of iron-eureka or other suitable components. The "grid" of junctions (about 8) is so mounted that the ends are nicely in line and spaced about 0·6 to 0·8 mm. apart. The cold junctions are about 1 cm. away. The active junctions should have the smallest possible amount of solder on them, or, better, may be fused together. A convenient mounting for these is one as made by the Cambridge and Paul Instrument Co., so that the height of the tips of the junctions above the heater may be adjusted. The heaters are carried on separate interchangeable frames in such a manner

that they may be removed and replaced in exactly the same position.

These heaters and thermojunctions are high-class standard current-measuring devices for currents from 10 milliamps. to 1 ampere when used with a good-quality galvanometer. The heater, being separate, does not allow earth capacity currents *via* the galvanometer (it has negligible capacity to the thermojunctions). A burned-out heater is easily repaired; such combinations are not so sensitive, however, as the vacuum directly connected thermojunction heater. In some of these separate junction heaters also there is a certain amount of negative creeping effect on the galvanometer — probably due to the slow slight warming of the cold junctions by conduction or convection of air inside the enclosure.

(*b*) By submerging a separate heater and thermojunction unit in oil the current-carrying capacity can be considerably increased. By suitably disposing the thermojunctions also, the law of the device may be made such that the deflection is nearly proportional to the current instead of its square; this is, of course, a great advantage.

In some experiments on oil-immersed heaters made at the National Physical Laboratory (24) the results given in *Fig.* 44 were obtained with a heater of 0·25 ohm resistance.

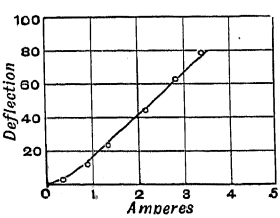


FIG. 44.—Calibration Curve—Oil-immersed Heater.

Such heaters produce a considerable magnetic field in the immediate neighbourhood of the wire, and can cause errors due to hysteresis and eddy currents heating the wires of the thermojunctions if these are magnetic. The use of manganin-constantan thermojunctions overcomes this trouble, or, alternatively, the heater may be doubled on itself to form a narrow loop. In this case, however, there is some danger of the effective resistance of the heater wire changing with frequency owing to the proximity effect of the two parallel wires.

With such oil-immersed heaters currents up to 5 amperes may be conveniently measured, using wires whose diameter is sufficiently small to make their change of resistance with frequency inappreciable.

W. Duddell (5) has described a twisted strip milliammeter which belongs to the class in which expansion of a heated element is made use of. The instrument is indicated in *Fig.* 45, in which ABCD is the twisted strip with mirror M and thin mica damping vane fixed at the centre. The strip is mounted in a frame and kept taut by a spiral spring S. Wires W, W of the same material as the strip are also stretched taut by the cross-piece E. Deflection due to change in temperature of the instrument as a whole is thereby eliminated. The strip carries the current to be measured. The period of the mirror is very short (about 1/10 sec.) so that the mirror will very quickly follow temperature changes in the wire due to the current. The temperature changes will, of course, not be so quick as the current changes, but with wires of the thinness described (0·001″ Pt. Ag.) the steady state would be reached very quickly with the strips in air. A deflection of 25 cm. at 1 metre was obtained on a particular instrument having a resistance of 20 ohms with a current of 22 milliamperes, corresponding to an energy consumption of 0·01 watt. The instrument is not described as suitable for radio-frequency currents, but there seems no reason to doubt that it would quite successfully measure such currents. The lightness and portability of such an instrument is of great value in many cases; also the comparative ease with which it can be constructed, if the highest accuracy is not desired.

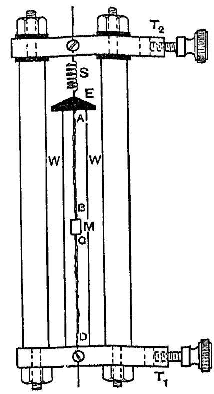


FIG. 45.—Duddell Twisted Strip Thermo-ammeter.

(iv.) *Hot-wire Ammeters.*—A large range of hot-wire ammeters similar in principle to those used for commercial frequencies are in common use, and as cheap portable instruments are fairly satisfactory.

They range from a sensitive instrument giving full scale deflection of 60 milliamperes with a resistance of about 8 ohms, up to single-wire instruments reading a few amperes.

An ingenious short-circuit contact switch is fitted to the delicate instrument of Messrs. Hartmann and Braun, to save burning out the wire in case the instrument receives too much current. It consists of fixed and spring platinum contacts arranged so that they can be closed by the pointer when it reaches a deflection somewhat beyond the full scale reading. A small piece of iron is attached to the pointer and when it reaches the contacts a minute permanent magnet pulls the pointer

still further and closes the short-circuit contacts with considerable pressure. To reset the pointer a small arm can be rotated from outside the instrument. It disengages the armature from the magnet and allows the pointer to return to zero.

If accuracy greater than 1 or 2 per cent is required from these hot-wire ammeters, they should be calibrated immediately after the high-frequency reading has been obtained by changing over to direct current and adjusting to the same deflection. The direct current is read by a standard instrument or potentiometer. By this means zero creep and deflectional creep are eliminated from the calibration.

§ (20) LARGE CURRENT MEASUREMENTS.—The measurement of currents above 10 amperes or so presents difficulties quite peculiar to radio frequencies, if any of the methods involving the heating effects of the actual current being measured are used. This is on account of the fact that a single element of small cross-section cannot be used to carry a large current. Resort must be had to some form of shunting; this again introduces possibilities of change in distribution of current in the various paths due to differences in the proportion of inductance to resistance in the various branches, and to circulating currents due to differences in mutual inductance between the various elements.

(i.) *Current Transformer.*—A method which overcomes these difficulties and is, perhaps, the most reliable means at present available for measuring large currents consists in the use of suitable current transformers (25). Various types of transformers may be used both with and without iron. Two such are shown in *Figs.* 46 and 47. In *Fig.* 46 is an

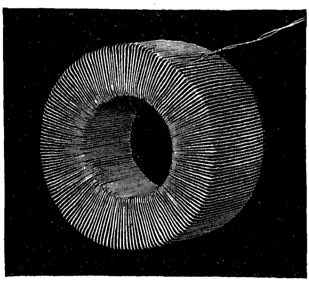


FIG. 46.—Air-core Current Transformer.

air-core transformer having 200 turns in the secondary winding accurately spaced round the circumference. With a single wire through the centre the ratio is very nearly equal to 200 : 1. By using two turns symmetrically disposed a ratio of 100 : 1 may be obtained. In *Fig.* 47 is shown an iron-cored transformer having 100 turns of stranded wire evenly wound on a bundle of thin stalloy rings with a mean diameter of about 5 cm.

The secondary may be connected to any radio-frequency measuring ammeter for small

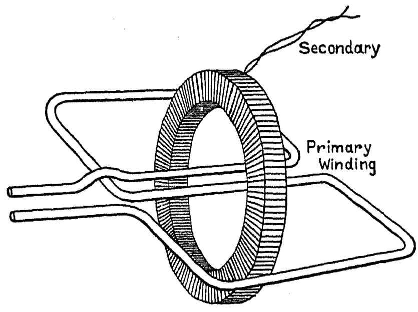


FIG. 47.—Iron-core Current Transformer.

currents, such as the Duddell thermo-ammeter, etc., mentioned above.

The advantages of such transformers are:

The ratio is very exact provided that certain factors are kept small (see formulas below).

In the case of the ring transformers no special calibration of the transformer is required; the ratio is that of the number of turns in secondary to those in the primary winding.

A number of different ranges may be obtained using one instrument by using various ratios of transformation.

The ratio is almost entirely independent of frequency.

The formulas for the current transformer are:

(a) For slightly damped waves,

$$\frac{I_1^2}{I_2^2}=\frac{L_2^2}{M^2}\left\{\frac{1-(R_2-L_2b)^2/L_2^2\omega^2}{1+b^2/\omega^2}\right\},$$

where I_1 and I_2 are root mean square values of primary and secondary currents respectively.

R_2 and L_2 are effective resistance and inductance of the secondary circuit, including the measuring instrument.

M is the mutual inductance between primary and secondary,

b = damping coefficient of the primary current waves,

$b = n\delta$, where n = frequency of oscillations, and

δ = log decrement per complete period.

For small values of $R_2/L_2\omega$ and $b/\omega=\delta/2\pi$ the above equation may be written

$$\frac{I_1}{I_2}=\frac{L_2}{M}\left(1+\frac{R_2(R_2-2L_2b)}{2L_2^2\omega^2}\right).$$

This equation will be true to 1 in 1000 if $R_2/L_2\omega$ and $\delta/2\pi$ are less than 0·05.

(b) For steady maintained oscillations $b=0$, and the equation simplifies to

$$\frac{I_1^2}{I_2^2}=\frac{L_2^2}{M^2}\left(1+\frac{R_2^2}{L_2^2\omega^2}\right).$$

To a close approximation therefore

$$\frac{I_1}{I_2}=\frac{L_2}{M}\left(1+\tfrac{1}{2}\cdot\frac{R_2^2}{L_2^2\omega_2}\right).$$

The correction term $\frac{1}{2}(R_2/L_2\omega)^2$ is small at ordinary radio frequencies (300 to 3000 metres), but at the very low frequencies coming into use more and more, this term cannot be neglected unless the transformer is made large.

For an air-core transformer in the form of a toroid of 8 or 10 sq. cm. cross-sectional area the value of L_2 for 200 turns will be about 300 microhenries. The effective value of R is rather difficult to obtain as it will not be quite the effective value which would be obtained if the secondary circuit were treated as a simple inductance and tested in one of the ways for measuring effective resistance of a coil at radio frequencies. The error, however, would not be large if it were so treated and would be on the high side. The average effective value of R in the case of the secondary considered above would be of the order 5 to 10 ohms, including the heater of the measuring circuit. Taking $R_2=10$, $L_2=300\ \mu$H, and $\omega=600{,}000$ ($\lambda=3000$), then

$$\frac{1}{2}\frac{R_2^2}{L_2^2\omega^2}=\frac{1}{628},$$

i.e. 1·5 in 1000.

With the iron-core transformers a compensating influence comes in, as ω becomes smaller owing to the increasing effective permeability of the iron giving a larger value to L_2.

In using such transformers it is desirable to try the effect of inserting, say, 10 ohms in series with the ammeter and observing what diminution of current results.

In the case of the toroidal form of secondary it should be revolved round to see that neighbouring conductors are not influencing the readings due to non-uniformity of winding or of variable effective permeability of the iron.

(ii.) *Hot-wire Cage Thermo-electric Ammeter* (26).—This shunt, as described by Fleming, has a number of parallel wires in a plane, and depends upon symmetry of parts and of construction for its invariability of calibration with frequency. In an improved form it (27) consists of a "cage" of parallel wires between two brass discs; the number of wires may be varied to suit the range of current being measured.

The current carried by each wire may be from 1 to 2 amperes as a maximum, since they should not be thicker than about 0·15 mm. diameter to preserve the necessary invariability of resistance with frequency. Such a shunt is shown in *Fig.* 48.

A thermojunction of iron-eureka wires about 0·05 mm. diameter is attached to one of the parallel wires near its middle point. The ends of the thermojunction are connected to a direct-current galvanometer in the usual way.

It is thus seen that the instrument really measures current in one wire only, and the

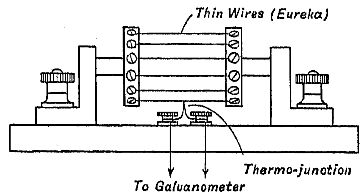


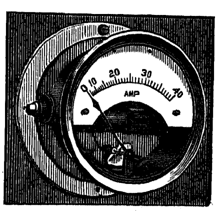
FIG. 48.—Fleming Parallel Wire Shunt.

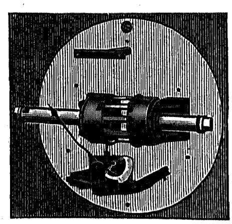
ratio of this current to the total current is assumed constant. The effect of a neighbouring conductor carrying the same current as the shunt is very serious, as it causes a large change in the distribution of current amongst the various wires. In an experiment on such a shunt the presence of a return conductor, first on the side near the wire carrying the thermojunction, and then on the far side, caused a change in reading of about 10 per cent on each side of the mean value.

Such effects might be eliminated or reduced by screening the wires with a copper tube. The writer is not aware whether such a method has been tried. Alternatively, a series of separate thermojunctions could be mounted closely above the middle of each wire on a light circular frame. All the junctions being then connected in series, a better resultant effect for the total current would be obtained.

The shunt is easily made and easily calibrated with reversed direct current.

(iii.).—A commercial ammeter of the parallel wire cylindrical cage type (28) is shown in *Figs.* 49 and 50; in this case, however,





FIGS. 49 and 50.—Commercial Heated Strip Ammeters.

the wires are replaced by thin strips symmetrically mounted to form parts of the same cylinder. The strips are only 3 cm. long, of platinum-iridium alloy. They are attached at their ends by short clamps and screws to two massive copper blocks with straight central thick stems coming out on either side of the instrument.

The heating unit is mounted by means of iron angle pieces on a marble base. The

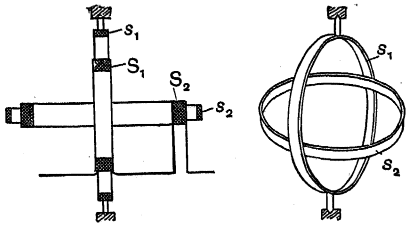
usual form of magnifying device converts the sag of one strip into angular reading of the pointer. The short length of the strips renders any temperature compensation unnecessary; the choice of the metal and the solid heat-conducting copper parts enables the high temperature (necessary to obtain the deflection) to be obtained.

The instrument dissipates a considerable amount of power (about 60 watts for full scale deflection of 80 amperes). It is stated to be accurate between 300 and 1500 metres.

The accuracy of such an instrument will increase as the wave-lengths become greater.

(iv.) *Ammeters of G. Keinath* (29).—Two types of ammeter suitable for large currents have been developed by G. Keinath at the works of Messrs. Siemens and Halske.

(*a*) *Dynamometer Type.*—This instrument is shown in *Figs.* 51 and 52. The moving element



FIGS. 51 and 52.—Keinath Dynamometer Ammeter.

consists of two coils in series forming a closed circuit. One coil has the axis of rotation as a diameter in its plane; this coil experiences the torque which turns the system. The other coil is mounted so that the axis passes through its centre at right angles to its plane.

The fixed current-carrying coils are in series and are similarly mounted with regard to the axis of the moving system, *i.e.* one coil on a meridian and the other equatorial.

The feature of the equatorial fixed and moving coils is that a comparatively large invariable mutual inductance is provided between fixed and moving systems. The induced short-circuit current is thus much less dependent on the position of the moving coil, so that, whereas in the simple dynamometer case the torque would vanish when the coils were at right angles to one another, in the compound circuit the torque can be operative through a theoretical 180° of angle. The scale of current is also much more uniform.

The torque will be proportional to $I_1I_2 \cos\theta \cdot dM/d\phi$, where I_1 = current being measured, and I_2 is the short-circuit secondary current. θ is the phase angle between primary and secondary currents, $dM/d\phi$ is the rate of change of mutual inductance between the two circuits at any deflection ϕ of the moving system. If R_2 is small compared to $L_2\omega$, then, approximately, $\cos\theta = 1$. Thus Torque $\propto I_1^2 \cdot (M/L) \cdot dM/d\phi$, which would give approximately a square law calibration if $dM/d\phi$ were constant.

An instrument reading up to 5 amperes is satisfactory, though rather highly inductive, since at wave-lengths of 300 metres the inductive drop may be of the order of 200 volts.

Keinath therefore developed shunted hot-wire ammeters on the principle of making the ratio R/L of the shunt equal to the ratio R_1/L_1 of the hot wire actually indicating the total current.

(*b*) *Hot-wire Type.*—The hot-wire system is similar to straight-through hot-wire ammeters. The compensation for temperature other than that due to current is as shown in *Fig.* 53. The hot wire carrying

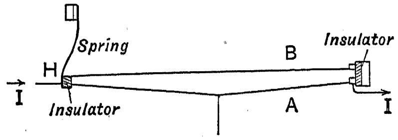


FIG. 53.—Principle of Hot-wire Ammeter.

the current is shown at A, B is an exactly similar wire but carries no current, S is a steel spring maintaining tension on B. If the general temperature rises, block H simply moves as a whole and no change of sag in A is produced. When current passes through A, sag is, of course, produced and operates the pointer.

On a moderate current instrument the shunt is an open spiral; an inductance consisting of a few turns of wire is connected in series with the hot wire and adjusted by stretching or compressing the turns until $R/L = R_1/L_1$, $R/L\omega$ is of the order 1/50 for a value of ω of 10^6. Considerable changes in the effective resistance of the shunt are therefore, relatively, unimportant. The change in inductance will already have taken place long before a value of $\omega = 10^6$ is reached, so that this quantity will be very nearly constant.

A test on an instrument reading to 20 amperes showed a change in calibration of +2·5 per cent when λ was changed from 2000 to 300 metres.

(v.).—For very large currents (500 amperes) the shunt is specially constructed as shown in *Fig.* 54. It consists of a large number of thin manganin strips hard-soldered to the blocks A, A_1, and to the massive block at E. They are separated from one another by mica strips between each. The whole electrical part is mounted on porcelain. A view of such an instrument is given in *Fig.* 55.

(vi.) *Precautions in using Large Current Radio-frequency Ammeters.*—The leads should run straight and axial for a considerable distance from each end of a straight-through

type of parallel wire or strip ammeter, so as to avoid non-uniform fields acting on the wires or strips. With strip instruments the

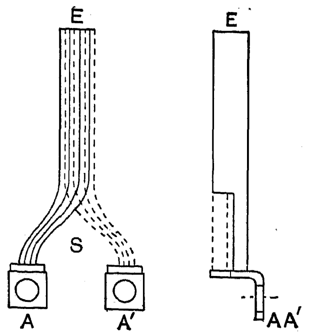


FIG. 54.—Shunt for Very Large Currents.

strip should occupy the same position when in use as when calibrated. The temperature of a strip is very appreciably altered when its plane is turned from horizontal to vertical.

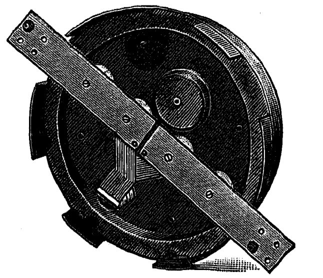


FIG. 55.—Ammeter with Special Shunt for Large Currents.

To overcome the troubles due to zero shift and creeping, it is necessary, if high accuracy is desired, to calibrate with direct current immediately after use, adjusting the current to give the same deflection as the radio current. A suitable subsidiary direct-current circuit and standard ammeter is, of course, necessary for this; a change-over switch can be arranged so that the pointer hardly moves from its position on throwing over.

Alternatively to this method, it is possible to work on a time basis, and to calibrate on the same time basis; this may be done as follows. Allow the radio current to flow for, say, half a minute without taking a reading. Switch off and after n seconds (say 30) set the zero. Switch on for the same number of seconds, *i.e.* about n seconds, and observe the reading. Switch off and observe the zero after the same n seconds. Repeat until a cyclic condition is arrived at. The direct-current calibration is made in the same way with the same value for n.

In instruments in which the terminals are close together, it is desirable to reverse the connections to these so as to eliminate disturbing effects from the external circuit, and to obtain an idea of the magnitude of this disturbance.

A very full discussion of the behaviour of high (radio) frequency ammeters is given by J. H. Dellinger (30), and a number of very valuable conclusions are stated in the work.

IV. Capacity and Condensers

§ (21).—A condenser is an electrical circuit so arranged that capacity is its main property. With a properly designed air condenser the ideal is very approximately reached, *i.e.* the condenser is a pure capacity.

Condensers with oil or mica as dielectric can, however, be used satisfactorily if suitable materials are chosen. The capacities should be measured over the range of frequencies for which it is desired to use the condensers.

Practically all the condensers which can be continuously varied consist of a system of fixed plates or vanes and a system of movable plates or vanes which can interleave the fixed system to any desired extent. Various other methods have been tried, such as winding off, from a drum, a flexible conductor and strip of insulator; varying the air gap between two cones by screwing one into and out of the other; but these systems have not survived, and the only type in practical use to-day is that having turning or sliding movable plates or tubes. There are a number of different forms of construction of both fixed and variable condensers according to the class of work for which they are intended. In general, a final design is the result of a compromise between various conflicting factors; the relative importance of these factors profoundly modifies the resulting form in the different cases.

These points will be made clear in the following descriptions of the various types of condenser for various measuring purposes.

Some details have already been touched upon in the section on wavemeters.[1]

§ (22) Standard Air Condensers.—These are required both in fixed and variable form for accurate measurements of every kind at radio frequency.

The qualities desirable in the fixed condenser are:

(*a*) Invariability of the value of the capacity with frequency, time, and temperature.

(*b*) Freedom from leakage and dielectric losses.

(*c*) Freedom from internal inductance and series resistance.

For a variable condenser the following additional features are desirable:

[1] See also "Capacity and its Measurement," § (32).

(*d*) The capacity should be the same for the angular setting of the plates, independently of the manner of arriving at the setting.

(*e*) The accuracy of reading should be as high as possible.

(*f*) The moving system should not move too easily, but should move with perfect smoothness; there should be no suspicion of jerkiness, even for the smallest movements, from any setting.

To satisfy these conditions great accuracy of workmanship is necessary, and careful choice and treatment of the materials.

Condensers of any type may be divided constructionally into three parts—(i.), (ii.) the two sets of plates, and (iii.) the insulating material.

Conditions (*a*) and (*b*) can more nearly approach the ideal in a fixed than in a variable condenser.

For invariability in the fixed condenser the plates should be annealed (1) and should be clamped together with accurately machined washers, or made up with some form of comb, with slots, in which the plates are securely fixed. The insulation should be not too long unless it has the same coefficient of expansion as the banks of plates.

The plates are better with a bright burnished finish, with well-rounded edges. In some condensers a fairly stout copper wire runs along the bank of plates in a groove and is soldered to each plate so as to ensure proper electrical contact.

The air gap should be as large as can be arranged without rendering the condenser too bulky. For fixed and variable condensers of not more than 0·001 μF capacity, the air gap between the two sets of plates may be as large as 4 or 5 mm. with advantage. With condensers of capacity of the order 0·005 to 0·01 μF, however, such an air gap would mean a very bulky condenser: an air gap of 1·5 to 2 mm. is more usual in these larger condensers. The plates must be very clean and free from dust, otherwise fine threads of fluff, etc., will be drawn across from plate to plate by the electrostatic field; losses will thereby be caused. The insulating material which, unfortunately, has to form a link in the mechanical location of one set of plates with regard to the other is the weakest part in the design of condensers.

In some forms of condenser it is common for the fixed bank of plates to be mounted on rods screwed into a thick ebonite top. In the centre of the top the bush is fixed, forming the bearing for the moving plates. This design is very unsound mechanically, because a considerable length of insulating material intervenes between the two sets of plates and this material is in a state of stress. Such condensers will not keep their calibration if subjected to any considerable rise of temperature. The choice of materials for the insulation is thus limited for the highest class of condenser.

(i.) *Bureau of Standards Pattern.*—In the standard condensers made at the Bureau of

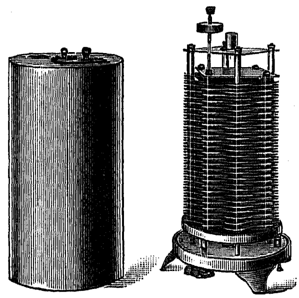


FIG. 56.—Standard Fixed Air Condenser (Bureau of Standards).

Standards (2) the insulation is quartz in the form of rods or pillars. A fixed and a variable condenser of this type are shown in *Figs.* 56

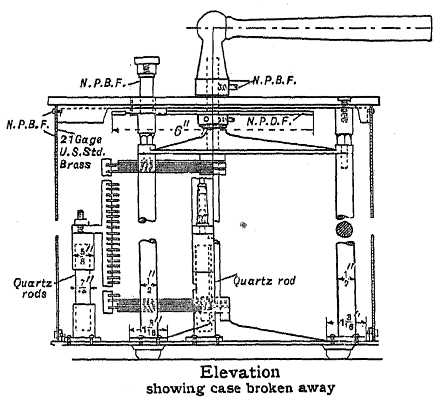


FIG. 57.—Standard Variable Air Condenser (Bureau of Standards).

and 57. The fixed condenser consists of two banks of square brass plates arranged as in *Fig.* 58.

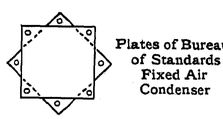
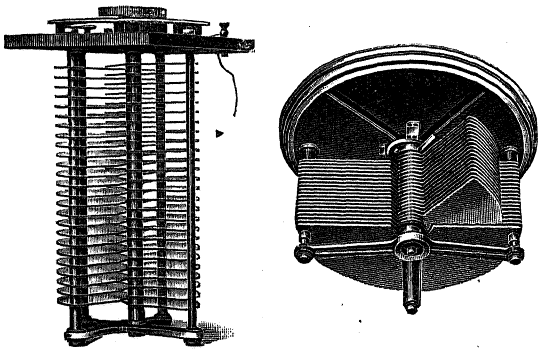


FIG. 58.—Arrangement of Plates—Standard Fixed Condenser.

One set is in direct connection with the base and metal shield. The insulated bank of plates is first fixed below on a rigid circular metal disc-shaped base, and this in turn is supported on three quartz pillars let into sockets on the main base.

The variable condenser has the moving plates in electrical connection with the main



FIGS. 59 and 60.—Standard Variable Air Condensers (N.P.L.).

frame and shield, thus obviating the necessity of insulating the bearings. The fixed bank of plates is held in position in combs which are in turn insulated with quartz rods in sockets attached to the base.

(ii.) *N.P.L. Pattern.* — In the variable air condensers in use at the National Physical Laboratory (3) the bank of fixed plates is insulated from the moving system and framework by amber bushes above and below on each of the three rods. Two such condensers are shown in *Figs.* 59 and 60. The plates are of brass in both condensers and are held together with distance washers by bolts passing through.

Figs. 61 and 62 show two fixed standard air condensers; the smaller of these, of 0·001 μF capacity, has circular aluminium plates interleaving one another. One set is fixed to a solid brass circular base and the other (insulated) set is supported on three amber rods about 3 cm. long resting in sockets. A brass shield encloses the whole in a dust-free manner. The condenser shown in *Fig.* 62 is of 0·01 μF capacity and has rectangular banks of plates arranged as in *Fig.* 63. The insulating washers are of quartz. The air gap, however, is only 1·5 mm. in this condenser.[1]

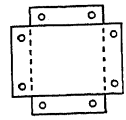


FIG. 63.—Arrangement of Plates in Large Fixed Condenser.

§ (23) TYPICAL CONDENSERS.—Of the many types of condensers used for wavemeters, decremeters, etc., a few are shown in *Fig.* 64. Condensers *a*, *b*, and *c* are of the usual type with semicircular fixed and moving plates, *d* is the Seibt[2] condenser in which the two sets of plates are carved out of the solid aluminium alloy by special machinery; the moving

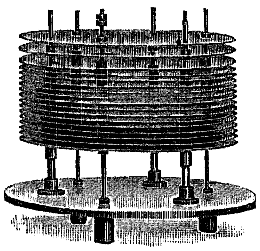


FIG. 61.—Standard Fixed Condenser, 0·001 μF (N.P.L.).

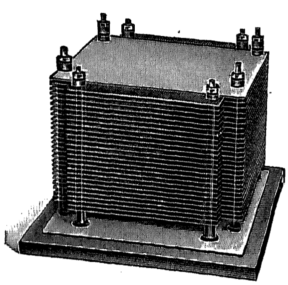


FIG. 62.—Standard Fixed Condenser, 0·01 μF.

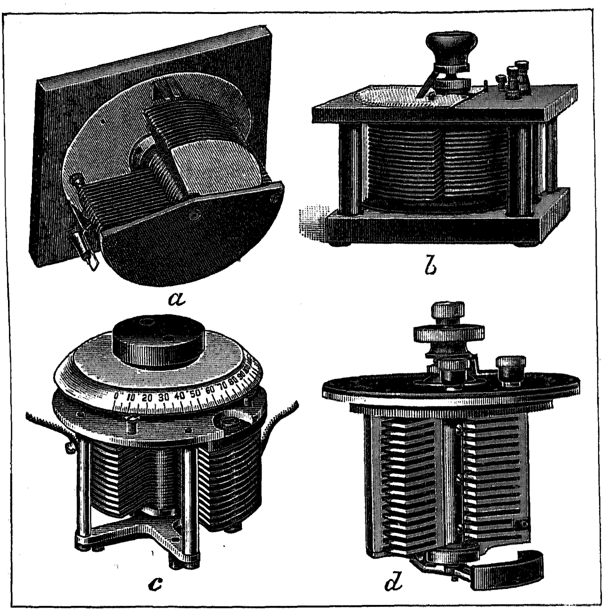


FIG. 64.—Group of Typical Condensers.

[1] See also H. Schering and R. Schmidt, "Standard Condenser," *Zeit. für Inst.*, 1912, xxxii. 253; and *E.T.Z.*, 1912, xxxiii. 1343.

[2] G. Seibt, "A Precision Turning Plate Condenser, etc.," *Jahrb. d. D. Tel.*, 1911, v. 407· and *E.T.Z.*, 1914, xxxv. 531.

system is mechanically balanced so that the condenser may be used at an angle. The air gap is extremely small in this condenser, resulting in a condenser of very small bulk for its capacity. Owing to the smallness of the air gap the calibration curve is frequently not a very straight line. The voltage limit of these condensers is, of course, low also.

§ (24) Special Condensers.—A group of special condensers is shown in *Fig.* 65. Condenser a is an interesting type, first suggested by Bethenod in 1909, and described by W. Duddell (4). The feature of this condenser is the shape of the moving plates. This is such that the capacity varies as the square of the angular movement. The result of this is that when used for wavemeters a wave-length scale is obtained which is approximately uniform. Kolster (5) has also described a somewhat similar type of condenser with specially shaped plates to read decrements directly; this instrument is described in the section on decrement and resistance measurements. b and c are condensers made by Lorentz (6) of Berlin. b is a mechanically balanced condenser, thus relieving the spindle of any side strain and enabling the condenser to be used tilted from the vertical if desired. Condenser c is one in which the spindle is horizontal; it really consists of two condensers, one giving a fine adjustment (at the left-hand side).

The shape of the moving plates for condenser of type a (*Fig.* 65) to give uniform wave-length scale must be such that the area common to both sets of plates (shown shaded in *Fig.* 66) must satisfy the equation area $= b\theta^2$, where $\theta =$ angle of the moving plates from the theoretical position, in which the capacity is zero. This zero cannot, of course, be realised in practice.

If the fixed set of plates is semicircular with a semicircular opening cut in them for the spindle, the equation connecting r and θ will be $r=\sqrt{4b\theta+r_1^2}$: r and θ are the polar co-ordinates of the boundary of the moving plate, and r_1 the radius of opening in fixed plates.

In the direct-reading decremeter type of condenser (5) the capacity must satisfy the equation $dC/C=bd\theta$, or $C=C_0\epsilon^{b\theta}$, where b is percentage change of capacity per division and is required to be constant for any setting of the condenser if a given angular displacement from resonance is to cause the same percentage change in current whatever the resonant position may be, damping decrement

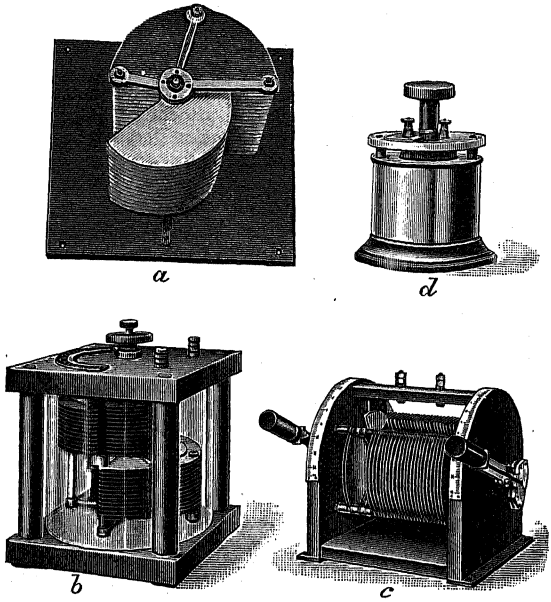


Fig. 65.—Group of Special Condensers.

being also constant. The diagram *Fig.* 67 shows the shape of vane for such a condenser.

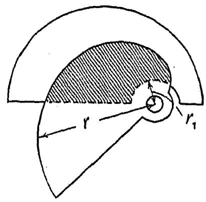


Fig. 66.—Shape of Vanes for Uniform Wave-length Scale Condenser.

The polar equation for the moving plates in this case is

$$r=\sqrt{2C_0b\epsilon^{b\theta}+r_1^2},$$

where r_1 has the same meaning as in the preceding equation.

Fig. 65 (d) is a very small condenser in a brass-shielded case.

The range of capacity is from 15 to 35 $\mu\mu$F, and the insulation is amber. The terminals are small, the plates have been machined out of the solid. Such a condenser is very useful for determining self-capacities of coils and for making small changes in the capacity of wavemeters when determining decrements by the change of capacity method.

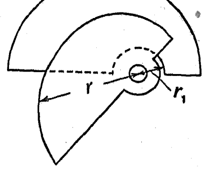


Fig. 67.—Shape of Vanes for Direct-reading Decremeter Condenser.

Some types of variable condensers have

a slipping worm-wheel device for making extremely small changes in the capacity when using difference tones in connection with various measurements. The device is of some value for this purpose, but it does not, of course, increase the accuracy of reading, nor can small changes, which may be made by it, be accurately measured. In attaching devices of this kind care must be taken that they put no mechanical restraint on the moving plate system.

Some air condensers have two sets of fixed and moving plates, with a view to utilising what would otherwise be waste space (Marconi Co.). The two banks of fixed plates are exactly similar and face each other in the same container. The two sets of moving plates are also mounted facing each other, and, of course, insulated from each other. If A and B are the two fixed banks of plates and C and D the two moving banks, then by connecting A and C and B and D a condenser will be obtained whose maximum value is approximately twice the value of that of an ordinary variable condenser whilst its minimum value is not much larger.

By connecting B to D and using A and C as terminals, a condenser having a range approximately one-fourth that of the previous case is obtained.

The satisfactory mounting of two sets of insulated moving plates, with every corresponding pair of plates in the same plane, presents difficulties.

Much information on various types of condensers is contained in a series of papers by E. Nesper.[1]

§ (25) Condensers with other Dielectrics than Air.—Various oils, ebonite, and mica have been used in variable condensers; the objects being, of course, to obtain larger capacities in a given space and to enable higher voltages to be used on the condensers.

Of the oils, paraffin is satisfactory if pure and thoroughly dried, the dielectric constant is about 2 and varies but slightly with frequency from telephonic frequencies upwards. On account of their higher dielectric constant, olive oil and castor oil have been tried. Experiments on castor oil made at the National Physical Laboratory have shown that whereas the power factor of such a condenser is very small at telephonic frequencies, it is several per cent at radio frequencies; this defect is very serious when the condenser is used in connection with measurements of effective resistance.

Thin ebonite discs have also been used, in condensers by the Marconi Co., between plates of very thin brass, with a view to crowding a great many plates into a small space and to greatly increasing the dielectric constant of the material between the plates. Such condensers certainly achieve this object, but they have no other virtue than that of large capacity for their bulk. Such condensers generally have very large dielectric losses, change their capacity considerably with frequency, and are uncertain in capacity in use, owing to variations in thickness of the discs, which usually are free to rotate. Variations also occur in the distribution of spacing of the plates, all of which are free to move up and down.

Any condenser having some other dielectric than air should be calibrated over the actual range of frequencies where it will be used.

§ (26) Electrical Properties of Condensers at Radio Frequencies. (i.) *Calculation of Capacity.*—For purposes of design of condensers the area of plates necessary may, with sufficient accuracy, be calculated from the well-known formula

$$C = \frac{AK}{0\cdot 9 \times 4\pi \times d}\,\mu\mu\text{F},$$

where A = total effective area of one set of plates,
d = length of dielectric in cms. between the plates,
K = dielectric constant of the medium between plates.

For very small condensers (below 50 $\mu\mu$F) the edge effects on the plates and the capacities of the various internal parts of the insulated system to the screen will become relatively large. In such cases, calculations are usually of less value than experiments. The minimum capacity of a screened variable condenser will be approximately of the order given by $C_{\text{min.}} = 1\cdot 5\sqrt{a \times C_{\text{max.}}}$, where a = air gap in millimetres, $C_{\text{min.}}$ and $C_{\text{max.}}$ are in $\mu\mu$F units.

The value will of course vary with other conditions than simply a and $C_{\text{max.}}$, but the above is a rough guide which will usually give results for $C_{\text{min.}}$ within about 20 per cent of the true value, for a normal design of condenser.

When separate condensers are used in parallel their capacities will of course add. Only screened condensers should be so connected, and the screen-connected terminals of each should be common. The condensers may then be quite close together and short leads used to connect them, so that the inductance and resistance of these may be very small or negligible. The capacity of the leads may be calculated from the simple parallel-wire formula. It will be of the order of 1 to 3 $\mu\mu$F in an ordinary case, it is directly additive to the capacity of the condensers.

(ii.) *Change of Effective Capacity with Frequency.*—In the case of high-class air

[1] E. Nesper, "Development of Apparatus in Wireless Telegraphy (Condensers)," *Jahrb. der D. Tel.*, 1908, ii. 92.

condensers there will be no measurable change of true capacity with frequency over the whole range from telephonic frequencies upwards. The change, if present at all, will be less than 1 in 1000.

If a condenser has a leakage resistance R considered as a fixed shunt resistance across its terminals, then, as regards a series circuit, the condenser and shunt resistance C_0 and R respectively may be replaced by an equivalent effective C and series r having the following values $C_{\text{eff.}} = C_0(1 + 1/R^2C_0{}^2\omega^2)$ and $r \doteq 1/RC_0{}^2\omega^2$.

(iii.) *Change in Effective Capacity due to Internal Inductance.*—All condensers have a small internal inductance due to leads and to the general internal circuit. The main part of the inductance is, however, due to the leads, and if a stout wire be temporarily used to connect the two banks of plates inside, the condenser may be measured as a loop at telephonic frequencies.

If l be the effective inductance of the condenser—assumed invariable with frequency and setting of the condenser — then the effective capacity will be given by

$$C = C_0(1 + lC_0\omega^2),$$

l and C_0 being in henries and farads respectively.

Usually the inductance of a condenser is only a few hundredths of a microhenry and may therefore be neglected.

The inductance can also be determined (or a maximum limit set to it) by connecting to it a standard inductance of small and known value. The wavemeter so formed is caused to resonate to a loosely coupled source of radio-frequency current.

Resonance may be determined by a series heater of small and known inductance or by a single loop loosely coupled to wavemeter and itself connected to a heater.

From the known frequency we have

$$(L + l)(C + c)\omega^2 = 1,$$

where L = Inductance of standard coil, c = Self-capacity of the same. The ratio c/C should be much smaller than the ratio l/L, so that no great accuracy is required in the knowledge of the self-capacity of the coil.

(iv.) *Change of Capacity with Frequency due to Dielectric Losses.* — In condensers with dielectrics other than gas there will be a change in capacity with frequency necessitating the calibration of the condenser at the radio frequencies by one of the methods described later (§ (28)). In general the change of capacity with frequency cannot be predicted from a knowledge of the phase angle or power factor of the condenser, although probably for a given dielectric there will be a law, more or less simple, connecting power factor and frequency coefficient of capacity.

In some cases the change of capacity is approximately proportional to $1/\sqrt{\omega}$. In all cases the larger the hysteresis loss per cycle in the dielectric the larger will be the change of capacity with frequency.

With good-quality well-dried paraffin oil the change in capacity from telephonic to radio frequency will be negligibly small except for the most refined experiments.

Such condensers may therefore be used as standards.

Mica condensers also, if properly made with thick metal foils, are sufficiently good to serve as standards and are very convenient; the bulk occupied is very small, and the voltage which may be safely used is quite considerable. The capacity of a high-class mica condenser will not change by more than 1 in 1000 over a range from telephonic to radio frequencies.

(v.) *Energy Losses in Condensers* (Refs. 7 to 10).—These are very important in radio telegraphy generally, and, from the point of view of measurements at radio frequencies—apart from direct measurements of the losses on various condensers and dielectrics—a knowledge of these losses in the standard condensers, used for all kinds of resistance measurements, is of vital importance.

The absolute measurement of dielectric loss in small condensers at any frequency is no easy matter; when measurements on good air condensers at radio frequencies are attempted the task becomes almost impossible.

In any absolute measurements of electrical quantities one is usually forced back ultimately, either (*a*) upon perfection of construction of apparatus from the standpoint of the end in view as far as that is possible, or (*b*) upon mathematical calculation.

The condenser is a case which appears to fall quite clearly into class (*a*) from the point of view of having no loss, and it may be said that a high-class air condenser, constructed with every care to avoid the known sources of losses, forms the most reliable foundation upon which to build up measurements of dielectric hysteresis. On this basis, therefore, a standard air condenser should have (i.) the insulation of as perfect a kind as can be used, (ii.) the insulation should not be in a strong electric field within the condenser, (iii.) its surface should be as perfect as possible to reduce leakage to zero, (iv.) the plates should have clean bright surfaces and rounded edges; the air gap should be as large as possible and no dust should be present; the air inside the condenser should be dry also. The internal connections should be thick so as to reduce internal resistance to a negligible amount.

Such a condenser will have so small a power factor that it is uncertain whether methods at present available can measure it.

Losses in condensers at radio frequencies

and at small voltages are of three kinds: (i.) conductor losses located in leads, etc., and proportional to I^2. These may be expressed as a simple fixed resistance. Such a loss will give the condenser a power factor which increases proportionally to the frequency. (ii.) Leakage conduction losses across and through the dielectric. Such a loss is represented as a more or less constant shunt resistance across the condenser. Let this effective shunt resistance be S, then the series resistance equivalent to it will be $r = 1/SC^2\omega^2$. Such a shunt will give the condenser a power factor which diminishes proportionally with the frequency. (iii.) True dielectric loss usually spoken of as dielectric hysteresis. This loss is a function of frequency and possibly also of voltage gradient in the dielectric, if this is not small. It cannot be said that the laws governing these losses are very well known. In many cases of good dielectrics, however, the dielectric hysteresis losses are roughly equivalent to a constant amount of energy being spent in the dielectric per cycle. This means that the power factor due to them is constant with regard to frequency. They may be represented by either an equivalent series resistance r or its equivalent shunt S; either r or S will be variable with regard to frequency; for most calculations the series resistance equivalent is the more useful representation.

The dielectric hysteresis losses are specially important at radio frequencies and are usually much the greatest, except at high voltages where brush discharges may become serious.

The whole losses may thus be lumped together as an equivalent series resistance s. The power factor of the condenser will then be

$$\text{Power factor} = \cos\phi = SC\omega,$$

s being in ohms, C being in farads.

If the equivalent total shunt is used, the power factor will then be given by $\cos\phi = 1/SC\omega$, where S = equivalent shunt in ohms.

The two values of power factor must, of course, be equal to one another for any given condenser; for small values the power factor is equal to the phase difference of the condenser in radians, *i.e.* $\cos\phi \doteqdot \pi/2 - \phi$, where ϕ is the angle between current and voltage on the condenser.

§ (27) Measurement of Capacity of Air Radio Condensers at Low Frequency.—In the case of air condensers it is, in general, more accurate to measure the capacity at telephonic frequencies by any of the methods suitable for such values of capacity at these frequencies.[1]

The measurements should be made with the condenser under the precise conditions of use, *i.e.* regarding connection of screen, leads, etc.; if a vacuum tube or crystal detector forms part of a wavemeter and is connected across the condenser, the leads to these should be included (the detector being, of course, open-circuited).

[1] See "Capacity and its Measurement," § (41).

A caution is needed here regarding certain kinds of leads forming a flexible connection to wavemeter coils. These are sometimes in cotton braiding, leather, or other materials, having properties which vary largely with frequency. Such leads should be tested separately at radio frequencies by connecting them in parallel with a calibrated air condenser forming part of a resonant oscillatory circuit. The difference between the resonant readings of C with the leads first not connected, and then connected, will give the true effective capacity of the leads at the radio frequency chosen.

Variable air condensers of the usual type with semicircular plates will have a calibration curve as in *Fig.* 68. The line should be

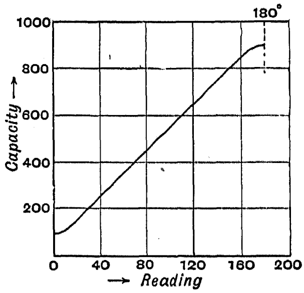


Fig. 68.—Typical Calibration Curve of Variable Condenser.

straight over 160° of the scale. Very commonly there will be an irregularity in the line near the 90° position owing to the front advancing edge of the moving plates approaching the middle rod and washers which usually hold the fixed plates. A number of readings should therefore be taken fairly closely together at this region.

§ (28) Measurements of Capacity at Radio Frequency.[2]—When the unknown condenser or capacity is within the normal range of standard variable air condensers the most accurate method of measurement is one of simple substitution. For this test a pair of leads is connected to the terminals of the standard condenser, and their free ends brought to a convenient position for attachment to the unknown condenser. The ends are so bent that they may be connected up without displacement.

The standard condenser with a suitable inductance and a current-measuring device

[2] For measurement of dielectric constants see R. Jaeger, "Dielectric Constants of Solids," *Ann. d. Phys.*, 1918, liii. 14, 409.

forms a simple wavemeter as in *Fig.* 69. The inductance and frequency are so chosen that the condenser reading for resonance is sufficiently great to enable reading C_2 to be taken. The first reading is now made with leads in position, but with the unknown condenser in position and disconnected. Let this reading be C_1. The unknown condenser is now connected up and the resonance again found by readjustment of the standard condenser to the smaller reading C_2. The value of the unknown condenser will then be simply $C_1 - C_2$. It is convenient to make a trial first with the unknown condenser connected and then to adjust frequency by the inductance coil so as to ensure that the readings C_1 and C_2 both come on the condenser scale.

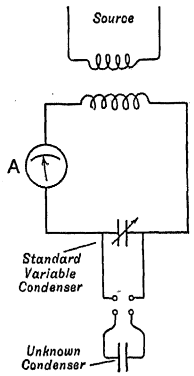


FIG. 69.—Measurement of Capacity at Radio Frequency.

The method is suitable for all kinds of fixed or variable condensers the capacity of which lies within the region $C_{max.} - C_{min.}$ of the standard variable condenser.

A slight correction may be necessary on account of the inductance of the leads in the case where the unknown condenser is in circuit.

For calibration of a large variable condenser at radio frequency, such as an oil condenser, in which the calibration at telephonic frequencies could not be assumed valid for radio frequency, the most direct and accurate method is to proceed by marking off the scale in increments, using a calibrated standard air condenser of smaller value (11). Calling the large condenser B and the known small condenser A the procedure is as follows: Condenser A is set to a convenient even value near the top of its scale (say C_{Am}), forming the capacity part of a simple resonant circuit with current-indicating instrument. The frequency of the oscillating source is adjusted until resonance occurs. Condenser A is now replaced by B, and resonance obtained by adjustment of B, leaving the frequency constant. This gives an accurate point low down on the scale of B (say C_{B1}).

The two condensers are now connected in parallel in the wavemeter circuit with their screen-connected terminals common. Condenser B is set to the calibrated point C_{B1}, and condenser A is set to some low reading C_{A1} (say 200 $\mu\mu$F). Resonance is then obtained by adjustment of the frequency of the source. Condenser A is now set at successive even values of capacity (300, 400, etc., $\mu\mu$F) and resonance re-established in each case by adjustment of B. The points so obtained mark off equal increments of 100 $\mu\mu$F from the point C_{B1} downwards to $C_{B\,min.}$. The accuracy of setting is of course very high in this region of C_B.

Condenser A is now set to C_{Am}, and condenser B to C_{B1}; the frequency is again adjusted to give resonance. Reducing the reading of A to C_{A1} again, and obtaining resonance by adjustment of B, a new point C_{B2} is obtained on B. The interval between C_{B1} and C_{B2} is subdivided by the process outlined above and the whole procedure repeated until the maximum of B is reached.

After the completion of a series of readings at any one frequency the settings at which the series commenced should be again obtained so as to check that the frequency has not changed.

The method is not quite rigorous, because the whole condenser has not been calibrated at one frequency; but changes in capacity, with frequency, are not usually large at radio frequencies, so that the error introduced from this cause will be far smaller than the error introduced by calibrating at telephonic frequency and assuming constancy right up to radio frequency.

Another method of measuring capacity of a large condenser applicable also to fixed condensers is to make use of two standard inductances of known values. The ratio of the inductances should be approximately equal to the ratio of the capacities of the unknown condenser to the standard variable air condenser against which it will be compared. Calling A_A and B_B the two inductances, including leads, etc., A_A (the larger) is set up with C_A as a standard wavemeter, and B_B similarly is set up with the condenser C_B being measured. Both are loosely coupled to the same source. When both are in resonance to the same frequency we have $L_A(C_A + c_A) = L_B(C_B + c_B)$, where L_A = total inductance of A_A + leads, etc.; L_B = total inductance of B_B + leads, etc. C_A is the reading of the standard condenser for resonance, c_A and c_B are the self-capacities of L_A and L_B respectively; c_B will be of small importance.

From the above expression

$$C_B = \frac{L_A}{L_B}(C_A + c_A) - c_B.$$

The inductance of the leads in the B circuit is of importance, since L_B will not be very large. Likewise the self-capacity of A_A—*i.e.* c_A—is of importance, because it is multiplied by L_A/L_B in the equation for C_B.

The above method can be used in the reverse

direction for the measurement of small capacities by choosing L_A large and L_B small; in this case, however, c_B becomes very important, as it may form 20 per cent of the capacity being measured. The method, however, can be used to measure quite small *changes* in capacity, since the constant terms of C_A and C_B cancel out.

We then have

$$C_{B1} - C_B = \frac{L_A}{L_B}(C_{A1} - C_A).$$

With care it is possible to measure small changes in capacity of small total amount to an accuracy of 0·03 $\mu\mu$F.

Another method which has been found very successful for measurement of very small capacities or changes in capacity is as follows. An oscillatory circuit possessing large inductance and small capacity is set up. There is also included a very low range condenser in parallel to the main condenser. A current-indicating instrument is also included in the circuit. Adjustment of the condenser or frequency is made so that one is working at the steep part of the resonance curve. The curve is calibrated for intervals of 1 $\mu\mu$F by the help of the low-range condenser. Extremely small changes in the capacity of any added part may then be measured by observing changes in resonant current. In a particular case where the total capacity was about 100 $\mu\mu$F, a change in deflection of about 4 cm. on the scale of the (Duddell) milliammeter was obtained for a change of 1 $\mu\mu$F. By taking the usual precautions accuracy to 0·01 $\mu\mu$F was obtained.

Methods making use of change in the beat tone produced between two generating circuits when changes in C are made in one of them are also susceptible of good accuracy. The beat tone produced by the interaction of two radio-frequency currents of slightly differing frequencies constitutes one of the most sensitive methods of measuring small changes in capacity in one of the oscillatory circuits.

The outline of an arrangement for making such measurements is shown in *Fig.* 70. A

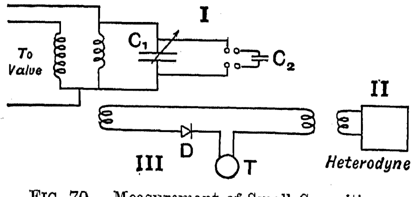


FIG. 70.—Measurement of Small Capacities.

self-generating circuit I is provided having a calibrated variable air condenser C_1. A heterodyne set II is also provided and is indicated at II in the diagram. Circuit III is an aperiodic circuit containing rectifier D and telephone T.

The capacity under measurement is connected in parallel across C_1; it is indicated by C_2.

With C_2 disconnected, the circuits I and II are adjusted to give a beat tone in the telephone T at any desired radio frequency n_1 in circuit I. Let the acoustic frequency of the beat tone be f_1.

C_2 is now connected to C_1. A change in n_1 will be produced and will be indicated by an alteration in the frequency of the beat tone. If f_2 is the frequency of the beat tone when C_2 is connected, we have

$$C_2 = 2C_1 \frac{f_2 \sim f_1}{n_1},$$

assuming C_2 to be small compared to C_1. The frequencies f_2 and f_1 may be measured by tuning-forks or any other method of determining acoustic frequency.

The usual precautions regarding earth capacities, etc., become of enhanced importance in such measurements.

The radio frequencies in I and II must also be in a condition of great steadiness. Repetition readings with C_2 successively connected and disconnected must be made.

§ (29) MEASUREMENT OF EFFECTIVE CAPACITY OF INDUCTANCE COILS (12 to 18).—This is a very common measurement to be made on all kinds of inductances. One of the most accurate methods of determining this quantity makes use of the integral ratio between the fundamental and a harmonic of a source which is not a pure sine wave.

(i.) *Harmonic Method.*—A convenient way to carry this out makes use of an amplifying set and two sources such as heterodyne generators. Instead of the amplifier set an aperiodic circuit with telephone and crystal detector may be used.

The circuits are as in *Fig.* 71, in which L

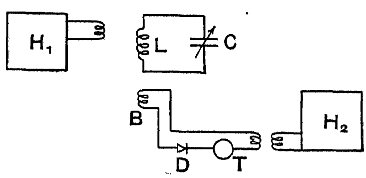


FIG. 71.—Capacity of Inductance Coil—Harmonic Method.

is the coil being tested, C is a standard calibrated air condenser, H_1 and H_2 are the heterodyne sets, and D is the valve amplifier set or crystal detector.

H_1 is loosely coupled to LC, and LC is likewise loosely coupled to a coil B of a few turns connected to the amplifier. H_2 may be a considerable distance away.

H_1 and H_2 are set going at frequencies near one another, and of values so that the

resonance for the fundamental on C is obtained high up on its scale.

By careful adjustment of B with the LC circuit out of tune it is possible to get the beat tone (heard in the amplifier or detector telephone T) to be inaudible or of very faint intensity.

Resonance is now obtained as judged by loudness of the beat tone in the amplifier telephone. This can be judged with certainty to about 1 $\mu\mu$F on the standard condenser at a reading of say 1500 $\mu\mu$F. The beat tone is set (by adjustment of H_2) to a value of about 600 ~ per sec. Condenser C is now adjusted to about one-quarter of its original reading, when a second beat tone will be found having *exactly* twice the frequency of the fundamental beat tone. This is the beat tone of the first harmonic (frequency $2n$) of the two heterodynes. When a maximum has been obtained by adjustment of C the LC circuit is now resonating at *exactly* twice its original frequency.

Let C_1 be capacity setting for resonance of the fundamental of frequency n,
C_2 be capacity setting for resonance of the first harmonic of frequency $2n$,

then $$L(C_1+c)=4L(C_2+c),$$

where c = coil capacity required,

or $$c=\frac{4C_2-C_1}{3}.$$

If the experiment is repeated, using the third harmonic giving resonance at reading C_3, then

$$c=\frac{9C_3-C_1}{8}.$$

For great accuracy it is desirable to make observations at two or three harmonics and with slightly differing values of the fundamental so as to average any small uncertainties and irregularities in the calibration of the standard condenser.

Accuracy to a few tenths of a micromicrofarad may be obtained by taking the mean of six readings.

(ii.) *Graphical Method of Measurement of Effective Capacity of Inductance Coils.*—This is one of the oldest methods of measuring effective capacities of coils, having been used by Diesselhorst, Campbell, Howe, Hubbard (12), and others.

The method is as follows: the coil is connected to a standard calibrated variable condenser forming a simple wavemeter. Another standard wavemeter is also set up, and both are loosely coupled to the same source of radio-frequency oscillations.

The method consists in plotting the square of the wave-length, as determined on the wavemeter, against capacity readings of the circuit containing the inductance coil being measured. The circuits are as in *Fig.* 72. The resonance obtained in circuit II. is that given by a current indicator in the condenser circuit as shown.

The wavemeter should be so proportioned that no reading is used below that corresponding to a capacity of say 500 $\mu\mu$F on its condenser; this can, of course, be done by choosing suitable inductances. The uncertainty, due to ignorance of the exact self-capacities of the wavemeter coils, is then lessened.

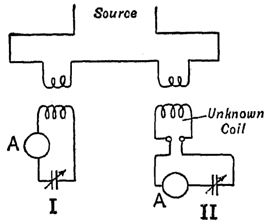


FIG. 72.—Wavemeter Method of measuring Self-capacity of Coil.

The wave-lengths observed should be so chosen that condenser readings in circuit II. containing the unknown coil cover a range from, say, 50 $\mu\mu$F up to 500 $\mu\mu$F. If a calibrated condenser of low range is available more accuracy can be obtained, unless the coil is of very small inductance.

On plotting λ^2 against C_{II} reading, a sloping straight line will, in general, result (*Fig.* 73). This line cuts the axis of capacity at a point F. OF is then the self-capacity of the coil and its leads.

When measuring single-layer or carefully constructed many-layer coils of stranded wire the straightness of the line is remarkable, even when the external capacity is only a very few times the coil capacity. With great care accuracy to 0·5 $\mu\mu$F may be obtained by this method.

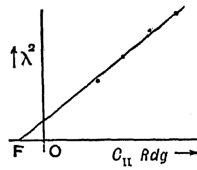


FIG. 73.—Graphical Determination of Self-capacity of Coil.

If preferred, the analytical treatment of the observations may be used to give c by applying the method of least squares to the observations. The formula given by Whittemore and Breit (242) is

$$c=\frac{(\Sigma C_1)(\Sigma\lambda_1^4)-(\Sigma\lambda_1^2)(\Sigma C^2{}_1\lambda_1^2)}{(\Sigma\lambda_1^2)^2-N\Sigma\lambda_1^4},$$

where C_1 and λ_1 are corresponding readings and N = total number of readings. This formula is based on the assumption that the accuracy of C_1 is proportional to its value, and that the accuracy of λ_1 is proportional to λ_1^2.

(iii.) *Oscillation Method.*—A third method of measuring coil capacity is to excite oscillations in it from a source of variable frequency

and to determine the self-resonant frequency of the coil by the use of a vacuum tube indicating maximum voltage at the coil ends.

We then[1] have

$$C = \frac{\lambda^2}{3{\cdot}553L} \quad . \; . \; . \quad \mu\mu F,$$

where λ is the resonance wave-length, L = inductance of coil in microhenries.

Experiments made by Howe (14) using this method have given good agreement with the plotting method.

(iv.) *Method of A. Meissner* (16).—In this method the coil is attached to a calibrated variable condenser, and resonance is obtained with a steady source of radio-frequency oscillations. The coil is now completely immersed in a large volume of an insulating liquid of known dielectric constant.

Resonance is again established by adjustment of the condenser. If the condenser has its capacity reduced by an amount c, then the self-capacity of the coil in air will be $c/k-1$, k being the dielectric constant of the liquid.

§ (30) Capacities of Antennas, etc. (i.) *Measurement.*[2]—The measurements of the capacity of antennas do not present any special difficulty, since the accuracy required cannot be very great owing to the variability of capacity with the condition of the earth below it.

The method consists in first determining λ_0, the fundamental resonant wave-length of the antenna, by either (a) direct observation by means of a wavemeter on the self-generated oscillations, when the antenna is excited by sparks or buzzer, etc., or (b) coupling loosely to the antenna a source of variable frequency and observing when maximum current is induced in the antenna.

If now a coil of known inductance L_1 is inserted at the base of the antenna, and the altered value of wave-length λ_1 determined, we have

$$L_0 = L_1 \frac{\lambda_0^2}{\lambda_1^2 - \lambda_0^2}.$$

By insertion of the value of L_0 in the fundamental equation

$$\lambda_0 = \sqrt{3{\cdot}553 L_0 C_0}$$

the value of C is determined.

Similarly a condenser of capacity C_1 may be inserted at the antinode of current of the antenna and the changed wave-length λ_2 observed.

In this case

$$C_0 = C_1 \frac{\lambda_0^2 - \lambda_2^2}{\lambda_2^2}.$$

An examination of the various methods of measurements on antennas has been made by A. Esau (19).

[1] For $\lambda^2 = V^2\tau^2 = 4\pi^2 V^2 LC = 3{\cdot}553LC$.
[2] See also H. Behnken (19a).

(ii.) *Calculation of Capacities.*—Various formulas are given for calculating capacities of many shapes and dispositions of conductors. These may be found collected together in Dr. W. H. Eccles's *Handbook of Formulas, Data, and Information*, in *Wireless Telegraphy and Telephony*, and also in J. Dellinger's "Radio Instruments and Measurements," *Circular* 74, *Bureau of Standards*.

G. W. O. Howe (20) has treated the more complicated cases of antennas of practically all the forms met with in practice. His method is a particularly powerful one for dealing with such cases. The principle of the method is as follows. Assume that the charge is uniformly distributed over the whole surface of the system by a fictitious insulation of unit portions all of which receive the same charge. The potential curve for the system is obtained, and the average of this potential is assumed to be the same as the actual potential taken up by the system when given the same total charge. The method is considered to be sufficiently accurate for all practical purposes and to be nearly as accurate as the experimental methods.

Other investigators who have developed formulas for the calculation of capacities of antennas are F. Braun (21) and L. Cohen (22).

V. Inductance and Inductance Coils

Inductance is the complementary part to capacity in an oscillating circuit, but, of course, it exists also in many circuits for quite other reasons than that of determining the frequency characteristics of a circuit.

The feature of inductive circuits at radio frequencies is the high reactance of the circuit, with the consequent high voltage between the ends of the inductive part for given currents.

As a consequence of this, coils for use at radio frequency usually consist of comparatively few turns and short lengths of wire wound with special regard to the conditions imposed and the factors arising from the rapid rate of alternation of the current.

As will be seen from the considerations which follow, inductance coils usually take a special form and design in order to minimise effects of distributed capacity and eddy currents. This fact involves, in the design of the whole coil as a piece of apparatus, suitable and sometimes special means of supporting the conductors or turns.

It cannot be said that everything is known about the design, use, and measurement of inductance at radio frequency, but certain broad lines indicate these in most cases, and in many cases the form to be given to the inductance for any particular use is quite clear.

§ (31) CALCULATION OF INDUCTANCE.—A great variety of formulas for the calculation of every kind of inductance likely to be met with in radio measurements have been evolved from fundamental principles.[1] These formulas are collected in various text-books and handbooks on radio-telegraphy. The whole subject has been dealt with very thoroughly in the *Bulletin of the Bureau of Standards*, 1911, viii. 1 (1).

The following books and papers contain valuable formulas and abacs especially suited to radio-frequency calculations:

Handbook of Formulas, Data, and Information (Wireless Telegraphy and Telephony), W. H. Eccles.—Formulas are given for straight, round, and rectangular wires, tapes, parallel wires, concentric conductors; rectangles, squares, and circles of one turn; short, intermediate, and long single-layer solenoids, ring solenoid of rectangular section (toroid); flat coils, circular coils of rectangular section, and many-layer long solenoids. A similar variety of formulas and tables is given for the calculation of mutual inductance.

Some formulas not included in the above are to be found in *Circular* 74, *Bur. Stands.* (2), "Radio Instruments and Measurements," where the following special cases are given, with the necessary tables to facilitate reduction of the results: Flat spiral of flat strip bent in the way easiest to it. Rectangular and square coils of rectangular section,[2] also single-layer prismatic and flat coils (useful for calculation of direction-finding coils).

The case of flat coils (disc coils) has been worked out by A. Esau (3), *Jahrb. der D. Tel.*, 1911, v. 212; also, more recently, by J. Spielrein (4), *Archiv für Elek.*, 1915, iii. 187, and *Jahrb. der D. Tel.*, 1918, xiii. 490—tables to facilitate the calculations are given. The formulas usually give the values of self-inductance for zero frequency except where specially stated to be otherwise. In the cases of stranded wire coils, however, the change of pure inductance with frequency is usually negligible.

§ (32) GENERAL CONSIDERATIONS REGARDING INDUCTANCE.—The electrical properties of inductance coils which are of importance at radio frequencies are:

(*a*) Geometrical Inductance.
(*b*) Distributed Capacity.
(*c*) Effective Resistance.

These quantities are all linked together in actual coils, but it cannot be said that the theoretical relationships between them are established; this is partly owing to mathematical difficulties and partly owing to uncertainties regarding the distribution of the current along the winding of the coil.

[1] See "Inductance (Mutual and Self), Calculation of Coefficients of."

[2] Also calculated by A. Esau (5).

Certain factors are, however, fairly well established, chiefly on the experimental side, in regard to each of these three quantities. These serve as a guide in designing inductances for use in radio circuits (6, 7, 8). The treatment of inductances which have to carry large currents, where insulation and heat dissipation become very important, lies outside the scope of the present article.

(i.) *Geometrical Inductance.* — Commencing with (*a*) the geometrical inductance, this quantity is the inductance obtained by calculation. This, except in a very few cases, neglects the effect of non-uniformity of current distribution over the cross-section of the wire due to eddy currents, and along the wire due to leakage currents and dielectric currents. Dielectric hysteresis is also associated with the dielectric capacity currents.

Fortunately, in most cases where thin or stranded wires are used, the pure radio inductance approximates extremely closely to the geometric inductance. In cases of thick solid wires the diminution in pure inductance due to displacement of the lines of flow of the current may amount to several per cent. Large changes in inductance may also be produced by the presence of circuits or solid metal in the field of the coils.

The effective pure inductance of a coil with a neighbouring circuit attached by mutual inductance coupling will be

$$L_{\text{eff.}} = L_0 + \frac{M^2\omega^2 C(1 - NC\omega^2)}{(1 - NC\omega^2)^2 + S^2C^2\omega^2},$$

where N, C, and S are the inductance, capacity, and resistance of the associated circuit, and M is the mutual inductance between it and the inductance coil. These quantities are not known in the case of solid metal near a coil. The effect is best determined experimentally.

If C is ∞ as in the pure eddy current case of metal near the coil

$$L_{\text{eff.}} = L_0 - \frac{M^2\omega^2 N}{S^2 + N^2\omega^2}.$$

The effect is therefore always to reduce the value of $L_{\text{eff.}}$. From the above expression, including C, it is seen that $L_{\text{eff.}}$ can be either greater or smaller than L according as $NC\omega^2$ is less than or greater than unity. When the coil forms part of a valve generating set, heterodyne, etc., and the attached circuit is, for example, a wavemeter, two frequencies are possible, but the valve can only oscillate with one frequency at one time. Within certain limits either frequency may be generated, and by slowly varying C through the region near that where $NC\omega^2 = 1$, the frequency generated will suddenly "click" over from one value to the other at settings of C approximately equidistant on each side of $C = 1/N\omega^2$. This click forms a useful

method of calibrating wavemeters and is described in the section dealing with calibration.

(ii.) *Distributed Capacity.*—This is one of the most important subsidiary properties of inductance coils. Though distributed throughout the coil between all parts of it, this capacity can, for most purposes and types of coils, be replaced by an equivalent capacity concentrated across the ends of the coil. Such theoretical capacity will be invariable with frequency even up to frequencies approaching the self-resonance of the coil.

The effect, on the coil, of the distributed capacity can be considered in different ways according to the manner in which the coil is used.

If the coil is considered as in series in a circuit, then its effective inductance in that circuit will be given by

$$L_{eff.} = \frac{L_0(1 - L_0 c\omega^2) - Rc^2}{(1 - L_0 c\omega^2)^2 + R^2 c^2 \omega^2},$$

where R, L, and c are the constants of the coil; its effective resistance is

$$R_{eff.} = \frac{R}{(1 - L_0 c\omega^2)^2 + R^2 c^2 \omega^2},$$

also $$Rc\omega^2\left[\frac{2L - L_0^2 c\omega^2 - R^2 c}{(1 - L_0 c\omega^2)^2 + R^2 c^2 \omega^2}\right].$$

For frequencies far from self-resonance, when $L_0 c\omega^2$ is small compared to unity, the equations reduce to

$$L_{eff.} \doteqdot L_0(1 + L_0 c\omega^2),$$

$$R_{eff.} \doteqdot R(1 + 2L_0 c\omega^2),$$

$$\Delta R_{eff.} \doteqdot 2R_{eff.} L_0 c\omega^2;$$

at self-resonance

$$L_{eff.} = -\frac{1}{c\omega^2}, \quad \text{and } R_{eff.} = \frac{1}{R_n c^2 \omega^2},$$

showing that $R_{eff.}$ becomes very great (about a million times its value at low frequencies in a practical case). For frequencies above the self-resonance

$$L_{eff.} \doteqdot -\frac{1}{c\omega^2}, \text{ thus approaching zero,}$$

$$R_{eff.} \doteqdot \frac{R}{(L_0 c\omega^2)^2}, \text{ also approaching zero,}$$

as ω is increased.

If the inductance coil is considered as shunted across a condenser, as in many oscillatory circuits, the effective capacity may be considered as thrown in parallel across the condenser as far as effective inductance is concerned, *i.e.* $L_{eff.} = L_0$ and for the circuit $C_{eff.} = C + c$, where c = capacity of coil.

As regards the resistance of the coil, however, this will be the same whatever its position in the circuit, since it is impossible to separate the inductance and capacity circuits in coils or to measure any other than the ingoing resultant current.

To make this clear we may assume the coil to be replaced by an equivalent circuit made up of pure inductance, capacity, and pure resistance, as in *Fig.* 74, in which R is in series with L, and c is across the terminals. The methods of measuring c all determine it as an equivalent capacity directly across the terminals.

As regards the theoretical quantity R, however, this is not what is measured by any method at present available, because it is impossible to separate the capacity current from the inductance current.

The effective resistance as measured by any experimental method is that which may be represented by *Fig.* 74A, in which R_1 is in series in the main circuit. This value is, of

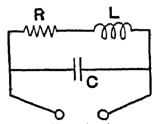


FIG. 74.

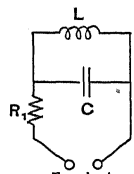


FIG. 74A.

Equivalent Circuit for Inductance Coil with Self-capacity.

course, the one usually required; in comparing experimental and theoretical results, however, it is the quantity R (*Fig.* 74) which is required. This is connected with R_1 by the expression $R_1 = R(1 + 2L_0 C\omega^2)$.

The capacity of the coil will, in general, have dielectric hysteresis, which will still further increase the effective resistance of the coil.

This will depend almost entirely on the nature of the insulation on the wire and on the support or frame on which the coil is wound.

No very accurate estimation can be formed of the actual loss involved, but it will be probably represented by some quantity rather less than the actual power factor of the dielectric immediately surrounding the wire. If a power factor for the coil capacity is assumed, then the equivalent shunt resistance "s" can be calculated from the relation $\cos\phi = 1/sc\omega$. This shunt being considered as across the terminals of the coil will cause an increase in effective resistance, given approximately by $\Delta R = L^2\omega^2/s$. For a coil of 1 millih. with self-capacity of 10 $\mu\mu$F, having a power factor of 2 per cent, the value of ΔR becomes 0·2 ohm at a frequency corresponding to $\omega = 10^6$. Substituting for s we find

$$\Delta R = L^2 \omega^3 c \cos\phi.$$

(iii.) *Resistance.*—The resistance of an inductive coil is dependent on the nature of the wire or cable of which it is wound, and the disposition of the turns to one another and to the whole winding, *i.e.* it is a function of the general magnetic field acting on every part of the wire. In addition to these pure eddy-current effects there may be circulating currents, in the case of stranded wire coils, owing to non-similarity of any strand to every other strand in regard to resistance, self and mutual inductance. There are then the effects due to self-capacity, true leakage, and dielectric loss. All these effects combine to produce increase in the resistance of a coil. A considerable amount of work has been done on the experimental side with a view to elucidating the various effects and to obtaining empirical laws connecting the effective resistance with the various factors.

Theoretical formulas have been evolved by various workers in this field, and fair agreement is obtained between these and the experimental results in the case of straight wires and long solenoids; more work is necessary before the agreement between formulae and experimental results on short solenoids, flat coils, and overwound coils is established.

The formula for straight round solid wires based on Maxwell's theory and reduced by Kelvin (9) to the form involving "ber" and "bei" functions becomes

$$\frac{R_n}{R_0}=\frac{q}{2}\cdot\frac{\text{ber}\,q\,\text{bei}'\,q-\text{bei}\,q\,\text{ber}'\,q}{(\text{ber}'\,q)^2+(\text{bei}'\,q)^2},$$

where $q=\pi d(\sqrt{2\mu n/\rho})$,
d = diameter of wire in cm.,
ρ = resistivity-absolute cm. units $=10^{-9}$ ohm (for copper at 16° C. = 1700),
n = frequency (~ per second);

ber′ and bei′ are differential coefficients of ber and bei functions.

$$\text{ber}\,q=1-\frac{q^4}{2^2 4^2}+\frac{q^8}{2^2 4^2 6^2 8^2}-\text{etc.}$$

$$\text{bei}\,q=\frac{q^2}{2^2}-\frac{q^6}{2^2 4^2 6^2}+\text{etc.}$$

These functions have been tabulated, and a short table (*Fig.* 75) is given below. For higher values of q (greater than 20) the simplified formula is sufficiently accurate:

$$\frac{R_n}{R_0}=0{\cdot}3536q+0{\cdot}25.$$

These formulae are for a simple periodic sine wave of current. For damped waves of logarithmic decrement δ a correction term must be applied to the values determined by the previous formulae.

This (10), for large values of "q," becomes

$$R'_n=R_n g\sqrt{g+k},$$

where $\quad g=\sqrt{1+k^2},\quad$ and $k=\dfrac{\delta}{2\pi}.$

For practically all values of δ likely to be met with the expression reduces to

$$R'_n=R_n\left(1+\frac{\delta}{4\pi}+\frac{\delta^2}{8\pi^2}\right).$$

$\frac{R_n}{R_0}$ for straight round solid wires for various values of q

q	$\frac{R_n}{R_0}$	q	$\frac{R_n}{R_0}$
0·0	$1{\cdot}000_0$	4·5	1·862
0·5	$1{\cdot}000_3$	5·0	2·043
1·0	1·005	5·5	2·219
1·5	1·026	6·0	2·394
2·0	1·078	8·0	3·094
2·5	1·175	10·0	3·799
3·0	1·318	15·0	5·562
3·5	1·493	20·0	7·328
4·0	1·678	50·0	17·93

FIG. 75.—Effective Resistance of Round Straight Wires.

In the case of coils of solid wire a few formulae are available. Sommerfeld (11) treats the case of a long solenoid of wire of rectangular section with insulation of negligible thickness.

His results give, for the ratio R_1 (effective) of the straight wire to R_2 (effective) when coiled up, the value $R_2/R_1=3{\cdot}54$ for very high frequencies. This ratio is, however, known to be much too large from experimental evidence, being, in fact, about twice the probable true ratio.

Cohen (12) and Wien (13) have examined the same case as Sommerfeld, and have obtained theoretically a formula which may be reduced to

$$\frac{R''}{R_0}=1+4{\cdot}8N^2d^3\sqrt{\frac{n}{\sigma}}, \quad \text{(Fleming)}$$

where R'' = effective resistance of solenoid at frequency n,
and R_0 = direct current resistance,
N = turns per cm.,
d = side of square section wire,
σ = resistivity.

Nicholson (14) and Lenz (15) have also contributed theoretical discussions on solenoids of solid wire.

The advantages of stranding the wires used for radio-frequency work were first pointed out by Dolezalek (16). Since then much mathematical and experimental work has been done on straight and coiled stranded wires; in particular, mathematical formulae have been developed by Möller (17) for straight and coiled stranded cable. Howe (18) has also contributed on the theoretical and experimental side for stranded and solid straight wires and solenoids.

On the experimental side much work has been done by R. Lindemann (19) on all cases—also by Black (20), using calorimetric methods, as have also L. W. Austin (21) and Fleming (22).

Regarding the comparison of solid with stranded wire coils each at various frequencies, one important fact, which has been established both experimentally and later theoretically, is indicated in *Fig.* 76. In this diagram curve A represents the change in ratio of R_n/R_0 plotted against n^2 (n = frequency). Over a very long range the law is a straight line. Curve B represents the case of a coil similar with regard to size, pitch of wire, and direct-current resistance. From this it is seen that, although at moderate frequencies the stranded wire coil has the advantage, at high frequencies the position is reversed so that the solid wire coil is the better.

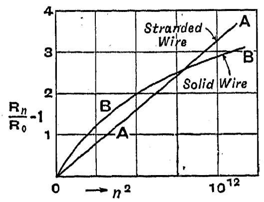


FIG. 76.—Comparison of Solid and Stranded Wire Coils.

One of the great virtues of inductance coils wound with stranded wire is invariability of inductance with frequency. The linearity of the law connecting resistance ratio with (frequency)^2 is also of considerable value in decrement measurements and other radio-frequency resistance measurements.

Various forms of stranding are employed; a common type is to multiply the strand wires in threes. The total number of strands in a cable is then some power of three; thus, a 27-strand cable would have three main stranded bundles each consisting of three, three wire bundles also twisted together. By this means every wire occupies in turn every part of the cross-section of the cable and every wire is exactly similar to every other wire. Usually the separation of the single wires obtained by using double silk covering on each results in a lowering of the effective resistance below that which would result if the strands were closer.

For detailed information regarding the many measurements which have been made on straight, solid, and stranded wires, also single-layer solenoids and short coils of both solid and stranded cable, the following additional original papers should be consulted (23-28). The outlines of some of the methods employed are described in the section on measurements of resistance.

§ (33) DESIGN OF INDUCTANCES.—Confining the attention to inductances used for measuring purposes, these are, of course, essential for wavemeters and as standards of comparison for measurement on other inductances and condensers.

(i.) *Single-layer Coils.*—For inductances up to about 1 millih. single-layer cylindrical coils of stranded wire are suitable. For the highest-class inductances the wire may be wound in a screw-cut track on ebonite tubing; the pitch of the winding should be rather larger than the diameter of the wire. For coils to be used at very high frequencies the pitch of winding may be considerably greater than the diameter of wire; this will result in an appreciable reduction of effective resistance. The terminals should be small and mounted some distance away from the field of the coil. For this reason it is convenient to mount the smaller coils on a strip of ebonite, say, 20 cm. long with the terminals at the end as in *Fig.* 77, showing a group of such coils. The coil will then also be removed a convenient distance from the condenser or other parts of any circuit in which it is introduced. The effective capacity will be increased by about 3 $\mu\mu$F by the parallel leads to the terminals.

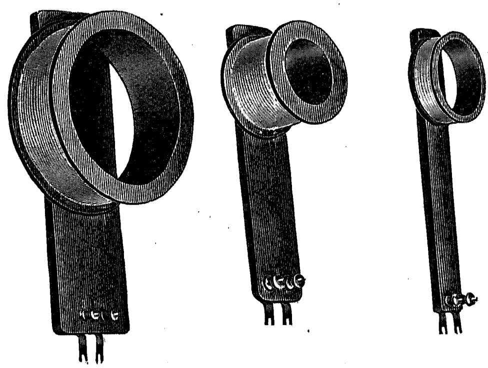


FIG. 77.—Group of Standard Single-layer Coils.

(ii.) *Disc Coils.*—For many purposes disc-shaped coils are preferable to the cylindrical type, because closer coupling can be obtained (as sometimes necessary in valve oscillatory circuits). They pack more compactly also, and can be wound with many turns on special machines by a wave-winding principle which produces a disc-shaped coil of any desired number of turns. The coil so wound is self-supporting, has spaced windings (if desired), and forms quite a permanent unit. The effective capacity in $\mu\mu$F of such coils is of the order of the mean diameter in cm. of the coil. Formulae are not at present available for the inductance of such coils, but it is probable that the formula for disc-shaped coils of ordinary type and of rectangular section will give a sufficient approximation.

Types of disc coils are shown in *Fig.* 78.

(iii.) *Many-layer Coils.*—When the inductance becomes greater than 1 or 2 millihenries the single-layer coils become rather large and are not so efficient as regards length of wire

for a given inductance as overwound coils. One guiding principle for such coils is, that no considerable fraction of the total voltage on the coil should be possible between any

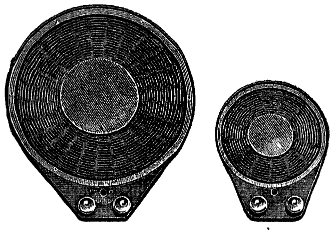


FIG. 78.—Disc Coils.

adjacent turns. Coils with long cylindrical layers on top of one another are thus excluded. The type most satisfactory from the point of view of small capacity, small decrement, and permanence appears to be a cylindrical or polygonal coil wound in slices. *Figs.* 79 and 80 show two such types, one made at the National Physical Laboratory and the other at the Bureau of Standards, Washington. The similarity of these independently designed coils is striking.

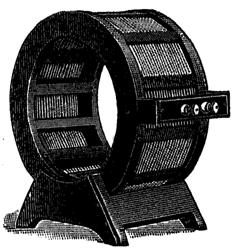


FIG. 79.—Standard Overwound Coil with Slice Winding (N.P.L.). (Photograph from original.)

The framework is a built-up, twelve-sided, skeleton ebonite bobbin with radial supporting combs. The winding is carried out by filling the first slot in the combs with the requisite number of turns (say 8 or 10). Slot No. 2 is now similarly wound and so on until the whole length along the bobbin has been filled.

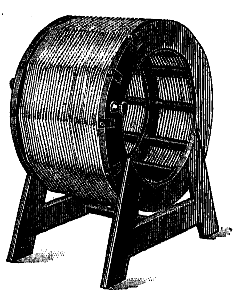


FIG. 80.—Standard Overwound Coil with Slice Winding (Bureau of Standards). (Reproduced from book.)

Strictly speaking, it would be better if the wires in any one slice were spaced so that each wire occupied a square of, say, twice its diameter. The gain would, however, not be very great, since at the rather long wavelengths at which such coils are required the effective resistance is only about 1·5 times R_0, and very considerable part of this rise is due to the simple skin effect and cannot be reduced by any means in a given stranded wire.

Such coils are very permanent, the main insulation is air, the self-capacity is quite small (about 9 $\mu\mu$F in the case of the coil shown in *Fig.* 79); the effective resistance low, increasing fairly accurately as the square of the frequency over the working range. Such coils are naturally rather costly and bulky. For more compact and cheaper coils of large inductance the disc-shaped, zigzag-spaced wound coils are quite good, and may for many purposes be wound with solid wire of, say, 0·4 mm. diameter.

The self-capacity of such coils having a mean diameter of 10 cm. is about 20 $\mu\mu$F.

A many-layered cylindrical coil can be wound with "banked" turns as in *Fig.* 81,

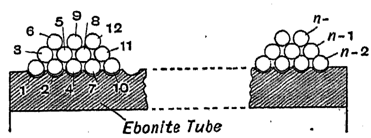


FIG. 81.—Banked Winding for Overwound Coils.

which illustrates a three-layered coil. In some coils of this type the wires are further held in position by shellacking the whole coil. This, however, cannot be considered good, owing to the large increase in capacity and the variableness introduced by the moisture-absorbing shellac.

§ (34) VARIABLE INDUCTANCES.—These have been made successfully for use at radio frequencies by Lorentz. One such is shown in *Fig.* 82. The effective capacity of such self-inductometers does not vary greatly with setting, but the effective resistance is, of course, much larger at low readings than would be the case with a suitably designed fixed inductance. They are of use in carrying out rapid measurements on fixed condensers at radio frequencies, since the replacement of the condenser with a variable air condenser is avoided.

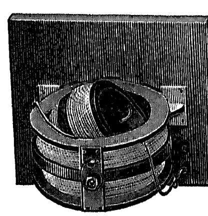


FIG. 82.—Lorentz Variable Self-inductance.

§ (35) INDUCTANCES WITH TAPPINGS.—These are sometimes used on ordinary wavemeters and heterodyne sets. Unless cheapness is the main consideration it cannot be considered good practice. Experiments made on such coils have shown that the effective capacity

is very roughly inversely proportional to the inductance up to the point tapped off, so that the correction term $LC\omega^2$ is nearly constant. Since this term is governed by the inductance of the whole coil, the correction becomes very large at the higher values of ω for which the smallest section of the coil is intended. A large increase in effective resistance also results from the attachment of a "dead" end coil having large mutual inductance to the section.

§ (36) INDUCTANCES FOR GENERATING CIRCUITS.—For a valve generating circuit in connection with measurements, it is generally satisfactory to use disc coils and mica or oil condensers. In calorimetric measurements, however, where considerable current is required, it is necessary to provide for dissipation of heat in the generating circuit. The usual type of compact disc coil is not low enough in effective resistance when, for instance, 10 amperes is being produced.

A suitable type of coil for this purpose consists of a disc spiral of copper tape similar to lightning conductor. The turns may be from 5 to 10 mm. pitch. Such a coil, having a self-inductance of several microhenries, is shown in *Fig.* 83. The copper has a cross-section of 15 mm. × 0·8 mm. Such a coil will just become warm when carrying a current of 20 amperes at a wave-length of 600 metres.

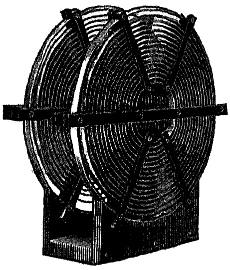


FIG. 83.—Flat Inductance Spiral of Copper Tape.

For large wave-lengths (3000 metres and upwards) it is generally necessary to have an inductance of at least 1 millih. This value cannot be very readily obtained by the flat strip spiral. A suitable coil for this purpose is a hank of vulcanised, rubber-braided electric light cable of, say, No. 16 S.W.G. wire. The spacing of the wires is a great advantage secured by the thick insulation on such cable, and very considerably lessens the effective resistance.

Both the above types of inductance may be used also with spark and arc generating outfits.

§ (37) MEASUREMENT OF SELF-INDUCTANCE AT RADIO FREQUENCIES.—The most generally useful method of accurately measuring the effective pure inductance of a coil is to form an oscillatory circuit by means of a standard calibrated variable condenser and current-measuring device of known or negligible self-inductance (a heater with thermojunctions has negligible inductance for all but the smallest coils).

The circuit and a standard wavemeter are each loosely coupled to a radio-frequency source the frequency of which can be varied. The method is shown in *Fig.* 84; circuit I is the wavemeter and circuit II contains the coil being measured. Resonance is obtained in the two circuits. Wave-length is determined from readings of the condenser in circuit I and C_2 is observed in II.

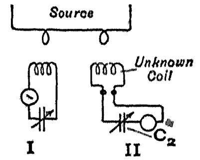


FIG. 84.—Circuit for Measurement of Effective Pure Self-inductance of a Coil.

Several observations are taken at various values of wave-length. C_2 is then plotted against λ^2. In general a straight or nearly straight sloping line will result. If this line usually cuts the axis of C in A (*Fig.* 85), then AO is negative and corresponds to c the self-capacity of the coil. The slope of the line measured by PN/AN gives λ^2/c or $4\pi^2v^2L$; hence L the effective self-inductance is equal to PN/3·553AN. The coils on the wavemeter should be so chosen that the condenser reading C_1 on the wavemeter is in the upper part of the scale so as to get as great accuracy as possible.

If the line is quite straight and the value of c is small, the coil may be presumed to be a good one so far as inductance is concerned. If very great accuracy is desired, the effective pure inductance may be obtained from the observations by the method of least squares in the same way as the determination of the self-capacity. The expression for inductance (29) is

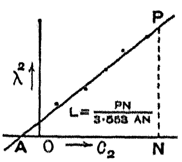


FIG. 85.—Appertaining to *Fig.* 84.

$$L = \frac{n\Sigma\lambda_1^4 - (\Sigma\lambda_1^2)^2}{3·553[n\Sigma C_n^2 - \Sigma C_n\Sigma\lambda_1^2]}.$$

The apparent inductance can, of course, be obtained from one observation. It is equal to $\lambda^2/3·553C_2$; but if separation of pure inductance and self-capacity is desired, then the least number of observations is two.

Variations on this method can, of course, be made if desired. It is not essential, for instance, to use a separate condenser for the unknown coil. The wavemeter condenser may be used, adopting the method of substitution. In any case, the quantity measured by the slope of the line will be the total effective pure inductance of the circuit. Correction for the rest of the circuit containing the coil must be made. This residual self-inductance can, of course, be very easily measured by carrying out observations on a standard coil of known

inductance. The residual inductance of the circuit is then obtained by difference.

The harmonic method may also be used with good accuracy to determine effective pure inductance.

If a source is chosen having harmonics (heterodyne for example), then taking readings C_1 and C_2 on the condenser at wave-lengths λ_1 and λ_2, where $\lambda_1 = 2\lambda_2$ (first harmonic), then we have

$$L(C_1 + c) = 4L(C_2 + c) = \frac{\lambda_1^2}{3{\cdot}553},$$

whence $$L = \frac{3}{4}\frac{\lambda_1^2}{3{\cdot}553} \cdot \frac{1}{C_1 - C_2}.$$

It is then only necessary to measure λ_1 on the standard wavemeter. As previously mentioned in the capacity section, c is also determined from the expression

$$c = \frac{4C_2 - C_1}{3}.$$

§ (38) Mutual Inductance at Radio Frequencies, Measurement of. — Mutual inductance between conductors occurs whenever a magnetic flux produced by a variable current in one conductor is linked with a second conductor.

Such mutual inductance exists in connection with nearly every circuit used at radio frequencies. This property is dealt with here from the point of view of measurement. In many cases the mutual inductance between two coils can be measured with considerable accuracy. If, for example, the mutual coupling is large compared with that due to capacity, then a measurement at telephonic frequency against a calibrated variable low reading mutual inductometer will suffice.

Another method is to measure the self-inductance of the two coils in series. Two measurements are made. If AB are the ends of one coil and CD the ends of the other coil, then with A connected to C the effective self-inductance between B and D is measured.

A is now connected to D, and the inductance with B and C as terminals is measured; calling these two inductances L_1 and L_2, we have

$$L_1 - L_2 = 4M.$$

In deducing M by the above method the effective pure inductances must be measured, with the coils separated, by the sloping straight lines obtained by plotting λ^2 against C readings.

If the mutual inductance is very small compared to L_1 the above method is not very accurate, because of capacity coupling altering the apparent mutual inductance. The mutual coefficient can be measured approximately by observing the resonant current in one circuit produced by a known current in the other circuit. If I_1, $I_{2\ \text{res.}}$, ω, and R_2 stand for the usual quantities, R_2 being the effective resistance of the secondary circuit, then

$$E = I_{2\ \text{res.}} \cdot R_2 = I_1 M\omega,$$

$$M = \frac{I_{2\ \text{res.}} R_2}{I_1\omega}.$$

This, however, includes any small-capacity coupling between the inductances or other parts of the two circuits, and gives what may be termed the equivalent effective mutual inductance.

Further information regarding methods of measuring the coupling coefficient and mutual inductance of coils and circuits may be obtained from the following papers:

P. Ludewig, "An Arrangement for the Direct Measurement of the Coefficient of Coupling of Oscillatory Circuits," *Phys. Zeit.*, 1912, xiii. 450; *Jahr. d. D. Tel.*, 1913, vii. 6.

F. Koch, "Measurement of Mutual Inductance," *Jahr. d. D. Tel.*, 1912–13, vi. 113.

F. Kiebitz, "Measurement of Coupling Coefficients and Self-inductance," *Verh. Deut. Phys. Ges.*, 1913, xv. 1240; *Ann. d. Phys.*, 1913 (4), xl. 151.

VI. Decrement and Effective Resistance

§ (39) Damping Decrement and Effective Resistance Measurements.—Measurements of decrement and effective resistance are amongst the most important of all radio measurements; they are also measurements demanding much skill and many precautions if accuracy is desired.

Mathematically, damping decrement, as applied to oscillatory circuits, may be interpreted as a coefficient of the decay of amplitude of successive oscillations. It is usually expressed as the logarithm of the ratio of two successive maxima in the same direction on the curve connecting current or voltage with time. In some books and papers the logarithm refers to the half-period and not the complete oscillation. Throughout the present article logarithmic decrement $= \delta = \log_e(A_1/A_2)$. (See *Fig.* 86, in which the curve represents

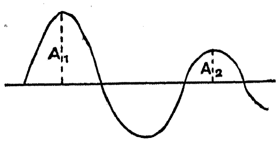


Fig. 86.—Damped Oscillation.

a decaying oscillation.) It is seen, therefore, that decrement is a term to be applied to a complete oscillatory system. Strictly speaking, the term should be limited to such use and not applied to an inductance, condenser, or other part of a circuit as is commonly done. In many cases it is, however, convenient to work in terms of the equivalent decrements of the various parts of a circuit.

The electrical quantity corresponding to damping in actual circuits is, of course, effective resistance. This quantity can quite rightly be split up and associated with the various parts of the circuit. In general in the present article the word decrement will be reserved for use in connection with circuits; in dealing with condensers and inductances effective resistance will be spoken of. The relations between decrement and effective resistance are, of course, quite simple, and either quantity can be expressed in terms of the other.

§ (40) Damping Relations in a Simple Oscillatory Circuit. — The fundamental equation for an oscillatory circuit on which no external electromotive force operates is

$$L\frac{di}{dt}+Ri+\frac{\int idt}{C}=0;$$

assuming the quantities L, R, and C to be independent of i and t, the solution for small values of R becomes

$$i=I_0\epsilon^{-at}\sin\omega t,$$

where I_0 is the initial current amplitude, i the current at any instant, and a the damping factor.

If the oscillations are started by the sudden closure of the circuit at the instant when C has a voltage V_0, then

$$I_0=\omega CV_0,\quad a=\frac{R}{2L},$$

and $$\omega\doteq\frac{1}{\sqrt{LC}}.$$

The expression for the current at any instant in terms of the constants of the circuit becomes

$$i=V_0\sqrt{\frac{C}{L}}\epsilon^{-\frac{R}{2L}\cdot t}\cdot\sin\frac{t}{\sqrt{LC}}.$$

The ratio of one maximum of current to the corresponding one in the next cycle is therefore

$$\frac{I_0\epsilon^{-(R/2L)\cdot t}}{I_0\epsilon^{-(R/2L)\cdot(t+T)}},$$

where $$T=\frac{2\pi}{\omega}.$$

Thus the ratio is

$$e^{\frac{\pi R}{\omega L}}.$$

Hence the logarithmic decrement $=\delta$ is given by

$$\delta=\pi\frac{R}{\omega L}.$$

Fully expressed, we have therefore

$$\delta=\pi\cdot\frac{R}{L\omega}=\pi R\omega C=\pi R\sqrt{\frac{C}{L}}.$$

§ (41). (i.) *Damping and Frequency Factors in Coupled Circuits.*—The fundamental equations for two coupled circuits are

$$L_1\frac{di_1}{dt}+M\frac{di_2}{dt}+R_1i_1+\frac{\int i_1dt}{C_1}=0,$$

$$L_2\frac{di_2}{dt}+M\frac{di_1}{dt}+R_2i_2+\frac{\int i_2dt}{C_2}=0.$$

These equations have been investigated by a number of workers. For details of the analytical methods employed and the assumptions made in order to arrive at a solution, the following original papers should be consulted: VI. (1 to 7). The results for particular cases of interest are clearly set out in Dr. Eccles' *Handbook of Radio Formulae and Data.* Using the notation given there, we have oscillatory currents of two frequencies and two damping coefficients.

Case (*a*). Circuits having different free frequencies when separated.

Coupling small <10 per cent and $g_1/g_2>1\cdot5$.

$$G=g_1\left(1+\frac{k^2g_1}{g_1-g_2}\right);$$

$$B=b_1+k^2g_1\left[\frac{(b_1+b_2)g_1-2b_1g_2}{(g_1-g_2)^2}\right],$$

$$G_1=g_2\left(1+\frac{k^2g_2}{g_2-g_1}\right);$$

$$B_1=b_2+k^2g_1\left[\frac{(b_1+b_2)g_2-2b_2g_1}{(g_1-g_2)^2}\right],$$

where $G=P^2+B^2$, $G_1=P_1{}^2+B_1{}^2$.

P, P_1; B and B_1 are the two angular velocities and damping coefficients respectively of the combined circuits.

$$P=2\pi n,\ B=n\delta,$$

$$\left.\begin{matrix}g_1=\omega_1{}^2+b_1{}^2\\ g_2=\omega_2{}^2+b_2{}^2\end{matrix}\right\},\text{ in which }\begin{matrix}\omega_1=\dfrac{1}{\sqrt{L_1C_1}}\\ \omega_2=\dfrac{1}{\sqrt{L_2C_2}}\end{matrix}$$

$$\text{and }\begin{matrix}b_1=\dfrac{R_1}{2L_1}\\ b_2=\dfrac{R_2}{2L_2}\end{matrix},\text{ also }k=\frac{M}{\sqrt{L_1L_2}}.$$

Case (*b*). Circuits having the same free frequency when separated.

$$(G,\ G_1)\doteq\frac{g}{1\mp k},\text{ and }(B,\ B_1)\doteq\frac{b_1+b_2}{2(1\mp k)},$$

$$g_1=g_2=g,$$

$$\omega_0=\frac{1}{\sqrt{L_1C_1}}=\frac{1}{\sqrt{L_2C_2}}$$

when the coupling is not too small.

From the above

$$(\omega,\ \omega_1)\doteq\omega_0\sqrt{1\pm k},$$

$$(\delta,\ \delta_1)\doteq\frac{\delta_1+\delta_2}{2\sqrt{1\mp k}}.$$

(ii.) *Root Mean Square Currents in the Secondary of Coupled Oscillatory Circuits with regard to Decrement.*—When the oscillations are maintained, either as successive trains of damped oscillations or as persistent sinoidal currents, the following are the most useful equations from the point of view of measurements.

Case (*a*). Sine wave impressed electromotive force

$$I_2=\frac{E}{L_2\omega_1\sqrt{(1-(\omega_2/\omega_1))^2+(R_2{}^2/L_2{}^2\omega_1{}^2)}},$$

where ω_1 = angular velocity of applied electromotive force,

$$\omega_2=\frac{1}{\sqrt{L_2C_2}}.$$

For I_2 at resonance this of course reduces to $I_{2\text{ res.}}=E/R_2$. For the resonance curve we have

$$\frac{I_2}{I_{2\text{ res.}}}=\frac{1}{\sqrt{1+\dfrac{(1-\omega_2/\omega_1)^2}{(\delta/2\pi)^2}}},$$

whence

$$\delta=2\pi\left(1-\frac{\omega_2}{\omega_1}\right)\sqrt{\frac{I_2{}^2}{I_{2\text{ res.}}{}^2-I_2{}^2}}.$$

When C_2 is varied this is equivalent to

$$\delta=\pi\left(\frac{C_{2\text{ res.}}-C_2}{C_2}\right)\sqrt{\frac{I_2{}^2}{I_{2\text{res.}}{}^2-I_2{}^2}}.$$

Three typical curves are given in *Fig.* 87.

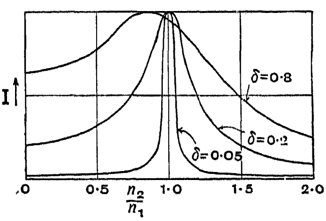


FIG. 87.—Resonance Curves for Various Degrees of Damping.

This is the formula for calculating δ from the reactance variation of the secondary circuit. δ is of course always to be taken as positive.

If the resistance R_2 of the secondary circuit is varied we have

$$\delta=\frac{\Delta\delta\,.\,I_B}{I_A-I_B},$$

where $\Delta\delta$ is the equivalent increment of decrement due to added resistance $R_2{}'$.

I_A = resonance current before adding resistance,
I_B = resonance current after adding resistance $R_2{}'$.

Case (*b*). Coupled circuit in which damped trains of waves are generated in the primary. The equations developed by Bjerknes (1) for the secondary currents in such circuits are:

(i.) For the two circuits in resonance

$$I_2{}^2=\frac{NE_0{}^2}{(2\omega^3/\pi^3)L_2{}^2\delta_1\delta_2(\delta_1+\delta_2)},$$

where N = number of spark trains per second, E_0 = maximum amplitude of impressed electromotive force.

(ii.) If the secondary circuit is detuned so that its free period is ω_2, we then have, for the ratio of I_2 at resonance to the value when the circuit is detuned, an equation which, when arranged to give $\delta_1+\delta_2$, becomes

$$\delta_1+\delta_2=2\pi\left(1-\frac{\omega_2}{\omega_1}\right)\frac{I_2}{\sqrt{I_{2\text{ res.}}{}^2-I_2{}^2}},$$

$$=\pi\left(\frac{C_{\text{res.}}-C}{C}\right)\frac{I_2}{\sqrt{I_{2\text{ res.}}{}^2-I_2{}^2}}.$$

If the resonance curve of I_2 and λ_2 is plotted, it may be shown that the locus of the middle points of chords parallel to the axis of λ_2 is a rectangular hyperbola (*Fig.* 88). It results that

$$\delta_1+\delta_2=\frac{\pi}{\lambda_0}\sqrt{\frac{abI_2{}^2}{I_2{}^2{}_{\text{res.}}-I_2{}^2}},$$

where $a=\lambda_0-\lambda_1$, and $b=\lambda_2-\lambda_0$.

If resistance $R_2{}'$ is added in the secondary circuit, we have, for the ratio of resonance currents before and after adding $R_2{}'$, the expression

$$\frac{I_0{}^2}{I_1{}^2}=\frac{(\delta_2+\Delta\delta_2)(\delta_1+\delta_2+\Delta\delta_2)}{\delta_2(\delta_1+\delta_2)},$$

where $$\Delta\delta_2=\frac{R_2{}'}{2L_2n}=2\pi^2R_2{}'Cn.$$

If, therefore, $\delta_1+\delta_2$ is first determined by the equation above, its value may be inserted in equation connecting $I_0{}^2$ and $I_1{}^2$, thus enabling the separate values of δ_1 and δ_2 to be determined.

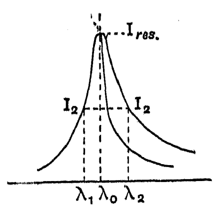


FIG. 88.—Property of Resonance Curve.

Case (*c*). *Impulse Excitation.*—In this case the excitation is in the form of an electrical impulse given to the secondary circuit, which therefore oscillates freely.

Quenched-spark oscillatory systems and some buzzer circuits are of this nature.

The reactance-variation method is not available in this case, since the frequency is determined chiefly by the electrical constants of the secondary circuit.

By varying resistance in the secondary, the equation for I becomes

$$RI^2 = (R + \Delta R)I_1^2.$$

On the assumption that the energy is equal to $\frac{1}{2}E_0{}^2C$, and is therefore the same in the two cases, we then have

$$R = (\Delta R)\frac{I_1^2}{I^2 - I_1^2},$$

while

$$\delta = \Delta\delta\frac{I_1^2}{I^2 - I_1^2}.$$

If ΔR is variable it can be so adjusted that the deflection, on an instrument whose readings are proportional to I^2, is reduced to one-half by insertion of ΔR. Under these conditions $R = \Delta R$.

It is rare, however, that the above conditions hold in practice, since the secondary circuit is not free during the whole of the time. During the time that the primary circuit is closed the case reverts to (*b*) that of two coupled circuits both having damping.

In all the equations whereby decrements are determined from the resonance curve the following assumptions are made:

ω_2 is not very different from ω_1,
$(\delta_1 + \delta_2)^2$ is small compared to $4\pi^2$.

Except where specially stated, the coupling must be loose.

§ (42) Apparatus for Decrement and Resistance Measurement.—In addition to a simple wavemeter, the apparatus required for decrement measurement is a calibrated current indicator, also some standard resistances of suitable type and (desirably) a small accurately calibrated auxiliary condenser in screened case.

(i.) *Resistance Coils.*—For measurement of effective resistance of coils a few standard coils of known effective resistance at various frequencies are very valuable for use in substitution methods.

Resistances should be of fine wire and short. Boxes or pots containing decade resistances may also be used, provided the wires are fine and short. For the most accurate measurements, resistance in the circuit may be changed without changing inductance or induced electromotive force by substituting a length of manganin or other high-resistance wire for an exactly similar and similarly disposed copper wire. Resistance over 100 ohms is seldom required in any measurements of decrement. Up to this limit wires only 20 or 30 cm. long can be used. If any single loop of wire is not more than 10 ohms resistance no error due to capacity of the ends or terminals will result; loops should be in series for larger resistances. It might not be safe, for instance, to have a resistance of 100 ohms in a single loop. At the Bureau of Standards short wires (6 cm.) up to 40 ohms are used mounted in glass tubes.

The current measurement may be made by any of the devices described in the section on current measurement (III.) suitable for small currents (10 to 100 milliamperes).

For measurements by calorimetric methods much larger currents have to be used (1 to 10 amperes), and either hot-wire or strip instruments or current transformers are suitable for measuring the current. In using direct-reading, hot-wire instruments an advantage is that they can be immediately switched on to direct current and calibrated at the deflection used, thus eliminating all zero errors, creep, etc.

A small screened condenser having a range up to 50 $\mu\mu$F and accurately calibrated is very useful for making small accurately known changes in capacity when applying the reactance variation method of measurement of decrement.

(ii.) *Precautions when measuring Decrement or Resistance* (8).—It is most important when varying the measuring circuit to arrange so as not to vary the net total induced E.M.F. during the observations. For this reason the observer should stand in the same position throughout or, better, keep far away when taking readings. For the same reason the resistance to be added should be connected in the earthed or screened side of the condenser as should also be the current-measuring device. If a small auxiliary condenser is used, its screen-connected terminal will of course be connected to the similar terminal on the main condenser. When any change in resistance has been made the resonance may be affected; the observations should be made to test this, and the resonance restored if necessary.

During and after a series of observations a check should be kept on the constancy of frequency of the source by observing that the resonance condition is preserved for a particular condition of the measuring circuit at the beginning and end of a series of observations.

The above remarks apply also to constancy of primary current. There may be considerable slow drift of current in the sources with time. When an observation has been made and then a change effected in the circuit for purposes of measuring, the circuit should be restored to its original condition to see that no drift of frequency or current is occurring.

Measurements by means of calorimetric methods are difficult and the technique must be learned for each special method employed when taking the observations. (See also Ref. VI. 9.)

§ (43) Instruments for Direct Measurement of Decrement.—A few decremeters

have been devised enabling direct readings of decrement to be obtained.

(i.) *Kolster Decremeter* (10).—The principle of this decremeter consists in constructing a variable condenser having such shaped moving plates that the absolute change in capacity due to a small angular displacement from any particular reading shall be proportional to the capacity at that reading. A subsidiary scale is rotated through gearing from the condenser spindle. This scale is a uniform one calibrated directly in decrements; it is held friction-tight on its mounting, so that it can be set to zero against a fixed index for any resonance position of the condenser. If now the condenser is turned from resonance through such angle that a current-indicating instrument reads some pre-chosen fraction of the resonance deflection, the decremeter scale will now read δ_2 or $\delta_1+\delta_2$ directly.

For semicircular fixed plates with a semi-circular opening of radius r_2 for the spindle of the moving plates the equation to the moving plates is

$$r=\sqrt{2C_0a\epsilon^{a\theta}+r_2^2}.$$

C_0 is the capacity when $\theta=0$. If this is a chosen quantity, and r for either $\theta=0$ or $\theta=180$ is fixed, the condenser is determined from the equation $C_0=C\epsilon^{a\theta}$, and

$$a=\frac{\log(C_{180}/C_0)}{180}.$$

In an actual instrument it is convenient to work so that $I_1^2=\frac{1}{2}I_{res.}^2$; the equation for decrement then reduces to

$$\delta_1+\delta_2=\pi\frac{C_{res.}-C}{C}.$$

This gives for the decremeter scale the law

$$\Delta\theta=\theta_1-\theta_{res.}=A\log\frac{\delta_1+\delta_2+\pi}{\pi}.$$

For a great many purposes the scale represented by this equation would be far

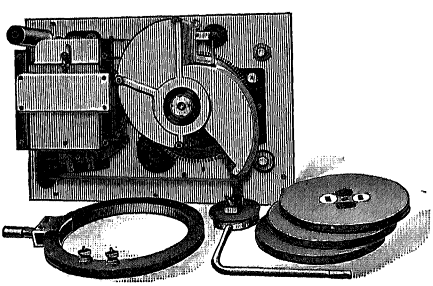


FIG. 89.—Kolster Decremeter.

too closed up to give accurate readings of $\delta_1+\delta_2$.

In the instrument, as indicated in *Fig.* 89, the decrement scale is geared up by means of toothed wheels as indicated. The shape of the movable plates is also shown.

(ii.) *Marconi Decremeter* (11).—This instrument is described in Fleming's *Principles of Electric Wave Telegraphy*. Referring to the diagram (*Fig.* 90), AB is the inductance of a

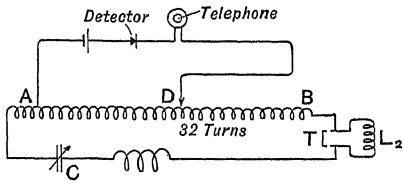


FIG. 90.—Marconi Decremeter.

wavemeter, and is in the form of a uniformly wound, long, straight spiral. Any potential difference between A and a slider D may be picked off and applied to a rectifier and telephone. By finding the positions of D for maximum audibility for various values of C and plotting 1/AD against C, a resonance curve is obtained from which the decrement may be found from the Bjerknes formula.

For this purpose an extra inductance L_2 of known value can be thrown into the circuit by opening switch T, thereby altering the free period of the decremeter by about 4 per cent. The procedure is to tap off a fixed number of turns (32) on AB and observe the loudness of sound produced in the telephone, with switch T inserting the inductance L_2 in the previously tuned circuit. The same loudness is then obtained by adjusting D to the smaller number n of turns necessary when L_2 is cut out and the wavemeter is in resonance.

The expression for $\delta_1+\delta_2$ becomes

$$\delta_1+\delta_2=2\pi\times 0{\cdot}04\sqrt{\frac{1}{(32/n)^2-1}}.$$

A decrement meter has also been described by P. Ludewig.[1]

§ (44) METHODS OF MEASUREMENT OF DECREMENT AND EFFECTIVE RESISTANCE.—The principal methods of making these measurements may be classed under five headings as follows:

(i.) Caloric methods.
(ii.) Substitution method.
(iii.) Reactance variation method.
(iv.) Resistance variation method.
(v.) Delineation of decay curve.

Of these methods (i.) may be regarded as absolute or semi-absolute, but is only applicable, in general, to parts of a circuit and not to the whole of an oscillatory circuit.

(i.) *Caloric Method.*—A very good example of this method is that devised by Fleming

[1] P. Ludewig, "A Decremeter for Practical Use in Wireless Telegraphy," *Phys. Zeit.*, 1911, xii. 763.

(VI. 12) for measurements on effective resistance of straight wires. Essentially the method consists in placing the circuit or apparatus being measured in a vessel and measuring the heat produced when the known high-frequency current traverses it. In the actual measurements made by Dr. Fleming a null method was used. The wires or helices were in duplicate, each in its own tube. The tubes were closed, but communicated with one another by a narrow-bore tube having coloured water and an air-bubble to serve as an index. The scheme is shown in *Fig.* 91. A and B

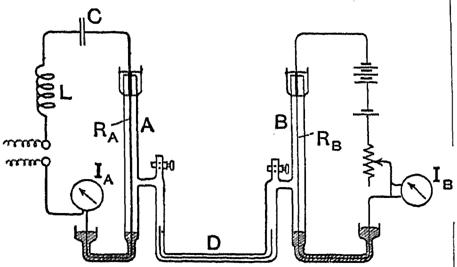


FIG. 91.—Thermal Method of measuring Effective Resistance of Wires and Spirals (Fleming).

are the two main tubes and D the communicating tube. The whole thus forms a differential air thermometer. The procedure consisted in passing the radio-frequency current through the wire in one tube, whilst at the same time a direct current was passed through a similar wire in the other tube. A temperature balance is obtained by adjustment of I_B. From the known direct-current resistance of R_B and current I_B the energy dissipated is known; this energy is equal to that dissipated by the radio-frequency current I_A in wire of unknown effective resistance R_A, so that

$$R_A I_A^2 = R_B I_B^2.$$

It is desirable to interchange tubes—*i.e.* place R_A in tube B and R_B in A—and repeat the measurements with radio-frequency current in R_B and direct current in R_A as before.

The apparatus can of course be modified to suit particular cases, *i.e.* various shapes of calorimeter may be used instead of the glass tubes. To avoid duplicating the coil, etc., being measured, a second set of observations may be made on the coil in the same enclosure, using direct current and merely using the B branch to preserve balance of the zero in the two cases.

Other methods of measuring the temperature or rate of temperature rise are to observe quickly the direct-current resistance, on a Wheatstone bridge, of the circuit under test. This is done at intervals and a curve connecting direct-current resistance with time obtained.

A similar set of observations is made with direct currents of various values. From these the equivalent direct current can be determined. The method has been used by H. Abraham (13).

Calorimetric measurements have also been made by L. W. Austin [1] (14), who used differential oil calorimeters. The two calorimeters were closed vessels as nearly identical as possible. Two similar coils were wound, and one was placed in each vessel. The high-frequency current (measured by ammeter) was passed through one coil and direct current through the other coil. Thermojunctions were immersed, one in each calorimeter, and they were connected in opposition to a sensitive galvanometer. The oil was kept slowly stirred during the observations, which consisted in adjusting the direct current until no deflection of the galvanometer was obtained. The same conditions then hold as in the Fleming differential air thermometer.

Thermojunctions at various parts of the circuit may also be used, but care in interpreting the results is necessary, because the heat developed with direct current will be differently distributed along the coil from that developed by the radio-frequency current.

Thermal measurements on coils have been made by T. P. Black (15), who compared spirals with straight wires, measuring the relative amounts of heat produced when the same radio-frequency current traversed both.

Thermal methods are capable of considerable accuracy and possess the advantage over other methods of measuring the localised resistance of part of a circuit that no assumptions are made regarding the rest of the circuit. They are, however, slow and laborious and require more or less specialised apparatus.

(ii.) *Substitution Method for Measurement of Effective Resistance.*—As its name implies, this method consists in substituting, for the part of the circuit being measured, a similar part of known value. If, for example, the effective resistance of an inductance coil is required, resonance is obtained with the coil in circuit and the maximum current observed. The coil is now removed and a known standard coil (with series resistance if necessary) is inserted in its place, resonance is again established and the current made the same as before by adjustment of the added resistance. It is not always possible or desirable to use continuously variable resistances in radio circuits, but by using two suitable resistances, and interpolating, the resistance difference between the two coils may be determined.

If small changes only in C are made, no considerable error due to this cause will arise

[1] *Ann. der Physik*, 1906, xix. 157, "Über den Widerstand von Spulen für schnelle electrische Schwingungen."

in the case of an air condenser, but if large changes in C are made, the change in effective resistance of the condenser may vitiate the results. The method demands invariability of the induced electromotive force, so that separate inducing loops should be included in the circuit for this purpose. The possibility of variation of the effective coupling between the circuit and source, when replacing one piece of apparatus by another, constitutes one of the chief uncertainties of the method. When damped oscillations are used a correction will be necessary if C_2 does not equal C_1.

In the case of measuring the effective resistance of a condenser, however, the method is probably as satisfactory as any other. The method simply consists in replacing the condenser under test by a variable standard air condenser and resistance; the added resistance, plus the small resistance of the standard condenser (if known), gives the effective resistance of the condenser being measured. The results apply equally to damped or undamped sources of oscillation.

The method has been used for condenser loss measurements by L. W. Austin (16).

A substitution method which has been found satisfactory for measuring effective resistance of the order of 10,000 to 10^6 ohms, such as are represented by grid leaks, anode resistances, and crystal rectifiers, consists in substituting for the high resistance — connected as a shunt across the condenser — its equivalent series resistance, as in *Fig.* 92. The current is made the same in Case B as it was in Case A, by adjustment of R.

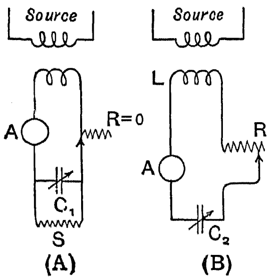


FIG. 92. — Method of measuring very High Resistances at Radio Frequencies.

We then have

$$R=\frac{S}{1+S^2C_1{}^2\omega^2}, \text{ and } C_2'=C_1\left(1+\frac{1}{S^2C_1{}^2\omega^2}\right).$$

C_2' = theoretical capacity in case B.

In practically every case $S^2C_1{}^2\omega^2$ is large compared to unity; the expressions then reduce to

$$S=\frac{1}{RC_1{}^2\omega^2}, \quad \text{and } C_2'=C_1.$$

The difference between C_2' and C_2 (obs.) gives the effective capacity of the shunt resistance S.

The difficulty due to change in the induced electromotive force, when one part of a circuit is substituted for another, may be overcome by a combination of the substitution method and one of the other methods which determine the total effective resistance of a circuit.

For example, in measuring effective resistance of a condenser, the condenser, shunted by a variable air condenser for purposes of tuning, forms part of a simple oscillatory circuit. The total resistance of this circuit is measured either by the resistance or reactance variation method. The unknown condenser is then removed and measurements of total resistance again made on the circuit, using a standard air condenser and the same frequency. The difference between the two values of total resistance so obtained gives the true effective resistance of the condenser under test on the assumption of perfection in the standard air condenser.

(iii.) *Reactance Variation Method for measuring Effective Resistance.*—This method really measures the sharpness of resonance, which may be defined as the ratio $\omega L/R$ or $1/RC\omega$. The method consists in determining at least two points on the resonance curve corresponding to different currents in the circuit varied. The equation of the resonance curve is then applied to find either δ or R, the decrement or total effective resistance respectively of the circuit. If damped oscillations are used in the source the value obtained for the decrement includes that of the source. Either inductance or capacity may be varied, but, in general, capacity variation is used. A small screened condenser is very satisfactory for the purpose, since variations of its capacity will not affect the total electromotive force induced into the circuit from the source. There is an optimum amount of detuning depending on the law of deflection of the current instrument; but, in general, if the deflection of the instrument is reduced from its maximum to one-half by change of C, good accuracy is obtained. The usual method is to observe two values C_1 and C_2, one on each side of $C_{res.}$, corresponding to the same value I_2 of current. In this case

$$R=\frac{1}{2\omega}\,\frac{C_2-C_1}{C_2C_1}\sqrt{\frac{I_2{}^2}{I_{res.}{}^2-I_2{}^2}}.$$

On an instrument whose deflections are proportional to I^2 it is further convenient to make $I_2{}^2=\frac{1}{2}I_{res.}{}^2$. The expression then reduces to

$$R=\frac{1}{2\omega}\cdot\frac{C_2-C_1}{C_2C_1}.$$

Since the value of R thus determined is the total effective resistance of the whole circuit, it is necessary to know the resistance of all the parts of the circuit external to the part being measured.

In some cases it is necessary to take account of the change, with frequency, in resistance

of the leads. This can always be done by calculation from the size of the wire and the frequency. The current-indicating instrument (heater, etc.) may, in general, be assumed to have resistance invariable with frequency. For most purposes also the standard air condenser can be assumed perfect. There is, however, some doubt regarding this; it cannot be said that certainty in the knowledge of effective resistance of high-class air condensers is established to a closer degree than perhaps 0·1 or 0·2 ohm in a condenser of 1000 $\mu\mu$F at radio frequencies of the order 3×10^5. Greater certainty is seldom required in ordinary radio measurements of effective resistance.

In all these measurements the deduced value of R will be that of the effective resistance referred to the current actually flowing through the current-measuring instrument. If there are leads connecting the part being measured to the rest of the circuit the effective resistance of it will be different according to the position of the ammeter. If the ammeter is at the far end of the leads their capacity will be thrown across the part under measurement and will alter its effective resistance. For an inductance the true effective resistance will be $R \doteq R_1(1 - 2Lc\omega^2)$, where R_1 is the apparent radio-frequency resistance. If the coil is connected in any circuit by means of leads, then, of course, the value R_1, as measured at the distant ends of the leads from the coil, is the effective resistance with regard to the circuit.

The method requires great care in taking the observations, since one is operating at the steep part of the resonance curve where very small changes in capacities to earth—the observer, etc.—will produce comparatively large changes in current. For this reason it is more accurate to make the small capacity change and then read the current from a distance. By trial and error the second value of capacity on the other side of resonance to that for the first reading can be set for equal current, and then the maximum can be found by setting capacity to the mean of C_1 and C_2.

The reactance variation method of measuring effective resistance requires very loose coupling for its successful use. If the coupling is too tight there will be an appreciable change in the frequency of the oscillations in the source when the circuit under observation is detuned. The frequency will change in such a direction that the true intervals $(C_2 - C_{res.})$ and $(C_{res.} - C_1)$ will be larger than the observed intervals, because the true $C_{res.}$ point changes in a direction opposite to that in which the change in capacity is made.

Tests for invariability of frequency should be made by means of a separate and loosely coupled wavemeter.

(iv.) *Resistance Variation Method.*—This method is essentially different from the reactance variation method in that, since resonance is maintained for all readings, a simplification results in deducing the value of effective resistance or decrement.

The most consistent results are obtained by arranging the circuit so that the added resistance and the ammeter are contiguous, and in the earthed lead (in the case of an aerial) or the screen-connected terminal side of any oscillatory circuit that may be under measurement.

The circuit is thus as shown in *Fig.* 93, in which R is the added known resistance. The measurement consists in obtaining resonance with no added resistance and with two or three values of resistance, such as 1, 2, 5, etc., ohms; the resonance current is carefully observed in each case. In general, the residual inductance of the added resistance is so small that it is unnecessary to retune when changing R, but trial of this should be made.

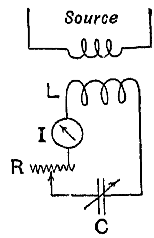


FIG. 93.—Resistance Variation Method of measuring Effective Resistance.

If added R is plotted against 1/I a straight line should be obtained cutting the axis of R at a reading—R_n for $1/I = 0$. R_n is then the required effective resistance of the circuit when the added resistance = zero.

If the effective resistance of some particular part of the circuit is required a second set of observations may be taken with the part removed. The difference in total effective resistance measured in the two cases gives the resistance of the part desired. If this part represents the whole or nearly the whole of either the capacity or inductance of the circuit this method cannot of course be carried out, since resonance cannot be preserved in both cases.

For the greatest accuracy the added resistance may take the form of a short loop or straight high-resistance wire. Comparative readings are taken with a precisely similar copper wire disposed in the same way in the circuit, so that the total induced E.M.F. is the same in the two cases.

If the current-measuring device gives readings proportional to I^2, then all calculations can be eliminated by adding such resistance as to reduce I_2^2 deflection to one-quarter $I^2_{max.}$. This method, however, requires a variable resistance, unless interpolation is used between two values of added R, giving deflections slightly greater and slightly less than one-quarter that due to $I_{max.}$.

For general information on practical measurement of damping the following papers may be consulted (VI. 17 to 26).

(v.) *Decrement Measurements by Direct Delineation of the Curve of Current.*—The Braun cathode-ray oscillograph has been used to measure decrements of circuits, and particularly of spark-gaps, by photographing the actual damped oscillatory current resulting from discharging a condenser through a spark-gap and inductance (F. Braun (1)). The form of the resonance curve has also been directly demonstrated by E. Marx and F. Bonneitz (2), who obtained photographs of resonance curves, using a Poulsen arc as source.

The use of such an apparatus requires considerable skill and care in interpreting the results. It is one of the few methods at present available, however, for giving records from which information may be obtained regarding circuits whose decrement varies with current, such as spark-gaps, arcs, and inductances containing iron. For the special technique required in the use of cathode-ray tubes the original papers should be consulted (VI. 1 to 13).

§ (45) Cathode-ray Tube.—A device of peculiar value for measurements and investigations at radio frequencies is the cathode-ray tube. In its general form the tube is as shown diagrammatically in *Fig.* 94. A long

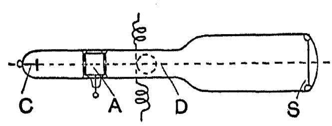


Fig. 94.—Hot Cathode-ray Tube.

bulb blown at one end of the tube contains a screen S coated with a fluorescent substance. At the other narrow part of the bulb are the cathode C and anode A respectively; this latter usually is of a hollow box or narrow-bore tube form, so that only a thin beam of electrons emerges. The beam may be kept from dispersing by an axial magnetic field due to a coil near C. The radio-frequency currents or potentials give rise to a magnetic or an electric field, as the case may be, in the portion of the path of the electrons at D. Deflections of the ray are thereby produced, causing the luminous spot to deviate and form an oscillatory band. If two deflecting forces of the same frequency be simultaneously applied at right angles to one another a closed curve results on the screen. From the shape of this and the knowledge of the nature of one of the applied forces much information can be obtained regarding the other unknown force.

By this means hysteresis loops for magnetic materials may be obtained, also the phase relations between current and voltage in various parts of a circuit.

In such a tube as described above many thousands of volts are required to obtain the necessary electrons; their velocity under these conditions is also extremely high. In some modern experiments cathode-ray tubes having a heated cathode have been used, (12) and (13). Such a tube with, at the same time, a very high vacuum enables a cathode beam to be obtained with a comparatively low voltage (400 to 1000). The velocity of the electrons is of course much lower. Two properties of the tube are thereby affected. (i.) the deflections are produced with a much weaker magnetic or electric field. This in general is an advantage. (ii.) The time interval between the arrival of the electron at D and at the screen is longer, and causes a phase displacement of the Lissajous or other figure on the screen. The shape of the figure is of course unaltered if the two fields act at the same region of the tube, but if there is an interval of several cm. between the two fields, then an alteration in the shape of the figure on the screen will result if the velocity is changed.

This may be either an advantage or a disadvantage according to the investigation being undertaken. Experiments on such tubes have been made by E. Lübke and C. Samson (12 and 13), who devised a point-by-point method of delineating radio-frequency

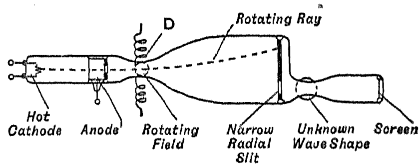


Fig. 95.—Special Hot Cathode-ray Tube.

current or voltage curves. Such a tube is shown in *Fig.* 95.

Some of the uses to which cathode-ray tubes may be put in connection with radio-frequency measurements (14):

Determining harmonic ratios between radio-frequency currents (Lissajous figures).

Obtaining hysteresis loops on magnetic materials at radio frequency.

Delineating transient discharges, such as with spark-gaps and discontinuous currents as in arcs.

Determination of phase relations between currents or current and voltage in a circuit, also between these quantities in coupled circuits.

Determination of wave shapes of current and voltage.

D. W. D.

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Rayleigh Balance, for absolute current measurement. See "Electrical Measurements," § (33).

Rayleigh's Method, for measuring the capacity of a condenser. See "Capacity and its Measurement," § (60).

Reactance: that part of the impedance of an alternating current circuit which does not contribute to the power dissipated. It is measured by $L\omega - (1/K\omega)$, where L and K are the conductance and capacitance, ω the pulsatance of the circuit. See "Inductance, The Measurement of," § (3).

Reactance Drop of Transformers: pressure drop in transformers due to leakage flux. See "Transformers, Static," § (16).

Receiver, Telephone, consideration of, as an element of a transmission circuit (*i.e.* as a transformer of electrical power into sound). See "Telephony," § (19).
Electromagnetic. See *ibid.* § (17).
Vibratory characteristics of. See *ibid.* § (18).

Recording Instruments. See "Direct Current Indicating Instruments," § (21); "Switchgear," § (27).

Reflection in Telephone Circuits due to irregularities in the electrical constants of the line. See "Telephony," § (36).

Registering Mechanism for Meters. See "Watt-hour and other Meters for Direct Current. II. Watt-hour Meters," § (28).
Frictional effects in dials of. See *ibid.* § (29).
Specification of B.E.S.A. for. See *ibid.* § (30).

Regulation of Transformers: pressure drop due to load current losses. See "Transformers, Static," §§ (15), (17).

Reichsanstalt Pattern of Standard Low-resistance Units. See "Potentiometer System of Electrical Measurements," § (13).

Rejector, The: a device for increasing the selectivity of aerials, by arranging the various impedances, so that only oscillations of a certain definite frequency pass through the receiving instruments. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (9).

Relay: Non-polarised, for telegraphy. See "Telegraph, The Electric," § (5).
Polarised, for telegraphy. See *ibid.* § (5).
Vibrating: a sensitive relay with a vibrating tongue employed in telegraph circuits. See *ibid.* § (13).

Relay Action: discussion of, in thermionic valves. See "Thermionic Valves," § (4).
Use of, in thermionic valves, for amplification purposes. See *ibid.* § (11).
Relays: resonance instruments as. See "Vibration Galvanometers," § (46).
Use of, in telegraphy. See "Telegraph, The Electric," § (5).
Reluctance, between two surfaces, is the ratio of the difference of magnetic potential or magnetomotive force between the surfaces to the magnetic flux. Thus if Ω be the difference of potential B, the flux; Reductance $=\Omega/B$.
Remanence: the magnetisation which exists in a material (expressed in terms of B or I) when the magnetising field is reduced from some maximum value to zero, the material being in a cyclic condition. See "Magnetic Measurements and Properties of Materials," § (1).
Repeater: a device employed in long-distance telegraph circuits for the automatic retransmission of signals, in order to overcome attenuation. See "Telegraph, The Electric," § (12).
In telephony, a device for amplifying the currents produced by the voice. See "Telephony," § (28).
Circuits, types of. See *ibid.* § (29).
Residual Charge: the charge in an electrical condenser which remains after the main charge has been neutralised by a momentary short circuit. See "Capacity and its Measurement," § (9).
Resistance:
An expenditure of energy is needed in order to transfer electricity through a conductor, and the property of the conductor to which this is due is called its resistance. It is constant for a conductor under constant physical conditions, and is measured by the ratio of the electromotive force to the current produced.

Unit resistance is that of a conductor in which one erg is expended in causing the transfer of unit quantity of electricity per second through the conductor.

The practical unit of resistance, the ohm, is 10^9 times this quantity, and is the resistance of a conductor in which one watt is expended by a current of one ampere.

The quantities electromotive force, current, and resistance are connected by the equation $E=CR$. See "Units of Electrical Measurement," §§ (7), (9); "Electric Measurements, Systems of," § (9).

Measurement of very high values by loss of charge of a condenser. See "Capacity and its Measurement," § (73).
Methods for the Measurement of Electrolytic. These fall into two classes: (1) Methods in which the effects of polarisation are avoided—(*a*) circuit wholly electrolytic and E.M.F. supplied by electromagnetic induction; (*b*) by the use of auxiliary or potential electrodes. (2) Methods in which the effects of polarisation are reduced to a minimum—(*a*) Horsford's method, in which the electrolyte is contained in a vessel of uniform cross-section between parallel electrodes, each perpendicular to its length, the vessel being connected in series with a battery and galvanometer, and a resistance whose value is adjusted so that the same current flows through the galvanometer for different distances apart of the electrodes; (*b*) Kohlrausch's method, in which alternating currents are used. See "Electrolysis and Electrolytic Conduction," § (17).
Resistance, Effective: the ratio of the total power dissipated (including iron and dielectric losses) to the square of the current for any circuit. See "Inductance, The Measurement of," § (2).
Measurement of, at radio frequencies. See "Radio-frequency Measurements," §§ (43) *et seq.*
(At radio frequencies), references to original papers on. See *ibid.*, end of Section VI.
Resistance, Electrical: permanence of, in the case of various metals and alloys. Matthiesen's investigation. See "Resistance, Standards and Measurement of," § (3).
Results of absolute measurements of. See "Electrical Measurements," § (20).
Values of, for metals and alloys at very low temperatures. See "Resistance, Standards and Measurement of," § (4) (iii.); "Superconductivity," § (1).
Resistance, Internal, of an Electric Cell. See "Batteries, Primary," § (10).

RESISTANCE, MEASUREMENT OF INSULATION

§ (1) Direct Deflection Methods.—For high resistances of the order of more than one megohm, methods such as the resistance bridge are unsuitable and the measurement of insulation resistance is usually made in the laboratory or test room by means of a direct deflection method, in which the insulation to be tested is connected in series with a high-resistance galvanometer and a battery, the leakage current being determined from the deflection of the galvanometer.

This simple method, however, does not allow of discrimination between the actual leakage through the dielectric and the leakage over the surface which, in certain cases, may be considerably greater than that through the insulation. Thus, in testing the insulation of a length of cable the coil is usually immersed

in water, except for short lengths of the ends. The resistance between the conductors and the water under these conditions would be affected by the leakage current over the short length of the projecting end, and it is necessary to correct for or eliminate the effect of surface leakage, and, generally, in the case of all insulating materials it is desirable that the methods of test should allow of these two effects being determined separately.

(i.) *Price's Guard Ring.*—W. A. Price's guard ring method for the elimination of surface leakage consisted essentially of taking a wire from an intermediate point between the two electrodes to that pole of the galvanometer which is connected directly to the battery (see *Fig.* 1). Thus, any current flowing over the surface of the insulation is led direct to the other pole of the battery and does not pass through the galvanometer. *Fig.* 1 shows also the connections for an ordinary insulation test, the calibration of the galvanometer scale being usually made by means of a dry cell connected in series with the galvanometer and

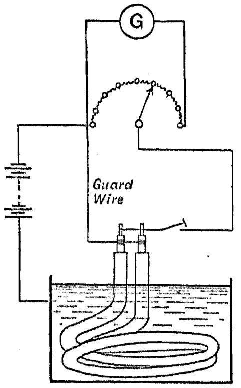


FIG. 1.

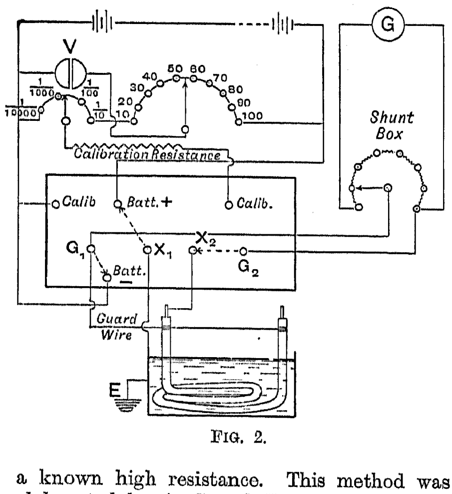


FIG. 2.

a known high resistance. This method was elaborated by A. Campbell, as in *Fig.* 2. In this case the battery of 500 or 1000 volts is connected across the ends of a subdivided resistance of 100,000 ohms. The measurement of pressure is made by means of an electrostatic voltmeter, reading to 130 volts, connected to the switch contacts, which permits of connection of the voltmeter to any convenient fraction of the total pressure. At one end of the resistance box there are subdivisions of 1/10th to 1/10,000th of the total resistance, any of which can be used for calibration of the galvanometer through a known resistance. This eliminates the necessity for a separate dry cell with the accompanying keys.

The switch is arranged so that

(1) The galvanometer may be connected through the known resistance for calibrating, and

(2) That the battery, galvanometer, and insulation under test may be connected in series.

The arrangement also permits of the polarity of the battery being reversed.

The galvanometer is connected across a shunt box of the Ayrton Mather pattern.

Thus, if, when calibrating,

v = the pressure used,
s = the reducing factor of the shunt box,
d = the galvanometer deflection,
r = the value of the known resistance,

and if, when measuring the insulation, the corresponding values and factors are designated

V, S, D, R,

then
$$R = \frac{V \times S \times d \times r}{v \times s \times D}.$$

The galvanometer used for the purpose should be of high resistance and may be either of the moving magnet or moving coil type. The moving magnet type can be made of higher resistance and more sensitive than the moving coil, but when adjusted to be very sensitive is somewhat difficult to work with, particularly in a laboratory where other work is going on. The moving coil galvanometer, having a resistance of about 1000 ohms, is therefore more generally used for the purpose. The order of sensitiveness of a galvanometer of this kind is as follows:

Resistance . . .	1200 ohms.
Scale distance . .	1 metre.
Deflection for one microampere	3500 mm.

Thus, with a battery of 500 volts, a deflection of 1 millimetre on the scale corresponds to a resistance of about one million megohms.

(ii.) *Electrodes and Shape of Samples.*—In the case of a cable, the coil is put into a tank of water for 24 hours maintained at constant temperature, and the resistance measured between the cable core and water. With other samples, such as varnish and hard insulating materials, the shape of sample and electrode will naturally vary with the type of material. Varnishes are usually tested by coating a smooth brass plate which can receive the

requisite number of coats and be stoved as required. The brass plate itself forms the lower electrode, and the upper is conveniently made by a pool of mercury confined in a ring resting on the upper surface of the varnish. Various alternative methods have been used, mainly with a view to dispensing with the use of mercury, generally by means of a semi-plastic material such as rubber or blotting-paper covered with tin foil, but as a rule the pressure required to ensure intimate contact over the surface to which it is applied is sufficient to compress and distort the sample.

For hard insulating materials, the tests are usually made on sheets, the electrodes are usually pressed or pasted on to the sheet, but more modern practice tends to the use of mercury electrodes. The question has been investigated by a Committee of the British Electrical and Allied Industries Research Association, which has adopted, with slight modifications, the form of electrode designed by Melsom and Booth. This is shown in *Fig.* 3, and consists of a steel cup which is filled

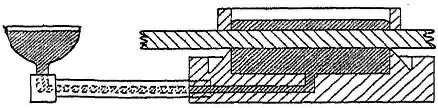


FIG. 3.

with mercury and on which the test sample is placed. The tube at the side provides for maintaining the level of the mercury slightly higher than the edge of the cup. The possible inclusion of air bubbles between the mercury and the sample is easily avoided by tilting the sample slightly when placing it in position. This electrode is suitable for testing any flat piece of insulating material, the upper electrode being formed by a pool of mercury in a ring resting on the upper surface of the sample.

For other samples and for extreme conditions, the electrodes must be specially arranged. For tests at high temperatures, such as are required for sparking plugs, electrodes of metal filings or fusible metal have been found satisfactory. For such samples as tubes, rods, porcelain insulators, etc., it is usually possible to arrange the electrodes so that a satisfactory test can be made.

The British Electrical and Allied Industries Research Association[1] has drawn up a specification, based on a large number of tests, of the most convenient forms of samples for composite moulded materials, and has specified the tests which are desirable. The form of sample, complete with electrode, is shown in *Fig.* 4. The annular groove at the top of the sample is used as a guard ring. The dimensions specified are shown in the figure. If, therefore, the measure of resistance between the upper

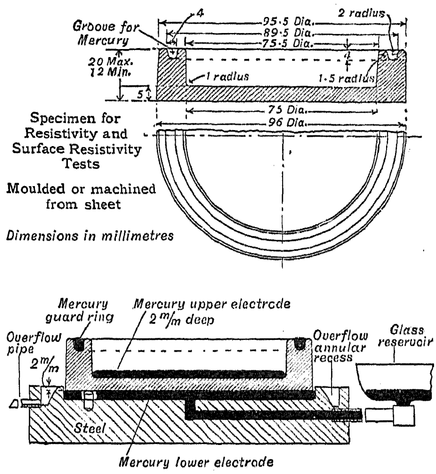


FIG. 4.

and lower electrodes is R (megohms), the specific resistance (megohm-cm.) is

$$\frac{R \times \pi(7\cdot5)^2}{0\cdot5 \times 4} \text{ megohm-cm.}$$

When the resistivity is calculated on the area of the upper electrode the result will be low, owing principally to the increased area in contact with the lower electrode. The error will be approximately 15 per cent, and if this degree of accuracy is desired the result should be corrected accordingly.

Sheet Materials.—The resistivity of sheet material shall be tested when possible on a specimen of the shape and dimensions shown in *Fig.* 4. When it is impracticable to obtain a specimen 20 mm. thick the over-all thickness may be reduced to a minimum of 12 mm., the thickness of the base being 5 mm. in every case. The other dimensions shall be in accordance with *Fig.* 4. The specimen shall be set up, with suitable electrodes, generally in the manner indicated in *Fig.* 4.

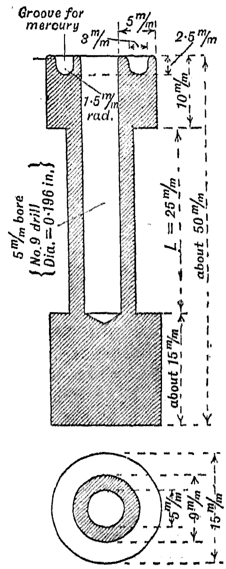


FIG. 4A.

[1] *Technical Publication Report B/51 of the British Electrical and Allied Industries Research Association.*

All the surfaces of the specimen shall be machined smooth, but not polished.

Rods.—The resistivity of a rod shall be tested by boring out the rod so as to form a deep cut and turning the outside surface. The dimensions shall be as shown in *Fig.* 4A.

The specimen shall be set up for testing, with suitable electrodes as in (*a*), generally in the manner indicated in *Fig.* 4B.

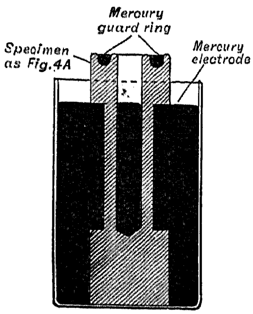


FIG. 4B.

If L = length in millimetres,
r_1 = inner radius in millimetres,
r_2 = outer radius in millimetres,
R = measured resistance,
ρ = resistivity,

then $$\rho = \frac{2\pi RL}{\log_e (r_2/r_1)}.$$

If the specimen is made accurately to the dimensions shown in *Fig.* 4A, then

$$\rho \doteq 1{\cdot}07RL.$$

Tubes.—For testing the resistivity of a tube, one end shall be plugged, with a good insulator, so as to prevent leakage of mercury. The specimen shall be set up and the test carried out in a manner similar to that specified for rods.

Conditions of Test.—The resistance shall be measured, at the end of each minute, over a period of ten minutes' electrification, at a potential difference of 500 volts. The resistivity shall be expressed in megohms for a centimetre cube after one, two, and ten minutes' electrification respectively.

(iii.) *Surface Resistivity* (*Surface Leakage*).—The resistance is measured between the annular groove and the mercury pool, the mercury inside the specimen serving as a guard ring to eliminate the effect of the major part of the leakage through the material itself (see *Fig.* 5). Thus, if the distance from the groove

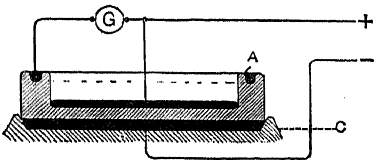


FIG. 5.

to the edge of the pool is L, the outer diameter of the specimen D, and the resistance between A and C, r, the surface resistivity expressed as the resistance between the opposite edges of a square of any dimensions is

$$\frac{r \times \pi D}{L}$$

to a close approximation.

Conditions of Test.—The surface resistance shall be measured, after one minute's electrification, at a potential difference of 500 volts, and the surface resistivity shall be expressed in megohms for a centimetre square.

For temperatures not exceeding 100° C. mercury electrodes shall be used. For higher temperatures suitable fusible metal electrodes shall be used, if necessary, according to the temperature.

In the case of rods and sheets all the surfaces of the specimen shall be machined smooth, but not polished. The specimen shall be tested with the surfaces as left by the tool, unless special circumstances render it necessary for them to be finished in a particular manner in service.

It is suggested in the specification for insulation resistance and surface leakage that the specimens shall be tested in the following conditions (*a*) to (*c*).

Except in the case of specimens to be immersed in oil (see (*h*) below), each specimen shall be given a preliminary treatment, being wiped carefully with petroleum spirit, specific gravity not exceeding 0·68 at 15° C.

(*a*) New condition; at a temperature from 15° C. to 20° C.

(*b*) New condition; after remaining in a desiccator for twenty-four hours. The specimen to be tested whilst in the desiccator at a temperature from 15° C. to 20° C.

(*c*) New condition; after remaining in a controlled atmosphere, relative humidity 80 per cent, for twenty-four hours. The specimen to be tested whilst in the controlled atmosphere at a temperature from 15° C. to 20° C.

(*d*) New condition; after heating in an oven for one hour at grade temperature, preparatory to testing in the oven at grade temperature.

(*e*) After immersion in distilled water for one week at a temperature from 15° C. to 20° C., and afterwards removing surface moisture by wiping, preparatory to testing at a temperature from 15° C. to 20° C. within ten minutes of removal from the reagent.

(*f*) After immersion in a solution of salt in water of approximately 10 per cent for one week at a temperature from 15° C. to 20° C., followed by swilling with distilled water, and then removing surface moisture by wiping. The test to be made at a temperature from 15° C. to 20° C. within ten minutes of removal from the reagent.

(*g*) After immersion in a sulphuric acid solution (specific gravity 1·25 at 15° C.) for one week at a temperature from 15° C. to 20° C. followed by swilling with distilled water, and then removing surface moisture by wiping. The test to be made at a temperature from 15° C.

to 20° C. within ten minutes of removal from the reagent.

(*h*) After immersion in mineral transformer oil (specific gravity approximately 0·86 at 15° C.) for one week at grade temperature (but never higher than 100° C.) preparatory to testing in the oil at grade temperature (but never higher than 100° C.).

(*i*) After immersion in any other reagent which may be required to meet special conditions.

(iv.) *Effect of Temperature.*—The change in resistance of insulating materials with temperature is very large, and it is desirable that in all measurements, and especially in those of which the results have to be compared, the temperature should be maintained constant, or, if this is not possible, the actual temperature noted. The actual temperature coefficient of various insulating materials is difficult to determine, since the effect is complicated by the fact that when the material is exposed to a high temperature several factors may be operating. Thus, with a composite insulating material the heating of the sample may result in driving off some of the more volatile constituents with a consequent change in the nature of the material, while in the case of any material which is capable of absorbing moisture, exposure to heat will result in the drying out of the moisture, a process which may be very protracted, and the effects of which are often much larger than the change of resistance of the material itself. Generally, the insulation of all materials falls with increasing temperature. E. H. Rayner [1] gives curves showing that with a coil of cotton-covered wire the insulation resistance, which was at first only 0·6 megohm, when subjected to a temperature rising finally to 130° C., fell rapidly during the first hour and after that rose to 200 megohms at the end of 9 hours. The same coil, tested after 21 hours at a mean temperature of 120° C., varied in the opposite direction, the resistance increasing, largely as a result of a small decrease of temperature. Thus, the increase of resistance during the first 9 hours is obviously due to drying.

Dietrich [2] determined the effect of temperature on various hard insulating materials, and considered that the formula of Koenigsberger and Reichenheim,[3]

$$W = W_0 \epsilon^{-\frac{qt}{(t+273)273}},$$

where W is the resistance at any temperature t, W_0 the resistance at zero temperature, and q a constant for the particular material, applied to all insulating materials.

Curtis [4] gives the following table for various hard materials:

[1] *Proc. I.E.E.* xxxiv. 657.
[2] *Phys. Zs.* ii. 187. [3] *Ibid.* vii. 570.
[4] *Bull. Bur. Stds.* xi. 373.

Table showing the Decrease of Volume Resistivity with Increasing Temperature

ρ_{20} = Volume Resistivity at 20° C.
ρ_{30} = Volume Resistivity at 30° C.

Sample.	ρ_{30}.	ρ_{20}/ρ_{30}.
Sealing wax	$1·9 \times 10^{16}$	0·9
Mica (India ruby, slightly spotted)	$1·0 \times 10^{17}$	1·0
Insulate No. 2	$8·4 \times 10^{15}$	1·0
Hemit (*a*)	$2·1 \times 10^{10}$	1·2
G.E. No. 55A	$1·3 \times 10^{15}$	1·2
Moulded mica	$2·0 \times 10^{15}$	1·2
Mica (brown African clear)	$1·7 \times 10^{15}$	1·2
Hemit (*b*)	$5·7 \times 10^{9}$	1·4
Tegit	$1·9 \times 10^{12}$	1·4
Gummon	$3·4 \times 10^{12}$	1·4
Shellac	$1·2 \times 10^{16}$	1·5
Ivory	$1·5 \times 10^{8}$	1·6
Vulcabeston	$1·2 \times 10^{10}$	1·6
Unglazed porcelain	$2·2 \times 10^{14}$	1·6
Yellow stabalite	$4·3 \times 10^{13}$	1·6
G.E. No. 40	$6·7 \times 10^{14}$	1·6
White celluloid	$1·4 \times 10^{10}$	1·8
Murdock No. 200	$3·2 \times 10^{15}$	1·8
Murdock No. 201	$5·2 \times 10^{15}$	1·8
Mica (clear)	$1·1 \times 10^{17}$	2·0
Parawax	$2·6 \times 10^{15}$	2·0
Redmonite No. 183	$14·7 \times 10^{13}$	2·0
Black electrose	$6·0 \times 10^{13}$	2·0
Vulcabeston (*a*)	$8·4 \times 10^{9}$	2·3
Yellow electrose (L)	$4·7 \times 10^{15}$	2·3
Bakelite No. 140	$7·5 \times 10^{6}$	2·4
Bakelite micarta	$2·7 \times 10^{10}$	2·4
German glass	$2·0 \times 10^{13}$	2·5
Halowax	$1·3 \times 10^{13}$	2·5
Yellow electrose (D)	$3·3 \times 10^{15}$	2·6
Bakelite No. G. 5074	$2·3 \times 10^{10}$	2·6
Red fibre	$7·8 \times 10^{9}$	2·6
Bakelite No. L. 558	$1·0 \times 10^{16}$	2·6
Mica (India ruby, stained)	$2·2 \times 10^{13}$	2·7
Opal glass	$5·0 \times 10^{11}$	2·8
Yellow condensite	$1·7 \times 10^{10}$	2·9
Black condensite	$4·8 \times 10^{10}$	2·9
Tetrachlornaphthalene	$1·7 \times 10^{13}$	2·9
Glyptol	$7·4 \times 10^{15}$	3·0
Dielectrite	$2·2 \times 10^{12}$	3·0
Hard fibre	$1·0 \times 10^{10}$	3·2
Plate glass	$1·0 \times 10^{13}$	3·2
German glass (special)	$5·0 \times 10^{13}$	3·5
Bakelite No. 4	$3·3 \times 10^{9}$	3·6
Bakelite No. 190	$4·2 \times 10^{10}$	3·6
Bakelite No. 150	$1·9 \times 10^{12}$	3·6
Paraffined maple	$1·9 \times 10^{10}$	3·6
Paraffined poplar	$2·3 \times 10^{11}$	3·6
Rosin	$1·7 \times 10^{16}$	3·6
Kavalier glass	$2·0 \times 10^{15}$	4·5
Sulphur	$3·9 \times 10^{16}$	4·9
G.E. No. 55 R.	$5·9 \times 10^{15}$	5·1
Bakelite No. 5200 R.G.R.	$1·2 \times 10^{11}$	5·3
Khotinsky cement	$2·1 \times 10^{14}$	11·0
Yellow beeswax	$4·0 \times 10^{14}$	16·0

The insulation resistance of cables, either impregnated paper or vulcanised rubber, has a temperature coefficient over a small range of temperature near 15° C., of from 10 per cent to

20 per cent for 1° C., and it is clear that the effect of temperature has to be considered in any measurement of insulation resistance.

(v.) *Effect of Voltage.*—The constancy of the relation between voltage and current for different values of the testing voltage has been the subject of much discussion. In an absorbent material, such as cotton, Evershed [1] found that a tenfold increase of the voltage resulted in a decrease of apparent resistance in the ratio of 2·95 to 1. It appears probable that this large change is due mainly to the presence of moisture. Appleyard,[2] working with materials such as celluloid and gutta percha, and using both tinfoil pressed into contact and mercury electrodes, found that while with tinfoil the changes were large, with mercury electrodes on the same samples they were very much smaller. He concludes that with tinfoil electrodes the effect of the increased voltage is to bring the surfaces into more intimate contact and so to decrease the apparent resistance, and that when mercury electrodes are used the dielectric resistance remains sensibly the same under wide variations of voltage. This result is confirmed by Bairsto [3] and later by Curtis.[4] Curtis agrees with Evershed that with porous material the changes are large but with other materials they are comparatively small. He gives a table showing the change of volume resistivity with voltage.

Material.	Thickness of Specimen.	Ratio of Resistance at 50 volts to the Resistance at 500 volts.
	cm.	
Yellow electrose . .	1·27	1·0
Hemit	·73	1·0
Tegit	·78	1·0
Gummon	·62	1·0
Red fibre	1·27	1·0
Hard fibre	1·23	1·0
Redmonite	·13	1·0
Paraffined maple . .	2·3	1·0
Paraffined poplar . .	1·8	1·0
Yellow condensite . .	1·32	1·0
Black condensite . .	1·28	1·0
Bakelite No. 140 . .	·96	1·0
Bakelite No. 141 . .	·99	1·0
Bakelite No. 150 . .	·97	1·0
Bakelite No. 151 . .	·98	1·0
Bakelite No. 190 . .	·96	1·0
Bakelite No. 192 . .	·99	1·1
Bakelite No. G. 5074 .	·95	1·9
Vulcabeston . . .	·60	2·0
Slate	..	1·1
Marble—		
Pink Tennessee . .	2·25	1·4
Blue Vermont . .	2·3	2·0
Italian	2·2	2·5
Opal glass	·17	·7

[1] *Proc. I.E.E.* lii. 64.
[2] *Proc. Phys. Soc.* xiii. 155, and xix. 724.
[3] *Ibid.* xxv. 301.
[4] *Bull. Bur. Stds.* xi. 371.

The case of absorbent materials is met in practice by a specification of the testing voltage, usually 500 volts.

(vi.) *Time of Electrification.*—In most cases there is a considerable difference in the apparent resistance of insulation materials, depending on the time that the voltage has been applied to the specimen. This arises from the fact that along with the true leakage current there is a dielectric displacement current, due to the capacity, and also an absorption current. The displacement current will become negligible in a few thousandths of a second, but the current depending on absorption will affect the values for some time and may not become negligible for several hours. In the case of a length of rubber-insulated cable, where the absorption current will be larger than in many materials, Evershed [5] gives curves (*Figs.* 6 and 7), which show the actual current passing through the galvanometer during test, as compared with the

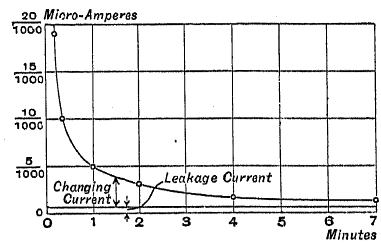


FIG. 6.

absorption current; *Fig.* 6 indicates the first portion of the change, and *Fig.* 7 the same extended to a period of 7 hours. At the end

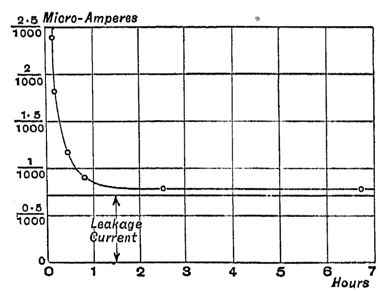


FIG. 7.

of 27 hours the current had fallen to a value which was assumed to be the true dielectric leakage current, and this value is shown on the curves by the horizontal lines.

It should be noted that in practice it is customary to specify that the observation should be taken one minute after the full voltage has been applied. According to

[5] *Proc. I.E.E.* lii. 56.

Evershed's results, which may be taken as typical of the behaviour of a rubber insulated cable, the absorption current at the end of one minute is about six times as great as the true leakage current, and hence the value of the insulation resistance so indicated is only one-sixth of the real value.

This effect of the absorption current depends largely on the order of resistance of the material: if this is low, the absorption current, being only a small fraction of the true leakage current, may be disregarded, but where the resistance is high, the absorption current will mask the real leakage current unless the test is continued for a long period.

Curtis[1] states that where the volume resistivity is less than about 10^{13} ohms, the absorption current gives no greater error than 10 per cent, provided that the resistance is measured at the end of one minute. If, however, the resistivity is about 10^{16} ohms, the absorption current at the end of one minute will probably be equal to the leakage current and may be much larger.

(vii.) *Polarisation Effect.*—In some types of insulating materials there appears to be a polarisation effect due probably to the presence of a small amount of acid, either in the material or on the surface. This is apart from the absorption effect and may affect the values very considerably. It is more usually on the surface of the material, and in this case, owing to the use of a guard ring, an E.M.F. over the surface is taken direct across the terminals of the galvanometer. Reference to *Fig.* 1 will make this clear.

The same effect appears in ceramic materials, such as porcelain, when exposed to a measurement at a high temperature, the effect of polarisation being at times as much as ten times the value of the indication of the leakage current. In such cases it is perferable to use an alternating current method and to measure the resistance by means of an A.C. voltmeter and milliammeter, the quotient of effective voltage divided by effective current being taken as the resistance. This method, however, is only suitable where, as in the case of heated porcelain, the resistance is comparatively low.

(viii.) *Source of Testing E.M.F.*—Owing to the influence of capacity on measurements of insulation resistance, it is essential that the source of E.M.F. should remain constant to a high degree. Great attention, therefore, must be paid to the battery and its connections used for the purpose.

§ (2) Condenser Methods.—For the measurement of very high resistance where the deflection method is not sufficiently sensitive, various methods of connecting the resistance across a condenser are used. The common plan is to connect the high resistance in parallel with a well insulated condenser and an electrostatic voltmeter (see *Fig.* 8). The condenser is charged from a battery and left discharging through the resistance until the reading of the voltmeter falls to one half of its initial value. Then

$$R=\frac{1\cdot442T}{K},$$

where K is the capacity in microfarads, T the time in seconds, and R the resistance in megohms.

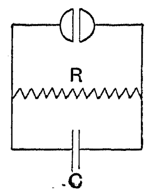


Fig. 8.

For if E be the potential difference at any moment, Q the charge in the condenser, and I the current at any time t, we have

$$\frac{Q}{K}=E=RI$$

and $$I=-\frac{dQ}{dt}=-K\frac{dE}{dt};$$

$$\therefore \frac{dE}{dt}+\frac{E}{KR}=0,$$

$$E=E_0e^{-\frac{t}{KR}}=\frac{E_0}{2}, \text{ when } t=T.$$

Hence $$\frac{T}{KR}=\log_e 2.$$

Thus $$R=\frac{T}{K\log_e 2}=\frac{1\cdot442T}{K}.$$

Sometimes the body to be measured, *e.g.* a short length of cable, may itself be used as a condenser. This gives maximum sensitiveness of test but involves a determination of the capacity of the sample.

It should, however, be pointed out that there are considerable difficulties in the way of obtaining reliable results with such methods. First, with material of such very high insulating properties the absorption current will be so very much greater than the actual leakage current that any results are valueless unless the observations extend over several hours. Again, the insulation of the condenser and the voltmeter should be at least as high and preferably higher than the resistance being tested. A separate determination of the effect of the leakage through the condenser and the voltmeter should be made and applied as a correction to the final result. An air condenser is best suited for the purpose; this, however, must be of large dimensions in order to have the necessary capacity.

§ (3) Portable Insulation Testing Sets and Instruments.—Apart from the tests of insulation that can be made in the Laboratory, the demand for a portable instrument which will enable tests of machines, cables, installations, etc., to be carried out quickly with a fair degree of accuracy has led to the development

[1] *Bull. Bur. Stds.* xi. 374.

of specially designed instruments of patterns most satisfactory for this purpose.

(i.) *The Ohmmeter.* — The first instrument of this type which enabled the resistance of a circuit to be measured by means of a single observation was the ohmmeter designed by Ayrton and Perry.[1]

This instrument (see *Fig.* 9) consisted of two coils placed usually at right angles to each other, and both acting on a pivoted magnet to which was attached the pointer. One coil A is connected through a resistance R across the battery or other source of pressure, and the other coil B across the same source of supply but in series with the unknown resistance X which is to be measured. Then, if there is no mechanical control the needle will take up a position depending on the ratio of the currents through the two coils. The instrument was intended for the measurement of comparatively low resistances, but in 1889 Evershed produced an instrument based on the same principle but for measurement of insulation resistance. In this instrument the testing E.M.F. was supplied from a portable hand generator and the effects of the earth's field were counteracted by means of a permanent magnet floating under the needle.

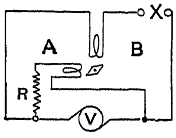


FIG. 9.

Further types were :

(1) An astatic system, and
(2) A moving soft-iron system.

(ii.) *The Megger.*—These types were in use for a number of years ; they had the limitation, however, that in the moving-magnet type the needle was liable to be demagnetised, and the readings of the soft-iron type to be affected by stray fields. The modern form, called the Megger, in which the moving-coil system is used, was introduced in 1904. In this case the two coils, fixed together at right angles to each other, work between the poles of a permanent magnet, the same magnet being used for the generator. A full diagram of this instrument is shown in *Fig.* 10.

One of the difficulties in a moving-coil system used in this way is to ensure that any control exerted by the leading-in wires or strips used to carry the current into the coil should be negligible. These strips, therefore, are very thin and flexible, and are supported by small drums to prevent entangling during transit.

To compensate for stray fields an additional coil is put in series with the pressure coil and wound in the opposite direction. This coil rotates outside the magnetic field of the instrument and, since it is equally affected but in the reverse direction by any stray fields, it thereby provides compensation for this effect.

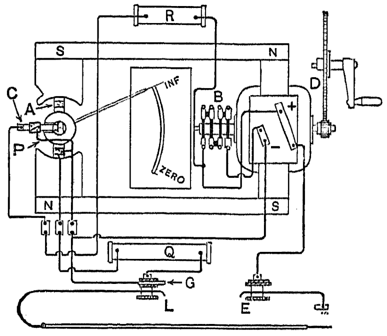


FIG. 10.

The hand generator provided is wound to give pressure up to 1000 volts when the handle is rotated at a speed of approximately 100 r.p.m.

To ensure the constancy of pressure necessary for testing circuits which may have a considerable capacity, the generator is pro-

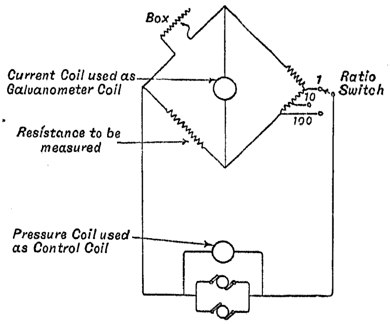


FIG. 11.

vided with a slipping clutch which maintains a constant speed when the handle is turned at any speed above that at which the clutch operates.

A further adaptation allows the use of the instrument as a resistance bridge, the

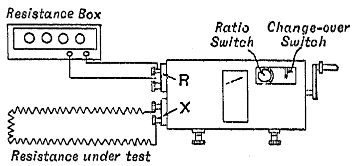


FIG. 12.

generator being the source of pressure, and the moving-coil system of the instrument the galvanometer (see *Figs.* 11 and 12).

The ohmmeter principle is used also by

[1] *Journal I.E.E.* xi. 254.

Evershed for the measurement of low resistance, the two moving coils being in this case connected one across a standard resistance and the other across the resistance to be measured, various ratios being provided to enable resistances from a few microhms to 1 ohm to be measured by direct deflection of the pointer. The arrangement is shown more clearly in *Fig.* 13.

The "Metrohm" sets use essentially the same principle as the Evershed instrument, but separate magnetic fields are provided for

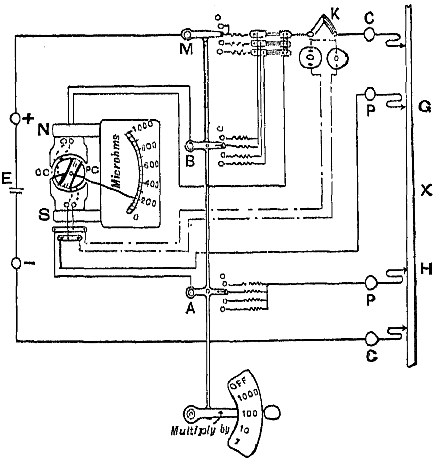


FIG. 13.

the ohmmeter and the generator, and a rather longer scale is obtained.

(iii.) *The Omega.*—The Omega set, while using the ohmmeter principle, embodies a modification of the ordinary practice made by G. W. Harris. Here the coils, instead of being narrow, are extended to cover the whole width of the air gap for the purpose of getting larger working forces and a greater useful angle of deflection. With this instrument a wide series of ranges can be obtained by means of switches which introduce a suitable series of resistance, and/or interchange the functions of the two coils. These ranges vary from a resistance of 0·01 ohm to 100 megohms, all of the measurements being made on the same instrument. A diagram of the connections of the instrument, when used on the insulation resistance ranges, is shown in *Fig.* 14.

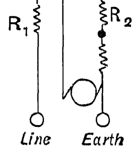


FIG. 14.

Here D and C represent the two moving coils, C in this case being the control coil. The resistances R_1 and R_2 are varied in order to provide different ranges. For measurement of low resistance, however, the connections are as in *Fig.* 15, in which case D is the control coil and is shunted by a resistance of 10 ohms, the resistance R in series with C being variable for different ranges.

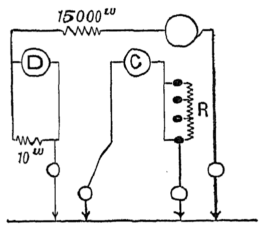


FIG. 15.

(iv.) *The Ohmer.*—The Ohmer, invented by Cox in 1902, employs the electrostatic principle, and is therefore distinct from the instruments already mentioned. The description given by the makers is as follows:

"The generator has one of its terminals attached directly to one quadrant A of the electrostatic ohmmeter and the same terminal connected through a resistance R (which is wound on porcelain insulators, and contained in the case of the instrument) to the other quadrant B of the ohmmeter. The other terminal of the generator is connected to the vane V. In actual practice, however, four sets of quadrants are used, the opposite pairs being connected together. The vane V is connected to the line to be tested, and the quadrant B to earth. When the insulation resistance between the earth and line is infinite there is no current flowing through the resistance R, and the vane V takes up the position shown in the diagram (*Fig.* 16). When a current flows from E to L there is a drop of potential due to the current flowing through R, the quadrants A and B are then at different potentials, and the vane V takes up a new position, which is determined by the difference of potential and the shape of the vane. The vane V is shaped approximately as shown, so as to be in stable equilibrium at all parts of the scale.

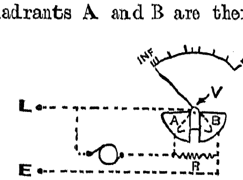


FIG. 16.

"It will be easily seen that the indications of the instrument are independent of the voltage of the generator, so that the exact speed at which the handle is turned is unimportant."

(v.) *Other Portable Instruments.*—Other portable instruments that are used for measuring insulation resistance consist essentially of a pivoted or suspended sensitive high-resistance moving-coil microammeter, the measurement being made by means of the instrument in series with a testing battery and the insulation to be measured. The sets are generally supplied with appropriate suitably controlled shunt boxes, etc., and can be used also

for a large variety of current, pressure, and resistance measurements.

§ (4) Leakage Indicators.—For an indication of the condition of the insulation of the main cables in a supply system, an instrument, generally called a leakage indicator, is used. This, for a D.C. circuit, consists of a moving-coil milliammeter, connected across a resistance, as in *Fig.* 17. It will be seen that the resistance $r_1 r_2$ is connected across the supply mains, and with both keys K_1 and K_2 closed, the moving coil is connected through a protective resistance to earth. Thus, if the insulation resistance of the two cables to earth is equal, no current will flow through the instrument, but if one is lower than the other, the pointer will deflect in one direction or the other. To measure the insulation of one cable, the circuit controlled by K_1 is broken, and I, the current indicated by the instrument, will be inversely proportional to the sum of the resistances r_2, R, and L_1. Hence

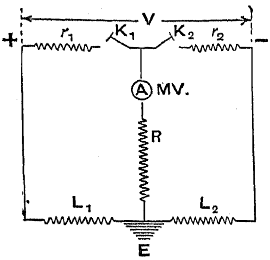


Fig. 17.

$$L_1 = \frac{V}{I} - r_2 - R,$$

and similarly for the other cable

$$L_2 = \frac{V}{I} - r_1 - R.$$

The instrument works off and is calibrated for the normal pressure of the supply, and tables are given whereby the value of L_1 and L_2 can be directly deduced from the reading I. It should be noted, however, that since L_1 and L_2 are not directly proportional to V/I a small error in the calibration of the milliammeter will affect the evaluation of L_1 and L_2 to a much larger extent.

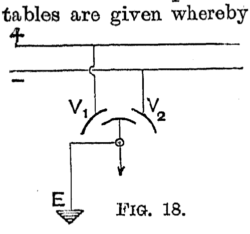


Fig. 18.

The diagram (*Fig.* 17) applies to a two-wire D.C. circuit with both poles insulated. The same principle is, however, applied to a three-wire circuit with the middle wire earthed, or a two-wire circuit with one pole earthed.

For alternating-current circuits, a system of electrostatic voltmeters is used (see *Fig.* 18), the two quadrants V_1 and V_2 being connected to the supply and the needle to earth. This system has the advantage of dispensing with the auxiliary resistance boxes required for the moving-coil type.

S. W. M.

RESISTANCE, PRACTICAL MEASUREMENT OF ELECTRICAL

For the measurement of electrical resistances where the great accuracy [1] necessary in the case of fundamental comparisons is not required, and where the range of resistance is from 1 ohm to 100,000 ohms, the arrangement known as the Wheatstone Bridge is in universal use. Direct deflection methods have been used in places where no other apparatus was available, but a bridge method is so much easier and quicker that a portable bridge is now a common piece of apparatus in everyday use.

For the higher resistances, say up to 10 megohms, the same method is used, but measurements of this kind require:

(1) High testing pressures.
(2) Special precautions with regard to the insulation of every part of the circuit.

For such tests, it is usual to build up a Wheatstone Bridge out of a number of highly insulated coils, using ratio arms of a high resistance, say 10,000 ohms, a high-resistance galvanometer and a testing pressure of 50 volts or upwards. Tests of this kind, however, may be regarded as somewhat special in nature, and are not covered by the methods and apparatus described below.

A simple direct deflection method is as follows:

A voltmeter of known resistance is connected across the supply pressure with the unknown resistance X in series (see *Fig.* 1). Thus if D_1 is the reading of the voltmeter when connected directly across the supply, and D_2 the reading when the resistance is put into series, and R the resistance of the voltmeter, the value of the resistance X will be

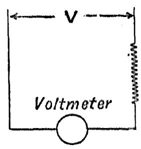


Fig. 1.

$$X = R\left(\frac{D_1 - D_2}{D_2}\right).$$

Conversely the method may be used for the measurement of the internal resistance of a voltmeter or galvanometer, the value of X in this case being known.

§ (1) Wheatstone Bridge. (i.) *Plug Pattern.*—The earliest arrangement of the network of the Wheatstone Bridge is shown in *Fig.* 2, where P and Q, the ratio arms, can be raised in powers of 10 from 10 ohms to 10,000

[1] For measurements of the highest accuracy see "Electrical Measurement, Systems of," §§ (9)-(22), and "Resistance, Standards and Measurement of."

ohms, and R can be continuously raised in steps of 0·1 or 1 ohm up to 10,000 ohms.

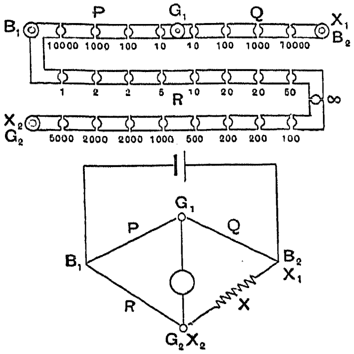


FIG. 2.

Two double terminals are provided at the points X_1B_2 and X_2G_2 for connection of the two wires required at these points. The coils are connected to the underside of brass blocks, and between the blocks is a conical hole which enables them to be short-circuited by means of a taper plug. The blocks are usually provided with central holes into which the plugs are put when not in use.

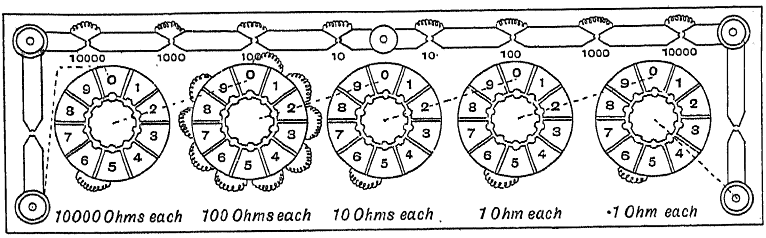


FIG. 3.

The value of the unknown resistance X is RQ/P.

Various modifications have been made from time to time with a view to obtaining a higher degree of accuracy and convenience in use.

(ii.) *Arrangement of Coils in Arm* R.—To eliminate the contact resistance of the large number of plugs in the ordinary bridge and to facilitate reading, Messrs. Elliott Bros. introduced a form of bridge (*Fig.* 3) in which the resistances in the arm R were arranged around a centre block in dial form. As will be seen from *Fig.* 3 the arrangement requires a larger number of coils than the older form, but has the advantage that all the coils in each decade are of equal value, and can therefore be easily intercompared, and that only one plug is required in each dial. A further modification was that introduced by Anthony with the object of facilitating intercomparison between successive decades. *Fig.* 4 shows one decade of this pattern, and it will be seen that the 10 coils are arranged so that the insertion of a single plug will put any number of coils in series, and that if a plug is put in every hole with the exception of that norm-

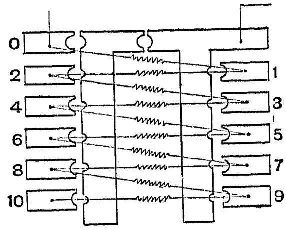


FIG. 4.

ally joining the two bus-bars, the coils are all connected in parallel. By this means it was intended that with the coils in series the sum of the coils in any one decade could be compared with one coil in the next higher decade, and when they were connected in parallel with one of the next lower decade. Other combinations can be obtained by varying the number of coils connected in parallel. Leeds and Northrupp have introduced methods of connecting the coils so that the decade system may be obtained with the use of only four or five coils. (This method is also claimed by Bombe.[1]) In the four-coil method by various combinations of coils of 1, 2, 3, and 3 ohms respectively (or multiples or submultiples of these) any value between 1 and 9 can be obtained with the use of a single plug. The scheme is shown in *Fig.* 5, from which it will be seen that by an ingenious arrangement of

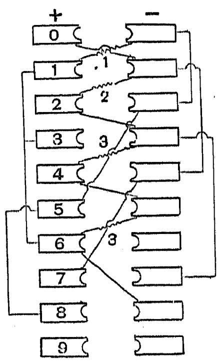


FIG. 5.

[1] *Instrumentenkunde*, xxviii. 301.

cross connections the insertion of the plug between any pair of blocks gives a combination of coils whose value corresponds to that indicated on the block.

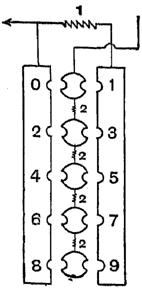


FIG. 6.

In the five-coil method (*Fig.* 6) any value of resistance up to 9 may be obtained by various combinations of coils of values 1, 2, 2, 2, 2 respectively. A similar arrangement was described by Feussner,[1] but in this case the order of the plug holes was somewhat complex.

(iii.) *Arrangement of Ratio Arms* P *and* Q.—The arrangement of the ratio arms shown in *Fig.* 2 is still used largely, but various modifications have been made from time to time mainly with a view to reducing the number of plugs in circuit. The first change was to arrange each arm as in *Fig.* 7, where the

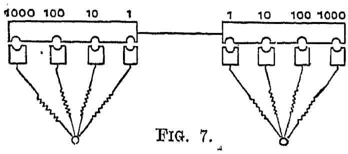


FIG. 7.

desired value of resistance is connected to a bus-bar through a single plug. Schone[2] described an arrangement shown in *Fig.* 8, where the ratio coils are connected to (i.) a central bus-bar, and (ii.) to individual blocks, the plugs making connections from the central blocks to the centre bus-bars. In the ordinary way only one plug is used for each ratio coil; to obtain an even ratio of, say, 100 : 100, the plugs would be inserted one from the block 100 to the bar A, and the other from 100 to the bar B. The scheme has the great advantage that the ratio arms can be reversed merely by changing over the plugs. Any uneven ratio can be obtained in four different ways, for example the ratio 10 : 1000 can be made up by using either of the two tens with either of the two 1000-ohm coils. Ratios of 1 : 5 and 1 : 20, etc., are obtained by the use of three plugs; for example, if the two 1000-ohm coils are plugged at the bar A, and a single 10-ohm coil to the bar B, the ratio is 50 : 1.

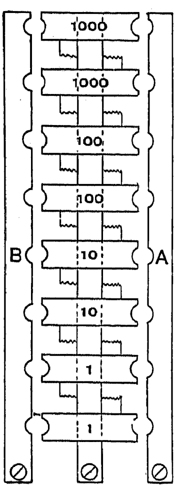


FIG. 8.

[1] *Instrumentenkunde*, xviii. 133.
[2] *Ibid.* xviii. 134.

(iv.) *Types of Plugs used.*—Practice varies considerably regarding the diameter and taper of the plugs used; ten plugs each of a different make had the following dimensions, L being the length of the surface, T_1 the lesser, and T_2 the greater diameter of the plug.

Type.	L.	T_1.	T_2.	Taper $=\frac{T_2 - T_1}{L}$ approx.
	cm.	cm.	cm.	
(1)	1·5	0·64	0·74	1-15
(2)	1·0	0·52	0·62	1-10
(3)	1·25	0·693	0·880	1-7
(4)	1·25	0·59	0·69	1-12·5
(5)	2·0	0·825	0·985	1-12·5
(6)	1·5	0·695	0·860	1-9
(7)	1·43	0·475	0·54	1-18
(8)	1·59	0·54	0·665	1-12·7
(9)	1·1	0·7	0·9	1-6
(10)	1·25	0·637	0·742	1-12

Generally the contact will be better with the finer taper plug since greater pressure can be exerted, but when the taper is too fine, say 1-15, the use of undue pressure when inserting the plug results either in slight loosening of the blocks, or else the plug becomes bound and is difficult to remove. On the other hand, if the taper is not sufficiently fine the plugs become loose with slight vibration. In the case of some plug boxes made at the National Physical Laboratory, the question of the taper was considered, and it was decided that a ratio of 1 to 12 was the most satisfactory. Some leading makers, however, use a considerably coarser taper than this with satisfactory results, and it is clear that the question of taper is largely one of precision in manufacture.

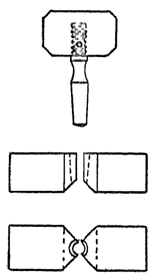


FIG. 9.

The construction of the ordinary plug and block is shown in *Fig.* 9. The plug is usually of harder brass than the block. The insulating handle is fixed in some cases by being moulded direct on to the plug and in others it is screwed and further secured by means of a pin as shown in *Fig.* 9. The blocks are usually rounded or chamfered on the underside to increase the leakage path and also to facilitate cleaning.

Gambrell invented a greatly improved plug and socket, which is shown in *Fig.* 10. Here the two halves of the sockets corresponding to the blocks in the ordinary pattern, are conical on the outer surface and have a

cylindrical hole. The plug consists of a metal cylinder conical inside to fit the outer surfaces of the socket, and has a centre pin which fits into the hole in the socket. Thus when the plug is pressed on to the socket, the socket is forced into contact with the inner pin and both inside and outside surfaces are in contact. Besides increasing the area of contact the inner pin prevents the two halves of the socket being forced together by undue pressure, and thus ensures stability of contact. The plug itself is covered with insulating material.

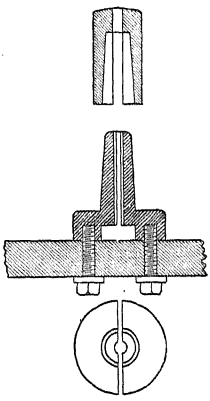


FIG. 10.

§ (2) WHEATSTONE BRIDGE, DIAL PATTERN BRIDGE WITH BRUSH CONTACTS.—As an alternative to the method of making contacts by means of plugs, the sliding brush contact type is now very largely used. The usual construction is to have brass studs and laminated copper brushes. The connection from the brush is sometimes made by means of a flexible connection inside the box from the spindle carrying the brush to the appropriate terminal, but more usually by a ring over which the other end of the brush moves. An example of this construction is shown in *Fig.* 11; the separate strips joining the brush are kept apart by means of distance pieces, thus allowing each strip to make independent contact on the stud. Various other metals are, however, employed at times for the brushes and studs. Thus, (i.) studs capped with gold and silver alloy, (ii.) phosphor bronze brushes, (iii.) silver-tipped brushes and contact surfaces are all used by various makers with uniformly good results for the purpose for which the boxes are designed.

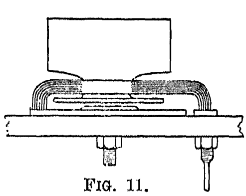


FIG. 11.

Another pattern is illustrated in *Fig.* 12. This has a double brush which makes contact between the studs and an upper plate, thus decreasing the total diameter of the dial and equalising the pressure on the contacts.

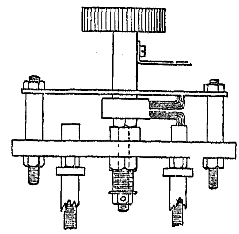


FIG. 12.

The use of a thin film of high-grade oil is recommended to maintain good contact between the brushes and the studs.

A convenient arrangement of dial form resistance bridge is shown in *Fig.* 13. Here the coils in arm R are controlled by five switch dials and the ratio arms by means of two plugs, and the connections are arranged to permit of easy checking of the coils. The centre terminal A on the ratio arm bus-bar and the terminals t_1 and t_2, etc., are not required for the ordinary use of the bridge, but are available for checking the value of any of the ratio coils or dials. In checking the resistance of the ratio coils connection would be made to the terminals A and X_2 or t_1, the value of the inside connecting pieces being included in the value obtained for any coil in the ratio arm, as it would be included in the normal use of the bridge. The value of all or any one of the dials can be measured from the appropriate terminals between t_1 and X_1. The importance of allowing proper means for checking is not always realised by the makers of bridges, and in some cases the additional terminals are not supplied and the battery and galvanometer keys, instead of being taken from the points shown in *Fig.* 13, are inserted between the points AG_2 and X_2B_2 respectively, with the result that a determination of the values of the individual coils must include the resistance of the connecting wire and the tapping key.

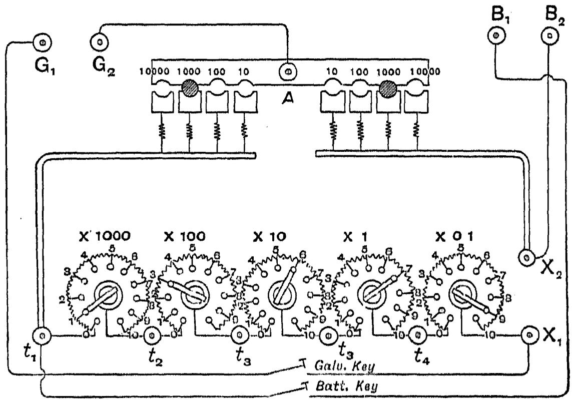


FIG. 13.

A further addition to the high-grade Wheatstone Bridge is a slide wire usually connected between the ratio arms and arranged so that it can be short-circuited when not in use. Looking at *Fig.* 13, if the block A were split at the point A and the slide wire connected between the two halves, the galvanometer connection to G_2 would then be taken to a sliding contact on the wire. The resistance of the slide wire is usually 0·1 ohm, and if this is used with the 100-ohm ratio the value of the change from the centre to one end or the other would be 0·1 per cent. The arrangement is useful for the rapid comparison of a number of coils which have nearly the same value, the small differences between the coils being read off on the slide wire without changing the setting of the dials.

§ (3) Contact Resistance of Plugs and Brush Contacts.—The contact resistance of both plug and brush contacts will naturally depend on the size, fit, pressure, and condition. Under the best conditions the resistance of a plug contact is of the order of 0·00005 ohm and of a brush switch contact 0·0002; in actual use, however, the resistance of the connecting pieces inside the bridge comes in, and the box resistance, that is the resistance with all plugs in or with all switches, is generally of the order of:

For plug dial bridge = 0·0003 ohm per dial.
For switch dial bridge = 0·0015 ohm per dial.

Thus while the contact resistance of the plug type is decidedly less than in the switch pattern, the total box resistance is in either case low in comparison with the resistance used in the R arm, and, except for special cases where a correction can be made, may be considered to be negligible. The switch dial type is much more convenient and rapid in use than the plug type.

§ (4) Low Resistance Measurements. (i.) *Ammeter Method.*—For a rough measurement of a low resistance the ammeter and voltmeter method is sometimes convenient. The resistance to be measured is connected in series with an ammeter and a steady current passed through it, the pressure drop across the resistance being measured by means of a millivoltmeter. Then R = E/I.

(ii.) *Potentiometer Method.*—An alternative and more accurate method is to compare the pressure drop across the unknown resistance and that across a known resistance connected in series with it by means of a potentiometer, or by adaptation of the potentiometer principle such as is shown in *Fig.* 14. In the first position, the points *ab* are connected through the galvanometer to the potential terminals of the standard R, balance being obtained by adjustment of the current by means of the regulating resistances, RR. The connections are then made to connect *cd* through the galvanometer to X, balance now being obtained by shunting the arm Q—that is if X is smaller than R: if X is the greater then the procedure is reversed, balance being first obtained on the arm Q. Inasmuch as shunting one or the other arm will make a slight change in the total resistance of the circuit it is necessary to go to and fro once or twice until an exact balance is obtained, in both positions. Then X = RQ/P, the value for P or Q being the value if and where shunted. To eliminate the effect of thermo-E.M.F. readings should be made with the direction of the currents in both circuits reversed, the mean of the two results representing the correct value.

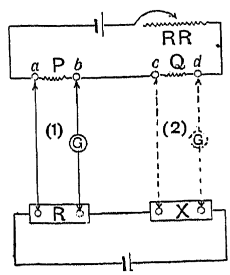


Fig. 14.

§ (5) Kelvin Double Bridge.—This bridge, the theory of which is dealt with in the article on "Resistance, Standards and Measurement of," § (8), is generally used for the measurement of low resistances, its advantages being (i.) that a steady current is not essential, (ii.) that in the form generally used the results can be obtained by a single direct reading and with a minimum of calculation. Two forms of the Kelvin bridge are in common use for general resistance measurements, (i.) in which balance is obtained by varying the resistance of the arms P, p (see *Fig.* 15), and (ii.) by varying the position of the connection from p by means of a slide wire which forms part of the resistance R. The essential feature of the bridge in both forms is that, referring to *Fig.* 15, if $P/Q = p/q$ the formula for the bridge reduces to the simple form X = RQ/P, and the effect of the connecting piece k is eliminated. The bridges here described are designed so that the above relation is maintained automatically during the operation of the bridge.

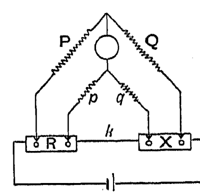


Fig. 15.

The first type for the general measurement of low resistances was developed at the Reichsanstalt. It consists (see *Fig.* 16) of two sets of ratio coils joining the arms P and p, and four double dials by means of which the equal resistances Q and q are simultaneously varied. The standard resistance R and the

unknown X are connected to the pairs of terminals marked R and X. Balance is effected first by setting the ratio coils P and

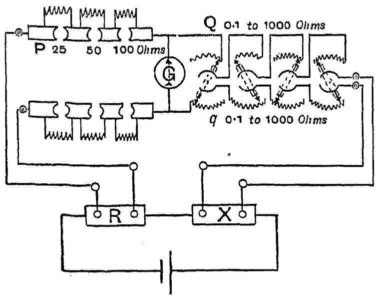


FIG. 16.

p to a suitable (equal) value, and then moving the dials until a balance is obtained. In order to keep the dials to a convenient size, Wolff introduced a special type of double dial and brush, which is shown in *Fig.* 17. The

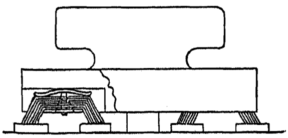


FIG. 17.

short laminated brush is pressed into contact with the studs by means of a coach spring, and is free to rock slightly so that the pressure may be equal on both contact surfaces. Tinsley, using somewhat the same type of contact and brush arrangement as Wolff, introduced a type of double bridge in which the standard resistance R is incorporated in the instrument, a series of values being obtainable. This bridge is shown in diagram in *Fig.* 18. Since the current is limited by the carrying capacity of the internal standard resistances, additional terminals are provided which allow of the bridge being used with auxiliary standards to carry any desired current.

The second type of double bridge is made both by R. W. Paul and Crompton & Co. In this case (see *Fig.* 19) the arms P, p and Q, q are combined on one dial to give a number of ratios, and the arm R is formed of a resistance of 0·01 ohm divided into a number of sections controlled by a plug or switch, and a slide wire of 0·001 ohm along which the sliding contact moves. The slide wire is necessarily of large section, and therefore the effect of wear due to the contact is very small. The current which may be used with this type of bridge is limited by the size of the resistance R to about 50 amperes.

The order of accuracy which can ordinarily be obtained under reasonable conditions with bridges of this kind is 0·02 per cent; some care, however, has to be used with regard to

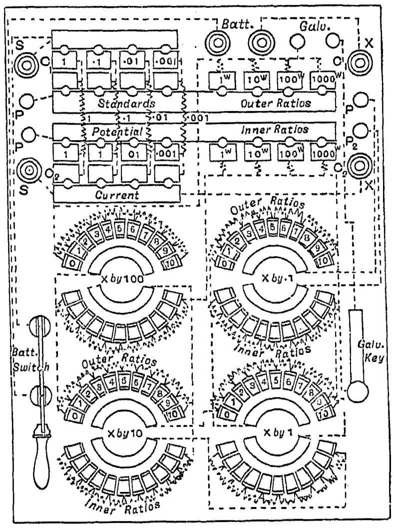


FIG. 18.

the connecting leads from the resistances to the bridge, and the link between the standard and the unknown resistance. Taking first the leads, the resistance of each pair should be small, compared with that of the ratio arms, and should have approximately the same ratio as that of the two resistances being

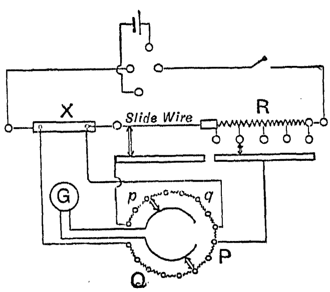


FIG. 19.

compared. Thus when comparing, say, a 0·01 ohm resistance with a 0·001 ohm resistance with ratios of 1000/100, if the connecting leads each had a resistance of 0·1 ohm this would cause an error of 0·09 per cent in the reading, whereas if the leads were 0·1 ohm and 0·01 ohm respectively this error would be eliminated. It frequently happens in practice that the resistance to be measured is necessarily at a considerable distance from the apparatus and the connecting leads are small; in such cases the potentiometer method is preferable.

The magnitude of the connecting link between the two resistances will not affect the readings if the adjustment of the bridge is such that the ratio $P/Q = p/q$ is exact. If, however, there is a slight inequality such as almost invariably exists in practice, the effect of the connecting link may be considerable. Thus, taking the equation of balance[1]

$$X = \frac{RQ}{P} - kp\left(\frac{q/p - Q/P}{p+q+R}\right),$$

if there is an error of adjustment in the coils amounting to 0·02 per cent when nearly equal resistances are being compared with a connecting link k of five times the resistance of either R or X, then the second term on the equation does not disappear and the error of reading would be 0·05 per cent.

The bridges described above are designed for the measurements of a wide range of resistances. Where a higher degree of accuracy than 0·02 per cent is required with resistances that are not suited for connection to the bridges described in the article referred to above,[1] it is usual to build up a bridge out of a number of standard coils, and to obtain a balance by shunting one or the other pair of arms. In special cases, such as a manufacturer's test-room where a large number of resistances, all of equal value, are being adjusted, it is the practice to employ a simple bridge which gives a reading over only a short range. This usually consists of the arrangement shown in *Fig.* 20, where the centre position of the ratio arms is varied over a small portion of the total range of the bridge; the total variation of the dial is ± 1 per cent, but this can be modified to suit individual requirements.

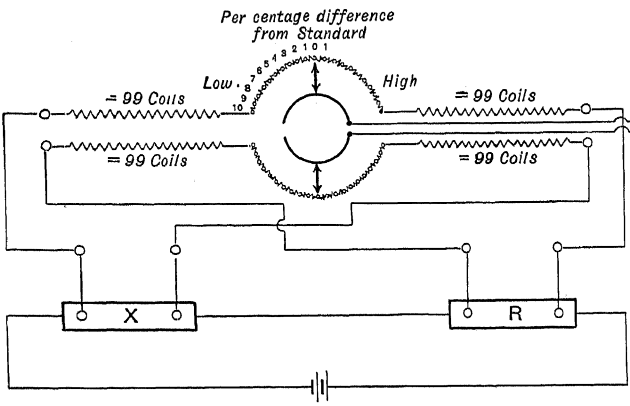


Fig. 20.

§ (6) Effect of Thermo-E.M.F.—In all measurements of resistance precautions have to be taken with regard to thermo-E.M.F. in the circuits. When a material such as manganin, which has a low thermo-E.M.F. against copper, is used for the construction of the coils in the bridge, this effect can almost be neglected, but in practically all measurements of low resistances it is necessary to take readings with the current flowing in opposite directions.

In the use of a Wheatstone bridge for the measurement of a resistance coil, it is the ordinary practice to make the key in the battery circuit first, making that in the galvanometer circuit a short time afterwards. This is to allow the effect of any self-induction in the resistance to subside before the measurement is made.

Thermo-E.M.F., however, also affects the reading, and the question of which of the two keys should be made first depends on the exact condition of use. With a resistance having thermo-E.M.F. but little or no self-induction it is preferable to make the galvanometer key first, when the effect of thermo-E.M.F. will result in a displacement of the zero point of the galvanometer: if the new zero point is taken as being correct, the reading of the bridge will be independent of the value of the thermo-E.M.F. The effect of self-induction when the battery key is made will cause only a momentary displacement of the galvanometer spot. If, however, the self-induction of the resistance is very high, it is desirable to make the battery key first, and if there is also an appreciable thermo-E.M.F., to take readings with the current flowing in both directions.

Generally, in most resistance coils the self-induction is low.

§ (7) Construction of Coils.—Manganin is now almost invariably used for high-grade resistance boxes, having superseded materials such as platinum-silver and platinoid. Constantan, or other forms of the 60 : 40 copper-nickel alloy, is used considerably, but, on account of its high thermo-E.M.F. against copper, not usually for the highest grade boxes, except, of course, where the coils are of very high resistance, when it presents considerable advantages, or in cases where the thermo-E.M.F. can be compensated.

Resistance coils up to 10,000 ohms are

[1] See "Resistance, Standards and Measurement of," § (8).

usually wound on brass tubes, two layers of wire being used, as a rule, except for the coils of higher resistance, when four or six layers are required. Various methods are used to render the coils reasonably non-inductive, the simplest being to loop the wire at the centre and to wind it in the form of a double helix, bringing the two ends out together. Thus the current is flowing in opposite directions through the adjacent turns, and any external magnetic field is almost entirely neutralised. This method is quite satisfactory for low-resistance coils, but since the total pressure across the insulating covering at the ends of the wire has to withstand the total voltage across the coil, it is not so suitable for higher resistance coils. For these the wire is generally wound singly over a complete layer; the direction of winding is then reversed and the second layer wound over the first but in the opposite direction: a procedure which is repeated in subsequent layers. This method is an improvement on the first, but can be still further improved if at the end of the first layer the wire is brought back in a straight length, with insulation between, to the starting-point, the second layer being then wound in the reverse direction to the first, the maximum voltage across any portion of the insulation being thus reduced to one-half of the total.

The brass tubes are usually covered with a double layer of silk ribbon, coated with shellac varnish baked to remove all moisture.

Manganin coils are impregnated after winding with shellac varnish to prevent oxidation, and subjected to the annealing process, recommended by the Reichsanstalt, of baking for ten hours at a temperature of 140° C. This treatment usually diminishes the resistance of the coil by an amount which does not exceed 2 per cent, and it is usual to allow this margin in winding coils. After baking, the resistance of the coils increases slightly, often by 0·05 per cent in one month. This effect, however, may be due to the absorption of moisture by the shellac, and, in view of the results obtained by Rosa,[1] manganin coils intended for use in resistance bridges are frequently boiled in paraffin wax after the annealing process, to prevent the entrance of moisture.

The ends of manganin coils are invariably jointed to copper terminal pieces with silver solder—it is impossible to make a satisfactory permanent joint with soft solder in the coils of finer wire,—the joints are bound into the coil, leaving only the thicker copper ends projecting.

Coils of constantan (eureka, etc.) do not require the same degree of protection from oxidation as is necessary in the case of manganin, but these are generally treated in the same way, the varnish serving to bind and improve the insulation, and the baking to dry the varnish and drive off any moisture.

[1] *Bull. Bur. Standards*, iv. 121.

The change in resistance due to the baking in the case of the copper-nickel alloy is not nearly so large as with manganin; the resistance of the coil usually increases with the annealing by not more than 0·2 per cent.

Double silk-covered wire is almost invariably used, although wires coated with cellulose enamel are now being tried with a fair measure of success.

In most resistance boxes the dimensions of the coils are usually designed to dissipate one watt per coil. The size of tube varies from 2 in. to 3 in. in length and ½ in. to ¾ in. in diameter, the average area being 30 square cm. This rating, however, probably represents the maximum amount of energy which can be dissipated without injury by individual coils. Where a number of coils are placed in one box the heating will depend on the size and ventilation of the enclosure, and under these conditions the rating per coil mentioned would certainly be excessive for the great majority of the resistance boxes in ordinary use.

§ (8) Methods of Fixing Coils.—Usual methods of mounting coils are shown in *Fig.* 21. (A) represents a construction suitable where the dimensions of the coils and studs allow of it. The brass tube is closed at one end with a brass washer soldered to the tube, the centre of the brass washer being threaded and screwed direct on to the under side of the stud. In (B) the same type of tube end is used, but the tube is screwed down to the insulating base, and in (C) the coil is fixed by means of a brass rod which passes up the inside of the tube and secures the coil by means of a nut and piece of brass strip recessed slightly to prevent movement of the coil.

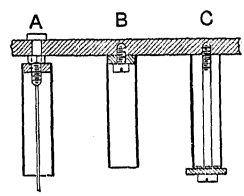


Fig. 21.

§ (9) Resistance Boxes other than Bridges.—The details given above for resistance bridges apply practically to other resistance boxes, with the exception of very high resistance units usually of the order of 1 megohm; for these, special conditions as regards winding and insulation have to be met. Taking the case of a megohm box subdivided into ten sections of 100,000 ohms, the coils will be made of constantan, which material is suitable for such high resistances, since it is more easily drawn to the finer sizes, and can be annealed between drawing without undue oxidation; the resultant wire is much more easily handled than fine manganin wire.

The coils, each of 100,000 ohms, are wound in some patterns on porcelain bobbins, and in others on wooden bobbins: in both cases the bobbins are boiled in paraffin wax before winding. Many layers are required, and it is usual to reverse the direction of the winding every four layers to keep the self-induction low; a supplementary layer of silk or waxed paper being interposed at each reversal. When wound, the coils are carefully boiled in paraffin wax at a temperature of approximately 140° C., until all the moisture and air are driven off. The wax is then allowed to cool somewhat, so that when the coil is withdrawn it retains a thin layer of wax.

§ (10) INSULATING MATERIALS USED FOR RESISTANCE BOXES.—For high-grade resistance boxes the use of ebonite is universal. Various substitutes, such as a compound of bakelite and paper, have been used with a view to avoiding the disadvantages of ebonite, but owing to slight absorption of moisture, and consequent low insulation resistance, these are not satisfactory for coils of high resistance, say of the order of 10,000 ohms.

Ebonite has the serious defect that prolonged exposure to light leads to the formation of acid on the surface as a result of the liberation of free sulphur. The insulation can be restored in a large measure by periodical treatment with a neutralising agent such as soda or ammonia, but this is a troublesome if not impossible operation with most types of boxes, and various means have been used to overcome the defect. The most common is to fit the box with a wooden cover which is kept on when the box is not in use; this prolongs the life of the ebonite considerably, but since the box is ordinarily used during the day it does not afford permanent protection. A method devised at the National Physical Laboratory which has been found to be completely satisfactory is the form of construction shown in *Fig.* 10, where the surface of the ebonite is completely protected from the entrance of light by an aluminium shield. This type of construction, however, is not well adapted for plug contacts, and in consequence these are replaced by switch dials. The terminals are brought up through the aluminium sheet, the gap between the terminal and the sheet being filled by an ebonite bush—these bushes will deteriorate, but can be readily cleaned or renewed.

Amberite or amberoid, a material composed of compressed fragments of amber, possesses a very high insulation resistance, which is apparently not affected by the action of light. This material cannot be obtained in large sheets, but is well adapted for use as bushes through which terminals or leads can be passed. In some forms of boxes made at the National Physical Laboratory, where only a small number of special coils are used, the external insulation consists of amberite washers, the supporting plate being of brass and the terminals carried to the inside through amberite bushes (see *Fig.* 22). This form of construction is, however, too costly where a large number of studs or terminals have to be brought out.

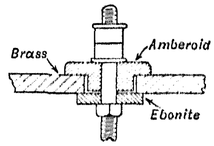


FIG. 22.

For high-resistance boxes, such as megohms, the individual terminals are usually mounted on ebonite pillars to supplement the insulation of the top. At one time it was considered that to corrugate the outer surface of such pillars was better since it increased the length of the leakage path, but later practice tends to the use of smooth pillars that do not easily collect dust. The form of petticoated pillar used on insulating testing keys at the National Physical Laboratory and shown in *Fig.* 23 would greatly improve the insulation of the ebonite pillars used for megohm boxes. As will be seen from *Fig.* 22, a deep annular cut is made which greatly extends the length of the leakage path. The inner surfaces being almost entirely shielded from light and dust, the insulation of these surfaces should remain perfect for a very long period.

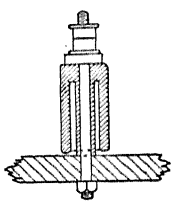


FIG. 23.

S. W. M.

RESISTANCE, STANDARDS AND MEASUREMENT OF

§ (1) TYPES OF STANDARD RESISTANCE COILS. (i.) *The British Association Pattern.*—The oldest form of standard resistance coil is that known as the B.A. pattern, which is illustrated in *Fig.* 1. It consists of a coil of resistance wire non-inductively wound on a metal bobbin 3 in. long and $\frac{5}{8}$ in. diameter. The coil is usually of platinum-silver and is connected to two long bent copper rods, about $\frac{1}{4}$ in. in diameter, which form the electrodes; the ends of these rods are amalgamated. The coil and bobbin are contained in an outer case, thus forming a double cylinder, and the space between the cylinders is filled with paraffin wax; an ebonite disc is fitted over the top. The Board of Trade Ohm, which is the legal standard of the British Isles, is a coil of this pattern.

When in use, such a standard is immersed in water within about half an inch of the ebonite top, and after being maintained at a constant temperature for about twelve hours, measure-

ments of resistance can be made which are reliable within about 0·001 per cent. The current employed in the measurements must

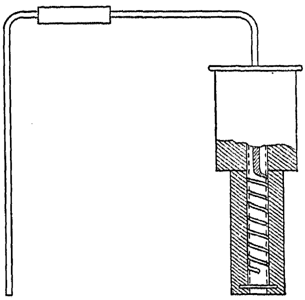


Fig. 1.—British Association Pattern Resistance Coil.

be small, or the heating effect must be allowed for.

(ii.) *Fleming's Pattern.*—The chief disadvantage of the B.A. type of coil is the temperature lag between the outer bath and the coil when the former changes in temperature. To overcome this, Fleming [1] wound the coil in the form of a ring and shaped the container so that the water was very near to every part of the coil. The general form of the leads was unaltered.

(iii.) *R. W. Paul's Pattern.*—In 1892 R. W. Paul [2] described a form in which the resistance coil, which is of a flat type, is enclosed in a hollow disc from the centre of which rises a metal tube containing the leads. In later years R. W. Paul employed manganin for the coil, and it is worthy of note that the construction ensures that the coil is not affected by changes of humidity of the air. Experiments have shown the temperature lag between the bath and the coil to be very small

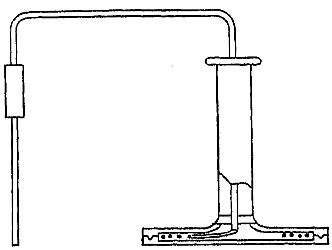


Fig. 2.—Paul's Pattern Resistance Coil.

indeed, and no difficulty is experienced in making accurate and reliable measurements. Paul's type is shown in *Fig.* 2.

(iv.) *Nalder Bros.' Pattern.*—Messrs. Nalder Bros. also introduced a form in which the temperature lag is small; in this case the coil is of large diameter and of short length, and is contained between two cylinders of metal close together. The thickness of the insulating medium being small, the temperature lag is also small.

(v.) *Reichsanstalt Pattern.*—The Reichsanstalt pattern of standard resistance coil is shown in *Fig.* 3. It was first described by Feussner [3] and later by Lindeck.[4] Manganin is used as the resistance alloy. The manganin is covered with two layers of silk and is wound on a metal bobbin *bb*, which is previously covered with a thin piece of silk, coated with shellac varnish, and heated, in order to secure good insulation. After an accurate adjustment of the resistance the bobbin is secured to the ebonite disc *d*. The resistance of the wire must be from 1 to 2 per cent greater than the ultimate nominal value of the coil to begin with, and after being wound in a non-inductive manner on the bobbin (when its resistance increases due to coiling),

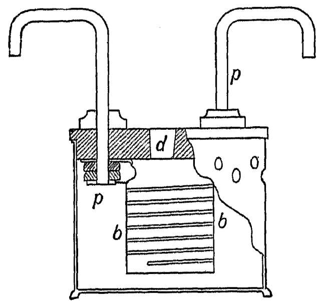


Fig. 3.—Reichsanstalt Pattern Resistance Coil.

it is heavily coated with shellac varnish and heated in an air bath at a temperature of 140° C. for about five hours. At this temperature the shellac is melted and becomes, after cooling, a hard highly insulating mass which at the same time protects the wire from any chemical action. The exact adjustment is made by means of a fine wire resistance coil, from 100 to 200 ohms in resistance, which is placed in parallel with the thicker wire. A comparatively great length of this fine wire corresponds to a very small change of the whole resistance, thus enabling the latter to be easily adjusted within a few thousandths of 1 per cent. To the ends of both wires are previously soldered, with silver, small copper rings, and corresponding rings attached to the two coils are screwed together and soldered to the stout connecting pieces *pp*. A wide perforated brass case serves to protect the wire from mechanical damage. The long thin bent rods of the B.A. type are replaced by short arms of greater cross-section, and

[1] *Phil. Mag.*, 1889. [2] *Phys. Soc. Proc.*, 1892.

[3] *Zeitschr. für Instrumentenk.*, 1890.

[4] *British Association Report*, 1892. The Reports of the B.A. Committee on Electrical Standards were published in a collected form in 1913, and should be referred to for much of the information in this article.

originally no potential terminals were used; however, in later years such terminals were fitted, and the general form of leads now used is that shown in *Figs.* 4 and 5.

(vi.) In 1907 the Reichsanstalt form of coil was modified by Rosa and Babcock.[1] They found shellac to expand when it absorbs moisture from the air or from oil, and a wire which is shellacked also stretches, its resistance being consequently increased. The Reichsanstalt type was therefore modified to prevent this action. The form recommended by Rosa and Babcock is shown in *Fig.* 4. The coil is wound in the usual manner on a brass tube 30 mm. in diameter and 70 mm. long, and is contained within a cylinder 40 mm. in diameter and 12·5 cm. high. The coil is shellacked, dried, and annealed in the manner specified by the Reichsanstalt; it is supported by a small tube, closed at the bottom, which serves as a thermometer tube. The ebonite top, through which the leads pass, is threaded and screws into the outer brass cylinder which forms the case. When the resistance of the coil is finally adjusted, the case is nearly filled with pure oil which has been freed from moisture, and the top screwed firmly into place.

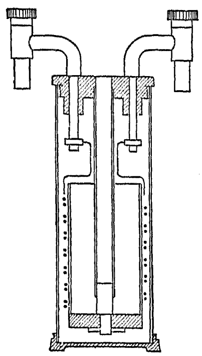


FIG. 4.—Rosa and Babcock's form of Resistance Coil.

To make the joint perfectly tight, shellac is usually put into the threads before screwing up. Shellac is also put into the joints in the top where the leads and thermometer tube pass through the ebonite.

To reduce the effect of humidity with coils not hermetically sealed, Lindeck suggested the use of a heavy paraffin oil which transmits and absorbs less moisture than the paraffin commonly used. He suggested also the use of a very thin tube on which to wind the resistances, slits being cut in the tube so that it yields readily to external pressure.

(vii.) To permit the circulation of the oil near the coil, and in order to retain the general features and design of the Reichsanstalt type, Smith modified the latter in the manner shown in *Fig.* 5. Essentially there are two concentric cylinders (the coil being wound on the smaller one); annular pieces of ebonite and brass secure the two cylinders together at the top and base respectively. The annular ring of brass is fitted last and is soldered in position to the outer and inner tubes. A hole about 1 cm. in diameter is drilled in the brass ring, and after all shellacked joints are secure and dry the coil is inverted and the space between the tubes is nearly filled with paraffin oil free from water. A brass disc is then fitted over the hole and soldered so as to seal the coil completely. Potential terminals are fitted as shown.

(viii.) *Drysdale and Burstall's Patterns.*—In order to obtain an exact knowledge of the temperature, two coils in one case have at times been used, one coil, the standard, having a low temperature coefficient of resistance, and the other, in a position close to the first, having a high temperature coefficient and serving as a resistance thermometer. Drysdale[2] used "new metal" (an alloy with a small negative temperature coefficient) for the standard coil, and platinum for the "temperature" coil. F. W. Burstall[3] also used a platinum coil to measure temperature, but employed platinum-silver for the standard resistance. As early as 1876 Chrystal suggested that resistance coils should have thermo-electric couples associated with them, one junction being buried in the centre of the paraffin surrounding the coil, and the other outside. Such systems are, however, of little practical importance, owing to resistance alloys being procurable with temperature coefficients not greater than a few parts in a million.

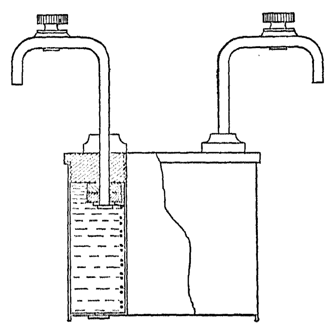


FIG. 5.—Reichsanstalt type as modified by National Physical Laboratory.

(ix.) The types so far described are suitable for resistances of 0·1 ohm and upwards. The best type is undoubtedly one of the modifications of the Reichsanstalt pattern, with potential terminals, the coil being hermetically sealed. The best resistance alloy to employ is manganin.

For resistances having values less than 0·1 ohm, such as are largely used for the measurement of current by potentiometer methods, it is essential for current and potential terminals to be fitted. Such resistances are built of wires, of tubes, of castings, and of strips.

[1] *Bureau of Standards Bull.*, 1909, v. 413.

[2] *Electrician*, 1907. [3] *Phys. Soc. Proc.* xiv.

§ (2) Desirable Properties of Standard Resistances.—A standard coil should possess, so far as practicable, the following properties:

1. Permanence.
2. Small temperature coefficient of resistance.
3. Good and constant insulation.
4. Such a construction as permits of its temperature being easily measured.
5. A robust form of construction.
6. Freedom from thermo-electric effects.
7. Non-inductive.
8. Reasonably large current-carrying capacity.
9. Terminals of such form that it is easy to define between which points the resistance is to be measured.

Of these qualities permanence, temperature coefficient, and the thermo-electric effect are dependent almost entirely on the resistance alloy of which a coil is constructed. All the types of coils so far considered may be regarded as robust in construction and possessing good and constant insulation; in all cases, too, the coils may be wound non-inductively. In general, for standards work the current-carrying capacity of a coil is dependent on the temperature coefficient of the alloy, on the resistivity and cross-section of the latter, and on the nature of the insulating medium. Manganin is the best of the alloys known, and it is better for the alloy to be immersed in an insulating oil than in a solid insulator. On the nature of the insulating medium depends also the ease with which the temperature is measured, a liquid insulator being preferred. In the case of resistances of nominal value less than 10 ohms, potential terminals should be fitted; the current and potential leads are interchangeable in use, *i.e.* the potential leads may be employed to convey the current and the usual current leads may be employed as potential ones. Searle [1] has shown that the leads should be at least four times as long as their diameter.

By far the most important desideratum for a standard coil is that of permanence, and the data available on the constancy of resistance standards are of considerable importance.

[1] *Electrician*, 1911.

§ (3) Permanency of Resistance of Metals and Alloys.—The first serious study of the relative constancy of resistance standards was made by Matthiessen.[2] The chief factors leading to inconstancy were thought to be (*a*) oxidation of the alloy by the oxygen of the air as well as by acids produced by the oxidation of the oil or grease with which a wire is almost always covered when drawn, (*b*) internal mechanical strains produced by stretching and bending during the process of covering with insulating material and winding, and (*c*) strains produced by expansion and contraction owing to variations of temperature. Annealing was found to be of prime importance, and in this connection a special study of copper was made; it was concluded also that in the case of hard-drawn silver-copper wires annealing by age increased the conductivity.

After the absolute determination by a Committee of the British Association of the resistance of a particular coil, a number of standard resistance coils were made by Matthiessen and Hockin in 1863 and 1864, and these coils have been intercompared by many observers, including Matthiessen and Hockin (1865–67), Chrystal and Saunder (1876), Fleming (1879–81), Lord Rayleigh (1881), Glazebrook and Fitzpatrick (1887–88), Glazebrook (1888–1900), and by Smith [3] (1900–20). The probable changes which have taken place in these coils are discussed by these observers, and as Glazebrook, Lord Rayleigh, and Smith also made comparisons with mercury standards of resistance, there is much data available. All of the coils are of the B.A. pattern, and as two of them are of platinum very accurate measurements of temperature are necessary. With regard to the other coils, two are of platinum-iridium, one of gold-silver, and four of platinum-silver. A summary of the results of the measurements in the period 1867–1908 is given in the above table, on the assumption that the two platinum coils D and E have not changed.

Coil.	Material.	1867.	1876.	1879-81.	1888.	1908.	Maximum Difference.
A . . .	Pt. Ir.	1·00000	1·00077	1·00056	1·00147	1·00122	147×10^{-5} ohm
B . . .	Pt. Ir.	1·00029	1·00121	1·00080	1·00104	1·00098	92 ,,
C . . .	Au. Ag.	1·00050	1·00141	1·00101	1·00146	1·00173	123 ,,
D . . .	Pt.	1·00092	1·00092	1·00092	1·00092	1·00092	0 ,,
E . . .	Pt.	1·00152	1·00152	1·00152	1·00152	1·00152	0 ,,
F . . .	Pt. Ag.	..	..	1·00016	1·00072	1·00160	144 ,,
G . . .	Pt. Ag.	1·00022	1·00030	0·99982	1·00025	1·00175	193 ,,
H . . .	Pt. Ag.	1·00020	..	..	1·00042	1·00044	24 ,,
Flat . .	Pt. Ag.	..	..	1·00079	1·00120	1·00125	46 ,,

It is evident from this table that the maxi-

[2] *British Association Reports*, 1862 and 1863.
[3] *British Association Collected Reports on Electrical Standards.*

mum number of coils which can have kept constant is two, and if the platinum coils have not remained constant, then one only of the other coils can have done so. However, direct comparisons with mercury standards of resistance in 1881, 1888, and 1908 leave little doubt that it is the two platinum coils which have kept constant in resistance. With alloys of platinum-iridium, gold-silver, and platinum-silver the tendency is for the resistance to increase with time; it will be seen later that this is in general true for all resistance alloys.

No other resistance coils are known which have been intercompared over such a long period of time, but resistance standards of manganin at the National Physical Laboratory, at the Bureau of Standards, and at the Physikalisch-Technische Reichsanstalt have been measured for a large number of years, and afford valuable evidence of the relative permanency of manganin alloys. The oldest coils of manganin are those of the Physikalisch-Technische Reichsanstalt; the values in ohms of four of these over a period of twenty-one years are given in the following table:

Date.	Coil 148A.	Coil 149A.	Coil 150A.	Coil 151.
1892	$1{\cdot}01215_7$	..	$0{\cdot}99856_1$	$0{\cdot}99767_9$
1893	4_6	$0{\cdot}99857_4$	4_8	8_2
1894	5_2	7_3	4_6	8_3
1896	5_3	7_6	4_1	9_0
1897	5_7	8_5	3_9	9_6
1898	5_6	8_8	3_3	70_3
1899	5_7	8_7	3_0	0_6
1900	5_7	9_3	2_1	1_0
1901	5_7	8_8	2_2	1_3
1902	5_7	9_4	1_8	1_3
1903	6_1	9_0	1_3	1_5
1904	6_1	8_9	1_3	1_7
1905	6_2	8_8	1_0	2_0
1906	6_3	8_7	0_7	2_3
1907	6_4	8_7	0_5	2_6
1908	5_7	9_4	0_5	2_6
1909	6_2	8_8	50_4	2_6
1911	6_3	9_0	49_7	3_1
1912	6_5	8_9	9_4	3_3
1913	6_6	8_6	9_4	3_5

These four coils have remained remarkably constant, the biggest variation in the resistance of any one of them over twenty-one years being 6 parts in 100,000. For the first two coils the biggest change is 2 parts in 100,000.

Before tabulating the variations of other manganin coils it is necessary to describe the changes undergone by many resistances due to varying atmospheric humidity.

(i.) *Variation of Shellacked Resistances with Humidity.*—In 1907 Rosa and Babcock [1] discovered that in the case of coils embedded in a heavy covering of shellac, thoroughly dried out by baking, the shellac absorbs moisture from the surrounding atmosphere and expands, stretching the manganin wire and thereby increasing its resistance. The amount of moisture absorbed depends on the relative humidity of the atmosphere, the moisture in the shellac gradually coming to equilibrium with the moisture outside when any given humidity is maintained constant. Submerging the coils in oil does not protect the shellac from atmospheric humidity, as the oil absorbs moisture and transmits it to the coils. Dipping the coils in melted paraffin wax seals them effectually against moisture, but the coils should not then be immersed in paraffin oil owing to the solubility of the wax. The coils of most Wheatstone bridges, potentiometers, and other resistance apparatus are not, however, intended to be immersed in any liquid, and in such cases they may be dipped in melted paraffin wax and so protected from the humidity effect. Coils intended as standards should be shellacked, immersed in paraffin oil, and hermetically sealed.

[1] *Bureau of Standards Bull.*, 1907, iv. 121.

The effect of humidity and the protection afforded by hermetical sealing is well shown by some standard manganin coils of the National Physical Laboratory. The values of these over a period of fifteen years are given in the following table: the coils were hermetically sealed in 1911.

Date.	Coil No. 780.	738.	2450.	2449.
1903	$0{\cdot}99997_7$	$9{\cdot}9989_1$	$100{\cdot}000_6$	$1000{\cdot}03_6$
1904	7_9	90_4	4_7	$\cdot 26_8$
1905	7_8	89_0	9_5	$\cdot 51_8$
1906	$1{\cdot}00000_3$	88_4	13_9	$\cdot 69_2$
1907	0_3	90_5	17_9	$\cdot 83_8$
1910	2_5	90_2	36_5	$1{\cdot}06_7$
Maximum change before sealing (1903–10)	$0{\cdot}004_8\%$	$0{\cdot}002_1\%$	$0{\cdot}035_9\%$	$0{\cdot}103_1\%$
1911	$0{\cdot}99999_7$	$9{\cdot}9991_4$	$100{\cdot}039_9$	$1001{\cdot}06_9$
1913	9_2	0_7	40_6	$\cdot 05_4$
1915	9_2	0_2	41_3	$\cdot 04_1$
1917	9_2	0_5	47_4	$\cdot 04_1$
1919	9_0	0_3	50_9	$\cdot 02_6$
Maximum change after sealing (1911–19)	$0{\cdot}000_7\%$	$0{\cdot}001_2\%$	$0{\cdot}011_0\%$	$0{\cdot}004_3\%$

It is clear that hermetical sealing is highly beneficial, and that manganin standards of resistance have been made which, while not quite constant in resistance, change by not

more than a few parts in a million per year. On the other hand there are a large number of manganin coils which have changed very rapidly with time and are practically useless. The exact causes of variation are not known. Smith [1] classifies the possible causes of change under the following heads: (1) Change in structure of the alloy. (2) Surface action. (3) Humidity effect. (4) Change in the soldered joints connecting the wires of high-resistance coils to the current leads. (5) Change at the junctions of the potential leads with the resistance standard. It is probable that the first only of these is operative in all standards. Causes (2) and (4) would have an inappreciable effect on very low resistances, yet some of these change considerably with time. Cause (5) has no effect on high-resistance coils, but such coils often increase in resistance at the rate of several parts per 100,000 per year.

(ii.) Appleyard [2] pointed out the great differences which often exist between samples of the same nominal quality of an alloy, and found that tropical heat and moisture usually accelerate failure. The alloys he experimented with were German silver and platinoid.

§ (4) Metals and Alloys for Standard Resistances.—Pure metals if carefully annealed and protected from corrosion probably keep constant in resistance. Unfortunately, however, for all pure metals the rate of variation of resistance with temperature is very large, and in general this renders it impracticable to construct resistance coils with pure metals. Standard resistance coils of platinum were, however, made in 1864,[3] and are still in existence. As early as 1862 Matthiessen [4] showed that the variation of resistance with temperature of all pure metals, except iron and thallium, was practically the same, the increase of resistance for a rise of temperature of 1° C. being about 4 parts in 1000. As a result of a large number of experiments on alloys, Matthiessen found that when a solid metal is alloyed with another (with the exception of lead, tin, cadmium, and zinc amongst each other), a lower conducting power is observed than that corresponding to the mean of the components of the alloy. With most alloys the law found to regulate this property is the following: "The percentage decrement between 0° and 100° in the conducting power of an alloy in a solid state stands in the same ratio to the mean percentage decrement of the components between 0° and 100° as the conducting power of the alloy at 100° does to the mean conducting power of the components at 100°."

If A_0, B_0, C_0 be the respective conductivities at 0° C. of an alloy A and of its two components B and C, and A_{100}, B_{100}, C_{100} the corresponding quantities at 100° C., then the law may be expressed in the form

$$\frac{(A_0-A_{100})/A_0}{\frac{1}{2}\{(B_0-B_{100})/B_0+(C_0-C_{100})/C_0\}}=\frac{A_{100}}{\frac{1}{2}(B_{100}+C_{100})},$$

and as $(A_0-A_{100})/A_0$ and the corresponding expressions for B and C are the temperature coefficients of conductivity, the law may be stated in the more convenient form: "The ratio of the temperature coefficient of an alloy to the mean temperature coefficient of its constituents is equal to the ratio of the conductivity of the alloy at the higher temperature to the mean conductivity of the constituents at the same temperature." It follows from the equation already given that if $A_{100}/\frac{1}{2}(B_{100}+C_{100})$ is very small, *i.e.* if the resistivity of the alloy is very great compared with the mean resistivities of its components, the alloy will have a small temperature coefficient of resistance. In order to obtain a suitable resistance alloy, it was consequently only necessary to experiment with alloys having a high resistivity compared with the mean resistivities of the metals of which it was composed. In all Matthiessen examined over 100 alloys, and of these he found an alloy composed of one part by weight of platinum to two parts of silver had the smallest temperature coefficient of resistance. This alloy has been in use since its introduction in 1862 and is known as platinum-silver. More modern resistance alloys containing nickel and manganese have been found to possess negative temperature coefficients, and this naturally prevents the general application of Matthiessen's law. The following results obtained by Matthiessen are of considerable interest:

	Conducting Power at 0° C.	Percentage Decrement in Conducting Power between 0° and 100° C.
Pure iron	16·81	39·2
Pure thallium	9·16	31·4
Other pure metals in solid state	..	29·3
Gold + 15% iron	2·76	27·9
Proof gold	72·55	26·4
Standard silver	80·63	23·2
Gold with 10% iron	2·06	17·5
Gold + 14·3% silver + 7·4% copper	44·47	15·5
Copper + 36·7% zinc	22·27	12·4
Copper + 25% zinc	22·08	11·5
Silver + 5% platinum	31·64	11·3
Silver + 9·8% platinum	18·04	7·1
Copper + 9·7% tin	12·19	6·6
Platinum + 33·4% iridium	4·54	5·9
Copper + 10·3% tin	10·21	5·2
Gold + 18·1% silver + 15·4% copper	10·6	5·2
Gold + 15·2% silver + 26·5% copper	12·02	4·8
German silver	7·80	4·4
Gold + 5% iron	2·10	4·3
Gold + 4·7% iron	2·37	3·8
Silver + 25% palladium	8·52	3·4
Silver + 33·4% platinum	6·70	3·1

F. E. S.

[1] *Brit. Assoc. Report on Electrical Standards*, 1908.
[2] *Phil. Mag.*, 1898.
[3] *British Association Reports*, 1865, 1908.
[4] *British Association Report*, 1862.

(i.) *Manganin and other Alloys of Low Temperature Coefficient.* — In 1889 Weston discovered that many alloys containing manganese and copper, and called by him manganin, possessed a very small temperature coefficient of resistance, in many cases not exceeding a few parts in a million per degree. Such alloys are now extensively

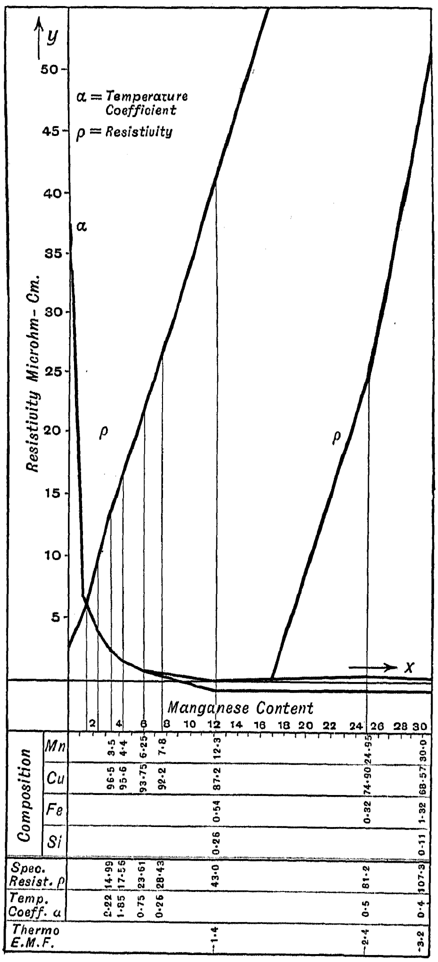


FIG. 6

used, and in recent years several investigations have been made with a view to discovering the relationship between the composition of manganin, its treatment in drawing and annealing, and its permanence. Lindeck in 1892 [1] made some measurements on manganin at the Physikalisch-Technische Reichsanstalt, and concluded that it was a most suitable alloy for resistance coils.

[1] *British Association Report*, 1892.

Then in 1898 Feussner and Lindeck [2] published an account of an investigation into the properties of alloys of this kind with varying proportions of copper and manganese and nickel, showing (see *Fig.* 6) that with a proportion of copper 86, manganese 12, and nickel 2, the temperature coefficient was extremely small, and the resistivity was of the order of 45 microhms-cm. In *Fig.* 7 Feussner and Lindeck show the curves of temperature coefficient for the various alloys. In the case of an alloy having the composition specified above the resistance increases up to approximately 18° C., from there to 55° C. the curve is practically flat, and from above that point the resistance diminishes. *Fig.* 8, which is also taken from Feussner and Lindeck's paper,

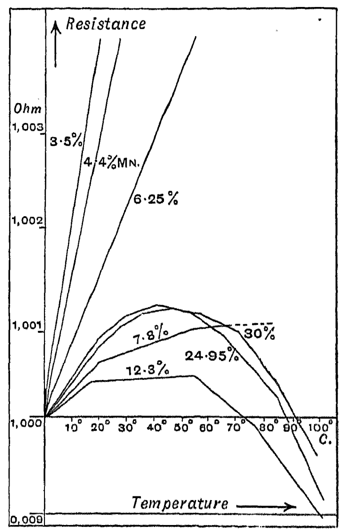


FIG. 7.

is a curve obtained with a standard one-ohm coil made from the same alloy. This curve may be taken as typical of the behaviour of manganin with varying temperature, although in different samples the change in resistance over the same range may be greater or less, and the position of the maximum point different. The guarantee of temperature coefficient, given by the Isabellen Hütte who made the alloy, was 0·00004 per 1° C.

Later publications of the Reichsanstalt at various times gave particulars regarding resistance coils constructed of manganin; these referred mainly to the permanence, and are dealt with in § (3), dealing with the permanence of resistance standards.

In later years the properties of manganin

[2] *Wissensch. Abh. P.T.R.* ii. 503.

have been investigated both in America and England. In each case the investigators made an analysis of material supplied by the Isabellen Hütte, with the following results :

	American.		British.
	F. E. Bash.[1]	Liddell.[2]	N.P.L.
Copper . .	82·62	81·12	85·1
Manganese .	12·82	15·02	10·9
Nickel . .	3·78	2·29	3·65
Iron . . .	0·72	0·57	0·34

Bash investigated the properties of the alloy more particularly with regard to the effect of the iron content, and found that by varying the amount of iron he could shift the point of maximum resistance and, if desired, make the temperature coefficient more negative. He finally produced an alloy containing :

Copper	83·0 parts
Nickel	2·5 „
Manganese	14·0 „
Pure Iron	1·5 „
Total	101·0 parts

This material gave the following coefficients :

15° C. to 30° C. . . . $+0{\cdot}0000030$ per 1° C.,
15° C. to 45° C. . . . $-0{\cdot}0000038$ per 1° C.,

the maximum point of resistance being about 25° C. The resistivity of the material is not specified, but the thermo-E.M.F. is stated as $5{\cdot}6 \times 10^{-6}$ volts per 1° C.

Hunter and Bacon[3] worked on much the same lines as Bash, making up a number of alloys and determining their temperature coefficient. The alloys made and the results obtained are shown in the above table.

Sample.	Analysis of Material.				Temperature Coefficient 18° to 24° C.	Resistivity in Microhm-cms. at 20° C.	Thermo-E.M.F. against Copper: Microvolts per 1° C.
	Cu.	Mn.	Ni.	Fe.			
9D	88·20	8·84	1·78	0·93	$0{\cdot}33 \times 10^{-5}$	$55{\cdot}6 \times 10^{-6}$	3
10D	88·02	9·93	1·74	0·24	$1{\cdot}2 \times 10^{-5}$	$34{\cdot}2 \times 10^{-6}$	4
11D	87·24	10·26	1·77	0·52	$1{\cdot}5 \times 10^{-5}$	$37{\cdot}4 \times 10^{-6}$	5
12D	83·60	12·03	3·41	1·04	$0{\cdot}22 \times 10^{-5}$	$47{\cdot}8 \times 10^{-6}$	8
13D	84·72	12·83	2·08	0·73	$0{\cdot}38 \times 10^{-5}$	$50{\cdot}8 \times 10^{-6}$	4
14D	84·07	12·98	2·60	0·82	$0{\cdot}57 \times 10^{-5}$	$51{\cdot}1 \times 10^{-6}$	..

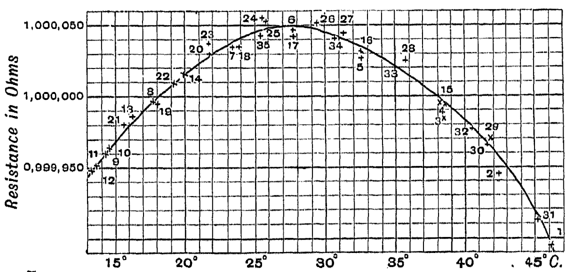


FIG. 8.

The actual curves of temperature coefficient are given in *Fig.* 9.

In the case of the alloys 10D and 11D, the iron present was due to original impurities in the materials used, but in the other samples iron was added to the alloy.

Hunter and Bacon conclude that :

(1) The percentage of manganese between the limits used affects the resistivity of the wire, but has no effect on the temperature coefficient of resistance.

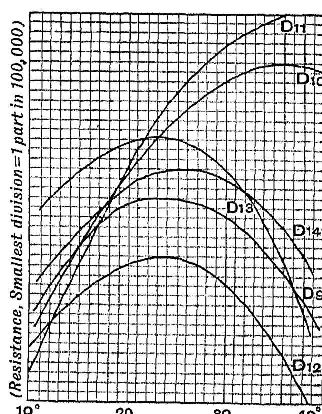


FIG. 9.

(2) The presence of iron affects the temperature coefficient to a considerable degree. Those wires in which the iron content was low did not show the temperature coefficient reversal which is characteristic of a manganin wire. The results show that the presence of iron up to 1 per cent reduces the temperature coefficient of the resulting alloy.

[1] *Bull. Am. Inst. Min. Metall. Eng.*, Sept. 1919, p. 1717.
[2] *Met. and Chem. Handbook*, 1916, p. 482.
[3] *Trans. American Electro-chemical Soc.* xxxvi. 323.

They consider that alloy No. 12D is the best material.

Rosa[1] states that variation in the amount of the manganese content affects the resistivity of the wire, but has no effect on the temperature coefficient, and gave the following table :

Sample.	Analysis of Material.				Temperature Coefficient 18° to 24° C.	Resistivity in Microhm-cm.
	Cu.	Mn.	Ni.	Fe.		
1	88·02	9·93	1·74	0·24	$1·2 \times 10^{-5}$	$34·2 \times 10^{-6}$
2	87·24	10·26	1·77	0·52	$1·5 \times 10^{-5}$	$37·4 \times 10^{-6}$
3	88·20	8·84	1·78	0·93	$0·33 \times 10^{-5}$	$55·6 \times 10^{-6}$
4	83·60	12·03	3·41	1·04	$0·22 \times 10^{-5}$	$47·8 \times 10^{-6}$
5	84·72	12·83	2·08	0·73	$0·38 \times 10^{-5}$	$50·8 \times 10^{-6}$
6	84·07	12·98	2·60	0·82	$0·57 \times 10^{-5}$	$51·1 \times 10^{-6}$

It is clear that a small temperature coefficient is required, but this can be equally well effected by a decrease in both the manganese and the iron content. The resistivity of the N.P.L. sample No. (2) is lower than the normal value for manganin, but very high resistivity is not essential for standard resistance work.

In the investigation at the N.P.L. it was assumed that the iron in manganin was present as an impurity, and by the use of specially pure materials an alloy was produced of similar composition to the German, but with a lower iron content. The analysis of the first sample made, alloy No. 1, was:

	Sample (1).
Copper	85·29
Manganese	10·85
Nickel	3·57
Iron	0·24

Subsequently, further alloys were made, as follows, in order to investigate the effect of varying both the manganese and iron content.

	Sample.		
	(2).	(3).	(4).
Copper	86·78	83·20	84·15
Manganese	9·10	12·77	10·75
Nickel	3·55	3·51	3·60
Iron	0·19	0·28	1·30
Silicon	0·10	0·16	..

The temperature coefficients of these alloys are shown in the accompanying curves (*Fig.* 10), where it will be seen that while the alloys Nos. (1) and (2) have a temperature coefficient less than $\pm 0·00001$ per 1° C., the effect of increase of manganese content in No. 3 is to make the temperature coefficient strongly negative.

The resistivity of these samples was as follows :

(1) . . 45 michrom-cms.
(2) . . 36 „ (Thermo-E.M.F. against copper =1·9 microvolts per 1° C.)
(3) . . 45 „
(4) . . 48 „

Thus, the results do not completely bear out the views of the American experimenters.

Working the Alloy.—In the investigation at the N.P.L. it was found that it was impossible to anneal the wire without affecting the surface. Feussner and Lindeck refer to the fact that when a manganin wire is annealed, even at

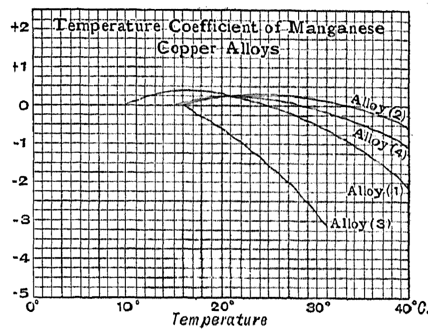


FIG. 10.

low temperatures, the manganese suffers a selective oxidation, and the results of a number of experiments at the N.P.L. with wires annealed at various temperatures and in different atmospheres appear to show that the manganese is diffused from the surface, leaving on the outside of the wire a tube of material with a high copper content, which has a large temperature coefficient. Hunter and Bacon confirm this and recommend annealing in a current of carbon dioxide.

The general practice has been to anneal in this way, and to remove the outer surface either with emery paper or by pickling the manganin in a solution of 5 parts nitric acid, 1 part sulphuric acid, 20 parts water. In manufacture, however, it is desirable to draw the material cold, and the effect of cold working on the alloys was investigated at the N.P.L. It was found that alloy No. (2) (9 per cent Mn) was quite ductile and could be

[1] *El. World*, April 24, 1920, lxxv. 941.

drawn without annealing, from a ½ in. bar to the finest wire, without difficulty. Alloy No. (1) (11 per cent Mn) was only rather more difficult, but with No. (4) (Mn 11 per cent, Fe 1·3 per cent) the material broke up badly in the drawing process. Thus, the 9 per cent manganese alloy appears to be most suitable for resistance standards, and it is recommended for trial for the purpose.

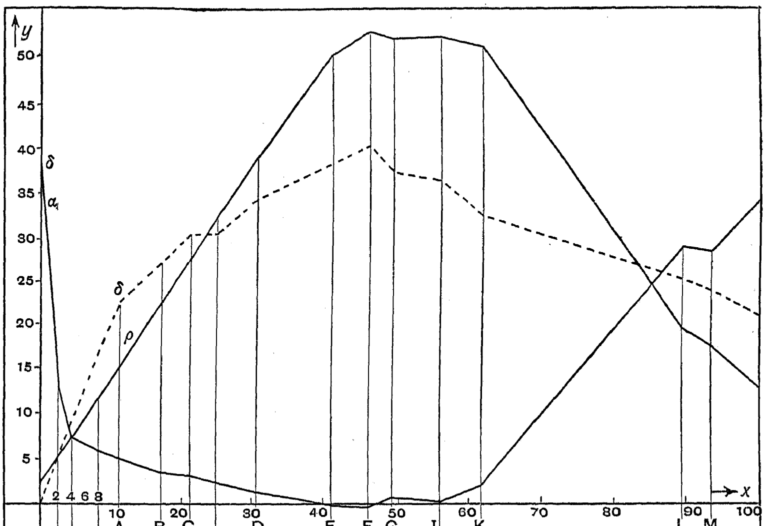
Observations extending over a period of eighteen months, of the secular change of the various alloys when made up into coils and including some samples of American wire, showed that they were all satisfactory as regards permanence over this period.

Jointing.—Manganin cannot be soft-soldered, and it is usual to make all joints with silver solder.

Before the introduction of manganin, alloys such as German silver and platinoid were used, and later, alloys of copper and nickel generally known as "patent-nickel" or "nickelin." In 1895, Feussner and Lindeck [1] investigated the properties of nickel copper alloys, starting with the then existing alloys "nickelin" and "patent-nickel," which latter was in use in the German Mint for coinage, and experimented with further alloys made up especially for the purpose of the investigation.

[1] *Wissensch. Abh. P.T.R.* ii. 503.

The results are given in *Fig.* 11, reproduced from the original paper, from which it will be seen that with an alloy of about nickel 40, copper 60, the temperature coefficient is very low and the resistivity of the alloy of the order of 50 microhm-cms.



		A	B	C		D	E	F	G	I	K	L	M	
Composition	Ni	10·04	16·01	20·04	24·07	29·94	41·25	46·18	49·36	56·37	62·02	89 46	93 66	
	Cu	89·85	83·97	79·76	74·92	69·68	58·61	54·05	49·76	43·96	38·44	7·95	3·86	
	Mn	SPUR	SP.	SP.	MN: 0·02 ZN: 0·70	0·36	0·70	—	0·33	SP.	SP.	1·40	1·45	
	Fe	0·15	0·18	0·13	0·58	0·36	0·37	0·32	0·54	0·34	0·34	1·19	1·03	
Spec. Resist.		15	21·5	26·5	31	38·5	50	52	51·5	52	51·3	20·0	18	
Temp. Coeff.		5	3	2·5	2	1·3	-0·4	-0·5	0·4	0·1	2	28·5	28·4	
Thermo E.M.F.		21·8	26·3	29·1	29·1	33·5	37·9	39·3	35·8	35·0	31·3	26·9	23·5	19·5

FIG. 11.

This alloy, which is sold under various trade names, such as constantan, eureka, etc., etc., is easily worked, can be readily soft-soldered, and is satisfactory as regards permanence. The large thermo-E.M.F., however, restricts its use for resistance standards mainly to coils of high value, where the thermo-E.M.F. is only a very small proportion of the volts required to make measurements with the coil or, as in some cases, where special precautions are taken to compensate for the thermo-E.M.F.

The permanence of the alloy may be judged from the series of observations given on the following page of a megohm box built in ten units, each of 100,000 ohms, the wire being wound on porcelain bobbins and the coils boiled in paraffin wax.

Another material which has been more lately introduced is a copper manganese alloy containing a small amount of aluminium. The temperature coefficient is small, but no very definite information is yet forthcoming as to the permanence. The alloy has the

VALUES OF MEGOHM BOX

1910. Temperature 15·0° C.	December 1911. Temperature 19·5° C.	February 1913. Temperature 16·5° C.	November 1914. Temperature 17·4° C.	February 1915. Temperature 16·6° C.	January 1918. Temperature 17·0° C.	January 1920. Temperature 18·0° C.
$99,98_8$	$99,99_5$	$99,98_6$	$99,98_6$	$99,98_6$	$99,98_3$	$99,99_0$
$100,00_8$	$100,01_1$	$100,00_5$	$100,00_1$	$100,00_8$	$99,99_9$	$100,00_0$
$100,00_8$	$100,02_4$	$100,01_5$	$100,01_1$	$100,01_6$	$100,01_2$	$100,01_8$
$100,00_8$	$100,01_7$	$100,00_8$	$100,00_7$	$100,00_8$	$100,00_4$	$100,01_1$
$100,01_8$	$100,01_6$	$100,01_1$	$100,00_7$	$100,01_4$	$100,00_4$	$100,00_6$
$100,01_8$	$100,02_1$	$100,01_5$	$100,01_3$	$100,01_8$	$99,99_8$	$100,00_9$
$100,00_8$	$100,01_6$	$100,01_0$	$100,00_7$	$100,01_3$	$100,00_3$	$100,00_5$
$100,00_8$	$100,01_5$	$100,01_0$	$100,00_6$	$100,01_3$	$100,00_3$	$100,00_3$
$99,99_8$	$100,01_1$	$100,00_5$	$100,00_3$	$100,00_9$	$99,99_9$	$99,99_9$
$100,00_8$	$100,01_6$	$100,00_9$	$100,00_7$	$100,01_1$	$100,00_5$	$100,00_9$
$1,000,07_0$	$1,000,14_2$	$1,000,07_4$	$1,000,04_8$	$1,000,07_6$	$1,000,01_0$	$1,000,05_0$

Note.—The accuracy of the observations here was only of the order of ±0·00005, and therefore practically all the differences are within the limits of errors of observations.

curious characteristic, due probably to its aluminium content, that it can be soft-soldered once, but the same piece of metal cannot be re-soldered. The thermo-E.M.F. of this material against copper is stated to be 0·3 microvolts for 1° C.

The approximate composition, resistivity, and temperature coefficients of the alloys principally used for the construction of standard resistances are given in the following table :

Alloy.	Composition (Approximate).	Resistivity. Microhm-cms. at 20° C.	Approximate Temperature Coefficient over small Range near 20° C.
Manganin *	Copper Manganese Nickel Sometimes iron	36 to 55	$0·00000_3$ to 0·00002
Nickel-copper, as constantan, eureka, etc. †	Copper, 57·5% Nickel, 41·9% Iron, 0·3% Manganese, 0·4%	50	+0·00001 to −0·00004
Platinum-silver	Silver, 66·6% Platinum, 33·4%	20	+0·00027
Therlo	Copper, 86% Manganese, 12% Aluminium, 2%	47	$+0·00000_5$

* For further details of manganin, see previous pages.

† Actual analysis of a sample. This is a fair average, but the values of either nickel or copper may vary slightly with different samples.

Particulars of other alloys are given in the section on Electrical Constants of Metals and Alloys.

S. W. M.

(ii.) *Thermo-electric Properties of Common Metals and their Alloys.*—When comparing resistances, if the galvanometer circuit includes junctions at which large thermo-electric forces come into action the galvanometer system is deflected, and if the junctions are poorly made or apt to vary in temperature, the thermo-electric forces also vary. Such effects are extremely inconvenient in measurements and often reduce the accuracy. Fortunately they are rarely of large magnitude except when making measurements of low resistances constructed with nickel alloys such as constantan. In general, however, a resistance alloy should be chosen in order to have a small thermo-electric force against the metal employed for making junctions, which is usually copper, but phosphor bronze has been used to a small extent.

Drysdale [1] and Warren and Murphy [2] have made measurements of the thermo-electric properties of many resistance alloys. Drysdale employed copper as the base metal for comparison and obtained the following results.

[1] *Electrician*, 1907. [2] *Ibid.*, 1908.

Alloy.	Thermo-electric Force Microvolts per 1° C. to Copper.
Constantan	36 to 39
German silver	36
Nickelin	19 to 37
Platinoid	24
Manganin	1·4

Warren and Murphy, who used phosphor bronze as the base metal, found the following values:

Alloy.	Thermo-electric Force Microvolts per 1° C. to Phosphor Bronze.
Platinoid	16
Platinum silver	9
Manganin	0·2 to 1·8
Copper	1·8

The marked superiority of manganin is evident. Warren and Murphy experimented also to determine the magnitude of the error which might be produced at a silver-soldered junction. For instance, in a tapping from a potentiometer coil the junction is usually silver-soldered and the physical properties of the wires near the junction may be affected. To test this, two lengths of manganin were silver-soldered together and the junction experimented with. The conclusion was that the wires had not been affected to an appreciable extent.

In general it is safe to conclude that resistance circuits made up of copper or phosphor bronze, and manganin, give rise to errors due to thermo-electric effects, which are practically negligible.

(iii.) *Peltier Effect in Alloys.*—The fact that the resistivities of all alloys are greater than those calculated on the assumption that the components conduct proportionally to the percentage of each present, led Lord Rayleigh [1] to advance a theory to account for such a fact. Subsequently Liebenow [2] advanced a theory which is practically identical with that put forward by Lord Rayleigh. The theory may be stated as follows: The flow of electricity across a junction of dissimilar metals leads to an absorption or emission of heat, a phenomenon which is known as the Peltier effect. The temperature gradient thus produced increases until the heat lost or gained by conduction balances the Peltier effect and an electromotive force is produced which tends to stop the current. This back electromotive force is proportional to the current and cannot be distinguished by means of ordinary experiments from a resistance, so that an alloy should, according to such reasoning, possess a spurious resistance. Lord Rayleigh's calculation shows that the value of this spurious resistance, R, per unit length is given by the expression

$$R = \frac{273E^2}{K/p + K'/p'},$$

where E is the thermo-electric force of a couple for 1° difference of temperature between the junctions; K and K′ are the thermal conductivities in ergs of the metals, and p and p' are the proportions by volume of the two metals. Hence $p + p' = 1$. The temperature is supposed to be near 0° C.

It is interesting to calculate on this theory the best constitution of some high resistance alloys. For maximum or minimum resistivity $dR/dp = 0$,

hence
$$\frac{K}{p^2} - \frac{K'}{(1-p)^2} = 0,$$

or
$$p = \frac{1}{1 + \sqrt{K'/K}}.$$

For platinum and silver $\sqrt{K'/K} = 0{\cdot}40$, so that the ratio of volumes of silver and platinum should be 0·7/0·3, and the ratio by weight is 0·35/0·3 or 1·2/1. Matthiessen used the ratio 2/1 for platinum silver; he does not appear to have experimented with a higher proportion of platinum.

For copper and nickel $\sqrt{K'/K} = 0{\cdot}85$ approximately, so that the ratio of volumes of copper and nickel should be about 54 to 46 and by weight the ratio is about the same. It is of interest to note that the composition of constantan approximates very closely to this.

R. S. Willows [3] attempted to measure the value of the spurious resistance in brass, platinoid, platinum silver and platinum-iridium. The alloy to be tested formed one of the arms of a Wheatstone bridge, the adjacent arm being a simple metal such as copper or lead. The bridge was balanced first for alternating, and then for direct current. The pure metal possesses no spurious resistance, but it was concluded that the apparent resistance of the alloy would decrease when the alternating current was used. The reasoning is as follows: Suppose a current passes through an alloy and sets up a back electromotive force; if the current is now reversed quickly, this back electromotive force will at first assist the current in its passage and more will flow in the second direction than in the first, or, what comes to the same thing, the resistance will appear to be less for the quickly reversed current than for the steady direct one. The skin effect, arising from the concentration of current in the outer layers, can be allowed

[1] *Scientific Papers*, iv. 232.
[2] *Encyklopädie der Elektrochemie*, x.
[3] *Phil. Mag.*, 1906, xii.

for. However, no certain differences could be detected between the resistances of an alloy to direct and alternating current at the temperatures 20° C. and 100° C.

Fleming and Dewar[1] found the resistivity of a pure metal very small at low temperatures, while that of an alloy is comparatively great. As the temperature is reduced it appears possible therefore that the true resistance of an alloy may decrease and the false resistance become relatively great. Willows therefore made experiments with alloys at the temperature of liquid air but in no case could any false resistance be detected.

(iv.) *Drysdale's Compensated System.*—Drysdale[2] gives resistance-temperature curves for a number of alloys, and also describes a compensated system in which a wire made of a resistance alloy with a negative temperature coefficient is plated with a pure metal, the thickness of the plating being such that the temperature coefficient of the combination of resistance alloy and pure metal is practically zero. The temperature coefficient of any resistance alloy can in general be represented by the expression

$$R_T = R_t\{1 + \alpha(T - t) + \beta(T - t)^2\}.$$

If R_T is the resistance of the alloy at temperature T and R'_T the corresponding resistance of the plating, then if R is the resistance of the combination

$$\frac{1}{R} = \frac{1}{R_T} + \frac{1}{R'_T},$$

and

$$\frac{1}{R_T} + \frac{1}{R'_T} = \left(\frac{1}{R_t} + \frac{1}{R'_t}\right) + \left(\frac{\alpha}{R_t} + \frac{\alpha'}{R'_t}\right)(T - t) + \left(\frac{\beta}{R_t} + \frac{\beta'}{R'_t}\right)(T - t)^2.$$

Hence, for compensation,

$$\frac{\alpha}{R_t} = \frac{-\alpha'}{R'_t} \text{ and } \frac{\beta}{R_t} = \frac{-\beta'}{R'_t}.$$

Compensation for temperature coefficient by plating with a pure metal imposes, therefore, the condition that the alloy or pure metal must have a negative temperature coefficient at ordinary working temperatures. For approximate compensation the amount of metal required to be deposited can be easily calculated from the equation

$$\frac{m'}{m} = \frac{\alpha\rho'\Delta'}{\alpha'\rho\Delta},$$

where α, ρ, and Δ are the temperature coefficient, resistivity, and density respectively of one of the materials and α', ρ', and Δ' the corresponding quantities for the other material. By such a plating process, resistance coils of an alloy plated with nickel have been made so as to have temperature coefficients less than 1 part in 100,000 per 1° C.

[1] *Phil. Mag.*, 1893, xxxvi.

[2] *Electrician*, 1907.

§ (5) Network of Conductors for the Measurement of Resistance. (i.) *Kirchhoff's Laws.*—Let there be any network of conductors, such as that comprising a Wheatstone bridge, a Kelvin bridge, or a potentiometer, and let P be the resistance of any one of the conductors, and G be the resistance of a galvanometer comprising another branch of the network. Let the current through P be i, and assume in the first place that the bridge or potentiometer network is a balanced system such that no current passes through the galvanometer. Let P be changed to $P + \delta p$. The currents through the various parts of the network are altered by this change and some current will in general pass through the galvanometer. The distribution of currents in such a network and the changes which occur when any one branch of the network is varied can be obtained by the application of Ohm's law round the several circuits of the network, together with the condition that there is neither creation nor destruction of electricity at the junctions. This statement concerning the distribution of currents is a modification of two generalisations known as Kirchhoff's Laws. These laws are usually stated in the following form:

(*a*) The algebraic sum of the currents which meet at any point is zero.

(*b*) In any closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the electromotive force in the circuit.

On applying these two laws there results an important rule which enables the sensitiveness of measurement of resistance by means of any given network to be compared with that of any other network. The rule is:

(*c*) In any network of conductors the current in one arm due to an electromotive force in another arm is equal to the current in the latter when an equal electromotive force is placed in the former.

(ii.) *Sensitiveness of a Network for the Measurement of Resistance.*—When P is changed to $P + \delta p$ the changes in the currents in the network are equal to those produced by an electromotive force $i\delta p$, introduced into the branch P, and by rule (*c*) the current through the galvanometer is equal to that in P when an electromotive force $i\delta p$ is placed in the galvanometer branch. If $\delta\gamma$ is the current through the galvanometer, then

$$\delta\gamma = \frac{i\delta p}{P + G + x},$$

where G is the resistance of the galvanometer and x is dependent on the other resistances of the network. In an extreme case, as with

the potentiometer, x can be made very small and then approximately

$$\delta\gamma=\frac{i\delta p}{P+G}.$$

Suppose now that i_m is the maximum current which P can carry with safety, then

$$\frac{\delta\gamma}{i_m}=\frac{\delta p}{P+G}.$$

This represents an important proposition which is given by Schuster [1] in the form:

"With a given resistance and galvanometer, the ratio of the smallest change of resistance which can be detected to the sum of the given and galvanometer resistances is equal to the ratio of the smallest current which can be detected by the galvanometer to the maximum current which can be sent through the resistances."

As an example, if a particular galvanometer of 10 ohms resistance can detect 1×10^{-8} ampere, and if a 1 ohm resistance can safely carry 1 ampere, then it is possible to compare the resistance with another one approximately similar to it within $1{\cdot}1\times10^{-7}$ ohm.

If there is a choice of galvanometers, and taking account of the fact that in galvanometers the coils of which are wound in similar channels, and contain the same mass of wire, the electromagnetic force on the needle, and hence the deflection, varies with the square root of the resistance, the last equation shows that the best galvanometer resistance is that for which P=G, *i.e.*

$$\frac{\delta p}{P}=\frac{2\delta\gamma}{i_m}.$$

"Hence with a given type of galvanometer having the most suitable resistance, the smallest percentage change of resistance which can be measured is given by twice the ratio of the smallest current which the galvanometer can detect to the greatest current which can be sent with safety through the resistance."

The energy W in the galvanometer branch is $(\delta\gamma)^2G$, so that $\delta\gamma$ may be written $\sqrt{W/G}$. When P=G the following relation therefore holds:

$$\frac{\delta p}{P}=\frac{2\sqrt{W}}{\sqrt{i_m{}^2P}}.$$

This has been expressed by Schuster in the form:

"The highest percentage accuracy with which a given resistance can be measured is directly proportional to the square root of the maximum electric work which can be done on it without overheating."

It is usually possible in measurements such as those of temperature by means of resistance thermometers, and that of air velocity by resistance anemometers, to choose the resistance of P so as to be suitable for the measurements. Frequently, however, the choice of the galvanometer resistance is very limited. It is useful, therefore, to know to what extent the percentage possible accuracy is reduced owing to the employment of a galvanometer which has not the most suitable resistance.

If in the equation $\delta p/(P+G)=\delta\gamma/i_m$, $\sqrt{W/G}$ is substituted for $\delta\gamma$, then

$$\delta p=\frac{(P+G)\sqrt{W}}{i_m\sqrt{G}},$$

and if nP is substituted for the actual galvanometer resistance, where P is the best galvanometer resistance, then we have

$$\delta p=\sqrt{P}\frac{n+1}{\sqrt{n}}\cdot\frac{\sqrt{W}}{i_m}.$$

If a galvanometer of resistance nP is used, it follows therefore that the sensitiveness is $2\sqrt{n}/(n+1)$ times that of the obtainable maximum. Moreover, the reduction is the same whether the resistance of the galvanometer is nP or P/n. If $n=20$ or 0·05 the sensitiveness is 0·426 times the maximum.

§ (6) WHEATSTONE BRIDGE.—The Wheatstone network, which is universally used, was invented by Christie [2] of the Royal Military Academy at Woolwich, and the notice of electricians was drawn particularly to this device by Sir Charles Wheatstone, who, however, gave Christie full credit.

The Wheatstone bridge is a network of six conductors and is usually represented as an arrangement of conductors in trilateral symmetry about the point D (*Fig.* 12). In this system a battery is inserted in the branch AB and a galvanometer in the branch CD.

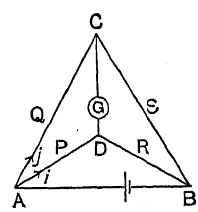


FIG. 12.—Wheatstone Bridge.

In practice adjustments of one or more of the branches are made until the current through the galvanometer is zero whether the battery in AB is connected or not. When this is so

$$\frac{P}{Q}=\frac{R}{S},$$

where P, Q, R, and S represent the resistances of the conductors AD, AC, BD, and CB respectively. For when there is no current in G the currents in P and R have the same value i, and the current j in Q has the same value as that in S. Hence by Ohm's law

$$Pi=Qj,$$

and

$$Ri=Sj,$$

[1] *Phil. Mag.*, 1895, xxxix. 176.

[2] *Roy. Soc. Phil. Trans.*, 1833.

from which there follows the well-known relation

$$P = \frac{QR}{S}.$$

Sensitiveness of the Wheatstone Bridge.—Let P be changed to $P + \delta p$. The current through it will change to $i - \delta i$ and the change in the potential difference at the extremities of P is $i\delta p - P\delta i$; of R it is $R\delta i$. If the galvanometer circuit is now completed the current through it will be equal to that produced by an E.M.F. $i\delta p - P\delta i$ in P and an E.M.F. equal to $R\delta i$ in R. If an E.M.F. equal to the latter is placed in the galvanometer branch, the current through R is $PR\delta i/(P+R)r$, where r is equal to

$$\frac{(P+Q)(R+S)}{P+Q+R+S} + G,$$

i.e. the resistance of the "external galvanometer circuit" plus that of the galvanometer. Similarly the current through P due to an E.M.F. $P\delta i$ in the galvanometer branch is equal to $RP\delta i/(P+R)r$. Now the current through the galvanometer due to an E.M.F. $R\delta i$ in R is equal to the current through the same due to an E.M.F. $P\delta i$ in P. As these must be in opposite directions through G, the only effect it is necessary to consider is the current due to an E.M.F. $i\delta p$ in P. The current through G due to this E.M.F. is found in a similar manner and is equal to

$$\frac{i\delta p}{G + \{(P+Q)(R+S)/(P+Q+R+S)\}} \cdot \frac{R+S}{(P+Q+R+S)}.$$

This, therefore, is the current through the galvanometer when the balance of the bridge is disturbed by an alteration in P of δp. The current is a maximum when the galvanometer resistance $G = \{(P+Q)(R+S)/(P+Q+R+S)\}$, *i.e.* the resistance of the "external galvanometer circuit," and this is the most suitable galvanometer resistance. In galvanometers the coils of which are wound in similar channels, and contain the same mass of wire, the electromagnetic force on the needle, and hence the deflection, is proportional to $x\sqrt{G}$, where x is the current through G. Substituting for G the most suitable galvanometer resistance, an expression for the deflection is obtained, which, from the conjugate relations between the arms of the bridge, may be reduced to the simple form

$$\frac{i\Delta\sqrt{P}}{2\sqrt{(1+Q/S)(1+Q/P)}},$$

in which $\Delta = \delta p/P$.

The best conditions for sensitiveness are here clearly indicated. The resistance Q should be small compared with S and with P, *i.e.* P, the resistance to be measured, should be connected to a comparatively large resistance R and a small resistance Q. If i is the maximum permissible current through P, R must be a resistance of large cooling surface and small temperature coefficient; if R is of the same type and dimensions as P, it should be of the same nominal value. In the latter case, which is very frequent in practice, $P = Q = R = S$ and the sensitiveness is proportional to $i\Delta\sqrt{P}/4$.

§ (7) STANDARDISING BRIDGES OF THE WHEATSTONE TYPE.—Such bridges may be divided into three main classes: (*a*) Slide wire bridge, in which the difference between two coils is measured in terms of a small length of slide wire connected between the ratio coils or the coils under comparison. (*b*) Reichsanstalt bridge, in which the ratio of two branches is altered by a definite amount and the difference of resistance between two coils is measured in terms of a galvanometer deflection. (*c*) Shunt bridge, in which coils are compared by shunting one or more of the arms of the bridge. In addition bridges have been devised which are combinations of these, and the special bridges designed for use with platinum thermometers can be used for the comparison of standard resistance coils.

(i.) *The Slide Wire Bridge.*—The first bridge to employ a slide wire was devised by Fleeming Jenkin in 1862 and was used to intercompare the standard coils made for the British Association Committee on Electrical Standards. The arrangement is shown in *Fig.* 13. The two coils to be compared are P and R, Q and S being the ratio arms. Between these latter a slide wire is connected and balance is obtained by adjusting the position of the contact on this wire. The coils P and R are next interchanged and the balance restored by altering the position of the contact.

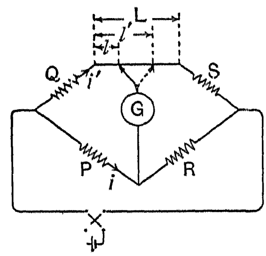


FIG. 13.—Fleeming Jenkin Slide Wire Bridge.

From the figure and the application of Ohm's law it follows that

$$Pi = (Q \times lr)i',$$

$$Ri = \{S + (L-l)r\}i',$$

where r is the resistance per unit length of the bridge wire. On reversal of P and R the new relation is

$$Ri = (Q + l'r)i',$$

$$Pi = \{S + (L-l')r\}i'.$$

From these four equations it follows that

$$P - R = \frac{i'}{i}(l - l')r.$$

The ratio i/i' can be sufficiently well estimated from the resistance coils and the bridge wire; r is directly measured.

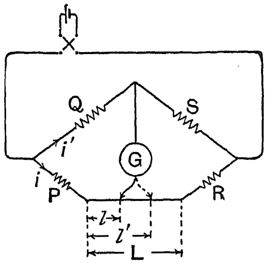


FIG. 14.—Carey Foster Slide Wire Bridge.

In employing this method copper blocks with mercury cups were used and excellent results were obtained.

(ii.) *Carey Foster Slide Wire Bridge.*—In 1872 Carey Foster[1] modified the above method by connecting the slide wire between the coils to be compared.

Fig. 14 shows the connections for comparing two coils.

The condition for the first balance is

$$i(P + lr) = Qi',$$

$$i[R + (L - l)r] = Si',$$

and after interchanging P and R in position,

$$i(R + l'r) = Qi',$$

$$i[P + (L - l')r] = Si',$$

from which equations it follows that

$$P - R = (l' - l)r.$$

In words, the difference between the two resistances P and R is equal to the resistance of the length of the slide wire included between the two points of contact at which balance is obtained.

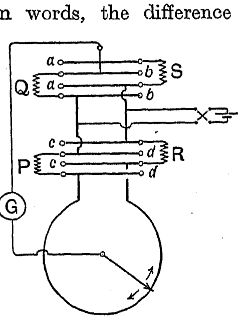


FIG. 15.—Fleming Standardising Bridge.

The slide wire is calibrated by shunting one of the coils to be compared with a known resistance and making another determination of the difference.

(iii.) *Fleming Form of Carey Foster Bridge.*—This is shown diagrammatically in *Fig.* 15. The bridge was designed by Fleming in 1880 for the intercomparison of the coils belonging to the British Association. The slide wire is of platinum-iridium and is protected from direct contact with the hand; it is 39 in. long, has a resistance of about one-twentieth of an ohm, and is in the form of a circle. The contact piece included in the galvanometer circuit is a small prism of platinum-iridium. The thick lines represent massive copper blocks with mercury cups at their ends and permit the interchange of coils in the bridge by moving each coil from one pair of mercury cups to a neighbouring pair. Thus when the leads of P are moved into the cups *cc* and those of R are moved into the cups *dd*, the coils P and R are interchanged in position in the bridge arms.

This form of bridge gives excellent results, but does not meet modern requirements. It is not suited for the comparison of resistances with potential terminals, nor can it be easily adapted for the measurement of very low resistances. In practice it is difficult to eliminate entirely the thermo-electric effects at the slide wire contact, because the galvanometer circuit must in general be made *after* the battery circuit. These defects are common to most slide wire bridges.

(iv.) *Nalder's Form of Slide Wire Bridge.*—This is an exceedingly compact form of bridge and is in considerable use. It is shown diagrammatically in *Fig.* 16. Massive bars of copper connect the coils to the bridge, the amalgamated leads of the coils dipping into mercury cups formed in the bars. The coils to be compared are connected to the cups *cc* and *dd* and the ratio coils are connected to *ee* and *ff*. The ratio coils are on one bobbin and in general are of 1, 10, 100, or 1000 ohms. The slide wire of the bridge is a very short platinoid wire, and in general the instrument is provided with a number of slide wires the resistances of which are suitable for various comparisons. The interchange of the coils P and R in position in the bridge is effected by a commutator turning about the centre O.

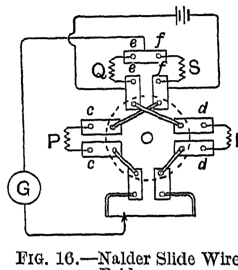


FIG. 16.—Nalder Slide Wire Bridge.

(v.) *Reichsanstalt Form of Wheatstone Bridge.*—This is intended for use with manganin coils of the Reichsanstalt pattern. One form of connections for coils without potential leads is shown in *Fig.* 17.

The special feature of this bridge is the ratio bobbin QS, which contains two coils of equal value between which an interpolation resistance is inserted. The complete ratio bobbin consists of four coils in series, and in *Fig.* 17 these

[1] *Society of Telegraph Engineers Journal*, 1872.

are shown as having resistances of 99·95, 0·05, 0·05, and 99·95 ohms respectively; three contact studs are connected to the junctions of these coils, and by moving the contact arm the ratio Q/S can be made 100·05/99·95, 100·00/100·00, or 99·95/100·00.

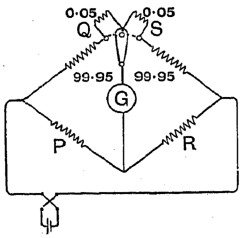


FIG. 17.—Bridge with Ratio Coils having Interpolation Resistances. (Reichsanstalt form.)

In the ordinary use of the bridge the difference P – R is measured in terms of a difference of two deflections of the galvanometer, first when the ratio contact piece is on the centre stud, and again when it is on an end stud. A movement from the centre stud to a side stud is equivalent to a change of 0·1 per cent in the ratio, and the resulting change in the deflection enables the difference P – R to be calculated. Thus if when Q = S = 100·00 the deflection due to a want of equality of P and R is + 20 divisions, and when Q = 100·05 and S = 99·95 the deflection is – 85 divisions, then a change in the ratio of 0·1 per cent produces a deflection of 105 divisions, so that P is greater than R by 20 × 0·1/105 per cent, which is equal to 19·4 parts in 100,000. To correct for possible inequality of the ratio coils, an interchange of position of Q and S, or P and R is necessary. In cases where the difference between P and R is appreciable the greater may be shunted by a known resistance to make the effective values more nearly equal.

The Reichsanstalt form of bridge is simple and cheap, and can be used for both high and low resistances. The bridge itself has massive copper blocks with mercury cups to provide good contacts, the blocks being adjustable in position so as to allow for differences in size of coils and to provide for slightly different arrangements. The coils are immersed in insulating oil maintained at a constant temperature of 20°·0 C., small centrifugal pumps being employed to keep the oil in constant circulation. The bath is electrically heated. The general arrangement is shown in *Fig.* 23, which shows six coils arranged to form a Kelvin double bridge for the measurement of resistances with potential terminals.

(vi.) *Wheatstone Shunt Bridge.*—In recent years the use of Carey Foster's method has been abandoned for measurements of the highest precision in favour of the method of shunting. This latter method necessitates the use of a resistance box with high-resistance coils the values of which are known with fair accuracy. As a general rule, the more nearly equal the two coils are which it is desired to compare, the less is the necessity for knowing the accurate values of the shunt resistances. The shunt method possesses the great advantage over the slide wire method inasmuch as resistances of many thousands of ohms are dealt with instead of the resistance of a few millimetres of slide wire. Moreover, the galvanometer circuit is kept closed and thermo-electric forces produce therefore the minimum disturbances. This method has been in use for many years at the National Physical Laboratory, not only for the comparison of coils with current leads, but also for the comparison of resistances with potential terminals.

Fig. 18 is a diagrammatic representation of the bridge, the selector switch K enabling

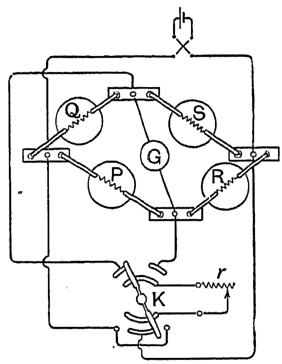


FIG. 18.—Shunt Bridge as used at National Physical Laboratory.

any one of the four coils P, Q, R, S to be shunted by a resistance r which can be adjusted to have any value up to one megohm.

Which of the coils will require shunting depends on the relative values of the coils, but in no case is it necessary to shunt more than one coil. However, if it is assumed at first that each of the coils P, Q, R, and S is shunted, the respective shunts being W‴, W, W″, and W′, then when no current flows through the galvanometer

$$\frac{P}{1+P/W'''}=\frac{[Q/\{1+(Q/W)\}].[R/\{1+(R/W'')\}]}{S/\{1+(S/W')\}},$$

or calling P_1, Q_1, R_1, and S_1 the effective values of the shunted resistances,

$$P_1=\frac{Q_1R_1}{S_1}.$$

In practice the ratio of P to R for good standard coils does not differ from unity by more than 2 or 3 parts in 10,000, and the values P_1, Q_1, R_1, S_1 do not in general differ from P, Q, R, and S respectively by more than 5 parts in 10,000. The calculation of a shunted value is usually very simple, for with standard coils of nominal values 10^n ohms,

where n is a whole number, the reciprocals of the shunt resistances W, W′, etc., enable the changes in the effective values of the arms to be rapidly calculated.

The general arrangement of the apparatus is very similar to that shown in *Fig.* 25, which shows six coils arranged as a Kelvin bridge.[1] The coils are of manganin and are connected together by massive copper blocks which can be moved along ebonite slides rigidly connected to a copper tank which contains paraffin oil in which the coils are immersed. The temperature of the oil is maintained constant at 20°·00 C. An electrical heating coil distributed over and about 1 cm. above the bottom of the bath is used and a toluene temperature regulator is employed. Stirring of the oil is effected by blowing dry air through a large number of fine holes in a lead pipe coiled at the bottom of the bath. This method of stirring is remarkably effective, the temperature throughout the bath not differing from 20°·00 C. by more than 0°·01. The galvanometer circuit is closed permanently, the condition for balance being that there is no change of deflection on reversal of the current through the bridge.

(*a*) *Comparison of Standard Coils of Unit Value without Potential Leads.*—In *Fig.* 18, let P be a coil of known value and R a coil the value of which is desired. S and Q may be coils of 1 or of 10 ohms resistance; in general the former resistance is preferable. Balance is obtained by shunting Q or S; then

$$P=\frac{Q'R}{S'},$$

where Q′ and S′ represent the effective values of Q and S.

P and R are now interchanged in position and the balance restored by adjustment of the shunting resistance, or, if necessary, by shunting the other coil; then

$$R=\frac{Q''P}{S''}.$$

Combining this with the previous equation, the result is

$$\frac{P^2}{R^2}=\frac{Q'S''}{Q''S'}.$$

When Q and S are unit coils and the resistance of each is very near its nominal value, sufficient accuracy is obtained by rewriting the equation in the form

$$P-R=\tfrac{1}{2}(Q'-Q''+S''-S').$$

One-half of the difference between the shunted values is equal, therefore, to the difference between the two coils.

For example, let $P=1{\cdot}000100$ ohms, and for the first balance let Q (of nominal value 1 ohm) be shunted by 8520 ohms. After the interchange of position of P and R let the balance be restored by shunting S by 9450 ohms. If $Q=1{\cdot}000300$ ohms and $S=1{\cdot}000100$ ohms, the value of R within 1 part in a million is 0·999988 ohm. By the approximate equation given above

$$\begin{aligned}P-R&=\tfrac{1}{2}(Q'+S'')\\&=\tfrac{1}{2}(1-\tfrac{1}{8520}+1-\tfrac{1}{9450})\\&=-0{\cdot}000112 \text{ ohm.}\end{aligned}$$

Hence $R=1{\cdot}000100-0{\cdot}000112=0{\cdot}999988$ ohm.

If the coils differ appreciably from their nominal values the simple equation involving reciprocals only is not permissible for work of the very highest precision, but in general the errors are very small.

This method is simple and very accurate, but it is necessary to point out that the amalgamated ends of a coil must be at least about 1 sq. cm. in area, and the positions in the contact blocks must be reproducible if differences within one or two millionths of an ohm are to be repeated in the measurements.

(*b*) *Measurement of Unit Coils with Potential Terminals.*—Most modern standard coils are fitted with potential terminals, and in general the difference between the resistance measured between the ends of the current leads and that between the potential terminals does not exceed 0·00006 ohm. The type of leads is shown in *Figs.* 4 and 5.

The practice at the National Physical Laboratory is to compare such coils with a standard by a substitution method. The bridge is arranged as shown in *Fig.* 19, P being a 1 ohm standard with potential terminals. A balance is obtained, first with P in position in the bridge, and again when the resistance coil, the value of which is required, is substituted for P. In each case, however, two measurements are necessary. In general

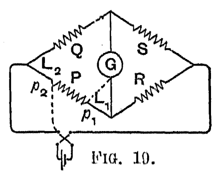


Fig. 19.

Q is a coil of nominal value			10	ohms.
R	,,	,,	,, 10	,,
S	,,	,,	,, 100	,,

Let the resistance of the standard between the potential terminals p_1, p_2 be P, and that between the ends of its current leads be $P+L_1+L_2$. Let the connections be those shown by the completed lines in *Fig.* 19. Then, assuming that R is not shunted,

$$P+L_1+L_2=\frac{Q'R}{S'} \text{ or } P=\frac{Q'R}{S'}-(L_1+L_2), \qquad (A)$$

where Q′ and S′ are the shunted values of Q and S respectively.

[1] See § (8).

The battery lead connected to the junction Q. L_2 is now disconnected and joined to the

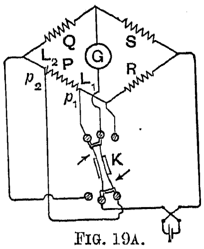


FIG. 19A.

potential terminal connected to the junction P . L_2, and the galvanometer lead joined to R . L_1 is transferred to P . L_1. In practice such changes may be made by a switch K, such as is shown in *Fig.* 19A. The new connections are indicated by the dotted lines in *Fig.* 19. After balancing we have

$$P=\frac{(Q''+L_2)(R+L_1)}{S''}, \quad . \quad . \quad . \quad (B)$$

or since Q''/S'' and R/S'' are both equal to 0·1 very nearly, and the term $L_1 . L_2/S''$ is in general less than one hundred-millionth of an ohm, we have, as a sufficiently accurate equation,

$$P=\frac{Q''R}{S''}+0{\cdot}1(L_1+L_2). \quad . \quad . \quad (C)$$

Combining this with the equation

$$P=\frac{Q'R}{S'}-(L_1+L_2),$$

an expression for the resistance of the leads is obtained. This is

$$1{\cdot}1(L_1+L_2)=\left(\frac{Q'}{S'}-\frac{Q''}{S''}\right)R. \quad . \quad (D)$$

The value of L_1+L_2 thus obtained includes all contact resistances and connecting pieces associated with the current leads, and is the value required for insertion in equation (C). In general the value of L_1+L_2 is about 0·00004 ohm, and the correction for the leads in equation (C) is one-tenth of this.

When a second coil is substituted for P similar expressions for the balancing conditions are obtained. If P_2 is the resistance between the potential terminals of this second coil, and $P_2+l_1+l_2$ the resistance between the ends of its current leads, then, as before,

$$P_2=\frac{Q''''R}{S''''}+0{\cdot}1(l_1+l_2),$$

and $$1{\cdot}1(l_1+l_2)=\left(\frac{Q'''}{S'''}-\frac{Q''''}{S''''}\right)R.$$

Hence

$$P-P_2=R\left(\frac{Q''}{S''}-\frac{Q''''}{S''''}\right)+0{\cdot}1[(L_1+L_2)-(l_1+l_2)].$$

For example, when P is in place in the bridge, let Q be shunted first by 127,000 ohms, and after the change over of galvanometer and battery leads let the shunt on Q be changed to 80,000 ohms. Then the current leads of P are given by the expression

$$1{\cdot}1(L_1+L_2)=\frac{QR}{S}[1-\tfrac{10}{127000}-(1-\tfrac{10}{80000})].$$

As QR/S is very nearly unity, $L_1+L_2=0{\cdot}00004_2$ ohm.

When P_2 is substituted for P, let the corresponding shunts on Q be 322,000 ohms and 121,000 ohms. Then similarly $(l_1+l_2)=0{\cdot}00004_7$ ohm, and

$$P-P_2=\frac{QR}{S}[1-\tfrac{10}{80000}-(1-\tfrac{10}{121000})]$$
$$-0{\cdot}000000_5 \text{ ohm}=-0{\cdot}00004_3 \text{ ohm}.$$

In the particular example quoted P was a manganin coil of $0{\cdot}99999_0$ international ohm at 20°·0 C., and hence P_2 was equal to $1{\cdot}00003_3$ international ohm at 20°·0 C.

This shunt method of comparing resistances with potential terminals is in practice both rapid and accurate. With care the differences between unit coils of manganin can be measured and the measurements repeated within one ten-millionth of an ohm. It will be observed that while Q, R, and S should be coils having resistances quite close to their nominal values of 10, 10, and 100 ohms respectively, the actual values are not needed. The galvanometer circuit is permanently closed to diminish any trouble due to thermo-electric effects, the condition for balance being no change of deflection of the galvanometer when the battery connections to the bridge are reversed. Standard coils rarely possess much inductance, so that the inductive kick of the galvanometer when the battery circuit is made is very small.

Other methods of comparing unit coils with potential terminals are by means of the Kelvin double bridge and by the potentiometer.

(*c*) *Comparison of Nominally Equal Coils greater than* 1 *Ohm.*—Standard coils of nominal value 10, 100, 1000, 10,000 ohms, etc., are readily compared by means of the shunt bridge, the general arrangement of the bridge being similar to that used for the comparison of coils of unit value. With increasing resistance in the bridge a high-resistance galvanometer should be employed, and since it is usual to employ a battery of 10 or 20 volts, electrostatic effects on the galvanometer should be guarded against.

(*d*) *Comparison of Coils of Unequal Values, the Ratios of Resistance being approximately equal to* 10^n *where* n *is any whole number.*—Such operations are usually termed "build-up" processes.

The first simple and accurate device for such a purpose was designed by Lord Rayleigh and is described in the 1883 Report of the British Association. There are three coils each of nominal value 3 ohms, and one coil of 1 ohm. Let the true values of the coils be $3+\alpha$, $3+\beta$, $3+\gamma$, and $1+\delta$, where α, β, γ, and δ are very small quantities. The resistance of the first three coils in series is $9+\alpha+\beta+\gamma$, and in parallel (if very small terms are neglected) it is

$$1+\tfrac{1}{9}(\alpha+\beta+\gamma).$$

If, therefore, the three coils are placed

first in parallel and compared with a 1-ohm standard coil, and afterwards placed in series, together with the 1-ohm coil referred to above, and then compared with a 10-ohm standard, the relative values of the 1- and 10-ohm standards can be obtained.

Lord Rayleigh wound the three 3-ohm coils on one bobbin, and devised an arrangement of mercury cups enabling the changes from series to parallel to be quickly made.

This principle has been used at the National Physical Laboratory for building up 100-ohm coils from 10-ohm resistances, and also 1000-ohm coils from 100-ohm resistances. For convenience ten 100-ohm coils are used, and these are arranged as shown in *Fig.* 20. The coils are of manganin and are immersed in paraffin oil. Mercury cups in heavy copper blocks terminate the coils, and by using short leads (which are allowed for) to connect to the bridge, the values of the ten coils in terms of a 100-ohm standard are obtained.

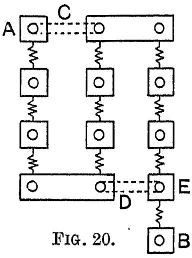


FIG. 20.

If the ten coils are placed in series by the insertion of leads at A and B (the connecting links C and D being omitted) a resistance of 1000 ohms (nominal) results, and this can be compared with a standard coil of 1000 ohms.

If the thick short amalgamated copper stirrups C and D are inserted, the resistance between A and E is 100 ohms nominal, and this is compared with a 100-ohm standard coil. As the relative values of the coils have been found to keep very constant, it is in general sufficient to compare the 3 three hundreds in parallel with a 100-ohm coil, and the 10 hundreds with a 1000-ohm coil. This method has proved to be accurate within less than 1 part in a million.

(*e*) Drysdale [1] has also used this principle in devising a combination of five coils of relative values 1, 3, 3, 3, and 1 ohm. These are wound on bobbins and contained within a single cylinder of the size used with standard coils of the Reichsanstalt type. The leads have heavy copper lugs with current and potential terminals and sliding mercury contacts. Between the lugs (*Fig.* 21) are four circular copper blocks, and these blocks and the ends of the lugs are faced off together so as to be exactly flat and level. The upper surfaces of these blocks are amalgamated, and two heavy copper bars with amalgamated lower surfaces make the necessary connections with the minimum contact resistance.

[1] *Electrician*, 1917.

With these coils combinations of nominal value 1, 2, 3, 4, 5, . . . up to 11 ohms can be obtained.

(*f*) Rosa [2] has designed a form of bridge for comparing the sum of five coils of one denomination with two coils in parallel of

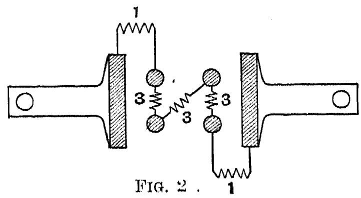


FIG. 2 .

the next higher denomination. However, it does not appear to possess any advantages over the paralleling of three coils as already described.

§ (8) KELVIN (OR THOMSON) DOUBLE BRIDGE.—The Kelvin double bridge, which was first described by Sir William Thomson [3] (later Lord Kelvin) in 1862, is a network of eight conductors and is diagrammatically represented in *Fig.* 22. It is essentially a bridge for the measurement of low resistances. In *Fig.* 22 P and R are two low resistances with potential terminals, Q and S are ratio resistances of a higher value (*e.g.* 1 to 10 ohms), and α and β are auxiliary ratio coils. G is the galvanometer and B the battery, the letters in all cases representing the values of the resistances in addition to designating them. The branch d is a link connecting P and R, and in general includes one current lead of R and one of P. d is of low resistance.

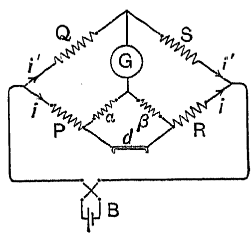


FIG. 22.—Kelvin Double Bridge.

When the battery is connected and there is no current through the galvanometer, let i be the current through R and i' the current through S. The current through α and β is $id/(\alpha+\beta+d)$, and through d it is $i(\alpha+\beta)/(\alpha+\beta+d)$. Then by Ohm's law

$$iP+\frac{i\alpha d}{\alpha+\beta+d}=i'Q,$$

and

$$iR+\frac{i\beta d}{\alpha+\beta+d}=i'S,$$

and hence

$$P=\frac{QR}{S}+\left(\frac{Q}{S}-\frac{\alpha}{\beta}\right)\left(\frac{\beta d}{\alpha+\beta+d}\right).$$

[2] *Bureau of Standards Bull.*, 1909, v. 423.
[3] *Phil. Mag.*, 1862, xxiv.

When $Q/S=\alpha/\beta$ the equation reduces to the simple form

$$P=\frac{QR}{S}.$$

Sensitiveness of the Kelvin Double Bridge.[1]—When P is changed to $P+\delta p$ the current through the galvanometer is

$$\frac{i\delta p}{G+(\alpha\beta)/(\alpha+\beta)+[(P+Q)(R+S)/(P+Q+R+S)]}\cdot\frac{R+S}{P+Q+R+S},$$

from which it follows that the best galvanometer resistance is

$$\frac{\alpha\beta}{\alpha+\beta}+\frac{(P+Q)(R+S)}{P+Q+R+S}.$$

As the deflection of the galvanometer is proportional to $\sqrt{G}$ and to the current, the expression for the deflection is

$$\frac{i\delta p\sqrt{G}.}{G+(\alpha\beta)/(\alpha+\beta)+[(P+Q)(R+S)/(P+Q+R+S)]}\cdot\frac{R+S}{P+Q+R+S},$$

and if for G, the best galvanometer resistance is substituted, an expression is obtained which, from the conjugate relationship of the arms of the bridge, may be written in the simple form

$$\frac{i\Delta\sqrt{P}}{2\sqrt{(Q+S)(P+Q+\alpha)/PS}},$$

in which $\Delta=\delta p/P$.

If $\alpha=0$ this expression for sensitivity is identical with that for the Wheatstone bridge. It is clear that for good sensitivity α should be kept as small as possible.

It has already been shown that the condition for balance is

$$P=\frac{QR}{S}+\left(\frac{Q}{S}-\frac{\alpha}{\beta}\right)\left(\frac{\beta d}{\alpha+\beta+d}\right).$$

In practice it is very desirable that the correction term shall be negligibly small, and it is clear that the correction vanishes when the main and auxiliary ratios are equal. Reeves[2] first showed how the ratios could be adjusted to equality and the bridge balanced at the same time. His method consists in adjusting the ratio coils Q and S until the bridge is balanced and then removing the link d (thus making a simple Wheatstone bridge), and adjusting the ratio $(P+\alpha)/(R+\beta)$ until equal to Q/S. The link d is then restored and a balance again established by adjustment of Q or S, after which d is removed and $(P+\alpha)/(R+\beta)$ again adjusted. Thus, by successive approximations, the three ratios P/R, Q/S, and α/β are made more and more nearly equal until there is no change in the balance when d is removed. Q/S is then equal to α/β within the limits of the errors of measurement.

This method is in use in the national standardising laboratories of England, the United States, and of Germany. It does not follow, however, because Q/S is equal to α/β within the limits of error of measurement, that $(Q/S-\alpha/\beta)[\beta d/(\alpha+\beta+d)]$ is negligibly small. It is so only if the value of $\beta d/(\alpha+\beta+d)$ does not exceed the value of P. If its value be NP and the probable error of an observation is 1×10^{-n}, the error of the final result is not less than $N\times10^{-n}$. It follows, therefore, that the current leads of standard resistances intended for measurement by means of the Kelvin double bridge should have a resistance not greater than that of the standard itself. This, however, is very difficult when P is 0·00001 ohm or less, and it is known that in many low-resistance standards the resistance of the current leads plus the connectors is greater than that of the standard. In cases where it is undesirable to remove d, Wenner and Wiebel[3] have devised two procedures which do not involve removing the link. One of these is described after a description of the bridges and connections actually employed.

§ (9) Kelvin Bridges of High Precision for the Comparison of Low Resistances. (i.) *Reichsanstalt Form of Kelvin Bridge.*—The

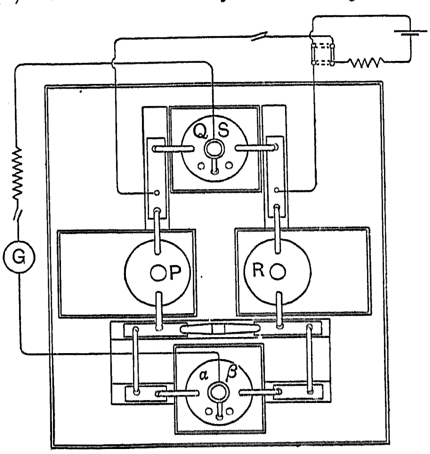


Fig. 23.—Reichsanstalt Type of Kelvin Double Bridge.

type is shown in *Fig.* 23 and the connections are indicated in *Fig.* 24.

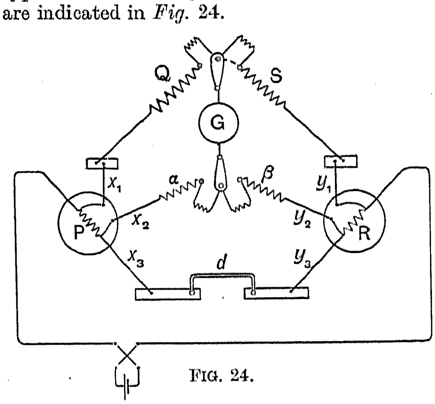


Fig. 24.

The bridge may be used to give P in terms of Q, R, and S by reading galvanometer

[1] Smith, *British Association Report*, 1906.
[2] *Phys. Soc. Proc.*, 1896, xiv.
[3] *Bureau of Standards Bull.*, 1914, xi. 65.

deflections only, or it may be used with Q or S shunted, and from the value of the effective resistances of Q and S together with the deflections given by the galvanometer the value of P in terms of Q, R, and S can be obtained. Two ratio bobbins are employed each of which contains two main coils, Q . S and $\alpha . \beta$ respectively, in addition to interpolation resistances. The resistances in any one ratio bobbin are in general 99·95, 0·05, 0·05, and 99·95 ohms. By preliminary adjustments the ratio α/Q is made equal to β/S and the interpolation resistances are also made equal; in practice this is conveniently done by shunting one of the coils. The resistances Q and S must in practice include the leads x_1 and y_1 which connect them to the potential points of P and R respectively, and the coils α and β are in series with x_2 and y_2. d includes the resistances of the current leads x_3 and y_3. The complete expression for P is therefore

$$P=\frac{(Q+x_1)R}{S+y_1}+\left(\frac{Q+x_1}{S+y_1}-\frac{\alpha+x_2}{\beta+y_2}\right)\left(\frac{d(\beta+y_2)}{\alpha+\beta+d+x_2+y_2}\right).$$

However, since x_1 and x_2, and y_1 and y_2 are in practice nearly equal, and also very small compared with Q, S, α, and β, the actual effect of x_1, x_2, etc., is practically negligible. Thus if Q, S, α, and β are equal and each of nominal value 100 ohms, and x_1, x_2, etc., are each of nominal value 0·0001 ohm but differ by amounts equal to 0·00005 ohm, then

$$\frac{Q+x_1}{S+y_1}-\frac{\alpha+x_2}{\beta+y_2}$$

cannot be greater than 0·000001. Hence, unless d is greater than P the effect of the leads x_1, x_2, etc., is in general negligible. However, such high values for Q, S, α, and β reduce the sensitiveness of the bridge.

By the deflection method the value of P in terms of $(Q+x_1)R/(S+y_1)$ can be obtained by moving both of the ratio contact arms from the centre studs to similar ends of the interpolation resistances and observing the galvanometer deflection before and after such change. The change of deflection represents a change in the ratio of 1 part in 1000. When the ratio $(Q+x_1)/(S+y_1)=(\alpha+x_2)/(\beta+y_2)$ is exactly unity, the galvanometer deflection is a measure of the difference P − R. By removal of the link d the ratio $(Q+x_1)/(S+y_1)$ can be made equal to $(P+\alpha+x_2)/(R+\beta+y_2)$ within the limits of measurement, by adjustment of Q and α, or S and β.

(ii.) *National Physical Laboratory Type of Kelvin Double Bridge.*—The arrangement of coils in the bath, together with the leads to the shunting resistances, etc., are shown in *Fig.* 25. *Fig.* 25A is a diagram of connections. Ratio bobbins with interpolation resistances are not employed; instead, the ratios Q/S and α/β are varied by equal amounts by shunting, the value of the correction term being thus made independent of the shunts.

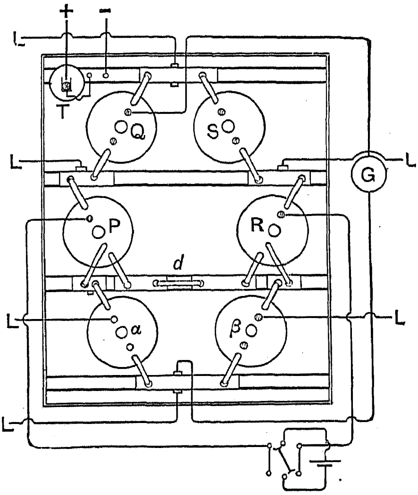


FIG. 25.—National Physical Laboratory Form of Kelvin Bridge.

Leads marked L are used to connect to shunting resistance. T is a toluene thermostat in series with a heating coil in the bath.

The most convenient way of effecting this is to have dial resistances for shunting, the switches being mechanically connected. Either Q and α or S and β may be shunted, but not both.

The comparison of two coils may readily be effected by substitution, but it is a little more

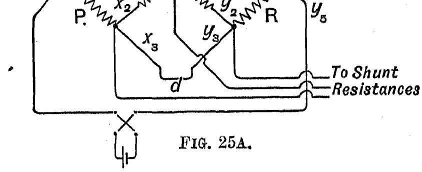


FIG. 25A.

difficult to evaluate a coil in terms of three coils Q, R, and S, each of which is of a higher denomination. It is, however, essential for standards work that this shall be done. In general, whatever be the value of P (the resistance of lowest denomination), Q is chosen to be of 1 ohm resistance and S of 10 ohms.

Similarly α and β are of 1 ohm and 10 ohms respectively. If, then, the value of P is 0·1 ohm, R is 1 ohm. In such a case the values of Q and R between potential terminals are known by direct comparison with 1 ohm standards and S has been determined by a build-up process. As shown in *Fig.* 25A, S has in series with it one current lead of R and one of its own current leads the summed resistance of which is y_1, and also one current lead of Q and the second current lead belonging to it, the combined resistance of which is y_4. Usually y_1+y_4 does not exceed 0·0001 ohm. In series with Q there is one of its own current leads and one belonging to P; the combined resistance of these leads is called x_1.

Now the final balance must ensure that

$$\frac{\text{P}}{\text{R}}=\frac{\alpha+x_2}{\beta+y_2}=\frac{\text{Q}+x_1}{\text{S}+y_1+y_4}.$$

Reeves's method of obtaining a balance whether or not d is in position is often employed; at other times the procedure is as follows:

(a) To make

$$\frac{\alpha+x_2}{\beta+y_2}=\frac{\text{Q}+x_1}{\text{S}+y_1+y_4}.$$

The potential leads to low resistances are so very similar that as a first approximation they may be regarded as equal in resistance. Thus in general $y_2=y_5$ and $x_2=x_5$ within 10 microhms. To ensure the above relationship, β is connected in series with y_5 instead of y_2 and α with x_5 instead of x_2; d is removed and the battery leads connected to y_3 and x_3. The general arrangement is then that shown in *Fig.* 26.

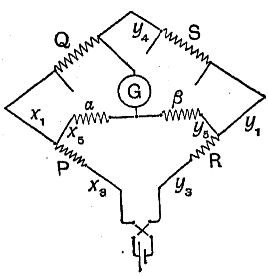


FIG. 26.

The bridge is balanced by adjustment of α or β by shunting. Then

$$\frac{\alpha+x_5}{\beta+y_5}=\frac{\text{Q}+x_1}{\text{S}+y_1+y_4}.$$

If $x_5=x_2$ and $y_2=y_5$ within 10 microhms the ratio $(\alpha+x_5)/(\beta+y_5)$ is equal to $(\alpha+x_2)/(\beta+y_2)$ within 11 parts in one million. If x_5 and x_2 *differ* by 0·0001 ohm, which difference is in general greater than the resistance of either, and y_2 and y_5 also differ in resistance by an equal amount, then if d is 0·0001 ohm the correction term

$$\left(\frac{\text{Q}+x_1}{\text{S}+y_1+y_4}-\frac{\alpha+x_2}{\beta+y_2}\right)\cdot\left(\frac{d(\beta+y_2)}{\alpha+\beta+d+x_2+y_2}\right)$$

cannot have a greater value than $\pm 0{\cdot}0011$ microhm. This may be regarded as a maximum value and may be neglected except when P is less than 0·001 ohm.

(b) To ensure that

$$\frac{\text{P}}{\text{R}}=\frac{\text{Q}+x_1}{\text{S}+y_1+y_4}.$$

The connections are restored to give the arrangement shown in *Fig.* 25A, d being in position. Balance is obtained by shunting Q and α or S and β simultaneously so as not to alter the equality of ratios already obtained. When balance results

$$\text{P}=\frac{\text{R}(\text{Q}'+x_1)}{\text{S}'+y_1+y_4}+\left(\frac{\text{Q}'+x_1}{\text{S}'+y_1+y_4}-\frac{\alpha'+x_2}{\beta'+y_2}\right)\left\{\frac{d(\beta'+y_2)}{\alpha'+\beta'+d+x_2+y_2}\right\},$$

where Q′, S′, etc., represent the shunted values. The correction term being negligibly small, then

$$\frac{\text{P}}{\text{R}}=\frac{\text{Q}'+x_1}{\text{S}'+y_1+y_4}.$$

d is usually removed as a check to see if

$$\frac{\text{Q}'+x_1}{\text{S}'+y_1+y_4}=\frac{\text{P}+\alpha'+x_2}{\text{R}+\beta'+y_2}.$$

If adjustment is necessary, which is very rare, d is restored in position and the bridge again balanced.

(c) To evaluate x_1, y_1, and y_4.

After procedure (b) the current lead to the junction R . y_1 is connected to the potential terminal p_1 of S. The change in the balancing condition enables y_1 to be calculated in the manner already described with the Wheatstone bridge. Similarly x_1 is obtained by moving the current lead to the potential terminal p_3, and y_4 is obtained by moving the galvanometer lead to p_2.

The process described is the most elaborate that is conducted with the Kelvin double bridge. The low-resistance standard is not compared with another one nominally equal to it, but is compared with a combination of standard resistances, each of which is at least 10 times greater than the one the value of which is desired. For measurements other than those of the highest precision it is often possible to ignore the values of x_2 and y_2 and to use for the values of y_1 and y_4 those resulting when R, Q, and S were originally evaluated.

(iii.) *Wenner and Weibel's Method of Balancing the Kelvin Bridge.*—Wenner and Weibel [1] suggested a method for balancing the Kelvin double bridge and eliminating corrections which does not involve disturbing the connector d. The method requires the use of a variable double ratio set (Q . S and α . β respectively) so adjusted that for any

[1] *Bureau of Standards Bull.*, 1914, xi. 65.

setting of the dial switches of the shunt resistances the lack of equality of the two ratios Q/S and α/β is so small that no appreciable error is introduced on this account.

Two switches S_1 and S_2 (*Fig.* 27) are required,

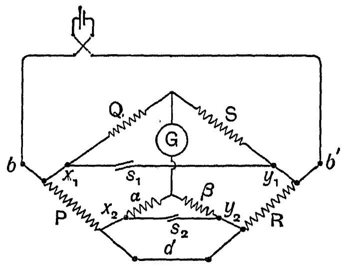


FIG. 27.

and special adjustable low resistances [1] are included in the connectors x_1 and x_2.

The procedure for the first method is as follows: (*a*) With S_1 and S_2 open, the bridge is balanced.

This makes P/R = Q/S approximately.

(*b*) With S_1 closed the bridge is balanced by an adjustment of the variable low resistance forming a part of x_1. This makes $y_1\alpha = x_1\beta$ very closely.

(*c*) The switch S_1 is opened and S_2 is closed, and the balance restored by adjusting the variable low resistance forming part of x_2. This makes $y_2\alpha = x_2\beta$ very closely.

(*d*) With the switches S_1 and S_2 both open, the bridge is balanced by an adjustment of the dial switches of the double ratio set. This makes P = QR/S with great accuracy.

If the fourth adjustment requires a considerable change in the setting of the shunts, the second, third, and fourth adjustments are repeated. The adjustable low resistances are in the form of a mercury slide; each consists of an ebonite tube about 12 cm. long and 3 mm. internal diameter. The terminals are at the upper and lower ends of the tube and an amalgamated copper plunger serves to displace the mercury and so vary the resistance.

§ (10) DRYSDALE'S UNIVERSAL STANDARDISING BRIDGE.[2]—This bridge serves for the interpolation and deflectional method of the Reichsanstalt, or for the shunting method. Its chief feature, however, is the adaptation of the Carey Foster slide wire method to the Kelvin double bridge. The connections are shown in *Fig.* 28. The bridge proper consists of eight copper bars arranged in the form of a cross, connection between the inner ends of these bars being made by an eight-pole mercury commutator. Two pairs of ratio coils Q . S and $\alpha . \beta$ are employed, which span across the front and back contacts respectively. The two resistances P and R to be compared are connected in series by their current

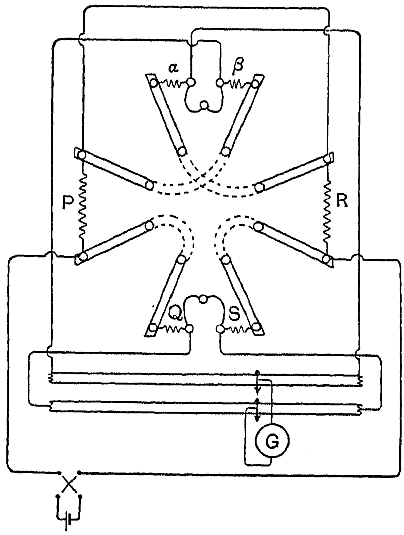


FIG. 28.—Drysdale's Standardising Bridge.

terminals, while their potential terminals make contact with the side bars of the bridge. By turning the commutator half round, the connections between the potential terminals of the coils under comparison and the ratio coils are interchanged.

The most convenient and accurate method of test is to employ ratio coils with interpolation resistances as in the Reichsanstalt bridge, but instead of working by deflections, to connect a slide wire across each pair of interpolation resistances. These slide wires can be adjusted to give the difference between the resistances directly in millionths of their value, by taking the difference between the scale readings with the commutator in the two positions. If Δ is the difference between the bridge readings in millionths (which by construction should be the same for both wires) and l, l' and m, m' are the small resistances at the front and back potential contacts, then

$$\frac{P-R}{P} = \Delta + \frac{l-m}{\alpha} + \left(\frac{l-m}{\alpha} - \frac{l'-m'}{Q}\right)\frac{X}{2P},$$

where X is the parallel resistance of the back connection shunted by the ratios α and β.

When the differences between the resistances of the potential contacts are negligible compared with α and Q

$$\frac{P-R}{P} = \Delta.$$

[1] Jaeger and Diesselhorst, *Wiss. Abhandl. P.T. Reichsanstalt*, 1904, iv.
[2] *Electrician*, 1917.

; (11) EXTENSION OF DOUBLE BRIDGE INCIPLE.—Smith[1] has extended the prinle of the double bridge to the battery arm shown in *Fig.* 29, the shunt coils a and b ving current-carrying capacities at least as

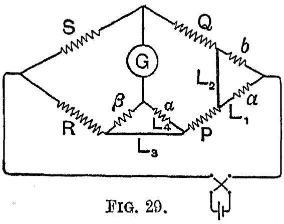


FIG. 29.

 əat as those of P and Q respectively. The ndition for balance is

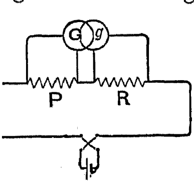
$$=\frac{QR}{S}+\frac{\beta L_3}{\alpha+\beta+L_3+L_4}\left(\frac{Q+\dfrac{bL_2}{a+b+L_1+L_2}}{S}-\frac{\alpha+L_4}{\beta}\right)+\frac{bL_2}{a+b+L_1+L_2}\left(\frac{R}{S}-\frac{a+L_1}{b}\right).$$

practice it is possible to make negligibly ıall the second and third terms on the rightnd side of this equation, and then

$$P=\frac{QR}{S}.$$

ıe bridge is of particular value for the easurement of resistances of small currentrrying capacity and with potential leads of nsiderable resistance, *e.g.* mercury standards resistance and platinum thermometers.

§ (12) DIFFERENTIAL GALVANOMETER ETHOD.—The ideal differential galvanometer ιs two distinct windings (preferably of ʻinned wire) of equal resistance and proıcing equal and coincident fields when equal əctromotive forces are applied to the ends the coils. In practice it is not possible to alise these ideal conditions, and the residual agnetic effect arising when equal and opposite electromotive forces are applied to the coils is often compensated by the effect produced by a small external coil.



:G. 30.—Differential Galvanometer Method.

The common method of using the differential galvanometer for the comparison of resistances is shown in *Fig.*). P and R are the resistances to be ɔmpared, and G and g are the resistances : the galvanometer circuits. The difference between the currents through the galvanometer coils is zero when $P/R=G/g$, but in general there will be a deflection owing to want of symmetry of the galvanometer coils. To compensate for this it is usual to make a preliminary adjustment. The galvanometer coils are connected in series and opposed magnetically when the maximum working current is sent through them, the external compensating coil being adjusted in position so as to ensure no deflection of the galvanometer. Such a procedure is not, however, entirely satisfactory for precision work, as it necessitates an adjustment of the compensating coil if the nominal values of P and R are altered. Moreover, when P and R are interchanged, the effective values of G and g are also changed, because these latter include the potential leads of the resistances, together with the contact resistances introduced. If G and g are comparatively large the error is reduced, but so also is the sensitiveness. For example, let $P=R=0\cdot1$ ohm, and suppose the resistance of the leads of P to be 0·0001 ohm and those of R to be 0·0002 ohm. Then, if $G=1$ ohm, and no correction is applied for the inequality of the leads, the error of measurement is 0·01 per cent. If $G=100$ ohms, the error is 0·0001 per cent, but the sensitiveness is reduced to one-fifth of its former value. Such errors are eliminated if the Kohlrausch[2] method of overlapping shunts is used. A diagram of connections is given in *Fig.* 31. Let $P=R$. Then, unless there is symmetry of the galvanometer coils and equality of resistance of their circuits, there will be a deflection. If, however, G and g are in effect exchanged in position by interchanging the battery arm and X, except that for X a resistance practically identical with it is substituted, then the deflection will be of the same magnitude and of the same sign as before. In the Kohlrausch method P is made equal to R by shunting the greater, the equality being determined by the equality in magnitude and sign of the deflection before and after interchanging G and g, in the manner indicated. The exchange is effected by a six-pole switch shown in *Fig.* 31A. In this method the resistances of the galvanometer circuits are constant, and it is apparent that this method of using the differential galvanometer is the only

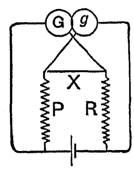


FIG. 31.

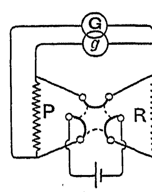


FIG. 31A.—Kohlrausch Method.

[1] *Phil. Mag.*, 1912, xxiv. 567.

[2] *Wied. Ann.*, 1883, xx. 76; also Jaeger, *Zeitschr. Instrumentenk.*, 1904.

one so far suggested that can be used for precision measurements.

§ (13) THE POTENTIOMETER.—A general arrangement for the comparison of resistances of unequal values is shown in *Fig.* 32. Let

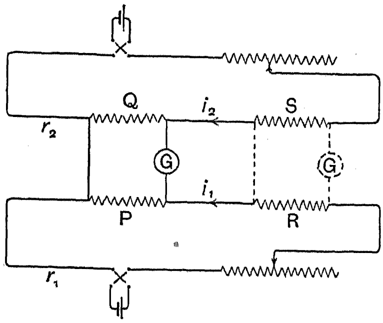


FIG. 32.—Potentiometer.

the resistances of the two circuits be $P+r_1$ and $Q+r_2$. If i_1 is the current through P and the balance is disturbed by changing P to $P+\delta p$, the current through the galvanometer is

$$\frac{i_1\delta p}{G+(Pr_1)/(P+r_1)+(Qr_2)/(Q+r_2)}\cdot\frac{r_1}{P+r_1}.$$

If, as is usual, r_1 and r_2 are great compared with P and Q, sufficient accuracy is obtained by writing the above as

$$\frac{i_1\delta p}{G+P+Q}.$$

In such a case the best resistance for the galvanometer is P + Q, and since the deflection of the galvanometer is proportional to $\sqrt{G}$ the expression for the sensitivity is

$$\frac{i_1\Delta\sqrt{P}}{2\sqrt{1+(Q/P)}},$$

where $\Delta=\delta p/P$.

If Q is small compared with P this becomes $i_1\Delta\sqrt{P}/2$, and the best resistance for the galvanometer is P. In practice Pi_1 is first made equal to Qi_2 by adjustment of i_1 or i_2 or by shunting one of the coils, and afterwards Ri_1 is made equal to Si_2 by shunting R or S. Unless P and R are nominally equal the galvanometer resistance cannot be the most suitable for both observations, and the sensitiveness of one of the measurements must be less than that of the other. If P=R and Q=S, the latter resistances being small compared with P and R, the sensitiveness is twice that of the Wheatstone bridge with equal resistances in the four arms. In practice, if P and R are nearly equal, only one resistance, say Q, is necessary in the second circuit, balance being obtained by shunting P or R. When comparing this method with the Wheatstone bridge it must be remembered that the currents in the potentiometer circuits are continuous and the heating effects more marked than in bridge measurements. Also the galvanometer circuit must be made after the current circuits are completed, and reversals are therefore necessary to eliminate thermoelectric effects in the galvanometer circuit. On the other hand, the resistances of current and potential leads are of little importance when measurements are made by means of the potentiometer.

§ (14) COMPARISON OF METHODS FOR THE MEASUREMENT OF RESISTANCE.—A strict comparison of the four main methods for measuring standard resistances is not possible. If in all cases the galvanometer has the most suitable resistance, and if the resistances to be compared, together with all ratio coils, are of unit value, then the relative sensitivities are as follows :

Wheatstone bridge	0·7
Kelvin double bridge	0·6
Differential galvanometer	1·0
Potentiometer	1·0

The sensitiveness is not, therefore, an important factor when a choice has to be made.

Standard coils of unit value and upwards, whether or not they possess potential terminals, are usually measured by the Wheatstone bridge. The method is simple, the measurements are rapid, and the probable error is small. When standard coils of nominal values 10,000 ohms and upwards are compared, it is often necessary to employ a battery having an electromotive force of about 20 volts, and as a result a small deflection of the galvanometer, due to an electrostatic effect, is sometimes obtained. However, with properly insulated circuits this is not a source of error. Standard resistances of nominal value less than 1 ohm are usually measured by the Kelvin double bridge, and conditionally that the resistance of the leads is not great compared with the resistance of the standard, the method is accurate and convenient. While the inductance of the standard itself is usually low, the ballast resistance in the battery circuit often possesses sufficient self-inductance to be a source of error unless the current is kept on for twenty seconds or more before reading the galvanometer. The maintenance of the current for such a period is not only desirable, but for all accurate measurements it is necessary in order that the resistances may acquire a steady state of temperature. In general, when it is suspected that the rise of temperature of the standard coils, due to the heating effect of the current, is sufficiently great to affect the resistances by an appreciable amount, the heating effect of the current

should be determined by making resistance comparisons with two or three different values of the current. As the heating effect is proportional to the square of the current, the resistance for a negligibly small current can be calculated.

The differential galvanometer is not employed to a great extent, and the method is not so accurate or so convenient as the other methods. This is largely due to the fact that when a balance is obtained there is usually a small deflection of the galvanometer, the condition for balance being no change in the deflection on reversal of the connections. The deflection is not produced by thermoelectric effects in the galvanometer circuit, but is due to want of symmetry in the resistances, etc., and is produced on making the battery circuit. The differential galvanometer has been used chiefly for the measurement of the resistance of platinum thermometers.

The potentiometer method is most convenient for the comparison of resistances with current and potential leads of relatively high resistance. The principal inconvenience of the method is that of maintaining steady currents, but when suitable provision is made for this the results obtained are very accurate. It is the method most used for the comparison of low resistances such as shunts capable of transmitting very large currents.

F. E. S.

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STANDARD CELLS: classified list of various types of. See "Electrical Measurements, Systems of," § (44).
Conditions to be compiled with, by. See *ibid.* § (44).
Effect of interactions between mercury and the sulphates of cadmium and mercury, on the electromotive force of. See *ibid.* § (45) (xv.).
Effect of short-circuiting. See *ibid.* § (45) (xix.).
Electromotive force of, and the International Volt. See *ibid.* § (43).
Influence of the size of the crystals of mercurous sulphate on the electromotive force of. See *ibid.* § (45) (xvi.).

STANDARD CONDENSERS: air condensers as primary and secondary standards. See "Capacity and its Measurement," § (32).
For radio work. See "Radio-frequency Measurements," § (21).

STANDARD MAGNETIC FIELD OF HIGH VALUE. See "Magnetic Measurements and Properties of Materials," § (3).

STANDARD RESISTANCE COILS, British Association pattern of. See "Resistance, Standards and Measurement of," § (1).
Comparison of 1 ohm coils without potential leads. See *ibid.* § (7).
Comparison of 1 ohm coils with potential terminals. See *ibid.* § (7).
Comparison of nominally equal coils greater than 1 ohm. See *ibid.* § (7).
Comparison of unequal coils whose resistances are approximately in the ratio $1 : 10^n$, where n is an integer. See *ibid.* § (7).
Desirable properties of. See *ibid.* § (2).
Drysdale and Burstall's patterns. See *ibid.* § (1) (viii.).
Fleming's pattern. See *ibid.* § (1).
History of the Reichsanstalt's oldest standards, with values from 1892 to 1913. See *ibid.* § (3).
Nalder Bros.' pattern. See *ibid.* § (1).
R. W. Paul's pattern. See *ibid.* § (1).
Reichsanstalt pattern. See *ibid.* § (1).
Rosa and Babcock's modification of Reichsanstalt pattern. See *ibid.* § (1) (vi.).
F. E. Smith's modification of Reichsanstalt pattern. See *ibid.* § (1) (vii.).

STANDARD RESISTANCES, various forms of air- and water-cooled. See "Potentiometer System of Electrical Measurements," § (7).

STANDARD SEARCH COILS, use of, for measuring magnetic fields. See "Magnetic Measurements and Properties of Materials," § (9).

STANDARD SOLENOID: a long, uniformly wound coil of wire, used for obtaining an approximately uniform magnetic field of calculable strength. See "Magnetic Measurements and Properties of Materials," § (2).

STANDARDISING BRIDGES: arrangements of conductors for comparing the values of standards of electrical resistance. See "Resistance, Standards and Measurement of," § (7).
Nalder's Slide Wire. See *ibid.* § (7) (iv.).
Smith's extension of the double bridge principle in. See *ibid.* § (11).

STANDARDS, ELECTRICAL: use of concrete standards of reference to represent ideal units. See "Electrical Measurements," § (7).

STARTING APPARATUS FOR ELECTRICAL MACHINES. See "Switchgear," § (10).

STATIONARY WAVES (ELECTRIC), measurements on, for wave-length determinations. See "Radio-frequency Measurements," § (3).

STEINMETZ COEFFICIENT: a magnetic constant for any sample of iron or steel, which is a measure of the hysteresis loss occurring in it. The hysteresis loss is approximately proportional to $\mathbf{B}^n$, where n is about 1·6 and is known as the Steinmetz coefficient. See "Magnetic Measurements and Properties of Materials," § (1).

STELGES SYSTEM: a low-speed system of telegraphy in which the receiving instrument prints the message in roman type on a paper slip. See "Telegraphs, Type Printing," § (1).

STOKES' LAW OF PHOSPHORESCENCE. See "Photoelectricity," § (6).

STRAY CURRENT ELECTROLYSIS

I. INTRODUCTION

§ (1) DEFINITION OF ELECTROLYSIS.—When an electric current passes from a metallic conductor to an electrolytic conductor, or *vice versa*, chemical changes are always produced at the surface of contact between the two classes of conductors. The process by which these chemical changes occur is called *electrolysis*. Electrolysis in this broad sense of the term is the basis of numerous useful industrial processes. In some cases, however, as when stray electric currents from electric railways or other power sources traverse underground metallic structures, such as pipe and cable systems, and are then discharged into the earth, the electrolytic action taking place at the surface of contact, where the current leaves the metal, results in more or less serious injury to the affected structures, and the mitigation of trouble from this cause is in many cases one of the serious problems connected with the distribution of electricity. This phenomenon is known as stray current electrolysis. In the present article only this destructive aspect of electrolysis is dealt with. The discussion which follows deals with the damage caused to underground pipe

and cable systems and to other earthed metallic structures due to the stray currents which have their origin chiefly in the earth return of electric railways. The subject is treated under three main heads, namely, (1) The Physical Laws governing Electrolytic Corrosion, (2) Electrolysis Testing, which deals with the making of electrolysis surveys for determining the danger to which such underground structures may be subjected, and (3) Electrolysis Mitigation, under which are discussed the methods best adapted in general and in individual cases for preventing or mitigating troubles from this source. These troubles arise as a rule from leakage voltages much greater than any which can occur in England, where the maximum potential drop on any uninsulated conductor is carefully limited.

II. General Considerations regarding Electrolysis

§ (2) Electrolytic Corrosion Proper.—Electrolytic corrosion of underground structures occurs in general wherever current flows from the metallic structure into the earth. The most common cause of these stray currents in practice is primarily the potential drop in the railway return circuit, as a result of which those portions of the track more remote from the power-house will be at a potential above the earth, whereas those portions of the track near the power-house will in general be at a potential below that of the earth. In consequence of this, current tends to flow from the tracks to the earth in the more outlying districts, and from the earth to the tracks in the region near the power-house. These currents, after getting into the earth, tend to flow, as a rule, in the general direction of the power-house and tend to accumulate on the underground pipe and cable systems buried in the earth; in the region near the power-house they leave these structures by way of the earth and return to the railway negative return, and it is in this region that electrolytic corrosion most frequently takes place. Very often, also,

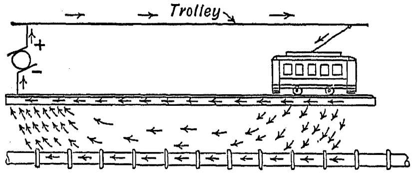

Fig. 1.

there may be an interchange of current between adjacent pipe systems that are at slightly different potential, and there may also be cases of current shunting around the high-resistance joints in the pipe network, thus giving rise to localised corrosion in widely distributed areas throughout the piping system. This general trend and distribution of stray currents in the earth is shown in typical form in *Fig.* 1.

§ (3) Positive and Negative Areas.—The region in which the currents are flowing from the pipes into the earth is called the *positive* or *anode area*, and the outlying region where the currents are flowing from the earth into the pipes is called the *negative* or *cathode area.* This definition of positive and negative areas is very general, however, and it is not safe to assume that trouble is confined exclusively to the so-called positive area. Places may occasionally be found where, owing to high resistance in the joints, currents will be forced to leak off the pipes and flow through the earth around the joints, thus making a small positive area on one side of the joint and a small negative area on the other side of the joint. Such local positive areas may be found in what is generally known as the neutral and negative regions, and where these occur corrosion of the pipes may take place although the pipe systems in general are found to be negative to the railway tracks. Experience indicates, however, that such cases are of relatively slight importance.

Further, two pipe systems occupying the same territory may both be decidedly negative to the railway return, but one more strongly negative than the other. In this case the two pipe systems will be at different potentials and one will discharge current into the other and thus give rise to corrosion on the pipe system at the higher potential, although readings might show that it is at all points negative to the railway return. In determining whether or not a pipe or cable is undergoing electrolytic corrosion, it is necessary to determine not only whether any portion of it is positive to the railway system, but also whether it is positive to any surrounding structure, preferably to the earth immediately adjoining the pipe.

§ (4) Pitting.—It is not possible to determine the amount of damage to the pipe structure

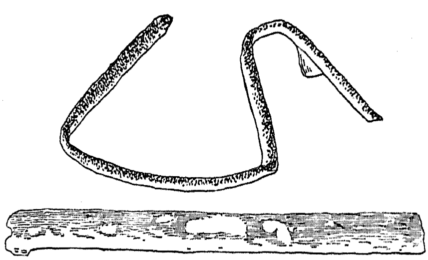

Fig. 2.

by determining the amount of corrosion which takes place. The reason for this is that the

corrosion is in practically all cases very unequally distributed over the surface of the iron, there being a marked tendency for the current to be discharged locally, producing the phenomenon commonly known as pitting. In this manner a small hole may be eaten entirely through the pipe, thereby destroying its usefulness, while the average corrosion over the surface of the pipe may be relatively small. *Fig.* 2 shows an example of a pipe that has been corroded by stray currents, and which exhibits the characteristic pitting that has been almost universally encountered.

§ (5) Self-corrosion.—Self-corrosion is very generally regarded as being due primarily to the presence of local galvanic currents at the surface of the corroded metal, due either to physical differences between adjacent points on the surface of the metal or to foreign conducting substances in the soil. For example, if a piece of carbon in the form of coke be embedded in the soil in contact with the pipe surface at one point, there will be a difference of potential between the coke and the pipe and a current will flow from the pipe through the moist soil to the coke and return to the pipe through the contact point between the two. The action here is exactly analogous to the action in a primary battery in which a piece of zinc and a piece of carbon are immersed in the electrolyte and connected together through an external circuit. A current flows through the electrolyte from the zinc to the carbon, thereby corroding the zinc. The electromotive force given by iron or steel when embedded in ordinary soils in contact with coke is usually about 0·6 volt, and this is sufficient to give rise to very rapid corrosion and pitting of the iron surface. Action of this kind is frequently met with where pipes are embedded in soils containing cinders in which particles of coke may be found. This phenomenon is frequently encountered and must be taken into account in electrolysis investigations.

§ (6) Self-corrosion similar to Electrolysis.—Unfortunately, self-corrosion generally manifests itself in a manner very similar to that caused by stray currents. Because of this similarity in appearance it is not possible in general to determine by inspection of a corroded pipe whether or not the corrosion was caused by stray currents or by local galvanic action. Owing to this fact it not infrequently happens that cases of pipe corrosion are charged to the railway companies when the damage was actually due to local causes arising from the nature of the soil or of the pipe, or both.

§ (7) Determination of Cause of Corrosion.—The only sure way of determining whether or not stray currents are causing corrosion in any particular case is by making proper electrical tests to determine whether or not the pipes are actually discharging current into the earth. In a case where serious corrosion has been caused by stray currents and the cause of these stray currents later removed, the only certain way of determining whether or not the previous corrosion was caused by stray currents or by local influences is by making actual corrosion tests in the soil under the same average conditions of moisture and using the same kind of iron as was previously found corroded. In the absence of a test of this kind it is not possible to fix with certainty the cause of the damage.

§ (8) Troubles from Stray Currents other than Corrosion.—While it is true that by far the greater portion of damage caused by stray currents takes the form of corrosion of underground pipes and cables, there are certain other dangers resulting from these stray currents that deserve consideration. Among these may be mentioned overheating of pipes in buildings due to the flow of excessively heavy currents therein. Another danger from the presence of stray currents on pipes is that due to the possibility of explosion of gases. Wherever any considerable amount of stray current is found on a gas pipe, it is necessary to bond around any contemplated break in the continuity of the pipe, otherwise the arc which would occur between the pipe sections when disconnected would be likely to ignite escaping gas with possibly serious consequences. These dangers have, however, been greatly overestimated in the past and they can in most cases be avoided entirely with reasonable care.

III. Physical Laws governing Electrolytic Corrosion

§ (9) Coefficient of Corrosion.—The *anode* is the terminal through which a current of positive electricity passes into the electrolyte, and the *cathode* is the terminal through which the current is led out of the electrolyte. When an electric current is discharged from the anode into the electrolyte there is, in general, corrosion of the anode surface. In some cases this corrosion is the sole reaction involved at the anode, while in other cases there is a certain amount of the electrolyte decomposed by the passing of the current. If the corrosion of the anode is the sole reaction involved, then according to Faraday's law, 96,500 coulombs are required to corrode one gram equivalent of the metal. If a portion of the anodic reaction is involved in breaking up the electrolyte, then a lesser amount of metal is corroded from the anode for a given quantity of electricity. In any case, the ratio of the actual anodic corrosion to the theoretical corrosion, according to Faraday's law, is called the

"coefficient of corrosion." The coefficient of corrosion of iron in soil is very greatly affected by a number of physical conditions, such as current density, moisture content of the soil, etc. The subject has been investigated by Hayden, Haldene Gee, and Ganz, who have reported coefficients of corrosion both below and above unity. More recently[1] this subject has been very fully investigated by McCollum and Logan at the Bureau of Standards, U.S.A., as a result of which the laws of electrolytic corrosion in soils have been quite definitely established.

§ (10) Effect of Current Density.—The relation between current density and coefficient of corrosion for a certain range of current density is shown in Table 1. The curve of

Table 1

Relation between Current Density and Efficiency of Corrosion

(Partial analysis, per cent of moisture-free sample $\left(\frac{Cl_1}{0{\cdot}002}\ \frac{NO_3}{0{\cdot}002}\ \frac{CO_3}{0{\cdot}003}\ \frac{SO_4}{0{\cdot}004}\right)$

No.	Density, Milliamperes per Square Centimetre.	Coefficient of Corrosion.
1	2·0	57·6
2	1·8	37·4
3	1·7	69·2
4	1·6	43·2
5	1·5	84·3
6	1·3	75·8
7	1·2	72·8
8	1·1	83·8
9	1·0	89·2
10	0·8	75·0
11	0·7	88·3
12	0·6	79·1
13	0·5	75·2
14	0·4	102·1
15	0·3	142·2
16	0·2	96·2
17	0·1	148·4
18	0·05	142·9

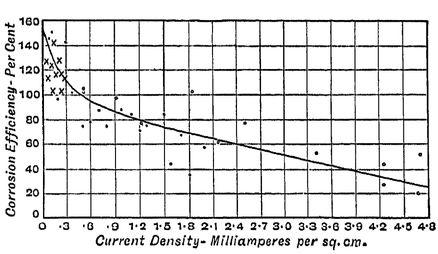

Fig. 3.

Fig. 3 also shows this relationship for a considerably wider range of current densities.

[1] *Bureau of Standards Technological Papers*, Nos. 18, 25, 26, 28, 32, etc.

These results apply to soils saturated with moisture. From these it will be seen that, in general, the higher the current density the lower the coefficient of corrosion. At a current density of 4·8 milliamperes per square centimetre the coefficient of corrosion is less than ·3, rising steadily to approximate unity at around ·5 milliampere per square centimetre, while for still lower current densities the coefficient of corrosion exceeds unity. This is due, no doubt, to the increase in self-corrosion caused by galvanic action induced by the oxidation products resulting from the electrolytic corrosion.

§ (11) Effect of Moisture Content on the Coefficient of Corrosion.—The effect of moisture content on the coefficient of corrosion of iron in soil is very striking indeed. This is shown by the curve of *Fig.* 4. From this it

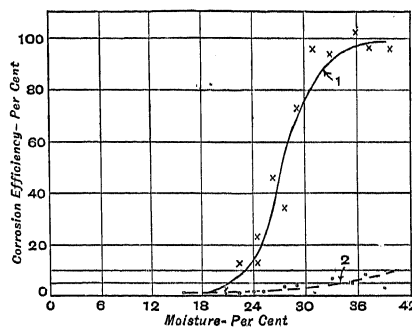

Fig. 4.

will be seen that so long as the moisture content does not exceed about 15 per cent by weight of the dry earth the coefficient of corrosion is not over ·01. It increases slightly with increasing moisture content until about 20 per cent of moisture is present, when it rises very abruptly with the moisture content, becoming practically unity when the moisture content is about 40 per cent.

§ (12) Effect of Low Voltages.—It has often been erroneously stated that iron will not corrode so long as the voltage impressed on the electrodes is less than that required for the dissociation of water. There is no valid theoretical reason supporting this view, and it has been experimentally shown by McCollum and Logan that the coefficient of corrosion is substantially independent of the voltage, even down to voltages of the order of ·1 volt.

§ (13) Effect of Chemicals.—It has long been known that certain chemicals added to an electrolyte, like strong alkali, for example, tend greatly to inhibit electrolytic corrosion in iron. It has been found that similar action takes place in soils, but the inhibition is in general much less complete than in water solution. In addition to the reduction of the total amount of corrosion it has been shown by

McCollum and Logan that the character of the corrosion is also affected in a marked degree by chemical composition of the soil. For example, the addition of a nitrate to the soil resulted in very uniform corrosion, leaving the corroded anode smooth, and while showing the fibrous structure there will be no pronounced pits. The anodes in earth containing sulphates were also quite smooth, and carbonates resulted in a nearly uniform surface of the anode, while chlorides produced the most pronounced pits of all. This has an important bearing on the damage to buried pipes in practice. The actual damage is a function of the depth of the pits rather than of the total amount of corrosion.

§ (14) Miscellaneous Effects.—The effect of other factors, such as temperature of the soil, depth of burial of the pipes, oxygen content of the soil, and different kinds of iron, have also been carefully investigated by McCollum and Logan. It has been found that the temperature of the soil has no direct effect on the coefficient of corrosion, but does greatly affect the total amount of electrolysis, due to its great influence on the resistivity of the earth, as will be explained later. Depth of burial was also found to exert very little influence. The oxygen content of the soil has no material effect on the coefficient of corrosion, but does affect in a marked degree the composition of the end products of the corrosion. If the corrosion takes place slowly, so that there is an ample supply of oxygen to oxidise the primary products of corrosion, the final product will be red oxide of iron. On the other hand, if the corrosion takes place very rapidly and the supply of oxygen is restricted, the tendency is to form the black or magnetic oxide. This has some importance in practice, because it affords some indication as to whether the corrosion is due to stray currents or self-corrosion by the soil. In general, where the end products of corrosion are largely black oxide, it may be presumed that the corrosion took place very rapidly, and this increases the probability that it was in a large part due to stray currents. Many different kinds of iron have been tested and found to possess substantially the same coefficient of corrosion when other conditions are kept similar.

Tests made on electrolytic corrosion in soils, from a large number of widely separated localities, indicate that in general the coefficients of corrosion that may be expected in practice under average conditions varies between ·6 and 1.

§ (15) Earth Resistance.—Since the amount of stray current that leaks into the earth from a railway system depends in a large measure on the resistivity of the earth, it follows that this factor is an important one in determining the amount of corrosion that will take place under given electric conditions in the railway negative return. This factor has been investigated by McCollum and Logan, as regards moisture content and temperature, both of which factors affect the resistivity of the earth to a very marked degree. The effect of the moisture content is shown in Table 2, from which it

Table 2

Relation between the Amount of Moisture in the Soil and its Specific Resistance

Per Cent Moisture (in Terms of Dry Earth).	Specific Resistance (Ohms per Centimetre Cube).
5·0	2,340,000
11·1	237,400
16·7	13,800
22·2	6,835
33·3	5,400
44·5	4,725
55·6	4,870
56·7	5,197
77·8	5,045

is seen that in a given earth the resistivity with 5 per cent moisture was 2,340,000 ohms per centimetre cube, and when the moisture content was increased to 33·3 per cent the resistivity dropped to 5400 ohms, or in the ratio of over 400 to 1. Further increase of moisture beyond this point produced little effect on the resistivity.

The effect of temperature on earth resistivity is shown in Table 3, and in *Fig.* 5, which is

Table 3

Effect of Temperature on Resistance of Soil

(Soil No. 32 ; moisture, 18·6 per cent ; specific resistance at 20°, 6260 ohms/cm.^{3})

Temperature.	Resistance.
° C.	Ohms.
18·0	224
13·0	286
8·5	398
1·5	458
1·0	462
0·0	542
− 2·0	940
− 3·0	1,185
− 5·5	4,340
−12·0	21,700
−13·0	24,600
−15·0	36,200
−18·0	45,000
−19·0	48,900

a plot of the data of Table 3. From these it will be seen that the resistivity increases very slightly with falling temperature between 20° and about 0° Centigrade. With further

reduction of temperature, however, the resistivity rises enormously, and at $-19°$ Centi-

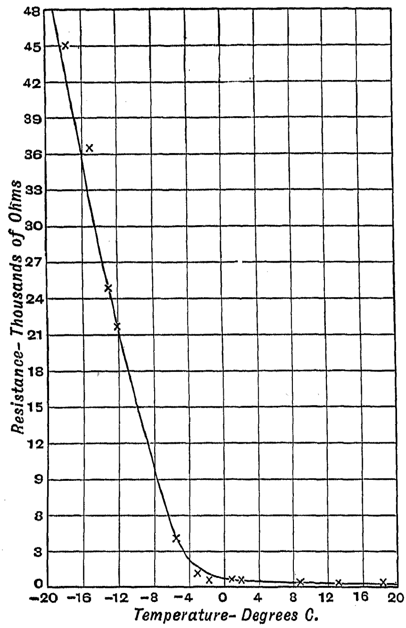

FIG. 5.

grade becomes nearly one hundred times greater than at 0° Centigrade. This has an important practical bearing, especially on electrolysis testing, and shows that tests made in very cold weather are almost certain to yield misleading conclusions.

§ (16) ELECTROLYTIC CORROSION OF IRON IN CONCRETE.—During the last few years it has been observed that when an electric current flows through reinforced concrete, certain effects are produced which may result in the destruction of the concrete structure. The first of these to be observed is known as the anode effect, first described by A. A. Knudson, which occurs under certain circumstances when current passes from the reinforcing material out into the concrete. The action here is essentially the same as when current is discharged from a pipe into the earth, namely, the surface of the iron is corroded; but the effects in this case are even more serious than in the corrosion of pipes, because of the secondary actions which take place.

As soon as iron is carried into solution by the electric current, it comes in contact with oxygen in the concrete, and there is formed a precipitate of iron oxide. This occupies a volume of about 2·2 times the volume of the original iron, and there results a swelling action which in time may become sufficient to split even very large masses of concrete. The character of the results of this action is shown in *Fig.* 6, which shows a block of concrete with an embedded iron electrode which has been exposed for some time to the flow of current from the iron into the concrete.

It is very important from the practical standpoint to bear in mind, however, that the disastrous results shown in *Fig.* 6 occur only under special conditions which are not likely to be frequently encountered in actual practice. Extensive investigations have shown that in the case of ordinary concrete this rapid corrosion of the embedded iron, with the resultant cracking of the concrete, is likely to take place only when the potential gradient impressed on the specimen is quite high—much higher in fact than would generally be encountered from stray railway currents. If the voltage on specimens of ordinary size, such as might be used in buildings, is kept as low as ·2 or ·3 volt or less, long-time experiments have shown that practically no damage to the concrete results. It is therefore only under very extreme conditions that serious trouble of this kind is likely to be met in practice. However, it may occur; as, for example, when an electric light wire becomes grounded on a metallic conduit embedded in concrete. The voltage here would generally be high enough to cause

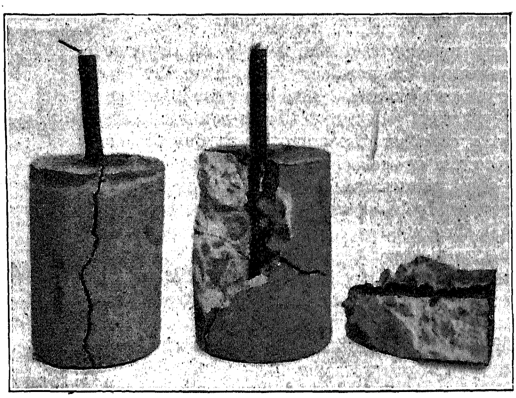

FIG. 6.

corrosion of the conduit and any reinforcing material that might be electrically connected therewith, which would result in ultimate splitting and more or less complete local destruction of the concrete structure; this is therefore a danger which under certain conditions should be guarded against. For this reason it may be well not to embed metal conduit in concrete structures where the char-

acter of the building is such that this is not necessary; nor should the conduit be electrically connected to the reinforcing material.

§ (17) Effect of Salt on Electrolytic Corrosion of Iron in Concrete.—It is important to call attention to the fact that while, as above stated, in ordinary normal concrete, with only low voltages acting on concrete structures of the sizes usually encountered in practice, serious corrosion or cracking of the concrete will not occur, such corrosion and cracking will develop rapidly if any appreciable amount of salt is added to the concrete either during or after construction. It has been conclusively shown by years of experience that ordinary concrete affords a good protection for iron against natural corrosion, and recent investigations have shown that under ordinary circumstances it also affords some degree of protection against electrolytic corrosion. The addition of a small quantity of salt, however, has been found to destroy completely the protective effect against electrolytic corrosion. For this reason it is very important that no salt be used in the erection of reinforced concrete structures wherever there is any likelihood of stray currents from any source getting into the reinforcing material.

§ (18) Cathode Effect.—Another effect of electric currents on reinforced concrete has more recently been discovered. This is the cathode effect, and it occurs only where the current flows from the concrete toward the reinforcing material. In this case there is no corrosion of the iron at all, but there is a gradual softening of the concrete at the surface of the reinforcing material, due to the gradual concentration of alkali at the cathode. This softening begins at the surface of the iron, and very slowly progresses outward into the concrete, usually requiring many weeks or months to progress a distance of one or two millimetres. The practical importance of this phenomenon lies in the fact that the softening of the concrete at the surface of the reinforcing material, although usually confined to a thin layer, is nevertheless sufficient practically to destroy the bond between the iron and the concrete, and in this way a structure may be weakened or destroyed.

The cathode effect has been found to occur not only on relatively high voltages, as in the case of the anode effect above described, but also on relatively low voltages, the rate at which it progresses being roughly proportional to the voltage applied in any particular case. Examples of trouble of this kind are shown in *Fig.* 7, which shows very definite regions of softened concrete surrounding the embedded iron. Because of the fact that this softening of the concrete near the cathode develops under a much lower voltage than is generally required to produce the anode effect previously described, the cathode effect is likely to prove of greater practical importance than the cracking of the concrete at the anode which had previously been observed.

In regard to the actual dangers to which reinforced concrete structures in practice are subjected as the results of these phenomena,

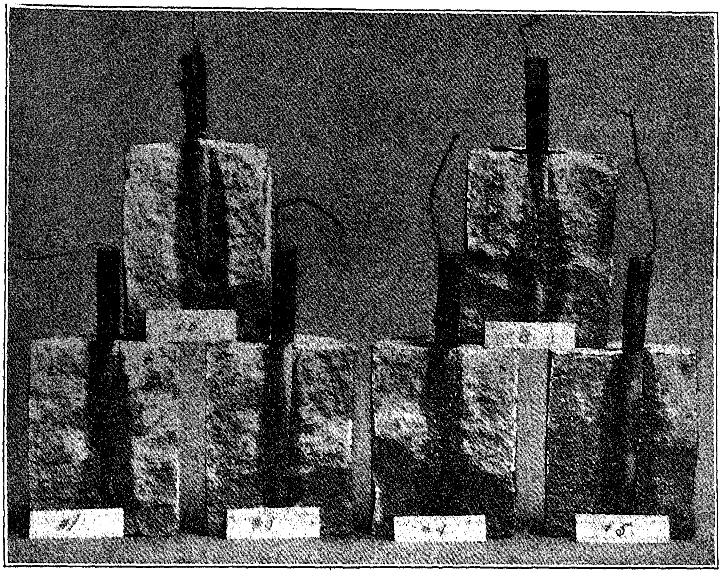

Fig. 7.

it should be emphasised that while cases of actual damage of this sort have been encountered in practice in a few instances, they are comparatively rare, and only in exceptional cases have conditions been such as to produce any appreciable amount of damage in actual building structures. There have been numerous cases in which reinforced buildings or bridges have been damaged, in which the damage was attributed to stray currents, but in most cases it has been found that the trouble was not in any way due to the presence of electric currents, except in certain instances where salt was present in the concrete in considerable quantity. This emphasises the importance of omitting salt in the construction of reinforced concrete buildings wherever there is any likelihood of stray currents getting into the structure.

§ (19) Alternating Current Electrolysis.—It has long been known that alternating current of ordinary commercial frequency produces only relatively small electrolysis effects. The actual amount of corrosion for a given number of ampere-hours is generally less than 1 per cent of the amount that would occur with a corresponding current strength in the case of direct current. If the length of the period is increased, however, the amount of corrosion increases according to a fairly definite law, but even where the period of the reversal is as long as several minutes or more, the total corrosion is still only a few per cent of what it would be if the current flowed continuously in one direction. Reversals of current of such long periods occur throughout a large portion of what is commonly called the neutral zone of a railway system, so that this fact is of great importance in determining the danger from electrolysis. If the current reverses every few minutes there is a redeposition of dissolved metal which, although it may be of little value mechanically, is nevertheless in the metallic state, and is again corroded when the metal becomes positive. Because of this partial reversibility of electrolysis, the actual amount of corrosion which takes place is more nearly proportional to the algebraic average of the current flow than it is to the arithmetical average during the time the pipe is anode. If the period of reversal is very long, however, such as a day or longer, the corrosion is likely to be considerable, although much less than if the current were continuous in one direction. This law holds for both iron and lead, the metals most commonly subjected to electrolytic corrosion in practice.

IV. Electrolysis Testing

The object of electrolysis testing is chiefly to determine the extent to which underground pipe and cable systems are being injured by electrolysis, to ascertain the location of the danger areas, and to determine what remedial measures are necessary for overcoming the trouble. In many cases, however, the sole object of the survey may be simply to determine whether an existing law is being complied with, in which case the procedure is much simpler, measurements being taken only of those electrical quantities that are fixed by the law.

In making a general test of electrolysis conditions, the following classes of measurements are usually made:

(*a*) Voltage surveys throughout the entire affected regions;

(*b*) Current measurements made chiefly on the subsurface structures; and

(*c*) Measurement of leakage current from pipes.

(*d*) Miscellaneous tests, of which there are a great number, depending upon local conditions and the scope and character of the survey.

§ (20) Voltage Surveys.—Voltage surveys comprise three different classes of measurements:

(i.) Overall potential measurements, or measurements of the maximum potential drop between the point of lowest potential in any feeding area and the more remote portions of the track in the same area;

(ii.) Potential gradient measurements or measurements of potential drop on definite relatively short intervals of the railway negative return, the unit of length usually being taken at 1000 feet; and

(iii.) Measurements of potential differences between rails, pipes, and other subsurface conductors.

The overall potential measurements and the potential gradient measurements are of value chiefly in giving information as to the condition of the railway negative return, both with regard to bonding and the use of supplementary return feeders. The potential difference measurements between pipes and rails are of value chiefly in locating areas in which trouble on the subsurface structures may be in progress. Neither the potential difference measurements, nor the voltage drops along the tracks, can be regarded in any sense as a quantitative measurement of the extent of the damage, since the actual amount of leakage current that may get on the pipe is determined not only by the voltage drops in the negative return and between the affected structures, but also by the resistance of the path followed by the leakage current. This resistance may vary between extremely wide limits under ordinary practical conditions, so that voltage measurements may be regarded in general as having only a qualitative significance.

§ (21) Current Measurements.—Current measurements on buried pipes and cables

form a much more accurate criterion of the total extent of corrosion in progress, provided it is known that all this current leaks directly from the affected structures to the earth. Where, however, a portion of this current is taken off through metal conductors, then the current measurements lose a great deal of their significance as a quantitative measure of the corrosion. Current measurements are made on pipes and cables usually by the potential drop method, namely, measuring actual drop of potential on the pipe or cable, usually including a few feet between contact points, and not including a pipe joint, and then calculating the current from this potential drop and the estimated resistance of the pipe or cable. As a rule the measurement obtained in this way can be depended upon to an accuracy of 10 per cent, which is sufficient for most cases of electrolysis testing. In special cases, however, as where measurements are being taken for use in court procedure, where it is desirable to eliminate all uncertainty as to the accuracy of the measurement, the pipe may be calibrated by sending a known current through the test portion and measuring the change in voltage drop due to this known current. A variety of methods have been developed for making this calibration, but it would involve too much detail to discuss them here.

§ (22) Leakage Current Measurements.—A still more accurate criterion of corrosion is the leakage of current from pipes into the earth. If it were feasible under practical conditions to measure the density of leakage current at any point in the pipe or cable, this would give the most accurate criterion of the hazard at that point, but unfortunately such measurements are extremely difficult to make under practical conditions.

(*a*) *Differential Current Measurements.*—One method of measuring leakage current from a pipe is to make measurements of actual flow of current at two different points on the pipe a short distance apart. The difference between these currents is of course the total amount of current that leaks from the pipe in the intervening space. This method can be made to give fairly satisfactory results where the total leakage between the points of measurement is a considerable fraction of the actual current on the pipe. If this is not the case, however, the true leakage is likely to be completely obscured by the relatively small errors in making measurements at either point.

(*b*) *Use of the Haber Earth Ammeter.*—Attempts have been made from time to time to measure the current density in the earth leaking from an affected pipe by the use of what is known as the Haber earth ammeter. This instrument has a current collector designed to be buried in the earth in such a manner as to disturb as little as possible the distribution of current thereon, and is designed to offer a measure of the actual current flowing through the section covered by the collector.

(*c*) *Potential-resistance Measurements.*—A third method of measuring leakage current recently developed consists in the measurement of the potential drop between two points in the earth very close to the affected structure and at right angles thereto, and the resistivity of the earth in the region between the two points in the earth. It will readily be seen that these two measurements permit the calculation of the current density in the earth leaking from the affected structure. The resistivity of the earth at any point can be measured most conveniently by placing four small terminals in contact with the earth on a straight line, each pair of terminals being spaced a few inches apart. By using two of these terminals for leading current into the earth, and the other two for potential terminals, the resistivity of the earth can be readily calculated. This measurement is probably the most useful single measurement that can be made where it is desired to determine the extent of danger locally at any point of a pipe network.

§ (23) Miscellaneous Measurements.—In many cases special problems arise calling for a number of miscellaneous tests to provide special information. Among these may be mentioned the location and testing of high-resistance joints in pipes, track testing, measurement of earth resistance, measurement of leakage resistance between tracks and earth, determination of the cause of corrosion and of the source of stray currents. These tests require special apparatus and special methods of procedure, a description of which cannot be given within the space here available, and reference must be made to the bibliography given at the end of this article.

§ (24) Selection of Instruments for Electrolysis Testing.—In general, in making electrolysis surveys, both indicating and recording instruments are required, the former being useful for taking short-time readings of a preliminary nature, which often assist materially in laying out the detailed plans for a more comprehensive survey. They may also be used for permanent test data where the load is rather steady, so that a short-time reading can be taken as typical of average conditions. Wherever it is practicable to use recording instruments, however, it is desirable to do so since in this way readings can be taken over a much greater length of time without unduly increasing the cost of the work, and a permanent record obtained in which the personal element is eliminated. On account of the variable character of the values to be measured, and the rough character of

the work, high accuracy in the readings of the measuring instruments is less important than ruggedness. All instruments should be designed to have sufficient ruggedness to yield moderate accuracy under the severe handling which such instruments inevitably receive in field service.

§ (25) Need for Supervision of Electrolysis Surveys.—It is not possible to over-emphasise the importance of having electrolysis surveys carried out under the direct supervision of a competent engineer thoroughly familiar with the planning of electrolysis surveys, the methods of making the tests, and the interpretation of the results. The very great value of the properties exposed to possible danger from electrolysis is such as to make this question one of great importance. The subject is a very complicated one, and not only are the possibilities of error in measurement and interpretation of results very great, but unless these results are studied and interpreted by one thoroughly familiar with the subject the conclusions may be misleading and a large measure of the value of the investigation may be lost. The tendency among progressive utility companies is to consider the electrolysis problem as one of their important engineering problems, and one which should not be dealt with by the more or less empirical and unscientific methods that have too often been followed in the past.

V. Prevention of Electrolysis Damage

A great many methods have been tried for mitigating electrolysis troubles. Broadly speaking, these methods may be divided into two classes—those applicable to pipe systems and those applicable to railway lines. Only the most important of these will be considered here.

§ (26) Methods applicable to Pipes.—Of the methods of electrolysis mitigation applicable to pipes only four have found any considerable application. These are: (1) surface insulation of pipes, (2) resistance joints in pipes, (3) pipe drainage, and (4) favourable location of pipes with respect to railway tracks.

(i.) *Surface Insulation.*—Many attempts have been made, especially in the early days of electrolysis troubles, to reduce the damage by the use of insulating paints. Such methods, however, have almost invariably failed. Explanation of the failure lies in the fact that none of the paints available are absolutely impervious to moisture, and when they are brought into the presence of water a slight trace of moisture ultimately permeates the coating. When this occurs at any point, the coating becomes slightly conducting, and if an electromotive force is applied, a trace of current flows; this gives rise to slight electrolysis, which is accompanied by the formation of more or less gas beneath the coating. As the gas increases in volume the coating is ruptured, after which the current flow is greatly increased at the point of breakdown; and in case the pipes are positive to earth, rapid electrolysis of the exposed portion follows. Pipes wrapped with cloth or paper impregnated with insulating compounds have given somewhat better results, but if sufficiently heavy coatings are used to give long life, the cost becomes considerable. Protection of pipes by laying them in conduit filled with pitch would appear to be effective for a much greater length of time, and in special cases this method would be useful, but the expense would be high if used on a large scale, and at best it appears very questionable whether or not the protection secured under average conditions would be worth the cost. In special cases, however, involving very important pipes, this method may prove very desirable.

(ii.) *Resistance Joints in Pipes.*—The method of protecting pipe systems against electrolysis by breaking up the electrical continuity of the pipes by means of insulating or high-resistance joints has been quite widely used during the last few years, and where such joints have been used with sufficient frequency the method has proved very effective. In numerous cases, however, where the joints have been used only very infrequently, they have not only failed to protect the pipe, but have actually aggravated the trouble. If the joints are used only infrequently, the current will accumulate on the pipes between the joints, and since it is compelled to leak around the high-resistance joints to the earth, rapid deterioration on the positive side of the joint may develop. Cement joints have proved very satisfactory when used in sufficient number, especially in case of gas mains.

(iii.) *Pipe Drainage.*—A system of electrolysis mitigation that has been widely used, especially in America, is what is known as the pipe drainage system. In its essential features this system involves connecting conductors at suitable points of the pipe system where the latter tends to become positive to the earth, to take off the current from the pipe through these conductors instead of permitting it to discharge directly into the earth. The conductors may take the form of bonds connected directly between the pipes and tracks wherever the pipes are found positive to the tracks or the conductors may take the form of special feeders running directly from the negative bus out to various points on the pipe system.

It will readily be seen that since damage takes place at the points where the current is discharged from a metallic conductor into

an electrolyte, if all of the current could be taken off the pipe through metallic paths without introducing new elements of danger into the system to which the cable is attached or to any other system, the pipe drainage method would prove very effective in relieving electrolysis troubles. Such a condition, however, is somewhat difficult to realise in practice unless suitable measures are taken to keep the voltage drops in the railway negative return down to relatively low values. The more important of such measures are discussed briefly later.

Wherever a pipe drainage system is used, it should always be through the medium of insulated negative feeders running directly from the bus bars to various drainage points in the pipe system, and the resistance of these cables should be so proportioned that the drop of potential should be about the same on all of them, as in this way the current collected in the pipe can be so subdivided as to eliminate in a large degree some of the most serious difficulties that have heretofore been encountered where pipe drainage has been applied by other means.

§ (27) Favourable Location of Pipes with respect to Railway Tracks.—The location of pipe lines with respect to railway tracks has a very important bearing on the danger of electrolysis. In general it may be said that the farther the pipes are from the tracks the less current they will pick up, and hence the total damage can in this way be greatly reduced. A more important matter, however, is the distribution of the discharge, it being desirable, as far as practical, to distribute the discharge over as large an area as possible. If the pipe in the positive area be brought near to the tracks at one point, as by crossing immediately under it, the tendency is to concentrate the discharge at that point and thereby cause rapid destruction locally. The location of the pipe is therefore more important within the positive area than outside this zone, although in both these it is important.

In laying new pipes or replacing old ones, therefore, the pipes should be placed as far as practicable from the rails, and the crossing of service pipes under the tracks should be avoided if circumstances permit. The practice of putting mains immediately under the tracks tends greatly to increase damage by electrolysis, and should be avoided wherever possible.

§ (28) Methods of Electrolysis Mitigation applicable to Railway Systems.—None of the systems of electrolysis protection mentioned above have to do with the nature or conditions of the street railway return circuits. Many engineers regard it as more logical to attack the problem by beginning at the source of the evil and preventing, to a large extent at least, the leakage of the currents from the railway return conductors into the earth.

A great many possible methods are available for this purpose, only a few of which need be discussed here. Double-trolley systems, while very effective in reducing electrolysis troubles, are prohibitively expensive, besides being objectionable from the operating standpoint. The most practicable means that can be applied to railway systems for reducing electrolysis troubles consists, broadly, in reducing potential drops in the uninsulated portion of the negative return to as low values as economic conditions will warrant.

The various possible methods of reducing track gradients to a satisfactory value all have for their primary object the taking of the current direct from the track through the agency of negative feeders, and they are therefore classed as "track-drainage systems." Numerous methods are available for accomplishing this, the chief of which are (i.) the proper construction and maintenance of track to secure full benefit of the conductivity of the rails, (ii.) the use of insulated return feeders, (iii.) the use of three-wire systems, and (iv.) the use of a large number of power-supply stations.

(i.) *Construction and Maintenance of Way.*—Proper maintenance of the track in order to secure a high conductance is everywhere recognised as a necessary condition in electric railway operation, but it does not always receive the attention that its importance justifies. Within recent years, however, more attention is being paid to this matter than formerly, and the standard of track conductance is being gradually improved. All joints should be well bonded. Cross bonding between the rails should be installed at intervals of a few hundred feet, and all special work should be shunted by heavy cables capable of carrying all of the current passing over the track at that point.

A properly constructed and drained roadbed is a very effective aid in reducing the leakage of stray currents from the rails. The conductance of the leakage path is mainly dependent on the amount of moisture which is contained in the material forming the roadbed, and on the intimacy of the contact between the rails and the roadbed, and any construction which tends to reduce the average moisture content therein will reduce in corresponding degree the magnitude of the leakage currents.

(ii.) *Insulated Negative-feeder Systems.*—The use of insulated negative feeders for the purpose of securing low potential drops in the negative return independently of the drops in the feeders themselves as a means of reducing electrolysis troubles, was first proposed and fully described by Isaiah H. Farnham in

Cassier's Magazine, in August 1895. In this paper the fundamental principle of the insulated return-feeder system without boosters was clearly set forth, and more recently the same system has been described[1] in somewhat greater detail by A. P. Trotter in England, and still more recently by George I. Rhodes in a paper before the American Institute of Electrical Engineers in 1907. Little use was made of this system until within the last few years, but since that time a large number of installations embodying the insulated negative feeder principle have been installed throughout the world, and this system is now quite widely used.

(iii.) *Principle of the Insulated Negative-feeder System.*—In the insulated negative-feeder system, instead of tying the tracks directly to the negative bus bar and depending on the track and such copper as may be in parallel therewith to return the current to the power-house, the connection at the power-house is either removed or given a suitable resistance, and insulated feeders are run from the negative bus to various points in the tracks, as shown diagrammatically in *Fig.* 8. By this means

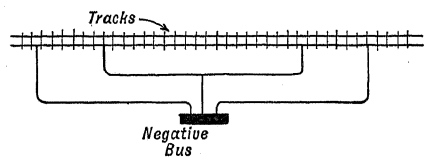

FIG. 8.

several important results may be achieved. In the first place, current being taken off the rails at numerous points, high-current densities and consequently high-potential gradients in the rails can be avoided to any desired degree. In the second place, by so designing the system that the drop of potential on all of the feeders is the same, the current flow in the tracks is so subdivided that the direction of the flow will be frequently reversed, thus preventing the accumulation of large potential differences between points on the tracks which are some distance apart.

Further, it will be evident that in this case the actual drop of potential in the different feeders is of little importance, so far as electrolysis protection is concerned, so long as it is nearly the same in all. We can thus impose any desired potential restrictions on the track and still be free to design the feeders to give maximum economy, which we cannot do when the feeders are connected in parallel with the tracks, as has been the common practice in the past.

(iv.) *Insulated Negative Feeders with Resistance Taps.*—A modification of the insulated negative-feeder system, as above described, will generally be found desirable. In this modification, instead of running several independent feeders in one direction from the power-house, a single large feeder is run along the line and connected to the track at suitable points by means of resistance taps. This system is shown diagrammatically in *Fig.* 9. It has the advantage of simplicity of line construction, cheaper first cost and maintenance (where the total feeder area is not great), due to running one large feeder instead of several small ones,

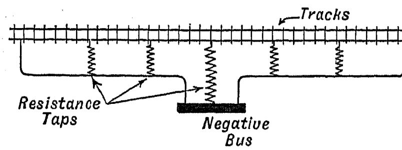

FIG. 9.

and it also has the advantage of bringing back a much larger load on a single feeder, which results in a less variable current, and this makes possible a more economical use of the negative copper.

(v.) *Three-wire Systems.*—Numerous attempts have been made at various times to reduce electrolysis troubles through the adoption of the three-wire system of distributing power to cars. The earlier attempts at this method failed because of operating difficulties, but within recent years a number of very successful three-wire installations have been put into operation in various parts of the world, particularly in America. When proper attention is given to the sectionalisation of the feeding areas, this method can be made quite effective in minimising electrolysis troubles, especially where chief reliance is being placed on the reduction of potential drops in the negative return. Owing, however, to the constant shifting of the positive and negative areas with changes of load, it is very difficult to apply pipe drainage as an auxiliary means of protection with the three-wire system, although this has been done with some success in a few cases.

(vi.) *Automatic Substations.*—The possibility of reducing feeding distances by the use of a large number of feeding stations has until recently been seriously limited by economic considerations, due in large part to the excessive labour costs of operating a large number of substations. During the last few years, however, there have been developed several types of completely automatic substation equipment, and a considerable number of very successful installations of this kind are now in operation in America. Owing to the great reduction in labour costs with such equipment a much larger number of feeding stations can be used before the economic limit is reached, and in

[1] *Journal I.E.E.*, 1898, xxvii. 457.

fact economic conditions alone generally would dictate a considerably larger number of feeding stations than when manually operated substations are used. In consequence of this the use of automatic substations in sufficient number to bring about the most satisfactory economic and operating conditions will in general reduce the feeding distances to such an extent that electrolysis troubles will be largely taken care of as a by-product of such installations. This appears at the present time to be one of the most promising developments bearing upon the problem of electrolysis mitigation.

§ (29) Prevailing Practice in Electrolysis Mitigation.—It cannot be said that there is at the present time any universal standard of practice in regard to the application of the electrolysis mitigative measures discussed above. In some countries the application of pipe drainage in any form is prohibited by law, chief reliance being placed on maintaining sufficiently low values of voltage drop in the railway return circuit to make any other measures unnecessary. This has been the case in Great Britain for many years, where the maximum potential drop in the negative return of the railway system is limited by law to seven volts. Experience seems to show that the results of this practice have in general been quite satisfactory, although many engineers contend that it involves unnecessary expense. In other places relatively little attention is given to the maintenance of the negative return, except such as may be necessary to maintain satisfactory car service, and the pipe drainage method is depended upon as the sole protection for the underground utilities. For many years this was perhaps the most common practice in America, but the haphazard and unscientific methods of drainage often followed brought this method into disrepute in many quarters. This question has been the subject of a great deal of systematic study during the last few years, especially in America, where extensive researches have been carried on over a period of years by representatives of all the utility interests concerned and the Bureau of Standards. The prevailing practice in America, and in some other countries at the present time, indicates that there is a decided trend toward the use of a combination of the methods applicable to the track network and to underground pipe systems, whereby such mitigative measures are applied to the railway return system as will reduce the potential drop therein to as low values as are economically practicable; and then, if further mitigative measures are found to be desirable, the residual trouble is largely eliminated by the application of a limited amount of pipe drainage. A considerable number of such systems are now in operation, and they are held by many engineers to offer the most effective solution of the electrolysis problem when viewed both from the engineering and the economic standpoint.

B. McC.

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Electrical Conductors. See *ibid.* § (30).

Isolating Switches. See *ibid.* § (31).

Oil Switches. See *ibid.* § (34).

Potential Transformers. See *ibid.* § (33).

Switchgear, Means for Reduction of. See *ibid.* § (35).

Transformers. See "Transformers, Static," § (19).

String Galvanometer. See "Vibration Galvanometers," § (7).

Subdivision of Generating Capacity in Electric Power Stations. See "Switchgear," § (36).

SUPER-CONDUCTIVITY

§ (1) EXPERIMENTAL DETAILS. — Super-conductivity is the name given by Professor Kammerlingh Onnes of the University of Leiden to the peculiar type of electrical conduction which is exhibited by certain metals at the extremely low temperatures obtainable by the use of liquid helium. This range of temperature is from 1·6° to 6° absolute and has so far only been obtained at the Cryogenic Laboratory, Leiden. On account of the extreme difficulty of obtaining these low temperatures, all of the experimental work on this subject has been done at the Cryogenic Laboratory.

At ordinary temperatures most pure metals show an approximately linear relationship between resistivity and temperature, the temperature coefficient being in almost all cases slightly greater than 1/273. At very low temperatures the resistance of most metals decreases less rapidly than the temperature and tends to approach a constant small value. Very small amounts of impurity greatly increase this residual resistivity, the amount of increase being roughly independent of the temperature. When investigating this effect in mercury, which was chosen because of the ease with which it can be obtained in a pure state, Kammerlingh Onnes found that instead of a continuous decrease of resistance with decrease in temperature there was a sudden disappearance of resistance at a temperature of 4·2° K. Within a temperature range of less than 0·02° the resistivity dropped from a perfectly definite value (about 0·0005 of its value at 0° C.) to a value less than he could detect with a sensitive galvanometer (certainly less than 10^{-9} times its value at 0° C.).

It was later found that other metals exhibit this property, tin at 3·8° K. and lead at about 6° K. It was noted further that amalgamated tin-foil showed super-conductivity at 4·29° K., which is higher than the critical temperature of either pure mercury or pure tin. On the other hand Pt, Cu, Fe, Au, and Ag show no indication of super-conductivity at the lowest temperatures yet attained. It is interesting to note that the metals which show super-conductivity also are the ones which were found by Matthiessen in 1860 to yield alloys the resistivity of which was an additive function of the resistivity of the constituents. Metals of the second group, on the other hand, are those which, when mixed with a metal of either the same or the former class, yield an alloy of much greater resistivity than would be computed from that of the constituents.

While the disappearance of resistance when measured with small current densities occurs at a very definite temperature, it was found that when very large current densities were used the critical temperature was considerably lowered. In other words, for each current density there is a certain critical temperature, and for each temperature a certain threshold current below which the resistance vanishes. In some cases current densities as high as 900 amperes per square millimetre could be used without producing appreciable resistance.

The most natural explanation of such a phenomenon is that the heavy current produces enough heating to raise the temperature of the metal above that of the bath. This is also indicated by the fact that a wire when wound into a compact coil, and with correspondingly reduced possibility of heat dissipation, showed a threshold current only about one-tenth as great as when the wire was stretched out straight. On the other hand, a straight wire in an evacuated vessel showed practically the same threshold current as when actually in contact with the liquid helium. Kammerlingh Onnes has shown, however, by measuring the rate of flow of heat through the walls of the capillary, that any uniformly distributed resistance would not account for the observed effects. That is, the heat generated by a heavy current could be dissipated through the wall with a temperature difference between metal and bath much less than the difference between the critical temperature for the heavy current and for a very small current. He also showed that the heat was not conducted

along the wire from parts of the circuit at higher temperature. This was proved by the very ingenious device of using three wires in series with the middle one of smaller cross-section than the ends. A current can then be passed of such magnitude that it is greater than the threshold value for the central part (which therefore shows resistance), while it is less than the threshold value for the end sections (which therefore remain super-conducting). This shows that if the appearance of resistance in the central part is due to its being at a higher temperature than the bath, then this difference of temperature cannot be due to heat conducted along the wire from the ends.

Some time after making the experiments mentioned above, Onnes investigated the effect of a magnetic field on the resistance of super-conductors. He found that the substances (tin and lead) remained super-conducting until a certain critical field strength was reached, and then suddenly showed a considerable resistance, which increased gradually with further increase in the field strength. At lower temperatures the threshold value of the magnetic field was greater.

It has been suggested that the critical magnetic field and the threshold current are directly connected by the relationship that the threshold value of current is that at which the magnetic field, due to the current itself, is equal to the critical magnetic field. This relationship seems to be confirmed by all of the rather meagre experimental data which is at present available on the point, and would indicate that the existence of a threshold magnetic field is the fundamental phenomenon to which the existence of a threshold current is merely a logical consequence.

Beginning in April 1914, Onnes performed a number of interesting and ingenious induction experiments involving super-conductors. These may be regarded either as interesting confirmations of Maxwell's electromagnetic theory under extreme conditions, or as a new method for measuring extremely low resistances.

The essential elements in the experiment were a super-conducting coil of lead wire closed on itself, and a magnetic field produced by a fairly powerful electromagnet. If we denote by r and L the resistance and inductance of the coil of n turns, i the current in the coil at any time t, and ϕ the magnetic flux linking the coil from the field, we have as the general equation

$$\mathrm{L}\frac{di}{dt}+ri=\frac{d\phi}{dt}. \qquad (1)$$

In order to perform the integrations let us assume that r is not a function of i either below or above the threshold value of i but has a discontinuity at that value. Also let us confine ourselves to cases where $d\phi/dt$ is either a constant or is zero. Then

$$i=\frac{1}{r}\frac{d\phi}{dt}+\mathrm{C}e^{\frac{-rt}{\mathrm{L}}}. \qquad (2)$$

If $i=i_0$ at $t=0$, and $d\phi/dt=$const.,

$$i=\frac{1}{r}\frac{d\phi}{dt}(1-e^{\frac{-rt}{\mathrm{L}}})+i_0e^{\frac{-rt}{\mathrm{L}}}. \qquad (3)$$

For rt/L very small and $i_0=0$

$$i=\frac{1}{\mathrm{L}}\frac{d\phi}{dt}t. \qquad (4)$$

For rt/L very small and $d\phi/dt=0$

$$i=i_0e^{\frac{-rt}{\mathrm{L}}}. \qquad (5)$$

For rt/L very large (as in the case above the threshold value)

$$i=\frac{1}{r}\frac{d\phi}{dt}+i_t, \qquad (6)$$

where i_t is the threshold current.

In most practical cases the time constant L/r of a circuit is so small that the exponential terms become negligible after a very short time. In the case of these super-conducting circuits, however, r is so small that L/r is to be measured in days rather than in seconds, so that a current dying out in accordance with equation (5) is practically constant over a considerable time.

The particular experiments illustrating these relations are:

(*a*) The coil of wire is cooled below the critical temperature while in the magnetic field. The magnetic field is then removed and currents are found to be induced in the coil proportional to the strength of the field until the latter is sufficient to produce a current equal to the threshold value. Further increase in the initial magnetic field produces no corresponding increase in the induced current.

(*b*) If the coil is first cooled to the super-conducting state and a magnetic field then applied and removed, the currents induced by the application of the field are neutralised by its removal and no resultant current is found at the end of the experiment. If, however, the value of the magnetic field is so great that the current induced by its application exceeds the threshold value, then the inverse current induced by the removal of the field is greater than that existing while the field was applied, and consequently the specimen is found to contain a residual current in the reverse direction at the end of the experiment.

The current measurement in these cases was made by suspending a compass needle near the coil and placing a geometrically similar coil in a symmetrical position on the other side of the needle. Sufficient current was then sent

through this second coil to neutralise the effect of the first coil at the needle, and this current measured.

The rate of decay of these induced currents was found to be less than could be detected by the apparatus used and was certainly less than 1 per cent per hour. This value would correspond to a value L/r of four days, or to a value of the resistance less than $0{\cdot}2\times10^{-10}$ times the resistance of the same coil at 0° C.

These permanent induced currents may be looked upon as analogous to those assumed in Ampere's theory of magnetism, and the more recent suggestions of Weber and Langevin on diamagnetism.

Since the results of the direct experiment as outlined above might have been due to some peculiar magnetic behaviour of the materials of the coil at these extreme temperatures, a number of check experiments were performed. In the first of these the axis of the coil was placed perpendicular to the direction of the magnetic field and the magnetic effect at the compass needle was found to be less than 10 per cent as great as in the original experiment. The experiment was then repeated as originally but with the coil open-circuited, and again only about 10 per cent of the original effect was observed. The next step was to connect a ballistic galvanometer in parallel with a short length of the lead wire and arrange a small knife to cut the wire between the galvanometer contacts. Current was then induced in the coil and measured by the compass. The wire was then cut and a ballistic throw obtained equivalent to 90 per cent of the electro-kinetic momentum of the observed current. The compass still showed the presence of a magnetic field equivalent to the remaining 10 per cent. At first this outstanding 10 per cent was attributed to a possible short-circuited turn in the winding, but Onnes now believes it is due to eddy currents induced in the thickness of the wire itself.

Later Onnes found that it was feasible to construct a super-conducting key using two lead blocks, one of which had three small conical points on its surface. By pressing these together under the liquid helium a contact of negligible resistance could be made or broken at will. With such an arrangement

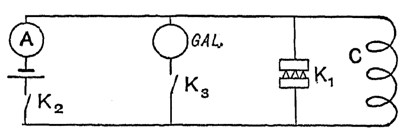

FIG. 1.

a large number of interesting experiments are possible.

As typical of these, consider the circuits shown in *Fig.* 1. With keys K_3 and K_1 open, current from the battery flows through the super-conducting coil C and is measured by both the ammeter A and the compass needle N. K_1 is then closed and no change occurs, since there had previously been no difference of potential across K_1. K_2 is then opened and the ammeter of course reads zero, but the compass needle is unaffected because the current in the coil still continues to flow, now through K_1. Next K_3 is closed and K_1 then opened. The galvanometer then responds to the current which had been started, perhaps some hours before, by the battery; and the compass needle indicates the stopping of the current in the coil.

The experiments outlined above cover practically all the measurements which have thus far been made on the electrical behaviour of super-conductors. Onnes has, however, also carried out some determinations of the specific heat and thermal conductivity of mercury just above and just below the critical temperature. Although he observed a considerable decrease in the former and increase in the latter, the changes are only by a factor of 2 to 4, and are of the order to be expected for most substances at very low temperatures. There therefore seems to be no correlation between these changes and the factor of 10^{-7} by which the electrical conductivity is increased.

§ (2) THEORY.—Coming now to the theoretical side of the subject, it is found that the classical theory of conduction[1] by free electrons is hardly able to account for the phenomenon. According to this theory the electrical conductivity is given by

$$\gamma=\frac{pn\lambda}{T},$$

where p is a constant. n is the number of free electrons in unit volume, λ is the mean free path of an electron, and T the absolute temperature. To account for the great increase in γ it is necessary to assume either a number of electrons very large compared to the number of atoms or a mean free path larger than the dimensions of the apparatus.

(i.) *Onnes.*—Onnes has suggested a modification of this theory which, combined with the quantum theory, yields a much more possible explanation. In the classical theory it is usually assumed that $\lambda\propto1/\sqrt{T}$, since thereby the conductivity becomes $\propto1/T$, which is in agreement with the experimental facts at reasonably high temperatures. Also at these temperatures the internal energy of a substance E_T is proportional to T. Onnes suggests that the assumption be changed so that

$$\lambda\propto\frac{1}{\sqrt{E_T}},\text{ and therefore }\gamma\propto\frac{1}{\sqrt{TE_T}}.$$

[1] See "Electrons," § (27).

Now at low temperatures, measurements of specific heat have shown that E_T is no longer proportional to T but quite closely follows the equation deduced by the quantum theory

$$E_T = 3R\frac{\beta\nu}{e^{\beta\nu/T}-1},$$

where R is the gas constant, $\beta = h/k = 4{\cdot}86{\cdot}10^{-11}$, and ν is the natural period of vibration of the atoms. Substituting this equation in the expression for γ we find that the result is a relation between γ and T which is nearly a straight line at moderate temperatures but at low temperatures becomes concave upward. It was in fact deduced by Onnes to explain his results on gold and similar metals, and it was in an attempt to verify this formula that he discovered the super-conductivity of mercury. To "explain" super-conductivity on this basis it is necessary merely to make the additional hypothesis that at the critical temperature there is a sudden increase in ν such as would be caused by a "freezing up" of the slower modes of vibration. Owing to the exponential relations only a moderate change in ν is necessary to account for an enormous change in conductivity. It is difficult to see, however, why the specific heat should not be similarly affected.

(ii.) *Lindemann.*—Lindemann[1] has advanced a theory based upon the existence of a definite space lattice of atoms in each crystal of metal such as those studied by Bragg in various salts. Lindemann further assumes that the electrons also form a similar but independent space lattice, and that an electric current results from the relative motion of these two interpenetrating lattices. Above a certain critical temperature the heat motion of the atoms will cause them to interfere with the free motion of the electron lattice and the metal will show resistance. Below this temperature the amplitude of the vibration is so small that the atoms no longer obstruct the motion of the electrons and super-conductivity results. This hypothesis is rather speculative and does not suggest lines for further experimental investigation.

(iii.) *J. J. Thomson.*—Perhaps the most detailed theory of metallic conduction which accounts for super-conductivity is that proposed by J. J. Thomson,[2] which is a modification of a theory which he developed back in 1886. He postulates the existence in the metal of a large number of electric doublets of moment M (probably each atom is such a doublet). When an electric field X is applied, these doublets experience a moment XM tending to orient them in line with the field. Thomson assumes that the forces tending to oppose this orientation are proportional to the kinetic energy W of heat motion of the atom. Consequently the electrical polarisation, or moment per unit volume, I, will be some function of (XM)/W and may be written

$$I = NMf\left(\frac{XM}{W}\right),$$

where N is the number of doublets per c.c. Little is known of the function $f(x)$, but when there is no directing field and $x=0$ we know $f(0)=0$; also for very large values of X all the doublets are lined up and $f(\infty)=1$. Therefore the curve connecting I with x must be of the general shape shown in *Fig.* 2.

Now the field X is made up of the external field X_0 and the field at the doublet in question

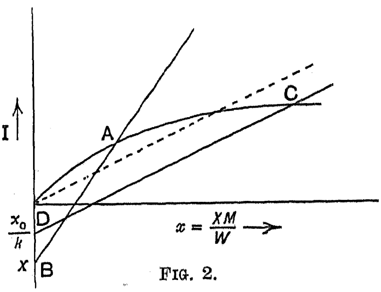

FIG. 2.

due to the adjacent doublets, which we may write as kI, so that

$$X = kI + X_0,$$

or substituting $x = XM/W$ and solving for I,

$$I = \frac{W}{Mk}x - \frac{X_0}{k},$$

which is the equation of the straight lines in *Fig.* 2. The actual value of I for any given X_0 and W is found at the intersection of the proper straight line with the curve, as at A.

The further assumption is now made that under the influence of the large electric forces which exist when the doublets are close together a certain number p of electrons are passed along by each atom per second. When I is zero as many pass in one direction as another and there is no current, but when the medium is polarised there is a resultant current density given by

$$i = epId,$$

where e is the charge on an electron and d the mean spacing centre to centre of the atoms. The conductivity is therefore

$$\gamma = \frac{epId}{X_0}.$$

[1] Lindemann, *Phil. Mag.*, 1915, xxix. 127.
[2] J. J. Thomson, *Phil. Mag.*, 1915, xxx. 192.

Now near the origin the curve $I = NMf(x)$ is nearly the straight line

$$I = NMxf'(0),$$

and the intersection of this with a straight line such as AB is given by

$$I = \frac{NM^2f'(0)X_0}{W - NM^2kf'(0)},$$

and the resistivity is therefore

$$\rho = \frac{W - NM^2kf'(0)}{epdNM^2f'(0)}.$$

Also except at low temperatures $W = RT$, so that ρ has the form

$$\rho = a(T + b),$$

and the resistance is seen to vary linearly with the temperature as required by experiment.

At low temperatures, however, W becomes very small and the slope of the straight lines CD eventually becomes less than that of the curve at the origin. In this case we see that X_0 may be reduced to zero and a current will still flow in the conductor. In other words, the material will be super-conducting. The temperature at which this will occur is that corresponding to

$$W = NM^2kf'(0).$$

To explain the absence of super-conductivity in some metals he assumes that the restoring force is proportional to W + D instead of W, where D is a constant independent of temperature. This force D would be large in the case of mixed crystals, and Thomson uses it to account for many well-known relations among the resistivities and temperature coefficients of alloys.

This theory seems to account qualitatively for a great many phenomena; and is sufficiently flexible in the number of unknown constants and functions to be able to fit a good deal of data quantitatively. F. B. S.

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SURFACE RESISTIVITY, method of measuring specified. See "Measurement of Insulation Resistance," § (1) (iii.).

SUSCEPTIBILITY, MAGNETIC (k): the ratio of the intensity of magnetisation to the magnetising force producing it, for any material. See "Magnetic Measurements and Properties of Materials," § (1).

SUSPENSIONS, GALVANOMETER. See "Galvanometers," § (9).

SWITCHGEAR

§ (1) FUNCTION AND LOCATION OF SWITCHGEAR.—The power generated by the electrical plant of a central station has to be collected and distributed to the various points at which it is required. This function is carried out by a so-called "switchboard," which is a device consisting of two or more main conductors called busbars, on which is collected the power, and the distributing apparatus proper, *i.e.* switches, circuit breakers, fuses, etc., which control the various circuits including those of the generating plant.

The regulation of the generators is usually also effected at this point, and the necessary regulating apparatus either fixed direct on to the switchboard or placed in a convenient position in its vicinity, the regulating handles being mounted on or near the board.

In the earlier days, when voltages were low and powers small, these switchboards—as their name indicates—consisted of panels usually made of slate or marble, the busbars being mounted on the back, and the control apparatus on the front. As the voltages and the powers collected increased it became more necessary to guard the operator against accidental contact with the current-carrying parts as well as to protect him against flashes produced by the operation of the various control apparatus. In order to effect this, the switching apparatus was placed at the back of the boards, thus removing it from the immediate proximity of the operator. With further increase of power collected, the apparatus became more unwieldy, and the danger of short circuits between the apparatus and the busbars and other conductors increased, and since with the increase of plant generally continuity of supply became more and more important, further precautions had to be taken to reduce to a minimum the chances of breakdown or failure of supply.

This led to the introduction of the so-called "cellular construction," which in its present form consists of chambers constructed of concrete or other non-inflammable material, in which the various pieces of apparatus are mounted, separate chambers or tunnels being employed for the busbars and connections. In order further to increase the factor of safety, this stone structure may be placed remote from the operator. In this case a separate control board is provided from which the operator manipulates the various pieces of apparatus required for switching and regulating purposes.

The control board is sometimes placed on the same level and in front of the stone structure, with a passage-way between the two for facilitating inspection of the wiring

on the back of the control panels, as well as inspection of the apparatus in the cells. In other arrangements the control panels, which may take the form of a control desk, may be placed on a gallery, with the stonework structure containing busbars and apparatus below the latter, or the stonework structure may be placed in an adjacent chamber, thereby reducing the possibility of troubles arising on the machines communicating themselves to the switchboard.

All these arrangements have in view one main feature, namely, that the operator of the switchboard should be able to overlook the engine-room while manipulating the switches on the board. When a power station is small this has certain advantages, as it tends to reduce the number of attendants and facilitates communication between these.

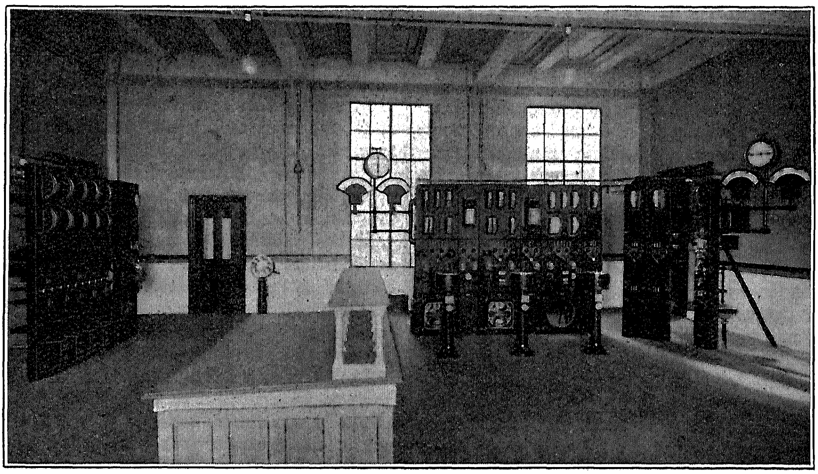


FIG. 1.—Buenos Ayres Western Railway—Control Room.

In power stations of later constructions, where 50,000 kilowatts or more are collected together, the dimensions of the engine-room become considerable, and communication by word of mouth increasingly difficult. Accidents to large machines may be alarming in their proportions and effect, and in order that the attendant should not be distracted by these, it is becoming more usual to place the control board, or control desk as the case may be, as well as the switching apparatus in an annexe to the power-house, or even in a building entirely separate from the engine-room (*Fig.* 1). Telegraph and telephone apparatus and remote controlled instruments are employed for communication between the two buildings and for indicating to the switchboard attendant the electrical position of the plant at a given moment, as well as for transmitting instructions from the attendant to the engine-room staff.

§ (2) ISOLATING SWITCHES. — Isolating switches are switches which are used for disconnecting a circuit or a piece of apparatus under no-load conditions, that is, when no current is flowing in a particular circuit. They are used for isolating individual pieces of apparatus for inspection purposes or repairs, or for dividing up the system into sections. They may be coupled to form two- or three-pole units (*Fig.* 2), and may be operated by means of a common operating handle. On higher voltages, however, it is usual to arrange them as single-pole units, to provide each isolating switch with an eye, and to operate them by means of an insulated pole with a hook or other suitable devices fixed at the end. Where large powers

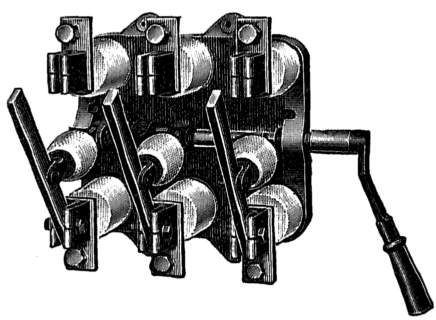


FIG. 2.—3300-volt Triple-pole Isolating Switch.

are connected to the system and heavy rushes of current may momentarily pass through isolating switches, these may have to be provided with locking devices to prevent them opening on such a current rush.

§ (3) Fuses.—Fuses are metal links inserted in a conductor for the purpose of interrupting the circuit in the event of the current rising beyond a certain predetermined value (*Fig.* 3). They are usually made of metal which has a low melting-point, such as lead or tin or some alloy, though fuses made from copper strands or silver wire are likewise often employed.

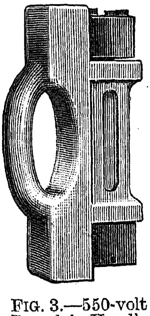


Fig. 3.—550-volt Porcelain Handle Fuse.

The "fusing-point" is, in the majority of fuses, unreliable, especially if the fuse has been in circuit for some time. This is particularly the case with lead fuses, which, through repeated heating, very soon become covered with a non-conductive oxide which in time, as it thickens, materially reduces the conductive cross-sectional area, thereby increasing the resistance and the watts absorbed, and consequently brings the temperature corresponding to normal load nearer the fusing value.

Draughts having a cooling effect may also materially affect the accuracy of an "open type" fuse, *i.e.* a fuse in which the strip or wires are exposed to the atmosphere.

It is claimed for the "cartridge type" fuse, particularly the one using silver strands, that it is more dependable with regard to its fusing-point.

Fuses have their use on smaller, particularly subsidiary, circuits. On larger circuits the amount of metal which would be melted on an overload would be considerable, and if the power behind these is also large this melting would be very rapid, *i.e.* vaporisation would take place almost at once causing pressure rises in the container, prolongation of the arc, and consequent damage.

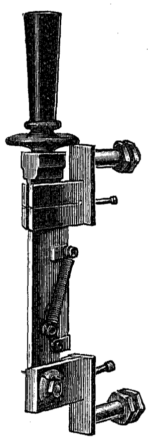


Fig. 4.—550-volt Quick-break Knife Switch.

§ (4) Switches.—Switches are mechanical devices suitable for breaking a circuit-carrying load when put into operation.

For direct-current circuits and on low-voltage alternating-current systems these consist of movable blades bridging two contacts (*Fig.* 4). For larger currents the blade may take the form of brushes. Handles are supplied for operating these, and where a larger breaking capacity is required, the blades are fitted with a quick-break device consisting of a follower blade actuated by a spring which is brought into tension by the act of opening the switch and produces a "snap" action of the follower blade.

Instead of, or in addition to, the quick-acting follower blade, carbon tips for the interruption of the final circuit may be provided.

For higher voltage alternating current the switch may be immersed in oil, and the make and break take place in this medium. When fitted with automatic opening features the switch is termed a "circuit breaker."

§ (5) Circuit Breakers.—These are divided into two main groups: air-break circuit breakers, usually termed "circuit breakers," and oil-immersed circuit breakers, usually termed "automatic oil switches."

Each group is further constructed in two types, viz. the "fixed handle" pattern and the "loose handle" pattern.

In the *fixed-handle* type the circuit breaker or oil switch can be held in the closed position by the action of the operator, that is, the automatic features are inoperative whilst the breaker is being closed and until the handle has been released.

Such being the case, ordinary knife switches have to be arranged in series with this type of automatic circuit breaker, where there is a risk of closing the circuit on a fault condition, as, for instance, in parallelling generators. When knife switches are employed the breaker is closed first and after that the knife switch, thereby permitting the breaker to open at once if a fault condition in the circuit demands this.

A *free-handle* circuit breaker, as the name implies, is arranged with a loose handle in such a manner that it can automatically open up again by means of its automatic features the moment it is closed and irrespective of the action of the operator.

An *air-break* circuit breaker (*Fig.* 5), employed on direct-current circuits and low-voltage alternating-current systems, consists of a circuit-closing mechanism operating a brush or brushes, provided with auxiliary circuit-breaking devices for taking the final break.

In air-break circuit breakers the various automatic features may be embodied in the circuit-breaking apparatus itself, or else relays may be employed for carrying out the functions of the different features.

To increase the breaking effect of a circuit breaker it may be fitted with a magnetic blow-out device, that is, a magnetic field is arranged across the breaking portion in such a manner as to cause the arc to lengthen and

to move in a predetermined direction, thus causing a rapid interruption of the arc formed at breaking. This has its usage mainly where

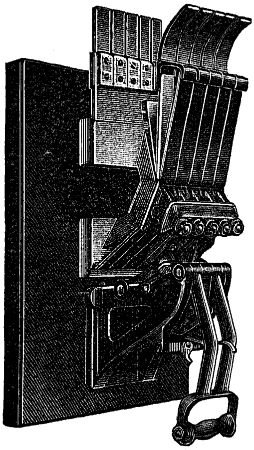


FIG. 5.—550-volt, 3000-amp., Single-pole Circuit Breaker.

the circuit breaker is mounted in a confined space and there is danger of the fumes causing short circuits to earth or between poles. It is particularly suitable in tramway cars or similar conditions, where the space available for mounting the circuit breaker is confined.

On the other hand, the very rapid interruption of the circuit may, particularly if the interruption takes place in a main circuit or near the source of supply, cause an undue potential rise at the terminals of the breaker, due to the stored magnetic energy in the circuit; and as this potential rise may obtain very considerable dimensions, damage may result to the apparatus and machinery connected to the circuit.

The danger of such potential rises is, however, less when the circuit protected forms one of a number of subsidiary parallel feeders, if the other feeders can serve to form a discharge path for the potential increase in question.

§ (6) OIL-IMMERSED CIRCUIT BREAKERS.—The most suitable apparatus for breaking alternating-current circuits is the automatically operated oil switch, which, in view of the fact that it may be used in connection with power stations of largely varying kilowatt capacity, is usually manufactured in a number of different sizes, irrespective of voltage and carrying capacity, but suitable for dealing with emergency conditions arising out of the powers collected behind them.

The mechanical form of such breakers may differ to some extent, but the main principle is the interruption under oil of the arc formed by breaking circuit, the oil being utilised for cooling and quenching the arc in question.

The form in which such quenching may take place varies in different designs, but in the majority of cases it depends on moving the oil by different means into the arc produced on interrupting circuit.

For stations of smaller kw. capacity and usually also for lower voltages the three poles in a three-phase system may be arranged in a common oil tank; on larger systems each pole is preferably arranged in a separate tank. Likewise, in the smaller switches two breaks are usually provided for each phase, but in switches for larger kw. four or even six breaks may be arranged per phase.

In this country and the United States the breaks for each phase are arranged in series, but in continental practice they are sometimes arranged in parallel, the idea in the latter case being to divide the current and to break part in each of the parallel paths. It may here be noted that in one form of balanced protection the nature of the protection necessitates interruption of the arc in parallel circuits.

In the simplest form of oil switch (*Fig.* 6) bushes are arranged in the top plate or " base "

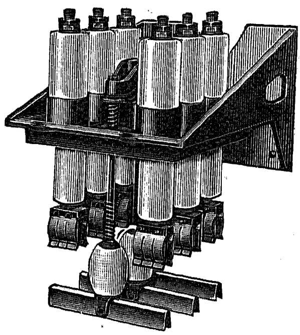


FIG. 6.—6600-volt Triple-pole Oil Switch.

of the oil switch, which project into a tank suspended below and carry the necessary stationary contact-making portions. A cross arm or connecting piece, operated by a rod, is moved up and down, thereby making and breaking the connection between the stationary contacts.

The tank with a wooden lining is suspended from the base and contains the necessary oil.

In the majority of switches a so-called " air cushion " is arranged above the oil level, that is, a space is left above the oil which will act as a cushion in the event of pressures being set up in the tank.

The arc under oil disintegrates the quenching

medium and produces hydro-carbon gases, which when mixed with air produce a highly explosive mixture, which if fired may cause an explosion.

Various forms of arrangements have been adopted to prevent, if possible, such explosions, or alternatively to prevent them doing damage to the switch itself and the adjacent apparatus. In one form which is mainly applicable to switches of light construction, and where the mass of oil and the tank are within reasonable limits, the air cushion is avoided altogether, and the apparatus arranged in such a manner as to exclude pockets where explosive gases can collect. The tank is suspended from springs which allow a certain vertical movement in order to relieve the pressures which may be suddenly produced in the oil tank through causes other than explosion of gases.

Where, however, the inertia of the tank is excessive, and would not allow of a rapid vertical movement, the tank and switch may be constructed to withstand the full force of the explosion taking place in the oil switch, a vent pipe being fitted to the latter and terminating outside the switch-house for carrying away gases and relieving an explosion if it occurs.

In the majority of switches the break is effected by drawing out the arc under oil, and depending on the latter effectively cooling the arc and extinguishing the same: occasionally devices are added, the object of which is to produce a flow of cool oil into the path of the arc and to facilitate quenching.

In another form of oil switch the fact that two currents flowing in opposite directions have a repelling effect on each other is utilised to produce a horizontal displacement of the arc, and to disperse the latter into the oil instead of forcing oil into the arc as in the previous instance.

§ (7) Operation of Switches.—For operating large oil switches, or oil switches which are arranged remote from the control board, and where a system of linkwork or wire rope becomes too cumbersome, compressed air has occasionally been resorted to in the past, but is now almost entirely superseded by an electric device which may take any of the following forms (*Fig.* 7):

(*a*) A motor winds up a spring, which, when it is required to operate the switch, is released to rapidly carry out the closing function; another spring wound up in a similar manner carries out the opening function.

(*b*) A solenoid employed for the closing function and a spring assisting gravity to cause the opening of the circuit breaker.

(*c*) A motor which only carries out part of a revolution, and directly effects the closing of the circuit breaker, gravity or springs, or both, being again employed for the opening function.

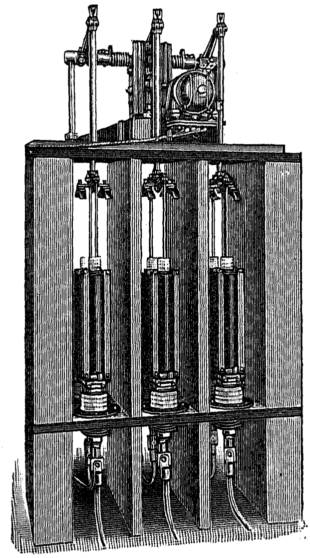


Fig. 7.—Motor-operated 1200-amp., 15,000-volt, H_3 Oil Switch in stone cellwork. Switch open.

§ (8) Protection of the System against Internal Faults.—Since all the power generated is brought together on the switchboard, that is, to one point, failure at this particular point may put the whole system out of commission, no matter how many reserve generators and reserve feeders there may be in the distribution system. It becomes clear, therefore, that every endeavour should be made and all possible precautions taken to ensure the satisfactory operation of the individual pieces which go towards the making up of a switchboard.

It is particularly important so to arrange the busbars that troubles in any part of the system cannot communicate themselves to these. Faults arising on the generating side or the distributing side must be cut off with the greatest despatch, and with the smallest amount of dislocation. The switchboard must further be arranged to localise these faults as much as possible, and to ensure that a fault occurring on any portion of the system may be effectively dealt with.

In the early days when only one generator was employed the protection against faults arising from defects in the system was a comparatively simple matter, but when several generators were connected to common busbars and the collected power increased, protective devices had to be introduced which under more severe overload conditions would effectively cut out a faulty generator, particularly a

generator which, instead of giving power, was taking power from the system, or a faulty feeder circuit.

For this purpose, the switches used in such switchboards are provided with devices which, under predetermined load conditions, automatically open out and disconnect the particular generator or feeder from the system in question. The simplest way of obtaining this end is by means of fuses, that is, links in the circuit which will melt if the current flowing through increases beyond a predetermined value, thus cutting off the circuit in question. This method of interruption has obviously certain limitations, which were soon found out when the power collected increased.

The different circuits connected to the busbars are designed and constructed for normally carrying a certain predetermined power, but in the event of a fault arising on, for instance, an outgoing feeder, and such a fault obtaining the dimensions of a "dead short circuit"—that is, the two or more conductors of opposite polarity becoming accidentally joined together without interposition of a resistance—the whole power connected to the busbars flows into the fault. A fuse installed to protect the circuit in question would in this case be called upon to interrupt a load many times in excess of its normal load capacity. This effect is further enhanced by the energy stored in the revolving parts of the machines, resulting in a generator giving from twenty to thirty times its normal current in the first instant of the short circuit.[1] Apparatus had therefore to be constructed which would effectively interrupt under emergency conditions powers far in excess of its normal carrying capacity, and, in order that the circuit which was protected by such apparatus might be reconnected with the shortest possible delay, protective devices had to be constructed of a type which do not destroy themselves as fuses do, but which, after carrying out their function of opening circuits, would be ready for reclosing on the shortest possible notice. To attain this object, the switches arranged in the circuits for disconnecting the latter are provided with automatic features which, under predetermined overload conditions, will cause the interruption of the circuit. A switch provided with such features is called a circuit breaker. The automatic features may be of different forms :

(i.) *Maximum or Overload Devices*—that is, devices or relays which will cause the switch or circuit breaker to open circuit if the current flowing exceeds a certain predetermined value. An adjusting device is usually provided by means of which the overload can be varied.

[1] Miles Walker, *Proceedings Inst. Elec. Eng.* xlv. 295.

(ii.) *Reverse Current Devices.*—These are relays or devices which will cause the breaker to operate in the event of the current flowing through the same in the reverse direction. They are largely used for disconnecting generators or other pieces of apparatus whenever these, instead of carrying out their proper function of supplying power to the system, take power from it.

In the case of continuous current circuits, where the potential and current are always "in phase," such a device is comparatively simple, but when dealing with alternating current, where the phase displacement is a variable, it is usual to make the relay react on a reversal of power in order to comply with the more usual fault conditions. It may be here noted that if the excitation of an alternator is cut off, the current will be at 90° to the pressure, thus a different type of protection has to be introduced to deal with this particular fault condition.

(iii.) *Low-volt Devices.*—Automatic circuit breakers are often equipped with a device which will cause them to operate on the potential falling below a certain predetermined value ; they are set to operate at about 65 per cent of normal potential. These devices may, however, be arranged to cut off only in the event of the potential falling to a lower value, about 25 per cent of normal, in which case they are termed no-volt features. Such low-volt or no-volt devices are largely used on circuits supplying power to motors, as it is essential that, should the voltage fall to a value which will cause the motor to pull up or stop, the motor should not again automatically and suddenly restart on the supply being restored, as this would entail danger to the motor itself and the men operating the machinery.

(iv.) *Minimum Relays.*—These are set to operate when the current falls to a predetermined value below normal, and have a few special applications, such as in connection with battery charging, etc.

The foregoing devices, which form the simplest means of protecting individual circuits, may be used singly or in combination with each other. Thus a circuit breaker fitted with overload and reverse protection is often placed in the generator circuit, particularly on smaller power switchboards, where both the number of units as well as the size of the individual unit is small. On switchboards for larger powers, the overload features are usually omitted, the idea being that the generators must "hang on" and supply power under momentary heavy overload conditions. The danger of shutting down a power station through overload protection on the generators may be considerable, particularly if the generators are of different sizes and different

characteristics; the whole of a certain overload coming on to the station may be momentarily thrown on to a particular generator, due to the greater responsiveness of the latter, and cause the overload circuit breaker to disconnect the same, with the result that the overload and the load of the generator in question will be taken up by another prime mover, which in its turn would be cut out by the sudden rush, and so on. The overload in question may be purely of a temporary nature, well within the guaranteed momentary overload capacities of the various units taken as a whole; thus it is best to dispense with overload protection.

There are a number of other circuit protecting devices, the functions of which depend on the conditions of the circuit.

(v.) *Time Limit Devices.*—For delaying the action of any one of the above features for a predetermined time interval, so-called Time Limit Devices may be employed. These may be arranged for causing a definite period of time to elapse before the apparatus they control functions, and independently of the value by which the "setting" is exceeded (in case of an overload feature the magnitude of the overload); in this case they are termed *constant time limits.* Alternatively they may be constructed to give a delaying action which is inversely proportional to the amount by which the setting is exceeded; they are then called *inverse time limits.* On an overload device, for example, the latter form of time limit would cause a shorter time interval to elapse on a heavy overload, and a longer interval on a smaller overload.

(vi.) *Balanced Protection.*—Some systems are based on the principle that the input into any one circuit must be equal to the output at the other end. On a circuit which has either a leak to earth or between poles, this condition would not be fulfilled, and relays are arranged to cause the circuit breaker to cut off the faulty circuit whenever this condition occurs, and to do so preferably when the fault in the circuit is of a small value, that is, before it has had time to develop into a magnitude which would place a strain on the system or the apparatus controlling this individual portion.

Other systems are based on the fact that the algebraic sum of the magnetic effects of the three phases is zero in a healthy circuit. If leakage takes place between any two conductors or to earth, this condition is not fulfilled, and the relays responding in a similar manner to the previous case again disconnect the faulty circuit.

These methods of protection are coming more and more into vogue, particularly on a widely distributed network where substations are interlinked with power stations and with each other, forming one or more ring mains. The power in this case would flow in one direction or the other. Momentary heavy overloads have to be sustained, and in order to inconvenience the distribution as little as possible the faulty cable and the faulty portion only should be disconnected.

If overload protection only were used in such instances the rush from the generating plant to the fault might pass through a number of substations, and cause the disconnection of these, thus throwing out of commission healthy substations and feeders.

The tripping of switches is sometimes graduated by means of time limit devices set to operate with a greater time lag as their position approaches the centre of the distribution, that is, the switches in the power house or near to it are set to trip after a longer interval of time, and the switches farther away after a short interval. This cannot readily be obtained with the inverse type of time limit when the generated powers are considerable, as, in consequence of the large powers available, the short circuit value may be of such a magnitude as to cause the time element to disappear and all the switches to open at the same instant.

The introduction of the constant time limit for this purpose achieves the required result, but at the same time prolongs the overload on the system unduly, and may cause very serious strains on generating plant and network.

§ (9) Insulators.—The insulators used in connection with switchgear and transmission lines may be divided into four principal classes:

(i.) *Pillar-type insulators,* which are used for supporting conductors or parts which are under pressure.

(ii.) *Pin insulators,* which carry out a similar function to (i.), but are used in the open, and consequently are exposed to rain, snow, dirt, etc.

(iii.) *String insulators,* which are used for suspending current-carrying conductors from ceilings or other suitable supports.

(iv.) *Bushings.—Insulators* which are employed for carrying conductors through a wall or a base which has a different potential to that of the conductor.

(i.) *Supporting Insulators.* — These are usually of cylindrical form. It was the practice in the early days of electrical engineering to arrange ribs on supporting insulators, that is, to provide these with a corrugated surface, the contention being that the creepage surface was increased and better insulation obtained by this means. The lodgment of dirt on the ribs and the weakness of these led, however, to the reintroduction of insulators with a plain surface for the lower voltages,

or where cleaning became an expensive item. Where very high voltages are employed, and the over-all dimensions are considerable, ribs or corrugations may still find a useful application.

In addition to the nature of the surface, the shape of the insulator is of considerable importance. The insulation efficiency, a term introduced to denote the ratio of the voltage at which the insulator actually flashes over to the voltage which would bridge the same distance in air, is materially affected through the shape. This is largely due to the capacity effects of the different parts of the insulator, which cause unequal potential stresses over the surface and thereby affect the break-down pressure. Shields arranged at the top of the insulator and connected to the conductor may, if suitably arranged, be utilised to affect advantageously the potential distribution in question, and may improve the efficiency of an otherwise inefficient insulator.

(ii.) *Pin-type Insulators.* — These being exposed to rain, dust, snow, etc., are usually provided with petticoats, which ensure a dry and comparatively clean surface on the underside, thereby improving the insulation properties under the cited adverse conditions. The efficiency of these is effected in a similar manner to that of the supporting insulators by the shape and form of the component parts, and the material employed in the construction. The number of petticoats arranged on an insulator depends on the line voltage, method of construction, and atmospheric conditions.

(iii.) *String Insulators.* — String insulators find their application in overhead transmission lines, particularly for pressures above 66,000 volts. They are usually built up from units to form a chain, the number of units depending on the voltage of the system. Thus, for instance, on a 110,000-volt line, six or eight units might be employed, and three or four on a 66,000-volt line. It has also to be borne in mind that the efficiency of these, particularly when metal is used for connecting two units to each other, is materially affected by the capacities of the component parts, and that it is important carefully to select the shape.

(iv.) *Bushings.* — Since bushings are used for carrying a live conductor through a base, which is usually at earth potential, the capacity effects are very pronounced, and consequently the distribution of the potential stress across the insulator varies considerably with the distance of the insulating element from the live conductor. Such being the case, bushings, if no further precaution were taken, would, for the higher voltages, assume very large diameters, as the potential stress per unit of length must be kept within the dielectric properties of the material used in the construction. For voltages up to 33,000, solid porcelain bushings give satisfactory results, but for higher voltages the bushings may take the form of concentric cylinders made from paper or porcelain, and filled with oil, or some other insulating material of high dielectric strength.

In another form a series of condensers is arranged round the conductor, and these form the bushings. By suitably choosing the capacities of the various condensers, even potential stress can be obtained, and the insulating material used to its full advantage. A common form of building up such condenser insulators consists in arranging alternate layers of paper and tinfoil. These layers of tinfoil are longest in the inside of the bushing, and become shorter as they approach the outside diameter of the bushing in question. The paper is then usually stepped off in a similar manner, thereby giving the whole a taper form.

Where a condenser insulator is exposed to damp a further insulating sheath may be placed over the whole, and the space filled with an insulating compound.

If the insulator has in addition to be guarded against rain and snow, porcelain rain-shields may be arranged on the outside of the tube.

§ (10) Starting Apparatus. (i.) *Direct-current Starters.*—When starting up a direct-current motor the voltage must be applied gradually in order to avoid heavy current rushes, and for this purpose starters are employed, which, consisting of a resistance, a number of contacts, and a moving arm, are arranged to cut out the interposed resistance in steps until the motor has obtained its final speed, when the resistance is cut out altogether. It is usual to provide such starters with no-voltage releases in order to ensure that if the motor is cut off for any cause the starter arm flies back into the off position, thus avoiding the possibility of closing the circuit on the motor without the interposition of the starting resistance. Overload releases are usually also embodied in the direct-current starter.

Where shunt-wound motors are employed the connections have to be further arranged to ensure that the field obtains the full voltage when starting up, and irrespective of the voltage applied to the armature. Also, if the starter arm for any reason should fly back, the arrangement must be such that the field finds a discharge path, in order to avoid excessive pressure rises in the same. It is usual to arrange the starting contacts for motor starter up to 50 or 60 h.p. in a circle or part of a circle, but for larger starters this construction becomes too unwieldy, and multi-

switch starters are often employed. These consist of a number of knife-switches, which carry out the function of cutting out the starting resistances step by step. Interlocking devices may be introduced, which will ensure the proper sequence of operation, and will interlock the step switches with the main circuit, which is usually employed for overload and no-voltage protection, as well as with the shunt regulator, in order to ensure that the motor when starting up has no additional resistance in the field circuit.

(ii.) *Alternating-current Starters.* (*a*) *Induction-motor Starters.*—For an induction motor the main circuit breaker with overload and no-voltage protection connects the stator windings to the source of supply. A step-by-step switch, connected to the slip rings, gradually reduces the amount of resistance in circuit with these until the motor obtains its proper speed, when the slip rings are short-circuited by means of a special device on the rotor. No-voltage and overload devices are not usually arranged on induction motor starters, but are generally embodied with the main circuit breaker.

(*b*) *Star-delta Starters.*—Where squirrel-cage motors have to be started up a switch is employed, which first places the stator windings in star, and by the next movement connects these in delta, thus applying the voltage to the motor in two steps. In this case also the starting switch may be provided with the necessary overload and no-voltage protection, though the practice is more common to arrange a separate circuit breaker for that purpose and place the protective features on the same.

(*c*) *Auto-starters.*—These consist of a small transformer with tappings, and a three-pole switch, which connects the motor terminals to successive tappings, thereby applying the voltage to the motor in steps. The overload and no-voltage protection, as in the previous case, is preferably also arranged on a separate circuit-breaking device.

§ (11) Liquid Starters.—These are used both for alternating- and direct-current motors, they dispense with the metallic starting resistance, and use a column of water in place of the latter. A moving blade or blades are arranged to dip gradually into the water and thus reduce the resistance in circuit. Metallic short-circuiting contacts are usually fitted at the end of the travel; these come into operation when the blades are fully immersed. Overload and no-voltage releases are not supplied as part of the liquid starter.

§ (12) Controllers.—Controllers serve the same purpose as starters, but take a different mechanical form on account of the heavier duty required of them; whereas a starter is only employed to start up a motor a limited number of times per day, the function of the controller is repeatedly and at short intervals to start and stop a motor. It may also be, and very largely is, used for speed-regulating purposes. Such being the case, the contact-making portions have to be more robust, the resistance has to be more ample, and the moving parts more substantial. In the controllers the contact-making portions are usually arranged on a drum. A number of stationary fingers come into contact with them as the drum is revolved round its axis. Overload and no-voltage releases are not embodied in controllers.

§ (13) Contactor Starters.—When the duty imposed on the starter is particularly heavy, or in cases where automatic starting and stopping from a distance has to be carried out, the starters may take the form of a series of contactors, arranged to cut out consecutive resistance steps.

§ (14) Accumulators.—Accumulators [1] are storage cells which are employed for storing electrical power, and are used on direct-current systems. They are divided into two main classes: (*a*) lead cells, consisting of positive and negative lead plates immersed in sulphuric acid, and arranged in an insulating container; and (*b*) nickel and iron cells in connection with which an alkali is used to form the path between the plates of opposite polarity.

The advantage claimed for the nickel cells is that they weigh less and will stand both heavy discharging and heavy charging, also that the gases given off from these are less injurious than gases given off from accumulators employing a sulphuric acid solution. The internal resistance, however, of the lead cells is smaller than that of the nickel cells, and consequently better regulation under varying loads is obtained from the former. The voltage of a lead cell, when fully charged, is 2·4 volts; it may, however, only be discharged down to 1·8 volts per cell, or 1·65 for some makes of lead cells. Further, discharge causes damage to the plates used in the construction of the cell. In the nickel-iron cell, on the other hand, the discharge may be carried on until the cell is entirely exhausted.

Accumulators built up into batteries find their application in power stations, where peak loads of comparatively short duration have to be dealt with, and where surplus power is available at other times. The generators, when running light, charge the battery, which if arranged "to float" on the line automatically supplies current in parallel with the generators in the event of a heavy demand being placed on the station.

In smaller installations, particularly those

[1] See "Batteries, Secondary."

employed for country house lighting, the battery often supplies the whole of the demand; the generator, being used solely to charge the accumulators, is not usually called upon to furnish power to the system.

Since the voltage of a given number of cells is higher when they are fully charged than when discharged, devices have to be introduced which will regulate the voltage supplied by a battery to the system. Such regulation may be automatic, or the devices may be controlled by hand. To effect such regulation battery switches or, alternatively, boosters may be installed.

§ (15) Battery Switch.—The battery switch consists of a number of contacts which

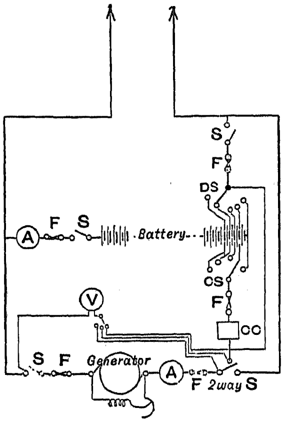


Fig. 8.

V, voltmeter; A, ammeter; F, fuse; S, switch; CC, automatic cut-out; DS, battery switch discharge side; CS, battery switch charge side.

are connected to the regulating cells usually arranged at one end of the battery. By moving a contact-making device along these terminals the number of regulating cells in series with the battery can be varied (*Fig.* 8). A small motor controlled by a voltage relay may be employed for causing the required automatic movement of the bridging piece.

§ (16) Excitation of Alternators.—The proper excitation of alternators running in parallel is a most important point in a power station.[1]

In the early days one or more direct-current generators or exciters, driven by separate prime movers, were installed for supplying the excitation current from common busbars to the various alternators in the power house. In this arrangement the regulation of the alternators had to be done on the respective alternator fields by means of regulators which, as a distinction from the regulators used on the field of the exciters, are termed "series regulators," the others being named for convenience "shunt regulators."

With the increase in the size of units these series regulators obtained large dimensions, and the power absorbed in them represented a considerable loss. It also became more difficult and costly to carry the heavy currents to the switchboard at which the regulation took place. The failure of any link in the common source of excitation spelt failure to the whole power supply. Duplication of the exciter generators did not very largely increase the factor of safety in view of the fact that the excitation switchboard and its apparatus could not be duplicated without introducing complications which in themselves might be the cause of further break-down.

Where several circuits are connected to a common supply (*Fig.* 9) it is customary and necessary to protect each individual circuit against carrying excessive overloads, which may cause trouble at the source. This arrangement, however, has its disadvantages, where the excitation of alternators is concerned, as, if the excitation is suddenly cut off without first disconnecting the alternator from the busbars, the current flowing back from the other sources of supply may destroy it. On the other hand, if the faulty excitation circuit is not cut off from the source of excitation, the generator or generators supplying current to the alternator fields may burn out, and all the alternators be left without excitation, in which case again other power stations feeding the same system would "pump" into the generating plant in question and cause a considerable amount of damage. Such consideration led to the introduction of separate exciters (*Fig.* 10) for the various alternators, these exciters being often mounted on an extension of the alternator shaft. They either supply the excitation for their particular alternator only, or else they may be so dimensioned that under emergency conditions they can supply the necessary excitation for a further alternator as well. Such an arrange-

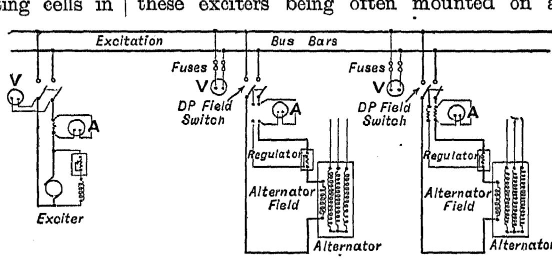


Fig. 9.

[1] See "Dynamo Electric Machinery," § (12).

ment also lends itself readily for a further protection of the alternator against faults occurring in its windings.

Should an alternator break down through

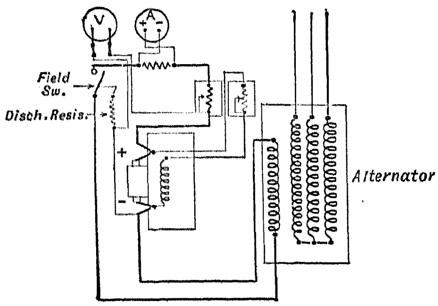


FIG. 10.

an internal fault, the automatic circuit breaker placed between it and the busbars, if equipped with suitable protective devices, will cut it off from the system, but the alternator will continue to revolve for a considerable time before it comes to rest; this is particularly the case on high-speed turbines. The excitation being "on" all the time will cause a heavy current to circulate in the alternator, which would cause a fault, the early interruption of which might have saved the machine from serious damage, to develop and be the means of the total destruction of the alternator in question.

Devices have been constructed which come into operation the moment the alternator is cut off from the busbars on the occurrence of an internal fault. Such devices automatically reverse the shunt circuit of the exciter, thereby causing it quickly to destroy its own field. As a consequence, the field of the alternator dies down more rapidly than would be the case if the usual field-breaking switch and discharge resistance were employed. A reversal of polarity of the exciter and the building up of the exciter voltage in the opposite direction cannot occur, as the shunt is now reversed with respect to the exciter terminals.

With suitably designed exciters, and where the range of required regulation on the alternator is limited, the whole of the regulation can be done on the field of the exciter, thereby dispensing with expensive and bulky series regulators and the losses they entail.

§ (17) EARTHING.—In order to fix the potential of a circuit with respect to the ground, earthing of one or other part of the system is resorted to. In the case of two-wire, direct-current systems, particularly on traction circuits, it is usual to earth the negative pole. In three-wire direct-current distribution the mid-wire is more commonly earthed. The application of this means of fixing the potential, when applied to a three-wire, direct-current supply, also ensures greater safety against shock to the consumer for a given voltage, as the shocks that can be obtained through accidental contact with one or other live conductor are produced by a pressure which is half the voltage impressed across the "outers," or positive and negative conductors.

In a three-phase system it is more common to earth the neutral point, and this may be carried out through a resistance. The object of inserting a resistance is to reduce the short-circuit current to a predetermined value, in the event of one of the lines coming in contact with earth. The resistance is then dimensioned to pass a current which would be slightly higher than the setting at which the feeder switches in the system are arranged to operate.

In order to avoid earth currents and interference with telephones, water mains, etc., it is only permissible to earth the system at one point. Opinions as to the advisability or otherwise of earthing a system differ, and are largely governed by local conditions.

In a three-phase distribution the stress on the insulation, when earthing is resorted to, is reduced, the maximum stress on the insulation, except for surges, being only the star voltage.

In single-phase or two-phase systems, where concentric cables are employed and one point is earthed, cables of a cheaper construction may be used. The concentric cables in such a case are usually constructed with full insulation for the high-tension or "inner" conductor, which is placed in the centre, and the low-tension or "outer" arranged to surround it; and since the latter is connected to earth at one point its insulation to earth need only be dimensioned to suit the voltage drop occurring in the outer conductor.

In an unearthed system the supply can be maintained, though one phase may have direct contact with earth, until it has been possible to remedy the defect. This of course is not the case in an earthed system. The Home Office Regulations for Mines prescribe earthing of the neutral point, largely to ensure that in the event of a fault occurring on the feeder it will be instantaneously cut off by the overload devices which control it.

When a system is earthed on one pole the switches must be arranged to ensure that the circuit is always broken first on the insulated, *i.e.* non-earthed, conductor. Fuses, or automatic circuit breakers, which may cause single-pole interruption on the earthed conductor, must be avoided, and the circuit-closing devices arranged to close this first. A reversal

of the operation of opening or closing would impress the high voltage on the lightly insulated conductor, and cause its break-down. It is therefore usual to avoid placing switches and automatic circuit-opening devices in the earthed conductors, and to connect these solidly to the system. For cable testing or similar purposes isolating switches may be arranged in such return circuits, but interlocked with the other apparatus in order to avoid their improper use.

In another form of earthed protection the system is normally insulated from ground, but an earthing device, consisting of two metal discs separated from each other by a thin layer of insulating material, is inserted between ground and the point which it is intended shall assume earth potential in the event of a pressure higher than normal being impressed on the system. By this means the system is normally insulated from earth, but on the occurrence of a surge, or in the event of the conductor coming into accidental electrical contact with a higher voltage supply, the interposed insulation breaks down, a direct contact with earth is established, and the strains produced by the superimposed higher voltage removed.

Where more than one generator is connected to the common busbars in a three-phase system only one neutral should be earthed at a time, in order to prevent circulating currents of higher frequency passing between the alternators.

Various devices have been constructed to ensure that only one generator neutral is connected to earth, the arrangement being such that if the particular generator is disconnected another neutral point is automatically connected to ground and the earthing of the system re-established.

§ (18) Protection against Atmospheric Disturbances and Surges caused by Switching.—In addition to protecting the generating plant and circuits against faults which may arise due to inherent properties of the generating plant or feeder circuits, it is customary on systems where power is transmitted to a considerable distance, and also on higher voltages, to protect against—

(*a*) Atmospheric influences, and

(*b*) Surges arising out of switching operations.

The atmospheric influences may be twofold.

(i.) The system may be struck directly by lightning, in which case none of the known protective devices are capable of dealing with the trouble; or

(ii.) These influences may be of a secondary nature. They may be inductive effects producing high-frequency oscillations in the conductors, or causing a single wave to progress along a conductor. These can be more or less dealt with by means of so-called lightning arresters, which usually consist of a horn gap in series with a non-inductive resistance connected between the conductor and earth. The horn gap, when bridged by a potential in excess of the line voltage, allows the higher pressure to take its path to earth over the resistance and the charge to flow away. The function of the resistance is to limit the flow of current while the discharge is taking place, the horn gap again interrupting the current flowing from the system when normal potential has been restored.

In the early days these resistances were made in the form of carbon or graphite sticks, and were later replaced by wire resistances immersed in oil or by columns of water. Another form is the "Brazil" carbon powder resistance, arranged in moulded concrete troughs.

In place of an ohmic resistance as described above, aluminium cells are often used, particularly in the United States (*Fig.* 11). These

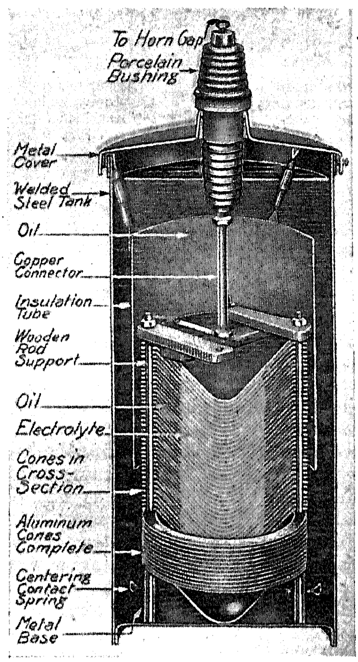
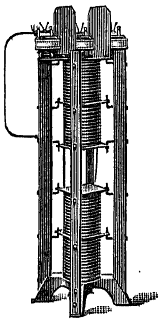


FIG. 11.—Cross-sectional view of Aluminium Lightning Arrester Cell.

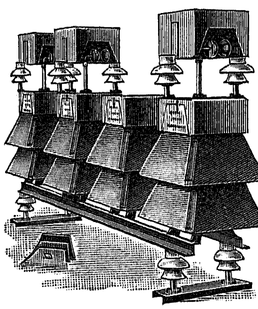
consist of cones made of aluminium and placed inside one another to form a column, the space between adjacent cones being filled with a conductive acid. The aluminium oxidizes on the surface under the potential stress, and forms a non-conductive film. The oxide film

is broken down by a pressure exceeding the normal working voltage, and allows the current to pass to earth. The current on its passage re-forms the oxide film, thereby preventing a further flow of current. The oxide film, however, deteriorates and has to be re-formed by a daily charging process, which is done by bringing the horns together and passing a current through the arrester for a short time.

A newer type of arrester, which it is claimed obviates this inconvenience, is the lead oxide arrester (*Fig.* 12), also produced in the United



Arrester for Indoor Service, Three-phase 5000/7500 Volts.



Arrester for Outdoor Service, Three-phase 15,200/25,000 Volts.

FIG. 12.—Oxide Film Lightning Arrester.

States, in which the elements are built up in a similar manner but without the interposition of an acid. This arrester does not require recharging.

Other forms of arresters, such as the Moscieki, depend on the interposition of condensers between the line and earth, the condenser acting as a valve, and allowing currents of higher frequency to pass while keeping back those of lower or normal operating frequency.

Protection of the line is also obtained by arranging a path of high resistance between the line and earth, which is always in circuit, and allows the wave to flow away. It may take the form of a jet of water. This form of discharge arrester permitting a constant flow to earth, however, has the disadvantage of causing a permanent loss of energy.

In order to prevent the wave or waves entering the station and apparatus, choking coils are usually placed at the end of the transmission line or cable. The choking coil consists of a few turns of copper, and presents an inductive resistance to high-frequency oscillations and prevents these entering the building, while allowing low-frequency waves to pass freely into the line.

§ (19) Voltage Regulation.—In a distributing network, particularly when dealing with lighting circuits, the regulation of the pressure plays a very important rôle. A slight variation in the supply pressure has a detrimental effect on the life of incandescent lamps, and at the same time causes a considerable variation in the illuminating power of these.

It will therefore be seen that the pressure in the power station must be kept as nearly constant as possible, and means must be provided for maintaining the pressure on fluctuation of load.

§ (20) Rheostats.—The regulation of the generators is effected by means of regulators or rheostats placed in the excitation circuits. These consist of metallic resistances connected to multi-way switches in such a manner that sections or steps of the resistance can be cut in or out as may be required (*Fig.* 13). The

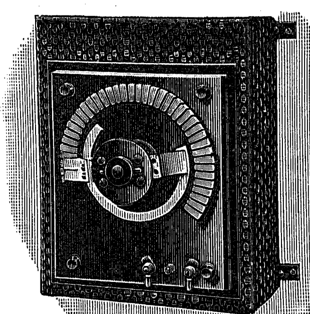


FIG. 13.—Shunt Regulator.

control of these regulators is effected by hand either direct or by means of a system of rods and chains in the event of the rheostats being placed in an inaccessible position. For larger regulators, and regulators placed at a greater distance, motor operation may be resorted to, in which case a small motor is fitted on the regulator, which, by means of a worm drive, causes the contact-making portion or brush to move along the contacts.

Such a motor may also be automatically actuated by means of a relay, which, responding to voltage fluctuations, will cause the motor to revolve in one or other direction and compensate for the rise or fall in the pressure of the system.

Such means of regulation are, however, of necessity slow, and the load fluctuations cannot be quickly dealt with in this manner, particularly where a number of machines are connected in parallel. Coupling the various regulators by means of a common spindle which can be actuated from one point has been resorted to, and often serves a useful purpose, but where very rapid and fine regulation is required, such as is now largely

demanded in power stations of larger size, more quick-acting and sensitive automatic regulators have to be resorted to.

§ (21) AUTOMATIC REGULATORS.—These regulators control the field of the exciter.

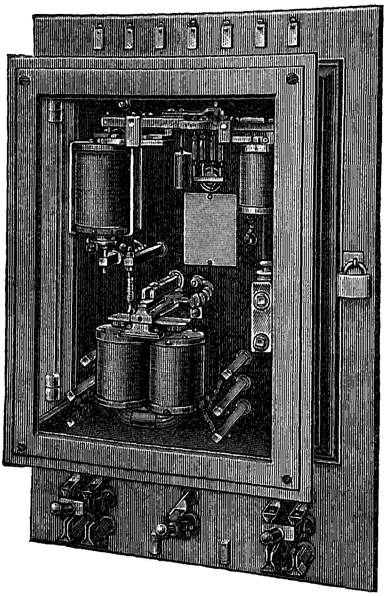


FIG. 14.—Type TA, Form AA, Tirrill Regulator.

In the "Tirrill Regulator" (*Fig.* 14) the operation is as follows:

On receiving an impulse from the line voltage, or if adjusted for constant current

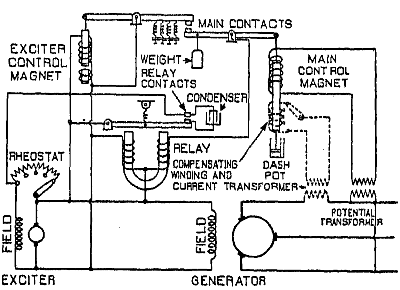


FIG. 15.—Elementary Diagram of Type TA Regulator, connected to Three-phase Alternator, showing (in dotted lines) Compensating Coil and Current Transformer.

from the current flowing through the circuit, a couple of contacts momentarily short-circuit a resistance placed in series with the field winding of the exciter (*Fig.* 15). This momentary cutting out of the resistance suddenly increases the excitation and restores the falling pressure, but would, if prolonged, raise the voltage beyond the desired value. Having restored the pressure, the contacts part and reinsert the resistance in the exciter circuit, causing the excitation voltage again to fall. This resistance is again short-circuited the instant the busbar pressure tends to fall below the desired value. Thus the play goes on, the contacts closing and opening at varying intervals to produce the required regulation of the pressure.

The curves shown in *Fig.* 16, taken of the busbar voltage controlled by regulators described above, show pressure fluctuations which only vary ½ per cent above or below normal.

In the "Brown Boverie" automatic regulator, the regulation is obtained by a segment which, operated by a solenoid, quickly short-

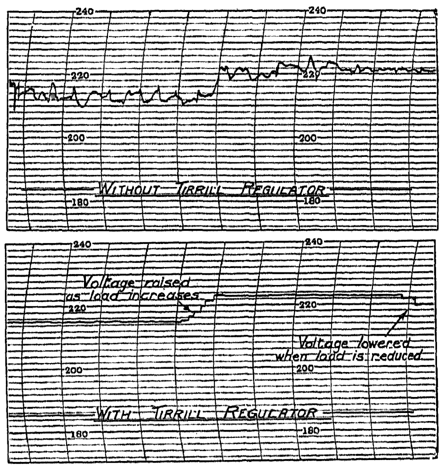


FIG. 16.—Voltage Records in connection with the Lighting and Power System of Ayr Corporation.

circuits one or more regulating steps. In its action and effect it is similar to the motor-operated regulator, but, due to its construction and the facility with which it can rapidly span a number of steps, it is quicker and more effective.

Such regulators may be applied to control groups of machines, or they may be installed to control the separate units. In smaller power stations, and particularly where the size of unit varies considerably, a regulator of this nature may be installed on the largest unit and the others left without automatic regulation, allowing them to "trail" on the former; that is, the larger unit is called upon to keep the smaller ones in step.

§ (22) BOOSTERS.—It is, however, not always sufficient to maintain constant voltage at the generating end. Load fluctuations on the outgoing transmission lines themselves some-

times have to be provided for in order to assure constant pressure at the far end.

For this purpose "boosters" are employed, which, on load fluctuations taking place in a particular transmission line or lines they are connected to, will raise or reduce the pressure corresponding to the amount of loss occurring, thus maintaining constant pressure at the supply or remote end.

Such a booster may be hand-operated or it may be automatic, that is it may be constructed automatically to follow load variations and compensate for these.

In direct-current systems it is usual to employ a motor generator set for this purpose, to energise the motor through the necessary starting apparatus from the busbars, and to connect the armature of the generating end of the set in "series" with the line that requires boosting in such a manner that the pressure generated in this armature will add to or subtract from the existing busbar voltage as may be required.

In traction circuits particularly, so-called "negative" boosters are often employed, *i.e.* boosters placed at the generating station end between the earthed point and the return rail or a particular feeder pertaining to the latter.

It will be readily seen that when such a booster gives a negative pressure, that is, produces a potential at this particular point of the return path below earth potential, not only will the potential difference at that point of the return rail be increased, but currents, which would otherwise return to the power station *via* the earth, gas or water mains, etc., will be "sucked" through the feeder in question and damage to the mains and pipes through electrolysis will be reduced.

In alternating-current systems the static type booster is used with advantage. This consists of the elements of a power transformer, the primary winding of which is fed from the main busbars and the secondary winding connected in series with the feeder or outgoing line. Suitable step switches are then arranged on either the primary or secondary side by means of which the number of turns in circuit can be varied, with a consequent variation of the additional impressed potential. By reversing the connections of either winding a negative boost can be obtained.

In another form the secondary winding is placed on the circumference of an armature, and the busbar potential is utilised for energising the "field magnets." By rotating the armature slightly in this field the number of turns exposed to the influence of the latter is varied, and with it the voltage produced in them.

§ (23) Paralleling and Synchronising.—Where the power station consists of more than one unit, these have to be connected or paralleled on to the busbars. In the case of continuous-current generators, this is a comparatively simple matter, the only requirement after ensuring correct polarity being to excite the "incoming" machine, that is, the machine which is to be connected to the busbars, to bring it up to the required voltage, *i.e.* a pressure equal to that of the busbars, and to close the main switch. The excitation of the "incoming" generator must be so arranged that the act of paralleling does not suddenly cause an alteration to the excitation of the machines, as this might result in a sudden transfer of load from one set to the other.

The following example will make this clear:

Where compound-wound machines are connected in parallel, equalising bars are run between the generator terminals which are connected to the compound winding, the object of which is to ensure a more equal distribution of load and to prevent one generator taking more than its fair share.

Suppose now that the incoming generator is brought up to the required potential by means of the regulator in the shunt winding, the compound winding not having previously been connected into circuit and paralleled, there will on paralleling be a sudden diversion of current from the compounding turns of the running generator or generators which will cause a strengthening of the field of the incoming machine, and a weakening of the excitation of the others, which will in its turn cause a sudden transfer of load from the running generators on to the incoming machine.

It follows, therefore, that when paralleling compound-wound generators the series winding must first be connected in parallel with the series windings of the running plant, whereupon the excitation of the incoming machine will be further regulated on the shunt, and the machine switched in when the voltages are equal. By this means the incoming machine when connected up will not participate in the load production. A further regulation of the shunt circuit will cause the incoming generator to take up its load gradually.

The equipment required for paralleling is:

(*a*) Two voltmeters, one connected across the incoming machine, and the other across the running machine or the busbars; or alternatively,

(*b*) A differential voltmeter connected across the terminals of the switch used for paralleling purposes. This differential voltmeter will read zero when the potential difference on

both sides of the latter is the same, *i.e.* the two machines have the same pressure.

Note.—When connecting a new generator to the busbars, it is always necessary to ensure that the polarity is correct, that is, the terminal of the incoming machines must have the same sign as the busbar it is intended to connect it to. Since the remnant magnetism ensures constant polarity, it follows that it is only necessary to verify the polarity when starting a new machine, or in the event of the machine being tripped out on a fault in the system which may have caused reversal of polarity.

Where alternating-current generators have to be connected in parallel, the operation called synchronising is rather more complicated and, after ensuring that the phase rotation of the alternators to be synchronised is the same in both cases, it consists in

(i.) Running up the alternator to the correct speed or frequency.

(ii.) Exciting the alternator to the correct voltage.

(iii.) Synchronising the periods of the running and incoming alternators.

In smaller stations where the control platform overlooks the engine-room, the running up of the alternator is done by signals from the switchboard attendant to the engine-room staff, and a synchroscope provided which indicates the proper time of closing the main switch, that is, when both machines are in synchronism. The speed regulation of the alternator in a power station where the switchboard is remote from the engine-room may, for greater convenience, be effected by a motor on the governor of the prime mover controlled by the switchgear attendant on the operating platform.

§ (24) Synchroscope.—In the early days of electrical engineering three lamps were employed, one of which was connected between phase A of the busbars and phase A of the incoming alternator, and the other two connected crosswise between B of one system and C of the other, and C of the one and B of the other (*Fig.* 17). When arranging these three

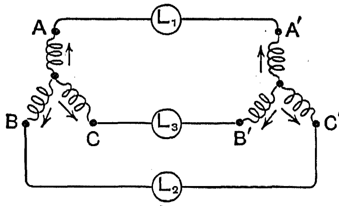


Fig. 17.

lamps in a circle they give the appearance of a wave of light revolving round the circle, which slows up as the machines approach synchronism and becomes stationary at the point where the wave-forms of the machines are in synchronism.

Later devices consist in a dial with a pointer (*Fig.* 18), the pointer usually arranged to point vertically when the machines are in synchronism, and to move to right or left depending on whether the incoming machine is leading or lagging. Sometimes a frequency indicator is added (*Fig.* 19) to the synchronising equipment for more readily ascertaining the speed of the incoming set. The addition of a voltmeter for the incoming machine and one

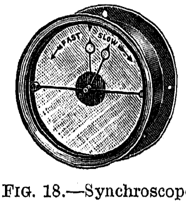


Fig. 18.—Synchroscope.

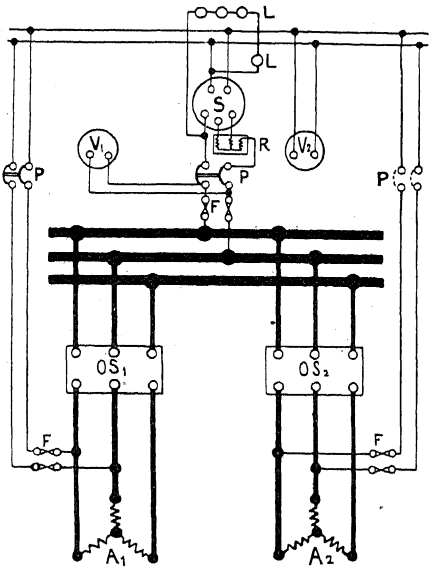


Fig. 19.—Synchroscope—Diagram of Connections.

S, synchroscope; R, resistance; L, synchronising lamps; P, synchronising plug; A_1 and A_2, generators; V_1, voltmeter for busbars or running generator; V_2, voltmeter for generator to be synchronised; OS_1 and OS_2, oil switches; F, fuses.

for the busbars, and the necessary plugs for connecting the synchroscope and voltmeter to the machine it is intended to connect the busbars to, completes the synchronising equipment.

§ (25) Metering.—For commercial as well as technical reasons, it is necessary to know what powers the individual sources of supply are transmitting to the collecting or busbar system. Similarly, the power transmitted by the different outgoing feeders also has to be known. This can be done by either metering

the power separately of each individual source and on each individual outgoing feeder, or it can be done collectively by arranging instruments[1] in the busbars between incoming and outgoing circuits. The individual arrangement, though rather more expensive, has obvious advantages, particularly where more than one set of busbars is employed, or where generators and feeders are arranged alternatively, and in addition it enables a record to be kept of each source and each feeder separately. It also ensures more accurate readings, as the instruments would, in this case, be reading at more nearly their own full load, that is, the point of their greatest accuracy, than would be the case if the meters were installed between busbar sections where the load variations are of necessity greater. A further disadvantage of the total output instruments is that when the station has at any time to be extended, these must be replaced by instruments reading larger values, thus causing a temporary shutdown of the power supply.

The apparatus used on switchboards or in connection with these for purposes of recording the power generated and supplied to or from a busbar system is divided into three principal classes:

(i.) Indicating instruments, that is, instruments such as ampere meters, voltmeters, power factor indicators, and wattmeters, which are intended to show momentary values of current, potential, power factor, and load.

(ii.) Recording, or graphic, instruments in the form of recording or chart drawing ampere meters, voltmeters, and wattmeters record the instantaneous values on a moving chart, thus providing a permanent record of the load fluctuations.

(iii.) Integrating meters or watt-hour meters; these, as their name indicates, integrate the power flowing in the circuit, and show at any moment the total supplied or consumed up to that point.

§ (26) AMPERE METERS AND VOLTMETERS.—In their simplest form ampere meters or ammeters and voltmeters consist of a pointer moving across a calibrated scale against gravity. In one type of instrument this pointer is connected to a piece of soft iron suspended in a coil and drawn into the latter by the current flowing in it, thereby indicating in the case of the ampere meter the amount of current flowing through the circuit, or, in the case of the voltmeter, the potential difference existing between the two poles. This type of instrument can be used in both alternating and continuous-current systems.

A more accurate instrument than the above is the so-called "moving coil" instrument, in which the effect of gravity is replaced by a spring, and the piece of soft iron by a coil without an iron core, energised by the circuit and, when used on a D.C. circuit, moving between the poles of a permanent magnet, thus dispensing with hysteresis losses and the consequent instrumental errors.

Since the current for operating an ammeter in this case must, of necessity, be of a small value, it is obtained by inserting in the circuit a resistance or "shunt," which is practically unaffected by temperature variations. The potential difference across this shunt being proportional to the current flowing through the latter will cause a deflection of the pointer corresponding to the magnitude of the current.

Where the current to be measured is alternating, the "field" must have the same frequency as the currents to be measured, and the permanent magnet is replaced by a laminated electromagnet energised from the current to be measured.

The transformation[2] of the current in alternating current circuits to a value more suitable for the instrument is obtained by means of the "current transformer" (*Figs.* 20 and 21), which consists of a closed iron circuit with two coils; the primary, composed of a few turns only, is connected in the circuit it is intended to measure, and the secondary—which usually has a greater number of turns (depending on the transformation ratio)—is connected to the instrument or instruments which require energising from this circuit.

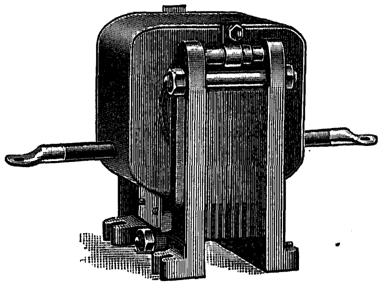


FIG. 20.—Multi-turn Current Transformer.

It is usual to choose the transformation ratio so that the secondary current corresponding to full-load is five amps.

It may here be noted that, in addition to the much smaller watt consumption, the current transformer has the very important advantage over the direct-current shunt that

[1] For a description of the various forms of meters in use see articles "Alternate Current Instruments," "Direct Current Indicating Instruments," and "Watt-hour and other Meters for Direct Current. I. Ampere Hour Meters, II. Watt-hour Meters."

[2] See "Transformers, Instrument."

the pressure of the system is not communicated to the instrument, but only the pressure of a secondary circuit which rarely exceeds

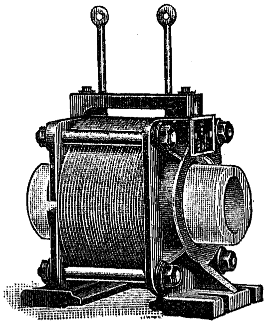


FIG. 21.—Current Transformer for Single Turn Primary.

ten volts, and then only attains this value if a number of instruments and relays are worked off the same current transformer.

For reducing the pressure which is applied direct to alternating-current voltmeters,

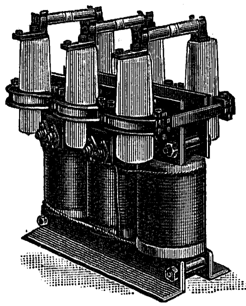


FIG. 22.—Potential Transformer, 6600 Volts Primary Three-phase; 110 Volts Secondary, with Fuses.

potential transformers (*Fig.* 22) are utilised. These are, in reality, power transformers of a small output, constructed to give a constant potential ratio, or, as nearly as possible, constant ratio between the limits of load.

§ (27) RECORDING INSTRUMENTS.—These consist essentially of a clockwork, a drum, and the elements of an ampere meter, a voltmeter or wattmeter, the pointer being provided with a pen and a container for ink. As the roll of paper controlled by the clockwork moves past this pen, it leaves a mark on the former corresponding to its position.

§ (28) WATT-HOUR METERS.—The later forms of watt-hour meter consist of a disc or armature which revolves under the influence of a potential and a current coil, or several current and potential coils in the case of a polyphase system. The rotation of the disc sets a train of wheels in motion, which, in their turn, actuate the counting mechanism, thereby registering the number of revolutions of the disc or armature, and since the revolutions of the latter are proportionate to the power flowing through the apparatus, they add up or integrate the power flowing in the circuit.

For measuring and recording the output of a continuous-current generator, an ammeter, a watt-hour meter, and, if only one generator is used, a voltmeter are usually considered sufficient. If there are several generators in parallel, it is not necessary to supply a separate voltmeter for each.

For single-phase alternating-current generators the equipment would be the same. In two-phase systems the instruments have to be duplicated. In three-phase systems, if the load is balanced, one indicating instrument only of each kind need be supplied per circuit.

For the excitation circuits, separate instruments may be installed to indicate the momentary excitation values. In alternating-current systems, power-factor meters or indicating wattmeters are often installed in the generator circuits in order to facilitate regulation and assist the distribution of load between the machines running in parallel.

The ammeters of alternating-current machines running in parallel may, at a given moment, show the same current flowing through them, but since the power factors of the two machines may be different, it follows that the output of the two alternators may be quite different in spite of the fact that the ammeters read the same value; a power-factor meter or wattmeter, however, will show which alternator requires regulation to make it take its proper share of the load.

§ (29) STRAINS AND STRESSES.—In the early days of electrical engineering the main difficulty encountered was the production of plant which would generate the necessary power for the requirements of those days. The collection and distribution of such powers, on account of the comparatively small size of the generators and the small number of machines connected in parallel, was an easy matter, but when the size of the individual unit increased and the number of units connected in parallel became larger, the stresses on the connecting system—that is, on the busbars and on the switches connected to them—also became of greater magnitude. Attention may again be directed to the fact that whereas the generator, in the event of trouble in the collecting system, has only to deal with its own overloads, this is not necessarily the case with a switch. The switch connected to an outgoing feeder may, for

instance, if it is provided with automatic release features, have to interrupt the whole power of the generator plant or plants at the back of it, together with any synchronous machinery in sub-stations which may at the time be connected to the system in question, and may feed back into the fault. Also, the fault current which has to be interrupted might, in the event of a dead short-circuit, be the maximum short-circuit value of the generating plant.

The initial value of the short-circuit current of a generator may be anything up to twenty or thirty times the normal current, depending upon the characteristic of the machines; as the force between conductors carrying this current increases with the square of the current, it will be seen that immense stresses may in the case of a short circuit be placed upon the switchgear, busbars, connections, etc., when the plant reaches considerable magnitude.

§ (30) STRESSES IN CONDUCTORS.—Dealing with the question of stresses set up between the different conductors, or conductors and structural work, these, as already intimated, may be very considerable.

Let us take, for example, a two-wire power station of 100,000 kw., 10,000 volts pressure. We should then have a normal current of 10,000 amperes on the busbars. In the event of a "dead short" occurring on a feeder, the generators would supply the whole power into the fault, and, since the momentary value may be twenty times the normal current, we should—on the assumption that there was no resistance in the circuit, *i.e.* that the fault occurred close up to the power station—have about 200,000 amperes flowing into the fault *via* the busbars, connection, etc.

The mutual repulsive force between two parallel conductors carrying current in opposite directions is given by the expression $2i^2L/a$ dynes, the current being in c.g.s. units. Thus we have

$$F=\frac{I^2L}{a}\times\frac{2}{g\times10^5},$$

in which F is the force exerted by the conductors on each other in kgm., I the current in amperes, a the distance in cm. between the conductors, L the length in cm. of the conductor under consideration, and g the acceleration due to gravity in cm./sec.².

Assuming now the conductors to be 1 metre, or 100 cm., in length, and the distance between the conductors to be 20 cm., we obtain for a possible maximum current of 200,000 amperes

$$F=\frac{200{,}000^2\times100\times2}{20\times981\times10^5}$$

$$=4080 \text{ kg. per metre of conductor.}$$

Take another example, viz. a generating station of 100,000 kw. but 20,000 volts single-phase distribution. As the voltage is greater the distance between the conductors will also be greater, which we will assume to be 40 cm. We now have 5000 amps. on the busbars at full load, which, on the assumption that the momentary maximum value is again twenty times as great, *i.e.* 100,000 amps., give a pull

$$F=\frac{100{,}000^2\times100\times2}{40\times981\times10^5}$$

$$=510 \text{ kg. per metre of conductor.}$$

When applying the above formula to a three-phase system in which the busbars are all in one plane (*Fig.* 23), we have to consider the following two alternatives:

(*A*) *A short circuit takes place between two phases.* The current in the two limbs is displaced 120° and we consequently have two components which act on each other, having a value of J × cos 30° = ·866J.

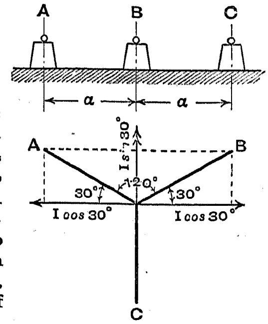


FIG. 23.

Since the two vectors tend to stretch to 180° on a heavy short circuit, the components we have considered will approach the vector values, and only a small error will be introduced if the calculation is carried out on the basis of the vector values.

Applying this to a 100,000-kw. three-phase power station at 20,000 volts, we obtain for the normal full-load current

$$I=\frac{100{,}000{,}000}{\sqrt{3}\times20{,}000}=2900 \text{ amperes (approx.)},$$

which, multiplied by 20, as in the previous case, gives 58,000 amperes for the instantaneous maximum value to be entered into the formula, and we obtain for a busbar spacing of 40 cm.

$$F=\frac{58{,}000^2\times100\times2}{40\times981\times10^5}=171 \text{ kg. per metre.}$$

(*B*) *A short circuit takes place between the three phases.* In this case, and in view of the fact that the vectors do not alter their relative positions, we have between two connections a fault corresponding to the components J cos 30°. At right angles to this we have two components J sin 30°, which are at 180° to the third vector, that is, the repulsive effect between conductors will be composed of the repulsive effect between two adjacent conductors A and B,

$$\frac{2L(I\cos30)^2}{981a\times10^5},$$

and the repulsive effect between the third conductor

C and the sum of the components of A and B which are at right angles to the components causing the strain between A and B. The distance being $1 \cdot 5a$, we obtain

$$F = \frac{2L(I \cos 30)^2}{981a \times 10^5} + \frac{2L(2I^2 \sin 30)}{1 \cdot 5 \times 981a \times 10^5}$$

$$= \frac{2LI^2(1 \cdot 5 \cos^2 30 + 2 \sin 30)}{1 \cdot 5 \times 981a \times 10^5}$$

$$= \frac{4 \cdot 25 LI^2}{1 \cdot 5 \times 981a \times 10^5}.$$

Thus for $L = 100$ cm.,
$a = 40$ cm.,
$I = 58{,}000$ amperes,
we obtain

$$F = \frac{4 \cdot 25 \times 100 \times 58{,}000^2}{1 \cdot 5 \times 40 \times 981 \times 10^5} = 243 \text{ kg. per metre.}$$

The stresses set up between current-carrying portions and the structural work may be ascertained in a similar manner after due allowance has been made for the nature of the material used in the structure.

A comparison of the results calculated in the first two examples shows a very interesting feature. They prove that such stresses can obtain very considerable values, and that the spacing and, in a similar manner, the height of the insulators have to be determined not only from the point of view of the voltage employed in the system, but also with reference to the kw. collected on the busbars and the maximum short-circuit values which may be obtained. In these cases the repulsion between the conductors is proportional to the square of the current, and inversely proportional to the distance between the conductors, and since the distance between the conductors is again approximately proportional to the voltage, we find that for a given supply of power the stresses are, roughly speaking, proportional to the square of the kilowatts and inversely proportional to the cube of the voltage.

§ (31) Stresses in Isolating Switches.—The isolating switches likewise will be exposed to considerable strains on the current rush, and they will tend to open if not locked in position.

It is a well-known fact that a conductor which forms an angle tends to straighten out on a current rush, *i.e.* it tends to take up such a position that it embraces the maximum number of lines.

The isolating switch blade with its contacts forms a right-angle turn in the circuit, and this turn tends to stretch out to 180° on a heavy rush of current. If the switch is back-connected this tendency is greater.

In a particular case which came to the writer's knowledge, approximately 40,000 kw. was connected to the busbars, the voltage being 10,000 volts, three-phase, and on a heavy short-circuit occurring, the isolating switches opened out with consequent damage to themselves and the surrounding structure.

Various forms of locking devices have been introduced for effectively holding such switches in the closed position against the action of such currents.

§ (32) Stresses in Current Transformers.—Current transformers which are used for energising the meters, instruments, and relays must be capable of giving a certain output. This output being for a given iron section proportional to the number of primary ampere turns, it follows that the number of primary turns for a given output is inversely proportional to the current for which the transformer is designed; in other words, for a smaller current there must be more turns than for a larger current if the same output is to be obtained. The number of primary turns is also governed by the voltage of the system, since for higher pressures the insulation becomes more bulky, and with it the length of the iron circuit and the losses in the latter are increased, necessitating additional primary ampere turns.

The rush of current occurring on a short circuit may pass through such a current transformer and, irrespectively of whether the transformer was designed for a normal current of 5 amps. or 1000 amps., it will be called upon to withstand the stresses set up by a current which may be many thousand times greater than the current it was designed for. Since the turns in the current transformer are in parallel and the currents flow in the same direction, and since such turns must of necessity be closely packed, it follows that the crushing strain on the insulation will obtain large values. The individual coils will also tend to open out and take up a straight position. Both these strains are set up in addition to the momentary heating due to overload on a conductor designed only to carry a very small fraction of the maximum possible current; this in itself will weaken the insulation and facilitate the breakdown through the mechanical strains referred to above. From this it will be seen how severely current transformers may be stressed when connected to a large power station, and how important it is to choose these very carefully, and to arrange them in such a manner in the switching scheme that, on failure, they will be immediately disconnected from the busbars by a power switch.

The safest and mechanically strongest form of current transformer, and consequently the most suitable where large current rushes have to be expected, is the one in which the primary winding consists of a single straight conductor, *i.e.* the so-called single turn or "straight-

through" current transformer. Unfortunately, however, the output of such a transformer for currents below 300 amps. is not always sufficient to energise without loss of accuracy the whole range of measuring and integrating instruments and protective relays often called for on individual circuits.

The difficulty of current measurement would at the moment appear to be the limiting feature, from the switchgear point of view, in fixing the maximum voltage permissible on the collecting system of a power station, if metering is to be provided for.

§ (33) Stresses in Potential Transformers.—The potential transformers in the power station are another feature which may, in view of the great number of fine turns confined in a small space, cause serious trouble through a breakdown. The transformer and its connections are in the same position with regard to the short-circuit values as, for instance, an outgoing feeder, since a dead short-circuit in the transformer may also attain values equal to the total output of the station.

In smaller switchgear installations it is usual, in order to facilitate synchronising, to connect the potential transformer which energises the synchroscope on behalf of the running plant direct to the busbars, and to rely on fuses to cut off the transformer in case of a breakdown of the latter. Where, however, large powers are available and fuses cannot be relied on, it is essential, where security is aimed at, to connect the transformers in such a manner that they can be disconnected from the busbars by an automatic switch, capable of dealing with the maximum rush that may occur.

The installation of a separate switch for the sole purpose of protecting the system from a fault which may occur in a potential transformer, would obviously, apart from the expense involved, not be fully satisfactory, in view of the fact that the normal current and overload currents which may occur on such a transformer will be too small to give sufficient impulse for tripping a large switch; consequently, should a small fault occur, the currents would not disconnect the transformer.

It is therefore becoming the practice to avoid connecting potential transformers direct to the busbars, and to arrange these on the "off" side of an oil switch, *i.e.* on the side remote from the busbars, in such a manner that should a serious fault develop, the oil switch protecting the generator or feeder will disconnect it from the busbars. The potential transformer has nevertheless to be further protected against small overloads as well as against placing an excessive load on the generator or feeder, and this, as in the case of smaller stations, is usually accomplished by means of a fuse. As, however, such a fuse, in the event of the potential transformer being connected to a generator, will have to interrupt the short-circuit current of that particular generator, the stress on the fuse may be considerable, and rise beyond the capabilities of the fuse; thus further protection must be provided.

The load on the potential transformer may be considered constant, *i.e.* a fixed number of instruments is permanently connected in the circuit; it is therefore feasible to insert a resistance in series with the primary of such a transformer which will limit the current passing into the same to a predetermined value. A resistance which would absorb, for instance, 5 per cent of the impressed voltage would reduce the current flowing on a dead short-circuit to 20 times the normal current. Such a resistance would at the same time represent an inconsiderable loss when compared with the total output of the power station. The small extra load put on the transformer when synchronising machines would only temporarily affect the accuracy of the measuring instruments, and can therefore be ignored.

In view of the fact that the potential transformer for higher voltages usually contains oil, it is imperative that the transformer be so placed in the system that fumes and smoke in the event of a fire cannot reach the conductors, insulators, and in particular the busbars.

§ (34) Stresses in Oil Switches.—The switches used for interrupting the supply of power in power stations utilising alternating current are, with few exceptions, of the oil-break pattern, the exceptions being switches employed in connection with small supplies at low distributing pressure.

These oil switches, if provided with automatic features, have to interrupt the available power under emergency conditions; it therefore follows that for a given voltage the oil switches will vary in size and larger switches will have to be employed where the power behind is considerable. The normal current-carrying capacity of the switches may be the same in both instances, but the work demanded of them under emergency conditions may be very different and will depend on the kilowatts connected "behind" the switch in question.

In addition to the stresses set up between adjacent conductors, we have the stresses set up by the gases produced by the action of the arc on the oil. These gases, when mixed with air, are of a highly explosive nature and can produce considerable pressures when fired.

Tests which were carried out on an oil switch with an open-ended, straight-vent pipe

of 6 in. diameter and 5 feet long showed a pressure rise of 80 lbs. to the square inch in the oil tank.

If the oil switch does not interrupt the current satisfactorily and quickly, so that a large amount of such gases is produced, these, when fired, may cause very serious results to both the switch proper and adjacent apparatus if no provision is made for their protection. Moreover, a bad contact may, through minute sparking, disintegrate the oil and produce gases which, if proper precautions have not been taken, would accumulate in the oil switch and even in the switch chamber; these, if accidentally ignited by, for instance, a relay operating, may cause a most serious explosion.

Where the power on the busbars is considerable, the amount of oil required for interrupting the arc under emergency conditions is likewise considerable. It therefore follows that, similarly to the potential transformer, the oil switch must be so placed that, should it catch fire or explode, even if this is a remote possibility, the burning oil cannot cause damage to other apparatus, or gases or smoke from it reach the conductors, particularly the busbars.

This being the case it is imperative to arrange the oil switches for large power stations in separate self-contained chambers which have no communication with the main building, *i.e.* chambers which are only accessible from the outside of the building, avoiding all passageways where smoke can enter and accumulate. It is further necessary, in view of the large amount of oil which may be freed by an explosion and catch fire in the chamber, to arrange drain pipes which will carry away the oil to a sump as quickly as possible, thereby reducing the fire and the consequent damage to a minimum. Furthermore, the leads connecting the oil switch to the busbars must be so arranged that they are not exposed to the above-mentioned fumes. This can readily be obtained by constructing the oil switch in such a manner that its top plates (usually termed the base, because of the fact that the structure is suspended from this) forms the roof of the oil-switch chamber. This base being made of non-inflammable material and of sufficient strength will confine all smoke to the oil-switch chamber proper, and prevent gases reaching the conductors.

It also follows that the component parts of the switch must be substantially constructed, and that ample and straight-vent pipes be arranged which will help to relieve the pressure, and will also carry away gases which may be produced under running conditions by, for example, a bad contact, *i.e.* contact which, due to bad alignment, may be sparking slightly.

§ (35) Means for reducing the Stresses.—The generating plant has in the last few years been developed very rapidly, and units of 30,000 and 40,000 kw. capacity are no longer unusual. Also running plant exceeding 300,000 kw. in power is in some instances collected in one power station. The development of switchgear, however, has not kept pace with the extraordinary progress in generators.

The problem before the switchgear engineer now is either to introduce means which will reduce these stresses on the switchgear in the event of serious trouble occurring on the line or connections, or, alternatively, to make both the connections and apparatus capable of withstanding the shocks and to produce switches which will, independently of such means, deal with the short-circuit values which may obtain under any conditions.

The various means now in use for reducing such stresses on the switchgear are:

(1) Subdivision of the main station into sections.

(2) Introduction of current-limiting devices.

(3) Introduction of some form of protective device which will cut out a faulty circuit when the fault begins to develop, and before it has attained a dangerous magnitude.

§ (36) Subdivision of the Main Station into Sections.—A simple form of busbar is shown in *Fig.* 24. The method first adopted

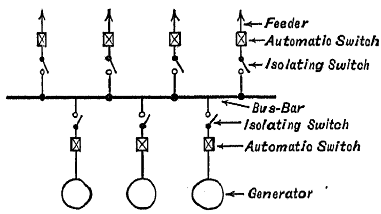


Fig. 24.—Simplest Form of Busbars.

for dividing a main station into sections was by means of duplicate busbars (*Fig.* 25). This, of course, is only equivalent to halving the power station. Besides reducing the stress upon the switchgear, it has the further

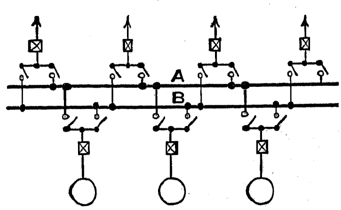


Fig. 25.—Power Station with Duplicate Busbars.

advantage that the two sections may be run at different pressures, thus providing means

of boosting certain feeders and ensuring a certain limited amount of regulation at the feeder ends.

As the total power to be collected increased, however, the duplication of busbars soon proved itself inadequate, and it was necessary to resort to further sectionalising of the station. This sectionalising was obtained by dividing the busbars and connecting one or more generating units and a certain number of feeders to each section (*Fig.* 26). This

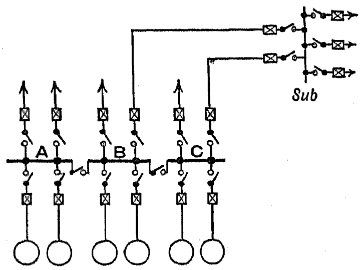


FIG. 26.—Power Station with Sectionalised Busbars.

arrangement, in a sense, divides up the station into a number of subsidiary stations, but obviously also introduces a number of disadvantages. It is not always possible to run the various generators at their best efficiency: generators on certain sections will be underloaded, and others will be overloaded. Nor is it possible, without introducing further complications, to transfer a generator from one section of the bus system to another section.

Furthermore, in a complicated network system, it becomes increasingly difficult to ensure that sections, which are divided from each other in the power station, are not inadvertently connected together through the network, particularly if they are out of synchronism with each other. This is a notable danger in sub-stations, as in order to ensure an uninterrupted supply to these it is usual to run two or more feeders, and in order to obtain the full advantage of this duplication of feeders it is again common practice to connect them to different sections in the busbar system.

It will also be obvious that should a heavy load or short circuit occur on a feeder of one section, another section could feed considerable power through such a station into the faulty feeder, if the resistance offered by the two feeders in series is not sufficient to prevent this.

Further, sectionalising is obtained by means of so-called "jumper" bars (*Fig.* 27). This method merely consists in arranging the switchgear in such a manner that a particular generator can be connected either directly to the common busbar or bars or, alternatively, straight through to a particular feeder. Provi-

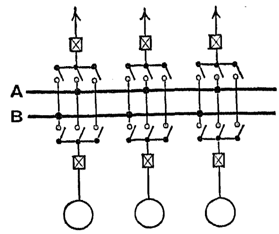


FIG. 27.—Power Station with "Jumper" Connections.

sion is also generally made to allow this feeder to be connected, when necessary, directly to the main busbars.

The difficulty of transferring the generators from one section of the busbars to another can be overcome by the introduction of still further complications, namely, transfer bars. As will be seen from *Fig.* 28, this method consists purely in the introduction of one or more, normally, idle continuous busbars. Any one of these bars can, at any moment, be connected to one or more of the generators

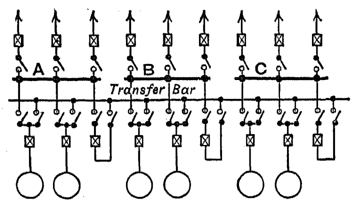


FIG. 28.—Sectionalised Power Station with Transfer Bar.

and at the same time to a particular section of the station which may need assistance at a critical moment. But it will at once be obvious that even this is very ineffective, unless we have a large number of transfer bars running the whole length of the switchboard, as each bar can only be employed to render assistance to one section of the station at a time.

In order to get greater flexibility, in any of the busbar methods of connection hitherto described, such busbars are often carried out on the ring main principle, that is, the two ends are joined together through another set of bars. It is usual in such arrangements to place isolating links in different parts of these busbars in order to facilitate cleaning as well as sectionalising the bars or cutting out faulty sections. In place of isolating links, circuit breakers are often employed, to enable the interruption to be made while transfer of load is taking place between section. It is then also usual to arrange feeders and generators

in such a way that each portion can be considered a self-contained unit or section of the station, the generator and feeder loads on a particular section being balanced as nearly as possible.

It was previously intimated that sectionalising of the busbars had considerable disadvantages from the point of view of efficient running of the power plant, and means were sought for interconnecting sections, but in such a manner that the various sections would not feed any considerable amount of load into a fault occurring on a feeder connected to another section.

It should here be noted that where large powers are concerned, and where the distributing system is of a certain magnitude, higher pressures have of necessity to be employed, as otherwise the connections and feeders would involve an excessive outlay in copper, and because the difficulties of interrupting large powers increase more rapidly with an increase of current than with an increase of pressure. This of necessity leads, in view of the present state of the art, to the employment of alternating-current plant. Consequently, in discussing lay-outs and arrangements of large power stations, we need only consider alternating-current systems, and when dealing with alternating current we have a ready means, in the choking coil or reactance, for preventing large amounts of power from flowing from one section into another. Such a reactance usually takes the form of a coil wound on an insulating base with or without an iron core, and is connected between two sections of busbars.

On reference to *Fig.* 29 it will be seen that, should a heavy overload occur on a particular

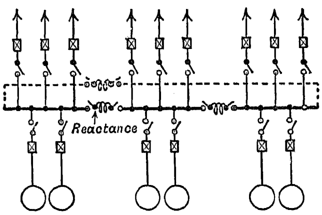


FIG. 29.—Power Station with Choking Coils in Busbars.

section, the generators connected to that portion of the busbars will feed into the fault, but the adjacent sections will only feed power into it to an extent depending on the reactance interposed between the sections. A choking coil of this nature is, however, a device that introduces a certain element of danger. It is a bulky piece of apparatus, and has to be suitably insulated for the pressure on which it is used, and since this pressure is usually high, a large amount of insulating material must be used. At the same time the mechanical stresses in such a reactance are very considerable; on a heavy current passing through it the turns will tend to straighten out and superimposed coils will try to come together.

There are, of course, quite a number of other ways of sectionalising power stations, but on analysing the various diagrams used, it will be found that they all reduce themselves in principle to one or other of the schematic forms instanced above.

Though these arrangements may, in certain cases, meet the full requirements of the power station as regards distribution and sectionalisation of the load, it must, nevertheless, be borne in mind that if the arrangement is complicated the chances of making mistakes in switching, and the difficulties in following out connections and dealing with a plant under emergency conditions, are increased. That is, unless the operator has a very clear diagram in front of him and can rapidly visualise the alternative paths produced by such a switching scheme, he is not in a position quickly and safely to alter his method of distribution or cut out faulty sections without interfering with the healthy ones.

Furthermore, all connections and busbars, circuit breakers, isolating switches, etc., are a potential danger. They must be supported by insulators which are exposed to considerable stresses under emergency conditions. In complicated arrangements it is also a very difficult matter to avoid the crossing of conductors and the consequent greater possibilities of short circuit between these. It cannot be sufficiently emphasised that these connections must be as simple and as straightforward as possible. The operator must, at one glance, be able to see the run of his connections. The number of insulators, the number of apparatus, and the number and lengths of such connections must be reduced to a minimum, and the operations to be carried out must be as simple as possible. Any complications in the method of connection, any hesitation in carrying out a switching operation, may have disastrous results.

§ (37) INTRODUCTION OF CURRENT-LIMITING DEVICES.—It has been proposed, and in a large number of cases the proposal has been carried into practice, to introduce additional choking coils into the generator and feeder circuits, which would (1) limit the power flowing from the generators to the busbars, or (2) limit the power flowing into a faulty feeder, or (3) a combination of both. Such limiting reactances, particularly in the feeders, can be made effectively to reduce the fault current. On the other hand, they introduce considerable permanent losses, which are

very noticeable on overhead transmission lines. In addition to this, they bring into use, as already stated above, a piece of apparatus which is in itself fragile, costly, and bulky.

When the generator voltage has to be raised to a higher value by means of a transformer, this is often used in place of the generator reactance, and is connected between the busbar switch and the generator, thus obtaining the same result as with the generator reactance, but without increasing the losses in the system. It is usual in such cases to treat the generator and transformer as one unit and not to interpose automatic switches between the two. This method has further advantages. It permits of the generator being wound for the most economical voltage irrespective of the busbar potential. It ensures that, since the generators are usually run at full load, the transformer likewise runs at full load, thus avoiding large transformer units being run only partially loaded, that is inefficiently. On the other hand, the flexibility is somewhat reduced in that it is not readily possible to connect a generator to another transformer in the event of one or the other having broken down. Break-downs on transformers, however, are now no longer very common, and transformers can as a rule be considered mechanically safe in comparison with generators. If, on the other hand, a generator is out of commission, and the total output of generators and transformers is equal, the services of the undamaged transformer will not be required.

In the foregoing we have dealt with means which will reduce the amount of the fault current should a fault occur, and we will now describe a method which aims at preventing the development of a fault before it reaches dangerous dimensions.

§ (38) Protective Devices which will cut out a Circuit when the Fault begins to develop.—Balanced protective devices (§ (8) (iv.)) for cutting out faulty circuits before a fault has had time to develop are applicable both to overhead and underground transmission lines. It is obvious from the nature of the devices, however, that their main application is on underground feeders. The cable invariably breaks down more or less gradually and a fault develops more slowly than would be the case on an overhead line. But with the overhead line the proposition is rather different, as in this case we are usually dealing with the breaking of a conductor and possibly a sudden and intense short circuit. Some of these devices have proved themselves eminently successful and have enabled stations to be run with considerable safety although the switches employed would under other conditions not have been considered safe for the magnitude of the installation in question.

It should, however, be noted that even such devices may under certain conditions be the cause of producing worse stresses on the switchgear than would be the case if they were not employed. For instance, if the fault which is gradually developing should suddenly show a considerable increase and develop into a "dead-short" at the moment when the switch, which has previously received its impulse, begins to open the circuit the total power may be interrupted at the peak of the short-circuit curve, thus placing immense strains on the automatic circuit breaker in question.

When a generator is short circuited, the characteristic curve shows a peak on the first half cycle, which usually dies down to the normal short-circuit value of the generator within the first 10 or 15 cycles or sometimes less. The switch receiving its impulse on the occurrence of the overload cycle requires time for its operation, and usually opens when the short circuit is decreasing in magnitude, and consequently interrupts the fault under more favourable conditions. Thus the stress on the automatic switch may be greater in the instance previously described than would be the case if the switch was provided with overload protection only. It should be noted that the possibility described above is very remote, but nevertheless, in a power station where a very large factor of safety is demanded, and where the switchgear arrangements are such that a break-down in a particular automatic switch may cause damage to the machinery in other parts of the system, such possibilities, even if remote, must be taken seriously into consideration, and arrangements made that even these remote possibilities are effectively dealt with.

Overload relays with definite time-limiting settings, that is, time-limit devices which cause the automatic switch to open when a certain overload is reached, but only after a predetermined time has expired, and irrespective of the magnitude of the current flowing into the fault, are occasionally employed to safeguard the circuit breaker from functioning at or near a peak value on the short circuit. Such an arrangement, though it reduces the load on the automatic switches under emergency conditions, can nevertheless not be considered a satisfactory solution from the point of view of the system as a whole, as it unduly prolongs the overload strain on the system.

§ (39) General Arrangement of Switch-house.—From the foregoing it will be seen that the general arrangement of the apparatus in the power house is a very important factor, and quite as important as the choice of the

individual pieces of apparatus which go towards making up a switchboard. With the increase in the size of power stations it has become increasingly difficult to obtain a satisfactory arrangement of the switching apparatus and connections in the engine-room, and, further, in view of the fact that the switching portion requires only a light structure, and also because it is always desirable to place the operator controlling the switchgear in such a position that his attention is not distracted by accidents to the machinery, it becomes more and more the practice to erect a separate switch-house, sometimes at a distance of several hundred yards from the engine-room. Indications of trouble on the running plant are transmitted to the operator through instruments, and his commands to the engine-room are conveyed by means of telephone, or preferably telegraph, apparatus, such as are used on board ship for transmitting orders from the bridge to the engine-room.

In the most modern power station the control-room is again set apart from the switch-room proper, and effectively divided off from the latter. This is particularly necessary where the oil switches cannot be absolutely relied upon to carry out their function. An explosion or the burning of an oil switch rapidly fills the building with smoke, and prevents the attendant from taking proper measures to confine the fire.

An accident of this nature in a power-house in the Rhineland, a year or two before the war, developed so rapidly that within two minutes access could not be obtained to the switch-room, which was placed on the fourth floor of the building and communicated through shafts and staircases with the chambers containing the switching apparatus. The result was that the attendants left their positions in a hurry and the running machinery had to look after itself. It was half an hour before they could enter the power-house and attend to their duties of shutting down the machinery.

In another case, in a certain station in South Africa constructed on the continental basis with oil switches and connections arranged in a common chamber, an oil switch controlling a feeder failed to open the circuit satisfactorily and blew to pieces, the oil catching fire. This was extinguished and the plant got under way again after a comparatively short interval of time. After a few hours, however, the smoke of the oil or soot, which was in suspension in the air, settled on the busbars and connections carried above these oil switches, causing the insulators to lose their insulating properties, with the result that the connections flashed over to earth. In the end the station in question had to be shut down and the staff employed for two whole days in cleaning insulators and connections before the station could be got under way again.

It may here be mentioned that these oil switches up to a certain point had proved quite satisfactory and capable of dealing with the emergency conditions arising in the station, but began to fail when the plant was extended beyond a certain limit.

Numerous other examples of a similar nature could be cited, but these two will suffice to illustrate the importance of so arranging busbars and connections that gases and fumes cannot reach these.

We will now see how the apparatus and connections have to be arranged in order to avoid disasters of this nature. The generators in a power-house, where the switching is done at, say, 20,000 or 30,000 volts, would be connected by cables to the switch-house. Since the cables are directly connected to the oil switch, it follows that the oil switch is most conveniently placed on the ground floor or suspended from the ceiling of the same. The connections from the oil switch running through isolating switches have to be connected to the busbars. We therefore obtain an isolating switch chamber on the first floor, and busbars either under the ceiling or preferably on a floor by themselves above this chamber. If, in order to obtain a short building, the switches are arranged in two rows, we obtain passageways between these, the lower one being conveniently utilised for the cable dividing boxes, and the so-called small wiring which connects the apparatus contained in the switch cells to the control switchboard. These "small wiring" connections, being preferably carried out as lead-covered armoured multi-core cables with a separate cable for each oil-switch cell, can be supported from the ceiling of the passageway in question. The passageway on the first floor can be utilised for the solenoids, motors, or other devices which are employed for closing the oil switches, thus effectively dividing off the control gear from the high-tension chambers and facilitating inspection and repair of the latter.

The control-room would be placed at the end of the building on the first floor, the chamber below the same on the ground floor being utilised for bringing the small wiring up to the control panels, for accommodating the battery required for operating the oil switches (in a separate ventilated compartment), as well as the motor-generator set for charging the latter.

On analysing this arrangement we find that the apparatus which may cause fire and smoke is arranged in chambers which are

only accessible from the outside. Apparatus and connections, which in themselves cannot cause damage or fumes of any magnitude, are arranged in chambers, as in *Fig.* 31, which may communicate with passageways.

The busbars again are divided off from the oil switches through two floors and arranged in chambers to which access cannot normally be obtained, the insulators being of sufficient height, and the mesh in the doors of these chambers of sufficient size to prevent vermin causing accidental short circuits.

The illustration shown in *Fig.* 30 refers to a power station which was designed to deal with 100,000 kw. on the busbars at 20,000 volts, three-phase, and to deal with any emergency which might arise from the collection of this large amount of power and without relying on the balanced protective devices, reactances, etc., or on the fact that because of the two

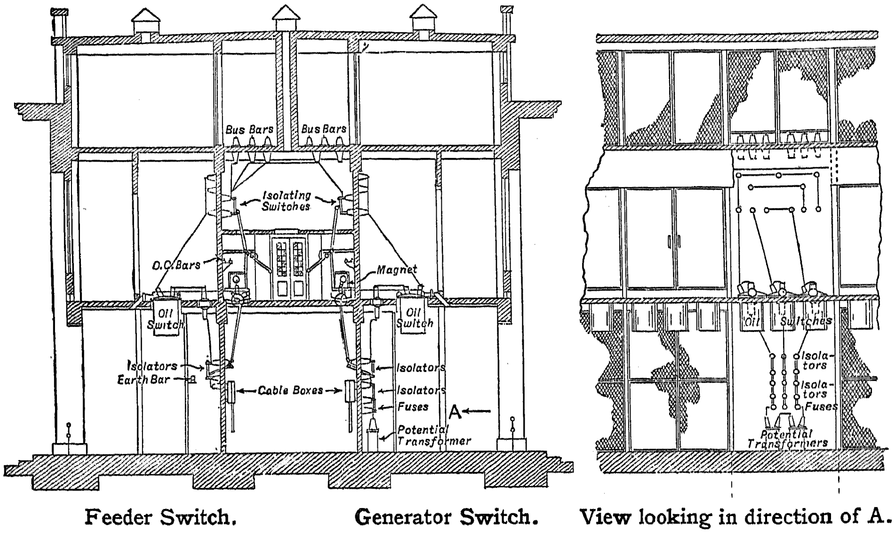


FIG. 30.—Buenos Ayres Western Railway Switch-house.

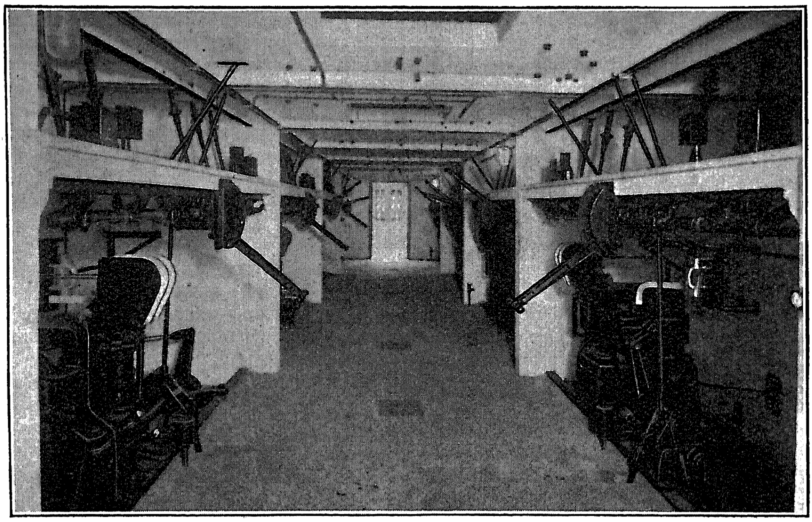


FIG. 31.—Buenos Ayres Western Railway. Oil Switch and Isolating Switch. Operating Mechanisms.

sets of busbars possibly only one half the power of the station may be concentrated on the fault.

The switches proper, it is claimed, are capable of dealing with power concentration in excess of the above values, and are suitable for interrupting 1,000,000 k.v.a.

It will also be noted that the spacing of conductors and the height of the insulators are greater than a consideration of the power only would require, but that these dimensions were chosen with a view to keeping the strains set up under emergency conditions within safe limits.

§ (40) DISTRIBUTION OF POWER.—The generated power which is collected on the busbar system in the power-house is, except for a few local feeders in the power station itself, distributed in bulk to other distributing centres.

The transmission of power to these subsidiary distributing centres or sub-stations may be carried out by means of overhead transmission lines or underground cables, the choice depending upon voltage, distance of transmission, nature of the country the lines or feeders traverse, and other local considerations.

At the present moment power transmission by cable is not usually resorted to for voltages above 33,000 volts, though cables for 66,000 volts have in some instances been employed with success. For overhead transmission lines pressures of 110,000 volts are now in common use in America, in the Colonies, and in some parts of the Continent, and little trouble is experienced with these when constructed on up-to-date lines.

In some parts of North America higher voltages are being employed, and 200,000-volt transmission lines are now in course of construction.

The question as to whether lines carrying current at extra high pressure should or should not be protected by lightning arresters forms the subject of much controversy.

The contention of those advocating the abolition of the lightning arrester for very high voltages is largely based on the assumption that the over-pressure caused by lightning or switching operations has a definite limited value, and that this value, when added to the high voltage already existing on the lines, comes within the factor of safety allowed for the material used in their construction, and consequently "spills" from this cause need not be feared. A badly constructed lightning arrester, or one not suitably attuned to the line may, in itself, be the cause of such "spills."

When lightning protection is employed, this is usually installed at the entrance of the incoming and outgoing lines of the sub-station.

Extensive observations carried out by the Edison Commonwealth Company have shown that on a low-tension network immunity against break-down through surges is increased by multiplying the number of lightning arresters installed, particularly in cases where a great number of transformers are placed in a small area. The contention is that the lightning arrester takes time to operate, also that it can only carry off a certain amount of energy at a time.

Transformers and motors, etc., can for a short time withstand considerable potential stresses, but if these are unduly prolonged they break down. If now the lightning arresters are not provided in sufficient numbers these stresses will be prolonged beyond the capacity of the apparatus in question to withstand them, and cause a break-down in the latter.

The power received in bulk in a sub-station is reduced to a suitable pressure by means of power transformers. Such sub-stations may in very large power schemes again form centres from which the power is transmitted in bulk, but at a lowered potential to other sub-stations, which in their turn again reduce the voltage for further transmission in different directions.

The power may, for instance, be generated in the power-house at 6600 volts three-phase. By means of transformers the voltage may be raised to 110,000 volts for transmission to a suitable supply centre in the neighbourhood of a town or an industrial centre requiring large amounts of energy. At the terminus of the line or lines transformers would be installed for reducing the pressure and distributing the current at, say, 11,000 volts to the outlying villages, where further sub-stations may be provided for reducing the pressure to perhaps 550 volts for local distribution. As this pressure, however, is still too high for direct application to the ordinary house service, transformer kiosks may be arranged in the streets to reduce the pressure to 220 or 110 volts. Short lengths of feeders would then carry the power from these kiosks to consumer's premises in the immediate vicinity.

High-pressure feeders may also feed large consumers, who, obtaining the power in bulk at a high pressure, reduce the potential to a value suitable for direct application to the apparatus, machines, lamps, etc., it is intended to feed; or, in other words, the sub-station may be the consumer's property and controlled by him, as distinct from sub-stations controlled by the supply company and arranged to feed a number of smaller consumers.

Where the power is received as alternating current, and direct-current supply is required, rotary converters or motor generators, with

or without the interposition of transformers between these and the line, have to be adopted.

There are other means, such as rectifiers, which transform alternating current into direct current; but these so far are only used where small amounts of power have to be dealt with, and find their most common application in garages for charging batteries used for car lighting, etc.

§ (41) Switching Operations.—When connecting a circuit which contains considerable capacity, or self-induction, to a source of supply, a large rush of current may take place. In the first case a large current may be derived from the source for charging the circuit in question. In the second case, the self-induction will have different effects depending on whether iron is present or not. If there is no iron present, the self-induction will keep down the current rush, which will gradually rise to its full value.

If, however, iron is present (*e.g.* a transformer is being switched on to the line) there may be a heavy current rush if the remanent magnetism is of such a direction with respect to the voltage curve that the current, on switching on, tends to increase the saturation of the iron; the additional ampere turns required to produce the necessary flux to balance the applied E.M.F. may be very large, as with higher saturation the permeability becomes very low. Many a transformer breakdown may be traced to such a current rush; coils have been known to move, with serious results to the apparatus, in consequence of switching operations. Also, and this is particularly noticeable on the higher voltages, the impressed potential may concentrate on the end turns of the motor or the transformer, as the case may be; and since the insulation of this apparatus for normal working conditions need only be proportioned for an even distribution of the potential stress across the windings, it follows that the end turns are in danger of breaking down, and precautions have to be adopted to safeguard these.

Such precautions may either consist (*a*) in strengthening the insulation of the end turns in question, in order to make them capable of resisting the extra pressure, or (*b*) introducing means which will avoid these potential or current stresses, as the case may be.

Various devices are employed for applying the voltage gradually to the apparatus. In the early days water resistances were sometimes used in which the length of the water column, and with that the resistance inserted, was gradually reduced, until it was finally short circuited. Modern practice, however, has shown that such step-by-step charging is not essential, and that one step usually suffices to attain the desired object. The operation being so far simplified, it is now usual to introduce an inductive or non-inductive resistance, which for the moment is placed in circuit before final contact is made. It is also common practice to combine these with the main circuit breaker, and to fit an auxiliary tip on to the main contact, which is connected to the latter through a resistance. The cross-bar or contact-making portion in its movement first makes contact with this auxiliary tip, thus placing the resistance in series for a short moment, and it then completes the circuit.

Where very long transmission lines are employed a separate generator may be installed for carrying out the charging process, which then consists in connecting the unexcited generator to the transmission lines or cable, and gradually bringing it up to voltage, after which it is synchronised with the system.

Where overhead transmission lines are employed a further danger arises from the humidity of the atmosphere. A line which is under pressure keeps up its insulation within limits, even in damp or foggy weather, particularly on the higher voltages. Any damp that lodges on the surface of the insulator will form a path which, when the line is under pressure, allows a minute leakage current to flow, and this quickly vaporises the damp and throws off particles of dirt as well.

When, however, a line has for some time been out of commission, and exposed to a damp atmosphere or fog, the amount of dirt and wet lodging on the surface of the insulators may be considerable, and sufficient to produce a path to earth, which allows a current of dangerous dimensions to flow over the insulator and destroy the latter. Care has therefore to be exercised when switching on transmission lines, and this operation should preferably be done at a time of day when the insulation is at its best. In a particular transmission line it was found that whenever the line was switched on in the morning break-downs were frequent, but if it was done before sunset practically total immunity against such accidents was obtained.

§ (42) Feeder Protection.—Where large amounts of power have to be carried by a transmission line or cable to an outlying district, and that particular sub-station is not interconnected with other sub-stations—that is, ring connections are avoided—the most common method of protecting the line is by means of overload protection on the outgoing feeder, and sometimes overload protection at the sub-station end. When, however, more than one transmission line connects the sub-station with the power station a fault on one of the lines would cause the automatic switches on the various feeders to operate, particularly if the fault occurred close to the sub-station, thus disconnecting the healthy

feeders as well as the faulty one. It is therefore usual to equip all incoming feeders at the sub-station end with reverse-power relays, and to provide the feeder switches at the power-station end with time-limit relays. If a short circuit or ground occurs on a feeder thus equipped, the reverse protection at the sub-station end will cause the switch controlling the faulty feeder to operate, due to the reversal of power, whereupon the overload device on the circuit breaker at the other end will cut off the feeder from the power station. It has, however, to be borne in mind that the power factor alters under heavy overload conditions, also that reverse power relays require pressure for their proper functioning.

If the fault should occur very close to the sub-station the line pressure will disappear, and the reverse power relay will not operate, thereby causing the overload circuit breakers on the feeders connecting the source of supply with the sub-station in question to open at the power end, thus completely isolating the former.

Where continuity of supply is a very important factor, and particularly where sub-stations are connected through ring-connections in which the power may under normal working conditions flow in one or other direction, overload and reverse power protection are no more suitable, and some form of balanced protection has to be resorted to.

§ (43) SUB-STATIONS.—Sub-stations may be divided into two main classes—outdoor sub-stations and indoor sub-stations.

In *outdoor sub-stations*, as the name implies, the transformers and the apparatus controlling the incoming power and outgoing supply, together with the lightning arresters, are arranged in the open, and only the rotating machinery, if any, placed in a building (*Fig.* 32).

In America and in the Colonies it is common practice, particularly in country districts and smaller cities, where small amounts of power have to be dealt with, to mount the transformers on poles, to connect them to the E.H.T. transmission line through isolating switches and fuses, and to arrange the distribution of the low-tension supply from the secondary side of the transformers by means of switch fuses.

In *indoor sub-stations*, largely employed where climatic or other considerations do not admit of the apparatus being erected in the open, the transformers and distributing apparatus are arranged inside a building, thus protecting them against the weather.

In addition to dividing sub-stations which transform power into outdoor and indoor types, they can also be conveniently divided into static transformer sub-stations—*i.e.* stations transforming the pressure by means of static transformers—and rotary sub-stations, which transform alternating current to direct current by means of rotating machinery.

(i.) *Static Transformer Sub-stations.* — In these the equipment is of a comparatively simple nature. Automatic circuit breakers are supplied for the incoming lines and connect these to the busbars. The busbars may be either sectionalised or arranged in duplicate, or there may be a combination of the two alternatives.

A power transformer or transformers may be placed either in the transmission line and

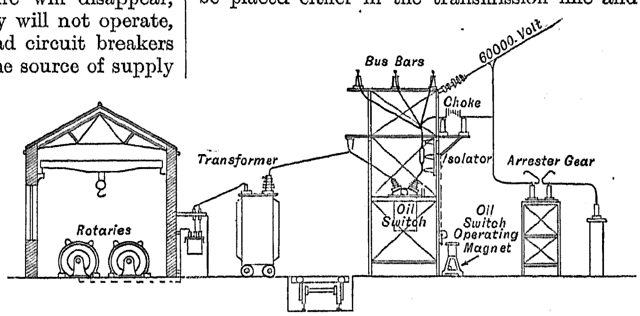


FIG. 32.

form a unit with the latter — that is, the transmission line may be connected direct to the power transformer—or, alternatively, the power transformer may be placed between the incoming oil switch and the busbars, in which case, however, a further switch has to be interposed between these, if more than one transformer is connected in parallel. In other cases, the transformers may be arranged on the sub-station side of the busbars, and either feed the outgoing circuit direct, or on to low-tension busbars, from which the distribution then takes place through suitable switchgear.

(ii.) *Rotary Sub-stations.*—In these, transforming alternating current to direct current, the switching arrangements are more complicated. In the first place, the busbars have to be supplied with the power received under high pressure. The power taken from these busbars may, by means of static transformers, have to be reduced to a pressure suitable for the machines which are to be driven. These rotating transformers, which require more care and attention than the static transformers used in the case previously mentioned, take the form of (*a*) motor generators, or (*b*) rotary converters.

Where motor generators are employed and

a battery is available at the sub-station, the set is usually run up by means of the generator, which for the time being carries out the functions of a motor, and receives its energy from the battery through a suitable starting switch. The alternating-current end, having been brought up to speed, is synchronised with the incoming supply, and takes over the function of driving the generator, which then supplies direct current to the low-tension busbars, from which the distribution takes place through the feeders equipped with the necessary direct-current, circuit-closing device.

When starting up rotary converters the method previously described may be employed. In addition to this, particularly where batteries are not available, and synchronising has to be avoided, the following two alternative methods are largely in use :

(*a*) *Tap Starting.*—The transformer is provided with one or more tappings, and the rotary started up from the alternating-current side, by connecting it successively to the different tappings, and running it up as an induction motor, the field being disconnected at the time, and the brushes in the larger sets lifted off the commutator. It is also necessary to split up the field into sections, in view of the fact that the potential rise on the field circuit may be considerable at the moment of starting up ; this is usually done by a so-called field-splitting switch (*Fig.* 33). When the rotary has attained full speed on the tapping, the field circuit is connected to the terminals of the rotary, but arrangements have to be provided to ensure that the polarity is correct. If it is found (by means of a suitable voltmeter) that this is not the case and the machine has "come up" incorrectly, the field is momentarily reversed by means of a double-throw field-breaking switch in order to cause it to slip a pole. This operation may have to be carried out several times in succession before the object aimed at has been attained. After correct polarity has been established, the rotary is connected to the full voltage terminals of the transformer and paralleled with the direct-current busbars in the usual manner, whereupon the field circuit is adjusted so that the rotary takes its fair share of the load.

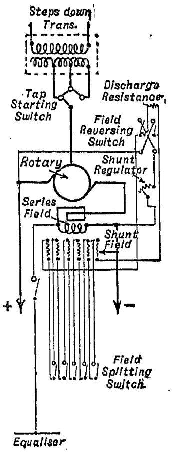


FIG. 33.

(*b*) *Pony Motor Starting.*—This method of starting up a rotary converter consists in connecting the stator winding of a small squirrel-cage motor, the rotor of which is mounted on an extension of the converter shaft in series with the main armature through the slip-rings. When the main switch is closed, the current passing through the stator winding of the auxiliary motor will start it up and bring the set up to speed and into synchronism with the supply, whereupon the direct-current field of the rotary converter is completed. The stator windings of the pony motor are then short circuited by means of switches, in order to avoid losses in these while running.

Correct polarity is obtained in the same way as when tap starting is employed. The remaining operations are the same as in the previous case.

(iii.) *Precautions against Flash Over.*—When a sub-station is connected to a large source of power, further precautions have to be adopted to prevent damage to the rotary transformers. A sudden very heavy overload or a short circuit across an outgoing feeder places a strain on the rotary converter, which may cause the latter to "flash over," with consequent severe damage to itself.

The ordinary circuit-breaking devices are not quick enough in action to prevent such an occurrence, and circuit breakers have had to be constructed which will open the circuit at a very much greater rate, and reduce the amount of current by inserting a suitable resistance. The time in which such a circuit breaker should operate must be less than the time taken for a commutator bar to move from one pole to another. This, in a fifty-cycle rotary converter, would be equal to ·01 of a second. Devices are on the market which will, in the time specified, insert a resistance in the rotary circuit, thereby limiting the current flowing from the same into the system. Under normal working conditions the resistance which is placed in series with the converter is short circuited by the switch in question, but on the occurrence of a rush of current the high-speed breaker opens, and the current is forced to take its path over the resistance. One or more devices of this nature may be installed in series with each other on the same converter in order effectively to deal with overloads of different magnitude. Such devices, with or without resistance, can also conveniently be used on the outgoing feeders. In rotary sub-stations supplying traction load the danger of heavy and unexpected overloads, short circuits, etc., is considerably enhanced by the fact that the negative pole is usually solidly connected to earth. The high-speed circuit breakers in these cases are arranged on the negative poles of the rotaries, but those on the outgoing feeders are arranged on the positive pole.

(iv.) *Automatic Sub-stations.*—Where the power requirements in a particular area fluctuate between considerable limits, as, for instance, in the case of a railway line with infrequent train service, a sub-station running all day on a low-load factor becomes an expensive matter. Automatic sub-stations have, therefore, been developed, the function of which is to come into operation only when the energy demand requires this.

In the early days these automatic stations took the form of what might more correctly be termed distant controlled stations—that is, the operation of connecting them to the high voltage supply, the operation of running up the rotary converters and paralleling with the direct-current busbars, was carried out from a distance, preferably from a large sub-station. Latterly the automatic devices in the sub-stations have been largely improved, and sub-stations are now in use in which the whole process of starting up the rotaries and paralleling them with the system is carried out automatically, and is governed by the demand on the direct-current system. When this demand falls the rotary converter is again automatically disconnected after a suitable time interval, which will allow of minor load fluctuations, as, for instance, the stopping and restarting of a train at a station in the immediate vicinity of the automatic sub-station. A number of sub-stations are therefore arranged at suitable intervals on a long-distance railway line, and when a train or trains run along this, the stations just in advance and just behind the load automatically come into operation, but as soon as the train has left the radius of supply of one of these, the latter is cut off and the next sub-station comes into operation in its stead. By this means the amount of attendance is considerably reduced, and cheaper buildings may be put up for the machinery. The losses in the transmission line are also, in view of the fact that the power is transmitted on the high-voltage side, less than would be the case with the longer low-voltage direct-current feeders.

§ (44) Switchgear in Sub-stations.—It has been previously intimated, in connection with the lay out of power station, that the choice of apparatus, the design, arrangement, etc., are largely governed by the power behind them. This also applies to sub-stations or subsidiary distributing centres. The stresses produced on the circuit-opening apparatus, connections, etc., under fault conditions will, however, be less severe, on account of the resistance of the line, and the reactance of the transformers, which are connected between it and the source of power.

It follows that the automatic switches, connections, etc., are exposed to a less heavy duty, as the distance from the power station increases as well as the number of transformers interposed in series between these and the source of supply, so that at the far end the smallest type fuses may give satisfactory protection even in a scheme where large powers are concentrated at the source.

Where sub-stations are linked together by ring-mains account has to be taken of these and due allowance made for power being supplied to the fault from the generating plant *via* other sub-stations, as well as through the feeder or feeders forming a direct connection between the source of the power and the sub-station under consideration.

If non-automatic switches are employed, *i.e.* switches which do not automatically interrupt on fault conditions, the rupturing stress placed on the circuit-opening devices may be, comparatively speaking, light, as the switches only have to interrupt the sustained short-circuit value of the generator or generators, or as much as the interposed feeders, transformers, etc., will allow to flow into the fault; but when these switches are fitted with automatic interrupting devices they may have to interrupt a much larger current.

The momentary short-circuit current of the generator, as has been shown, may obtain an instantaneous value of twenty to thirty times normal, which, however, quickly reduces to the sustained short-circuit value of the generator, *i.e.* the current which would be produced if the generator terminals were short-circuited and the generator run up fully excited. The magnitude of the momentary and the sustained short-circuit values and the time in which the latter is reached vary with the design of the alternator. It is fairly common practice, where only rough figures are required, to assume the momentary or peak value of a turbo-alternator as fifteen times normal and the sustained value as 2·5 times normal, and to assume that this latter value is reached in five or six cycles. In older slow-speed generators, driven by reciprocating sets, the corresponding values would be about ten for the peak current, three to five times normal for the sustained value, and the time interval twelve cycles.

The circuit breaker requires time to come into operation from the moment it receives its impulse until it finally interrupts the circuit, and this time interval depends on the method of moving the parts, weight and acceleration of these, number of breaks in series, methods of extinguishing the arc, etc. Also the time taken by the relays which transmit the impulse, and cause the auxiliary current source which carries out the opening operation to be closed, very materially affects the total time taken to interrupt the circuit.

The addition of time-limit devices further

affects the time of opening circuit, particularly if these devices are of the constant time-limit pattern.

It will be seen from the above that the automatic circuit breaker may be called upon to interrupt at any point between the maximum and the sustained short-circuit values, but it has to be borne in mind that even if the delay caused by the time limits and the time element of relays normally ensures interruption of the circuit at or near the sustained short-circuit value of the plant, it is nevertheless possible that under certain conditions — for instance, when the fault develops gradually and at a rate that it reaches its peak value at the moment when the contacts begin to part—the interruption of the circuit may place an exceptionally heavy strain on the circuit breaker

A further very severe condition arises when a circuit breaker is closed on a feeder which has previously been short-circuited. The direction of movement of the contact-making parts has in that case to be suddenly reversed at the moment of making circuit, and since the reversal of the movement takes time, and the current will be at its peak, the stress is considerable.

Let us take, as an example, the case of a sub-station fed from a 100,000 kw. power station by a 5000 kw. feeder, which supplies a 5000 kw. transformer. If we assume the reactance of the line to be 3 per cent and the reactance of the transformer 6 per cent, we obtain a total reactance of 9 per cent, *i.e.* the feeder switch on the "off" side of the power transformer will be called upon to deal with a current, the value of which corresponds to

$$\frac{5000 \times 100}{9} = 45{,}000 \text{ k.v.a.}$$

It will be noted that this value is well within the capabilities of the assumed generating plant, and could be supplied by it for a considerable time. Had we assumed a source of supply which could not maintain 45,000 k.v.a. the stress on the switches would be smaller, *i.e.* the figure in question could be reduced to the values the generating plant could supply. On the other hand, if the feeders and transformers were duplicated, the short circuit k.v.a. would be twice the above figure, or 90 per cent of the full-load output of the generating plant in question.

R. A. R. B.

SWITCHING OPERATIONS, PRECAUTIONS IN CARRYING OUT. See "Switchgear," § (41).

SYNCHRONISING OF ALTERNATORS. See "Switchgear," § (23).

SYNCHROSCOPE: instrument for indicating the attainment of the correct conditions for the paralleling of alternators. See "Switchgear," § (24); "Alternating Current Instruments," § (48).

Weston: a combination of a dynamometer synchroscope with a synchronising lamp. See "Alternating Current Instruments," § (50).

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TANGENT GALVANOMETER, use of, for absolute measurement of current. See "Electrical Measurements," § (25).

TELEGRAPH, THE ELECTRIC

§ (1) HISTORICAL.—The electric telegraph (telegraph from τῆλε, "distant," and γράφω, "to write") may be defined as a combination of apparatus by means of which intelligence may be communicated to a distance by the agency of an electric current flowing along an insulated wire.

The earliest known attempts at the transmission of electricity to a distance were made by Grey in 1727 and Dufay in 1733 with a view to ascertaining the maximum distance to which electrical manifestations could be observed in an insulated wire. The discovery of the Leyden jar in 1746 gave an impetus to such experiments, and in 1747 Dr. Watson in England succeeded in discharging a Leyden jar through nearly two miles of iron wire supported by wooden poles. Franklin in 1748 conducted similar experiments in America, but, like his predecessors, he does not appear to have entertained the idea of transmitting signals for practical purposes. There is no doubt, however, that the results obtained by these investigators subsequently suggested the possibility of controlling one or other of the phenomena of electricity for the purpose of telegraphy.

Charles Morrison in a letter to the editor of the *Scots Magazine*, in 1753, was the first to publish the idea of an electric telegraph. The letter described in detail a system of electrically selecting at a distance any desired character. A separate insulated wire was to be used for each character, and for the operation of the system the particular wires allocated to the letters forming the word to be signalled were to be charged electrically by means of a frictional machine. At the receiving station pieces of paper inscribed with the various characters were to be placed in proximity to the ends of the corresponding wires. Morrison suggested that the pieces of paper

under the charged wires would be attracted and thereby indicate to the receiving operator the word transmitted. The letter contained other ingenious suggestions, relating to the insulation of the wires and reading signals by sound of bells. As a tribute to Morrison's foresight it should be mentioned that these suggestions, in modified forms, were years afterwards brought into practical use.

The first attempt at a practical electric telegraph was made by Lesage at Geneva in 1774, and was almost identical with that suggested by Morrison. The letters signalled were indicated at the receiving end, however, by the divergences of pith-ball electroscopes, one of which was connected to each line wire. Lomond experimented with a similar system in France in 1787 and introduced the important feature of using one line wire only, by adopting a preconcerted code of signals in which a certain number of successive divergences of a pith-ball electroscope were fixed upon for each character.

Reusser (1794), Cavallo (1795), and Don Silva (1798) each suggested systems in which the signals were to be identified at the receiving end by means of electric spark discharges across parts of a broken conductor.

All of the foregoing systems were useless from a commercial point of view, owing to the inherent difficulties in using the high potential frictional electricity for such a purpose. In spite of this, however, Mr. Ronalds of Hammersmith recurred to its use in a telegraph system invented by him in 1816. His method was similar to that of Lomond's, but by using in conjunction with a pith-ball electroscope, rotating discs moving isochronously at each end, for exposing the characters successively, signalling could be effected much more rapidly.

A great step forward in the history of the electric telegraph was the discovery of the Voltaic battery in 1800 by Volta. It was not applied to telegraphy until 1807, when Sommering of Munich invented a system based upon the principle of the decomposition of water by the voltaic current. Thirty-five line wires were required, and at the receiving station each wire terminated in a gold pin arranged in a glass trough containing water. Two wires were used for each character, one for the outgoing and the other for the return path of the current. Each pin represented a certain character, and, when the system was being operated, the particular pin from which hydrogen gas was being evolved indicated the letter signalled. Schweigger of Halle suggested a number of improvements to Sommering's system, one in particular being a method of attracting the attention of the operator at the distant station by firing, by means of the current, a pistol charged with a mixture of hydrogen and oxygen.

In 1820, Oersted of Copenhagen made the important discovery that a magnetised needle could be deflected by a current of electricity. This led to Schweigger's invention of the Multiplier, which consisted essentially of a magnetic needle suspended within a vertical coil of wire, and forms the basis of a large number of modern telegraph instruments.

Immediately following the publication of Oersted's discovery, Ampere investigated the laws of electromagnetism, and as a result proposed to employ magnetic needles and coils for the reception of signals. He made the vital mistake, from a practical point of view, of using a separate line wire for each character, otherwise his system might have been brought into extensive use.

Baron Schilling in 1832 exhibited a telegraph system similar in principle to that proposed by Ampere, but requiring fewer line wires. The latter desideratum was attained by using a signalling code based upon the separate deflections of a magnetic needle to the right and left effected by reversing the direction of the current through a coil encompassing the needle.

Gauss and Weber in 1833 adopted a code similar to that of Schilling, but employed the movements of a suspended bar magnet as signals. A mirror was attached to the suspension wire, and the indications were observed through a telescope placed about 12 feet away. They also utilised Faraday's discovery (1831) of electromagnetic induction for the production of the signalling currents. Steinheil of Munich brought the Gauss and Weber system to a high state of perfection, and incidentally discovered during his experiments that the earth could be used instead of a return wire. He used two magnetic needles for the reception of signals, one of which responded to positive currents and the other to negative currents, and invented a convenient telegraphic code in which the letters of the alphabet were represented by combinations of the movements of the two needles. Steinheil constructed a variety of receiving instruments, arranging for the messages to be received in the following ways: (*a*) in a code of dots on a paper slip moved at a uniform rate by clockwork; (*b*) acoustically, by causing the needles to strike bells; (*c*) visually, by observing the motions of the needles by eye. A great deal of credit is due to Steinheil for the successful development of the electric telegraph; his many inventions have had a great influence upon the labours of subsequent inventors.

The invention of the electromagnet in 1825 by Mr. Sturgeon[1] of London ranks as a very

[1] See Fleming, *Fifty Years of Electricity*.

important step in the history of the electric telegraph, yet it was not utilised for the purpose until many years after the publication of its invention. At the present day it forms the essential part of the receiving apparatus of nearly all telegraph systems.

The partnership of Wheatstone and Cooke began in 1835 and resulted in the invention of the first practical electric telegraph. This was a five-needle telegraph requiring six line wires and using a very ingenious method of indicating the letters signalled. It was arranged that for each signal two of the five needles should converge and point out the letter corresponding, on a dial. A great novelty that was introduced was the employment of an electromagnet at the receiving station to work an alarm. The system was tried on the London and Birmingham and Great Western Railway lines, but was abandoned ultimately after an extensive trial owing to the prohibitive cost of the provision and maintenance of six line wires. This system was succeeded by a double-needle telegraph which required two line wires only, and this in turn gave way later to the single-needle system which is still in use on some of the less important circuits in this country.

The various electric telegraph systems that had been tried up to the time of the Wheatstone and Cooke partnership were of more scientific interest than practical utility. The successful demonstrations with the systems designed by these inventors settled conclusively any doubt that had been entertained of the practicability of utilising the electric telegraph for commercial purposes, and from thence onward the progress of the art was very rapid.

Professor Morse of America exhibited his practical telegraph in 1837 and was the first to make use of an electromagnet for the reception of signals. In his original instrument the electromagnet worked a movable armature carrying a pencil which traced, in a prearranged code, signals on a moving paper band. In 1844 Morse effected a number of improvements and for the first time used the now familiar " dot and dash " code, arranging for the signals to be embossed on the paper band instead of traced in pencil. At the present day the system founded by Morse forms the backbone of all telegraphic organisations.

Edward Davy in 1838 published a method of recording signals chemically on a band of prepared paper. Bain patented a similar system in 1846, using a paper band saturated with a solution of ferrocyanide of potassium and ammonium nitrate; the passage of a current through a steel style pressing on the paper decomposed the solution, depositing Prussian blue and recording the signals by long or short blue marks according to the duration of the current. A system known as the *Telepost*, based upon the above principle, survives at the present day.

Wheatstone invented his first alphabetical dial instrument in 1840. In this the signals at the receiving station were given by a pointer pivoted at the centre of a dial around the periphery of which were arranged the various characters. The transmitting apparatus was designed to send out a certain number of current impulses for each character, and the passage of these impulses through an electromagnet at the receiving station caused the pointer to be moved step by step round the dial to the required character.

The foregoing is a brief survey of the history of the electric telegraph up to the time when its commercial utility was firmly established. Space will not permit of the enumeration of the large number of subsequent improvements. An outstanding invention was that of the Wheatstone automatic system in 1858; it is still extensively used, in a modified form, and will be described later. The possibility of economising in the use of telegraph lines by increasing their message-carrying capacity led to the invention of Duplex telegraphy (see article " Telegraphy, Duplex ") by Gintl in 1853. Quadruplex telegraphy (see article " Telegraphy, Quadruplex ") was independently invented by Heaviside in 1873 and Edison in 1874. Multiplex telegraphy (see article " Telegraphy, Multiplex ") was suggested in 1852 by Farmer, and was afterwards considerably developed by Meyer (1873), Baudot (1881), and Delany (1884).

The latest developments in telegraphy have been in the direction of high-speed type-printing systems. The most important of these are described in the article " Telegraphs, Type Printing."

§ (2) Present-day Telegraph Systems.—All electric telegraph systems consist essentially of four parts:

(i.) *A Battery or Dynamo* for generating electricity at one end. In this country batteries are exclusively used, generally made up of a form of Leclanché cell at small offices and secondary cells at large offices.

(ii.) *The Transmitting Apparatus* by means of which the battery is applied to the line for sending the signalling currents.

(iii.) *The Line* along which the signalling currents may flow and which may consist of open wire supported on insulators attached to wooden poles, or of underground cable, with either an earth or a metallic return.

(iv.) *The Receiving Apparatus*, which is actuated by the incoming signalling currents and which gives a particular kind of signal, transient or permanent, audible or visual, according to the construction of the apparatus adopted.

In modern telegraph systems, the separate letters of which a word is composed are transmitted according to a prearranged code of signals. There are two principal codes in use in practice, known respectively as the Morse code and the Five-unit code. The latter is used exclusively for Type-printing systems (see article "Telegraphs, Type Printing"), while the former is used as a necessary adjunct to all systems.

The Morse code is made up of two elementary signals which are distinguished from each other by their duration. One signal called a "dot" is taken as the unit; the other signal, which is called a "dash," is three times as long. Each letter of the alphabet is indicated by a particular combination of "dots" and "dashes" as shown in *Fig.* **1,** which represents

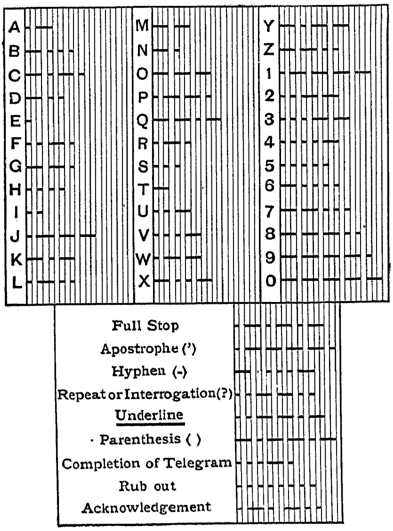

FIG. 1.

the International Morse Code. The spaces between the elements forming a letter are each equal in length to one dot, while the space between the letters of a word is equal to three dots, and that between two words is equal to five dots.

The Morse code as used in America differs in many respects from the International code, the spacing intervals between the elements forming certain letters being of different lengths; and more than four elements are used to form some of the letters.

Modern telegraph systems may be broadly classified as follows:

(*a*) Systems using the Morse Code.

(*b*) Type-printing Systems (see article "Telegraphs, Type Printing").

(*c*) Writing Systems (see article "Telegraphs, Writing").

The most important developments of the first class will be treated in this article, the other two classes being dealt with in separate articles as indicated.

§ (3) SYSTEMS USING THE MORSE CODE.—In the simplest Morse telegraph system the battery is connected to the line for a period of time represented by a dot or dash by means of a Morse key. This, which is sometimes known as a single current key, is shown in *Fig.* 2, and consists of a brass lever A, pivoted near

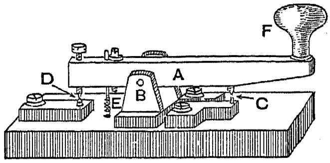

FIG. 2.

its centre at B. The up and down movement of the lever is limited by the contact stops C and D. In the position of rest, the adjustable spring E holds the lever in contact with the back stop D to which the receiving apparatus is connected. One pole of the signalling battery is connected to the front stop C, and the line is joined to the lever A. The knob F is grasped by the fingers and, when depressed, causes the lever A to make contact with the stop C and the battery is thereby connected to the line.

For receiving the signals a Morse Sounder, shown in elevation and plan in *Fig.* 3, is used. It consists essentially of an electro-magnet A A wound with two bobbins joined in series, the ends of the windings being connected to the terminals T, T. The brass bell-crank lever K is pivoted between two screws E, E and is held normally, against the adjustable stop screw C, by a spring S. The tension of this spring can be regulated by the screw F. B is a soft iron armature fixed to the lever K transversely so as to lie just above the cores of the electromagnet. When a current passes through the coils, the cores of the electromagnet become magnetised, the armature is sharply attracted and the lever K pulled down. The latter, when its stop screw D strikes the

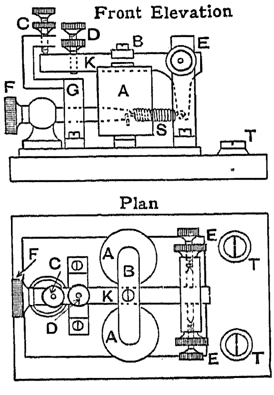

FIG. 3.

bridge piece G, gives a very distinct sound. When the current ceases, the spring S causes the lever to rise, drawing the armature away from the electromagnet. The impact of the lever against the upper stop C gives rise to another sharp sound. The time interval between these sounds depends upon the duration of the current flowing in the electromagnet; a short interval representing a "dot," and a long interval a "dash." With this instrument, therefore, the incoming signals are read by sound; considerable practice is required by operators, however, before they become efficient in distinguishing between the signals.

The manner in which the Morse key and sounder are joined up in a circuit is shown in *Fig.* 4, which represents a complete direct sounder circuit between two telegraph stations. In any telegraph circuit one terminal station (usually the more important station) is termed the "up" station and the other the "down" station. The line leading from

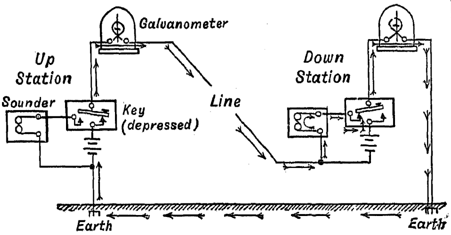

FIG. 4.

an "up" to a "down" station is called the "down" line, whilst the line leading from a "down" to an "up" station is called the "up" line. Intermediate offices on a circuit are "down" stations with respect to the "up" station and "up" stations with respect to the "down" station. An "up" station always connects the positive pole of the battery to line when signalling. In *Fig.* 4 the key at the up station is shown depressed, and the path taken by the current is indicated by the arrows. A galvanometer is introduced into the circuit at each station in order that the sending operator may be able to see that his signals are passing to the line. The type used is known as a single current galvanometer and is represented in *Fig.* 5. The coils are wound upon two bobbins and encompass a pivoted needle which is magnetised inductively by permanent magnets. The construction of the latest type of instrument is due to Spagnoletti, who made the needle of soft iron and in two parts as shown, separated magnetically from each other by a thin layer of spelter. The axle on which they are mounted is also in two parts, the front part being an extension of the lower half of the needle, and the back part an extension of the upper half. Two permanent magnets of horseshoe form are used, mounted as shown with their like poles adjacent. A non-magnetic pointer is fixed at the end of the axle carrying the needle, and moves over a dial. Two terminals project horizontally at the back of the instrument, and the pointer is deflected

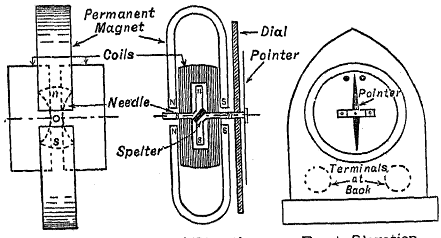

FIG. 5.

to the right or left according to the direction of the current passing through the coils.

Instead of using a sounder, which gives transient signals only, the Morse ink-writer may be used, which gives a permanent record of the incoming signals in ink on a paper slip, so that they may be interpreted at leisure. The principle of the Morse inker is shown in *Fig.* 6. It consists of an electromagnet E, with an armature A. The latter, when attracted by the electromagnet, brings a small disc M, which revolves in an ink trough, into contact with a paper band moved forward at a uniform rate by clockwork. Long and short

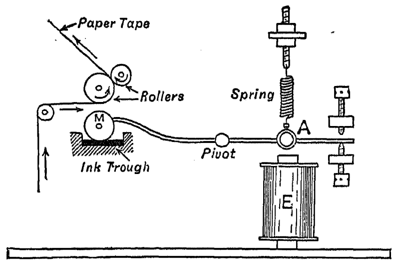

FIG. 6.

ink marks, corresponding to dashes and dots, are in this way made upon the paper according to the duration of the contact of the ink wheel. The instrument is connected in a circuit in exactly the same way as the sounder.

The type of circuit described is called "open" or "intermittent," because a current flows in it only during the time that a signalling key is depressed. There is another method of direct working, known as the "closed circuit" or "continuous current" system, which is extensively used in America. In this the connections are so arranged that a current

flows through the circuit normally. Signalling is effected by breaking and making the circuit again by means of a key. It is claimed as advantages for this method of working that any disconnection in the line is at once evident to all stations and that a battery is not required at every station. On the other hand, the batteries are more quickly exhausted than with the "open" circuit method, and the long continuance of the current in one direction tends to increase the effect of residual magnetism in the cores of the receiving electromagnets.

§ (4) THE SINGLE-NEEDLE SYSTEM.—In this system a code of signals based upon the Morse code is used; the dots and dashes, however, do not differ in length, but are distinguished from each other by the direction of the deflection of a magnetic needle, a deflection to the left representing a dot, and a deflection to the right a dash.

For the reception of signals a galvanometer similar to the single-current galvanometer is used. The reversals of current required when signalling are effected by means of a commutator which consists of two tapper keys. Each key is of wood and has a metallic extension at the back end which moves between two contacts. Fixed to each wooden key in front of the bearing is a contact screw which makes connection with the negative pole of the battery when the key is depressed. The commutator has six terminals, and the

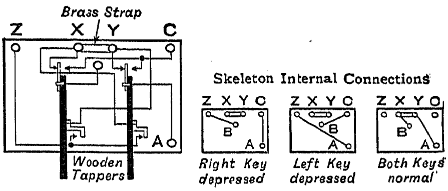

FIG. 7.

internal connections made by the keys in various positions are given in *Fig.* 7.

The connections of a circuit are shown in *Fig.* 8, in which the left-hand tapper at the up station is assumed to be depressed. This corresponds to sending a dot, and the direction and path of the current is shown by the arrows. If the right-hand tapper at the up station is depressed, as is required for sending a dash, the positive pole is put to earth and the negative pole of the battery to line, thus the direction of the current is reversed.

The galvanometer coils are each wound to a resistance of 200 ohms, and a current of 15 to 20 milliamperes is required for satisfactory working.

The needle of the galvanometer is sometimes transformed from visual to sound reading by arranging for the needle to strike pieces of tin shaped to give out two differing sounds according to the direction of deflection. A specially constructed dial, known as Neale's acoustic dial, is sometimes used where a louder sound is required.

The single-needle system has long since been relegated to unimportant circuits and is being rapidly replaced by the Morse sounder.

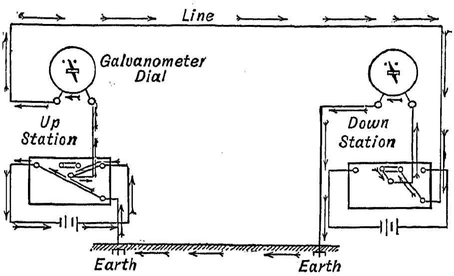

FIG. 8.

§ (5) RELAYS.—In the case of long lines of high resistance and low insulation it would be impossible to get enough current through the line to cause the sounder at the distant station to give out audible signals unless an inconveniently large battery power was used. The latter is not desirable for practical reasons, and it therefore becomes necessary to introduce a sensitive form of receiving apparatus, which by making and breaking an electric contact will work the sounder from a small local battery. Such a piece of receiving apparatus is termed a *relay*, and consists of a specially designed electromagnet having a very light and finely set armature, which, although unable to give audible signals, is capable of moving sufficiently to make and break a circuit.

The relays used in telegraphy may be divided into two classes:

(i.) *Polarised Relays*, in which the direction of movement of the armature is dependent upon the direction of the current through the electromagnet.

(ii.) *Non-polarised Relays*, in which the direction of movement of the armature is independent of the direction of the current through the electromagnet.

The fundamental differences between the two types will be understood from the following descriptions of the standard instruments used by the British Post Office.

(i.) *Polarised Relay*.—This type of relay is shown in *Fig.* 9. It consists of two electromagnets A and B, each wound with two separate coils of equal resistance and having the same number of turns. The ends of the coils are led as shown to four external terminals lettered U, (U), D, (D). Two soft-iron armatures, *ns* and *n's'*, are fixed to the vertical spindle L, and are magnetised inductively

by means of the powerful permanent magnet P. To the upper end of the spindle is fixed a German silver tongue T, tipped with platinum, and capable of moving between two contact screws S and M. The latter are insulated from each other and are mounted in such a manner that they can be moved together, for adjustment purposes, by means of a regulating screw (not shown). These contact screws are also separately adjustable, so that the play of the tongue may be varied to suit requirements.

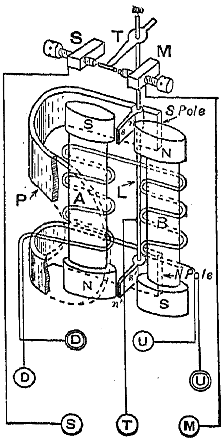

FIG. 9.

A current flowing through a coil from U to D (or (U) to (D)) produces polarity, as shown in the electromagnets. The soft-iron armatures are therefore caused to move in such a direction as to bring the tongue T to the contact screw M (marking contact). If the current is reversed the polarity of the electromagnets is reversed and the soft-iron armatures move in the opposite direction so as to bring the tongue T to the spacing contact screw S. The relay coils may be joined in series or in parallel by suitably connecting the terminals by means of brass straps. Each coil is wound to a resistance of 100 ohms. This type of relay will respond to a current of one-half milliampere, but a working current of 15 to 20 milliamperes is usually adopted.

(ii.) *Non-polarised Relay.*—This form, although somewhat similar in appearance to the polarised relay, differs from it in a number of important details. It has no permanent magnet and each soft-iron armature is made in two parts brazed together by spelter in order to prevent the electromagnet cores forming with them a closed magnetic circuit. The movement of the tongue is controlled by an adjustable spiral spring, and is normally held against one of the contacts. The pole-pieces (PP) of the cores and the armatures are shaped and arranged as shown in *Fig.* 10. The passage of a current of a certain strength through the coils in either direction causes the armatures to be attracted to the pole-pieces, and the tongue is moved to the opposite contact against the tension of the spring.

The connections of a single-current circuit in which relays are used is shown in *Fig.* 11. The key at the up station is represented depressed and the path of the current is indicated by the arrows. At the down station the current passes through the relay in the direction (U) to D, so that the tongue will move to the marking contact, closing the local circuit and operating the sounder. If polarised relays are used the relay tongues

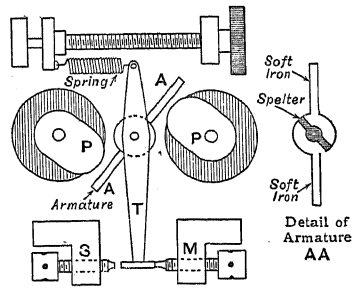

FIG. 10.

must be given a fairly strong spacing bias, so that they will return smartly to the spacing contact and thus break the local circuit when the current ceases.

The resistance of the sounder coils used in the local circuit is 21 ohms shunted by 420 ohms, making a joint resistance of 20 ohms. The function of the shunt coil is to provide a path for the currents induced in the sounder coils when the local circuit is broken, and thus

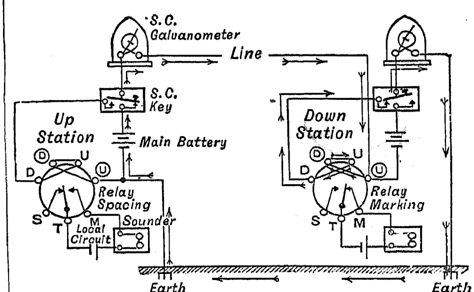

FIG. 11.

prevent sparking at the relay contacts, which would in time corrode them and cause imperfect signals. With universal batteries (see article "Telegraphy, Universal Battery System") the sounder coils are wound to a resistance of 1000 ohms shunted by 9000 ohms, thus giving a joint resistance of 900 ohms.

§ (6) THE DOUBLE-PLATE SOUNDER.—This instrument is used where the sound emitted by single-needle instruments with sounding pieces cannot be heard distinctly above the noise of a large instrument-room. The louder sound is attained by using two electromagnets, the armatures of which are each fitted with a

brass knob. These knobs strike on a brass and steel plate respectively giving out different tones, one representing a " dot " and the other a " dash " of the Morse code. The two sounders are controlled by a polarised relay, whose tongue is fitted with two adjustable spiral springs so as to keep it central between the contact stops, without touching either, when no current passes through the relay. A current flowing from U to D through the relay causes the tongue to move to the marking contact and to actuate the dash sounder by means of a local battery. A current in the reverse direction through the relay operates the dot sounder. The same type of commutator as used with the single-needle instruments is employed for signalling, but is connected in a slightly different manner as shown in *Fig.* 12, in which the up station is shown

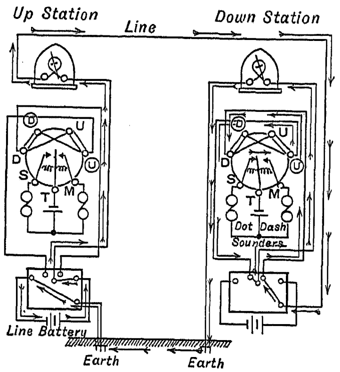

FIG. 12.

sending a " dot," *i.e.* with the left-hand key depressed.

§ (7) DOUBLE-CURRENT WORKING.—In the single-current Morse system already described it was pointed out that the tongue of the relay is given a bias in order to ensure that the local circuit will be broken when the signalling current ceases. On long lines, this bias, combined with the retarding effects of the electrostatic capacity, causes a great reduction in the speed of working, and in order to signal at a fair rate of speed it is necessary to employ the double-current system of working. This system arranges for a reverse current to be sent to line during the intervals between the " marking " currents ; the use of this reverse or " spacing " current not only tends to wipe out quickly the lingering effects of the previous marking current, but also allows the relay tongue to be set neutral, which is its most sensitive adjustment. These advantages enable a greater length of circuit to be worked at a given speed with a double current set than with single current apparatus under the same conditions. The system is described more fully in the article "Telegraphy, Double Current System."

§ (8) MULTIPLE TELEGRAPHY.—The average speed of an operator on a key-worked circuit rarely exceeds thirty words per minute, which is equivalent to about 60 average messages per hour. The number of messages that can be sent between two stations in a given time is therefore limited, unless additional circuits can be provided. This latter is not an economical proposition, and as an alternative, telegraph inventors have devised methods of multiple telegraphy by means of which the message-carrying capacity of key-worked circuits may be increased. The various systems that are used in practice to achieve this object are as follows :

(i.) Duplex working which provides for the simultaneous transmission of two messages, one from each end of a single wire (see article " Telegraphy, Duplex ").

(ii.) Diplex working, which provides for the simultaneous transmission of two messages in the same direction over a single wire (see article " Telegraphy, Quadruplex ").

(iii.) Quadruplex working, which provides for the simultaneous transmission of four messages, two from each end of a single wire (see article " Telegraphy, Quadruplex ").

(iv.) Multiplex working, in which the use of the line is given exclusively to a number of operators in succession for short recurring periods of time (see article " Telegraphy, Multiplex "). With methods (i.) and (ii.) a total of about 120 average messages could be transmitted in one hour between two terminal stations connected by a single line. With method (iii.) a total of about 240 messages could be dealt with in one hour, whilst with method (iv.) a total of 120 to 800 messages per hour according to the number of operators employed and channels available.

§ (9) THE WHEATSTONE AUTOMATIC SYSTEM.—As already pointed out, the average speed of an operator on a key-worked circuit is about 30 words per minute. To work at a speed much in excess of this entails a great strain on the operator ; he becomes tired, mistakes are made, and time is therefore lost in making corrections. It was early recognised that a mechanical method of transmission would be necessary if the traffic-carrying capacity of a circuit was to be increased by increasing the speed of signalling.

An effective method of doing this was devised by Wheatstone in his automatic system, in which the Morse key was replaced by a machine capable of sending Morse signals at a maximum speed of 600 words per minute, the signals at the receiving end being recorded in ink on a paper ribbon. The dot and dashes forming the letters composing the words of the message to be transmitted were represented

by holes punched in a paper slip which in passing through the transmitter determined the signals sent out by it.

A Wheatstone automatic set consists of three special instruments, namely:

(i.) A perforator by means of which holes are punched in a paper slip to represent Morse signals.

(ii.) A transmitter which sends out the signals to line in accordance with the holes punched in the paper slip.

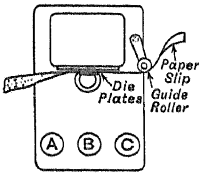

FIG. 13.

(iii.) A receiver which records the incoming dot and dash signals in the Morse code on a paper ribbon.

In addition to the above, key-worked Morse apparatus is required for speaking purposes, so that corrections and acknowledgments may be interchanged.

The plan of the perforator is shown in *Fig.* 13. It contains five steel punches, which are operated by purely mechanical means to punch the groups of holes corresponding to the signals to be transmitted in a slip of oiled paper. A, B, and C represent the heads of three plungers which the operator strikes with rubber-tipped mallets. When plunger A is struck, perforations representing a dot, viz. $\overset{\circ}{\underset{\circ}{\circ}}$ are punched in the slip. The operation of B causes the centre or feed-hole only to be punched, viz. $\circ$, which is equivalent to a space. The depression of C punches $\overset{\circ}{\underset{\circ}{\circ\ \circ}}$ in the slip and represents a dash. The centre holes serve as a means for propelling the paper by star-wheels through the perforator and transmitter. When the perforated slip is passing through the transmitter, the holes above the centre line determine the commencement of the marking currents and those below determine their termination.

Where the amount of perforating is considerable, compressed air is sometimes used for actuating the plungers in order to reduce the labour required to punch a number of slips. The arrangement adopted consists of three small cylinders, one for each plunger, containing pistons controlled by valves which are worked by three pianoforte keys. The depression of a key opens one of the valves and admits air to the corresponding cylinder, thereby driving down the piston which strikes the perforator plunger placed immediately underneath. In this manner as many as four slips may be perforated simultaneously. By working two perforators from one set of three keys, as is the usual practice, eight slips may be perforated simultaneously by one operator.

The latest development for the preparation of the slips consists in using keyboard perforators. With these instruments an operator merely depresses the keys of a keyboard which is similar to that of a typewriter. The operation of a single key selects, through a combined mechanical and electrical arrangement, the appropriate set of punches and forces them through the slip. It will be readily understood that the use of such instruments increases enormously the speed of preparing the slips and reduces the labour of operation to a minimum. The Gell and Kleinschmidt perforators are the most extensively used types for Wheatstone purposes.

Fig. 14 shows a portion of a slip perforated to represent the word "Telegraphy."

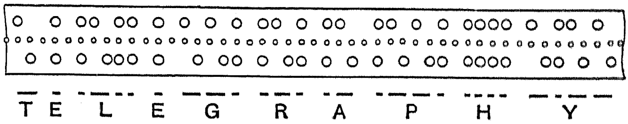

FIG. 14.

The arrangement of the electrical mechanism of the transmitter is shown in *Fig.* 15. The perforated slip is drawn through the instrument by means of the star-wheel W, which gears with the centre row of holes and is driven by the transmitter mechanism. DU is a compound lever consisting of two parts, D and U, insulated from each other

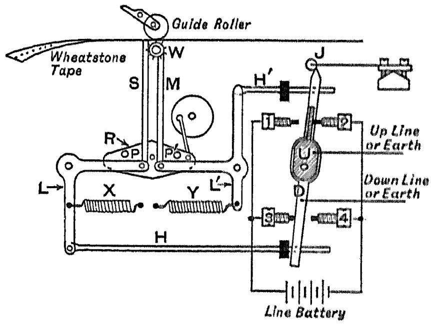

FIG. 15.

and connected respectively to line and earth at an up station. The four contact screws, 1, 2, 3, and 4, between which the compound lever moves, are connected to the line battery as shown. Their adjustments are such that when D is in contact with 3, U is in contact with 2, and when D moves over to 4, U moves to 1, so that alternating marking and spacing currents are sent to line when the

lever oscillates. The steel roller J, fitted at the end of a flat spring, exerts pressure against the upper end of the compound lever and thus ensures its quick action. R is a rocking lever, pivoted at its centre and actuated by the mechanism of the instrument. Fixed to this lever are two projecting pins, P and P′. The bell-crank levers L and L′, which control the movements of the compound lever through the rods H and H′, are normally held against the pins P and P′ by the spiral springs X and Y. To the ends of the bell-crank levers are attached the vertical rods S and M, placed one on each side of the star-wheel. When the transmitter is running without slip the rocking lever R oscillates; the rods S and M are free to move up and down, and as a consequence the bell-crank levers L and L′ follow the up and down movements of the pins P and P′. The motion is communicated to the compound lever by the rods H and H′, and reversals of current are sent to line. The insertion of unperforated slip in the transmitter presses down the rods S and M, which then hold the levers L and L′ in such positions that they are not actuated by the pins P and P′. The compound lever DU, as a consequence, does not move, resulting in a permanent current being sent to line. If a prepared slip is passed through the transmitter, the perforations determine the upward movements of the rods S and M. When M passes through a hole in the slip, above the centre line, the lever L′ is free to rise and the rod H′ moves the top of the compound lever to the right so that a marking current is sent to line *via* contact 3. The duration of this marking current depends upon the position of the succeeding perforation below the centre line; if it is directly opposite, as for a dot, then on the reverse movement of the rocking lever R, M will be withdrawn and after one oscillation of R, S will be free to rise and L will cause the rod H to push the lever DU to the opposite contacts so that the marking current will be terminated and a spacing current sent to line. If, however, the succeeding hole below the centre line is not opposite, as for a dash, the marking current is kept on until S is free to rise, which in this case occurs after three oscillations of R, a length of time equivalent to that of three dots. This is the standard length of a dash, so that the instrument automatically sends accurate Morse signals to the line.

The power required to drive the transmitter mechanism is often provided by the gradual descent of a heavy weight acting through a train of wheels. Sometimes clockwork is used, but the latest practice is to employ a small electric motor.

The instrument is provided with a lever for varying the speed and with a stop and start switch. *Fig.* 16 shows diagrammatically the internal connections of a Wheatstone transmitter. The switches I_1, I_2, I_3 are actuated simultaneously by the stop and start switch. In the start position shown the switches

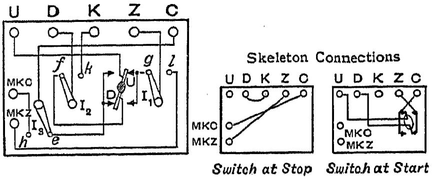

FIG. 16.

I_1, I_2, I_3 are on contacts *g, f, e* respectively; when the switch is put to the stop position the switches I_1, I_2, I_3 leave contacts *g, f, e* and move over to contacts *l, k, h* respectively. The skeleton connections for each position are also given in *Fig.* 16.

The Wheatstone receiver is similar in principle to the polarised relay and is represented in *Fig.* 17. The two electromagnets

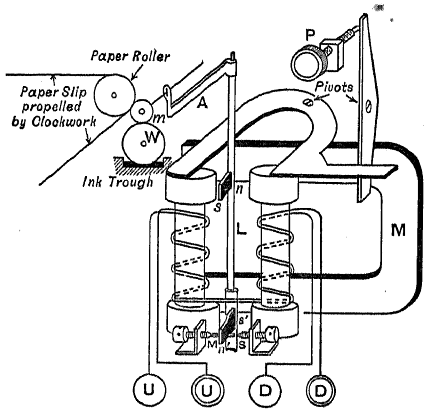

FIG. 17.

have cores of carefully annealed soft iron and are wound with two separate coils each of resistance 100 ohms. The terminals U (U) D (D) may be joined as desired by brass straps so that the coils may be connected in series or parallel. The armatures ns and n_1s_1 are polarised by the permanent magnet M. The lower armature is fitted with contact points and moves between contact screws S and M, to which a local sounder may be joined for key working. To the upper part of the armature spindle L is fixed an arm A carrying the marking disc m, which is geared to the clockwork of the receiver and takes up ink from the periphery of the inking disc W, which revolves in an ink trough. The passage of a current from U to D through the coils

causes the armature to be moved towards the left; the disc m is thus brought into contact with the moving paper slip and a dot or dash registered according to the duration of the current in the coils. The slip is drawn through the receiver past the inking disc by two friction rollers, one of which is driven by the mechanism of the instrument. The motive power for the latter may be a coiled clock-spring, a descending weight, or a small electric motor, having its speed regulated by a fly-governor situated in the receiving instrument. P is an adjusting screw which, acting through a system of levers, is able to move the electromagnets bodily and thus vary their positions with respect to the armatures. By this means the instrument may be given a spacing or marking bias or set neutral as desired.

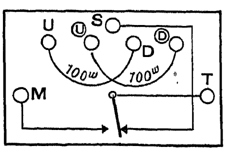

FIG. 18.

The instrument is provided with two levers, one for stopping and starting the mechanism and the other for controlling the speed. For diagrammatic purposes the internal connections of a receiver are represented as in *Fig.* 18.

The speed at which the Wheatstone system may be worked is limited, apart from line considerations, by the rate at which the receiver will record. This rate is limited because the self-induction of the coils of the electromagnets prevents the rapid magnetisation and demagnetisation of the cores. It is necessary to neutralise the effects of self-induction where a greater speed than 300 words per minute is required. This is done in practice by introducing a condenser shunted by a resistance into the circuit at the receiving end, as shown in *Fig.* 19. When the current is first made and the battery E.M.F. applied the resulting current in the electromagnet is increased above its permanent value by that required to charge the condenser and reduced by the self-induction of the coils. These two effects tend to balance each other and the current reaches its permanent value more rapidly. Immediately the line current ceases the self-induction of the coils sets up an E.M.F. tending to prolong the current in the direction of the arrow a; the discharge from the condenser, however, is in the opposite direction, as indicated by the arrow b. By suitably adjusting the capacity, K, of the condenser and the resistance, R, of the shunt it is possible to make these effects neutralise one another and the break is made more sudden. The general rule adopted is to make $KR^2 = L$ where L is the coefficient of self-induction of the receiver coils.

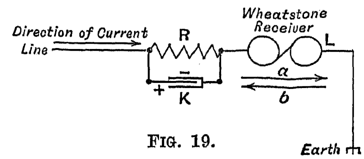

FIG. 19.

The connections of a Wheatstone automatic simplex circuit are shown in *Fig.* 20, in which the arrows show the path of a spacing current from the up station. In order to receive on the Wheatstone receiver at the down station it is necessary for the switch of the double-current key to be in the "receive" position. For speaking purposes sounders are connected in the local circuits of the receivers, but for simplicity are not shown in the figure.

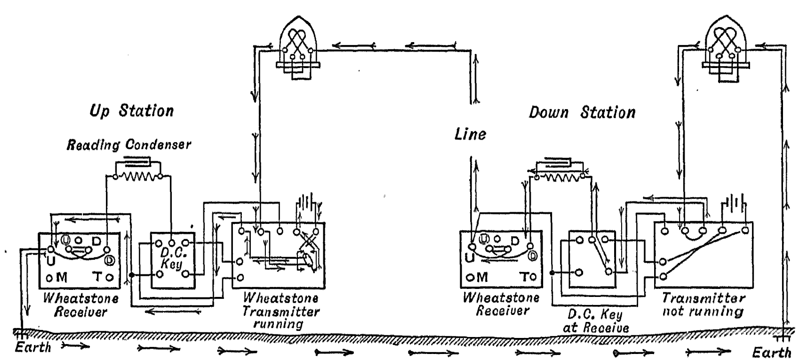

FIG. 20.

Wheatstone simplex circuits are used extensively for the transmission of press telegrams to all parts of Great Britain, and are known as "news" circuits. As a rule one transmitting office serves a number of offices on one wire. The terminal and intermediate receiving stations require a Wheatstone receiver only, but Morse apparatus is

fitted for giving acknowledgments to the transmitting office.

Either the bridge or differential method may be used for duplexing the Wheatstone system, but on long-cable circuits the former is invariably used, experiments having proved that under equivalent conditions the bridge method gives a greater speed than the differential.

During the last few years a form of Gulstad vibrating relay has been designed by the British Post Office, which has enabled the Wheatstone system to be worked differential duplex at a greater speed than bridge duplex. This form of relay will be described later.

§ (10) THE LONDON INTERCOMMUNICATION SWITCHING SYSTEM.—It is evident that to supply each circuit at a large telegraph office with a separate battery would require a large number of cells, the prime cost and maintenance of which would be very high. The number of cells may be considerably reduced by working a number of circuits in parallel from one battery of comparatively low resistance. This method of working is known as the Universal Battery System and is described more fully in article "Telegraphy, Universal Battery System." At small offices this system cannot be applied advantageously; at such offices, however, the introduction of the Central Battery System (see article "Telegraphy, Central Battery System") has allowed batteries to be dispensed with on those circuits converted to central battery working, resulting in a considerable saving in annual charges.

Most of the telegraph offices in the London Metropolitan area are connected to the Central Telegraph Office on the Central Battery System. To expedite the transmission of messages from one such office to another, and avoid the delays due to the internal circulation of messages at the central office, the whole of the local circuits are connected to an intercommunication switch by means of which the local offices may be connected together to signal their messages direct to one another.

The principle of the intercommunication switch is similar to that of the multiple telephone switchboard.[1] It consists of a number of sections each containing what are termed "home" and "multiple" panels, and subdivided vertically into three parts corresponding to the positions of three operators. To the "home" panel is brought a certain number of lines sufficient to afford a convenient load for three operators. These lines terminate on switch-springs (sometimes called "jacks"), and are associated with calling lamps. In the "multiple" panel are the switch-springs of all the lines connected to the intercommunication switch, and each operator has easy access to the switch-spring of any circuit.

Fig. 21 shows the connections of a local office to the switchboard. A polarised sounder (see article "Telegraphy, Central Battery System") wound to a resistance of 4500 ohms is used at the local office, and normally a current of 6 milliamperes flows from the 36-volt battery B at the central office to the line *via* the line

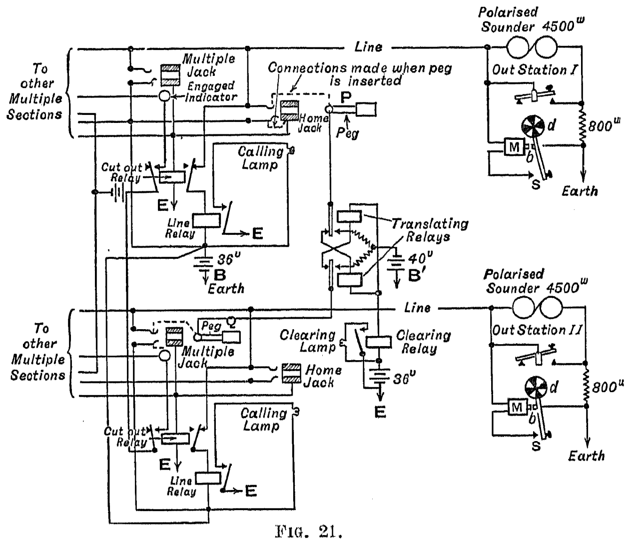

FIG. 21.

relay. This current is insufficient to operate the latter. An electromagnetic key indicator is provided at the local office for attracting the attention of the operator at the switchboard. It consists of a key attached to the armature *b* of an electromagnet M, and to a disc *d* which is divided into alternate black and white sectors. Above this disc is mounted a black plate cut with spaces to show normally the black sectors of the key disc. When the key is depressed it makes contact with a stop *s*, completing a circuit through the electromagnet M, and rotating the disc so that the white segments show through the spaces of the plate above. The current flowing through M keeps the armature attracted until the circuit is broken. When the armature returns to its normal position the disc rotates and the black segments are exposed.

A local office requiring connection with another office depresses his electromagnetic

[1] See "Telephony," § (4), etc.

key indicator and by so doing short circuits the polarised sounder and the 800-ohm resistance coil, thus reducing the resistance of the line circuit and increasing the current sufficiently to actuate the line relay. This relay closes a local circuit at the central office and causes the calling lamp to glow. The operator on observing this inserts one peg, P say, of a pair into the "home" jack corresponding to the calling lamp; the tip of the peg is thereby connected to the line and its shank connects the lower spring and socket of the jack. The latter connection completes a circuit from battery B through the cut-out relay, operating it and thus bringing about the disconnection of the line relay circuit. The calling lamp thus ceases to glow and a circuit through the engaged indicators is completed, corresponding to the particular circuit, on all the multiple panels. During this operation the line is automatically momentarily disconnected and the armature of the electromagnetic key indicator at the calling office is released, allowing the disc to show all black. This indicates to the calling office that the switch operator is ready to receive the necessary instructions for a connection. A speaking key and sounder, not shown in the figure, is provided to allow the operator to speak and ascertain the requirements of the calling office. Each office is denoted by a particular number, and having obtained the number of the office required, the operator, if the line desired is disengaged, puts the other peg Q of the pair into the required jack on the multiple panel. The calling office is then in a position to call the other office by means of his Morse key.

When thus connected the normal current flowing to each local office from the 36-volt battery is insufficient to actuate the translating and clearing relays. The depression of the Morse key at one of the offices reduces the resistance of its line circuit by short-circuiting the polarised sounder, thus increasing the line current to the value required for operating the corresponding translating relay. The operation of this relay repeats the signals to the other station from the 40-volt battery B′. When the offices have finished either may clear the line by depressing the electromagnetic key indicator. This action increases the current sufficiently to operate the clearing relay, which closes a local circuit, causing the clearing lamp to glow, thereby attracting the attention of the operator, who immediately removes the pegs, thus momentarily disconnecting the line and releasing the electromagnetic key indicators at the local offices. The offices are thus warned that the connection has been severed.

§ (11) CONCENTRATORS.—In large telegraph offices where there are many minor circuits carrying little traffic during the day it is the practice in Great Britain to connect the circuits to what is termed a "Concentrator." The object of so doing is to save space and operating charges by distributing the traffic of all the circuits connected to it among a relatively small number of operators and apparatus sets. Thus one operator is able to deal with messages from several offices in succession without leaving his instrument, the necessary circuit changes being made by a switching clerk at the concentrator.

Fig. 22 shows the connections of a circuit to a concentrator: each concentrated line is passed through a separate switch spring and lamp indicator. When an out office key is

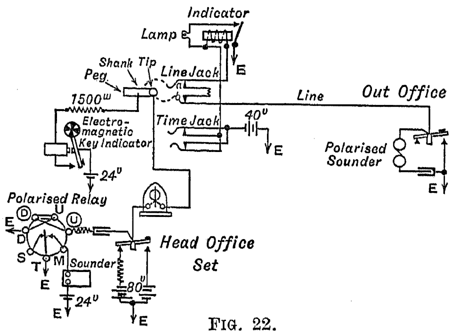

FIG. 22.

depressed, current flows from the 40-volt battery at the head office through the lamp indicator coil and line. The indicator attracts its armature, thereby closing a circuit through the lamp and causing it to glow. On observing the latter, the switching clerk inserts a disengaged peg, which is connected through a flexible cord to a set of apparatus, into the corresponding jack. This operation causes the outer springs of the jack to break from the inner springs *a* and *b*, thus disconnecting the circuit through the lamp indicator coil, which therefore allows its armature to fly back and extinguish the lamp. The tip of the peg connects a head office set of apparatus with the spring of the jack joined to the line and enables signals to be exchanged. The shank of the peg connects an electromagnetic key indicator to the spring joined to the coil of the lamp indicator. When the head office operator wishes to give the clearing signal to the switching clerk, he actuates the electromagnetic key indicator. This completes a circuit through the lamp indicator coils and the lamp glows. The switching clerk on observing this removes the peg from the jack, which by breaking the circuit through the electromagnetic key indicator restores the latter to normal, thus changing the appearance of its disc and warning the operator that his set has been disconnected.

§ (12) Repeaters.—To enable a high-speed telegraph system to be worked satisfactorily on a long circuit it is often necessary to introduce at an intermediate point a set of apparatus known as a Repeater. A repeater consists essentially of a sensitive polarised relay capable of being actuated by small currents at a high rate of working. The tongue of this relay retransmits signals of the same direction and duration, but of increased strength to those received by the repeater, to the distant terminal station. The introduction of a repeater practically divides a line into two independent circuits, and as the speed of a circuit is inversely proportional to the square of its length, a repeater at the centre of a circuit increases its speed nearly fourfold (see article "Telegraphy, Repeaters").

§ (13) Vibrating Relays.—The use on high-speed circuits of a modified form of polarised relay, known as the Gulstad relay, has in a number of cases allowed the withdrawal of repeaters. The construction and electrical connections of the Gulstad relay are such as to render it considerably more sensitive than the Post Office Standard relay, and as a result it is capable on difficult circuits of giving a higher working speed. The principle of the instrument is illustrated in *Fig.* 23. It is of

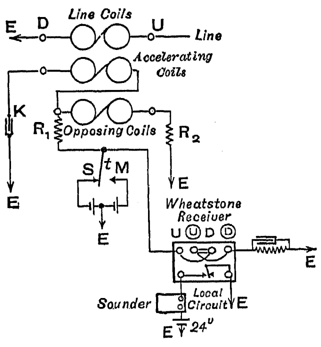

Fig. 23.

the polarised type and in addition to the "line" coils has two extra windings, known respectively as the "accelerating" and "opposing" coils, on the same cores. These windings are joined to external resistances and condensers as shown. The play and bias of the tongue may be varied by adjustable contacts, and in addition the width of the air-gap between the electromagnets and the armatures may be easily decreased or increased. A current through any of the coils from right to left tends to move the relay tongue *t* to the marking contact, resulting in a momentary rush of current through the accelerating coils to charge the condenser K. The direction of this current is such as to tend to hold the tongue to "marking" and thus to ensure a good contact. At the same instant, however, a current flows through the opposing coil and resistance R_2 in a direction which tends to move the tongue to spacing. As soon as the latter current exceeds the rapidly diminishing current that is charging the condenser, the relay tongue will commence to move towards the spacing contact. Immediately it leaves the marking contact the condenser K discharges in a spacing direction through both coils, thus hastening the motion of the tongue and reducing its time of transit from one contact to the other. A similar sequence of events take place when the tongue reaches the spacing contact. The relay tongue therefore vibrates between the contacts under the action of the currents derived from the local battery. The rate of vibration may be varied by altering the values of the resistances R_1 and R_2 and the capacity in K. It is usual to adjust these resistances so that the tongue vibrates at approximately the same speed as it would under the influence of reversals from the distant station's transmitting apparatus. In these circumstances the line currents merely control the movements of the tongue when actual signalling is taking place. For instance, if a marking current is passing through the line coils, it overpowers the effect of the current passing through the opposing coil, thus retaining the tongue on the marking contact. When the marking current in the line coils falls below the value of the local current in the opposing coil, the latter causes the relay tongue to move towards the spacing stop. It is this action of the local current in causing the relay tongue to start moving towards the opposite contact before the line current has fallen to zero, that enables the Gulstad relay to give a greater working speed on a difficult line than the Post Office Standard relay. Briefly it may be stated that when dots, that is, reversals, are passing through the line coils the relay tongue vibrates in unison with the transmitting apparatus; and when dashes or spaces are passing the vibration is controlled.

The coils of the Gulstad relay are so joined that it is only possible to use the instrument with simplex or bridge duplex circuits. A modified form of the instrument has been devised by the British Post Office for use on differential duplex circuits by the addition of another winding to the Post Office Standard relay so as to allow it to be connected in the usual way on differential sets and thereby increasing the rate of working.

§ (14) Line Construction.—For the maintenance of uninterrupted telegraphic communication between two stations it is highly essential that the line should be well constructed and properly insulated throughout its length. There are two classes of line construction,

viz. *overhead* and *underground*. For overhead work bare wire is used, attached to insulators supported by poles, which are generally erected along the sides of roads, railways, and canals. In large towns overhead wires are often necessarily carried over house-tops; in such cases the wires are supported by iron standards fixed on the roofs. Underground lines, or cables as they are termed, consist of line wires surrounded by an insulating covering throughout their entire length, and, as a rule, are protected by being laid in earthenware ducts or iron pipes. Overhead lines are less costly to construct than underground and possess the great advantages of being easy of inspection and repair. On the other hand, they are affected by climatic conditions and storms, so that the traffic over them is more liable to interruption than is the case with underground lines.

The poles for supporting overhead line wires may be of iron or wood. The latter is extensively used in this country and is much cheaper than iron, but in some tropical countries where timber is subject to insect depredations it becomes essential to use iron poles exclusively. The timber used for telegraph poles is generally Scandinavian red fir, which is treated by a preservative process to prevent decay. The most satisfactory method of treating timber is known as the Ruping process, which consists in forcing creosote into the pores of the timber under hydraulic pressure, the quantity injected being about 6 lbs. per cubic foot. Care must be taken to use only perfectly dry and well-seasoned timber.

The point of attachment of the line wire to the pole must be carefully insulated. This is done by binding the wire to an insulator fixed to a wooden cross-arm which is bolted to the pole. For efficiency the material of the insulator should possess a very high specific resistance and mechanical strength, but should be non-porous and non-hygroscopic so as to minimise any tendency to absorb or condense moisture from the air. In Great Britain insulators of glazed porcelain and of earthenware are exclusively used. The latter is the cheaper, but its insulating properties are not so good as porcelain so that its use is restricted to minor lines. Glass possesses a very high insulation resistance and a smooth hard surface, but as it is very brittle and hygroscopic it is not so suitable for countries that have a damp climate; it is extensively used in countries possessing a comparatively dry atmosphere, such as the United States of America. Insulators are very carefully designed so that they may be mechanically and electrically efficient. The standard type, known as the Cordeaux Insulator, is shown in half elevation and section in *Fig.* 24. It is of the double-shed pattern, a form which gives a large surface area over which the current leaking from the line must travel before it can reach the earth through the fixing bolt. The outer surface B is kept clean from dust or dirt by rain, while the inner surface A is kept dry during wet weather so that the electrical efficiency is high in all circumstances. The insulator is screwed on to the steel spindle S, a rubber washer W being inserted as shown to allow any unequal expansion of the spindle and insulator to occur without fracturing the latter. Special shaped insulators are on terminal poles used for leading in wires from poles to buildings, and where sharp turns occur along a route.

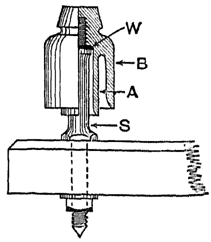

FIG. 24.

It is necessary to fix an earth wire to a telegraph pole in order to provide a direct path to earth for currents leaking over the insulators. In the absence of an earth wire, a current leaking over one insulator may pass to earth along other wires on the pole and so interfere with their working. The earth wire is carried a few inches above the pole roof so that it may serve as a lightning conductor; it is secured to the pole by staples and is fixed between the bolt-head and washer which secures each arm to the pole. A good contact with the earth is made by coiling the earth wire into a flat spiral at the base of the pole. Each arm is earth-wired by taking one turn of the wire round the arm between the insulators and connecting with the main earth wire under the washer of the arm bolt. Galvanised iron wire is generally used, but where acid fumes are present copper wire is substituted.

For use as an overhead conductor a wire should possess tensile strength, durability, and electrical conductivity to a high degree. The most suitable material is hard drawn copper wire, but owing to its high cost its use is restricted to high-speed circuits and districts where chemical impurities in the air would cause serious deterioration in iron wire. The copper wire generally used for the line conductor has a weight of 150 lbs. per mile and a resistance of 5·9 ohms per mile. The majority of overhead telegraph circuits are of galvanised iron, of which there are two gauges in extensive use, viz. 400 lbs. and 200 lbs. per mile, having resistances of 13·32 and 26·64 ohms per mile respectively.

It is very important that a telegraph line should be stable and able to withstand any fluctuating forces to which it may be subjected, and to this end it is necessary that the poles

should be firmly erected. They are usually inserted in the ground to a depth of about one-fifth of their length, and the latter may vary from 16 to 80 feet according to the number of wires required on the route. To keep the poles in a vertical position, and to counteract the effect of forces acting upon them due to curves along the route or to wind pressure in exposed regions, stays and struts are used. A stay consists of stranded galvanised iron wires firmly anchored in the ground and attached to the pole at that point at which the resultant of the forces upon it (due to tension of the line wires) may be supposed to act. Double stays, one on each side of the pole, are often required. It is usual on important lines to fix stays on each side of the pole and along the direction of the route in order to limit the extent of damage in the event of a breakdown in any one span. Wooden struts serve the same purpose as stays and are fitted where it is impossible to use stays. Along routes carrying a large number of wires H poles and A poles are often used; each being built up of two poles which are braced together by iron rods, forming a structure somewhat similar in appearance to the letters by which they are known.

The stress to which a line wire is subjected depends upon the distance between the poles, the sag or dip of the wire, and its gauge. Temperature variations affect the stress, an increase of temperature increasing the sag and decreasing the stress. It is of very great importance to secure that the stress on a wire does not nearly approach its breaking stress, hence it is usual, in order to provide a suitable margin against contingencies such as wind pressure and snow accumulation on the wires, to regulate the tension so that at low winter temperature the stress shall not exceed one-quarter of the breaking stress of the wires.

For the jointing of aerial telegraph conductors practically only two kinds of joints are used. The most effective is that known as the Britannia joint, see *Fig.* 25. In this the ends of

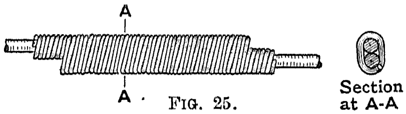

FIG. 25.

the wires to be jointed are laid side by side and a binding wire is whipped round the two until the right-hand portion is finished, then the left hand is dealt with in the same way and the joint finally soldered. If the conductors are of large section the interstices between them and the binding wire are filled up with small lengths of wire. Line wires of small gauge are often jointed by twisting their ends together for a distance of about 1¾ inches and closely wrapping them round the other wire for a few turns, finally soldering. All joints are given a coating of black varnish to prevent electrolytic action between the line wire and solder.

For underground telegraph circuits lead-covered paper-insulated cables are now invariably used. In this type of cable the conductors consist of annealed copper wire covered with a loose wrapping of paper, which, with the enclosed air, forms the insulating medium. There are many types of cable employed, designed to meet various requirements. In twin cables the conductors are each covered with two layers of paper, applied in strips wound spirally round the wire and bound with thread. The conductors are arranged in pairs, the two wires constituting a pair being uniformly twisted together so as to minimise inductive disturbances between adjacent circuits. In multiple twin cables the above principle of twisting or "twinning" is extended, two pairs being twisted together to form a core of four wires; two 4-wire cores are twisted together to form a core of eight wires, and two 8-wire cores twinned to form a 16-wire core. A definite colour scheme of paper wrappings is adopted to allow the wires of any particular pair to be readily identified. The interstices between the 8-wire cores are filled by insulated single wires or by pairs, and the whole is covered with two wrappings of paper and finally by a seamless sheathing of lead.

A special type of cable known as "screened" is employed where it is desired to use single wires with earth returns for telegraph circuits. Each conductor is covered with three wrappings of paper, one longitudinally applied and the others spirally. A helical winding of copper tape is lapped over the paper so as to completely cover it. All the screening tapes are connected together and to the lead sheathing which is earthed. Each conductor is therefore entirely surrounded by a continuous conducting shield which eliminates the major portion of the inductive disturbances between it and other circuits.

In some cases "composite" cables are used which are designed to accommodate both telephone and telegraph circuits. For telephonic purposes a large number of twisted pairs are provided, around which are placed a number of "screened" conductors for use as telegraph circuits.

Underground cables are laid either in cast-iron pipes or in ducts of suitable diameter made of earthenware or stoneware. The pipes or ducts are laid at a depth of from 14 inches under footways to 2 feet under roadways, and are generally arranged in sections of about 175 yards in length, openings

covered by joint boxes being left between the sections to facilitate the operations of drawing in and jointing the cables. No special foundation is required for iron pipes, but ducts require a firm support, and in some cases concrete is used for the purpose.

It is necessary with pipe lines to preserve their metallic continuity at joint boxes in order to prevent damage to the cable sheathing due to stray currents from other systems setting up electrolytic action. This is effected by fastening lead strip to the ends of the pipes and nailing it round the sides of the joint box, a method which is known as "bonding."

Each cable length is some yards longer than the pipe sections to allow for wastage in jointing. Exceptional care and cleanliness are necessary in the jointing of cables. Before the cable ends are opened out for jointing a lead sleeve is slipped over one of them, and when the joints have been made and soldered, the paper insulation is wrapped round each. The joints are then warmed by a charcoal brazier to eliminate all traces of moisture, and an outer layer of insulating paper is wrapped round the completed joint. Finally the lead sleeve is pulled over the joint and secured to the cable sheath at each end by a wiped plumber's joint.

As already mentioned, the earth is used for the return path for single-wire circuits between two telegraph stations. The earth is regarded as having no resistance, and its use instead of a second wire saves battery power and minimises the liability to stoppages due to faults. To secure a good earth connection at telegraph stations, earth plates of galvanised iron 2½ feet square are buried in the ground in a moist situation. A stranded copper wire is soldered to the plate, and to this the earth connections from all the circuits are led. At large offices several earth plates connected together are used, but at small offices earth plates are often dispensed with, the earth connection being made by means of clips screwed on to iron water mains. For efficiency an "earth" should have a resistance of less than 10 ohms, otherwise mutual interference between the circuits connected to it will ensue and satisfactory working rendered impossible. It is essential that the plates at all stations on a circuit should be of the same material, otherwise electrolytic action will take place and create a potential difference between them, causing what is termed "earth" currents to flow in the line wire. Earth currents are also caused by differences of earth potential due to causes not generally known, although they frequently accompany magnetic storms. They are often of such magnitude as to make working impracticable; in these circumstances communication may be maintained by discarding the earth return and converting the system into a metallic circuit. Although this method completely overcomes the disturbances arising from earth currents, it can only be adopted at the expense of some other circuit, and cannot be readily introduced where the universal battery system is in use. In such a case working may be maintained by interposing a condenser in the circuit which allows the rapid signalling impulses to pass along the line, but prevents the comparatively steady earth currents from flowing.

Steps must be taken to secure the immunity of telegraph plant from damage due to lightning. Open lines are chiefly subjected to the injurious effects of lightning, underground circuits being immune unless they are connected to overhead sections. To prevent the poles from being struck, the earth wire, as already explained, is carried above the pole roof to serve as a lightning conductor. The apparatus is protected from damage when the high-tension oscillatory currents due to a lightning discharge pass along the line, by the use of lightning protectors, which consist essentially of two plates of carbon separated by a small air gap, connected as a shunt to earth across the apparatus. The inductance of the coils of the apparatus offers a very great impedance to the passage of high-frequency lightning discharges, as compared with the resistance of the narrow air gap, so that the lightning discharges go direct to earth through the gap instead of damaging the instruments.

In localities where electric lighting and power circuits exist in the immediate neighbourhood of telegraph circuits it is necessary to safeguard the general public and telegraph plant from injury. The contact of bare overhead wires with a high-voltage power wire might have very serious consequences and involve loss of life and damage to property. To reduce such occurrences to a minimum, telegraph circuits are usually erected above power circuits, and "guard wires" efficiently earthed are provided between them, to prevent a broken telegraph line from falling directly on to the power wires. All telegraph circuits within the area affected by power circuits are fitted with "fuses" and "heat coils" to prevent damage to apparatus and risk of fire in the event of a contact occurring. The fuses consist of fine phosphor bronze wire enclosed within a glass tube and are fitted on the line side of the apparatus. A current exceeding one ampere would blow the fuse, thus disconnecting the circuit and preventing continued contact with the power wire. As telegraph instruments would be damaged by currents much smaller than one ampere, and it is not practicable to design fuses that will work with currents of much less than this amount, heat coils are provided in the line circuit, which are designed to disconnect the

apparatus from the line when a current of about one-quarter of an ampere flows through it for thirty seconds. The heat generated by this current flowing through the heat coil is sufficient to melt a soldered connection and thus to disconnect the instrument.

There are three classes of faults to which every telegraph circuit is more or less liable, viz. :

(i.) Disconnections which cause a total cessation of current. Intermittent disconnections may be caused by loose joints, dirty contacts, etc.

(ii.) Earths, which have the effect of increasing the outgoing current and decreasing or entirely stopping the received current. Full earth is the result of the line wire coming into contact with a good conductor connected directly to earth, such as a stay. Defective insulators may cause "partial earths," and branches of trees, etc., by blowing against the line wire, "intermittent earths."

(iii.) Contacts, which have the effect of causing currents flowing along one wire to pass into another. "Metallic" contact is caused by one wire becoming connected to another wire by a good conductor. "Partial" contacts are caused when wires are bridged by imperfect conductors. "Intermittent" contacts are produced by wires touching each other at intervals, often caused by the faulty regulation of the wires in the spans between the poles.

Underground wires are not so liable to the above faults as overhead, and when faults do occur they are chiefly due to imperfect manufacture and carelessness in the jointing and laying of the cables.

At telegraph offices into which a considerable number of line wires are led, test-boxes are fitted to facilitate the operation of testing for faults. A test-box provides a ready means of disconnecting and earthing wires, or of looping two wires, and in addition contains a galvanometer and battery for proving the continuity of circuits, for ascertaining the distance of a fault outside the office by systematic localisation, or for proving the apparatus in the office itself. In one form of test-box each line wire is joined to a metallic socket, known as a test-hole, and the wire from the corresponding instrument to another test-hole immediately below. A metallic U-link is inserted into the test-holes to connect the instrument to the line. Pegs and cords are also provided so that the lines and instruments may be crossed with one another.

For maintenance purposes periodical tests are made of telegraph wires to ascertain their conductor and insulation resistances. Such tests are valuable in indicating the presence of incipient faults, so that steps may be taken to prevent them from developing and causing a breakdown. To carry out the tests an instrument known as a "Bridge Megger" is now employed by the British Post Office. It combines the functions of the Wheatstone Bridge and Megger, the latter being a direct-reading ohmmeter. To measure the conductor resistance of a line wire, its distant end is earthed, whilst at the testing-office it is joined to the testing instrument to form the unknown resistance arm of a Wheatstone Bridge network. For the insulation test the distant end of the wire is disconnected, while the home end is connected to the "Megger," which, when operated, indicates, by means of a pointer moving over a graduated scale, the total insulation resistance of the line in megohms.

The insulation resistance of open lines depends upon the condition of the insulators and the state of the weather; it may vary from 400 megohms per mile in dry weather to 200,000 ohms per mile or less in wet weather. When the insulation resistance of an open line falls below the latter value it is generally assumed to indicate the presence of specific faults which need attention. Underground wires under normal conditions possess high insulation resistances usually of the order of 5000 megohms per mile; should the value fall below 500 megohms per mile, the cable is in need of attention.

A. E. S.

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TELEGRAPHS, TYPE-PRINTING

EVER since the establishment of the first practical telegraph in 1837 numerous inventors have striven to produce systems of telegraphy in which the messages could be automatically recorded in Roman characters. In this article no attempt will be made to trace the historical development of type-printing telegraphy, and only the most important systems that have been developed and are in use at the present day will be described.

Type-printing telegraph systems may be divided into four classes, viz. :

(1) Step-by-step systems.

(2) Systems in which a single-current impulse is transmitted for each character.

(3) Systems using the Morse code.

(4) Systems using the five-unit code.

§ (1) STELJES SYSTEM.—Apart from Stock tickers, the principal step-by-step system in use is the Steljes, which is a development of the Wheatstone A B C non-recording instrument. As a rule, step-by-step systems employ alternating current for signalling purposes, each character being represented by a certain number of alternations. The Steljes system consists of the original Wheatstone A B C transmitting apparatus, known as the communicator, and a special form of instrument invented by Steljes for recording incoming signals in Roman type on a paper tape.

The communicator which furnishes the alternating current required for signalling purposes, consists of a magneto-electric machine worked by turning a handle, so that batteries are not required. A dial is fixed above this generator, and its circumference is divided into thirty equal spaces, upon which are inscribed the letters of the alphabet and other characters. Arranged around the dial and opposite the spaces are thirty keys, each key representing a certain character. The signalling of a particular word is effected by turning the handle of the generator and depressing the keys in the order in which the letters occur. The armature spindle of the generator is geared to a pointer which is capable of moving step by step round the dial; the gearing is such that fifteen complete revolutions of the armature will cause the pointer to make one complete revolution of the dial. The depression of a key arrests, at that part of the dial, the motion of the pointer and at the same instant disconnects the generator from the line; the pointer is released immediately a second key is operated. When signalling a word the pointer is always started from the space marked zero, so that a number of current alternations is sent out for each letter signalled.

The Steljes recorder contains two electromagnets connected in series with the line. The armature of one of the electromagnets is polarised and therefore responds to the incoming current alternations; by so doing it acts upon an escapement wheel and moves a type-wheel round step by step until the letter signalled is opposite the paper slip upon which the record is to be made. The other electromagnet is non-polarised and its armature remains attracted until the alternations cease; when this occurs the printing mechanism is released, the paper brought into contact with the periphery of an inked type-wheel, and the required letter which has been previously brought step by step to the correct position is printed. The restoration of the printing mechanism, which takes place immediately after the printing operation, steps the paper slip forward a distance of one letter space, so that it will be in the correct position for the printing of the next letter.

A great advantage of the Steljes system is that no technical skill in sending and receiving messages is involved on the part of the operators. Its speed of working is comparatively slow, however, being limited by the number of alternations that have to be sent and the rate of manipulation of the communicator, which at the best will probably not exceed twenty words per minute.

§ (2) HUGHES SYSTEM.—The chief representative of the second class of type-printing telegraphs is the Hughes system, which was invented as far back as 1854 and is still extensively used, especially on continental circuits. Only one current impulse is required to secure the record of each character; but the spacing interval of time between characters varies and depends upon the preceding signal. The apparatus is almost entirely mechanical in its action, the only electrical operation being the release of the armature of an electromagnet. The characters are printed in Roman type by bringing a paper tape into contact with the periphery of an inked type-wheel, and the latter, unlike those of the step-by-step systems, revolves continuously.

Fig. 1 shows diagrammatically the transmitting arrangements. Signalling is effected

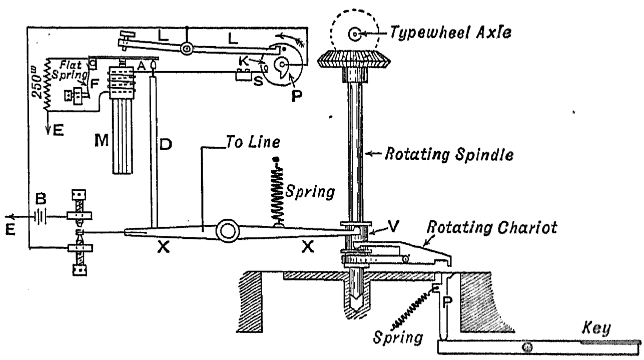


FIG. 1.

by means of a keyboard, consisting of 28 keys arranged in two rows like those of a piano. Each key is lettered to correspond with the characters on the type-wheel. To send a

particular character it is necessary to depress the corresponding key, and by so doing a pin P is raised in the path of a rotating arm called the chariot. The pins corresponding to the different keys are arranged round the circumference of a plate over which the chariot passes during its revolutions. When the chariot encounters a raised pin, the chariot lever causes a sleeve V on the vertical spindle to move downwards, and this sleeve by acting upon the end of the pivoted lever XX moves it in such a direction as to connect the signalling battery B to the line. The spindle carrying the chariot gears with the type-wheel axle, which is driven by an electric motor. The type-wheel is placed in such a position on its spindle, that when the chariot passes over a raised pin the corresponding character on it has been brought into the correct position above the paper tape, so that a "home" record of the outgoing signal may be printed. The printing mechanism is released when the lever LL is operated by the upward movement of the armature A of the electromagnet. For printing the "home" record the armature is moved mechanically by the rod D, which rests on the lever XX.

The sending and receiving instruments are exactly similar, but for successful working it is essential that the type-wheels at the two stations should run at practically identical speeds and keep in phase. The speed of each instrument is regulated and kept constant by a centrifugal governor, and any small deviation from phase is corrected by every current sent.

Incoming signals pass *via* lever LL, stud K, spring S, through the electromagnet to earth. The Hughes electromagnet consists of a coil of wire wound upon soft-iron cores attached to the poles of a powerful permanent horseshoe magnet M. A is a soft-iron armature which is normally attracted to the cores against the force of an antagonistic flat spring F. The direction of the current from the line through the coils is such as to oppose the magnetism of the cores sufficiently to allow the spring to overcome the attraction of the electromagnet and to pull the armature away. The upward movement of the armature depresses the right-hand end of the lever LL and by so doing causes the cam P to make one revolution by coupling its axle with a shaft geared to the type-wheel axle. The cam P carries with it the stud K, which as a result breaks contact with the spring S and disconnects the electromagnet until the contact is remade, when K resumes its normal position. This occurs after the cam P has replaced the armature A on the cores of the electromagnet through the medium of the lever LL. As the path through the electromagnet is disconnected immediately after its armature has been operated, an alternative path is necessary in order to facilitate the discharge of the line. This is provided by arranging for the armature to earth the line through a resistance equal to that of the coils directly it comes into contact with the lever LL. The axle of the cam P carries a number of other cams, by means of which the position of the type-wheel is corrected, the paper raised against the type-wheel for printing the latter, and finally stepped forward so as to be in position for the printing of the next character.

The maximum working speed of the type-wheel is about 150 revolutions per minute, but as a rule the speed does not exceed 120 revolutions per minute. At this speed the average output is about 30 words per minute; it is owing to this low output that the Hughes is now being replaced by multiplex systems.

§ (3) Morse Code System.—The only printing telegraph system using the Morse code, in extensive use at the present day, is that invented by Creed. It is used in conjunction with Wheatstone automatic apparatus, but arranges for the received signals to be produced in the form of ordinary perforations on Wheatstone slip, instead of dots and dashes on a paper tape (see article "Telegraph, The Electric," for Wheatstone Automatic System). The received perforated slip is afterwards passed through a Creed printer which mechanically translates the signals and prints in Roman type on a paper slip. By such means the loss of time incurred in writing up or typing the received Morse signals at terminal stations is greatly reduced, whilst at retransmitting stations the labour of receiving the signals and then preparing a new Wheatstone slip for retransmission purposes is entirely saved. From the foregoing it will be gathered that the Creed apparatus itself is used at the receiving end only and consists of two distinct items, viz. a perforator and a printer. Both instruments are worked by pneumatic power, the pressure used being about 30 lbs. per square inch. At the transmitting station an ordinary Wheatstone transmitter is used.

The incoming signals at the receiving station actuate a Post Office standard relay which repeats the signals to another relay within the Creed receiving perforator. This Creed relay is similar to the Post Office standard relay, but instead of a tongue its spindle carries a light arm A (see *Fig.* 2) which operates a small air valve V. This valve controls the supply of air to the piston P_1, and causes it to move in accordance with the line signals. The piston P_1 is mechanically connected to the piston-valve P_2, which admits compressed air to one side or other of the piston P, according to its position. The connections are such that when a marking current passes through the relay

coils the large piston P is driven to the right. The rods connected to this piston transmit the movement by means of the bell-crank levers X and Y and cause a striking rod S_1 to thrust the end of a correcting rod L_1 between the teeth of correcting wheel W_1, and also force the punch M_1 through the paper tape T. The movement of the piston P continuing, now causes the projection H on the rod N to come into contact with S_1, forcing it from L_1 and M_1, thus allowing the latter to withdraw to their normal positions under the action of retracting springs. When the line current is reversed the arm A moves in the opposite direction and causes the piston P to move to the left. This causes striking rod S_2 to act upon the right-hand correcting rod L_2 and punch M_2, and, as before, the projection H brings about their restoration to normal. The feed and correcting wheels are driven through a friction clutch by an electric motor. The object of inserting the end of a correcting rod between the teeth of the corresponding correcting wheel is to adjust the position of and momentarily hold the feed-wheel stationary, and therefore the paper tape, so that the paper may be perforated in the correct position. The right-hand punch is the spacing punch, and is actuated after the left-hand punch, which is the marking punch. Owing to the motion of the tape between successive punching operations, the spacing punch and the corresponding correcting wheel are given a "lead," so that the spacing perforation may, in the case of a dot, appear opposite the same centre hole as the marking perforation.

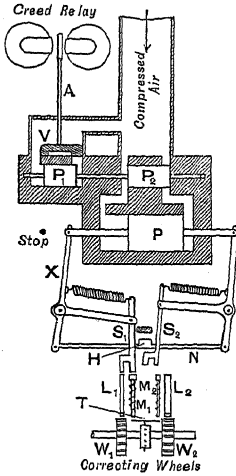


FIG. 2.

The machine will work at 200 words per minute, but for reliable working a speed of about 150 words per minute is regarded as the maximum.

The function of the Creed perforator, therefore, is to reproduce at the receiving station a facsimile of the transmitting tape at the sending station. In order that the received signals may be translated and printed in Roman type, the tapefrom the Creed perforator is passed into the Creed printer. In this the tape is drawn upward, letter by letter, by means of a star-wheel to pass in front of ten pairs of selecting needles. One needle of each pair is mechanically connected to a thin perforated plate which can take up one of two positions by longitudinal displacement, the latter arising when the corresponding selecting needle passes through a hole in the tape. The various positions that it is possible for the ten plates to take up provide a number of different combinations, and for each combination one particular passage is formed through the plates from a compressed air chamber B to a small cylinder C (see *Fig.* 3 for the principle of the Creed printer). Each character that has to be printed is provided

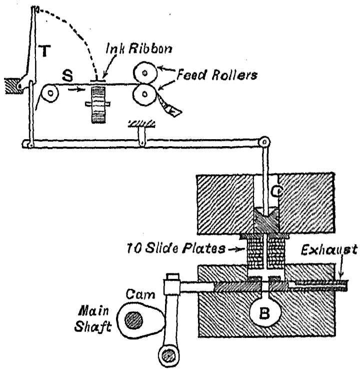


FIG. 3.

with a separate cylinder, whose piston is attached through a system of levers to the corresponding type-bar. The admission of compressed air to the cylinder C drives the piston outwards, causing the type-bar T to descend and print the character on the slip S.

Owing to the varying length of letters when represented by Wheatstone perforations in a paper tape, a variable vertical paper feed is necessary. The length of the feed is determined by the first blank space in the tape. The star-wheel for feeding the paper is actuated by a rack the vertical motion of which is normally limited by a group of ten space-levers. Any one of these levers is pressed out of the path of the rack, when either one of the corresponding pair of selecting needles pass through a hole in the tape, but where no holes occur in the tape, as for example between two letters, the space-lever at this point remains in its normal position and stops the downward movement of the rack. The subsequent upward movement of the rack to its normal position causes the paper to be fed forward a distance which will depend upon the length of tape occupied by the letter.

An electric motor is used to drive the main

shaft of the Creed printer, which carries a number of cams to perform certain functions. One cam gives the up-and-down movement to the paper-lifting rack, and another controls the admission of compressed air to the chamber B. Other cams actuate mechanism for moving the paper slip on which the message is printed, and for feeding the typewriter ribbon. The machine is capable of printing satisfactorily at a speed of 125 words per minute.

The Creed system is largely used for commercial work on long-distance circuits and is very suitable for heavy newspaper traffic.

§ (4) THE FIVE-UNIT CODE.—Most modern printing telegraph systems are based upon the use of a five-unit code for signalling purposes. The characters to be signalled are represented in this code by permutations of positive and negative current impulses. All characters are of equal length, the number of impulses for each being five, so that they all occupy the same time in transmission. Thirty-one permutations of positive and negative impulses are available for representing characters in a five-unit code. These suffice to provide for all the letters of the alphabet, but not the figures and secondary characters such as punctuation signs, etc., that are required. To accommodate all characters it is necessary to use "letter" and "figure" shift signals so that one particular combination may represent two distinct characters. The average word in the five-unit code is shorter than its equivalent in the Morse code in the ratio of about 5 to 8; hence a circuit whose maximum speed might be 160 words per minute with the former, would allow a Morse speed of only 100 words per minute.

(i.) *Baudot.*—The most important printing telegraph system using the five-unit code at the present day is that invented by Baudot in 1874. The Baudot system is remarkable for the ingenuity displayed in its mechanism, although for a number of important details the indebtedness to the Hughes apparatus is very apparent. The arrangement of the Baudot five-unit code is given in *Fig* 4, and its adoption made it possible to use the multiplex principle with complete success (see article ".Telegraphy, Multiplex"). The elementary principle of the Baudot system is represented diagrammatically in *Fig.* 5. At the sending station A, the five tapper keys comprising the keyboard of one operator are connected to five segments of a distributor. The latter is merely a segmented ring traversed by a brush which connects the segments successively to line. The five keys are manipulated by the fingers, sending out negative current impulses when depressed, and positive when in their normal positions. At the receiving

		Keys I	II	III	IV	V			Keys I	II	III	IV	V
A	1	●					P	%	●	●	●	●	●
B	8			●	●		Q	/	●		●	●	●
C	9	●		●	●		R	–			●	●	●
D	0	●	●	●	●		S	;			●		●
E	2		●				T	!	●		●		●
É	&	●	●				U	4	●		●		
F	F̲		●	●	●		V	'	●	●	●		●
G	7		●		●		W	?		●	●		●
H	H̲	●	●		●		X	,		●			●
I	o̲		●	●			Y	3			●		●
J	6	●			●		Z	:	●	●			●
K	(	●			●	●	t̲	.	●				●
L	=	●	●		●	●	[illegible]	[illegible]				●	●
M	)		●		●	●	FIGURES					●	
N	№		●	●	●	●	LETTERS						●
O	5	●	●	●									

FIG. 4.

station B, five successive segments of a distributor similar to that at A, are connected to a set of five electromagnets. The electromagnets form part of a Baudot receiver and when actuated they operate its translating

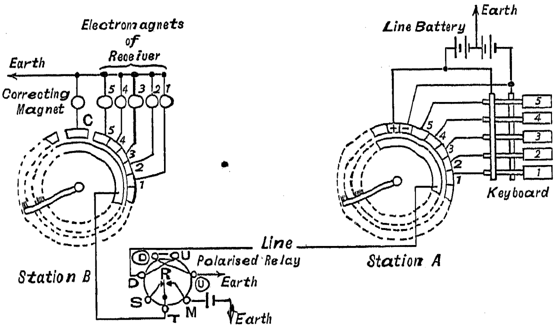


FIG. 5.

mechanism. A polarised relay must be inserted in the line at the receiving station B, because if the line were joined directly to the continuous ring of the distributor it would be impossible to operate the non-polarised electromagnets of the receiving instrument selectively by means of the negative and positive current impulses. By using a polarised relay joined up as shown, the electromagnets are worked only when a negative current passes from the line through the relay. In order that the signals may be transmitted and correctly received, two conditions must be complied with: (1) The sending operator must operate his keyboard just before the distributor brush reaches the segments allocated to his keys, and the latter (*i.e.* those keys depressed) must not be allowed to rise until after the brush has passed over the last

of the segments; (2) the brush arms at the two stations must revolve uniformly and keep in phase, so that when the sending brush is on segment 1 at A, the receiving brush is on segment 1 at B, and so on for successive segments, due allowance being made for line retardation (see article "Telegraphy, Multiplex"). This phase relationship must be maintained revolution after revolution.

The first of the above conditions is satisfied by providing a warning signal to the sending operator indicating to him the correct moment to depress his keys. This signal is known as the "cadence" and is given by a tap of the armature of an electromagnet forming part of the sending instrument and actuated by a current from the distributor. When any of the keys are depressed a mechanical locking device holds them down until they are released automatically by the next "cadence" signal so as to be in readiness for the next signalling operation.

To comply with the second condition it is necessary first of all to ensure a uniform speed of rotation of the distributor brushes by methods which will be described later. The manner in which the phase relationship between the two stations is maintained is as follows.

At station A, *Fig.* 5, two additional segments are provided on the sending ring for the transmission of a correcting signal, consisting of a negative impulse followed by a positive impulse. For their reception at station B, one large segment C, equal in size to two receiving segments, is provided. The station sending the correcting signal is called the "correcting" station, and the other the "corrected" station. The brush arm at the former is arranged to run at 180 revolutions per minute, which is equivalent to a speed of 30 words per minute; that at the corrected station is set to run at a slightly faster rate, usually about 182 revolutions per minute. The tongue of the polarised relay R at station B moves to the marking stop only when the brush at station A is on the segment marked –, so that the correcting electromagnet connected to segment C is operated only when the brush arms at the two stations are simultaneously on segments – and C respectively. The brush arms are in phase when they are on segments + and C respectively, in which case no correction occurs because the relay tongue is then on the spacing contact which is disconnected. The receiving brush arm, however, gains on the sending brush arm, and after a time is on segment C while the latter is on segment –; when this occurs the brushes are slightly out of phase, but not sufficiently so to cause indifferent working. A current, therefore, passes through the coils of the correcting electromagnet, attracting its armature which is arranged to uncouple, momentarily, the brush arm of the receiving distributor from the driving mechanism. The lag of the brush thus caused is sufficient to restore its phase relationship with regard to the brush arm at the sending station.

A set of Baudot apparatus consists of three principal items—viz. the Keyboard, the Distributor, and the Receiver.

The keyboard (see *Fig.* 6) has five tapper keys similar to those of a piano. To each key is fixed a vertical flat spring V joined to

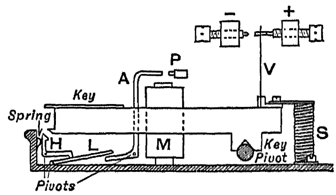


FIG. 6.

one of the segments of the sending ring. In its normal position the spring makes contact with the busbar connected to the positive pole of the line battery, and when the key is depressed it makes contact with the negative busbar; in that position the key is held down by means of the hook H. M is the cadence electromagnet which, on the passage of a current through it, causes its armature A to strike the stop P and warn the operator. At the same time the other end of the armature causes the pivoted lever L to trip the hook H and release the key, which is restored to its normal position by the spiral spring S.

The distributor contains the contact rings, rotating brush arms, and driving mechanism. In the actual instrument a distributor plate has six contact rings traversed by three pairs of brushes set at an angle of 120° with respect to each other. The various pairs of brushes and contact rings are insulated from each other. *Fig.* 7 shows a portion of a plate.

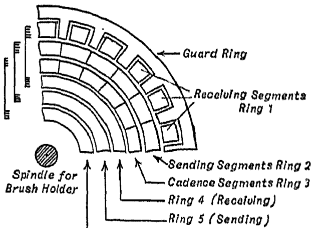


FIG. 7.

Ring 1 is the segmented receiving ring connected by one pair of brushes with the continuous ring 4, which is connected to the tongue of the receiving relay. Another pair of brushes connect rings 2 and 5, which are used for sending. The third pair of brushes connect rings 3 and 6, known as the "cadence and

brake" rings. The actual receiving segments on ring 1 are only one-half the length of the sending segments, the object of this being to allow only the crest of an incoming current wave, representing a signal, to be received on an electromagnet of a receiver. This arrangement to a large extent prevents disturbances from slightly inaccurate speeds causing false signals. Usually two similar distributor plates are used, one on the back of the distributor for reception and the other on the front for sending. The whole of the receiving and sending plates are mounted so that they may be rotated independently of each other; this allows the plates to be "orientated"—that is, rotated clockwise or anticlockwise, in order to compensate for line retardation and also to move the receiving segments into the best position for reception of signals.

There are two methods in use for driving the spindle which carries the brush-holders. In the first and older method the necessary energy required is provided by a falling weight. Theoretically this should give a constant speed to the brushes, but it is necessary in practice to provide a centrifugal governor to prevent fluctuations due to variations in the frictional resistances in the driving mechanism, otherwise satisfactory multiplex working would be impracticable. The second method, which is superseding the weight drive, consists in using a vibrating reed and phonic wheel, the skeleton connections of which are shown in *Fig.* 8.

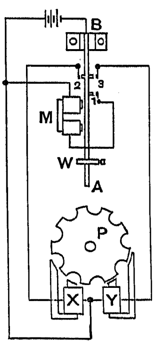


FIG. 8.

AB is the reed, which consists of a steel rod rigidly fixed at one end B. When the reed touches contact 1, the circuit through the electromagnet M is completed, causing the reed to be attracted. The attraction of the reed disconnects the circuit at 1, the current ceases to flow, and the electromagnet being no longer magnetised, the reed flies back. In this way the reed is maintained in a state of vibration, the rate of which depends upon the dimensions of the reed, the mass and the position of the weight W. The phonic wheel P consists of a toothed iron wheel which has an internal concentric cavity filled with mercury and iron wire. This method of construction is adopted to minimise speed fluctuations. The wheel when started by hand at the proper speed is kept in rotation by the magnetic attractions of the electromagnets X and Y, which are alternately energised by the vibrating reed through contacts 2 and 3. The phonic wheel is epicyclically geared to the spindle carrying the brush-holders. The armature of the correcting magnet when attracted interposes a stop pin in the path of one of the gear-wheels and momentarily uncouples the phonic wheel from the driving spindle of the brush-holders, allowing the latter to slip and restore phase relationship with regard to those at the distant station. The speed of the phonic wheel is independent of current fluctuations in the magnets X and Y, and depends only on the rate of vibration of the reed, which is constant for a definite position of the weight and adjustment of the contacts.

The modern Baudot receiver is belt driven from a small electric motor. It is necessary to control the speed of the receiver in order to secure a definite phase relationship with respect to the distributor receiving brush, otherwise the received signals will not be properly translated by the receiver mechanism. To arrange for this the receiver is run somewhat faster than the distributor brush spindle, but is prevented from getting out of phase by a friction brake. This friction brake is operated by an electromagnet which is energised by a current flowing from a particular segment on the cadence ring of the distributor. The circuit through this electromagnet is incomplete until the passage of the brush over the cadence segment coincides with a contact closed by a cam on the receiver axle. The receiver at first gains on the distributor, but in a very short time reaches a stage where it can gain no further owing to the brake coming into action; it will therefore run at approximately the same speed as the distributor and keep in phase with it. To prevent speed fluctuations that would be caused by the variations in the frictional resistances when the receiver is printing or running free, a centrifugal friction governor is fixed to the fastest running axle. The principal axle of the receiver carries the type-wheel, situated in front and outside the frame, and the combiner wheel inside. The object of the latter

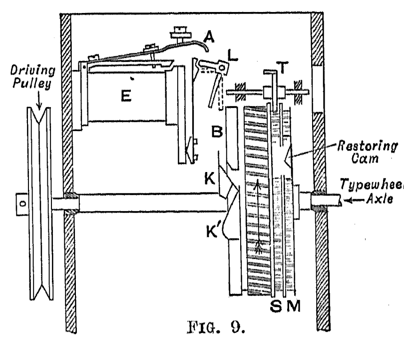


FIG. 9.

is to translate the five current impulses into a movement of the paper tape against the type-wheel. For this two discs S and M (see *Fig.* 9)

are used. On the rims are cut, in definite positions, a series of notches; those in the front disc correspond to marking units of the Baudot code and those in the back disc to spacing units. When no signals are being received the ends of five selectors T (*Figs.* 9 and 10), which correspond to the five receiving electromagnets E, rest on the rim of the spacing disc. When an electromagnet is operated an extension A of its armature acts upon the end of the lever L (one for each electromagnet) and presses it against the edge of the disc B. The end of lever L is subsequently engaged by the cam K which moves it to the right, thus causing it to act on the axle of the selector T and force it from the spacing to the marking disc. The selectors T and the levers L are restored to their normal positions by means of cams. When a character is being received, the five electromagnets, according to whether they are operated or not, cause the ends of their corresponding selectors to ride on the rims of the spacing or marking discs respectively. At a particular part of the revolution of the discs, there is for each character a combination of notches on the two rims into which the ends of all the selectors may fall. *Fig.* 10 shows all selectors on front disc, for example when translating the letter "P." The unevenness of the succeeding portions of the rims immediately jerks the ends out again and gives the top heads of the selectors a movement towards the left. This movement is communicated to the lever C, which, by lifting the rod P, releases the printing mechanism, bringing the paper slip into contact with the type-wheel and printing the desired character. The printing mechanism is restored to its normal position by means of a cam roller.

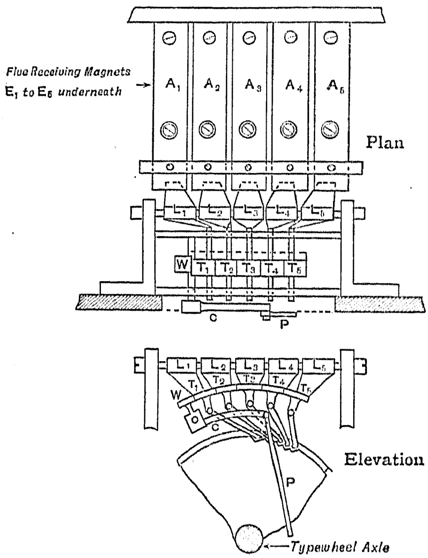


FIG. 10.

The type-wheel is of steel and the characters on its rim are inked by contact with a felt pad on the rim of an inking wheel. The characters are arranged so that letters and fiugres, etc., occur alternately. When it is desired to print figures or other secondary characters, a special signal, known as the figure shift, must be transmitted over the line; this signal causes the type-wheel to be angularly displaced on its axle so as to bring the figures into the positions previously occupied by the letters. The type-wheel is brought back to its normal position by a letter-shift signal.

The Baudot system may be worked either simplex or duplex. The former is chiefly adopted on the Continent and gives one or more channels in each direction on one wire. Busy lines are often worked quadruple simplex, giving four channels any of which may be used in either direction as required. In Great Britain the simplex Baudot is confined to Continental circuits, while duplex Baudot is used on inland circuits, usually quadruple duplex, which gives a total of 8 channels, 4 in each direction. In some cases where the line conditions allow, quintuple duplex and even sextuple duplex are used.

The principle of the Baudot duplex, devised by Col. A. C. Booth, is shown in *Fig.* 11, which represents diagrammatically the connections at one station. The outgoing current from

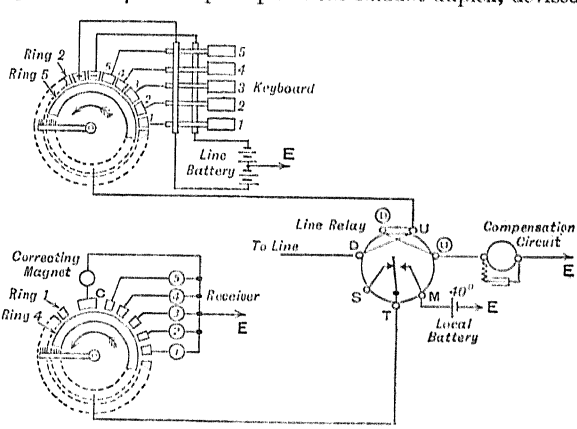


FIG. 11.

the keyboards passes to ring 2, through the brushes to ring 5, and thence to the split of the line relay, where it divides equally between the line and compensation coils of the relay and consequently does not affect the receiving apparatus which is connected to the local circuit of the relay. The incoming marking currents move the tongue of the line relay to marking, thus completing the local circuit from the 40-volt battery to ring 4, through the brushes to ring 1, and thence through the electromagnets of the receivers or the correcting electromagnet of the distributor according to the position of the receiving brush arm.

A great advantage possessed by the Baudot system, and which is common to all multiplex systems, is its flexibility in regard to providing direct communication between a number of offices on one line. For instance, if there are three stations A, B, and C (see *Fig.* 12 (*a*)) on one line, each fitted with a quadruple duplex set, it is possible to provide the following channels:

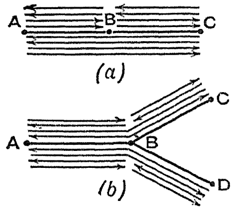


FIG. 12.

4 channels,	2 in each direction	between	A and B
4 „	2 „	„	A and C
4 „	2 „	„	B and C

By introducing a simple switching arrangement at the intermediate station B it is possible to vary the disposition of the channels according to traffic requirements. Thus the channels could be arranged thus:

8 channels, 4 in each direction between A and C

or

6 „	3 „	„ „	A and C
2 „	1 „	„ „	A and B
2 „	1 „	„ „	B and C

or any other combinations required.

It is also possible to arrange intercommunications between towns on a forked circuit. Thus if A, C, and D are the terminal stations and B the intermediate station at which the wires are forked (see *Fig.* 12 (*b*)) the disposition of the channels could be arranged as follows:

4 channels,	2 in each direction	between	A and B
2 „	1 „	„ „	A and C
2 „	1 „	„ „	A and D
2 „	1 „	„ „	B and C
2 „	1 „	„ „	B and D

Each channel of the Baudot is worked at a speed of 30 words per minute, which is equivalent to 60 messages of average length per hour. With a quadruple duplex installation this gives a traffic-carrying capacity of 240 messages per hour in each direction.

(ii.) *The Murray Automatic.*—Although the excellent results obtained by Baudot proved the practicability of working a five-unit system on the multiple principle, a number of inventors sought to solve the problem of increasing the traffic-carrying capacity of printing telegraphs by designing high-speed systems in which two channels only, in opposite directions, were provided by means of a duplex balance. Each channel on such systems was arranged to work at a speed of something over 100 words per minute, and it was therefore necessary to abandon the direct-sending manual keyboards such as Baudot employed on his relatively low-speed channels, and adopt automatic transmission arranged somewhat similarly to that of the Wheatstone system. The systems in which a high rate of sending is adopted on each of two channels, one in each direction, are designated "automatic" systems in order to distinguish them from the "multiplex" systems such as the Baudot.

One of the earliest high-speed automatic printing telegraphs, using a five-unit code, was that invented by Murray as far back as 1901. In the Murray Automatic System the message was prepared for transmission by means of a perforator worked by an ordinary typewriter keyboard. The depression of a key punched holes in a paper tape to represent the corresponding letter in Murray's adaptation of the five-unit code, and each letter occupied a length of one half-inch on the tape. The use of the five-unit code rendered the mechanism of the Murray perforator much less complex compared with instruments used for perforating Wheatstone slip, chiefly because the latter type required a complicated differential paper-feed mechanism. For the transmission of signals the prepared tape was passed through an automatic transmitter, similar in principle to the Wheatstone, but the Murray instrument, however, was driven by a phonic wheel motor in order to keep the speed perfectly steady. At the receiving end the signals worked a polarised relay in the local circuit of which was connected an electromagnetic perforator, consisting essentially of a punching and a spacing magnet operated through contacts closed alternately by a vibrating reed. The speed of the latter was governed by a relay so that it kept in unison with the reed of the phonic wheel at the transmitting station. The function of the electromagnetic perforator was to produce a facsimile of the transmitting tape. The received perforated slip was fed through the Murray printer, step by step, the length of a character on the tape, viz. one half-inch, and passed in front of five pins which were mechanically connected with five letter-selecting combs. The latter consisted of

metal plates each containing notches along one edge and capable of being displaced longitudinally. For each possible combination of positions of the five combs, a particular set of notches on them came into alignment. A comb was displaced whenever its corresponding pin came opposite to and passed through a hole in the tape in front of it. Each group of perforations in the tape, representing a character, therefore set up its own distinctive alignment of notches. Resting against the five combs were a number of pivoted levers, each mechanically connected with a particular key lever of a typewriter. For each alignment of notches one of the levers was free to fall into grooves thus formed, and in this position was acted upon by a motor-driven cam in such a manner as to cause the type lever connected to it to strike the paper and record the corresponding character. The printer typed the message in column form necessitating the use of special signals for causing the return of the paper carriage, and for turning it through a distance of one line space. An excellent feature of the system was an arrangement by means of which the keyboard operator at the sending end could correct known errors in perforating so that they would not appear on the printed message at the receiving end. To achieve this, the letter-shift key was arranged to perforate five holes in the tape, and this particular combination, apart from operating the letter-shift mechanism, had no effect upon the printer at the distant end. If an operator knowingly struck a wrong key, the error could be rectified by operating a back-spacing lever, which stepped back the tape until the mispunched letter was over the punches. The letter-shift key could be then operated and the error obliterated by the five holes punched in the tape. The working speed of this instrument was about 150 words per minute.

(iii.) *Siemens-Halske.*—A high-speed automatic system in use at the present day, chiefly in Germany, and giving excellent results, is one developed by Siemens and Halske. These inventors adopted the 5-unit code and devised a keyboard perforator which perforated a paper tape transversely to its length instead of longitudinally as in the Murray instrument, thereby effecting a great saving in the length of tape required for a message. The perforator is entirely electromagnetic in its action and has five distinct punching magnets, one or more of which are selected for operation according to the key struck. Distributor plates traversed by rotating brush-arms driven by shunt-wound electric motors are used on both the sending and receiving instruments, whilst unison between the two is maintained by the actual working signals. The speed of the two motors is adjusted by rheostats connected in series with the field magnets. On the motor spindle of the sending instrument a heavy flywheel is fixed, to prevent speed fluctuations. The sending distributor plate has five segments (see *Fig.* 13) which are connected respectively to five contact levers, operated selectively by the passage of the perforated tape through the transmitter, and playing between two contact bars which are connected to positive and negative supply mains. According to whether a lever is on the positive or negative contact-bar, when the brush-arm is passing over the segment connected to it, a condenser K is charged or discharged through a polarised transmitting relay R. This relay transmits the signals to the distant station, where they are received by another polarised relay R_1, which in turn, by means of condenser impulses, works a distributing relay D, and two regu-

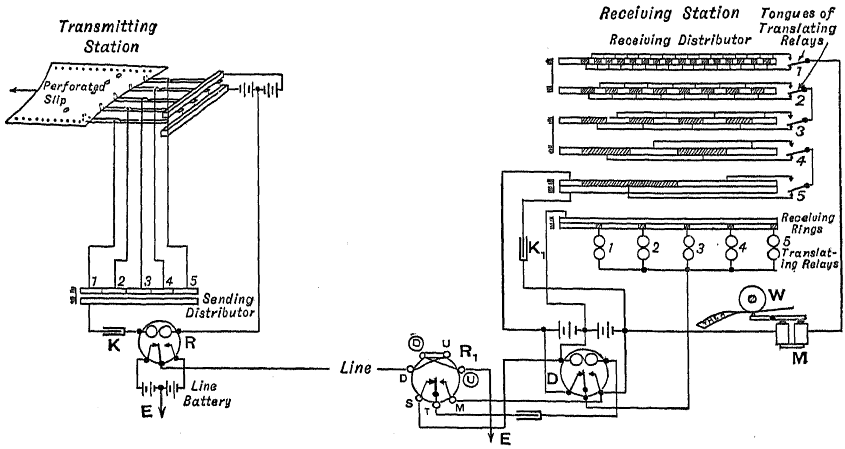


FIG. 13.

lating relays (not shown in figure). The distributing relay controls the position of the tongues of the five polarised translating relays, the coils of which are connected respectively to five segments on the receiving ring of the distributor. The two regulating relays act together to control a small-speed correction motor which rotates alternately in either direction to cut out or insert resistance, as required, into the circuit of the driving motor, thus ensuring the uniform rotation and definite phase relationship of its brush-arm with regard to that of the sending distributor. The translating arrangement consists of five distributor rings, each divided, as shown in *Fig.* 13, into a number of segments which are alternately connected to the contacts of one particular translating relay. The tongues of these relays are connected with one another and with a printing magnet M in such a manner that during the revolution of the brush-arm there is one particular position where the various brushes traversing the rings complete an electrical circuit through the five relay tongues and printing magnet. At the instant this occurs a continuously revolving type-wheel W has brought the required letter into its correct position for printing, and the discharge of the condenser K_1 through the abovementioned circuit causes the armature of the printing magnet to strike a paper tape against the inked type-wheel to secure a record of the signal in Roman type. Two groups of five translating relays are required, one set for the actual translation of signals, while the other is receiving the impulses of an incoming letter signal. A rotary switch is provided for changing the two groups over alternately at each revolution. The type-wheel has two rows of characters, one of letters and the other of figures, etc. The change from one to the other is effected by shifting the type-wheel axially by means of an electromagnet which is operated by a relay controlled by a particular signal, depending upon whether letters or figures are required. The receiver is also arranged so that in addition to translating and printing messages on a tape, the incoming signals may, if required, simultaneously operate a keyboard perforator and reproduce a replica of the transmitting tape. This arrangement saves considerable time and labour when automatic retransmission is required over circuits equipped with similar apparatus. The instrument prints well at a speed of 166 words per minute, which is equivalent to a traffic-carrying capacity of about 320 average messages per hour simplex or 640 duplex.

§ (5) Multiplex Automatic Transmission Systems.—The great success and development of the Baudot system after its duplexing by A. C. Booth in 1906, and its suitability for the convenient handling of traffic, had a great influence upon the labours of telegraph inventors. The result was a considerable development of multiplex type-printing systems based upon the principle of the duplex Baudot.

The multiplex systems possess a number of advantages over the automatic systems for ordinary commercial work, especially over short lines. The use of a number of transmitting channels, for instance, greatly facilitates the handling of traffic and minimises the delay on messages necessitated by inquiries and corrections. Also, owing to the high speed at which the instruments comprising an automatic system work, serious stoppages and delays are much more frequent than with multiplex systems where the speed at which each instrument works is comparatively low; for these reasons the cost of maintenance of a multiplex system is lower and less spare plant is required. Multiplex systems are also more economical in regard to labour and office equipment.

(i.) *Murray.*—The Murray multiplex system uses phonic wheel-driven distributors and the Baudot method of correction. The essential differences between it and the Baudot system consists in the use of keyboard perforators and automatic transmitters in place of the Baudot direct-sending keyboards, and in the use of column printers instead of slip printers. The speed is also increased to 40 words per minute on each channel, the higher speed being possible owing to the adoption of automatic transmission. The perforator is designed for the 5-unit code and perforates the tape crosswise similarly to the Siemens instrument. The principle of the perforator is indicated in *Fig.* 14. When a key is depressed, one or

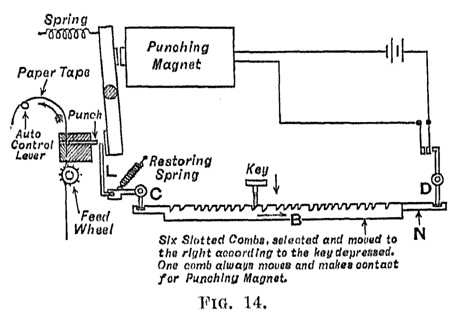


Fig. 14.

more of the 6-slotted combs move to the right. A projection N on one of the combs actuates lever D and closes a circuit through the punching magnet. Those of the other five combs selected act on bell-crank levers C, and cause rods L to be removed from between the corresponding punches and the hammer of the armature of the punching magnet, so

that when the latter is operated the remaining punches are forced through the tape. The back stroke of the armature acts on a feed wheel, causing the tape to be fed through the instrument. The signals are sent out to line by means of a transmitter, which is operated by the tape prepared by the perforator. The principle of the Murray transmitter is shown in *Fig.* 15.

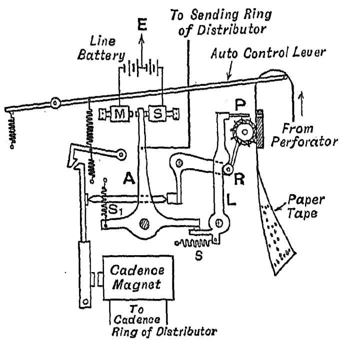


FIG. 15.

It contains five pivoted contact levers A (one only shown), each connected to its corresponding segment on the sending ring of the distributor and capable of moving between the contact bars M and S which are respectively connected to the negative and positive terminals of the line battery. Each lever A has its movements controlled by a pivoted rod L which carries at its upper end a pin P. R is a roller which is moved up and down, in contact with the rods L, by means of the armature of an electromagnet operated from a segment on the cadence ring of the distributor. As R moves up and down it encounters the projections on the faces of the rods L, giving the latter a to-and-fro movement which is communicated to the pins P, and also to the contact levers A, causing them to make alternately positive and negative contacts. Springs S and S_1 control the movements of A and L as R moves up and down. The tape is fed through the transmitter step by step in front of the pins P by means of a star wheel which is on the same axle as a ratchet-wheel actuated by the same electromagnet that works roller R. When a group of perforations come into position in front of the pins P, those of the latter that come opposite holes in the tape can move forward into them, and their corresponding levers A are placed on the marking contact bar M. The pins that do not come opposite holes cannot move, and the levers A controlled by them remain on the spacing contact bar S. Immediately after the setting of the levers A, the impulses are sent to line by the passage of the brushes over the corresponding distributor segments; the cadence electromagnet then operates, moving the roller R upwards, thus withdrawing the pins that have entered holes in the tape and allowing the latter to be fed forward a distance of $\frac{1}{10}$th of an inch. The perforated tape is fed direct into the transmitter from the perforator, and is not torn off in lengths of five messages as is generally the case in automatic high-speed systems in which a number of perforators feed one transmitter. To prevent mutilation of the tape in the event of the transmitter overtaking the perforator during working, the loop of the tape is passed over an automatic control lever fitted to the transmitter so that when the loop straightens out the lever is pulled down to prevent the armature of the cadence electromagnet from working and feeding the paper through the transmitter. When the perforator is worked again the tension on the tape is eased, the lever rises and allows the transmitter to continue working.

The Murray multiplex printer is similar in some respects to the one used with his automatic system, but differs from it in the important detail that the five selecting combs are operated by electromagnets instead of by perforations in a tape. Incoming signals are received on a polarised relay which repeats them *via* the continuous and segmented receiving rings of the distributor to the five electromagnets as in the case of the Baudot. The printer is belt driven by means of a small electric motor and runs faster than the distributor brush-arms. Immediately after the combs have been positioned, an electromagnet is operated from a segment on the cadence ring of the distributor. The armature of this electromagnet allows a spindle carrying a number of cams to make a single revolution. One of these, known as the printing cam, brings about the printing of the selected letter, another restores the selected combs to their original positions, while a third operates at the correct instant the letter-spacing mechanism of the typewriter carriage. The "basket" shift similar to that in use on some modern typewriters is utilised when it is required to print figures instead of letters, the mechanism required to effect the change being operated by the "figure" and letter-shift signals sent from the distant station. A number of special signals are used to bring about certain operations in the printer. The "line" signal runs the typewriter carriage back at the end of a line and turns the platen through a distance equal to the space between two lines. The "page" signal sets a train of wheels in motion, at the end of a message, and rotates the platen with the paper so as to bring the latter into position for printing the next message. The "column" signal allows

the message to be turned up to a new line at any point in order that the arrangement of short lines such as occur in addresses may be correctly spaced and aligned for printing.

		1	2	3	4	5			1	2	3	4	5
A	BELL	●	●				Q	1	●	●	●		●
B	/	●			●	●	R	4		●		●	
C	(		●	●	●		S	'	●		●		
D	²	●			●		T	5					●
E	3	●					U	7	●	●	●		
F	1/	●		●	●		V	)		●	●	●	●
G	3/		●		●	●	W	2	●	●			●
H	5/			●		●	X	£	●		●	●	●
I	8		●	●			Y	6	●		●		●
J	7/	●	●		●		Z	.	●				●
K	9/	●	●	●	●		LINE PAGE					●	
L	PERF		●			●	ERASE		●	●	●	●	●
M	’			●	●	●	COLM ==			●			
N	—			●	●		FIGURES		●	●		●	●
O	9				●	●	LETTERS				●		
P	0		●	●		●	START						

FIG. 16.

The Murray adaptation of the 5-unit code for multiplex page printing is shown in *Fig.* 16.

(ii.) *Western Electric*.—The Western Electric multiplex system, though similar to the Murray in principle, differs from it in a number of important details. The keyboard perforators and the transmitter are practically the same as those used by Murray, but the printer is constructed on different lines, being almost entirely electrical in its action. An important difference between the two systems is that in the Western Electric multiplex no special correcting signals are required for the maintenance of unison between the distributors, as the correction is effected by the actual working signals. To arrange for this, two extra rings are required on the distributors, and the brush-holders have four pairs of brushes instead of three as in the Murray and Baudot systems.

The distributors are driven by phonic wheel motors controlled by electrically vibrated tuning-forks. The manner in which correction is brought about is shown in *Fig.* 17, which represents the connections of the various relays required for the purpose at the corrected station. The brush-holder at the correcting station is arranged to run at a speed of 240 revolutions per minute, giving 40 words per minute per channel. At the corrected station the motor is set to run at a slightly lower speed, so that the two distributors are always tending to get out of phase. Before, however, the phase difference between them reaches a degree which would render satisfactory working impossible, correction takes place. This is brought about by causing a rubber-tipped buffer to strike one of the tines of the fork at the corrected station, a proceeding which increases its rate of vibration, and therefore the speed of the motor it controls, thus preventing the tendency towards loss of phase. A correcting impulse is generated only when the tongue of the line relay moves from the spacing to the marking contact. Now, although the tongue of the leak relay moves in accord with that of the line relay there is necessarily a time lag between them. It follows that when the line relay tongue touches the marking contact an instantaneous current will pass from the M of the line relay to S of the leak relay and *via* its tongue to the continuous correcting ring, thence through the brushes and one coil of the correcting relay, moving its tongue to S or M according to whether the brush is on a "B" or an "A" segment of the segmented correcting ring. If it is on the former the motor at the corrected station is running too slow; the correction impulse at this instant passes through the D to U coil of the correcting relay and moves its tongue to S. This brings about the

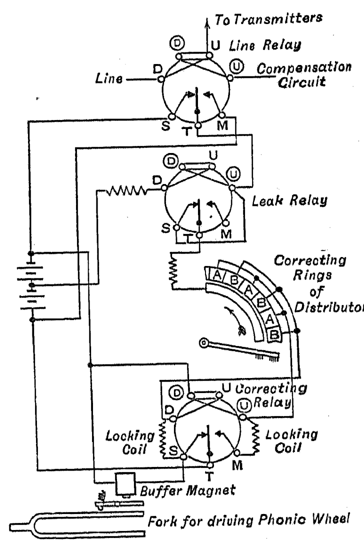


FIG. 17.

operation of the buffer magnet, which holds the buffer against the fork and increases its rate of vibration and consequently the speed of the motor. A locking coil joined from S to D maintains the tongue of the correcting relay against the spacing stop, so that the speed of the motor continues to increase until a correcting impulse arrives over an "A" segment. This impulse by passing through the (U) to (D) coil of the correcting relay causes its tongue to move over to contact M, and thereby breaks the circuit of the buffer magnet, which withdraws its buffer from the tine of the fork and allows the latter to vibrate at its normal speed.

On all systems in which correction is brought about by the working signals, special arrangements have to be made for bringing the brushes into phase before working can commence. It is generally arranged for the correcting station to send out a special correcting signal from definite segments of its sending ring. By means of a switch at the corrected station it is rendered impossible for this signal to produce correction there until the two distributors are in phase for correct working. In passing it may be remarked that systems, such as the Baudot, in which special correction currents are used, come into phase automatically.

The automatic control switch used with the Western Electric system is distinct from the transmitter, but is controlled by the tape in a similar manner to the Murray control lever. In addition, the auto-control switch is fitted with a means of enabling the sending operator to send automatically to the distant station any number of bell signals up to five.

A. E. S.

TELEGRAPHS, WRITING

Writing or autographic telegraphs, which reproduce automatically at one end of a line an exact copy of words written or sketches made on a sheet of paper at the other end, have been considerably used in America. The practical application of such systems in Great Britain, however, has not been extensive, owing chiefly to their low rate of working. They possess an advantage over other systems of telegraphy in that no skilled labour is required for signalling, the original copy being prepared by the sender of the message and the received copy being in condition for immediate delivery as it comes from the instrument.

The earlier forms were based on a writing telegraph invented by Bakewell in 1850, which used synchronously rotating cylinders, one at each end of the line. At the sending station the message for transmission was plainly written on a sheet of tin-foil with a pen dipped in non-conducting ink or varnish, and was laid round the cylinder. When the latter was set in motion it was traversed, after the manner of a phonograph record, by a metallic stylus, which was connected, as shown in *Fig.* 1, to the line and also to one pole of the sending battery, the tin-foil being connected through the cylinder to the other pole of the battery. At the receiving station an earth-connected rotating cylinder carried a sheet of chemically prepared paper, which was traversed by a metallic stylus connected to

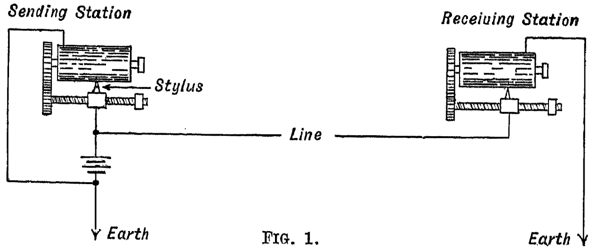


Fig. 1.

the line. When the sending stylus touched the tin, the battery was short circuited so that no appreciable current flowed along the line and no mark was made on the paper by the receiving stylus; but when the sending stylus crossed the insulating ink on the tin-foil, the current from the battery passed to line and through the receiving stylus to earth, producing a mark on the prepared paper there by electrolytic effect.

A widely used and successful instrument for the automatic transmission of writing is the telewriter, sometimes known as the telautograph, which requires two line wires and is based on entirely different principles to the Bakewell type. Referring to *Fig.* 2, it will be seen that the pencil of the transmitter is connected to two rods A and B, which communicate the motions made by it, when

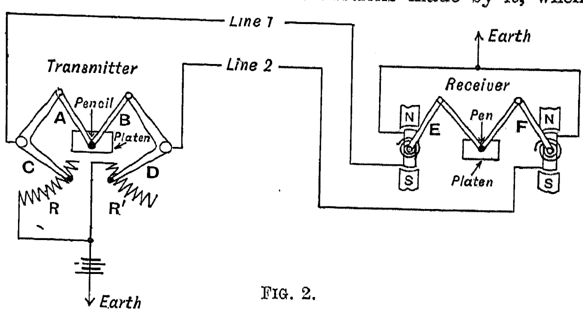


Fig. 2.

writing on a paper roll passing over a platen, to two levers C and D. As these levers move they vary the amount of resistance in the rheostats R and R′ respectively, so that, the resistances of the lines being constant, the currents along them will vary according to the motion of the pencil. At the receiving station each line is connected to a pivoted coil which is capable of rotating between the pole-pieces of a permanent magnet. These coils take up positions depending upon the strength of the currents passing through them, and

are controlled by spiral springs so adjusted that the turning movements of the rods E and F connected to the coils correspond exactly with the movements of the transmitting rods C and D. The recording pen is connected to the rods E and F by a linkwork precisely similar to the rods A and B, so that it will reproduce the motion of the transmitting pencil and record the message on a paper band passing over a platen. The movements of the transmitting pencil and receiving pen are limited in extent, so that arrangements are made for feeding the paper bands over the available writing space. This is done by operating a finger lever at the transmitting end, which mechanically shifts the transmitting paper and simultaneously works an electrical device for automatically feeding the receiving paper. The motion of the transmitting pencil is not confined to the plane of the paper because in the ordinary course of writing the pencil is moved both on and off the paper. It is necessary therefore to provide means for lifting the receiving pen off and lowering it to the paper as required. Accordingly, it is arranged that when the transmitting platen is depressed by the pressure of the pencil, a contact is closed completing a circuit and allowing a superimposed alternating current to be sent over the lines. This current causes a pen-lifting magnet at the receiving instrument to attract its armature, thus causing the receiving pen to rest upon the paper. When the transmitting pen is raised the contact made by the platen is opened, the alternating current ceases, and the pen-lifting armature on retracting removes the pen from the paper.

The telewriter may be attached to an ordinary telephone wire without in any way interfering with the telephone service. The arrangements are such that the mere lifting of the telephone receiver connects the telephone for use, and hanging it up again allows the telewriter to be used.

A. E. S.

TELEGRAPHY, CENTRAL BATTERY SYSTEM OF

THE Central Battery System of Telegraphy is one in which the transmission of signals between all stations on one telegraph circuit is effected by using a single battery located at one of them. The station containing the battery is termed the Head Office, in order to distinguish it from the others, which are known as Out-Stations.

The great advantage of the system is the saving of battery maintenance at all except the head office; but another and almost equally important advantage is the simplification of the apparatus employed, which eliminates to a large extent the liability to faults due to incorrect adjustments by the operating staff and lessens the amount of technical knowledge and experience required at the out-stations.

The closed circuit system (see article on the "Telegraph, The Electric") is based on the above principle, and is still extensively used in America. It was abandoned, in this country, many years ago, partly owing to the difficulty and high cost of maintaining in good condition the primary batteries for supplying the fairly heavy continuous current required, and partly because of the difficulty experienced in maintaining the lines at the required insulation resistance. The application of secondary cells to telegraphic purposes and the great advantages accruing from their use, coupled with a generally higher standard of insulation of the lines, have led to a revival in the use of central battery systems, and the developments during the last fifteen years have been fairly extensive.

It is proposed in this article to deal with the principal systems in use in this country at the present day.

§ (1) SIMPLEX C.B. SYSTEMS.—On simplex circuits a particular type of instrument known as the "Polarised Sounder" is used for the reception of signals. The most extensively used pattern of polarised sounder is that designed by Mr. C. C. Vyle. It is similar in appearance and in general construction to the ordinary pony sounder (see article "Telegraph, The Electric"), but differs from it, essentially, in having a permanent magnet which polarises the cores of the electromagnets and the armature. By this arrangement the up-and-down movements of the armature depend upon the direction of the current passing through the coils of the electromagnet, thus allowing the sounder to be connected directly in the line circuit and worked on the double-current method. The principle of the instrument will be understood by referring to *Fig.* 1. A and B are soft-iron cores wound as shown with two separate coils, the ends of which are led to four external terminals marked U, (U), D, and (D) respectively. By means of these terminals the coils may be joined in series or parallel, or connected up differentially as may be required. The cores A and B and the soft-iron armature are magnetised inductively by the permanent horseshoe magnet NS. This is placed in the base of the instrument with its poles immediately below the cores. The up-and-down movement of the lever (not shown) fixed to the armature is limited by stops, and controlled by a spiral spring T, exactly as in the case of the ordinary sounder. The tension of this spring, however, is adjusted to counter-

balance the attraction of the cores on the armature, due to the induced magnetism, thus allowing the lever fixed to the armature

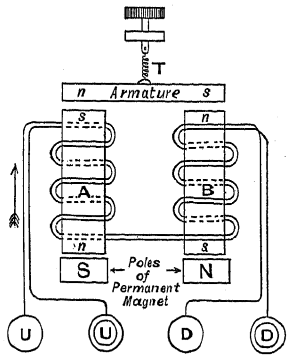


FIG. 1.

to remain against the upper or lower stop according to where it is placed by hand. If a current is passed through the coils in the direction of the arrow, that is, from U to D or (U) to (D), the magnetism in both cores is strengthened, making the attractive force of the cores upon the armature greater than the tension of the spring. The armature is therefore attracted and causes the lever to which it is fixed to strike the lower stop. A current in the opposite direction weakens the magnetism in the cores, and the tension of the spring then exceeds the magnetic attraction. The armature, in this case, is therefore pulled smartly away from the cores and the lever strikes the upper stop. The coils are wound to various resistances to suit particular requirements. The Vyle sounder may be adjusted to work with a current of 0·5 milliampere, but the working current is usually about 10 milliamperes. It is therefore much more sensitive than the ordinary 900-ohm sounder used with the universal battery system, which requires a working current of about 25 milliamperes.

The connections of a central battery simplex circuit serving three stations is shown in *Fig.* 2. It is generally known as an "Omnibus Circuit." The 80-volt central battery B is joined to line through a non-inductive feed resistance of 1000 ohms. S represents the head office polarised sounder, and S_1, S_2, those at the out-stations. Each is wound to a resistance of 2000 ohms. C, C_1, and C_2 are condensers each of 4 microfarads capacity, and k, k_1, and k_2 are single-current Morse keys. One side of each condenser is joined to earth, while the other side is joined through the respective sounders direct to the line. The head office set is connected in a slightly different manner, the sounder being joined to the back stop of the key instead of to the lever, as is the case at the out-stations. This prevents the head office sounder from being actuated by the operation of the head office key, thus lessening the noise in a busy office where there may be a number of such sets. The galvanometer is inserted in the line at the head office to indicate the outgoing current and to serve as a guide to the working condition of the line in respect to insulation.

Assuming that the line is perfectly insulated and that all keys are at rest, then the ordinates of the rectangle *ahed* represent the potentials at points along the line, viz. 80 volts since the distant end of the line is disconnected. The difference of potential between the plates of each condenser will be therefore 80 volts. If the key k_2 at Station II. is depressed it connects the line directly to earth, altering the curve of potential along the line to that represented by the figure *ahlmd*. The potential difference between the plates of condenser C is therefore reduced from *bg* to *bl*, that between the plates of C_1 is reduced

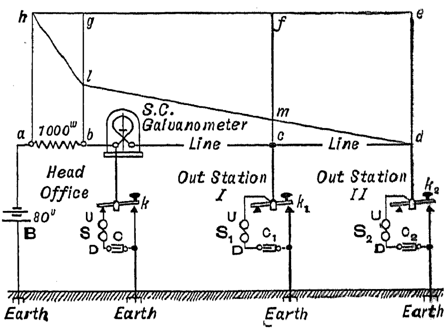


FIG. 2.

from *cf* to *cm*, and that between the plates of C_2 from *de* to zero. The condensers C and C_1 consequently partially discharge and C_2 completely discharges through their respective sounders, in the direction U to D. The armatures of all the sounders are therefore attracted to their cores and remain so until the key k_2 is released. When this occurs the line is restored to its original electrical condition, and current flows through the sounders from D to U into the condensers to recharge them to the full potential of 80 volts. The spacing currents passing through the sounders allow their levers to return to their upper stops. In this manner all the sounders are caused to emit signals when a key at any of the out-stations is actuated. The 1000-ohm feed resistance at the head office is necessary for two reasons. Firstly, it prevents the short-circuiting of the battery when the head office key is depressed, as well as when an earth fault occurs on the line near the head

office. Secondly, it *ensures* the partial discharge of the head office condenser through its sounder when an out-station key is depressed.

The limiting distance over which it is possible to work such a circuit is dependent chiefly upon the insulation resistance of the line. Low insulation conditions cause a loss of voltage along the line, thus reducing the charging and discharging currents that operate the sounders, and consequently lessening the efficiency of the circuit. Owing to the fact that most of such circuits in this country are worked on overhead wires, the insulation resistance of which are considerably reduced, more especially in wet weather, the maximum distance over which it is found practicable to work rarely exceeds fifty miles.

Other central battery simplex circuits are given in the descriptions of the intercommunication switch and the concentrator dealt with in the article on "Telegraph, The Electric."

§ (2) Central Battery Duplex Systems.—There are a number of methods by which central battery duplex working may be effected. The two principal methods in use in this country are described below:

§ (3) Hay's C.B. Duplex System.—The connections of this system are shown diagrammatically in *Fig.* 3. With both keys at rest the resistance from B to earth is 1250 ohms, and from D through the line and distant apparatus to earth, 1850 ohms. The point D will therefore be at a higher potential than B, causing a current to flow through the head office polarised relay from D to (U), that is, in a spacing direction. A current also flows through the line to the distant station, but the various resistances are such that it is of insufficient strength to attract the armature of the polarised sounder against the tension put on the spring. When the key at the out-station is depressed, one coil of the polarised sounder and the 500-ohm resistance is short-circuited, thus reducing the resistance from D to earth to 850 ohms. As this is less than the resistance from B to earth, B will now be at a higher potential than D, and current will flow through the head office relay in a marking direction, viz. from (U) to D, actuating the local sounder. The reduction in the resistance of the line circuit brought about by the depression of the out-station key practically doubles the line current. This increased current, however, passes through one coil only

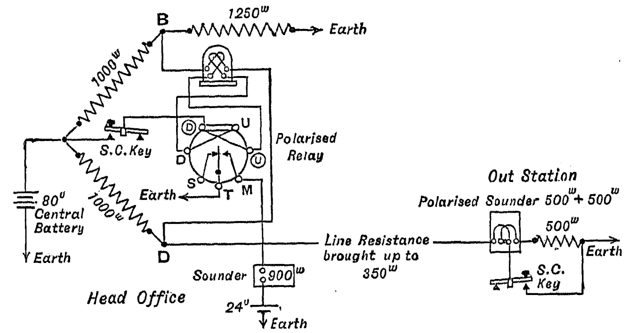


Fig. 3.

of the polarised sounder, and its effect is the same as the normal current flowing through both coils, the sounder is therefore not actuated. If the head office key is depressed the resistances AB and AD are shunted by the relay and galvanometer coils, thus increasing the line current sufficiently to actuate the polarised sounder at the out-station. The movement of the key at the head office does not upset the equality of the resistances between A and B, and between A and D, hence the potential of D remains higher than that of B and the polarised relay remains at spacing. To sum up, only the depression of the out-station key is capable of reversing the potential difference between B and D, while the line current can only be increased sufficiently to operate the out-station sounder by a depression of the head office key.

§ (4) Vyle and Smart's C.B. Duplex System.—*Fig.* 4 indicates the principle of this system. A non-polarised relay with its coils joined up differentially is used at the head office and a polarised relay at the out-station. The resistance in the compensation circuit at the

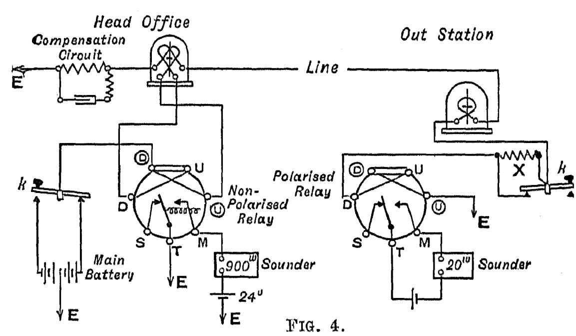


Fig. 4.

head office is adjusted so that the current through it is less than that through the line circuit, the difference in strength between the two being sufficient to produce a deflection of about 45 divisions to the right on the galvanometer. The key at the out-station is now kept permanently depressed and the rheostat X adjusted until the galvanometer needle at the head office is vertical, thus indicating that the circuit is balanced as regards resistance. The capacity balance is now obtained at the head office in exactly the same way as for an ordinary duplex circuit. When these preliminary adjustments have been made and the keys at both stations normal, the resistance of the line circuit is less than that of the compensation circuit, owing to the short-circuiting of the rheostat X by the out-station key. The current through the line coil of the non-polarised relay therefore exceeds that through the compensation coil, and the preponderance is sufficient to hold the relay tongue, against the tension of a spring, to the spacing contact stop. The operation of the head office key does not affect this condition, but by reversing the line current operates the distant polarised relay. The depression of the out-station key balances the circuit by introducing the rheostat X into the line. This causes the outgoing current at the head office to divide equally between the line and compensation coils of the non-polarised relay, and there is consequently no force of magnetic attraction between its cores and armature. The spring controlling the latter therefore pulls the relay tongue to the marking contact and thereby actuates the sounder in the local circuit.

When both keys are depressed simultaneously, the balancing of the circuit enables the non-polarised relay to close its local circuit, while the reversal of current over the line works the polarised relay at the out-station.

A. E. S.

TELEGRAPHY, DOUBLE-CURRENT SYSTEM

A TELEGRAPH circuit may be regarded as a condenser in which the wire forms one conductor, the earth (or in the case of a loop circuit, the return wire) the other, and the intervening air or other insulating material, the dielectric. It is owing to the fact that a telegraph circuit possesses electrostatic capacity as well as resistance, that the application of a battery at one end, as in single-current working, does not result in an immediate rise of the current at the receiving end to its full working strength. A certain interval of time elapses between the first application of a signal at the sending end and its arrival at the receiving end, depending directly upon the capacity and resistance of the circuit, and also upon the sensitiveness of the receiving instrument. If the battery is applied for a very short period, as in the case of signalling a dot, the received current over lines of high capacity may not reach a value sufficient to actuate the instrument, because the major portion of the current may be used up in charging the line. When the sending key is raised, the charge on the line flows to earth at each end, and, as the direction of discharge at the receiving end is in the same direction as the signalling current (see

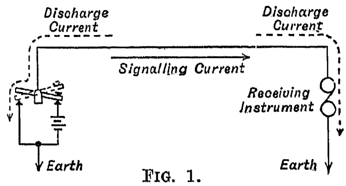


FIG. 1.

Fig. 1), the result is to cause a prolongation of the signal.

The above effects are inappreciable on short aerial lines which have small capacity, but on long telegraph lines and especially underground cables possessing considerable capacity, distortion of received signals may be caused. In such circumstances comparatively slow sending must be resorted to, if single-current working is used, in order to allow time for the line to completely discharge between successive signals. If the rate of sending is too fast, a fresh signalling current may reach the receiving end before the discharge current resulting from the previous signal has ceased, thus causing the received signals to run together and become unintelligible.

It is to overcome these difficulties and allow a higher speed of working to be attained that double-current working is adopted. In this system it is arranged for a spacing current to flow along the line, equal in strength but opposite in direction to the marking current, immediately the latter ceases. This reverse current (see *Fig.* 2) flows in the opposite

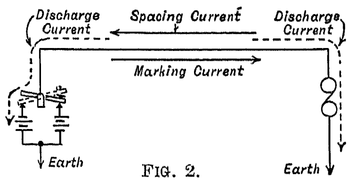


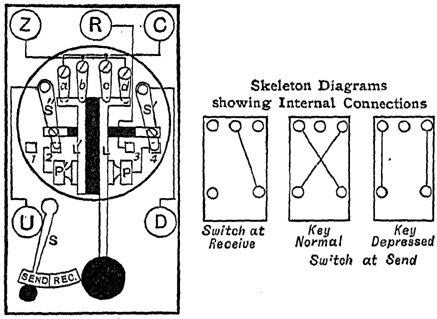
FIG. 2.

direction to the discharge current at the receiving end, and therefore nullifies the prolongation effect upon the received signals. It also hastens the discharge of the line and the recharging when the marking current is again

applied, so that signals may be transmitted much more rapidly than in single-current working, without causing distortion at the receiving end.

There are a number of other advantages derived from using the double-current system of working. The continual reversing of the current, for instance, tends to prevent the development of residual magnetism in the cores of the relays, and, owing to the fact that the latter are controlled by steady currents both when "marking" and "spacing," they are less liable to be affected by disturbing currents from the line. Again, in single-current working the relay tongue is given a decided bias in order to ensure its restoration to the spacing contact at the termination of a signal. This bias is not necessary when working double current because the spacing current brings the tongue to spacing when the marking current ceases. It is, therefore, possible to adjust the relay to the neutral condition —*i.e.* for the tongue of the relay to remain on either contact without a holding current, which renders it much more sensitive and thus enables a higher speed of transmission to be attained with a smaller working current. Another advantage is that a relay set neutral does not require adjusting to suit variations of current because the spacing current is affected by the variations to the same extent as the marking current. In single-current working, however, the bias of the relay must be altered if the current varies very much, in order that the controlling forces in the spacing and marking directions may be kept equal.

The direction of transmission is not automatically reversible in the double-current system, but is controlled by switches, which form part of the signalling keys at each end and are operated by hand. This type of key is known as a Double Current Key and is represented in *Fig.* 3. It has five terminals, of which Z and C are connected to the signalling battery. Terminals U and D are joined respectively to line and earth or *vice versa*, according to whether the key is at a down or up station. The key itself consists of two levers L, L′, insulated from each other by ebonite and pivoted at PP′. These levers are furnished with contact points which play between the contact springs *a*, *b*, *c*, and *d*. S′ and S″ are switch levers, insulated from each other, but connected mechanically and controlled by the switch S. When S is in the "send" position the switch levers are in the positions shown on studs 2 and 4, and if the key lever is at rest, levers L and L′ are in contact with springs *a* and *d* respectively, thus connecting Z through the key to D and C to U. The depression of the knob of the key causes L and L′ to break from springs *a* and *d* and make contact with *b* and *c*, Z is then connected to U and C to D, so that in



Plan of Double Current Key

FIG. 3.

this manner the current flowing to line may be reversed in direction by operating the key. On moving the switch S over to the "receive" position, the switch levers S′ and S″ leave studs 2 and 4 and make contact with studs 1 and 3, thus disconnecting the key levers L and L′ and joining terminal R to D.

The receiving apparatus is connected externally to terminal R, hence the former may be joined at will, in place of the battery, to the line by means of the switch S.

The connections of a Double Current Sounder simplex circuit are shown in *Fig.* 4. The key

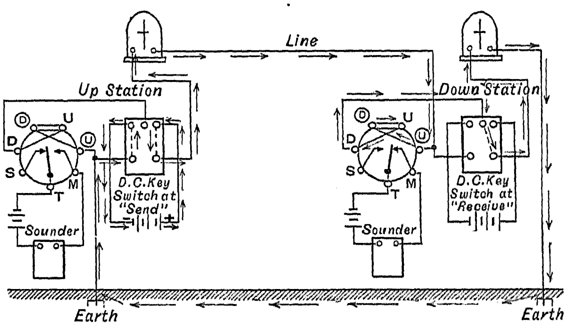


FIG. 4.

at the up station is shown with its switch in the "send" position, and its lever depressed, while that at the down station has its switch in the "receive" position. Under these conditions a current flows from the positive pole of the battery at the up station, through the line and the receiving apparatus at the down station to earth, and thence back to the negative

ole of the up station battery. This current asses through the relay at the down station a "marking" direction, thus closing the ocal circuit and actuating the sounder. The rrows show the path of the current.

The Double Current Duplex arrangement is lescribed in the article on "Telegraphy, Duplex."

A. E. S.

TELEGRAPHY, DUPLEX

Duplex telegraphy provides for the simultaneous transmission of signals from both ends of a single wire, without mutual interference. Such a system is almost equivalent to two separate circuits between two stations, but is clearly more economical as regards first cost and maintenance. There are several methods by which duplex telegraphy may be effected, the principal being the *differential*, the *bridge*, and the *central battery* systems.

The first two methods only will be dealt with in this article, as the latter is described in the article on "Telegraphy, Central Battery System of."

§ (1) Differential Duplex.—The differential duplex system is the one in most general use, and the principles upon which it is based will be understood by referring to *Fig.* 1.

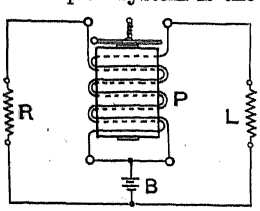


Fig. 1.

P is an electromagnet wound with two separate windings, each having the same resistance and number of turns, but one wound in the opposite direction to the other. By joining the windings externally as shown, the current from the battery B will divide equally between them if the resistance R is equal to the resistance L. As, however, the two coils are wound in opposite directions, the magnetism set up by one will be neutralised by that set up by the other, so that the resultant effect upon the core of the electromagnet will be *nil* and its armature will not be attracted. If the circuit containing L is disconnected or increased in resistance the excess of current through the winding connected to R will produce magnetism in the core and the armature will be attracted. The Post Office Standard relay (see article "Telegraph, The Electric") is in effect a differentially wound electromagnet, currents of equal strength flowing simultaneously through one coil from U to D and through the other from D to U, producing no effect upon the tongue. Such an instrument is also very sensitive, and is therefore used in practice in the application of the foregoing principles to duplex telegraphy.

The connections of the apparatus at the two stations for a duplex circuit are shown diagrammatically in *Fig.* 2.

A and B are relays, one at each station, and R and R′ are resistances, each equal to the line resistance plus that of the distant apparatus. When the up station key K is depressed, the current divides equally into two parts at the relay A, one half passing through the D to U coil to line, and the other passing through the U to D coil and the resistance R back to the battery E. The relay A will therefore be unaffected by the outgoing signal, and the local sounder will not be operated. The current flowing along the line traverses the U to D coil of the relay B at the distant station (a small portion of the current also passes through the other coil from U to D), and therefore the tongue of B is moved to the marking contact and the signal registered on the local sounder. Similarly, if the down station key K′ is depressed, relay B is unaffected and the up station relay A marks. When both stations depress their keys simultaneously no current will pass through the D to U coil of each relay because the E.M.F. applied to the line by battery E is equal and opposite to that applied by the battery E′. The current through the U to D coil of each relay remains unaffected, and both relays are therefore operated and signals simultaneously produced.

The circuits containing the resistances R and R′ are called *compensation* circuits; r and r' are resistances equal respectively to the internal resistances of the batteries E and E′. The object of introducing them is to ensure the same conditions as regards the resistance of the line circuit, whether the distant key is at rest or depressed.

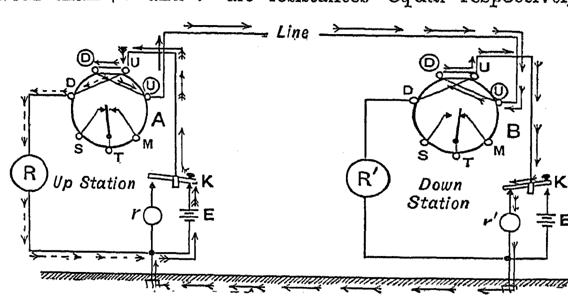


Fig. 2.

The full and dotted arrows in *Fig.* 2 show the complete paths of the line and compensation currents respectively when the up station key K is depressed.

§ (2) Combination Duplex.—The system outlined is known as the *opposition* method, because the batteries *oppose* each other in the line when both keys are depressed. The standard practice in Great Britain is to arrange the batteries to act in *conjunction* in the line when both keys are depressed, so that the line current in such a case is double the current in each of the compensation circuits. This system is known as the *combination* method.

A differential galvanometer is introduced at each end of a duplex circuit for the purpose of facilitating the adjustment of the compensation circuit (this will be referred to in detail later). The differential galvanometer is similar in construction to the single-current galvanometer (see article "Telegraph, The Electric"), but has four terminals at the back instead of two, and is differentially wound with two wires of exactly the same length and resistance. Its needle will deflect to the right or left according to the direction of the current through either of the wires. The conventional method of representing this galvanometer is shown in *Fig.* 3, in which it is connected differentially.

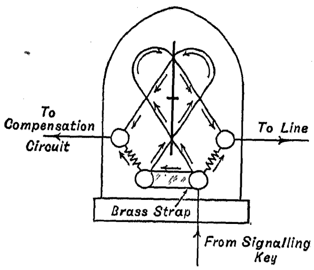


Fig. 3.

On all important circuits the double-current differential duplex is invariably used. This is similar in principle to the single-current system, but differs from it in that the conditions are modified by the use of a spacing current when either key is at rest. To enable both stations to work simultaneously the following conditions must be satisfied:

(i.) With both keys at rest, the tongues of both relays should be on their spacing contacts.

(ii.) With only one key depressed, the tongue of the relay at the distant station should be on its marking contact, while the relay tongue at the home station should be on its spacing contact.

(iii.) With both keys depressed, the tongues of both relays should be on their marking contacts.

Fig. 4 shows the theoretical connections of a double-current differential duplex circuit which fulfils the above conditions. The switches of the double-current keys must be kept in the "send" position while working duplex.

With both keys at rest the batteries help each other in the line, and therefore the line current is *double* the compensation current at either end. The preponderance of the line current which flows from (D) to (U) through both relays moves their tongues to spacing.

With only one key depressed, for example, at the up station, the positive pole of the up station battery is put to line, and at the down station the positive pole of the battery there is also to line, hence the batteries are in opposition and no current flows along the line. At the up station the compensation current flows from D to U through the relay and moves its tongue to the spacing contact. At the down station the compensation current flows from U to D through the relay and moves its tongue to the marking contact, thereby actuating the local sounder.

With both keys depressed the conditions are the same as for both keys at rest, except that the batteries at each end are reversed, and therefore also the directions of the line and compensation currents at each end. The preponderance of the line current which flows from (U) to (D) through each relay causes them both to "mark."

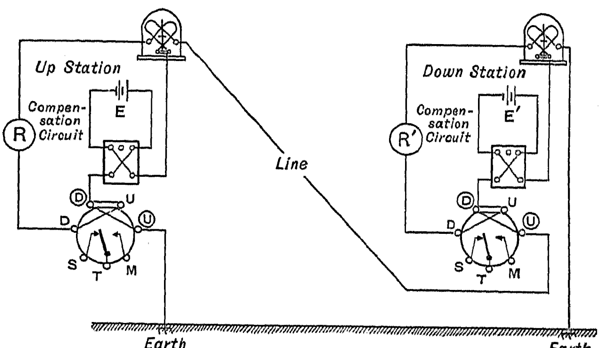


Fig. 4.

From the foregoing it will be readily understood that in order to ensure that the relay at the sending station shall not be affected by outgoing signals it is necessary to adjust the compensation circuit so that the current

passing through it is exactly equal to the current passing to line. The process of adjusting the compensation circuit to effect this is termed "balancing."

Now a telegraph line possesses electrostatic capacity, which causes a momentary rush of current into the line when the key is depressed. Similarly, when the key is raised, there is a momentary rush of current in the opposite direction due to the discharge of the line. On short aerial lines these effects are inappreciable, but on long lines the mere adjustment of resistance in the compensation circuit is incapable of counteracting the disturbances on the relay due to the momentary rushes of current, and it becomes necessary to include a condenser therein. Moreover, the act of charging and discharging the line circuit cannot be effected so quickly as in the case of a condenser, owing to the distributed capacity and resistance of the former; the times of charge and discharge of the condenser in the compensation circuit must therefore be delayed by introducing a suitable resistance, known as a "retardation coil," in series with it.

Hence in the process of balancing a duplex circuit two adjustments must be made, viz.: (1) the resistance balance; (2) the capacity balance. It is usual in some cases, when balancing, to request the distant station to replace his battery by an equivalent resistance, a process which is conveniently done by means of a two-way switch. In other cases the balance is obtained against the incoming spacing current from the distant station. The first step is to balance the line resistance before dealing with the capacity balance. Sufficient resistance is placed in the rheostat of the compensation circuit until the needle of the home galvanometer gives the same steady deflection to both positions of the signalling key. There will be movements of the needle of the galvanometer at the instant the key is raised or depressed, but this is a "capacity" effect and is not due to imperfect resistance balance. The capacity balance is more difficult to obtain, as it involves the adjustment of the retardation coil as well as the condenser. A general rule is that the capacity in the compensation circuit should be approximately equal to one-third of that of the line. On long aerial lines it is often necessary to construct the compensation condenser in two portions between which is inserted an additional timing resistance, known as a condenser coil (see *Fig.* 5). For long underground circuits and short submarine cables a "triple" condenser and three timing coils are required.

The resistance and capacity in the compensation circuit must be adjustable because of the variable nature of the resistance and capacity of lines, due to leakage and weather changes. These changes are particularly felt on long aerial lines, and the compensation circuit must be frequently readjusted to preserve the balance. On underground wires the balance when once obtained rarely requires altering.

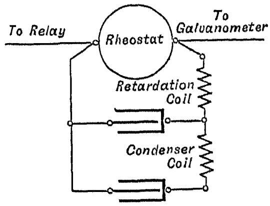


FIG. 5.

§ (3) BRIDGE DUPLEX. — This system of duplex working is based upon the principle of the Wheatstone bridge, and is represented diagrammatically in *Fig.* 6.

Referring to the up station, AC and AE are fixed resistances equal in value, known as the bridge arms, and the receiving apparatus is connected between them as shown. R is the compensation circuit, and, as in the case of the differential duplex, contains a condenser in series with a resistance shunted across a rheostat in order to imitate the distributed capacity and resistance of the line. The connections at both ends are identical.

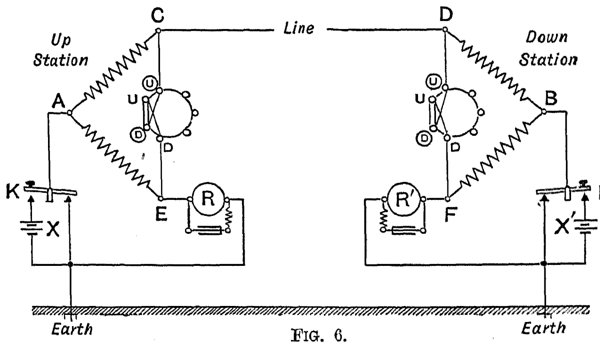


FIG. 6.

When the up station key K is depressed the current from the battery X divides into two equal parts, part going *via* resistance AE, compensation circuit, back to the negative pole of the battery X, and the other part going *via* AC, and the line to D, where it splits, part going *via* DB to earth, there uniting with the part going *via* DF to earth, and back through the earth to battery X. The part flowing through DF actuates the receiving

apparatus at the down station. At the up station the equal resistances AC and AE are traversed by equal currents, so that the points C and E are at the same potential and no current will flow through CE. The up station apparatus is therefore unaffected by outgoing signals.

Similarly, when the down station key only is depressed, the receiving apparatus at the up station responds, but that at the sending station is unaffected.

When the keys at both stations are depressed simultaneously both batteries are applied to the line and no current will flow along it between C and D. The current from the up station battery X now divides unequally at A, part going *via* ACE, thus actuating the receiving apparatus, and part going along AE to E, where the two parts unite and flow back through the compensation circuit to the battery. The operation of the down station receiving apparatus may be explained in a similar manner.

For maximum speed working on long cable circuits the bridge system is invariably used because under equivalent conditions a greater speed is attainable than with differential duplex. The bridge resistances are shunted by what is known as the "signalling condensers," which are usually approximately equal in capacity to that of the line. These condensers increase the rate of charge and discharge of the line by effectively short-circuiting the bridge arms at the commencement of a signal, thereby allowing a smaller battery power to be used to attain a required speed than would otherwise be the case.

A. E. S.

TELEGRAPHY, MULTIPLEX

The Multiplex System of Telegraphy is one in which a number of messages may be sent simultaneously over a single line, either in the same direction or in opposite directions, so that its full working capacity may be utilised. The duplex and quadruplex systems of telegraphy allow of this being done to the extent of two and four messages respectively, by employing duplex balances and currents of varying strength (see articles "Telegraphy, Duplex," and "Telegraphy, Quadruplex"); the term "multiplex," however, is reserved to denote systems that are based upon the entirely different principle of allowing several telegraph operators to have exclusive use of the line, periodically, for short intervals of time. The idea of such a system originated with Farmer, of Boston, U.S.A., in 1852, and was subsequently considerably developed and rendered practicable by Meyer in 1873, Baudot in 1881, and Delany in 1884.

The underlying principle of multiplex telegraphy is indicated in *Fig.* 1. At the terminal stations X and Y are segmented metallic rings which are traversed, respectively, by the rotating contact arms A and B. The latter are connected electrically with the line, and, assuming that they are running at exactly

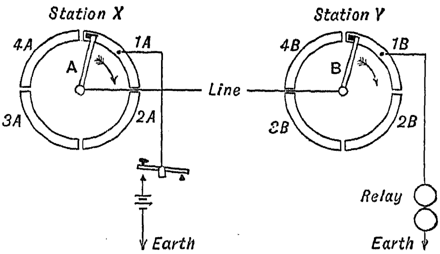


Fig. 1.

the same speed and that they start together from the beginning of corresponding segments, it follows that during the time arm A is in contact with segment 1A, arm B is in contact with segment 1B, and similarly for the other segments. Thus segments 1A and 1B, 2A and 2B, etc., are successively electrically interconnected once per revolution. If a Morse key and battery is joined to segment 1A and a relay to segment 1B, as shown in *Fig.* 1, then if the time of transit of the signal along the line is assumed to be negligible, the relay may be actuated by means of signals sent by the key during the time the segments 1A and 1B are electrically connected by the passage of the contact arms over them. By joining up the other segments to similar sets of apparatus it would be possible to work four channels independently. The combination of the rotating arm and segmented ring is known as a distributor, and the actual contact of the arm with the segmented ring is made by means of a wire brush.

It is evident that in such an arrangement as described the correct reception of Morse signals on a relay connected to a particular segment will depend upon the instant at which the corresponding key is depressed during the revolution of the arms. If the key is depressed before or after the contact arm has traversed the segment to which it is connected, the signals may be missed altogether or mutilated. The actual missing of signals may be prevented by increasing the speed of rotation of the arms and by subdividing each segment into a number of smaller segments, electrically connected, but rearranged equidistantly around the ring, as shown in *Fig.* 2. This does not, however, overcome the trouble due to mutilated signals, which has been a great drawback to all Morse multiplex systems. In the Delany system, which was first tried in England in 1885, the trouble due

to the above cause was greatly minimised by employing for the reception of signals a relay with coils possessing high resistance and

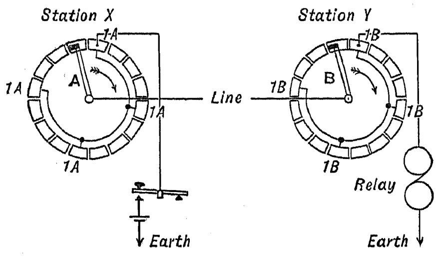


FIG. 2.

inductance, shunted by a condenser of high capacity, in order to render it very sluggish in its action. A mutilated signal such as a dash passing through the relay coils consisted of a number of rapidly succeeding impulses, but owing to the sluggishness of the relay it was restored more or less into a continuous signal on a local sounder.

The multiplex systems, prior to the Delany, failed chiefly because of the difficulties encountered in keeping the arms rotating at a uniform speed and in step with one another. Delany overcame this difficulty by driving the arms by phonic-wheel motors worked by intermittent currents sent out by electrically vibrated reeds, a method now used on all modern multiplex systems (see article "Telegraphs, Type-printing"). The arms at the two stations were kept in step by means of correcting currents sent out from special segments set apart on the rings at each station. These correcting currents were automatically transmitted over the line to whichever station was running slow, and, by operating a "correcting relay," brought about the short-circuiting of the electromagnet for driving the reed. This had the effect of increasing the rate of vibration of the reed and therefore the speed of the phonic wheel driven by the latter. In this way the contact arms were prevented from getting out of step.

The Delany system was not entirely satisfactory, but nevertheless it gave some good results and was not abandoned in England until about twenty years after its introduction. It was eventually abandoned because of the high line-speed required, which was only attainable on a few aerial line circuits which had very short underground sections.

The adoption of a 5-unit code for signalling purposes enabled the multiplex principle to be utilised with complete success. In the 5-unit code all the letters are of the same length and are represented by permutations of five current impulses; the time occupied in transmitting each letter is therefore the same. If the continuous quadrants of *Fig.* 1 are each divided into five segments insulated from one another it will be possible to send one character in the 5-unit code per quadrant per revolution. For setting up the permutations of current impulses the operator may be provided with five tapper keys (see *Fig.* 3) worked by the fingers. By warning the operator when to make a permutation, which should be just before the contact arm reaches the first of the segments allocated to the keys, and by preventing the keys depressed from rising until the contact arm has traversed the last of the five segments, the signal will be sent out unmutilated. At the receiving end the selective operation of five electro-

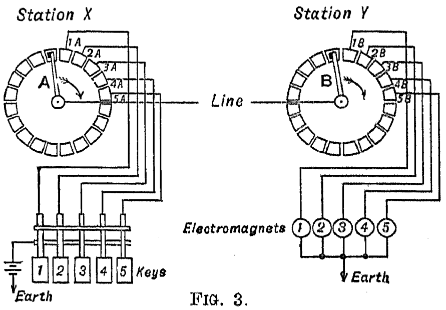


FIG. 3.

magnets records the signal for a particular channel. The preceding is what is actually done in the Baudot multiplex system, which is described in the article on "Telegraphs, Type-printing"; double current working is adopted, however, in practice.

So far, the time of transit of a current impulse from one station to the other has been assumed negligible. On all but very short lines the capacity and resistance are such as to cause an appreciable time to elapse between the sending of a signal and its reception at the distant station. This time interval may be of such duration that if the contact arms A and B in *Fig.* 3 are on segments 1A and 1B respectively when a current impulse is sent from the former, then by the time the impulse has reached the receiving station the arm B may have left segment 1B and be passing over segment 2B or even 3B. The result is that the impulses are received on the wrong segments and signals are rendered unintelligible. To overcome this difficulty the receiving segments are rotated or, as it is generally termed, "orientated" in a clockwise direction a number of segments equivalent to the retardation.

Fig. 4 shows diagrammatically the effect of retardation on a double simplex circuit, over which two channels are being worked, one in each direction. Thus keyboard A

works to receiver B and keyboard B to receiver A. If the retardation is equivalent to one segment then for the correct reception at B

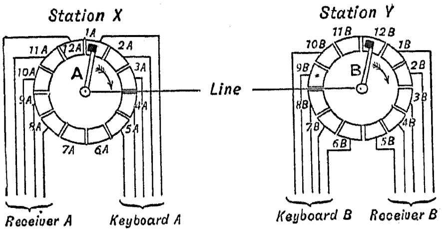


FIG. 4.

the relative positions of the segments must be as shown, that is, those at B are one segment in advance of those at A, so that when the contact arm at A is on 1A that at B is on 12B. The keyboard B is shown joined to segments 6B to 10B, but as these are one segment in advance of the corresponding segments 6A to 10A at A, then owing to a further retardation of one segment in a signal passing from B to A a signal sent out from 6B will be received on 8A, hence receiver A must be joined to segments 8A to 12A. It will be noticed, therefore, that two extra segments are required on the distributor rings on account of the retardation. If the retardation is equal to more than one segment additional idle segments will be required, and these can only be provided by dividing the ring into smaller segments. The effect of decreasing the size of the segments is to raise the actual speed of transmission of signals, but it does not alter the number of words sent per minute, which depends solely upon the rate of rotation of the contact arms. The number of channels that can be worked over a circuit is limited by its capacity and resistance, because the larger the number of channels the smaller the segments and the greater the speed of transmission of the signals.

Multiplex systems may be duplexed by either the differential or the bridge method (see article "Telegraphy, Duplex"), but the former is the method adopted in practice. The principle of the duplex multiplex is shown in *Fig.* 5. Two metallic segmented rings with corresponding rotating arms are required at each end, one for receiving and the other for sending. The arm traversing the sending ring is connected to the split of a polarised relay R; one coil of the latter is joined to the line and the other to the compensation circuit. The arm of the receiving ring is connected to the tongue of the relay. Assuming the compensation circuit to be the exact equivalent of the line as regards capacity and resistance, the outgoing currents from the sending ring will divide equally between the line and compensation coils of the relay and the latter will not be operated. Incoming currents from the line traverse the relay in a marking direction, causing its tongue to complete the local circuit and work the receiving apparatus connected with the segments of the receiving ring. To distinguish multiplex working from other systems of simultaneous transmission the following notation is adopted:

Simplex.—Two channels in the same direction or two channels one in each direction is termed "double" simplex.

Four channels, the directions of which may be varied to suit traffic requirements, is termed

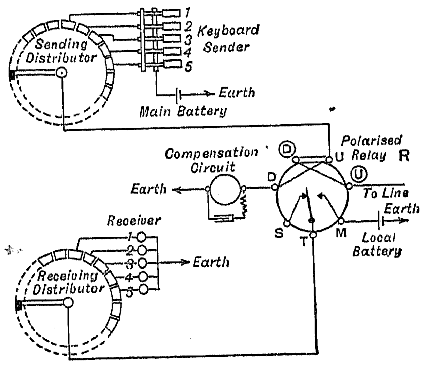


FIG. 5.

"quadruple" simplex. Similarly six channels is termed "sextuple" simplex.

With a Duplex Balance.—Four channels, two in each direction, is termed "double" duplex. Eight channels, four in each direction, is termed "quadruple" duplex, and so on.

There are other systems of multiplex working based upon the simultaneous transmission of alternating currents of different frequencies. In the Mercadier system, which is typical, each operator at the sending station transmits, in the Morse code, groups of long and short series of current alternations of definite frequency. At the receiving end the currents of different frequencies actuate tuned diaphragms of telephone receivers, so that the actual reception of signals is by sound.

A. E. S.

TELEGRAPHY, QUADRUPLEX

QUADRUPLEX telegraphy provides for the simultaneous transmission over one wire of four messages, two in each direction. The principle of the system is based upon the fact that electric currents may differ from each other in strength and also in direction.

Two signalling keys and two receiving relays are necessarily required at each terminal station. One key, called the A key, reverses

the direction of the current in the line, and operates a polarised relay at the distant station. The other key, called the B key, increases the strength of the current flowing in the line, when it is depressed, and actuates a non-polarised relay at the distant station. This second relay is biased against the smaller current from the A key, but responds to the larger current sent by the B key, whatever its direction.

The application of the above principle to quadruplex working is indicated in *Fig.* 1, which shows the skeleton connections of a circuit. For simplicity the connections of the local circuits of the relays have been omitted. A local battery and sounder is connected in the usual manner to each polarised relay, but a special method of joining up the local circuit of the non-polarised relay is adopted for reasons which will be explained later. The apparatus is duplexed on the differential principle (see article "Telegraphy, Duplex"), so that the arrangement may be worked in both directions simultaneously. Assuming that the compensation circuits R and R′ are properly adjusted, the outgoing current at a station divides differentially through both relays and therefore has no effect upon them. The incoming current operates the relays selectively according to the key depressed at the distant station.

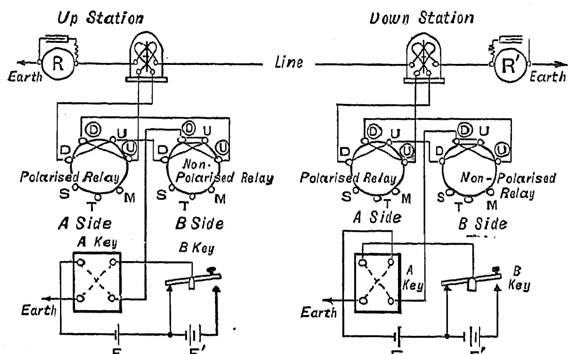


FIG. 1.

There is a number of possible combinations of positions of the four keys; the following represent the principal combinations and the actions brought about by them:

(i.) *All Keys at rest.*—At each station the battery E is of sufficient voltage to supply a current of about 15 milliamperes through both the line and compensation circuits. With all keys at rest, the batteries E are connected in series as regards the line circuit but act independently in the compensation circuits. At the up station the line current of 30 milliamperes passes through the D to U coils of both relays and the compensation current of 15 milliamperes passes through the U to D coils. The direction of the *effective* current of 15 milliamperes is from D to U through both relays, but as the non-polarised relay is biased against a current of this strength both relays will "space." In a similar manner it may be shown that both relays at the down station will also "space."

(ii.) *Both* A *Keys depressed,* B *Keys at rest.*—The currents are of the same strengths as in case (i.), but are reversed in direction. The effective currents of 15 milliamperes therefore flow through the U to D coils of both relays at the up station and through the U to D coils at the down station. The tongue of each polarised relay will consequently move to the marking contact and actuate the sounder. The non-polarised relays will not be affected because the current is of insufficient strength.

(iii.) *All Keys depressed.*—The conditions, as regards direction of currents, are the same as in case (ii.). The depression of the B keys, however, results in the application of the combined batteries E and E′ at each station to the line. Each combined battery is capable of producing a current of about 45 milliamperes in both line and compensation circuits, the voltage of E′ being twice that of E. The effective current in this case therefore becomes 45 milliamperes, and as it flows through the relays in the directions indicated in case (ii.) and is of sufficient strength to overcome the tension of the springs on the tongues of the non-polarised relays, all relays will mark and register signals.

(iv.) A *Key only depressed at the Up Station and* B *Key only depressed at the Down Station.*—In this case the current in the line due to both down batteries E and E′ is 45 milliamperes and the opposing current from the up battery E is 15 milliamperes, so that the net current in the line is 30 milliamperes. At the up station the line current passes through the relay coils from D to U, and the compensation current of 15 milliamperes passes through the relays from D to U. The combined effect is equivalent to a current of 45 milliamperes through one coil, from D to U of each relay. The polarised relay consequently spaces and the non-polarised relay marks. At the down station the line current is 30 milliamperes and flows from D to U through both relays; the compensation current is increased by the depression of the down B key to 45 milliamperes and flows from U

to D through both relays. The combined effect is equivalent to a current of 15 milliamperes through one coil from U to D of each relay. This is of insufficient strength to operate the non-polarised relay, but its direction is such as to cause the polarised relay to "mark."

The actions resulting from other combinations may be traced out in a similar way.

As already mentioned, sounders and batteries are connected in the local circuits of the polarised relays in the ordinary way; with the non-polarised relays a different method has had to be adopted in order to eliminate the effects of what is known as the "B" kick. This term is applied to the break in the continuity of signals received on the non-

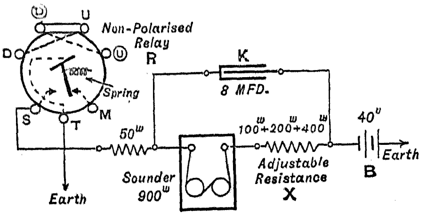


FIG. 2.

polarised relays, due to the momentary demagnetisation of their cores when the current in the line is reversed. The method of connecting the apparatus in the local circuits of the non-polarised relays in order to overcome this defect is shown in *Fig.* 2. When the local circuit is closed the condenser K is charged by the local battery B. If, during the time the local sounder is registering a signal, the A key at the distant station is depressed, the current through the non-polarised relay R is reversed and its cores are momentarily demagnetised. When this occurs the relay tongue is pulled by its controlling spring away from the contact and the local circuit is momentarily broken. At this instant the condenser K discharges through the sounder coils and the adjustable resistance X, prolonging the magnetisation of the sounder cores for a time sufficient to bridge the momentary disconnection, thus preventing a break in the continuity of the signal. The resistance coil of 50 ohms is inserted in the local circuit to prevent damage to the relay contacts, which might otherwise result from the powerful discharge of current from the condenser.

On short lines of low resistance and capacity but of high insulation, difficulty is often experienced in working quadruplex owing to the inductive discharges from the apparatus at the distant station. The effect is to cause the relay tongues at the sending station to "kick" and produce false signals. This difficulty is overcome in practice by inserting a suitable inductance coil in the compensation circuit or by connecting to the line at the sending station a condenser, as a leak to earth. Inductive disturbances may also be satisfactorily nullified by artificially increasing the line resistance by the insertion of suitable resistance coils at each end of the line.

The quadruplex system that has been described in the foregoing is known as the *Increment System*, because the current is increased when the B key is depressed. There is another method of quadruplex working known as the *Decrement System*, in which the whole battery is applied to line when the B key is at rest and part of it cut out when the B key is depressed. The apparatus used and the main connections are practically the same as for the increment system. It possesses a great advantage in that the "B" kick effects are not felt, so that the local sounders may be joined up in the ordinary manner to the non-polarised relays and the condensers and resistance coils used in the increment system are not necessary.

In practice it sometimes happens that the bulk of the traffic over a circuit is in one direction only, so that three channels of a quadruplex are sufficient to meet the requirements, two in one direction for the disposal of the traffic and one in the opposite direction for giving acknowledgments. It is usual in such circumstances to modify the connections and the apparatus by dispensing with the non-

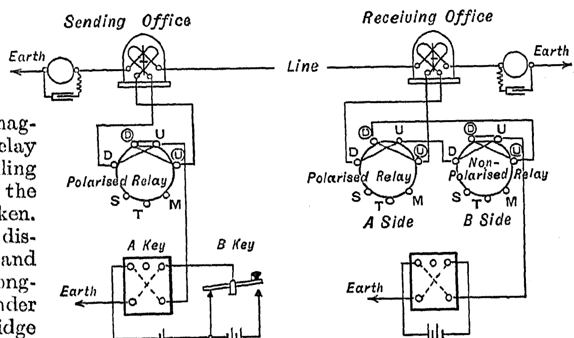


FIG. 3.

polarised relay at the sending station and the B key at the receiving station. This arrangement, which is known as *diplex working*, is shown in *Fig.* 3.

Other systems of quadruplex working are comprised as follows:

FORKED QUADRUPLEX.—Up station fitted with quadruplex set works to an intermediate station, which repeats the signals

on the A and B sides respectively to two separate down stations provided with duplex apparatus.

Split Quadruplex.—Quadruplex sets at up and down stations work duplex to each other on the A side. The B side works duplex from each terminal station to an intermediate repeating station.

Quadruplex A Side Relayed Duplex.—Full quadruplex set at one station works B side duplex to an intermediate repeating station. The A side signals repeated to another station equipped with duplex apparatus.

Quadruplex A Side Relayed Simplex Central Battery.—Full quadruplex set at one station. A side relayed at the repeating station to a central battery omnibus (see article "Telegraphs, Central Battery System of") or other simplex C.B. circuit.

The A side of a quadruplex circuit is often worked at a high speed by substituting a Wheatstone automatic transmitter for the A side key and a Wheatstone receiver for the polarised relay at each end, the B side being worked at the ordinary key speed (see article "Telegraph, The Electric," for Wheatstone Automatic System). A. E. S.

TELEGRAPHY, REPEATERS

Owing to the considerable cost of constructing long telegraph lines it is desirable, for economic reasons, to work them at the highest speed possible. The maximum speed of a circuit is limited, however, by its resistance and capacity, and although, with a particular type of receiving instrument, the retarding effects due to these causes may be overcome to a certain extent by increasing the battery power, there is a limit to the latter beyond which it is not safe to go for practical reasons. When such is the case a "repeater" is used to increase the working speed of the circuit.

A repeater, as its name implies, is simply an arrangement of apparatus introduced into a long circuit at an intermediate point, for retransmitting to the receiving station signals of greater current strength than those received by the repeater from the sending station. The station containing the repeating apparatus is generally known as the "repeater office."

The apparatus constituting the repeater will be described later; it will suffice at this stage to mention that it is carefully constructed and adjusted in order that it may be capable of working at a high speed with small current. The following considerations will serve to indicate the theory underlying the application of repeaters for increasing the speeds of circuits:

If K and R are the total capacity and the total resistance respectively of a telegraph circuit, then its maximum speed of working is equal to A/KR, where A is a constant depending upon the type of receiving instrument used. The speed of a circuit of one-half the length will be four times as great, because the total capacity and the total resistance, in such a case, are both half of what they were for the whole circuit. If therefore a repeater is introduced at the centre of a circuit, it will work at four times the speed of the whole circuit and automatically retransmit signals at this increased speed along the other half of the circuit to the receiving station. It is not always practicable to insert the repeater exactly at the electrical centre of the circuit so that the maximum speeds possible over the two sections will differ; the maximum working speed of the whole circuit will then be equal to that of its slowest section.

The general arrangement of a fast-speed, double-current simplex repeater is shown in *Fig.* 1.

R and S are ordinary standard polarised relays (see article "Telegraph, The Electric"). They are known as "transmitting relays," because, when either is operated by signals from a terminal station, its tongue transmits similar signals, but of increased strength, from the repeater battery to the receiving station. For fast-speed working it is essential that the pivots and bearings of these relays should be properly burnished, so that frictional resistances to the motion of their tongues may be minimised, and also that the play of their tongues between the contacts should be as small as possible. The relays P and Q, which are called "automatic switch relays," are similar in construction to the standard polarised relay, but the tongue of each is fitted with adjustable springs which hold it midway between the contacts and clear of both. T and V are "automatic switches," each consisting of an electromagnet which has two armatures that play between contact stops. These armatures are held normally against their outer stops by means of spiral springs, but when a current passes through the coils in either direction the armatures are attracted to their inner stops. Each automatic switch corresponds to one of the terminal stations, and its function is to respond automatically to the movements of the key switch of that particular station. If the key switch at a station is put to the "send" position the armatures of the corresponding automatic switch are attracted; if it is put to the "receive" position, the armatures are pulled by springs to the outer stops. In the first case the result is to connect the section of the line leading to the distant station to the repeater battery, while in the second case the line to

the distant station is joined to the repeating instruments but disconnected from the repeater battery.

The action of the repeater will be best understood by following the operation of signalling from the down to the up station. Suppose the down station key switch is put to "send," a current will flow over the line from the down to the repeater station and will pass through the down-side galvanometer, the right-hand armature of the automatic switch V, relays R and P to earth, as shown by the continuous arrows. The tongues of relays R and P are therefore placed on their spacing contacts. The operation of relay P completes the circuit of a local battery B through the automatic switch T, and the latter is are shunted by a resistance Z of the same value as the coils themselves. This makes the automatic switch sluggish in its action by providing an easy path for the self-induced currents in the coils when the momentary breaks occur in the local circuit, and consequently preventing the demagnetisation of the cores.

The dotted arrows show the path of a repeated spacing current to the up station from the repeater battery X. The armatures of the automatic switch T are shown in the attracted position dotted; the repeated current splits at the inner contacts, part going to the up station and part through a leak coil and Wheatstone receiver (see article "Telegraph, The Electric"). The object of the latter

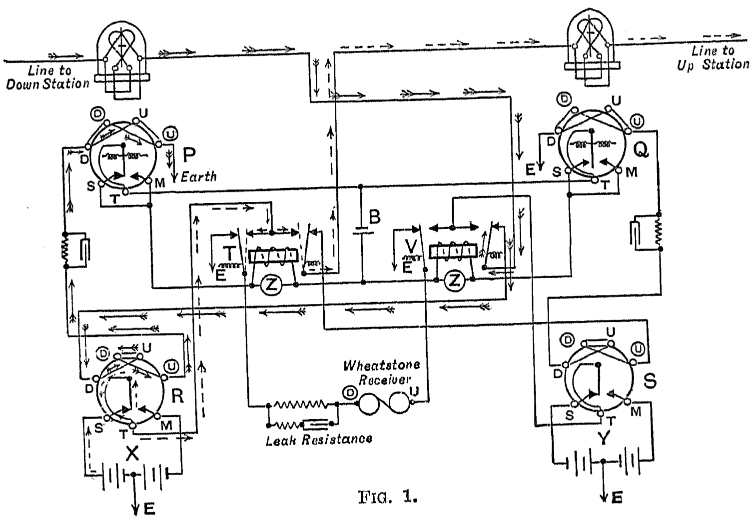


FIG. 1.

energised, thereby attracting its armatures to the inner stops. This connects the tongue of the transmitting relay R, *via* the right-hand armature of the automatic switch T to the line leading to the up station, and a spacing current is sent to the latter because the tongue of relay R is on the spacing contact. If the down station key is depressed the line current is reversed in direction and the tongues of both relays R and P are attracted to their marking-stops, and a marking current is sent to the up station.

During the operation of signalling from the down station the tongues of the relays P and R vibrate in unison with the signals passing through. The circuit through the automatic switch T is therefore being continually broken and closed again, and in order to prevent the armatures from breaking contact during the passage of the tongue of relay R from one stop to the other the coils of the automatic switch circuit is to enable the repeater clerk to check the working of the repeater.

When the down station has finished sending, the key switch there is put to "receive" and the line current is thereby cut off. The armatures of the automatic switch T are then pulled back by the springs and the apparatus is ready for the up station to send. In addition to the apparatus shown two single-current keys with switches (see article "Telegraphy, Universal Battery System") are provided to enable the repeater clerk to communicate with the terminal stations whenever necessary.

For duplex circuits a repeater arranged for "duplex" working is required. Owing to the fact that in duplex working the key switch is kept permanently to "send," it is possible in the duplex repeater to dispense with the neutral relays, automatic switches and the connections of the apparatus to be simplified. The skeleton connections of a duplex repeater are shown in *Fig.* 2. A and

B are standard polarised relays set neutral. R and R′ are compensation circuits for the up and down lines respectively. With the

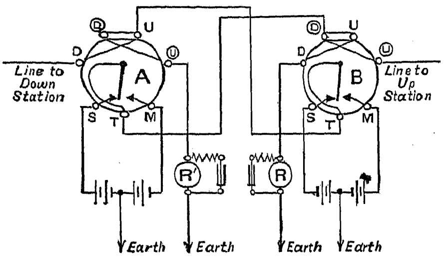


FIG. 2.

keys at both terminal stations at rest, the tongues of both relays rest on their respective spacing contacts. The current from the tongue of relay A passes to the split of relay B and divides equally between the compensation coil U to D and the line coil (D) to (U). The current through the line coil is supplemented by a current of equal strength from the up station, so that for relay B we have the current through the line coil from (D) to (U) double that through the compensation coil from U to D, therefore the relay "spaces." Similar remarks apply to the current passing from the tongue of relay B to the down station. When both terminal keys are depressed the currents are everywhere reversed in direction, with the result that the tongues of relays A and B both move to "marking" and transmit marking currents to the terminal stations. Briefly, relay A responds to the key at the down station and retransmits the signals to the up station; and relay B responds to the key at the up station and retransmits to the down.

A Wheatstone receiver in "leak" and a switch are provided so that the repeater clerk may check the signals passing through the repeating apparatus to the terminal stations. Sounders and signalling keys are also provided to enable communications to take place with the terminal stations whenever necessary.

Special forms of repeaters have been arranged to meet various requirements; the "forked news repeater," for instance, enables an up station to transmit signals to two down stations on separate lines "forked" at the repeater station, and arranges for any of the three stations to signal correctly to both the others.

A. E. S.

TELEGRAPHY, UNIVERSAL BATTERY SYSTEM

To provide a separate battery for each circuit at a large telegraph office would be a very costly procedure owing to the number of cells required for the purpose. Moreover, the space occupied by the batteries would necessarily be considerable and the cost of maintenance very high. In order to effect economies in these respects, it is now the practice to arrange for a number of circuits to be worked from one battery. This particular method of working telegraph circuits constitutes what is known as the Universal Battery System.

The principle of the method and the factors governing its successful application will be understood by referring to *Fig.* 1, where B

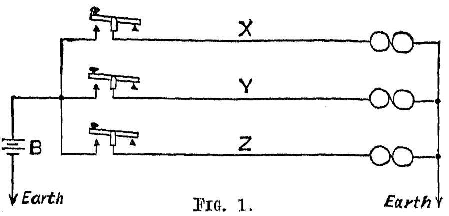


FIG. 1.

denotes the universal battery of E.M.F., E volts and internal resistance R ohms, connected as shown to a number of circuits the resistances of which, including the receiving instruments, are X, Y, Z, etc., ohms.

If the key in circuit X only is depressed, the current flowing in that circuit will be equal to

$$\frac{E}{R+X}. \quad . \quad . \quad . \quad . \quad (1)$$

When a number of keys are depressed simultaneously, the total current flowing from the battery will be equal to

$$\frac{E}{R+J},$$

where J denotes the joint resistance of all the circuits that are closed by the depression of the corresponding keys. The current flowing through circuit X will in this case be equal to

$$\frac{E}{R+J}\times\frac{J}{X}, \quad \text{that is} \quad \frac{E}{X(R/J+1)}. \quad (2)$$

On comparing (1) and (2) it will be evident that the current in the circuit X will vary in amount according to the number of circuits that are worked simultaneously from the battery. If, however, R is very small compared with X (and therefore small also compared with J), both (1) and (2) reduce to E/X, so that in this circumstance it will not matter whether one circuit alone is worked or whether several circuits are operated simultaneously, the current in circuit X will be the same in both cases.

It follows that for the successful working of the universal battery system it is essential that the internal resistance of the battery should be as low as possible compared with

the resistances of the circuits connected to it, otherwise the current in any particular circuit will vary according to the number of other circuits that are being worked at the same time. These variations may, under certain circumstances, depending upon the relation between the various resistances and also upon the nature of the receiving instruments, be of such magnitudes as to render satisfactory working impossible. If primary cells are used to form the universal battery the latter will have an appreciable internal resistance, and it becomes necessary in practice to restrict the number of circuits to be worked by it to five or six. Even with this restriction the variations in the currents may be too great, and in order to keep them within certain limits, so that good working may be possible, it is further stipulated that the total internal resistance of the battery must not exceed one-half of the joint resistance of all the circuits connected to it, and that the resistance of the circuits must not differ by more than 25 per cent between the highest and lowest. Circuits of low resistance connected to such a battery must therefore have equalising resistance coils added, so as to bring the total resistance of the circuits up to the minimum value. The position for these coils is in the battery leads so as to be out of circuit for received signals, otherwise they would entail an increase in voltage at the out station.

The internal resistance of secondary cells is practically negligible, thus enabling a large number of circuits to be worked from one secondary battery. The latter, however, must be of sufficient size to supply the maximum current required and be able to maintain a sufficient output so as to avoid the necessity of frequent charging.

As already pointed out, where primary cells are used for universal battery working, the circuits are all brought up to approximately the same resistance so that the same E.M.F. may be used for all the circuits. With secondary cells a different method is adopted, and the various circuits are not brought up to the same resistance, but each is supplied with a voltage suitable to its particular resistance. This is effected by arranging the circuits in groups, each consisting of circuits having approximately the same resistance. In practice only four groups have been found necessary, worked from voltages of 24, 40, 80, and 120 respectively, obtained by tapping the battery at suitable points. On the universal battery system a single battery can only be used for single-current working because one pole of the battery is permanently earthed. For double-current working it is necessary to employ two separate batteries which have opposite poles earthed.

It is usual to divide a universal battery installation into a main and a local battery. The former is used for supplying the voltages of 120, 80, and 40 required on long circuits. The local battery is of 24 volts and is employed for local sounder circuits and lines of low resistance. There is generally a great number of these minor circuits so that the current output of the local battery must be high: for this reason it is made up of larger cells than those of the main battery and kept separate from it.

A different type of signalling key, known as a "Single Current Key with Switch," is employed at the terminal stations on double-current circuits worked from universal batteries. It consists of a single current key with a two-way switch mounted together on one base. *Fig.* 2 is a diagrammatic representation of the

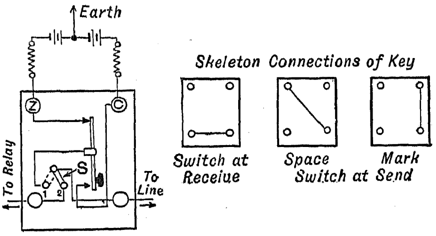


FIG. 2.

key and shows the internal and external connections for an up-station. By means of the two-way switch S the battery or receiving apparatus may be connected to the line as required. When S is on stud 2, the battery is disconnected from the line and the latter joined through to the receiving apparatus. By moving S over to stud 1, the receiving apparatus is disconnected and the signalling key joined to the line so that positive or negative currents may be sent out as desired.

With a secondary cell installation, the sounders used are wound to a resistance of 900 ohms instead of 20 ohms as for the ordinary sounder (see article on "Telegraph, The Electric"). In construction the two sounders are identical, but the higher resistance is adopted in the former so that the resistance of the battery and leads may be negligible in comparison and not cause unsatisfactory working.

It is necessary to insert fuses in the leads from a secondary cell installation, owing to the excessive currents that may result should contacts or earth faults occur. To prevent any of the fuses from blowing should a fault develop such as a short-circuit in a signalling key, metal-cased resistances, or, where the voltage exceeds 80, lamp resistances are inserted in the battery leads to the key, in order to

keep the resulting current from exceeding a certain value. The arrangement of the cells

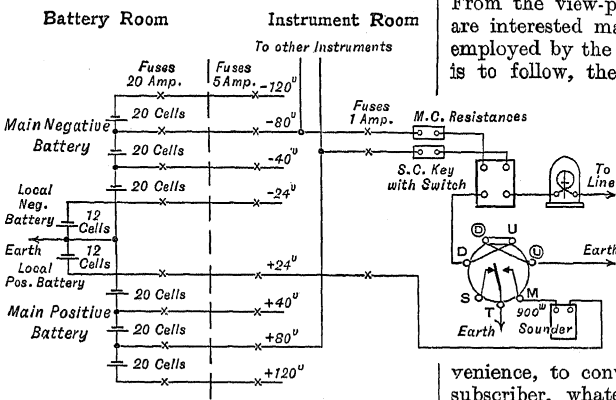


FIG. 3.

and the connections of a set of apparatus on a universal battery installation are shown in *Fig.* 3.

A. E. S.

TELEPHONE, use of, as detecting instrument in alternating current bridge measurements. See "Inductance, The Measurement of," §§ (31)-(35).

Humming: an arrangement of a telephone receiver and a microphone which gives a sustained musical note. Use of, as a source of audio-frequency current. See *ibid.* § (18).

TELEPHONE CIRCUITS, simultaneous use of, for telegraphy and telephony. See "Telephony," § (30).

By Composite Sets: a method involving the separation of the low-frequency telegraph currents from the high-frequency telephone currents. See *ibid.* § (32).

Simplex Method: a method involving the use of "phantom" repeating coils. See *ibid.* § (31).

TELEPHONY

§ (1) DESCRIPTION OF A COMMERCIAL TELEPHONE SYSTEM. (i.) *General.*—The telephone is the quickest means of direct communication between persons separated from one another. Of the three available systems for distant communication—the mail, the telegraph, and the telephone—the last is the only one in which the originator of the communication is enabled to speak directly to the desired person and to receive an immediate answer. It is the fastest and most complete intercommunication service required by the needs of business and social life. Correspondingly, it is usually somewhat more expensive than communication by mail or telegraph.

The problem of telephony involves commercial, traffic, and plant considerations. From the view-point of Applied Science, we are interested mainly in the physical means employed by the telephone system. In what is to follow, therefore, we will confine our attention almost entirely to the plant equipment or apparatus.

In order that a telephone system may meet satisfactorily the demands imposed upon it, it should include facilities such that at any time on request of a subscriber the latter may be enabled, without delay or other inconvenience, to converse freely with any other subscriber, whatever the distance involved, and the charges for the service should be as low as is consistent with the giving of a high grade of service. The most important economic and physical problems underlying the building of the plant may be grouped under (1) the lay-out of the telephone system as a whole, (2) the means employed for interconnecting subscribers, and (3) problems involved in the electrical transmission of speech.

(ii.) *Lay-out of System.*—The fundamental principle underlying the lay-out of the telephone system is that each subscriber must be able to reach every other one. For economy this requires grouping in some way, and the most economical way is naturally to group together the subscribers in small geographical localities. Moreover, the subscribers in a small local area will in general wish to talk more frequently to one another than to those at a remote distance. The subscribers' lines in a given locality are brought together at a common point called a local central office. The length of the average subscriber's loop—*i.e.* the distance between the subscriber's set and the local central office—is usually under one mile. If the locality is a small one (less than 10,000 subscribers) there is usually only a single local central office, and the district is known as a single-office district. If the city or area served is a large one, several local central offices will be required. Each of these is located as near as possible to the subscribers served by it. Such a district is called a multi-office district. The various central offices in such a district are interconnected by junction lines or junctions, as is shown on the diagram below, *Fig.* 1.

Engineers provide facilities in a given district according to a fundamental plan study, in the course of which an estimate is made of the probable growth of the district during the next twenty years or more. Such studies

are made to enable the plant to be expanded in the most economical manner to meet the needs of the district.

Each local central office is also connected to a trunk exchange by means of trunk junctions. These trunk exchanges may be

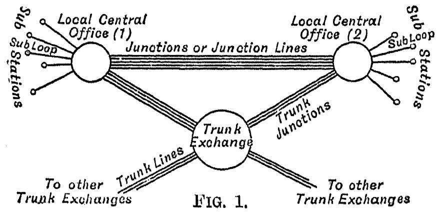


FIG. 1.

located anywhere from a few miles up to several hundred miles apart, and they are connected by a network of long-distance or trunk lines.

The subdivisions outlined above have simply resulted from the meeting in the most natural and economical way of the service requirements. The costs of the service in any local office district are usually lumped together and a uniform charge made for all the local calls. On the other hand, a graduated charge is usually made for long-distance calls, the amount depending upon the distance covered.

§ (2) OUTSIDE PLANT.—The lines connecting telephone exchanges with various subscribers' stations may be a combination of, or entirely one of the following alternatives:

(i.) Open wire.
(ii.) Aerial cable.
(iii.) Underground cable.

Pole lines are employed for supporting either open wires or aerial cable. In the case of open wires, poles are equipped with brackets or arms. The size of arm varies from two-way to ten-way, depending upon the anticipated number of wires which may be carried. When poles are used for aerial cables, a suspension wire is attached to the poles by means of a clamp and a bolt which passes through the pole.

Conduits are required for the mechanical protection of underground cables, unless an armoured cable is used, which may be laid direct in the ground. The general practice adopted is to lay down a system of ducts which will be sufficient for several years ahead, and to draw in cables from time to time as required. Cast iron, vitrified clay, concrete, wood, or fibre ducts may be used, depending upon local conditions. Access to the duct for feeding in the cables and making the necessary joints is provided by manholes, surface boxes, or jointing pits. The distance apart of these points varies with the circumstances, but would probably average 150 yards.

(i.) *Open Wire.*—Bronze, copper, and iron are used. A copper-clad steel is also being used in some countries, on the ground that this combines the advantages of tensile strength with high conductivity. The British practice is to use 40-lb. bronze for short lines for subscribers' circuits. Copper wire in larger sizes (100 lbs. and upwards) is used for junction and trunk lines, where better transmission is essential. The necessity for having heavy conductors for open-wire circuits, however, has been modified since the advent of loading coils and repeaters. The present tendency is to run these long-distance lines in cable of small gauge conductor.

Iron wire is not used in Great Britain as an overhead conductor, as the life would be so short on account of the climatic conditions. In some countries iron wire has a life of eight years and over, and if transmission requirements can be met, it is sometimes economical to use iron for open-wire circuits.

Open wires are carried on insulators of a suitable material. In Great Britain a high-grade porcelain is used. In America there is an objection to porcelain, as insects build their nests between the sheds of the insulator, and this lowers the insulation resistance of the insulator very considerably. A glass insulator is used practically universally in that country.

The connection from a pole line to a subscriber's premises may be either by means of open wires terminated on suitable brackets at the house, or by covered wires run direct from the pole.

(ii.) *Aerial Cables.*—Where the number of lines required is excessive for pole lines, and not sufficient to justify the large initial expenditure required for placing them underground, aerial cables are adopted. A telephone cable now generally consists of paper-insulated copper conductors twisted together in pairs and stranded in layers, to make up the total number of pairs forming the cable. The cylindrical core thus obtained is covered by wrappings of paper or tape and then lead-covered. An alloy of lead and tin, or lead and antimony, is used in the case of aerial cables in order to give the sheath the required strength. The thickness of the lead sheath depends upon the diameter of the cable, but varies from $\frac{1}{12}$ in. to $\frac{1}{8}$ in. Cables which are designed for phantom[1] working are made up of units of two pairs to form a quad. These cables are known as "quadded," "multiple twin," or "duplex" type, and are used for long-distance trunk service.

An aerial cable is attached to the suspender wire by means of metal, leather, or marline

[1] See § (27).

suspenders. If proper precautions are taken during the installation of the cable, considerable economy is effected in adopting an aerial cable instead of open wires, and, furthermore, there is much less likelihood of breakdown in the service. At the distribution or opening-out points it is very important that an efficient type of cable terminal shall be used. This should be air-tight in order to prevent dampness entering the dry-core cable and thus to maintain the high insulation resistance of the cable.

(iii.) *Underground Cables.*—In the majority of large towns it is the usual practice to have an underground system; all the plant is out of sight, and the cables can be installed when required at a minimum expense. The external diameter of the largest cable manufactured is under 3 in.; such a cable would contain 1200 prs. of 6½-lb., 900 prs. of 10-lb., 450 prs. of 20-lb., or 150 prs. of 40-lb. conductors. A combination of wires of different sizes to meet transmission requirements is frequently made to form what is known as a composite cable.

In the more congested sections of a city where a large number of lines is required in the same building, a "house cable" system is adopted. One or more feeder cables is led into the basement and smaller cables are taken to the several floors. Branch cables are run to suitable points for distribution, from whence internal wiring is taken to the subscribers' instruments. Another method of distribution which is very much used in America is known as "block wiring." Distribution cables are taken along the walls of a building, either internally or externally, and terminated on cable terminals at central points. Covered leads are run through bridle rings to the subscribers' premises. This is a flexible system, and after the initial installation has been completed additional connections are quickly made.

Where armoured cables are required for any special local reason, the lead sheath of the cable is protected by means of one or two layers of steel tape or wire. The cable can then be laid direct in the trench made in the ground with a reasonable degree of security. The same type of cable, but with heavier wire armouring, is used for short water crossings, such as navigable rivers, canals, etc. The paper-insulated cable is employed where a relatively large number of circuits is required. A rubber-insulated cable would only be economical if the number of circuits required in a submarine cable is very small. Long submarine cables are provided with loading coils[1] if the depth of water is not excessive. The alternative is to load continuously by means of specially annealed iron wire or tape, which is wrapped round the conductor before insulating.

[1] See § (24).

§ (3) The Central Office.—The central office is designed for the area in which it is employed. In areas having a small number of telephone subscribers a local battery central office is employed; in areas having a large number, usually from about 1000 upwards, a central battery system would be used. If the area is a single-office area only one type of switchboard section to terminate subscribers' lines is necessary. This is termed an "A" switchboard, and the operators are known as "A" operators. If the area is a multi-office area, then two types of switchboard sections are necessary, the "A" switchboard above referred to, and in addition a "B" switchboard for terminating the junction lines from the other exchange or exchanges in the area. The operators who attend to this switchboard are known as "B" operators.

(i.) *The Operations to be performed in a Telephone Exchange are*: To place a conducting link between the terminal of a calling party's line and the terminal of any party wanted.

To signal the wanted party and to return the apparatus to normal condition after the conversation. This switching, as it is termed, is performed by the operators in manual systems, and by machines under the control of subscribers in automatic systems.

In performing switching the operator observes the signal of the calling party, connects her telephone with the calling line, tests the wanted line to determine whether it is in use or not, and, if free, rings the bell at the wanted station, observes the call for disconnection, and removes the connecting means.

(ii.) *Classes of Calls.* — If the wanted party is connected to the same exchange as the calling party, the call is known as a "local call." If the wanted party is connected to another exchange in the same area, the call is termed a "junction call." If the wanted party is connected to another exchange in a distant area, which has to be reached over trunk lines, the call is known as a "trunk call."

§ (4) Telephone Exchange Arrangements.—At the central exchange the lines from the subscribers' stations are brought into the building and led through protective devices to the switchboard.

Fig. 2 shows a schematic diagram of a typical local battery or magneto switchboard.

Fig. 3 shows a schematic diagram of a typical common battery switchboard.

A photograph of a portion of a telephone office, showing a number of switchboard sections, is given in *Fig.* 4. These are "A" sections in which the answering jacks of sub-

scribers' lines are located. A photograph of another portion of the office, showing a junction board with "B" sections, is given in *Fig.* 5.

It will be noted that the operator's switchboard resembles in general outline an upright piano. The subscribers' lines are terminated in small sockets called jacks, which are mounted in the vertical part of the switchboard. The connecting links or "cords" are mounted on the horizontal keyshelf of the switchboard. Each subscriber's line has associated with it an answering jack, which is used solely for the purpose of answering the calls made by that subscriber, and one or more multiple jacks which are for the purpose of connecting other subscribers to his line when he is called. Each "A" operator has a certain number—on the average about 125 subscribers' lines—assigned to her position to answer. Hence in a local exchange the answering jacks are distributed in groups along the switchboard. This makes it possible to distribute the work of answering calls in flexible fashion, according to the abilities of various operators and the character of the subscribers' demands, and thus to give as good service as possible to the public. At each position the answering jacks assigned are mounted in the lower portion of the vertical section. Just over each jack there is a tiny electric lamp, or other visible signal, which lights, and thus signals the operator whenever the subscriber wants to make a call.

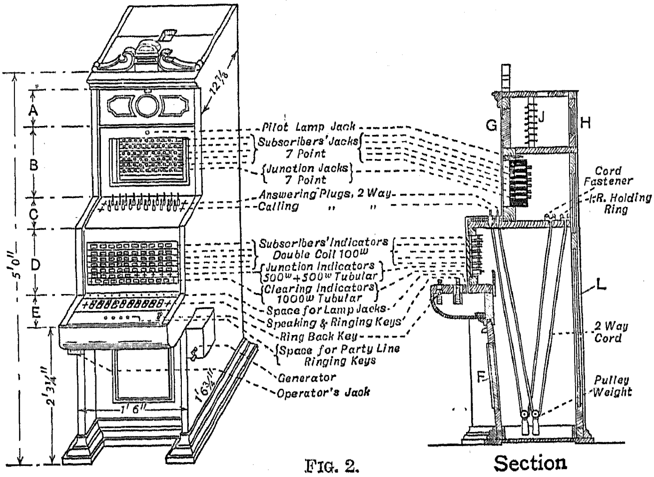


FIG. 2.

In addition to an answering jack there are several multiple jacks connected to each subscriber's line. These multiple jacks are arranged at definitely spaced distances along the switchboard, so that one jack for each subscriber in the local exchange is within reach of each operator. This arrangement enables the operator to connect the answering jack of any subscriber assigned to her to any other subscriber in the exchange directly, and without moving from her chair. The jacks in one of the multiple groups are on

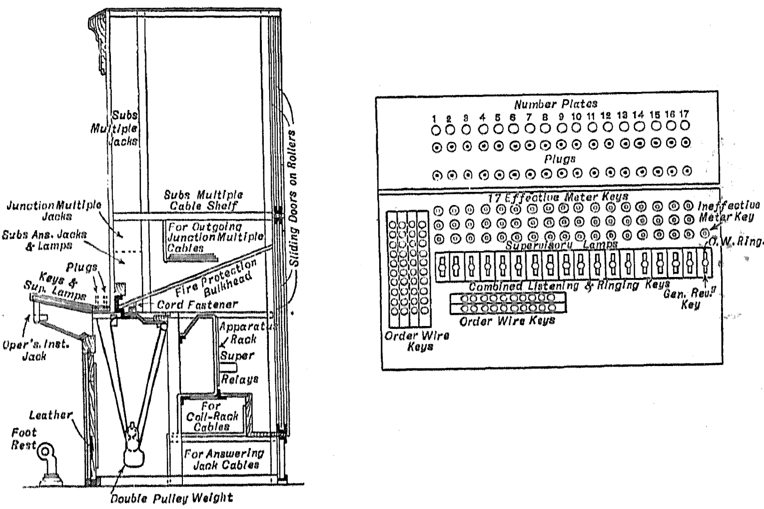
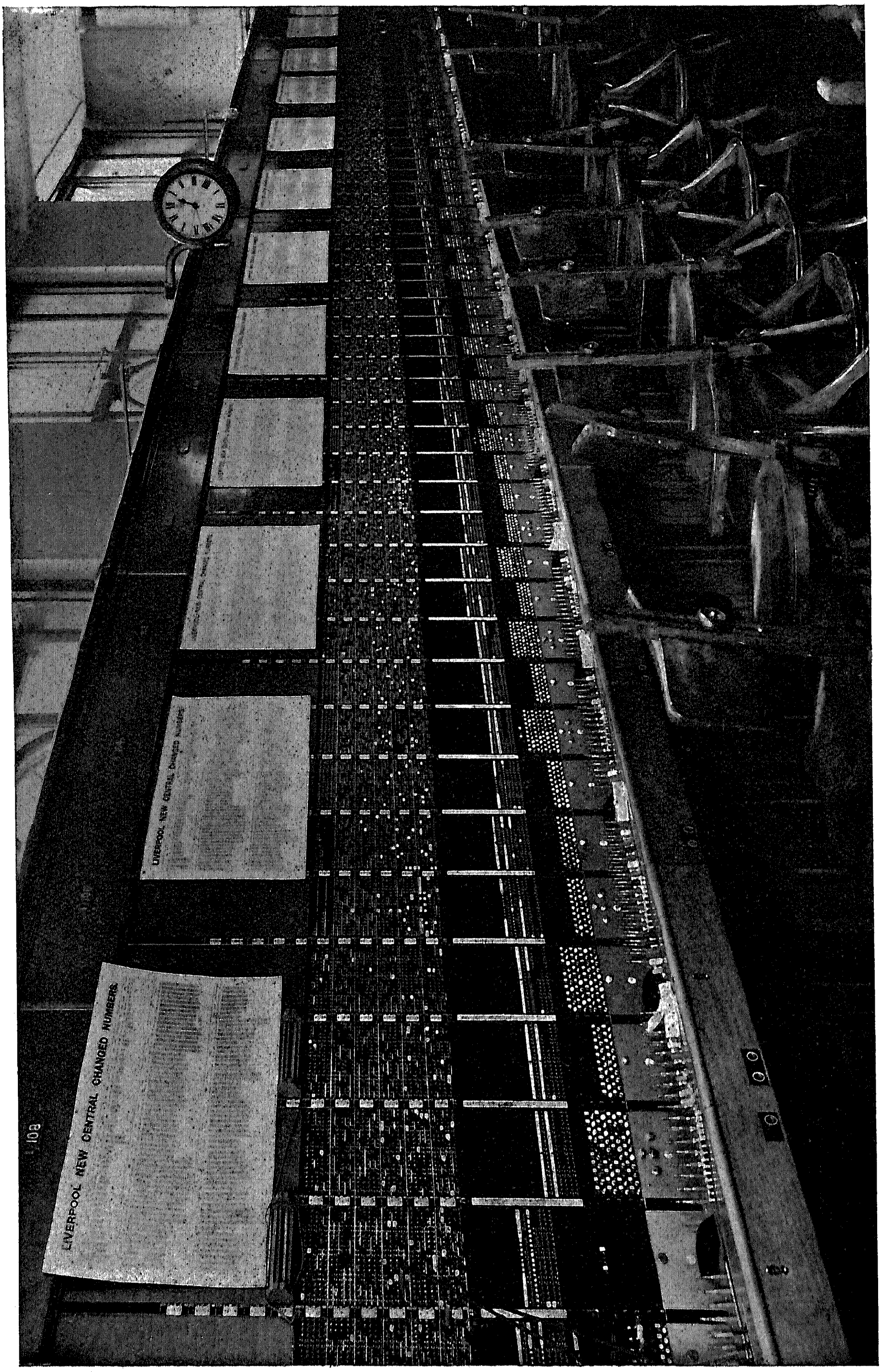


FIG. 3.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LIVERPOOL NEW CENTRAL CHANGED NUMBERS

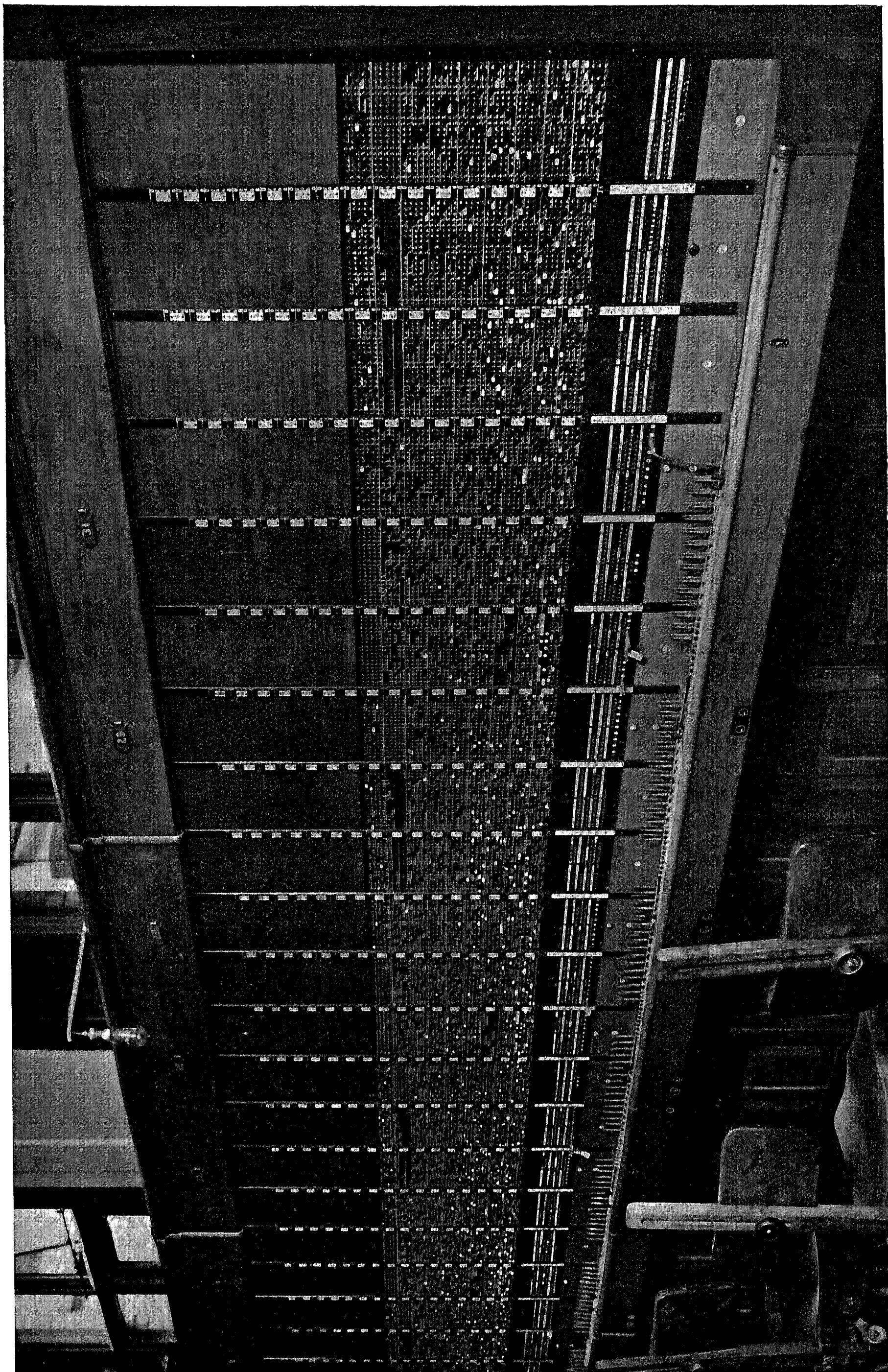


FIG. 5.

the upper part of the vertical sections of the board, and they are all arranged in order according to the subscribers' telephone numbers.

The "A" operator has seventeen links or cords mounted upon the key-shelf in front of her. Each "B" operator has from twenty-seven to thirty-six incoming junctions to attend to; these are mounted upon her keyboard, and she also has the subscribers' multiple within her reach.

The "A" operator's cord circuit consists of a flexible electrical conductor, terminated at each end in a plug-like electrical conductor, which is made to fit into the jack of the subscriber's line and to form the link or electrical connection. The plug at one end of the cord is used for answering calls, and is called the "answering plug." The plug at the opposite end of the cord is known as the "calling plug." At the centre of this flexible cord a two-way switching device, termed a key, is placed, which, when operated in one direction, connects the operator's telephone to the line so that she may speak to the subscriber and ascertain his requirements, and, when operated in the reverse direction, connects ringing current to the line to ring the wanted subscriber's bell; also either one or two small electrical signals are fitted in each cord circuit to enable the operator to supervise the call. These are known as supervisory signals, and if there are two, one is called the answering supervisory, the other the calling supervisory signal.

The "B" operator's cord circuit is slightly different from that of the "A" operator's circuit in that it forms a connecting link or junction between two exchanges in an area. One end of this link is terminated in the switchboard at exchange "A" in a small socket-like jack, similar to the answering and multiple jacks, but described as an outgoing junction jack; the other end is terminated in a cord and plug on the "B" operator's position at exchange "B." Each cord, in addition to the plug, is fitted with a ringing key and a supervisory signal.

Having described in a general way the exchange arrangements, we shall now describe in greater detail the switching operations in manual systems of the local battery and common battery types, and in the most modern type of machine switching or automatic telephone system.

(i.) *Local Battery Exchanges.*—The D.C. power which it is necessary to supply to the subscriber's transmitter may be supplied either from a local source—such as dry cells—at each telephone sub-station, or it may be supplied over the subscriber's line from the central office. The first type of system is called a local battery system or exchange, and the second type a common battery system or exchange. Local battery exchanges are used mostly in very small localities or in localities where the subscribers are situated a relatively long distance from the central office.

In local battery exchanges the subscriber usually signals the operator at the central office by turning the crank of a magneto or generator. Turning this crank, in the first place, closes a spring contact which connects the generator across the line terminals of the subscriber's set; and, secondly, generates from 50 to 100 volts at a frequency of from 10 to 20 cycles. Across the subscriber's line at the central office there is mounted in front of the operator a relay or drop, the armature or shutter of which normally presents a dull black surface to the view of the operator. When the magneto is turned, however, the current generated operates the drop and the shutter releases, exposing a bright surface to the view of the operator. The operator then takes a cord, consisting of a flexible conductor and terminated at the two ends in plugs, and puts one of the plugs into the jack associated with the subscriber's line or drop. This action of the operator restores the shutter to its normal position and connects the operator's telephone circuit to the subscriber's line.

The subscriber, in the meantime, has removed the receiver from the switch-hook—which process operates spring contacts and connects (1) the battery or source of primary energy with the subscriber's transmitter, and (2) the sub-station circuit with the line. The operator then ascertains with whom the calling subscriber wishes to talk. If the called party is located in the same exchange as the calling party, the operator plugs the other end of the cord directly into the jack associated with the drop of the called subscriber. On the other hand, if the calling subscriber wishes to talk to a subscriber in another exchange, the operator plugs the other end of her cord into a trunk leading to the exchange of the called subscriber, and the distant operator, called a B operator, completes the connection.

(ii.) *Common Battery Exchanges.*—In larger communities, where common battery exchanges are practically always used, the direct current power for the transmitter is fed from storage batteries at the central office out over the subscriber's line. The diagram (*Fig.* 6) shows the conditions which maintain at the subscriber's station and at the central exchange when the telephone is not in use. At the subscriber's station the bell and condenser are in series across the line. The transmitter has one terminal connected to the line and the other to a switch-hook which, when the telephone receiver is removed from it, operates under a spring control to close the

circuits of the transmitter, receiver, and induction coil, and thus to place the sub-station apparatus in the talking condition.

At the exchange the diagram shows the answering jack, the cut-off relay, the line relay, the line lamp, and the battery. The answering jack is a small socket device with spring connections, the relays are both electromagnetic devices which, when a current passes round the magnet coils, are energised to attract their armatures and to open contacts in the case of the cut-off relay, and to close a contact in the case of the line relay. The line lamp is switched on and off by the operating and releasing of the line relay. It will be noted that when the receiver at the sub-station is on the switch-hook the apparatus at the exchange is inoperative, but when the telephone is removed and the switch-hook contacts are made, there is a complete path for direct current from the exchange battery to flow. This passes through the coil of the line relay, causing it to operate and to light the line lamp, which indicates to the operator that the subscriber is calling the exchange.

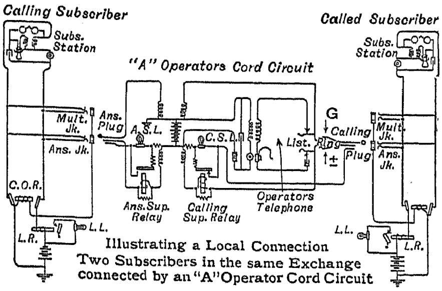


FIG. 6.

The operator, noting this lamp to be lighted, picks up the answering plug of an idle cord and inserts the plug into the answering jack associated with the lamp; this causes the cut-off relay to operate and so disconnects the line relay from the circuit, and puts out the lamp.

§ (5) "A" OR LOCAL CONNECTIONS.—The operator then operates her listening-key and connects her telephone with that of the calling subscriber and obtains from him the number he requires. If the call is for another subscriber in the same exchange, she then takes the calling plug at the other end of the cord which she is using, and before inserting this into the wanted subscriber's multiple jack, touches the sleeve or socket contact of the jack with the tip of her plug. If the line is busy, she will get a click in her receiver; if the line is free, silence. She then inserts the plug to its full extent and presses her ringing-key, which rings the bell of the wanted subscriber. When the calling plug is inserted in the jack of the wanted line, the calling supervisory lamp glows. When the called party answers, the calling supervisory relay operates and the lamp is shunted and ceases to glow. During a conversation both the supervisory signals are out, but at the termination, when either subscriber restores his telephone to the hook, the corresponding supervisory lamp will light. When both supervisory lamps are glowing the operator knows that the conversation is completed, and she then removes the plugs from the jacks and restores the apparatus to normal.

Fig. 6 shows the connections of two subscribers in the same exchange using a repeating coil cord circuit. If the call is for a subscriber in another exchange, then the circuits used are shown in *Fig.* 7.

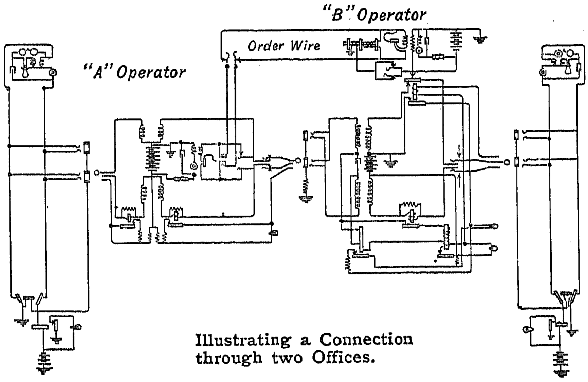


FIG. 7.

§ (6) A-B CONNECTIONS.—If the called subscriber is in a different exchange from the calling subscriber, the A operator who answers the calling subscriber first obtains the required exchange and number of the called subscriber. She then operates a key which connects her set to an order wire running to the desired exchange. At the other end of this order wire or call wire is a B operator, to whom the A operator gives first her exchange designation and then the number of the desired subscriber. The B operator then assigns an idle inter-office junction to the A operator (who connects the other end of her cord to this junction) and then makes a busy test by touching the tip of the plug associated with the inter-office junction to the socket or sleeve of the jack associated with the desired subscriber's line. Since such a board is a

multiple board—*i.e.* the same subscribers' lines are multiplied in front of different operators—the circuit is so arranged that the B operator will get a click when she touches the sleeve of the jack with her plug if the desired subscriber is busy. In such a case she inserts the plug into the "busy back" jack, which operation puts a distinctive tone on the line and thus gives the calling subscriber the information that the line is busy. If, on the other hand, the operator does not get a click when she touches the tip of her plug to the sleeve of the jack, she inserts the plug into the jack, and ringing takes place—usually automatically.

When either subscriber hangs up his receiver or operates his switch-hook in any other manner the corresponding supervisory relay in the A cord circuit releases and relights a light in front of the A operator. If this light is a flashing light—indicating a desire on the subscriber's part to talk with the operator—the latter pushes her listening-key, which puts her set across the line, and asks the subscriber what is wanted. If, on the other hand, the light is a permanent one—indicating that the subscriber has finished talking—the A operator pulls down the cord, which operates a relay and lights a light in front of the B operator, giving the latter the signal to pull down the corresponding cord at her end of the circuit.

§ (7) Machine Switching Systems.—The problem of connecting two subscribers together telephonically is essentially the same whether human operators or machines are employed. Several types of machine switching or automatic exchange systems have been devised, but in each of them the principle is to move the terminal of the calling line to that of the called line, which is fixed. In the system described, the calling lines are connected to contacts which are moved vertically and the called lines are connected to fixed horizontal terminals.

Reference to the diagram (*Fig.* 9) will show that the number of switching frames which may enter into the completion of a call is five:

1. The line-finder frame.
2. The district frame.
3. The office frame.
4. The incoming frame.
5. The final frame.

The lines entering the exchange are terminated upon horizontal rows of terminals on the line-finder frames; these terminals correspond to the answering jacks in a manual exchange. Each line-finder terminal has 60 contact points. In front of the line-finder terminals, line-finder elevators are arranged, 60 in all, so that any one of the 60 line finders may make contact with the calling line. The line finder corresponds to the answering plug in a manual exchange.

The other end of the conducting link in the machine switching system terminates in a district selector, which is a moving brush contact. There are 5 such contacts on each district selector, and there are 60 selectors, having access to 500 outgoing junctions, which are arranged in five horizontal banks of 100 lines each. Each outgoing junction is multipled 60 times, so that any one of the 60 district selectors may make contact with it.

The other switches are similar to the district selector frame in the arrangement of their moving and fixed terminals. The calling line is extended successively from the line-finder frame through the district, office, incoming, and final frames, where it is connected with the called line.

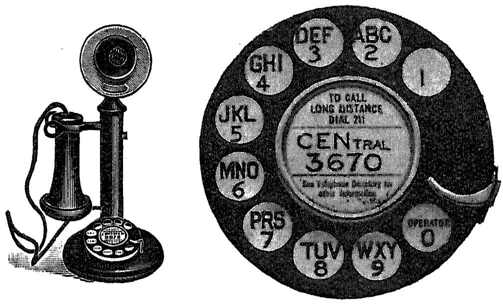
§ (8) General Description.—In full mechanical or automatic systems the central office machinery is under the guidance of calling devices or dials located at the subscribers' station. A subscriber, instead of giving a desired number verbally to an operator, so manipulates a calling device or dial as to cause the switches at the central office to build up the connection he wants.

A common form of subscriber's calling device is a dial arranged with the ten digits, 1 to 0, placed beneath ten finger-holes. This dial is operated by inserting the forefinger in the proper hole and rotating the dial against the tension of a spring until the finger comes in contact with a metal stop. The subscriber then releases the dial, which rotates back to normal. As it returns to its normal position it sends to the central office a series of electrical impulses corresponding in number to the number of units in the digit appearing in the finger-hole chosen. The dial is rotated to the stop and released as many times as there are digits in the called subscriber's number. *Figs.* 8 (*a*) and 8 (*b*) show a full mechanical or full automatic subscriber's station.

For example, when a subscriber calls a four-digit number, as 9653, the first movement of the dial by the calling subscriber will send nine impulses to the central office corresponding to the thousands digit, the next movement will send six impulses corresponding to the hundreds, the next five impulses corresponding to the tens, and the last three corresponding to the units digit.

The method of building up a connection by mechanical means in a full mechanical or full automatic central office varies somewhat in the systems produced by different manufacturers. In general, a system of junctions is used whereby the calling line is extended link by link until it reaches the terminals of the called line. Two methods of controlling the

central office switches have been used rather extensively. One has come to be known as the "direct



(a) (b)

FIG. 8.—Full Mechanical Subscriber's Desk-stand.

impulse control," the other as the "reverse impulse control." When the direct impulse control is used, the central office switches are operated directly by electrical impulses, the duration, character, and speed of which are determined solely by the calling device at the subscriber's station. When the reverse impulse control is used, the impulses created at the subscriber's station are first received and stored in a device located at the central office, known as a "sender," "register," or a "recorder," and this device then controls the central office switches in their movements toward the terminals of the trunk group or line wanted. In general, the "sender" starts the switches toward the desired line, permits them to go to the desired point, and then stops them by removing their driving power. This is usually accomplished by having the switches equipped with commutators which will transmit back to the sender the number of impulses which it (the sender) has been set to receive by the original manipulation of the calling device at the subscriber's station.

(i.) *Operation of a Typical Full Mechanical or Full Automatic System.*—Each subscriber's station is provided with a calling device or a dial which permits the calling subscriber to pull, in succession, the office [1] code letters and numerical digits in the number of the subscriber he wishes to call.

A call originating at a station for another station connected to the same office or to any other office in the city, unless it happens to be outside the no-extra-charge zone, is completed by means of the mechanical selectors directly to the line wanted without the aid of an operator. The ringing of the called station is done automatically, and an audible ringing tone is given to the calling subscriber while the ringing is in progress. A busy tone is sent back to the calling subscriber in case the called line is busy. In case the calling station is a coin box or a measured service station, the central office equipment may be arranged to automatically collect the coin or operate the message register if the call is completed, or to return the coin or not operate the message register if the call is not completed.

[1] When full mechanical systems are installed in large cities having a number of offices the subscriber's dials are usually arranged with letters as well as numerals.

The figure below shows schematically the principal central office members in a typical system through which such a call passes. The subscriber's line terminates in the central office on a main distributing frame, and is cabled to an intermediate distributing frame in the same manner as in the manual offices. From the intermediate distributing frame the subscriber's line is connected to the multiple contacts representing that line on the line-finder banks which corresponds to the answering jack in a manual office. The line and cut-off relays are mounted on racks located near the line-finder frames, so that no cable need be carried directly from the intermediate distributing frame to the relay rack.

Since the progress of a call through a full mechanical system is somewhat parallel to

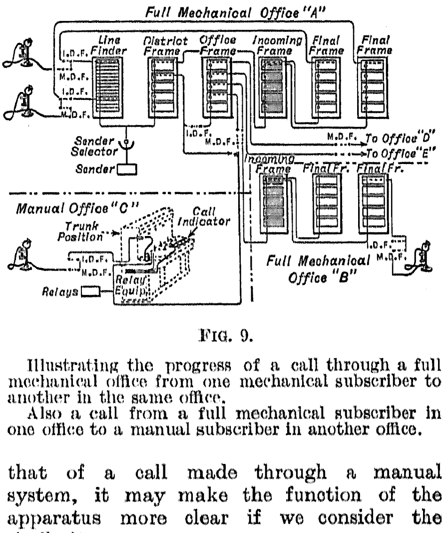


FIG. 9.

Illustrating the progress of a call through a full mechanical office from one mechanical subscriber to another in the same office.

Also a call from a full mechanical subscriber in one office to a manual subscriber in another office.

that of a call made through a manual system, it may make the function of the apparatus more clear if we consider the similarity.

(ii.) *Analogy between Manual and Full Mechanical Systems.*—

Manual	Full Mechanical
When a subscriber in a manual system removes his receiver to make a call, he causes the line relay at the central office to pull up and light a lamp associated with an answering jack. An operator takes up the	In a full mechanical system the subscriber removes his receiver and causes his line relay to pull up, and in place of lighting a line lamp it puts battery on a row of contacts corresponding to his line on the line-finder

answering plug of a cord pair, plugs in, and answers.

banks and also sets in motion an elevator, called a "line finder," which goes upwards in search of the contacts on which his line relay has placed battery. When the line finder reaches his contacts it is caused to stop by having its driving power removed.

Thus, there is a marked similarity between the answering jack and the contacts on the line-finder bank between the line finder and the answering cord, and between the operator finding and plugging into an answering jack, and the line finder finding and attaching itself to the contacts on the bank.

The operator, after plugging in, throws a listening-key which puts her in a position to receive the subscriber's order, and she notifies him that she is ready by saying, "number, please."

The "line finder" has associated with it a "sender selector" which proceeds to find and attach an idle "sender" during the period that it is finding the calling subscriber's line, and this "sender" notifies the calling subscriber that it is ready to receive his order by giving him a distinctive buzz called a "dial tone."

The calling subscriber gives his order verbally to the operator.

The calling subscriber dials his order to the "sender" by pulling the letters and numerical digits of the number desired in succession.

Assuming that the call is for another subscriber in the same office, and that there is no subscriber's multiple before the "A" operator, the operator knows from the office name that has been given that the call is for a subscriber in her own office, and she gives the number of the desired subscriber to the "B" operator in her own office over the call wire, gets a junction assigned, and extends the calling line to the "B" operator by plugging the calling cord of the pair she has previously used in answering into the outgoing junction leading to the "B" position.

The "sender," upon receiving the office code letters from the subscriber's dial, recognises their significance and causes the "district selector" (elevator) which is definitely tied to the line finder previously used to start up and select an idle path or junction leading to an "incoming selector" (elevator) in the same office.

The analogy between the "calling cord" of the manual system and the "district selector" of the full mechanical system, as well as the likeness of the manual outgoing junction multiple jacks which terminate in plug-ended incoming junctions at the "B" operator's position to the outgoing junction multiple contacts of the district selector banks which terminate in incoming selectors on the incoming frame, will no doubt be evident.

The "B" operator locates the desired subscriber's number in the multiple before her by first locating the hundred, then the strip, then the particular line. She then tests the line to see if it is busy. If it is not busy she inserts the incoming junction plug into that multiple jack and the ringing is started automatically. If the line had been busy she would have plugged the junction plug into a busy back jack.

The "sender" causes the incoming selector to locate the group of junctions leading to the particular 500 lines in which the desired number is located, causes a non-busy junction to be selected, and then causes the "final selector" (elevator) on the end of that junction to locate the hundred, then the ten, and finally the particular unit line desired. The "final selector" tests the line to see if it is busy, and if it is not, establishes the connection. Ringing is started automatically. If the line had been busy the selector would not have established the connection but would have given a busy signal to the calling subscriber.

The analogy between the subscriber's multiple on a manual "B" board and the combination of the incoming and final frames, while not perfect, is still rather striking.

Considering the two systems broadly, it will be evident that the line finder, district, and office selector apparatus corresponds closely to the manual "A" board, and that the incoming and final selector apparatus performs the same function as the "B" board in the manual system.

If, on the full mechanical system, the call had been for a subscriber who was connected to another full mechanical office (see the preceding figure) the "sender" would, upon receiving from the calling subscriber's dial code letters which designate the office wanted, have so controlled the "district selector" as to cause it to select a junction in the group of junctions leading to that office, and if the number of offices was too large to be accommodated on the district frame, as in New York City, the outgoing junction paths or junctions leading from the "district selector multiple" would have terminated in an office selector which would have been so controlled by the "sender" as to pick out a junction leading to the particular office desired. At that office the path or junction selected would terminate on an "incoming selector" leading to the group of 500 lines, then cause the "final selector" on the end of that junction to locate the hundred, then

the ten, and finally the unit line wanted. Ringing would be started automatically.

(iii.) *Full Mechanical to Manual.*—If the call had been to a subscriber connected with a manual office, the "sender" would have so guided the district and office selectors as to have caused them to select a path or junction leading to the manual office desired, where the junction would have been terminated in a single-ended cord before an operator at a "call indicator" position. A lamp would have been lighted on the plug-ended incoming junction, and the operator, upon pressing a key called a "display key," associated with that junction, would have allowed the distant "sender" to cause the numerals of the line desired to be displayed before her on a "call-indicator," whereupon she would take up the plug-ended junction, test the line and, if not busy, insert it in the multiple jack of the line desired. Ringing would be started automatically.

(iv.) *Manual to Full Mechanical.*—If a call is originated by a subscriber connected to a manual switchboard for a subscriber connected to a full mechanical office, it may be handled in any of three ways, depending on what equipment is installed.

The figure below illustrates the scheme of using "cordless B" operators at the full mechanical office. This plan contemplates that the call will come in to the manual switchboard in the regular way and be handled

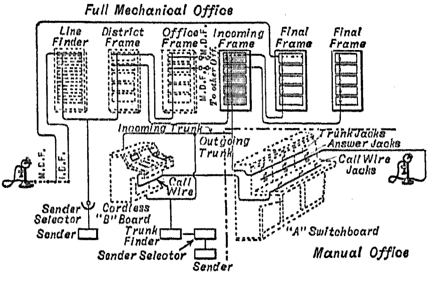


FIG. 10.—Call from a Manual Subscriber to a Full Mechanical Subscriber in another Office.

by the "A" operator exactly the same as a call to a manual office. At the incoming end the "cordless B" operator, after receiving the number of the desired party over the call wire, will set up that number on a set of keys before her. A "sender" is provided to receive and register this number and then so control the incoming and final selectors as to cause the desired line to be found and rung. This plan has the advantage that no changes are necessary at the manual offices.

When full mechanical offices have been installed to serve a section of a city and all remaining districts of that city are still manual, with a prospect of remaining on that basis for only a short time, it frequently happens that it is more economical to instal "cordless B" operators' positions at the full mechanical offices than to make either temporary or fairly permanent changes at the manual offices.

The figure below shows the "key-indicator" plan which permits the "A" operator at any manual office to complete a call very speedily within any full mechanical office without the

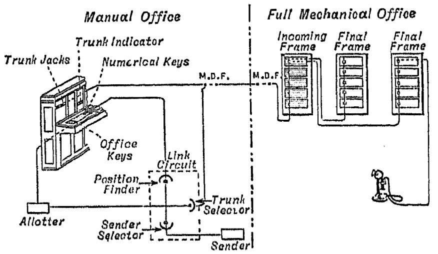


FIG. 11.—Key-indicator Plan.

aid of any other operator. The method of operation is, briefly, that after any operator has answered a call and has received the desired number from the calling subscriber, she depresses a key corresponding to the office desired and a junction to that office is automatically selected for her. The number of the junction which she is to use is displayed before her on a lamp indicator and she plugs into the junction jack which corresponds to the number displayed, having previously depressed the digits of the desired subscriber's number on a ten-button "numerical key" with which she is provided. A "sender," which has been attached automatically, registers the digits of the number and then proceeds so to guide the incoming and final selectors in the distant full mechanical office as to connect to and ring the desired subscriber's line.

This plan is the better one, particularly when it is proposed to retain the manual equipment for a number of years in offices which originate a large volume of traffic, which has to be completed in full mechanical offices. The economy through the use of this plan is made by the elimination of the "cordless B" operators and their equipment at the full mechanical offices and by the saving in operating expense which can be made due to the speed with which calls can be handled with this arrangement.

In the next figure is shown an outline of the plan of equipping all "A" positions at manual offices with dials, whereby they can complete calls within any full mechanical office without the aid of any operator. The plan, in general, is to provide each "A" operator with an ordinary ten-point dial which she can attach to any cord by means of a key. She tests the

outgoing junction jacks by means of an audible busy test and selects a non-busy junction to the full mechanical office to which she desires to connect. She then plugs in and pulls the digits of the desired number, one after the other, on the dial. These digits are received and registered by a "sender" at the full mechanical office, which then acts to control

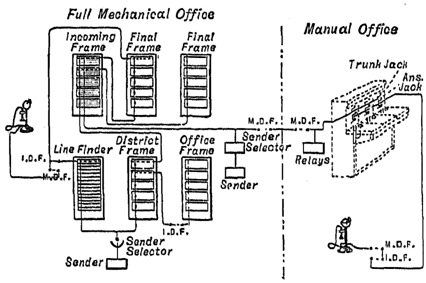


FIG. 12.—Manual Switchboard equipped with Dials.

the incoming and final selectors so as to connect to and ring the desired line.

The advantage of this scheme is that it eliminates the B operator, is low in the first cost and does not involve much apparatus, but it has the disadvantage, from a traffic standpoint, that where the number of calls to be handled is large it slows down the "A" operators and sometimes makes more "A" positions necessary.

In making a decision as to which of the plans is most suitable for a given project, the conditions, such as the length of time manual equipment will remain in service, and the amount of traffic from the manual offices to the full mechanical offices, must be carefully considered, along with the advantages and disadvantages of all schemes.

§ (9) MEASURES OF TELEPHONIC EFFICIENCIES. *Differences between Telephonic Transmission and Power Transmission.* — In order to get a clear conception as to the reason for expressing telephonic efficiencies in a different way from that employed in power work, it is necessary to consider the difference between the problems of the telephone engineer and those of the power engineer. The power engineer generally deals (1) with currents and voltages of relatively large magnitudes (100 to 200,000 volts), (2) a single low-frequency current—in the neighbourhood of 15 to 60 cycles, (3) a line that is electrically short, and (4) a condition where transmission of power is required in one direction only. The object of the power engineer is, in general, to obtain a large ratio for the power at the receiving point in a transmission system, as compared with the power given out at the generator. In a power system it is economical to achieve this object, but in a telephone system this condition cannot be realised without unwarranted expense.

The object of the telephone transmission engineer is to transmit the speech currents as far as possible and to preserve the clearness of intelligibility of speech. In telephone work we have to deal with extremely small voltages and currents and we have to cover a very high and wide frequency range.

The range of frequencies which is audible to the human ear varies to quite an extent with different individuals, but may be taken roughly as from 16 to 16,000 cycles,[1] or approximately a ratio of 1000-1. The more important telephonic frequencies, however, lie in the range of 250 to 2500 cycles, or a ratio of 10-1. While it is possible to carry on a conversation over a telephone circuit which is not efficient even for the small range of frequencies mentioned above, such a circuit will not in general be satisfactory from an articulation standpoint—that is, while sufficient volume of sound may be transmitted over such a circuit, it will be difficult to understand the conversation.

With regard to the electrical power efficiency of telephonic circuits, it may be stated that a fairly efficient transmitter will, when vigorously agitated, generate at its terminals from 2 to 3 volts. This voltage is stepped up by the induction coil or transformer at the subscriber's station, so that a maximum voltage of from 6 to 8 volts may be impressed across the line terminals of the set. This corresponds to a maximum A.C. current of approximately 10 milliamperes, or possibly $\frac{1}{10}$ watt entering the outgoing line. At the end of a long-distance line this current may be reduced to as low as $\frac{1}{10}$ milliampere. This reduction in current is, of course, in marked contrast with that existing in power circuits. In other words, instead of getting efficiencies of about 70 to 80 per cent — as is customary in power work on an extremely long line —the receiver in a telephone circuit often receives only $\frac{1}{10,000}$ of the original power given out by the transmitter.

§ (10) SYSTEM REFERENCE STANDARD.—Due to the radical differences which exist between the problems and requirements of power and telephone transmission, the same terms used for each field have grown to have different meanings. In telephone work, volume efficiency is usually expressed as a "loss below" or as a "gain above" some standard condition, and as a result the terms "efficiency" and "loss," or "gain," have essentially the same meaning. This loss or gain has ordinarily been expressed in miles of "standard cable" (l_s)—due to the fact that the number of miles or distance to which a person can

[1] Notes of much higher frequency are in some cases audible. See "Sound," Vol. IV.

talk with satisfactory volume is, when other things are equal, the logical measure of the efficiency of any telephone system.

It is customary to measure the volume transmission efficiency of any telephone system by comparing it with the volume obtained over the System Reference Standard. This standard is shown below:

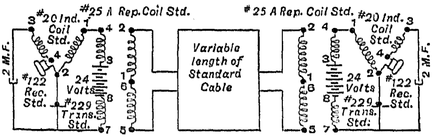


FIG. 13.

This System Reference Standard is seen to consist essentially of two common battery subscribers' circuits directly connected to each other by repeating coil cord circuits and a variable length of trunk of standard cable.[1] The System Reference Standard is therefore electrically similar to a system in which two common battery subscribers on zero loops are talking to each other over a non-loaded cable trunk. The circuit is simplified as much as possible by the omission of all apparatus such as ringers, supervisory relays, etc., which is not necessary from a transmission standpoint. The transmitters and receivers used in the System Reference Standard are required to meet certain volume and quality requirements, and together with the repeating coils and induction coils must have certain definite electrical constants at telephonic frequencies.

The comparison of the volume efficiency of any given system with that of the System Reference Standard is made by adjusting the amount of standard cable in the System Reference Standard until the two observed volumes are equal. The number of miles of standard cable which have to be inserted in the System Reference Standard in order to make the volume of sound equal to that obtained over the system under test is said to be the transmission equivalent of the system in question.

The limiting transmission equivalent which it is feasible to impose on a commercial telephone system depends upon a good many factors. For local service an equivalent of eighteen to twenty miles is usually regarded as reasonable. On the other hand, for trunk or long-distance service, the equivalent may be permitted to go as high as thirty miles. This latter figure is usually regarded as being approximately the upper limit of commercial transmission.

[1] Standard cable is cable with uniformly distributed constants of $R=88$ ohms resistance per loop mile and $C=\cdot054\times10^{-6}$ farads mutual capacitance per mile. The leakage G and the inductance L are ordinarily taken as zero—although it is the practice of some countries to assume an inductance of ·001 henry per loop mile.

The transmission equivalents of typical trunks may vary from ten to twelve miles—the remaining eighteen miles or so being allowed for terminal losses at the two ends of the circuit.

Although the standard mile, or mile of standard cable, is used for the measurement of telephonic volume efficiency almost universally, the logarithmic attenuation of a uniform line is sometimes taken as a measure of efficiency or loss. Unit attenuation is equivalent to the loss in that part of a uniform line of infinite length in which the current has decreased to $1/e$th of its initial value. The relation between miles of standard cable (l_s) and attenuation units (A) is as follows:

$$l_s=\frac{A}{\sqrt{\frac{1}{2}\omega RC}},\quad \text{or } l_s=\frac{259}{\sqrt{f}}A,$$

where R is 88 ohms, and $C=\cdot054\times10^{-6}$ farads, and f is the frequency under consideration. Also $\omega=2\pi f$.

§ (11) ARTICULATION TESTS.—The ability of a telephone system to transmit the meaning of the connected and organised speech sounds of conversation is called the intelligibility of the system. Intelligibility is measured, then, by the percentage of the total ideas which are successfully conveyed. It varies with the volume efficiency, frequency distortion, room and line noises, etc. Since it is very difficult to measure directly the intelligibility of a telephone system, it is customary to obtain a kind of intelligibility measure by means of "articulation tests." The articulation of a telephone system is the capability of the system for transmitting fundamental speech sounds. In making articulation tests, detached speech sounds are scientifically arranged and pronounced at the sending end of the apparatus under test. The articulation is indicated by the percentage of the total sounds spoken, which are correctly received.

§ (12) TRANSMITTERS. *Functions of the Transmitter.*—In ordinary conversation sound-waves are conveyed through the medium of the air from the lips of the person speaking to the ears of the one listening. In telephony the speech sound-waves enter the telephone system through the transmitter, are conveyed over as great a distance as desired through the medium of circuits, which are the electric channels of communication, and are finally transformed into acoustical form by the receiver. The transmitter is a machine or instrument which produces electric waves in the circuit corresponding to the sound-waves falling upon it. The receiver is a machine or instrument which produces sound-waves in

the air at the listener's ear corresponding to the electric waves delivered to it by the circuit. The system fulfils its purpose if the sound-waves reproduced at the receiving end are sufficiently good copies of those impressed upon the transmitter to be easily and clearly understood by one listening. The primary interest from a physical standpoint lies in the quality and volume of the sound from the receiver. The problem of the telephonist is to obtain sufficient received volume without undue distortion of the original wave-shape.

Each part of the telephone system, in general, will produce its own form of distortion, and perfect transmission may be approached by endeavouring either to make the algebraic sum of the component distortions zero, or — what is the more logical method—to make that of each part zero. It is practically necessary that the transmitter shall be capable of giving out a relatively large amount of electrical energy when spoken into with a moderate tone, and that the electric waves shall be faithful copies of the sound-waves. In addition, the instrument should be small, durable, and of low cost.

§ (13) Types of Transmitter.—Transmitters may be either vibratory electrical generators converting the mechanical energy of the sound-waves impressed upon them into electrical energy, with an efficiency somewhat less than 100 per cent, or they may be amplifying valves controlled by sound-waves in such a way that they admit to the circuit, from a battery, an amount of electric current which is proportional at each instant to the pressure of the sound-wave playing upon them. Instruments of the former type may be magnetic, electrostatic, electrocapillary, or thermal in their action. Instruments of the latter type are more important practically, and in general their operation depends upon variation of resistance.

The original magnetic telephone of Alexander Graham Bell consists essentially of a diaphragm connected to a movable armature, and a permanent magnet surrounded by a spool or wire. Its action is practically reversible, so that it operates either as transmitter or receiver. When sounds fall upon the diaphragm and set it in motion the air-gap between the armature and the pole-face of the magnet varies in tune with the sound-wave, and under the magnetomotive force of the magnet corresponding variations in the flux of the magnetic path take place. These flux variations are accompanied by the generation of corresponding electromotive forces in the turns of wire comprising the spool, the net result being that electric waves which are copies of the sound-waves are produced. The reverse action, as a receiver, is discussed in § (17). Instruments of the magnetic type can be designed to give very good quality. In general, even if they could be designed to approach 100 per cent in efficiency, however, transmitters of the magnetic type would not deliver sufficient energy to make them adaptable to the requirements of commercial telephone systems.

The capacity transmitter depends for its operation upon the small changes in the capacity of the instrument, which changes follow very closely the pressure variations in the sound-waves. On account of the high internal impedance, and because of the high voltage required for efficient operation, this instrument is not adapted for commercial use, although it is of considerable importance in experimental telephone work, because it can be so designed as to have a response over a wide frequency range; in other words, because under these circumstances it is an instrument of exceptionally good quality.[1]

Other instruments, such as the thermal transmitter—which requires extremely large currents for efficient operation—and the capillary transmitter—which is unstable and unsatisfactory mechanically—are also appreciably below commercial requirements in respect to volume efficiency.

§ (14) Carbon Button Transmitter.—The telephone transmitter universally used in practice is of the type which modulates the current from a battery, producing in the circuit electric waves which are greatly amplified copies of the sound-waves impressed upon its diaphragm. A familiar example of this type is shown in the attached *Fig.* 14. Very large numbers of instruments like the one shown are in use in Great Britain, the United States, and other countries. The current-varying element is a granular button or cell, containing two carbon discs or electrodes, between the polished surfaces of which a mass of granular carbon is held. One of these electrodes is fixed and the other rigidly attached to the centre of the diaphragm so as to move with it. The outer edge of the movable disc is flexibly joined to the edge of the button cup by means of a mica washer or annulus. A thin, circular, aluminium plate serves as the diaphragm which receives the sound-waves. Sound-waves are directed upon the centre of the diaphragm by the horn-shaped mouthpiece. The motion of the light diaphragm in response to sound-waves serves to agitate the carbon button. The terminals of the transmitter are connected to the two electrodes, and the resistance from the one to the other, principally made up of the

[1] See E. C. Wente, "The Condenser Transmitter as a Uniformly Sensitive Instrument for the Absolute Measurement of Sound Intensity," *Phys. Rev.*, July 1917, x. 1; also I. B. Crandall, "The Air-damped Vibratory System-Theoretical Calibration of the Condenser Transmitter," *Phys. Rev.*, June 1918, xi. 6.

resistance between the various granules, and between the granules and the electrode surfaces, is made to vary very nearly in pro-

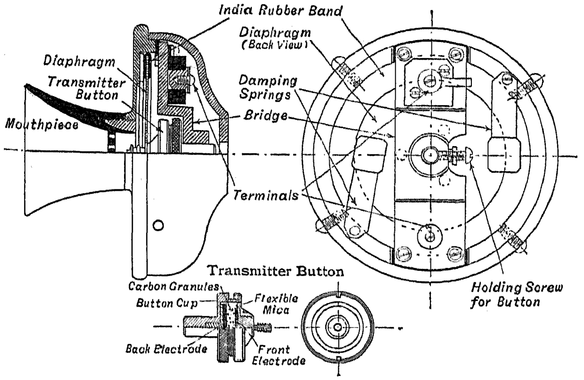


FIG. 14.

portion to the pressure of the sound-waves on the diaphragm. In this way, when direct current is supplied to the transmitter, from a battery, this variable resistance effectively generates variable electromotive forces which correspond closely to the original sound-waves. In the type of instrument shown the diaphragm is supported flexibly and the sound-chamber in front of the diaphragm effectively sealed by covering the edges of the diaphragm with a rubber ring. The diaphragm is further held against its seat and its mode of vibration somewhat controlled by two damping springs, the one pressing against its rubber-covered periphery and the other resting on its vibratory area. The button cup is rigidly supported from the transmitter bridge by clamping the stem of the button to the bridge centre.

The efficiency of the transmitter depends upon the amount of direct current supplied to it, and this, in the ordinary common battery system, varies with the distance of the subscriber's station from the central office, or as it is usually termed, with the length of the subscriber's loop. With the subscriber's station very close to the central office, the current may be as high as about ·22 ampere, while with the subscriber at a remote distance it may be as small as ·05 ampere. The resistance of the transmitter depends somewhat upon design and circuit conditions, but a common figure is approximately 50 ohms. The instantaneous speech voltage generated by the transmitter may vary from several volts for loud vowel sounds to a minute fraction of a volt for certain weak, high-pitched consonants. As previously stated, the maximum value of the electrical speech energy delivered to the line is approximately ·1 watt, when the transmitter is on a short loop.

§ (15) DISTORTION.—In general there are two types of distortion in such a transmitter: first, that due to the mechanical resonance phenomena within the instrument and the failure of the dynamical properties of the effective values of mass, resistance, and elasticity (particularly of the granular carbon), to remain constant for different amplitudes and frequencies; and, secondly, that which is due to the electrical characteristics of the granular carbon.

(i.) *Mechanical Distortion.*—To illustrate the method in which the first type of distortion arises it will be necessary to limit the discussion to the simplest case, in which the aluminium diaphragm is considered as being acted upon at its centre by a force F sin ωt, which we may write symbolically as $F\epsilon^{j\omega t}$, where ϵ is the base of the Napierian system of logarithms. If the effective values of the mass, resistance, and elasticity of the moving system be represented by m, r, and s respectively, the displacement "x" of the point of application of the force can be shown to be given by the equation

$$x=\frac{1}{j\omega}\frac{F\epsilon^{j\omega t}}{r+j(m\omega-(s/\omega))}+\text{(terms expressing the transient motion).}^1$$

Let us assume for the instant that a sine wave displacement of the front electrode produces a sine wave modulation of the resistance of the granular carbon. It is evident that the response for the value of ω equal to $\sqrt{s/m}$ is very large as compared to the corresponding response for other values of ω. This phenomenon, therefore, gives rise to a resonance type of distortion. By properly selecting the magnitude of the ratio s/m it is theoretically possible to obtain a comparatively uniform region of response over wide ranges of frequency. This may be accomplished, for example, by using stretched diaphragms of small effective mass. The form of the response curve may be also modified by changing the factor "r," but this of course decreases the amount of useful energy for producing the modulation. The value of "r" may be modified by such obvious methods as clamping the edges of the diaphragm in a rubber seat or by placing a rubber-covered damping spring against the centre of the diaphragm. Such methods also change s and m, though usually in a lesser degree, and have been commonly employed in practical commercial designs.

[1] See article "Simple Harmonic Motion," Vol. I.

It should be noted here that one of the theoretically simplest methods of controlling both s and r is by air damping. For example, if an auxiliary plate is mounted directly behind the transmitter diaphragm, so that the two are parallel and separated by only a few mils, the air contained between the two changes both the effective s and r of the system. At very low frequencies the air has time to escape and through viscous dissipation the value of r is increased, whereas at higher frequencies the air does not have time to escape, and increases the effective stiffness of the system.

Any one of these methods of securing a wide range of uniform response also decreases the average displacement "x" within this range, and therefore decreases the modulating effect. Furthermore, any of the known methods of securing a wide uniform response, such as those above mentioned, have not thus far been so modified that they can be commercially applied.

In all cases it is of course necessary to choose the factors r and m in such a way that the necessary damping of the free oscillations of the diaphragm is secured. It should be noted in this connection that the effects of resonance distortion are also produced by the other parts of the instrument, such as the mica discs, the face plate, etc.

Furthermore, experiments have shown that the effective mass and stiffness of the granular carbon chamber are not constant for all amplitudes and for all frequencies, and this gives rise to another factor of distortion. In general it has been found that a transmitter having a single resonant frequency of about 1500 cycles is the best practical solution of the problem.

(ii.) *Electrical Distortion.*—In addition to the above type of mechanical distortion there are electrical distortions in the modulation; *i.e.* even though the front electrode should respond uniformly over the entire commercial range of both amplitude and frequency there would still remain an electrical distortion in the modulation. This may take several forms, some of the more important of which are listed below. Due to the expansions of the different parts of the carbon chamber or any other part of the transmitter, on account of the heating of the granular mass by the direct current through it, the resistance of the instrument undergoes a comparatively slow change. In many cases the change may be cyclic, and in most cases it produces a marked change in the magnitude of the modulated resistance. This phenomena is known technically as "breathing." At times the resistance of the transmitter becomes very unstable when current is passing through it, and due to the rapid fluctuation in the conductivity of the granular mass a frying or burning noise is heard, particularly in the local receiver of the telephone set. These phenomena must produce a certain amount of distortion in the modulated resistance. This rapid fluctuation of resistance, as contrasted with the slow-breathing action, has been technically called "burning."

Due to the fact that the resistance *versus* pressure, and resistance *versus* displacement, curves for two carbon granules in microphonic contact are not straight lines, there is a certain amount of distortion depending upon the range of either the amplitude or pressure over which the contacts are worked under actual service conditions. It is of course true that in the commercial transmitters employing a granular mass the single-contact effect may be somewhat masked, but the statistical effect in most of these instruments appears to be quite similar.

The modulated resistance is also a function of the voltage across the instrument. For example, if the circuit conditions are such that the maximum voltage that can be obtained across the granular carbon chamber is in the approximate range of 0 to 3 volts, the instantaneous resistance of the instrument under forcing corresponding to that of loud talking may approach infinity, whereas, if the voltage is higher, it appears to be impossible to obtain resistances of more than a few hundred ohms—for the maximum agitation that a transmitter will undergo under service conditions.

In order to show that the electrical output of the transmitter is influenced by the circuit it will be sufficient to consider the effect of a sine wave of resistance in a pure-resistance circuit, which is the most simple case. Assume that the equation for the instantaneous resistance (R) of the transmitter is

$$R = R_0 + R_1 \sin \omega t.$$

Then if the instantaneous value of the variable output current be denoted by i, the mean resistance of the transmitter by R_0, the periodic variation of the resistance by $R_1 \cos \omega t$, the mean transmitter current by I_0, the effective A.C. impedance looking away from the transmitter by R_x, and the mean voltage across the instrument by V_0, it can be shown that

$$i = \frac{I_0R_0 - V_0 + I_0R_1 \cos \omega t}{R_0 + R_x + R_1 \cos \omega t},$$

and the variable part of this will be approximately a sine wave output only when R_1 is negligible in comparison with $R_0 + R_x$. Then

$$i = \frac{I_0R_1 \cos \omega t}{R_0 + R_x}, \text{ approximately.}$$

It may therefore be said that the resistance modulation suffers both resonance distortion

and distortion due to the changes in the effective values of r, s, and m, and to voltage and temperature effects, and that, even though the resistance modulation is an exact copy of the sound-waves, the electrical wave generated within the circuit may be considerably distorted.

The complete problem considered in detail becomes very complex, and a large amount of experimental and theoretical work remains to be done upon such problems as (1) the analysis of speech characteristics in order to determine the character of the complex wave forms to be transmitted, (2) the study of the different types of distortion, and (3) the development of practical methods of improving the transmitter.

The volume efficiency of a transmitter is ordinarily compared in practice with that of a transmitter standard, and is expressed in terms of miles of standard cable, as explained under § (10). In giving the volume efficiency of such a transmitter it is necessary not only to give the amount of D.C. flowing through the transmitter, but also the circuit conditions—especially if the two transmitters under comparison have materially different resistances.

§ (16) THE TRANSMITTER CONSIDERED AS AN ELEMENT OF A TRANSMISSION CIRCUIT.—What has previously been said deals with the physical problems involved in the design of a transmitter. In making circuit calculations it is customary to think of the transmitter as an A.C. generator. In other words, from the standpoint of the external circuit a transmitter may be treated as a source of electromotive force acting through an impedance. For a given degree of agitation, the electromotive force generated varies with the direct current, and the internal impedance is also a function of the current supplied by the battery. In the case of the granular carbon transmitter this impedance is a pure resistance, and is approximately 80 per cent of its resistance, as measured with the ordinary D.C. instruments.

Within certain limits the effective alternating voltage generated by the transmitter may be considered to vary with the D.C. passing through it. A curve showing this variation is termed a current-supply loss curve. The curve used for a typical common battery transmitter is given very closely by the relation

$$l_s = 23{\cdot}2 \log_{10}\left({\cdot}854 + \frac{{\cdot}0415}{\mathrm{I}}\right),$$

where l_s is the so-called efficiency of the transmitter, expressed in miles of standard cable, and I is the direct current through the transmitter. The efficiency is arbitrarily referred to the condition where $\mathrm{I} = {\cdot}278$, which is approximately the current which flows through the transmitter in the System Reference Standard.

§ (17) RECEIVERS. *General.*—The ordinary form of magnetic receiver is a vibratory electric motor receiving electrical speech energy from the circuit and giving out mechanical (acoustical) energy, which corresponds in wave form, into the air enclosed by the ear-piece of the instrument and the auditory canal of the listener. The reverse operation of the instrument, as a transmitter, is discussed under § (13).

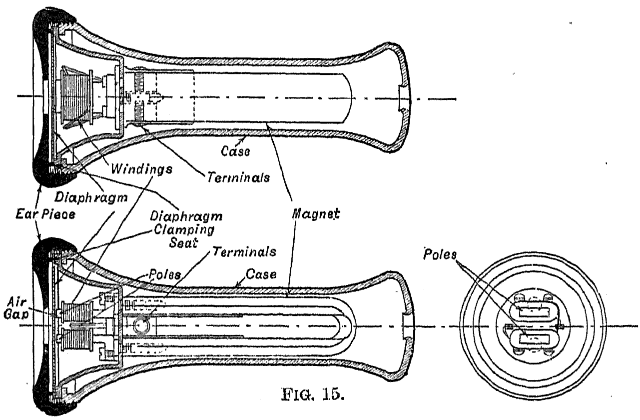


FIG. 15.

The attached *Fig.* 15 shows the type of receiver most commonly used in this country. In its principal features and dimensions it is similar also to the receiver in general use in America. It has a long U-shaped permanent magnet, the poles of which are extended by mild steel pole-pieces about which the telephone windings are placed. The end-faces of these pole-pieces are separated from the central portion of the diaphragm by narrow air-gaps, the actual separation being about 10 mils. The diaphragm is of ferrotype iron, 12 mils thick and a little over 2 in. in diameter. Under the influence of the permanent magnet a steady flux is maintained in the magnetic circuit, lines of force passing through the magnet and pole-pieces, and across from one pole-face to the other *via* two air-gaps and the central portion of the diaphragm, and it is therefore constantly deflected a small amount. Speech currents received in the windings superpose small variations of magnetomotive force, and of flux, in the system and give rise to vibratory forces which cause the diaphragm to emit the sound-waves which reproduce speech. It

is necessary that a large, steady flux be supplied, as otherwise speech waves of a given frequency would produce vibrations of double frequency and the resulting sounds would be practically unintelligible. The windings of the receiver shown have a D.C. resistance of about 70 ohms and an A.C. impedance at 800 cycles of about 250 ohms, with a positive (inductive) phase angle of about 44 degrees. The resonant frequency of the diaphragm is about 900 cycles. The speech energy delivered to the receiver on a long telephone connection is on the average about 10 microwatts, although under quiet conditions the minimum audible energy is less than one-billionth of this small value (10^{-14} watts).

Receivers may be classified according to the manner of supplying the steady flux, into permanent magnet or electromagnet types. The instrument described above is an example of the former. The electromagnet receiver is similar in action to the permanent magnet instrument, except that it derives its steady flux from direct current flowing through its windings simultaneously with the alternating current. Electromagnet receivers are used to some extent commercially, and with proper amounts of direct current flowing usually have an efficiency substantially equal to that of the permanent magnet type. Since in any given receiver there is an optimum value of flux there is in the electromagnet receiver an optimum value of direct current. If a direct current of greater or less than this best value flows through the receiver its efficiency will be decreased.

Receivers may also be classified according to the design and shape of case. The example described above is of the long or hand type—the type most commonly used at subscribers' stations. A second type, called the head type, uses a small circular magnet, and is mounted in a flat cylindrical case. It is often used in sets, such as operator's sets, where the receiver is kept at the ear for extended periods. Under such conditions the receiver is usually held by a head-band.

Other types of receiver than the magnetic are possible, though they have not been found to be practically desirable. The electrostatic receiver is one in which the potential variations of incoming speech waves are used to vary the electrostatic attraction on a diaphragm or membrane forming one of the charged surfaces of a parallel-plate condenser. The thermal receiver is one in which the heating of a resistance element undergoes minute variations due to the passage of the speech current. The air enveloping the resistance element expands and contracts with the variation of the heat, and thus sound-waves are produced. The reproduction is of good quality but feeble.

A further discussion of the magnetic receiver will now be given. It is to be noted that any receiver to be commercially satisfactory should be small and light in weight, rugged in design, and low in cost. It should convert as much of the incoming electrical energy into sound energy as is possible without an undue amount of distortion. The last requirement is equivalent to saying that the receiver should give a fairly uniform response over the range of frequency used in speech. If the natural or resonant frequency falls in this range, as is found practically desirable, the diaphragm vibrations should be well damped when the receiver is held to the ear.

§ (18) VIBRATORY CHARACTERISTICS OF A RECEIVER. (i.) *Impedance.*—The usual and most satisfactory means of determining the vibratory characteristics of a receiver is from its impedance analysis. For most purposes it is sufficiently correct to assume that the receiver is a system having but one degree of freedom; that is, that the diaphragm can be replaced by an equivalent mass m, concentrated at its centre, constrained by an elastic restoring force sx, and by a dissipative force $r\dot{x}$, where x is the displacement of the mass from its position of rest, and $\dot{x}$ its velocity, $\ddot{x}$ being the acceleration. If this mass be acted upon by an external force of F sin ωt, written symbolically $\mathrm{F}e^{j\omega t}$, the differential equation of motion [1] is

$$m\ddot{x}+r\dot{x}+sx=\mathrm{F}e^{j\omega t},$$

which gives the familiar solution for $\dot{x}$,

$$\dot{x}=\frac{\mathrm{F}e^{j\omega t}}{r+j(m\omega-(s/\omega))}=\frac{\mathrm{F}e^{j\omega t}}{z},$$

or in absolute magnitude

$$\dot{x}=\frac{\mathrm{F}e^{j\omega t}}{\sqrt{r^2+(m\omega-(s/\omega))^2}}.$$

The above equation shows that the locus of F/z as ω is varied from 0 to ∞ is a circle with its principal diameter horizontal.

Assuming that the force on the receiver diaphragm is proportional to the square of the magnetic flux in the air-gap between the pole-pieces and the diaphragm, we have $f=k\phi^2$, where ϕ is the flux in the gap and f is the instantaneous value of the force and k is a constant of proportionality. If we denote by ϕ_0 the steady value of flux produced by the permanent magnet, and by i, the instantaneous value of the current through the coils, we obtain

$$f=k(\phi_0+p\mathrm{N}i)^2=k\phi_0{}^2+2k\phi_0 p\mathrm{N}i+kp^2\mathrm{N}^2i^2,$$

in which N is the number of turns on the coil or coils, and p is the permeance of the magnetic

[1] See "Simple Harmonic Motion," Vol. I.

circuit. If i varies sinusoidally with the time, this last equation becomes

$$f = k\phi_0^2 + 2k\phi_0 pNi(\sin\omega t) + kp^2N^2i^2(\sin^2\omega t)$$
$$= k\phi_0^2 + 2k\phi_0 pNi\sin\omega t + \frac{kp^2N^2i^2}{2}(1-\cos 2\omega t).$$

The first term of this equation is a steady pull on the diaphragm, the second is the useful force corresponding to the current, and the third term is made up of a steady pull and a double frequency term. In normal operation the flux pNi is small in comparison with ϕ_0, so that the double frequency effect is negligible.

With an alternating current, the flux pNi will lag behind the current i by an angle β, on account of hysteresis and eddy currents in the iron. The velocity of the diaphragm $\dot{x}$, when a current of the form $i\sin\omega t$ is supplied to the coils, is therefore

$$\dot{x} = \frac{2k\phi_0 pNi\sin\omega t}{z} = \frac{Ai\sin\omega t}{z},$$

where A, the force per unit current, is the "force factor." The principal diameter of the velocity circle is therefore inclined at an angle of β_1 to the reference axis of the current.

When the diaphragm vibrates, a voltage of $N\dot{x}(d\phi/dx)$ is generated in the coil, and substituting in the last equation we obtain

$$E_m = NA\frac{d\phi}{dx}\,\frac{i\sin\omega t}{z},$$

where E_m is the voltage produced by the motion of the diaphragm. This flux, however, and hence the voltage, lags behind the velocity by an angle β_2 on account of the iron losses. It can be shown that for all known commercial types of receivers and within the requirements of practical analysis the voltage per unit velocity is equal to the force per unit current with the sign reversed, or $N(d\phi/dx) = -A$. Therefore we have

$$E_m = \frac{-A^2\,i\sin\omega t}{z}\,|\underline{-2\beta}.$$

If we define the motional impedance Z_m as the ratio of the impressed voltage, overcoming the motional voltage of the diaphragm, to the current we obtain [1]

$$Z_m = \frac{A^2}{z}\,|\underline{-2\beta}.$$

The motional impedance can be obtained by calculating the complex difference between the total impedance when the diaphragm is allowed to vibrate, and that when the diaphragm is held stationary in its position of rest. The former is known as the free impedance and the latter the damped impedance. The effective resistances and inductances for these impedances can be readily measured by means of an alternating current bridge.

(ii.) *Impedance Analysis.*—To obtain an impedance analysis, the free and damped impedances are measured, at suitable frequency intervals, over a range extending well to each side of the natural frequency—the natural frequency being that frequency at which the velocity of the diaphragm is a maximum for an impressed force of a given magnitude. The whole series of measurements is made with the same current through the receiver. In making measurements of the damped impedance, care must be taken not to change the clamping of the diaphragm or the separation between it and the pole-pieces. The diaphragm can be damped satisfactorily by carefully adjusting the point of a screw until it is heard to touch the vibrating diaphragm. However, after a little experience, a curve for the damped impedance can be drawn without measurements, from a visual inspection of the curve showing the free impedance.

An impedance analysis of a common type of receiver is given in the accompanying *Fig.* 16.

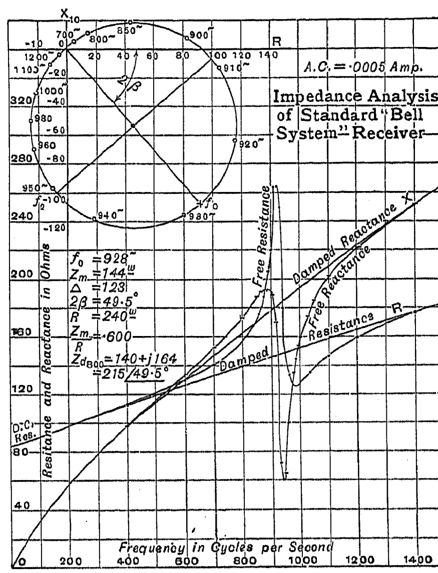


FIG. 16.

To obtain the impedance circle, the motional resistance and reactance for each value of frequency are laid off horizontally and vertically, respectively, from a common origin, each point being marked with its proper frequency. The locus of these points should be a circle passing through the origin with its principal diameter depressed below the horizontal axis by an angle of 2β. This angle is an indication of the iron losses, and varies roughly between 10 and 90°. The natural frequency $f_0 = (1/2\pi\sqrt{s/m})$ of the diaphragm is given at the remote end of the principal diameter. The damping constant $\Delta = (r/2m) = (x^2 - \omega_0^2/2\omega\tan\alpha)$ (where α is the phase angle of z) is the logarithmic decrement per second of the free oscillations of the diaphragm, and may be obtained from the expression $\pi(f_2 - f_1)$, where f_1 and f_2 lie at the ends of the diameter perpendicular to the principal diameter. The ratio of the maximum motional impedance to the free resistance at resonance gives a measure of the apparent efficiency of the receiver at resonance, and is an important quantity

[1] The symbols $|\underline{\phi}$ and $\overline{|\phi}$ are used to denote positive (lagging) and negative (leading) phase angles respectively.

in comparing two receivers. The force factor A is another important constant, and its value at resonance can be obtained from the expression

$$A=\sqrt{2mZ_m\Delta 10^9},$$

the units being dynes per absampere, Z_m being taken at the natural frequency.

The coefficients r, s, and m of the diaphragm cannot be obtained from the impedance analysis without an independent determination of the value of one of them. All of these quantities are difficult to measure; however, the effective mass can be determined with some degree of accuracy by adding a known mass to the centre of the diaphragm and noting the change in the quantities f_0 and Δ. Near resonance, the effective mass of a clamped metal diaphragm is approximately 25 per cent of the mass of the part inside the clamping ring. As soon as r, s, or m has been determined, the other two can easily be derived.

While the foregoing theory is incomplete, it will serve as an approximate statement of the phenomena of the telephone receiver.

§ (19) The Receiver considered as an Element of a Transmission Circuit.—An ideal receiver is one which converts all of the electrical power it receives in the form of alternating current of audible frequency, into sound power, this conversion being independent of the magnitude of the electrical input. In an actual receiver, however, the efficiency conversion is rather low, only a few per cent being converted into actual sound-power. The problem of transmission circuit design is in general to get the maximum amount of electrical power into the receiver, thereby obtaining maximum volume of sound output. Although at telephonic frequencies the impedance of the instrument, when held to the ear, differs somewhat from its damped impedance, the latter can be used without serious error for circuit calculations. The phase angle of commercial receivers varies from +40 to +75 degrees—a large phase angle usually, but not necessarily, connoting an efficient design. This relatively high-phase angle makes it difficult to get the maximum amount of power into a receiver except at one frequency.

It is always assumed in telephone circuit problems that a receiver can be wound to any desired impedance without affecting the phase angle. Such a result can be accomplished approximately by choosing the proper size of wire to keep the coil the same size and shape. This is equivalent to the assumption that we can change the characteristics of the receiver in the same manner as by associating with it an ideal transformer of any desired ratio.

The impedance, at normal telephonic frequencies, of a magnetic receiver may be from three to ten times its direct current resistance. Since the proper winding for a receiver is that which makes the effective A.C. ampere turns a maximum, and since an ideal coil would have a negligible D.C. resistance, the fairly common practice of specifying a receiver by its D.C. resistance has no logical justification. Practically its effect is seen in the custom, which ordinarily is entirely inexcusable, of winding receivers with nickel silver or other high-resistance wire where a high resistance is specified. In such cases, where the desired impedance cannot be obtained by winding the receiver with copper wire, it would be better to waive the resistance requirement and use a receiver of lower resistance. The absolute magnitude of the damped impedance, at 800 cycles, is a much more valuable constant and should be specified wherever possible.

§ (20) Transformers. (i.) *General Characteristics of Transformers.*—If we are given certain telephone instruments and a line or circuit having known A.C. constants, a problem which is frequently of interest is to determine how we can best associate these elements with a transformer so as to give the highest possible transmission efficiency. Consequently, a problem of great interest in telephone transmission work is to consider the general characteristics of transformers and how actual transformers compare with ideal transformers in efficiency.

Since a transformer may be defined as "any structure with two or more windings between which there exists mutual impedance," it is important first to define what is meant by mutual impedance. As it is ordinarily used in telephone work, the mutual impedance between one pair of terminals and a second pair of terminals is the vector ratio—with sign reversed—of the electromotive force produced between either pair of terminals on open circuit to the current flowing between the other pair of terminals.

With the above definition of mutual impedance (M) it can be shown that the impedance of two windings connected in series aiding (*i.e.* so that the flux which is produced by the current in one winding is aided or increased by the current flowing in the other winding) is

$$Z_{S.A.}=Z_1+Z_2+2M,$$

where Z_1 and Z_2 are the self-impedances of the two windings. If the direction of either the Z_1 or Z_2 winding is reversed we have what is commonly called a series opposing connection (*i.e.* flux produced by the current in one winding is opposed or decreased by the current flowing in the other winding). The effect of changing the direction of any transformer winding is to change the sign of any mutual impedance associated with that winding. In other words, if we reverse the direction of one of the windings, or if we have a series opposing connection instead of a series aiding connection, the impedance is

$$Z_{S.O.}=Z_1+Z_2-2M.$$

An ideal transformer is one which neither stores nor dissipates energy. Consequently, in an ideal transformer there is perfect flux linkage between the windings, and the mutual impedance M is equal to the square root of the product of the self-impedances of the windings, or

$$M=\sqrt{Z_1Z_2}.$$

Hence it is evident that the series aiding impedance of an ideal transformer is

$$Z_{S.A.}=Z_1+Z_2+2\sqrt{Z_1Z_2}=(\sqrt{Z_1}+\sqrt{Z_2})^2,$$

or if the two windings are equal,

$$Z_{S.A.}=4Z_1=4Z_2=4M.$$

Similarly, the series opposing impedance of an ideal transformer is

$$Z_{S.O.}=Z_1+Z_2-2\sqrt{Z_1Z_2}=(\sqrt{Z_1}-\sqrt{Z_2})^2,$$

which in the case of a 1 : 1 transformer (*i.e.* $Z_1=Z_2$) becomes zero.

Consider the case of two windings (*Fig.* 17) which are in parallel aiding or in parallel opposing connections. The direction of winding in the parallel aiding connection is analogous to the series aiding connection, *i.e.* it is such that the flux which is produced by the current in one winding is aided or increased by

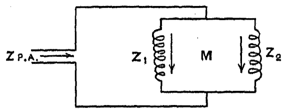


FIG. 17.

the current flowing in the other winding. In such a circuit the impedance of the two windings in parallel is

$$Z_{P.A.}=\frac{Z_1Z_2-M^2}{Z_1+Z_2-2M}.$$

Consequently, as previously explained, the impedance of the two windings in parallel opposing connection is obtained by reversing the sign of M, or is

$$Z_{P.O.}=\frac{Z_1Z_2-M^2}{Z_1+Z_2+2M}.$$

If, as before, we assume an ideal transformer, *i.e.* $M=\sqrt{Z_1Z_2}$, the parallel opposing impedance is zero, no matter what may be the relative magnitudes of Z_1 and Z_2. On the other hand, if $Z_1=Z_2$ the parallel aiding impedance, in the case of an ideal transformer, is equal to the mutual impedance between the two windings, or

$$Z_{P.A.}=\sqrt{Z_1Z_2}=M.$$

In any ideal transformer (*Fig.* 18) the currents flowing through the windings are always inversely proportional to the number of turns on the windings. Similarly, the voltages across the windings are directly proportional to the number of turns in the windings. It can also be shown that the impedance looking into an ideal transformer is equal to the impedance of the load multiplied by the ratio of the self-impedance of the "input" winding to that of the "load" winding. This may be seen by reference to the two cases shown below, in which

$$Z_a=\frac{Z_2}{Z_1}.Z,\quad\text{and } Z_b=\frac{Z_2}{Z_1}.Z=\frac{Z_1+Z_3+2M_{13}}{Z_1}.Z.$$

(ii.) *Transmission Losses in Actual Transformers.*—If a two-winding transformer is

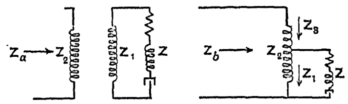


FIG. 18.

connected between two impedances, as shown below (*Fig.* 19), and an E.M.F. E is assumed

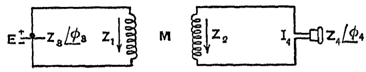


FIG. 19.

to be acting through one impedance, the current flowing in the other impedance is

$$I_4=\frac{-ME}{(Z_1+Z_3)(Z_2+Z_4)-M^2}.$$

In the case of an ideal transformer which has the best possible ratio for its windings, *i.e.* one in which $Z_2/Z_1=Z_4/Z_3$, the received current is

$$I_4=\frac{E}{2\sqrt{Z_3Z_4}\cos\frac{1}{2}(\phi_4-\phi_3)}.$$

The ratio (K) of the received current with the actual transformer to that of the current received with the ideal transformer gives a means (see § (31)) of determining how many miles of standard cable any actual transformer is less efficient than the ideal transformer.

In the foregoing equation, and in the diagram above, ϕ_3 and ϕ_4 are the phase angles of the transmitter and receiver impedances, respectively.

§ (21) SUB-STATION CIRCUITS.—The circuit used in connecting the subscriber's transmitter, receiver, ringer, etc., together is called the Sub-station Circuit. If by any method the circuit of a sub-station set is changed so that (1) when transmitting, the receiving element is effectively removed from the circuit and (2) when receiving, the transmitting element is removed from the circuit, the transmission efficiency of the circuit can be materially improved. A sub-station circuit that is capable of such a change is called a Variable Sub-station Circuit in contradistinction to the Invariable Sub-station Circuit, in which the circuit is identically the same electrically, whether transmitting or receiving.

(i.) *Ideal Variable Sub-station Circuits.*—An ideal variable circuit is one in which, when receiving, all the power it is possible to draw from the line is absorbed in the receiver. Similarly, when transmitting, all the external power that can be generated by the transmitter is sent out on the line. If the transmitter be regarded as a source of constant electromotive force E, the current entering the line from an ideal variable sub-station circuit is

$$I_5 = \frac{E}{2\sqrt{R_5 R_3}}.$$

Similarly, on receiving, with an electromotive force e acting in the line the current in the receiver of an ideal variable sub-station circuit is

$$I_4 = \frac{e}{2\sqrt{R_5 R_4}},$$

in which R_3, R_4, and R_5 are the effective resistances respectively of the transmitter, receiver, and line. The side tone current—or current in the receiver when transmitting—is evidently zero in such an ideal variable circuit.

(ii.) *Ideal Invariable Circuits.*—An ideal invariable sub-station circuit is one which has the maximum possible over-all efficiency and which is electrically the same in both the transmitting and receiving condition. In an ideal invariable sub station circuit the current entering the line when transmitting (*i.e.* with an electromotive force E acting in the transmitter) is

$$I_5 = \frac{E}{2\sqrt{2R_3 R_5}}.$$

Similarly, with an electromotive force e acting in the line, the current in the receiver when receiving is

$$I_4 = \frac{e}{2\sqrt{2R_4 R_5}}.$$

The product of these equations, which is

$$I_4 \times I_5 = \frac{Ee}{8R_5\sqrt{R_3 R_4}},$$

gives a means for determining the combined transmitting and receiving or over-all efficiency of any actual invariable sub-station circuit, as compared with that of the ideal invariable circuit.

Assuming an electromotive force E acting in the transmitter, the side tone current in the receiver of an ideal invariable side tone sub-station circuit is

$$I_{4\,\text{S.T.}} = 4\frac{E}{\sqrt{R_3 R_4}}.$$

(iii.) *Efficiencies of an Actual Sub-station Circuit in Terms of the Ideal Invariable Circuit.*—The transmitting, receiving, or side tone efficiency of any given sub-station circuit can be compared with that of the ideal invariable sub-station circuit by computing the ratio of the currents flowing in the actual circuit with the corresponding currents in the ideal circuit. The ratio K of these currents may then be expressed in miles (l_s) of standard cable by means of the relation:

$$l_s = \frac{2\cdot3026}{\sqrt{88 \times \cdot054 \times 10^{-6}\pi f}} \log_{10}\frac{1}{K} = \frac{596}{\sqrt{f}} \log_{10}\frac{1}{K}.$$

When the frequency $f = 796 (\omega = 5000)$ this becomes

$$l_s = 21\cdot12 \log_{10}\frac{1}{K}.$$

These relations are those which exist between the ratio K of the current at a given point in an infinitely long length of standard cable (having the constants $R = 88$, $C = \cdot054 \times 10^{-6}$) and the current at another point, l_s, miles nearer the source of the electromotive force.

The over-all efficiency of well-designed commercial common battery subscribers' circuits is—at the most important telephonic frequencies (800 to 1000 cycles)—only from two to three miles below that of the ideal invariable circuit. Hence no changes in induction coil design, capacity of the condenser, etc., can improve the over-all efficiency of such common battery subscribers' sets by more than approximately three miles. Any greater improvement in the over-all efficiency of such subscribers' sets must therefore come from improvements either in the transmitter or in the receiver.

(iv.) *Side Tone Circuits—Parallel* v. *Series Types.*—Practically all sub-station circuits that are in commercial use are side tone circuits, *i.e.* circuits of the type in which there is a relatively large amount of power dissipated in the receiver when an electromotive force is generated in the transmitter. Side tone circuits are of two types, the Series Type and the Parallel Type—depending upon whether the three elements, the line, transmitter, and receiver, are effectively in series or in parallel with each other. A complete list of all series and parallel types of side tone sub-station circuits which do not use more than two induction coils, and which employ only two terminal elements (line, transmitter, and receiver) has been given in a paper by Messrs. G. A. Campbell and R. M. Foster.[1]

Invariable circuits of both the series and parallel types of circuits have theoretically the same transmitting, receiving, over-all, and side tone efficiencies—as indicated by the formulae under 5.12. Due to the fact, however, that a carbon button transmitter has, in general, a higher effective resistance when agitated than when in a quiet condition, the series type of circuits has some practical advantages over

[1] Entitled "Maximum Output Networks for Telephone Sub-station and Repeater Circuits," read before the American Institute of Electrical Engineers on February 19, 1920.

the parallel type for most commercial uses. In ideal circuits of both types when transmitting, the power dissipated in the receiver—which is a measure of the side tone volume—is one-half of that dissipated in the line. A distinguishing characteristic between the two types of circuits is that in the ideal series type of circuit the impedance looking away from the transmitter (or receiver) terminals is three times that of the transmitter (or receiver), while in the parallel type of circuit the impedance is one-third of the transmitter (or receiver) impedance.

(v.) *Anti-side Tone Sub-station Circuits.*—An anti-side tone sub-station circuit is one in which, in the ideal case, no power is dissipated in the receiver when there is an electromotive force in the transmitter. All efficient invariable anti-side tone sub-station circuits require four elements (L, R, T, and N), or one more element than is required in efficient invariable side tone sub-station circuits. The necessity for this extra element, which is the balancing network N, may be seen by considering the anti-side tone sub-station circuit as a development of the Wheatstone bridge circuit.

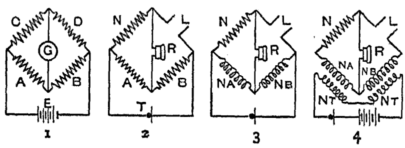


FIG. 20.

In the four figures shown above, *Fig.* 1 is a typical Wheatstone bridge circuit, in which if A/B = C/D no power from the battery E will be dissipated in the galvanometer G. If, as is shown in *Fig.* 2, the battery E is replaced by a source of A.C. electromotive force such as the transmitter T, the galvanometer G by the receiver R, the resistance D by the line impedance L, and the resistance C by the balancing impedance N, it is evident that there will be no power dissipated in the receiver, provided A/B = N/L. In such a circuit, however, less than one-half of the total power given out by the transmitter is dissipated in L (if A = B = N = L only one-quarter of the total power given out by the transmitter would be dissipated in L). Therefore, in order to increase the efficiency of such a circuit the resistances A and B are replaced by two windings on an induction coil or transformer as is shown by N_A and N_B in *Fig.* 3. If the induction coil is ideal—in which case the self-impedances of the windings N_A and N_B are pure reactances—all the power from the transmitter will be dissipated equally in N and L. In an actual transformer or induction coil there is only a slight loss in windings such as N_A and N_B.

Using a separate winding for the transmitter, as is shown in *Fig.* 4, does not change the "Wheatstone bridge action" of the circuit, but does allow the impedance of the transmitter to be effectively stepped up or down in any desired ratio by a proper choice of N_T. This is a desirable feature in that it enables a transmitter of any resistance to be used efficiently. If the line impedance L is nearly pure resistance over the telephonic range of frequencies, N will also be a pure resistance, in which case it can be incorporated in the winding N_A by making the latter of a fine gauge of wire, or winding it with wire of some high-resistance material.

When receiving, in an ideally designed anti-side tone circuit no power is dissipated in the balancing network N—half of the total incoming power being dissipated in the receiver and half in the transmitter—which is identically the same condition as exists in an ideal invariable side tone sub-station circuit. The transmitting efficiency of the ideal side tone and anti-side tone sub-station circuits is also identical. In the former, due to the fact that the transmitter is working into either 3 times or $\frac{1}{3}$ of its own impedance, only $\frac{3}{4}$ of the maximum power is given out by the transmitter, $\frac{2}{3}$ of this power being dissipated in the line. In other words, only $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ of the total power which it is possible for the transmitter to give out is dissipated in the line. In the ideal anti-side tone circuit, the transmitter works into its own impedance and hence delivers the maximum possible power of which it is capable – half of this power being dissipated in the balancing network and the other half in the line.

Therefore, the ideal side tone and anti-side tone circuits each have the same transmitting and receiving efficiencies, but the latter has no side tone. In the actual case, the anti-side tone circuit is, when other things are equal, slightly less efficient than the side tone circuit, due to the fact that the two conjugacy conditions — of no current in the receiver when transmitting and no current in the balancing network when receiving—cannot be made to hold rigorously for all the different line conditions, frequencies, etc., encountered in commercial practice.[1]

§ (22) TRANSMISSION OVER LONG LINES. *General Theory of A.C. Transmission.*—When an alternating voltage is applied at one end of a telephone line, a part of the power entering the line is dissipated as heat in the line, a part of it is stored in the inductance and capacity of the circuit, and a part is transmitted to the apparatus at the distant end.

In power lines the frequency is so low that the ratio of the actual lengths of circuit used to the wave-lengths is small, that is to say, power lines are electrically short. In such lines, a large part of the power impressed on the line can be delivered at the receiving end. Therefore, the power input to the line at a given voltage depends largely upon the power taken by the receiver. That is to say, the effective circuit impedance can be increased by increasing the impedance of the terminal apparatus. This is common practice in power transmission systems, the impedance of the terminal apparatus as measured from the line being greatly increased by the use of high-ratio transformers.

[1] A complete list of all the anti-side tone sub-station circuits, as well as side tone circuits, is given in the paper by Campbell and Foster referred to in note above.

In the ordinary telephone line, however, transmitting currents at high frequency, the actual length is long compared with the wave-length, that is, the lines are electrically long. Most of the power impressed on the line is dissipated in the line and it is possible to transmit but a small fraction of it to the receiving end. Under these conditions it is evident that the power input to the line at a given voltage is very nearly independent of the power taken by the receiving apparatus; that is, the effective line impedance is affected but very slightly by the impedance of the terminal apparatus and is practically fixed by the constants of the line itself.

In a uniform line long enough so that for points under consideration terminal reflections may be ignored, the voltage and current may be obtained by multiplying the voltage and current at the sending end by a single exponential factor. The exponent is composed of two factors, one the length of the circuit, and a second which is characteristic of the type of line. The second factor is complex and is called the Propagation Constant.

$$P=a+jb.$$

The propagation constant (P) per unit length of a uniform line or per section of a line of periodic recurrent structure is the natural logarithm of the vector ratio of the steady state currents at two points separated by a unit length in a uniform line of infinite length or at two successive corresponding points in a line of recurrent structure of infinite length. The ratio is determined by dividing the value of the current at the point nearer the transmitting end by the value of the current at the point more remote. The real part of the propagation constant (a) is called the Attentuation Constant and imaginary part (b) is called the Wave-length Constant.

The attenuation constant indicates the rate of dissipation or damping which the current (and voltage) undergoes as energy is propagated along the line.

The wave-length constant indicates the rate of change of phase, in radians, which the current (and voltage) undergoes as energy is propagated along the line. When $bl=2\pi$, l is the wave-length for the particular line and frequency involved.

The characteristic impedance (Z_0) of a line is the ratio of the applied electromotive force to the resulting steady state current in a line of infinite length and uniform structure, or of periodically recurring structure. The Characteristic Impedance and the Propagation Constant are the fundamental constants of a given type of the telephone circuit.

If a uniform transmission line has distributed constants of R ohms resistance per unit length, L henrys inductance per unit length, G mhos leakance per unit length, and C farads capacity per unit length, the characteristic impedance is

$$Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}};$$

and the propagation constant of the line is

$$P=a+jb=\sqrt{(R+j\omega L)(G+j\omega C)}.$$

This last equation can also be written:

$$P=a+jb$$
$$=\sqrt{\tfrac{1}{2}\sqrt{(R^2+L^2\omega^2)(G^2+C^2\omega^2)}+\frac{GR-LC\omega^2}{2}}$$
$$+j\sqrt{\tfrac{1}{2}\sqrt{(R^2+L^2\omega^2)(G^2+C^2\omega^2)}-\frac{GR-LC\omega^2}{2}},$$

in which the first term "a" is the attenuation constant and the second term "b" is the wave-length constant.[1]

It follows from these equations that the impedance of a telephone line can be increased by increasing the inductance L, and that the attenuation constant is decreased by increasing the inductance L. Adding inductance to a line is called loading the line, and such a line or circuit is called a Loaded Circuit.

It may be noted that the addition of inductance not only increases the impedance but also raises the power factor in the circuit, thereby permitting a still further reduction of current for a given amount of power. The improved transmission efficiency which may be attributed to this second effect is quite appreciable in the case of cable circuits.

The power factor of a circuit is the cosine of the angle between the current and voltage in the line, that is, the cosine of the angle of the characteristic impedance. This angle varies in different types of standard non-loaded circuits from $-5°$ to $-45°$, the latter value holding very closely for small gauge non-loaded cable circuits. This angle becomes practically zero when either open wire or cable circuits are loaded, and the power factor in such circuits therefore becomes practically unity.

The increase of voltage resulting from the increased impedance—when inductance is added to the circuit—increases the leakage losses, and these set a limit to the possible improvement in transmission efficiency by loading. If the inductance could be increased without increasing the resistance the most efficient transmission would be obtained when the inductance had been increased to such a value that the leakage losses were equal to the resistance losses. The maximum possible improvement is in practice obtained with a smaller value of inductance, however, because of the resistance of the inductance coils, and

[1] The derivation of the characteristic impedance and the propagation constant in terms of the constants of the line can be found in many standard articles on telephone transmission, for example, in Heaviside's *Electromagnetic Theory*, i. section 221, and Fleming's *Propagation of Electric Currents*, pp. 66-72.

the amount of inductance which it is economical to insert in the line is still less because of the cost of the loading coils.

The decrease in power of the wave as it is propagated along the circuit consists in general of a decrease both in the voltage and in the current. As in a long telephone line the ratio between voltage and current is the same at different points of the line, the rate of decrease of the voltage and current must also be the same. Therefore the rate of decrease of the power, which is the product of the voltage and current, is twice that of either. It is customary for convenience to measure the losses in a circuit in terms of the fractional decrease of current per unit length, and this fractional decrease is called the attenuation of the circuit. For practical telephone work it is customary to use, not the attenuation constant itself, but the ratio of the attenuation per mile of the type of circuit under consideration to that of Standard Cable. This ratio is known as the Transmission Equivalent of the type of circuit and expresses the length of Standard Cable equivalent as regards transmission loss, to one mile of the given type of circuit.

The equation for the characteristic impedance shows that the impedance of a line is not in general independent of the frequency. For example, in a well-insulated, non-loaded cable circuit the leakance and inductance are both so small that they may be neglected, and the characteristic impedance can be written:

$$Z_0=\sqrt{\frac{R}{jC\omega}}.$$

In this case the line impedance decreases with increasing frequency. Therefore, since the resistance remains practically constant, for a given amount of power transmitted, the power loss in the line for higher frequencies is greater than the loss for lower frequencies. As currents of a wide range of frequency are necessary for the transmission of intelligible speech, this unequal attenuation of the different frequencies distorts the speech. If this distortion is great enough the speech becomes unintelligible because of the excessive attenuation of the harmonics of higher frequencies. When such a circuit is loaded, however, the inductance term is large compared with the resistance term, and the line impedance becomes almost exactly:

$$Z_0=\sqrt{\frac{L}{C}}.$$

That is, the line impedance is practically independent of the frequency. Hence, were the resistance and leakance constant, the attenuation on such a circuit would be practically the same for all frequencies, and the circuit would be distortionless. Transmission over actual loaded circuits is not entirely without distortion, however, for several reasons. The effective resistance of the loading coils increases with the frequency and the losses for the higher frequencies are increased. The lack of absolute regularity in practical loaded lines also prevents the complete elimination of distortion in the line by loading.

The loading of cable circuits results in a marked improvement in distortion as well as in transmission efficiency.

Open-wire non-loaded circuits are very different in characteristics from non-loaded cable, and they have only a small amount of distortion. The loading of open-wire circuits improves their efficiency, but the distortion is increased. In the higher frequencies of the telephone range important distortion and transient effects are caused on the ordinary types of loaded lines, due to the fact that lumped, rather than uniformly distributed loading is used.

§ (23) TRANSMISSION EQUIVALENTS OF TYPICAL OPEN-WIRE AND CABLE CIRCUITS.—For the sake of reference, we give below the Transmission Equivalents of some of the types of open-wire and cable circuits commonly used:

CABLE CIRCUITS

APPROXIMATE TRANSMISSION EQUIVALENTS

(Copper Conductors)

Weight per Mile. (pounds.)	Type.	Non-Phantomed Circuits.	Sides of Phantoms.	Phantoms.*
10	Non-Loaded	1·51	1·51	1·42
	Light	·90	·90	·72
	Medium	·69	·70	·57
	Heavy	·51	·52	·44
20	N.L.	1·11	1·11	·91
	L.	·46	·46	·38
	M.	·36	·36	·30
	H.	·27	·28	·23
40	N.L.	·74	·74	·65
	L.	·24	·24	·19
	M.	·19	·19	·16
	H.	·15	·16	·13
70	N.L.	·55	·55	·47
	L.	·15	·15	·12
	M.	·12	·13	·10
	H.	·094	·11	·085
100	N.L.	·44	·44	·37
	L.	·10	·11	·085
	M.	·086	·094	·076
	H.	·075	·083	·069
150	N.L.	·33	·33	·28
	L.	·067	·071	·060
	M.	·060	·066	·055
	H.	·056	·063	·053

* See § (27) for an explanation of the term "Phantom circuit."

The above table assumes that a higher grade of loading coils is used on the larger-gauge circuits than is employed on the 10-, 20-, and 40-pound circuits.

The table also assumes that the damping constant, G/2C, is (at 800 cycles) equal to 7 for the larger-gauge circuits and is equal to 12 for the smaller-gauge circuits.

The A.C. capacity of the side circuits in the above table is assumed to be ·0665 mf. per mile and that of the phantom circuits to be ·106 mf. per mile. Paper cable is now manufactured which has an A.C. side circuit capacity of approximately ·062 mf. per mile and a phantom circuit capacity of approximately ·100 mf. per mile.

Light loading is now seldom used since the transmission gain resulting from such a type of loading rarely justifies its cost. Similarly large-gauge loaded circuits can very seldom be justified from an economic standpoint. 20-pound and 40-pound loaded circuits, with or without repeaters, are the most commonly used types of trunk cable circuits. On repeatered cable lines up to about 250 miles in length, medium or heavy loading is commonly used. Lines from 250 to 500 miles in length are medium-heavy loaded and those over 500 miles are extra light loaded. Both the latter types of loading lead to higher velocities of transmission than are characteristic of the other types of loading mentioned in the above table. This is found to be more and more desirable as the length and efficiency of long repeatered lines is increased.

The approximate particulars of loading for cable circuits are given in the following table:

Loaded Circuits

Type of Loading.	Spacing of Coils. (miles.)	Inductance of Coils. Side Circuit.	Inductance of Coils. Phantom Circuit.	Characteristic Impedance of Circuit. Side Circuit.	Characteristic Impedance of Circuit. Phantom Circuit.
Light . . .	2·20	·135	·083	1000	600
Medium . . .	1·66	·175	·107	1300	800
Heavy . . .	1·14	·250	·155	1800	1150
Medium-heavy .	1·14	·175	·107	1500	1000
Extra light . .	1·14	·044	·025	800	500

Open-wire circuits are assumed to be loaded by placing open-wire-type loading coils at such intervals that the capacity of the side circuit between coils is approximately ·0655 microfarads. With types of open-wire construction in common use this gives a spacing of about 8 miles. The side circuit coils add an inductance of ·25 henry to the side circuit and the phantom coils ·16 henry to the phantom.

The transmission equivalents of some of the more commonly used types of open-wire copper circuits are given below:

Open-Wire Lines

Approximate Transmission Equivalents

(Copper Conductors)

Weight per Mile. (pounds).	Type.	Non-Phantomed Circuits.	Sides of Phantoms.	Phantoms.
100	Non-Loaded	·109	·109	·094
	Loaded	·048	·050	·041
150	N.L.	·080	·080	·065
	L.	·036	·037	·031
200	N.L.	·062	·062	·051
	L.	·030	·031	·026
300	N.L.	·046	·046	·042
400	N.L.	·036	·036	·029
500	N.L.	·029	·029	·023
600	N.L.	·025	·025	·019

§ (24) Lumped Loading.—In order to obtain the benefits of increased line inductance by means of loading coils distributed along the circuit, it is necessary to have the coils uniformly distributed at distances that do not exceed certain maximum amounts. The spacing of loading coils ordinarily varies from 1·14 miles on heavily loaded cable circuits to approximately 8 miles on open-wire circuits, and the inductance of each coil may be as low as ·025 henry or as high as ·25 henry, depending upon the constants of the circuit.

The effect of loading a line in such a way is approximately the same as though inductance were uniformly distributed along the circuit. The conditions for this equivalence are fully developed in a paper by M. I. Pupin in the *Proceedings of the American Institute of Electrical Engineers* for May 19, 1900, and in a paper by G. A. Campbell in the *Philosophical Magazine* for March 1903, and are briefly summarised in the following:

If l is the length, in miles, of a loading section, *i.e.* the distance between adjacent loading coils, and Z_c is the impedance of a loading coil, the propagation constant of the loaded line is

$$P' = \frac{1}{l}\cosh^{-1}\left[\cosh lP + \frac{Z_c \sinh lP}{2Z_0}\right],$$

where P is the propagation constant per mile of the circuit without the loading coils and Z_0 is the characteristic impedance of the non-loaded line.

The impedance of a line that is loaded with coils evidently varies, depending upon the portion of the loading section that is entered. At mid-section, *i.e.* half-way between loading coils, the characteristic impedance is

$$Z_L''=Z_0\sqrt{\frac{2Z_0+Z_C\coth(lP/2)}{2Z_0+Z_C\tanh(lP/2)}}=Z_0\coth\frac{lP}{2}\tanh\frac{lP'}{2}.$$

The characteristic impedance at mid-load or mid-coil (*i.e.* when the line commences with a loading coil of half the normal inductance) is:

$$Z_L'=\sqrt{\left(Z_0+\frac{Z_C}{2}\tanh\frac{lP}{2}\right)\left(Z_0+\frac{Z_C}{2}\coth\frac{lP}{2}\right)}$$

$$=Z_0\frac{\sinh lP'}{\sinh lP}.$$

§ (25) Loading Coils.—"Loading coils" consist of coils wound on a core in the shape of an annular ring. Cores are usually made up of a very large number of turns of fine iron wire, or of fine iron dust mixed with a binder and pressed into ring-shaped form. Coils which are used to load the side circuits of a phantom circuit have two balanced windings, one of which is placed in series with each wire of the circuit. These windings are connected in such a way that currents flowing down one line wire and back over the other will tend to build up flux in the core of the coil. Coils which are used for phantom loading have four balanced windings, one of which is put in each of the four wires of the phantom circuit. These windings are so connected that currents flowing in the side circuits will not build up any flux in the core of the coil. On the other hand, currents flowing through phantom circuit will aid each other in building up flux in the core of the phantom coil. Consequently, the side circuit coil loads or offers inductance to the side circuit and simply adds a small D.C. resistance to the phantom circuit. Similarly, the phantom circuit coil adds inductance to the phantom circuit but offers only D.C. resistance to the side circuits. Other things being equal, a loading coil which has at telephonic frequencies a higher time constant than another loading coil is the more efficient of the two.

Loading coils are encased in iron cases and are mounted in manholes—if the circuit which is loaded is a cable circuit—or are mounted at the cross-arms of poles if the circuit which is loaded is an open-wire line.

§ (26) Continuous Loading. — Continuous loading or the introduction of uniformly distributed inductance in a telephone circuit has been confined largely to submarine cables. This is due to the fact that while continuously loaded circuits give theoretically a more uniform frequency-attenuation characteristic, they do not, in general for a given cost, give as low an attenuation constant over the telephonic range of frequencies as do circuits which are loaded with coils.

The method usually employed for continuously loading a circuit is that commonly known as the Krarup method, and consists in winding wire or tape of iron or other magnetic material around the conductor to be loaded. The objections to continuous loading by such methods are (1) its excessive cost, (2) the relatively small amount of loading (ordinarily less than ·02 henry per mile) that can be introduced into the circuit, (3) the relatively large increase in effective resistance accompanying such loading, (4) the difficulty of predicting the constants and hence the efficiency of such a circuit,—small differences in mechanical treatment or pressure between the tape and the conductor making a large difference in the A.C. constants of the circuit. This last objection is particularly serious on repeatered circuits where a definite and smooth impedance-frequency characteristic is necessary.

Continuous loading is frequently used on submarine cables for the reason that in deep water the difficulty of making water-tight joints at the loading points makes coil loading undesirable. Moreover, repairs on a deep-sea cable with lumped loading introduce irregularities which are practically unavoidable and which may be large, so that for such cables continuous loading is preferred for uniformity also.

When a circuit is loaded its inductive reactance (ωL) is usually large as compared with its resistance (R) and the susceptance (ωC) is large as compared with the leakance (G). Under these conditions the formula for the attenuation constant as given in § (23) reduces to

$$a=\left(\frac{R}{2}+\frac{GL}{2C}\right)\sqrt{\frac{C}{L}}=\left(\frac{R}{2L}+\frac{G}{2C}\right)\sqrt{LC}.$$

When $L/R=C/G$ the attenuation constant is a minimum—assuming L to be the variable—the propagation constant then being:

$$P=a+jb=\sqrt{RG}+j\omega\sqrt{LC}.$$

Such a circuit is said to be distortionless since the attenuation constant is independent of the frequency.

The velocity of the wave propagation is in general $V=\omega/b$ which in the above case of a distortionless line becomes $V=1/\sqrt{LC}$.

§ (27) Phantom Circuits.—Ordinarily only one telephone circuit is obtainable over each pair of wires. If, however, four wires run between two points, three telephone circuits may be obtained over these wires. Two of these telephone circuits are called "Side Circuits" and the other one a "Phantom Circuit."

The operation of such a circuit (*Fig.* 21) is as follows: Current flows from one wire (C) of the

phantom circuit to the midpoint (P_1) of the balanced winding of the Phantom Repeating Coil No. 1. If this point P_1 is at the electrical

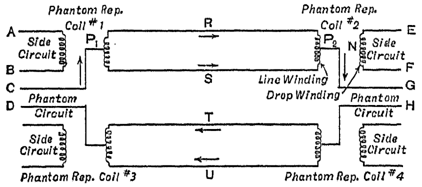


FIG. 21.

centre of the Line Winding of the repeating coil and if the line wires R and S are identically alike—*i.e.* have the same impedances—the current will split equally between the two wires R and S. At the distant end of the circuit, if P_2 is at the electrical centre of phantom repeating coil No. 2, the currents will pass in equal amounts through the windings L and M and out on the phantom wire G. The return of the phantom current from wire H to wire D is by an identical process.

The phantom current will evidently not produce any interference with the side circuit, provided the former passes equally through the balanced windings L and M, etc., as under this condition no flux is produced in the core of the repeating coil and hence no electromotive force from current flowing in the phantom circuit is induced in the side circuit. Any unbalance in impedance between the line wires R and S or T and U, or any unbalance in the balanced windings of the phantom repeating coils will unbalance the phantom circuit and tend to produce cross-talk between the phantom and the side circuits.

The "phantoming of a phantom circuit" or a "double phantom"—which is evidently possible theoretically—is usually found undesirable on account of the difficulties in properly balancing all of the eight wires.

A cable which is designed for phantom working is called a "quadded cable." The two wires of the side circuits are first twisted together into pairs—one pair being given one length of twist and the other pair a different length of twist. These pairs, in groups of two, are then twisted into quads, with a still different length of twist. Such processes

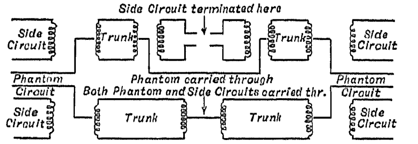


FIG. 22.

to secure symmetry are necessary—especially with loaded cables where the tendency to cross-talk is greater than with non-loaded cables.

It is sometimes desirable to take one or both of the side circuits out at an intermediate point. The circuit in *Fig.* 22 shows one side circuit taken out at an intermediate point and the other side circuit carried through.

The method of loading a phantom circuit is shown schematically below :

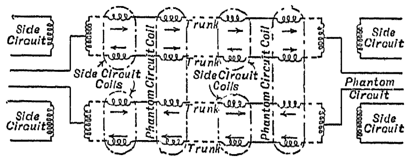


FIG. 23.

As explained under § (25), the side-circuit loading coils add inductance only to the side circuit and the phantom coils only "load" the phantom circuit. The D.C. resistance of the phantom coils, however, enters into the side circuit, and hence the efficiency of a side circuit is not quite as high as that of a similar circuit which is not phantomed.

§ (28) REPEATERS.—A telephone repeater is a device for amplifying voice currents. In its usual form it consists essentially of a repeater element for amplifying the voice currents and a repeater circuit which automatically enables this amplification to be accomplished in both directions over a telephone circuit.

Its use enables telephone communication to be carried on over distances for which the costs of efficient line conductors would otherwise be prohibitive, and also permits of more economical circuits for more moderate distances.

(i.) *Description of Elements.* — All forms of repeater elements which have so far been adapted to commercial requirements are of the unilateral mutual impedance type. In this type a current flowing in the input circuit induces a potential in the output circuit, but a current flowing in the output circuit induces no potential in the input circuit.

(ii.) *Requirements for an Ideal Repeater Element.*—1. The ratio of output to input power must be independent of both frequency and magnitude.

2. The input and output impedances must also be independent of the magnitude or frequency of the impressed power.

3. The element must give a large amplification. It is ordinarily desirable to have a power amplification for speech, frequency currents of 100 times or more, although for certain uses elements giving smaller amplifications might be employed.

4. The local sources of energy must not involve potentials unsafe for use in the telephone plant.

5. The element must be insensitive to external electrical disturbances and should not produce external fields which will cause cross-talk with adjacent circuits.

6. The element should be of long life and so constant and reliable in service as not to demand exceptional maintenance service.

7. In size, first cost, cost of power and of maintenance, the element must conform to the economic conditions of an established plant.

(iii.) *The Mechanical Repeater Element.*—The first repeater element to go into commercial service was the mechanical type, consisting essentially of a telephone receiver efficiently coupled to a sensitive microphone supplied with direct current in the usual manner. The weak incoming currents thus are able to control a much larger local source of energy and the element sends out the received currents with renewed strength.

The chief bar to its more extensive use is its failure (due to initial friction and fixed losses) to respond to inputs below a critical magnitude. The tendency of microphone buttons to "breathe" due to heating and packing and the distortion inherent in carbon buttons make even the best-designed of such devices difficult to maintain when several are operated in tandem.

(iv.) *Vacuum Tube Type of Element.* — The thermionic[1] vacuum repeater element, or "audion," as it is called, consists of an evacuated vessel containing three electrodes, from one of which, the filament, a thermionic emission of electrons is obtainable. The filament is heated by the passage of a current as shown in *Fig.* 24. The other two electrodes are a plate and a grid. If a battery is connected to the filament and plate, as shown, so as to make the latter positive with respect to the former, then a current will flow in the circuit so formed. The electrons emitted at the filament are drawn across the intervening vacuum by the electric field which the battery B in the plate circuit establishes. If an electromotive force is now applied between the grid and the filament as by the source marked V in the figure, the field between plate and filament is altered and the current in the plate circuit is correspondingly altered. If the grid is made positive with respect to the filament more electrons are urged across the space between grid and filament. While some of these electrons strike the grid, and thus result in a current in that circuit, by far the greater number continue through the meshes of the grid to the plate. The result is an increased current in the plate circuit. Conversely, if the grid is made negative there results a decrease in the plate current. In this case, however, no current flows in the grid circuit because electrons can be drawn to an electrode only if it is positive with respect to the source of the electrons.

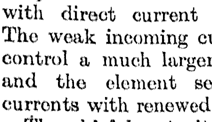


FIG. 24.

The characteristic relation between the grid voltage V and the plate current I_B is that shown below in *Fig.* 25, A.

If the plate voltage is altered the form of the curve is not altered, but the magnitude of the current is changed as illustrated in *Fig.* 25, B, which shows a family of such characteristics. It is evident from this figure that the number of negative volts which must be applied to the grid in order to reduce the plate current to zero is always the same fraction of the volts applied in the plate circuit. Hence it appears that the current in the plate circuit may be altered either by altering the voltage there applied or by a much smaller alteration of the voltage applied to the grid circuit. The device thus gives a voltage amplification.

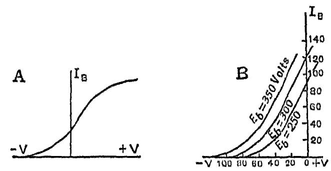


FIG. 25.

As long as the grid is kept negative no current can flow in that circuit, and any alterations in its voltage are unaccompanied by any current variation and hence are entirely wattless. Such variations are accompanied by current variations in the plate circuit and result in an energy expenditure in that circuit. The telephone efficiency is thus seen to be practically infinite, since an energy output may be obtained by a wattless variation of the input voltage. The practical limitations to a complete realisation of this ideal efficiency lie in the design of voltage transformers and the possession by the tube of a finite leakage conductance and capacity reactance. A discussion of the action of the vacuum tube element is given in a paper entitled "Theory of the Thermionic Amplifier," by H. J. van der Bijl in the *Physical Review* of September 1918.[2]

For use in amplifying telephone currents the voltage source V is replaced by an input transformer whose primary is connected to the source of speech. The galvanometer is replaced by the primary of an output transformer whose secondary is connected to a line or the receiving device.

§ (29) DESCRIPTION OF REPEATER CIRCUITS. (i.) *One-way Repeater Circuit.* — If one telephone line is connected to the input of a

[1] See article "Thermionic Valves."

[2] See also *The Thermionic Vacuum Tube*, H. J. van der Bijl, McGraw-Hill Book Company, 1920.

repeater element of the unilateral mutual impedance type and another line to its output, we have a repeater circuit capable of transmitting and amplifying speech in one direction only, or a one-way repeater circuit.

(ii.) *The "21" Repeater Circuit.* — Unless two pairs of wires (or two channels) are run to every subscriber, it is necessary that repeating systems give amplification in both directions. All commercially operative forms of such circuits depend on the Wheatstone bridge arrangement, which is also shared in principle by duplex telegraph and invariable anti-side tone sub-station circuits. The simplest of these circuits is the two-way — one element or "21" repeater circuit, illustrated below:

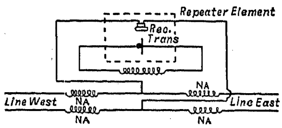


FIG. 26.

In this figure the receiver and transmitter correspond to the input and output of any type of repeater element. This circuit has many interesting properties, of which the one most fundamental to two-way repeating is that if the lines west and east are of equal impedance (both as to angle and magnitude) the output from T produces no input into R.

It follows directly that energy coming in on either line will in part go into R, which, by controlling T, sends out amplified energy to both lines, giving us the desired function of two-way amplification. If the two lines are not equal in impedance, that is, if the repeater is unbalanced, the outgoing energy does produce a potential across R, and if the amplification is great enough a circulating current is set up and the repeater is said to "sing." The amount of amplification which any "21" type repeater can introduce between two lines is a function simply of the vector ratio of the line impedances of the two circuits. It is therefore not possible to compensate for singing by any arbitrary modifications of the repeater circuit itself.

(iii.) *The "22" Repeater Circuit.* — For more general use a combination of two "21" repeater circuits is often necessary. This is called a two-way—two element or a "22" repeater circuit.

The path of energy travelling from west to east may readily be traced by the solid arrows and reversely by the dotted arrows, *Fig.* 27.

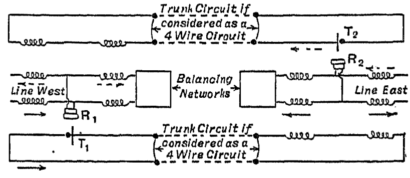


FIG. 27.

Singing is prevented by balancing the lines in each half of the circuit with specially designed balancing networks. With very high grade lines these balances can be maintained so as to permit of an energy amplification of 100 times. This corresponds to 21 miles of standard cable at 800 cycles. It requires that, at all frequencies of efficient amplification, the line impedances do not depart from those of their networks by more than approximately 10 per cent. Actual amplifications must be appreciably less than that which will actually produce singing, or a decided impairment of telephonic quality will result from a transient circulating current.

A typical "22" repeater circuit using vacuum tube repeater elements is shown below (*Fig.* 28):

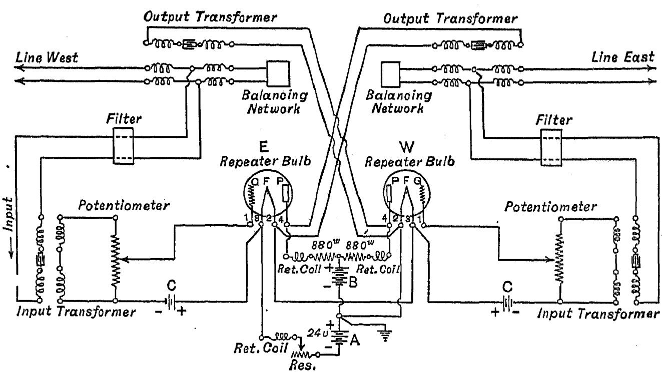


FIG. 28.

A comparison of the "21" and "22" type repeaters will establish the following points:

1. The "21" type repeater requires only half the power and less than half the apparatus and space needed by the "22" type repeater.

2. The "22" type repeater may be so equipped as to balancing networks that it will operate satisfactorily between any types of lines.

The "21" type repeater requires additional apparatus in order to permit it to be used between lines of dissimilar character.

3. The "21" type repeater is not suited for tandem operation, as it feeds part of the amplified energy back on the incoming line. Two adjacent repeaters would therefore produce "echo" effects.

4. The "21" type repeater is largely limited to cases where a single repeater is needed near the centre of a uniform trunk circuit of medium length.

5. With lines of a given degree of uniformity, approximately 3 or 4 miles more amplification may be obtained with a "22" circuit than can be secured with a "21" circuit.

(iv.) *Four-wire Repeater Circuits.*—Imagine the dotted lines in the schematic diagram of the "22" type of repeater (*Fig.* 27) to be telephone lines hundreds of miles in length. If the repeater elements make up the loss in this line and a little more, it is evident that amplification may still be introduced between the lines east and west. Such a system is called a four-wire repeater circuit. For cross-talk reasons it is not desirable to produce all of the amplification with the two terminal amplifiers, but to insert a number of one-way repeaters at intervals in each of the two pairs of the four-wire circuit.

Such a system may contain hundreds of miles of small-gauge cable with one-way repeaters introducing energy amplifications of roughly 1000 times (30 miles) at uniformly spaced intervals. Although circuits of this type use twice as many wires as other systems, they use smaller wires, and for long-distance cable operation there is a certain distance above which they are economically advantageous.

(v.) *Application of Repeaters.*—The successful application of repeaters to the telephone plant has completely revolutionised the methods of long-distance telephony. It is in most cases more economical to use repeaters than to load the heavier gauges of open-wire line. Furthermore, connections which formerly required open-wire conductors of the largest gauge may now often best be handled over smaller conductors of loaded cable. The present outlook forecasts great networks of loaded small-gauge cables equipped with repeaters. The use of heavy-gauge conductors in cables is largely a thing of the past.

On the other hand, the repeater is not a device which can be thrown in indiscriminately on any circuit and be expected to function properly. Circuits of uniform character are required, as is shown by a consideration of the requirements imposed by the balance conditions. Loading coils must possess magnetic stability as well as high inductance and low resistance, and they must be quite uniformly spaced in the circuit. In general repeatered telephone circuits call for a higher grade of plant maintenance and for special testing methods and equipment.

§ (30) Simultaneous Use of Circuits for Telephone and Telegraph Service.—The simultaneous operation of both telephone and telegraph services over the same conductors is generally accomplished by one of two well-known methods. The simpler of these is known as "simplexing." Fundamentally this scheme involves the complete balance of the telegraph circuit with respect to the telephone circuit and *vice versa*. Telegraph currents may then pass through the line wires in parallel by way of the telegraph branch without interfering with the voice currents which have their source in the telephone branches and which pass through the line wires in series. The return circuit for the telegraph may obviously be either the earth or another simplexed circuit. Differences of potential in the telegraph circuit thus have no tendency to produce currents in the telephone circuit and *vice versa*, and the frequency bands used in the telephone and telegraph circuits may overlap if desirable. The second of the customary methods of obtaining simultaneous telephone and telegraph operation is known as "compositing." This method involves no mutual balance of the telephone and telegraph circuits, but necessitates the use in the two systems of frequency bands which do not overlap. It involves fundamentally the arrangement of the several branches of the resulting circuit in such a way that the telephone branch or branches will select, and so far as possible, will admit to or from the line only those frequencies necessary to telephone conversation of good quality, while the telegraph branch or branches will in a corresponding manner select and admit to or from the line only those frequencies necessary to satisfactory telegraph operation.

§ (31) Simplex Sets.—A simplex set consists essentially of a phantom repeating coil—the line portion of the telegraph circuit or channel being the same as that used for one side of a phantom telephone circuit (*Fig.* 29).

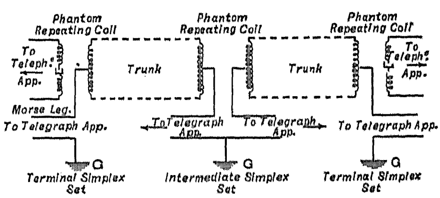


FIG. 29.

The return portion of the telegraph circuit is ordinarily through the ground. If the line wires and line windings of the phantom repeating coils are properly balanced, it is evident that the telephone and telegraph currents will pass over the line simultaneously

without mutual interference. The use of intermediate simplex sets makes it possible to introduce intermediate telegraph stations without interfering with through telephone service, excepting in so far as transmission and signalling losses are caused by the additional coils required.

§ (32) Composite Sets.—A composite set is an apparatus arrangement whereby the telegraph currents which involve frequencies from approximately zero to 100 cycles are separated from the voice currents which involve higher frequencies.

The use of simplex sets makes it impossible to use simultaneously a phantom telephone circuit, and only gives one telegraph channel for each pair of wires.

By means of composite sets, a pair of wires normally used as a telephone line may be divided at each terminal or intermediate station so that each wire of the pair may be used as an independent telegraph circuit at the same time that the pair is used as a telephone circuit. The branches of the composite sets which are connected to the telephone apparatus are known as telephone branches, and those connected to the telegraph apparatus are called telegraph branches or Morse Legs.

A schematic diagram of a typical composite set is shown below:

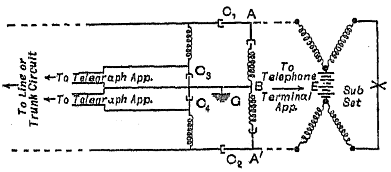


Fig. 30.

Composite sets are so designed as to reduce to a minimum any possible interference between the telephone and telegraph systems due to their simultaneous operation in their respective branches. The impedance of the telegraph apparatus and that of the coil winding in series with the telegraph branch, together with the capacity of the intervening grounded condenser connected across each telegraph branch, serve to reduce the suddenness in changes of potential, due to the operation of the telegraph apparatus, at the point where each telegraph branch joins the line, thereby practically eliminating the noise known as " Morse thump " which the operation of the telegraph apparatus has a tendency to produce in the telephone circuit. The possible tendency to unbalance the circuit due to differences in impedance between the telegraph circuits which may be connected to the two telegraph branches is overcome by the condensers (C_3 and C_4) connected across these branches. These condensers serve to maintain the impedance of the telegraph branches, to telephone currents, at practically a constant value, regardless of differences in the condition of the telegraph circuits which may be connected to the two branches.

In order to reduce the momentary impulses known as " cross-fire " which pass from one telegraph branch to another through the telephone branches and to reduce the effect of these impulses upon the signalling apparatus, each telephone branch of the composite set is connected directly to ground through a path A – B (or A′ – B) which has a low impedance to such impulses. The condensers in these grounded branches and in the telephone branches (C_1 and C_2) prevent interference with the proper operation of direct-current supervisory signals in cord circuits or trunks which may be connected to the line.

In order to eliminate cross-talk between phantom circuits and their side circuits when composited, it is essential that the composite sets should not introduce any capacity or inductance unbalance into the side circuits with which they are associated. To prevent this the condensers C_1 and C_2 are especially selected with a view to avoiding wide variations in their capacity and the windings of the associated coils are closely balanced electrically.

Either a side circuit or a phantom circuit may be composited. To avoid unbalancing the phantom telephone circuit, however, both side circuits must be composited, even if there is a demand for but one or two of the four telegraph circuits which are thus rendered available.

§ (33) Signalling on Long-distance Lines.—On ordinary local lines the frequency of the signalling currents used is about 16 to 20 cycles per second. At each local office a ringing generator of this frequency is provided. The application of signalling current from this source to subscriber's lines and junction circuits by means of the operator's ringing-key has already been referred to in describing the arrangements in the manual exchange. This same frequency is often used to transmit the ring or signal over long-distance trunk lines.

On very long trunk lines where the circuits are often phantomed and composited for simultaneous telegraph working, it is usually desirable to relay the ringing current at each end of the trunk and use a higher frequency signalling current over the trunk line. The frequency chosen should be high enough to be transmitted through telephone transformers efficiently and high enough to avoid interference with telegraph signals. A frequency of 135 cycles is commonly used.

Composite Ringer Sets.—The sets employed at trunk offices for relaying the ringing currents at the terminals of the trunk are called composite ringer sets. Such a set usually consists of two circuits with relays, bridged across the terminal, one sensitive to 16-cycle currents, the other to 135-cycle currents. A source of 16-cycle current and one of 135-cycle current are

also provided at each trunk office. When a 16-cycle signal comes in from one of the local offices the 16-cycle relay of the composite ringer set responds, and through the agency of other co-acting relays the 135-cycle generator is applied to the sending end of the trunk. At the distant terminal the 135-cycle relay of the composite ringer set responds, and through the agency of co-acting relays the 16-cycle generator at that terminal is applied to the junction line leading to the circuit of the distant subscriber.

§ (34) High-frequency Carrier-current Telephony and Telegraphy. — Recent developments in multiplex telephony and telegraphy have greatly increased the message-carrying capacity of long-distance telephone lines. Several telephone conversations or telegraph messages over one pair of wires are simultaneously transmitted in addition to the telephone conversation and telegraph messages provided by the ordinary methods. There is no interference between these various messages, and the subscriber is not aware that the line is being used by other subscribers at the same time. Due to the complexity and expense of the apparatus required for a system of this type, it is not practical to equip lines which are less than 150 or 200 miles in length.

In high-frequency multiplex transmission the voice current or telegraph signals are superimposed on a high-frequency current which carries them to the other end of the line. These high-frequency currents, commonly called carrier currents, are sustained oscillations generated by vacuum tubes. The carrier current and the voice current, or the telegraph signals, are impressed on a device known as a modulator. This may be a vacuum tube or any other electrical device which has a non-linear relation between input voltage and output current. In the process of modulation the band of frequencies representing the voice combines with the carrier current in such a way that the entire band is shifted upward in the frequency scale to a position adjacent to the carrier frequency. In a multiplex system the voice current or telegraph signal for each channel modulates a carrier current of different frequency. Each carrier frequency, then, represents an individual circuit or channel. These bands of high-frequency currents, allocated in different parts of the carrier frequency range, are then impressed on a telephone line and transmitted to the distant station. Here they enter a system of selective networks more commonly called wave filters. Each wave filter is so designed that it admits only that band of high-frequency currents representing a given channel. After the high-frequency bands have thus been separated, each one is again impressed on a detector or demodulator, whereby the original band of voice currents or telegraph signals are restored and transmitted to the subscriber in the usual way.

The frequency range used for carrier transmission extends approximately from 3000 to 30,000 cycles. The lower limit is determined by the fact that the ordinary telephone conversation transmitted over the line employs frequencies up to about 2500 cycles. The range between 2500 and 3000 cycles is used for effecting complete separation between the ordinary voice channel and the carrier channels. The upper frequency limit is determined largely by the increased attenuation of the line and by the transposition requirements necessary to prevent cross-talk or interference at high frequencies. In this frequency range as many as four two-way telephone channels or ten duplex telegraph channels have been obtained on one pair of wires.

Carrier systems are best adapted for operation over open wire non-loaded lines. Means have not yet been developed such that cables or ordinary loaded circuits can be made suitable for carrier transmission, the attenuation and cross-talk being too great for currents of high frequency. Phantom circuits are also considered unsuitable for carrier systems due to the extreme difficulty of maintaining a sufficient degree of balance between phantom and side circuits at high frequencies.

§ (35) Foreign Inductive Interference and Cross-talk.—Foreign inductive interference is a disturbance induced into communication circuits from neighbouring power lines or sources of electrical energy. Cross-talk is interference between adjacent telephone circuits due to the transmission of speech energy from one circuit to another. This interference is also largely of an inductive character.

Inductive disturbances and cross-talk are both the results of two different phenomena, electromagnetic and electrostatic induction. Both effects may be present in a circuit and may be nearly equal in their effects, although it is more usual to find one effect stronger than the other.

(i.) *Electromagnetic Induction.* — When a current flows in a conductor, there is set up a magnetic flux or field in the region surrounding the conductor, the strength of this field being directly proportional to the magnitude of the current flowing in the conductor. If a second conductor is placed parallel to the energised circuit, every change in the field of the first conductor will cause the field to cut across the second conductor and thereby produce a voltage in the latter. The magnitude of this voltage will not only be proportional to the current flowing in the energising conductor, but will also be directly proportional to the frequency of the

current and the length of the parallel or exposure of the two circuits.

The effect of electromagnetic induction may be seen by considering two grounded circuits which parallel each other for a short distance. If the current flowing in and the voltage to ground in a short element of one circuit be represented by I and E respectively, the induced voltage which this element of circuit No. 1 produces in circuit No. 2 is

$$V_2 = -j\omega(lM)I,$$

where ω is 2π times the frequency, l is the length of the element of the circuit considered, and M is the mutual inductance between the circuits per unit length.

In the case of a perfectly conducting earth bounded by a plane, M can be computed approximately from the formula:

$$M = \cdot 0003706 \log_{10} \frac{D^2 + 4H_1H_2}{D^2},$$

in which M is the mutual inductance, expressed in henrys per mile, H_1 and H_2 are the heights of the two wires above the conducting ground, and D is the distance between the centres of the two wires. Under actual conditions the mutual inductance between such circuits is appreciably affected by the fact that the earth is of finite conductivity. This is usually taken into account by assuming that the plane from which H_1 and H_2 are measured is situated at a distance below the actual surface of the earth. This distance is different for different localities. 300 to 500 feet has been found to be a proper assumption in certain cases. In others it may be as great as 2000 feet.

(ii.) *Electrostatic Induction.* — Considered electrically, any two wires which are supported in the air and insulated from each other and the earth may be regarded as two plates of a condenser — the air being the dielectric or insulating medium. Similarly each wire forms one side of a condenser, of which the ground forms the other side. When, therefore, an electromotive force is applied to one of several wires—as is shown in the figure below—the difference in potential between that wire and ground will cause an electrostatic charge to be induced on other adjacent wires. When the source of energy changes, or reverses as in the case of an alternating current, the charges induced on the other wires will change accordingly. It is the flow of current resulting from these changes in static charges that gives rise to the so-called electrostatic disturbances.

The electrostatic charge on a conductor varies directly with the voltage applied to the disturbing wire. The potential of the charge depends on the ratio of the wire's respective capacities to the disturbing wire and to earth. The magnitude of the electrostatic disturbance, however, depends on this potential and also on the magnitude of these capacities. Consequently, the static potential induced in a conductor is independent of the length of the exposure, assuming perfect insulation, while the charging currents, and therefore the disturbances, vary directly with the length of the exposure. Static disturbances, *i.e.* the currents flowing in the exposed circuit, also vary in direct proportion to their frequency.

Consider the effect of electrostatic induction in the case of the two grounded circuits as discussed in (i.) of this section.

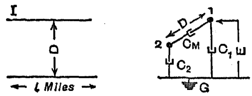


FIG. 31.

The electrostatic voltage induced in a short section of wire 2 (*Fig.* 31), due to the voltage (E) between wire 1 and ground, is

$$e_2 = E\frac{C_M}{C_2 + C_M},$$

in which C_M and C_2 are the direct capacities from wire 2 to wire 1 and to the ground respectively, *i.e.* they are the capacities measured by the charges on wire 1 and on the earth when wire 1 is connected to the earth and unit potential is applied between wire 2 and the combination of wire 1 and earth.

If a short section of wire 2 is connected to earth, the current flowing to ground is

$$i_2 = j\omega L(C_2 + C_M)e_2.$$

§ (36) METHODS OF PREVENTING CROSS-TALK. —In order to prevent cross-talk between open-wire circuits it is usually necessary to transpose the wires of each circuit at frequent intervals. A transposition consists in interchanging the pin positions occupied by the two sides of a given circuit. Induction into equal sections of line on the two sides of the transposition from an untransposed paralleling circuit will be approximately equal and opposite. The interval between successive transpositions in the same circuit is usually not less than 1300 feet, nor greater than two miles.

Where phantom circuits are used it is necessary to transpose these circuits as well as the two-wire circuits. The addition of phantom transpositions to an existing line ordinarily requires rearrangement of the transpositions in the two-wire circuits, since the relative positions of these circuits are changed by the phantom transpositions.

Cross-talk in cable circuits is largely due to electrostatic induction. In a given length of cable electrostatic coupling, usually called

"capacity unbalance," exists between various combinations of cable circuits. The two sides of a given circuit are twisted together to minimise this coupling, and the residual effect is largely due to irregularities in the twist or in the wires. Quadded cables, *i.e.* those suitable for phantom working, have the two pairs forming a phantom twisted together as well as the two wires of each pair twisted together. Four wires twisted in this manner are called a "quad." In splicing cable lengths together it is, of course, necessary to keep the quad construction intact at the splices. This results in the two pairs in a given quad being adjacent in all lengths, and, therefore, there is much more probability of cross-talk between the two pairs in a quad or between a phantom and its side circuits than between circuits of different quads, since the latter are only occasionally adjacent. For this reason, and because the "phantom-to-side" capacity unbalance is usually somewhat higher than the unbalance between other combinations of circuits, it is often desirable to make measurements of the capacity unbalance between circuits in the same quad when installing the cable and to connect successive lengths of cable together in such a way as to minimise the total unbalances (between circuits in the same quad) in several lengths after they are spliced together. A capacity unbalance may be considered as an admittance shunted between two circuits. For the given amount of energy transmitted, the voltage impressed across such an admittance, however, varies directly with the impedance of the circuit. Loaded circuits have higher impedance than non-loaded circuits, and for this reason the cross-talk for a given unbalance is more serious with the former type of circuit.

§ (37) Methods of preventing and overcoming Foreign Inductive Disturbances.—Distance is the surest preventative of induction troubles, and in the construction of either a new power line or a new telephone line consideration should always be given to the avoidance of parallels.

Balance of both telephone and power circuits is important, and where there is exposure of the former to the latter, measures to maintain the balance of both in good condition should be taken wherever possible. Electrified railways are examples of power circuits which are inherently unbalanced to ground. Certain types of power distribution circuits are also unbalanced; for example, in the three-phase four-wire system it is not uncommon to find single-phase connections. The unbalance existing in such connections, as well as the effect of this unbalance on the balance of the entire system, can be very greatly reduced by isolating the connected single-phase circuit by means of a transformer. The balance of three-phase circuits can in general be improved by transpositions, which tend to equalise the capacities of the three conductors to ground. Unbalance due to poor maintenance, such as contact with the limbs of trees or defective insulators, whether in the telephone or in the power circuits, contributes largely to noise induction; series unbalances in telephone circuits also produce important effects in some cases.

The use of co-ordinated transposition systems to reduce noise induction is of wide application. Power transpositions reduce the induction in exposed circuits due to the balanced components of the power currents and voltages, while telephone transpositions tend to equalise the effects in the two sides of the line due to induction from any extraneous sources. It is obvious that a co-ordination of the two systems of transposition, taking into account also the irregularities and discontinuities in the parallel, is essential if the greatest benefit is to be obtained.

Where noise in the telephone circuit is due chiefly to a single fixed frequency in the power circuit, it can usually be greatly reduced by the application of a shunt to the source of power, the shunt being designed to resonate at the frequency in question. This device has been applied to reduce induction from direct current railroads and also from relatively low voltage A.C. circuits. Resonant shunts have also been applied to telegraph circuits to reduce interference from 25-cycle railways. Drainage connections to ground embodying the same idea may also be applied to telephone circuits to get rid of low-frequency induction. In this case each wire is drained to ground through a circuit resonating at the disturbing frequency. Unless carefully balanced, however, such drainage connections are likely to make the telephone circuits noisy. They also cause transmission loss, and owing to the resonance conditions, both the coil and the condenser must usually be designed to withstand high voltages. For these reasons they are not adapted for use except in special cases where only a small number of circuits have to be taken care of.

Where noise is produced by the commutators of motors on electric locomotives or cars, some degree of relief can frequently be obtained by the use of series reactance in the connections to the motors.

In the compensating or neutralising transformer, which is sometimes useful in reducing electromagnetic disturbances, the wires to be relieved are connected to a number of windings, one for each wire, upon a single magnetic circuit. As many of the wires as may be required (depending upon the conductance needed) are used as primaries, and connections to ground are made at the ends of the circuit

or outside the parallel. The remaining circuits, which may be called "secondaries," are not connected to ground. The voltage directly induced in the primary wires by the paralleling power or railway circuit causes current to flow through these wires to ground, and this current induces voltage in the primaries and in the secondaries which balances to a considerable extent the voltage induced by the interfering power circuit. The efficiency of the device is limited by various practical considerations, the maximum efficiency being perhaps 90 per cent. The primaries cannot be used for grounded telegraph, and appreciable transmission losses are introduced in the secondary circuits.

Where induction is produced by A.C. railways, booster transformers spaced at suitable intervals may be used to reduce it. These transformers have substantially a one to one ratio with the primary connected in the trolley wire and the secondary connected across a joint in the tracks or in a return feeder, the latter being preferable. The effect of the transformers in either case is to tend to keep the return current out of the earth and thus to reduce induction.

Return feeders are, of course, applicable with either A.C. or D.C. railways.

With high-voltage direct-current electrifications, a quick-acting circuit breaker has been found to be of decided benefit in reducing the severity of inductive effects. Breakers of this type, capable of interrupting a short-circuit current in an interval of time of the order of 0·01 or 0·02 second, are in use in the sub-stations of an important direct-current railway system in America.

§ (38) TRANSMISSION COMPUTATIONS—TRANSITION LOSSES. (i.) *Losses due to any Apparatus inserted in a Circuit.* — The problem of finding the transmission loss due to a piece of apparatus inserted in or bridged across a telephone line is the problem of finding out how the current entering the receiving side is altered by the inserted or bridged apparatus. This "apparatus" may consist of series or bridged impedances, transformers, lengths of cable or open-wire lines, etc.

There are in general four terminals to a piece of inserted apparatus, *i.e.* two connected to the telephone line going in one direction and two connected to the line going in the other direction. A simple bridged impedance may be considered as having four terminals by including with it a foot of line on each side of it. The above statement of four terminals assumes the usual condition of the two sides of the circuit and also of the inserted apparatus being nearly symmetrical, so that the presence of the ground or other neighbouring bodies need not be considered. If this condition does not hold, the apparatus has another terminal to be considered, *i.e.* the ground.

If we are interested only in knowing the ratio of the current in a circuit before and after inserting any apparatus, it is simply necessary to take the line at the place where the apparatus is to be inserted, find the impedance of the line in both directions, and then assume that a constant E.M.F. is acting *through* one impedance, thus causing a current to flow in it and in the second impedance, and then compute how the current flowing in the second impedance is altered by the insertion of the given apparatus. The ratio of the currents with and without the apparatus will give a factor K, which in turn may be translated into miles of standard cable as described below. The only assumption made in the above is that the change produced in the magnitude of the currents by the inserted apparatus does not affect the constants of the line or the magnitude of the E.M.F. acting in the first impedance.

If the piece of apparatus changes the current by the factor K, and l_s is the loss expressed in miles of standard cable, then $K = e^{-l_s a}$ or

$$l_s = \frac{2 \cdot 3026 \log_{10} (1/K)}{a},$$

where a is the attenuation constant per mile of standard cable at the frequency considered. At 796 cycles, ($\omega = 5000$) "a," which is proportional to the square root of the frequency, $= \cdot 109$.

If in any case K comes out larger than unity, it is necessary to take the reciprocal of K before using the above equation, and the "loss" in such a case will actually be a *gain*. That is, in such a case a greater current will flow into the receiving circuit when the apparatus is inserted than when it is omitted, and, consequently, there will be more power dissipated in the listening receiver after inserting the apparatus than there was before. It will be noted that the "loss" simply depends upon the absolute numerical value of K and that the phase difference between the currents does not enter into the problem.

It is usually assumed in rough work that the transmission loss calculated for a frequency of about 800 to 1000 cycles is a proper value to use in estimating the transmission loss with voice currents. This is not a safe assumption, however, when the loss varies widely for different frequencies between 200 and 2000 cycles, as is apt to be the case where the losses under consideration are due to the insertion of condensers or to the omission of loading coils, etc.

(ii.) *Reflection in Telephone Circuits.* — In addition to simple attenuation in telephone circuits there is usually a dissipation of energy due to the reflections which take place wherever there is an irregularity in the impedance due to a transition from one type of line to another, or due to intermediate or terminal apparatus. Let us consider the simple case of a junction between two long circuits, one having the characteristic impedance Z_1, the other Z_2.

Then it can be shown that the current in passing the junction encounters partial reflection in such a manner that the coefficient of the transmitted portion is $2Z_1/(Z_1+Z_2)$, and that of the reflected portion is $(Z_1-Z_2)/(Z_1+Z_2)$, the difference of the two being equal, of course, to the incident current. Similarly the voltage encounters reflection, the transmitted and reflected portions having coefficients $2Z_2/(Z_1+Z_2)$ and $-(Z_1-Z_2)/(Z_1+Z_2)$ respectively.

The reflection loss occurring at such a junction is measured by the ratio of the speech energy transmitted to the distant receiving terminal, to the energy which would be transmitted if the line were all of a single uniform type. The reflection loss can be expressed in standard miles by a proper conversion of the ratio. As stated in (i.) above, it is common practice to make computations in terms of current ratios. This has come about as a matter of convenience, and is justified by the fact that at any point in a long uniform line the current (and voltage) is proportional to the square root of the power transmitted. In case of a transition from a circuit of one characteristic impedance to one of another, it is necessary to consider power or energy ratios. An equivalent current ratio may, however, be employed by taking the magnitude of the square root of the power ratio.

The power ratio for the simple junction Z_1, Z_2, is $4Z_1Z_2/(Z_1+Z_2)^2$, and the equivalent current ratio is

$$K=\left|\frac{I_2'}{I_1'}\right|=\left|\frac{2\sqrt{Z_1Z_2}}{Z_1\underline{|\phi_1}+Z_2\underline{|\phi_2}}\right|,$$

where the bars are used to denote the scalar magnitude of the enclosed complex quantity.

Putting $r=|Z_1/Z_2|$, and $\theta=(\phi_1-\phi_2)$, the reflection loss may be computed in standard miles, for any frequency, from the following formula, which is derived from the one above:

$$l_s=\frac{298}{\sqrt{f}}\log_{10}\left(\frac{1+2r\cos\theta+r^2}{4r}\right).$$

(iii.) *Transition Losses.*—The transition loss at any point in a circuit is defined as the maximum gain by which the circuit could be improved in transmission efficiency by the insertion of a passive network, *i.e.* a network which connects the two parts of the circuit as efficiently as possible without the introduction of energy from an outside source. Such a transition gain might be obtained in a particular case, for example, by the insertion of a transformer in combination with reactance elements.

The transition loss at a simple junction Z_1, Z_2, is expressed by the equivalent current ratio

$$K=\left|\frac{I_2'}{I_1'}\right|=\left|\frac{2\sqrt{Z_1Z_2}\cos\phi_1\cos\phi_2}{Z_1\underline{|\phi_1}+Z_2\underline{|\phi_2}}\right|,$$

and may be computed in standard miles from the following formula:

$$l_s=\frac{298}{\sqrt{f}}\log_{10}\left(\frac{1+2r\cos\theta+r^2}{4r\cos\phi_1\cos\phi_2}\right).$$

While reflection losses are of great interest in the theory of telephone transmission, the conception of transition losses is the one more generally used in computing losses and in devising means for the most efficient connection of lines and apparatus.

(iv.) *Transformer Gains.*—The gain which may be obtained in a telephone circuit by inserting an ideal transformer of the best possible ratio at a junction point is called the transformer gain of the point under consideration. If the impedances looking in the two directions from the point are $Z_1\underline{|\phi_1}$ and $Z_2\underline{|\phi_2}$, the transformer loss, which is the converse of this gain, is

$$K=\left|\frac{I_2'}{I_1'}\right|=\left|\frac{2\sqrt{Z_1Z_2}}{Z_1\underline{|\phi_1}+Z_2\underline{|\phi_2}}\right|\cos\frac{(\phi_1-\phi_2)}{2}.$$

If, as before, $r=|Z_1/Z_2|$, and $\theta=(\phi_1-\phi_2)$, the transformer loss or gain may be computed in standard miles from the formula:

$$l_s=\frac{298}{\sqrt{f}}\log_{10}\left(\frac{1+2r\cos\theta+r^2}{2r(1+\cos\theta)}\right).$$

In order to get a clearer idea of the order of magnitude of the transformer gains for various conditions, we are giving below a table of these values calculated for a frequency of 796 cycles ($\omega=5000$).

Impedance Ratio. "r."	l_s.		
	$\theta=\pm 0°$.	$\theta=\pm 45°$.	$\theta=\pm 90°$.
1·5	·19	·22	·37
2·0	·54	·63	1·02
3·0	1·32	1·52	2·34
4·0	2·05	2·32	3·46
5·0	2·70	3·03	4·38
7·0	3·79	4·21	5·84
10·0	5·08	5·58	7·44
15·0	6·66	7·23	9·26
20·0	7·84	8·43	10·6
25	8·78	9·40	11·6
30	9·55	10·2	12·4
40	10·8	11·5	13·7
50	11·8	12·4	14·7
75	13·6	14·3	16·6
100	14·9	15·6	18·0
200	18·0	18·7	21·1
300	19·8	20·5	23·0
500	22·2	22·9	25·3
1000	25·3	26·0	28·5

An inspection of the foregoing table shows that unless the impedance looking in one direction, from any point in the telephone circuit, is at least twice that of the impedance looking in the other direction, the gain to be obtained by inserting a perfect transformer is not more than ·5 to ·6 mile unless the difference in the phase angles of the impedances is very great. It is, moreover, evidently useless to insert a transformer simply for the purpose of improving the transmission efficiency of a circuit, unless the impedance in one direction is considerably over 50 per cent greater or less than the impedance in the other direction.

As an example of the use of the formulae for transformer gains and transition losses, suppose we have given a telephone circuit, as shown below:

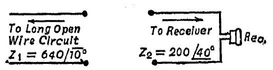


FIG. 32.

and the questions arise as to (1) how much the efficiency of this circuit could be improved if we insert at the point an ideal transformer of the proper ratio, and (2) what is the maximum possible transmission gain which could be obtained in such a circuit by inserting the best possible passive network at the point; in other words, what is the transformer gain and the transition loss at the point under consideration.

Since the impedance looking in one direction is $Z_1 = 640\overline{|10^\circ}$, and the impedance in the other direction is $Z_2 = 200\underline{|40^\circ}$, we have

$$r = \frac{Z_1}{Z_2} = \frac{640\ \overline{|10^\circ}}{200\ \underline{|40^\circ}} = 3{\cdot}2\ \overline{|50^\circ}.$$

Hence "r," which is always taken as the ratio of the higher impedance to that of the lower impedance, is 3·2, $\theta = -50^\circ$, $\phi_1 = -10^\circ$, and $\phi_2 = +40^\circ$.

If we assume a frequency of 796 cycles, we find, by putting those values in the equation for the transformer gain, that the transformer gain is 1·74 miles of standard cable. Similarly we find that the transition loss is 2·13 miles of standard cable.

Consequently, if we enter such a circuit as that shown above, and insert an ideal transformer of the proper ratio, we can increase the efficiency of the circuit (at 796 cycles) by 1·74 miles of standard cable. On the other hand, the absolute maximum gain which we could obtain by inserting the best possible passive network at the junction shown is 2·13 miles of standard cable.

F. B. J.

TEMPERATURE COEFFICIENT OF AMMETERS AND VOLTMETERS. See "Direct Current Indicating Instruments," § (16).

TEMPERATURE EFFECT ON RESISTIVITY OF DIELECTRICS. See "Resistance, Measurement of Insulation," § (1) (iv.).

TEMPERATURE RISE OF TRANSFORMERS. See "Transformers, Static," § (20).

TERMINALS, POTENTIAL, form and adjustment by means of. See "Potentiometer System of Electrical Measurements," § (12).

TERRESTRIAL MAGNETISM, SECULAR VARIATION OF: a change in the direction of the earth's magnetic force at any place, slow but effecting great changes in the course of time. See "Magnetism, Theories of Terrestrial and Solar," § (2).

TEST SPECIMENS, forms of, used in magnetic testing. See "Magnetic Measurements and Properties of Materials," § (17).

TESTING DIELECTRICS:
Bridge Methods. See "Dielectrics," § (11).
Wattmeter Methods. See *ibid.* § (12).

TESTING SETS. See "Direct Current Indicating Instruments," § (20) (iii.).

TESTS OF METERS FOR MEASUREMENT OF ELECTRICAL ENERGY. See "Watt-hour and other Meters for Direct Current. II. Watt-hour Meters," §§ (44), (51).

THERMAL DETECTORS, connections for, when used for wireless telegraphy. See "Wireless Telegraphy," § (20).
Use of, in wireless telegraphy. See *ibid.* § (19).

THERMAL EFFECTS IN INSULATED CABLES

THE results given in the article on "Cables, Insulated Electrical," have been considerably extended by recent work published since that article was completed. A preliminary report on this work was communicated [1] to the Institute of Electrical Engineers in February 1921.

Under the guidance of a representative Committee the experiments were carried out by Mr. Melsom and Miss Cockburn at the National Physical Laboratory, by Prof. Marchant at Liverpool, and Mr. Fawssett at Newcastle-upon-Tyne.

§ (1) NATIONAL PHYSICAL LABORATORY EXPERIMENTS.—At the N.P.L. a large number of cables of modern designs were tested and measurements taken of the current required to raise their temperature 50° Fahr. (= 27·8° C.) under various conditions. In the first series of experiments the cables were laid on the Laboratory floor, the room being maintained at a fairly constant temperature. This enabled the rise of temperature for a number of currents to be measured; it was also possible to determine with accuracy the electric and

[1] *Journal I.E.E.*, 1921, lix. 181.

thermal constants of the cables. The temperature rise for a given current depends partly on the thermal resistivity of the dielectric, and it is possible, if the permissible current corresponding to one value of the resistivity be known, by the aid of the formula given in § (5) of the article on "Cables," to determine its value if the resistivity be altered.

Table I. gives the results of Melsom's work for this series, showing the values of the current required to produce the rise of 50° Fahr. for the actual value of the resistivity and also when the latter is reduced to some standard value.

In these experiments alternating current was used to heat the cables, and a small direct current was superposed for the purpose of measuring the resistance.

The experiments were repeated with the cables buried, and the results are given in Table II., which also indicates the method of laying.

It appears that cables of the class tested will carry some 22 per cent more current when laid solid, or when armoured and laid direct in the ground, than when exposed to the air; if, however, they are drawn into stoneware ducts they carry some 2 to 3 per cent less.

The results for the concentric low-pressure cables are also shown in *Fig.* 1, which gives for comparison a curve drawn from the German tables for similar cables.

Table I

Cables reduced to Two-standard Values of Thermal Resistivity

Cable No.	Sectional Area.		Remarks.	Pressure.	Armoured.	Cables laid in Air. Current to produce a Rise of 50° F. (=27·8° C.).			
	Nominal.	Actual.				K=Actual Value.		K=500.	K=1000.
(1)	(2)		(3)	(4)	(5)	(6)		(7)	(8)
			Single Cables.						
	sq. in.	sq. in.				K	amp.	amp.	amp.
1	0·1	0·094	Single	L.P.	No	1200	190	206	197
2	0·2	0·196	Single	L.P.	No	800	304	353	294
			Concentric Cables.						
3	0·1	0·108	Concentric	L.P.	Yes	620	180	181	165
4	0·1	0·099	Concentric	L.P.	No	1060	159	181	164
5	0·1	0·100	Concentric	L.P.	No	1160	161	186	168
6	0·2	0·192	Concentric	L.P.	No	1060	245	270	247
7	0·2	0·196	Concentric	L.P.	No	1000	243	271	243
8	0·2	0·197	Concentric (jute insulated)	L.P.	(Jute covered)	870	265	286	259
9	0·2	0·205	Concentric	L.P.	Yes	720	268	274	240
10	0·5	0·480	Concentric	L.P.	No	1090	404	445	405
			Three-core Cables.						
11	0·025	0·029	3-core	L.P.	No	1050	66	71	67
12	0·05	0·048	3-core, shaped cores	10,000 V	Yes	730	100	111	90
13	0·1	0·105	3-core	6,000 V	No	720	143	161	132
14	0·1	0·100	3-core, shaped cores	L.P.	No	670	156	159	145
15	0·1	0·101	3-core, shaped cores	6,000 V	Yes	420	158	156	136
16	0·15	0·150	3-core, shaped cores	6,000 V	Yes	500	195	195	167
17	0·15	0·145	3-core	E.H.P.*	Yes	460	208	206	174
18	0·15	0·145	3-core	E.H.P.*	Yes	550	204	211	176
19	0·15	0·147	3-core	E.H.P.*	Yes	470	219	217	178
20	0·15	0·145	3-core	E.H.P.*	Yes	550	208	213	178
21	0·15	0·144	3-core	E.H.P.*	Yes	460	208	204	170
22	0·15	0·145	3-core	E.H.P.*	Yes	570	195	202	170
23	0·1	0·120	6-core } Split conductor system, each core circular	E.H.P.*	Yes	600	284	298	250
24	0·1	0·116	6-core }	E.H.P.*	Yes	600	264	281	238
25	0·2	0·208	3-core, split conductor shaped cores	3,300 V	Yes	650	219	227	206
26	0·2	0·200	3-core, oval concentric split conductor	E.H.P.*	Yes	710	258	278	238
27	0·25	0·250	3-core	10,000 V	Yes	580	255	262	231

* E.H.P. cables insulated as frequently employed for 20,000-volt workings.

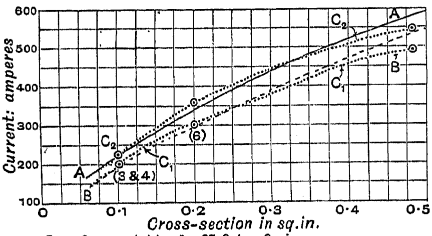
TABLE II

I_1 = Current required to produce a given temperature-rise when tested in air under the conditions stated for Table I.
I_2 = Current required to produce the same temperature-rise when buried.

No.	Cable.	Method of Laying.	I_2.	I_1.	I_2/I_1.
(1)	(2)	(3)	(4)	(5)	(6)
1	0·1 Single	Solid in bitumen	266	216	1·23
2	0·2 Single	Solid in bitumen	432	350	1·23
4	0·1 Concentric	Solid in bitumen	199	165	1·21
6	0·2 Concentric	Solid in bitumen	360	293	1·23
3	0·1 Concentric	Armoured, direct in ground	215	176	1·22
9	0·2 Concentric	Armoured, direct in ground	335	275	1·22
5	0·1 Concentric	Drawn into stoneware ducts	130	133	0·98
7	0·2 Concentric	Drawn into stoneware ducts	243	252	0·96
10	0·5 Concentric	Drawn into stoneware ducts	370	378	0·98
14	0·1 3-core L.P.	Drawn into stoneware ducts	157	162	0·97
13	0·1 3-core H.P.	Drawn into stoneware ducts	162	170	0·95

The factor given in column (6) is based on at least four determinations at different currents on each cable In columns (4) and (5) are given the values for one current only.

Table III. gives the thermal resistivities of the cables, expressed in terms of the difference of temperature in degrees Centigrade, required



AA From German tables for 27·8 deg. C. rise
BB From I.E.E. tables (factor 0·33)
C_1C_1 Actual determinations corrected for thermal resistivity 1000
C_2C_2 " " " " " " 500

FIG. 1.

to cause a flow of heat per second equivalent to 1 Joule between the faces of a centimetre cube, while in Table IV. the resistivity results obtained by some other observers will be found.

TABLE III

Thermal Resistivities of the Dielectric of Paper-insulated Cables

Cable No.	Paper-insulated Cable.	Mean Thermal Resistivity in Electrical Measure.
	Single Cables.	
1	0·1 Single L.P.	1200
2	0·2 Single L.P.	800
	Concentric Cables.	
3	0·1 Concentric L.P.	620
4	0·1 Concentric L.P.	1060
5	0·1 Concentric L.P.	1160
6	0·2 Concentric L.P.	1060
7	0·2 Concentric L.P.	1000
8	0·2 Concentric L.P.	870*
9	0·2 Concentric L.P.	720
10	0·5 Concentric L.P.	1090

* Jute insulated.

TABLE III—*continued*

Cable No.	Paper-insulated Cable.	Mean Thermal Resistivity in Electrical Measure.
	Three-core Cables.	
11	0·025 3-core L.P.	1050
12	0·05 3-core H.P.	730
13	0·1 3-core H.P.	720
14	0·1 3-core H.P.	670
15	0·1 3-core H.P.	420
16	0·15 3-core H.P.	500
17	0·15 3-core H.P.	460
18	0·15 3-core H.P.	550
19	0·15 3-core H.P.	470
20	0·15 3-core H.P.	550
21	0·15 3-core H.P.	460
22	0·15 3-core H.P.	570
23	0·2 3-core H.P.	600
24	0·2 3-core H.P.	600
25	0·2 3-core H.P.	650
26 *	0·2 3-core H.P.	710
27	0·25 3-core H.P.	580
Bitumen		511
Paper (not impregnated) . . .		960
Vulcanised bitumen		486
Bitumen cables: values determined by Dr. Marchant		510

* Jute insulated.

TABLE IV

Atkinson and Foster . .	1000—833
Powell	1235—877
Teschmuller and Humann	
Single L.P. cables . .	650
Multicore L.P. cables . .	600
Multicore H.P. cables . .	550
Opt and Maurstin's . . .	359—678

§ (2) NEWCASTLE EXPERIMENTS.—Mr. Fawsett's experiments dealt with the effect

depth on the temperature rise of a 0·1 sq. in. three-core, lead covered and armoured low-pressure cable; sections of the cable were laid at depths of 1, 2, and 4 ft. respectively. The tests showed that the average increase in temperature for a given current over that at 1 ft. was for the cable at 2 ft., 4 per cent, and for that at 4 ft., 15·7 per cent.

§ (3) Liverpool Tests.—These, carried out at the University, consisted in the determination of the temperature rise in four cables

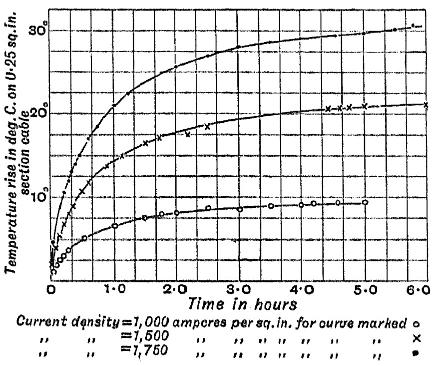


Fig. 2.

of nominal sizes—0·1, 0·25, 0·4, and 0·6 in.—laid at a depth of about 3 ft. in stoneware troughs, filled with bitumen and covered with flat tiles; the ground was filled in and covered with 3-in. wooden planks. The cable was insulated by two layers of fibre with a dielectric of vulcanised bitumen, then two layers of strong bitumen tape. *Fig.* 2 shows the rate of temperature rise on the 0·25 sq. in. cable, while *Fig.* 3 gives the amount of rise after a six hours' run with various currents.

The thermal resistivity of the various cables was measured and varied between 487 and 540 as the value of the temperature difference to cause a flow of heat equivalent to 1 Joule between the opposite faces of a 1-cm. cube.

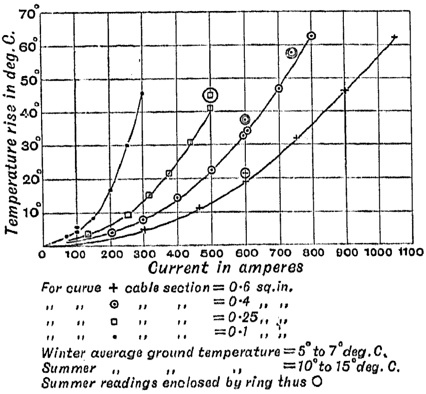


Fig. 3.

§ (4) Seasonal Variations.—Attention was called during the discussion at the Institution of Electrical Engineers to the fact that it was the actual temperature of the cable and not the temperature rise on which its life depended, and that in consequence of the variation of ground temperature a greater rise in the cable, at any rate when not buried deep, was permissible in winter than in summer; the current required to produce an actual conductor temperature of 80° was therefore worked out by Melsom for a 0·1 sq. in. three-cored, lead-covered and armoured cable laid direct in the ground; the results are given in Table V. and show striking differences.

Table V

Currents for an Actual Conductor Temperature of 80° C. with Various Depths and Seasons

Month.	Depth of Laying.								
	1 Foot.			2 Feet.			4 Feet.		
	Soil Temperature.	Available for Cable Rise.	Current.	Soil Temperature.	Available for Cable Rise.	Current.	Soil Temperature.	Available for Cable Rise.	Current.
	° C.	° C.	amperes.	° C.	° C.	amperes.	° C.	° C.	amperes.
February . .	3	77	293	5	75	284	6	74	268
May . . .	11	69	278	10	70	275	9	71	262
August . .	17	63	266	16	64	263	15	65	252
November . .	8	72	284	9	71	277	10	70	261

Thermionic Emission, connection with photoelectric effect. See "Photoelectricity," § (3).

Thermionic Phenomena in Highly Evacuated Enclosures: rectifying action and saturation current. See "Thermionics," § (3).

THERMIONIC SATURATION CURRENT AND TEMPERATURE, relation between. See "Thermionics," § (4).

Thermodynamic theory of relation between. See *ibid.* § (4) (iii.).

THERMIONICS: ELECTRICITY, DISCHARGE OF, FROM HOT BODIES

§ (1) THERMIONICS.—The term "thermionics" is applied to the phenomena associated with the discharge of electricity from hot bodies. Some of these phenomena have been known for a very long time; the fact, for instance, that air in the neighbourhood of hot bodies has the power of conducting electricity was observed upwards of 200 years ago.[1] Guthrie [2] made the important discovery that a red-hot iron ball could retain a negative charge but not a positive one, and that at higher temperatures this difference vanished, electricity of either sign being conducted away rapidly. Thermionic phenomena have been exhaustively studied in recent times, and have turned out to be of great practical utility, especially in connection with wireless telegraphy and telephony.

The main facts of thermionics are as follows: Bodies, especially metals, discharge electricity when heated, and cause the air in their neighbourhood to acquire electrical conductivity. A platinum wire, for example, maintained at a dull red heat in air will cause an electrode in its neighbourhood to become positively charged. If the temperature is raised to a white heat the electrode will lose its charge. If the experiment is done in an atmosphere where the pressure is very low or zero, the electrode becomes positively charged when the temperature of the wire is raised to a red heat and negatively charged when the temperature is sufficiently high. These phenomena are now known to be due to the emission from the hot body of positive and negative ions. Measurements of the ratio of charge to mass indicate that the positive ions are those of metals, chiefly potassium and sodium, probably contained in traces of salts present as impurities in or on the surface of the hot body. The rate at which the positive ions are discharged is found, in general, to diminish with continued emission. The negative ions are undoubtedly of the special type known as electrons,[3] *i.e.* small "atomic" charges of electricity ($1{\cdot}59 \times 10^{-20}$ electromagnetic units [4]) not associated with matter at all.

[1] Du Fay, *Mémoires de l'Acad.*, 1733.
[2] *Phil. Mag.*, 1873, (4) xlvi. 257.
[3] See "Electrons and the Discharge Tube," §§ (25), (26).
[4] Since 1 coulomb $=10^{-1}$ cm. units this is equal to $1{\cdot}59 \times 10^{-19}$ coulombs.

§ (2) METHODS OF EXPERIMENTAL INVESTIGATION. (i.) *The Apparatus.*—Metals and materials which are moderately good conductors of electricity are usually investigated in the form of wires or filaments which are heated electrically. The general type of apparatus used is illustrated in *Fig.* 1. The filament A to be tested is welded to stouter leads B and C. These in turn are welded or hard soldered to platinum wires sealed into the glass bulb D. The filament A lies on the axis of a cylindrical electrode E of metal foil, or, preferably, gauze supported by the sealed-in lead F. The tube H enables the bulb to be exhausted and sealed off or connected to apparatus for supplying various gases, for measuring the pressure, and so on. If the filament A is of platinum it is best to make all the metal parts inside D of platinum. The whole apparatus can then be thoroughly cleaned with boiling nitric acid and water. Tungsten filaments may be electrically welded in an atmosphere of hydrogen to stout iron or copper leads. Carbon filaments have to be joined with paste as in constructing incandescent lamps. The materials used in thermionic experiments should be clean and no traces of gas should be liberated in the bulb D during the course of the experiments. To attain this the bulb D is exhausted by a Gaede pump or other efficient pumping device assisted by liquid air and charcoal. The arrangement of the exhausting apparatus is shown diagrammatically in *Fig.* 2. An oil pump P is usually employed to "back" a Gaede rotary pump G. A side tube L containing small pieces of charcoal made from coconut shell is connected

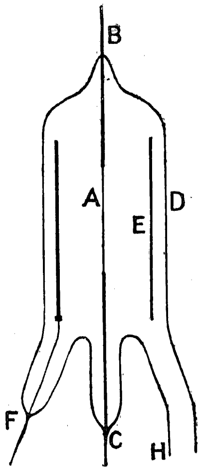


FIG. 1.

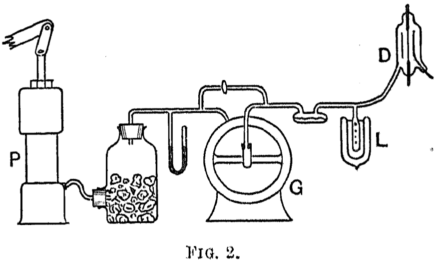


FIG. 2.

with the tube leading to the bulb D and is kept surrounded by liquid air during the process of exhaustion. The charcoal at the low temperature of the liquid air (about −180° C.), and especially if it has been previously heated to expel absorbed gases, assists very greatly in removing the last traces of gas from the apparatus. During the exhaustion the bulb D is maintained at a high temperature. The filament A is at the same time heated to incandescence by passing a current through it. The cylindrical electrode can also be heated by "bombarding" it with electrons emitted from the glowing filament. This is effected by applying a high positive potential to the cylinder. An alternative exhausting arrangement which is particularly efficient is to use a mercury vapour pump with suitable backing pumps and a connecting tube dipping in liquid air to prevent access of mercury vapour to the apparatus which is being denuded of gas.

To prevent the bulb D from collapsing under the atmospheric pressure it may with advantage be heated inside a vacuum furnace. A suitable form of furnace may be constructed with a heavy water-jacketed brass base provided with holes for the tube H and the leads B, C, and F (*Fig.* 1). The holes can be made air-tight with glass and sealing-wax and an additional hole for the insertion of a platinum thermometer or thermo-couple is desirable. On the base rests a large cylindrical brass bell-jar, the line of contact being made air-tight with a rubber gasket. The brass cylinder is balanced by weights attached to cords passing over pulleys so that it can easily be moved up and down. The furnace itself is inside the brass cylinder and rigidly attached to it. It consists of nichrome strip, with a suitable resistance and current-carrying capacity, wound on a nickel cylinder from which it is insulated by a mica wrapping. The leads to the nichrome strip and exhaust can be let in through the cover of the brass cylinder. This, as well as the brass base, should be water-cooled. Such a furnace is shown in *Fig.* 3.

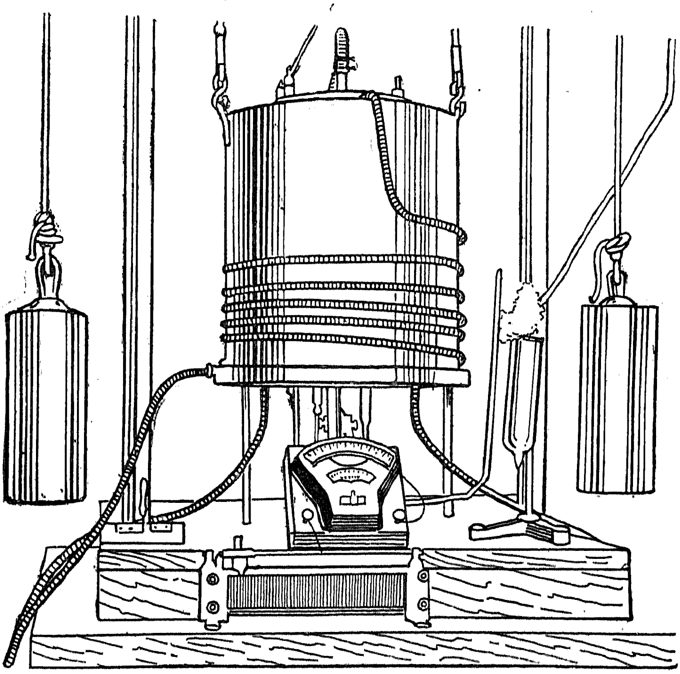


FIG. 3.

(ii.) *Methods of controlling the Temperature.*—The experimental arrangements for controlling the temperature of the filament and measuring the emission from it are illustrated in *Fig.* 4. The filament forms one arm of a Wheatstone bridge, which is worked by the battery supplying the heating current. The thermionic current between the cylinder E

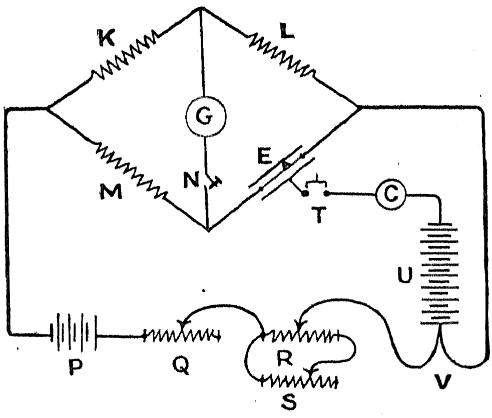


FIG. 4.

and the filament A is measured by the galvanometer C. The heating current is supplied by the battery P and can be regulated by the rheostats Q, R, S. A fine adjustment is obtained by placing two of the rheostats R and S in parallel. K, L, M are the resistances in the remaining arms of the bridge, and M must be of the order of magnitude of A and capable of carrying a large current without heating appreciably. The bridge is adjusted initially and the balance maintained by altering the resistances Q, R, S.

In the case of the metals platinum and tungsten it is most convenient to deduce the temperatures from the measured values of the resistance. The resistance temperature calibration should be made under the conditions of temperature obtaining in the experiments. This is secured by placing minute fragments of salts or other substances of known melting-point on the central portion of the wire or filament after it has been removed from the experimental tube. The filament is then heated electrically in a suitable atmosphere and the resistance at which the salt melts is then determined. There is a large number of suitable substances whose melting-points are accurately known, ranging from tin, melting at 232° C., to tungsten, melting at $3267° \pm 30°$ C.

The resistance method of measuring the temperature has the advantage that it does not involve the introduction of complications into the experimental tube, but on the other hand it does involve an investigation of the resistance temperature relations of the substance under investigation. Langmuir has published a very complete investigation of this kind for tungsten.[1]

§ (3) THE PHENOMENA IN HIGHLY EVACUATED ENCLOSURES. RECTIFYING ACTION. SATURATION CURRENT. (i.) *The Use of a Vacuum.*—Thermionic phenomena are much simplified when the experiments are conducted in evacuated enclosures. Not only is the influence of the gaseous atmosphere on the emission eliminated but the complication due to the ionisation of the gas by impact with the carriers is avoided. When a hot body, *e.g.* the platinum filament in *Fig.* 1, is maintained at a positive potential relatively to the surrounding electrode positive carriers only can leave it, the applied field preventing the negative ones from escaping. The positive emission, when continued for a sufficient length of time, is reduced ultimately to negligible dimensions. When a negative potential is applied to the filament only negative carriers or electrons will leave it. This emission, unlike the positive one, is quite steady and characteristic of the emitting substance and its temperature. The practical utility of thermionic phenomena is largely due to the fact that if an alternating potential difference is applied to a hot body and surrounding electrode placed in an evacuated enclosure a current will flow in one direction only, namely from the electrode to the hot body; since when the field is directed from the hot body to the electrode only positive ions can pass across and the number of these emitted is very small, whereas when the field is in the other direction there will be a more or less considerable current due to the emission of negative ions or electrons from the hot body.

(ii.) *Ionisation—The Saturation Current.*—When a moderate potential difference is applied to electrodes in air at atmospheric pressure no current, generally speaking, passes from one to the other. A current begins to flow when, by any means, ions are formed in the intervening space. When ions are produced at a steady rate independent of the potential difference, *e.g.* by a beam of X-rays, a current will begin to flow and the relation between the current and the applied potential is that illustrated by the left-hand portion of the curve in *Fig.* 5. This relation, it will be noted, is almost linear when the potential applied is small, while when it is sufficiently great the current is almost independent of the voltage. These facts are simply explained. Ions are being produced at a steady rate independent of the applied potential. As this potential increases from zero the ions moving under its influence will reach the electrodes at a progressively greater rate until a stage is reached when they get to the electrodes as fast as they are formed and no further increase in the current can occur so long as the rate of production of the ions remains constant. If, however, the voltage is made very much greater, and especially if the pressure of the air is low, say only 2 or 3 mm., it is found that the current begins to increase again very rapidly with the applied potential difference. This stage is illustrated by the right-hand portion of the curve in *Fig.* 5 and is explained[2] by the hypothesis that the ions attain sufficient momenta under the great applied voltage to cause the formation of new ions by impact with neutral molecules. When the current has the value corresponding to the horizontal part of the curve (*Fig.* 5) it is said to be saturated, and it provides a measure of the rate at which ions are being produced. Ionisation by impact may set in, in some cases, before the saturation stage is reached and it is then impossible to know what value of the current measures the rate of the primary production of ions. When ions are emitted by a hot body placed in a

[1] *Phys. Rev.*, 1916, vii. 302. See also Stead, *Journ. Inst. Elec. Eng.*, 1920, lviii. 107.

[2] M'Clelland, *Camb. Phil. Proc.*, 1901, xvi. 296.

vacuous enclosure there is no impact ionisation and saturation can always be attained if a sufficiently large potential difference is applied between the hot body and the surrounding electrode.

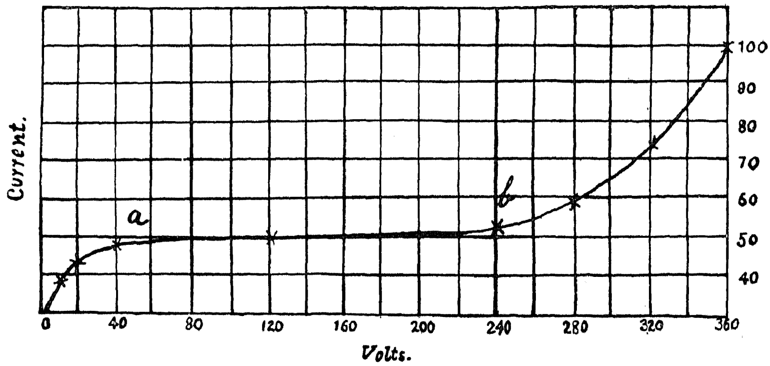


Fig. 5.

§ (4) Relation between Saturation Current and Temperature. Effects of Gaseous and other Contaminants. (i.) *Experimental Results.*—The relation between the saturation current from a hot body and its temperature has been the subject of an immense amount of experimental research. In all cases it has been found that if the material experimented on is in a condition which does not change with lapse of time, the rate of emission increases with enormous rapidity as the temperature is raised. The extreme rapidity of this variation is shown in *Fig.* 6, which represents the results of early experiments with sodium.[1] The observations recorded extend over a range of temperature from 217° C. to 427° C., whilst the corresponding currents increased from $1{\cdot}8 \times 10^{-9}$ amp. to $1{\cdot}4 \times 10^{-2}$ amp. Thus with a rise of a little over 200° C. the current increased by a factor of 10^7. In order to exhibit the results conveniently on the same diagram the curve is

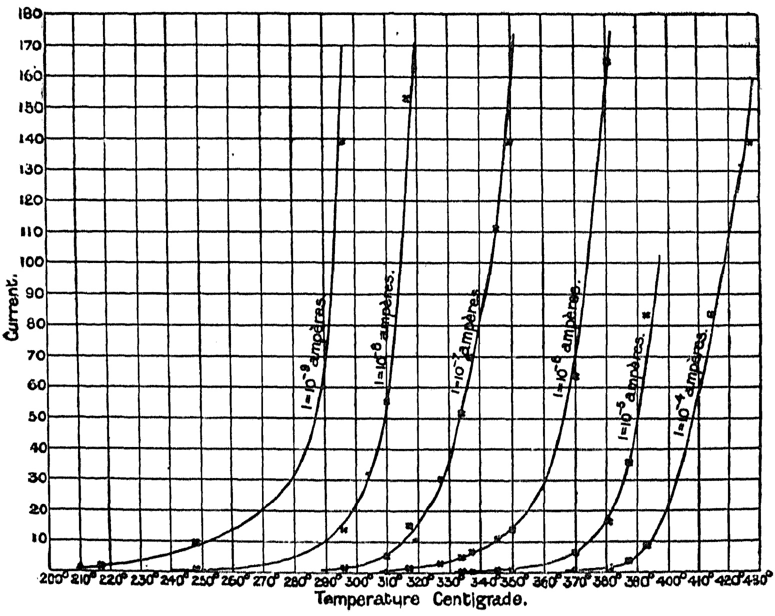


Fig. 6.

[1] Richardson, *Phil. Trans.* A, 1903, cci. 497.

shown by a number of branches in each of which the scale of the ordinates is successively reduced by a factor of 10 proceeding from left to right. Results given by Smith give values for the current in amperes per sq. cm. which increase in the case of a tungsten filament from $2\cdot35 \times 10^{-13}$ at 1050° K to 1·23 at 2520° K.

In studying the theory of the phenomenon it will be convenient to consider a vacuous enclosure surrounded by the emitting surface, the whole system being maintained at the uniform temperature T. There will be an accumulation of electrons in the vacuous space arising from the emission. This accumulation will not go on indefinitely since some of the electrons, on account of their heat motion, will return to the walls of the enclosure. A balance will ultimately be established when as many electrons will return to the hot body (the walls of the enclosure) as leave it in a given time. If N′ represents the number of electrons returning to the unit area per second and n the number per unit volume in the enclosure when the balance has been established, then it can be shown that [1]

$$\mathrm{N}' = n\sqrt{\frac{k\mathrm{T}}{2\pi m}}, \quad . \quad . \quad . \quad (1)$$

where m is the mass of an electron, T is the absolute temperature, and k is the gas constant, often known as Boltzmann's constant. This constant is equal to PV/NT, where P, V, and T are the pressure, volume, and temperature respectively of a given quantity of any gas and N is the number of molecules it contains. The relation (1) is obtained by an application of the kinetic theory of gases which there is good reason for believing applies exactly to this electron atmosphere when it is sufficiently attenuated.

(ii.) *Theoretical Considerations.*—The earliest attempt to obtain a theoretical basis for the relation between the saturation current and the temperature of the hot body involved the assumption that the hot body contained free electrons and that the principles of the kinetic theory of gases were applicable to them.[2] If, as before, n represents the number of electrons per unit volume in the vacuous enclosure and n_1 the number per unit volume [3] inside the hot body, we can show that

$$n = n_1 e^{-\frac{\phi_0}{k\mathrm{T}}}, \quad . \quad . \quad . \quad (2)$$

where ϕ_0 is the amount of work that must be done to remove an electron from the interior of the hot body to the enclosure. Eliminating

If we multiply this expression for the number of electrons emitted per second from the unit area of the hot body by the charge on an electron we get

$$i = n_1 e\sqrt{\frac{k}{2\pi m}}\mathrm{T}^{\frac{1}{2}} e^{-\frac{\phi_0}{k\mathrm{T}}} \quad . \quad . \quad (4)$$

for the current per unit area. This formula, which may be written

$$i = \mathrm{A}\mathrm{T}^{\frac{1}{2}} e^{-\frac{b}{\mathrm{T}}}, \quad . \quad . \quad . \quad (4')$$

where A and b are independent of the temperature but vary from one substance to another, represents exceedingly well the results of measurements made with many different materials over very wide ranges of temperature.

(iii.) *Thermodynamic Theory.*—It is instructive to study the temperature variation of the electronic emission from a hot body from the point of view of thermodynamics. As we have already seen, when we have a hot body in a vacuous enclosure with insulating walls a state of equilibrium is soon reached when electrons return to the hot body as rapidly as they are emitted. The state of things is similar in all respects to that which exists when a liquid or solid is in equilibrium with its vapour, and the following equation [4] must hold:

$$\mathrm{L} = v\mathrm{T}\frac{dp}{d\mathrm{T}}, \quad . \quad . \quad . \quad (5)$$

where L may be called the latent heat of "vaporisation" of the electrons, v is the change in volume of the system accompanying their reversible transference to the enclosure, *i.e.* in this case the volume they occupy in the enclosure, and p is the equilibrium pressure of the electrons. If we adopt the hypothesis, for the correctness of which there is strong evidence, that this pressure is the same as that of an equal number of molecules of an ideal gas at the same temperature, we may write

$$p = nk\mathrm{T}, \quad . \quad . \quad . \quad . \quad (6)$$

where n is the number of electrons per unit volume and k is the gas constant for a single molecule. If ϕ is the change in energy accompanying the transference of an electron from the hot body to the enclosure, the change in energy corresponding to the "evaporation" of the electrons occupying the volume v in the enclosure will be $nv\phi$. Further, an amount of work pv will be done against the constant pressure p, so we obtain for the latent heat of vaporisation

$$\mathrm{L} = nv\phi + pv.$$

[4] See "Thermodynamics," Vol. I. § (41).

We have therefore

$$nv\phi + pv = v\mathrm{T}\frac{dp}{d\mathrm{T}},$$

or

$$n\phi + p = \mathrm{T}\frac{dp}{d\mathrm{T}}. \quad . \quad . \quad . \quad (7)$$

Combining this result with equation (6) we get

$$n\phi = k\mathrm{T}^2\frac{dn}{d\mathrm{T}}.$$

Finally, on integrating this last equation we arrive at the result

$$n = n_1 e^{\int^{\mathrm{T}} \frac{\phi}{k\mathrm{T}^2} d\mathrm{T}}, \quad . \quad . \quad . \quad (8)$$

where n_1 is independent of the temperature. Combining this with equation (1) we get

$$\mathrm{N}' = n_1 \sqrt{\frac{k}{2\pi m}}\mathrm{T}^{\frac{1}{2}} e^{\int^{\mathrm{T}} \frac{\phi}{k\mathrm{T}^2} d\mathrm{T}}$$

for the number of electrons emitted per unit area per second and for the corresponding current

$$i = n_1 e \sqrt{\frac{k}{2\pi m}}\mathrm{T}^{\frac{1}{2}} e^{\int^{\mathrm{T}} \frac{\phi}{k\mathrm{T}^2} d\mathrm{T}} \quad . \quad . \quad (9)$$

If we write $\phi = \phi_0$, and suppose it to be independent of the temperature

$$\epsilon^{\int \frac{\phi}{k\mathrm{T}^2} d\mathrm{T}} = e^{-\frac{\phi_0}{k\mathrm{T}}},$$

and this result reduces to equation (4) obtained above. There are thermodynamical reasons for supposing that ϕ is not strictly independent of the temperature. The relation between them is represented approximately by the following equation

$$\phi = \phi_0 + \tfrac{3}{2}k\mathrm{T}. \quad . \quad . \quad . \quad (10)$$

When we substitute this value for ϕ in equation (9) we get a formula which may be written

$$i = \mathrm{AT}^2 e^{-\frac{b}{\mathrm{T}}}. \quad . \quad . \quad . \quad (9')$$

Both equations (4′) and (9′) represent the experimental results equally well, but since the latter is largely based on thermodynamical reasoning the presumption is that it is the nearer to the truth of the two.

(iv.) *Effect of Impurities in the Gas: (a) Emission from Platinum.*—The emission of electrons from hot bodies is greatly modified by the presence of gaseous and other impurities. The effect of an atmosphere of hydrogen on the emission from platinum has been investigated by H. A. Wilson[1] and others. At constant pressure the emission is found to follow the law (4′), the constants A and b being functions of the pressure of the hydrogen and in general quite different from those of the corresponding quantities appropriate to the emission from platinum wires in a vacuum. H. A. Wilson arrived at the following conclusions. When a wire whose temperature is constant is allowed to remain for some time in hydrogen at different pressures the emission assumes steady values which are represented by the formula

$$i = \mathrm{B}p^z,$$

where B and z are independent of the pressure, but depend on the temperature. Throughout the range of temperature used z was always between 0·5 and 1·0, and increased as the temperature diminished. When a change from one pressure to another was made the emission did not immediately assume its final value. A similar time lag was observed when the temperature was changed at constant pressure. These effects are explained if the emission is not directly determined by the pressure of the external hydrogen, but by the amount of hydrogen dissolved in the wire. The equilibrium amount diminishes as the temperature rises. The effect of hydrogen on the constants A and b is shown in the following table from a paper by H. A. Wilson:

Gas.	Pressure. Mm.	A.	b.
Air (1) . .	Small	$7{\cdot}14 \times 10^{26}$	$7{\cdot}25 \times 10^4$
Air (2) . .	Small	$4{\cdot}38 \times 10^{26}$	$6{\cdot}9 \times 10^4$
Hydrogen .	0·0013	$6{\cdot}25 \times 10^{24}$	$5{\cdot}5 \times 10^4$
Hydrogen .	0·112	$3{\cdot}13 \times 10^{23}$	$4{\cdot}5 \times 10^4$
Hydrogen .	133·0	$1{\cdot}25 \times 10^{21}$	$2{\cdot}8 \times 10^4$

Other observers and Wilson himself have obtained results quite different from those described above. In fact the emission from platinum in an atmosphere of hydrogen shows two distinct types of behaviour under conditions which are at first sight identical. Sometimes the emission is quite insensitive to changes in the pressure of the hydrogen atmosphere. According to Wilson the condition in which the emission is sensitive to changes in pressure occurs in fresh wires, *i.e.* wires which have not been heated in hydrogen for any appreciable length of time. The other condition is characteristic of wires which have been subjected to prolonged heating in the gas. He points out that the observed facts are consistent with the view that in the former case the hydrogen exists in solution in the wire, while in the latter case it is present in the form of a chemical compound which is formed with extreme slowness.

The emission from platinum in hydrogen appears to be susceptible to changes in the

[1] H. A. Wilson, *Phil. Trans.* A, 1908, ccviii. 248; *Phil. Trans.* A, 1903, ccii. 263; *Roy. Soc. Proc.* A, 1909, lxxxii. 71; *Phil. Trans.* A, 1906, ccvii. 1.

intensity of the electric field.[1] It has been observed that when the applied potential difference is increased considerably beyond the saturating voltage the emission diminishes, and increases again when the original potential difference is restored. There is a time lag in the effect, and it is probably due to the destruction, by the positive ions produced by impact ionisation, of a structure which is formed near the surface of the platinum which facilitates the escape of the electrons.

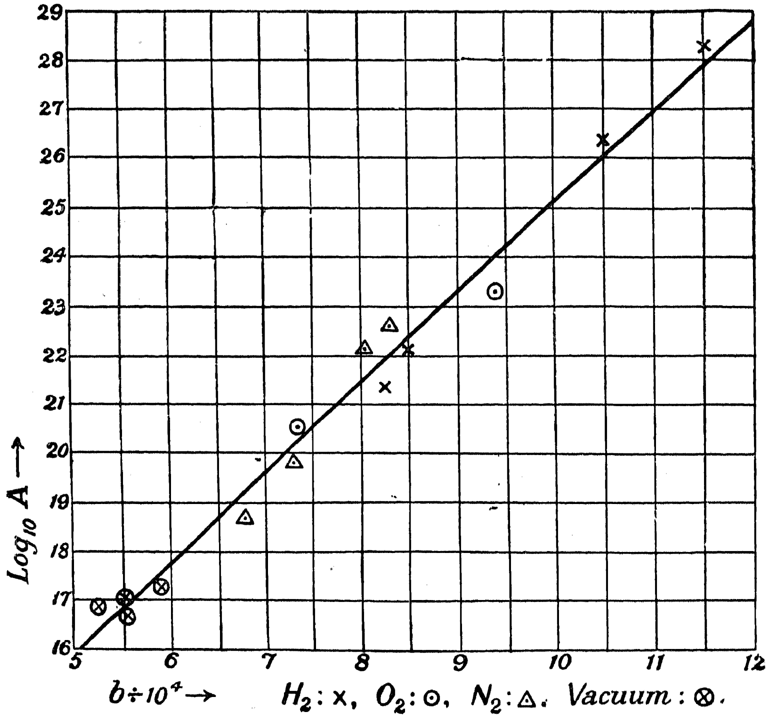


Fig. 7.

(*b*) *Emission from Palladium.*—Hydrogen has been found to increase the emission from palladium [2] and sodium.[3] On the other hand, Langmuir [4] found that it caused a great reduction in the emission from tungsten, an effect which he attributes to the action of water vapour formed by secondary reactions.

(*c*) *Emission from Tungsten.*—The effect of different gases on the emission from tungsten at about 2000° C. has been investigated by Langmuir.[5] Here again the emission is represented by (4′) when a steady state has been attained. The constants A and *b* were found to have different values in different gases, and very small amounts of gas were found to cause large changes in the constants. All the gases tested, with the exception of argon, were found to *increase* the values of *both* constants. Argon had no effect, except that it facilitated the attainment of saturation —the positive ions due to impact ionisation reducing the effect due to the repulsion of the electrons. In all cases the changes in the emission persisted some time after the removal of the gases, so that the effect was not directly due to some action of the external gas on the filament, but to a semi-permanent change produced in the character of the tungsten surface.

The magnitudes of the changes in A and *b* suggest that all these changes are due to a common cause; at any rate there are important features common to different cases. It is found that A and *b* always increase together and diminish together. In *Fig.* 7, $\log_{10}$A is plotted against $b \times 10^{-4}$. It will be seen that there is a linear relation between them.

There is a general resemblance between the effects of different gases on tungsten and

[1] O. W. Richardson, *Phil. Trans.* A, 1906, ccvii. 46.
[2] H. A. Wilson, *Phil. Trans.* A, 1908, ccviii. 265.
[3] J. J. Thomson, *Cond. of Electricity in Gases*, 2nd ed. p. 203.
[4] Langmuir, *Phys. Rev.*, 1913, ii. 463.
[5] *Ibid.* 450, and *Phys. Zeitsch. Jahrg.*, 1914, xv. 516.

that of hydrogen on platinum. The chief difference is that hydrogen causes both constants A and b to *diminish*. It may be said that the two types of effect are due to similar causes acting in opposite senses. If in the case of platinum the cause lies in the difference of concentration of positive hydrogen ions inside and outside the metal, it would be natural to attribute the effects with tungsten to the difference in concentration of the negative ions (of the electro-negative elements O_2 and N_2). On the other hand, if the platinum effects arise from the action of the positive hydrogen ions on a double layer at the surface, it is reasonable to ascribe the tungsten effects to a similar action of the negative ions of O_2, N_2, etc.

In more recent experiments Langmuir has investigated the effect of the presence of slight traces of thorium in a tungsten cathode, and finds that by heating the cathode in a high vacuum the emission can be made to change from that of pure tungsten to that characteristic of pure thorium.

§ (5) THE KINETIC ENERGY OF THE EMITTED ELECTRONS: THE LATENT HEAT OF EMISSION AND ABSORPTION OF ELECTRONS. (i.) *Maxwell's Distribution Law, Theoretical.*—In deducing the relation between the electron emission and the temperature of the hot body it was assumed that the electrons in a vacuous enclosure (provided their density was so small that their mutual repulsion could be ignored) behave like the molecules of an ideal gas. Experimental evidence of the correctness of this assumption has been furnished by testing its consequences. One of these is that the average kinetic energy of translation of the electrons in a *given volume* is $\frac{3}{2}k$T. Another more important one is Maxwell's[1] law of distribution of velocities, which may be described as follows: If N is the total number of electrons (molecules) in an enclosure, and if u, v, and w represent the velocity components of an electron parallel to the x, y, and z axes respectively, then the number of them whose x velocity component lies between u and $u+du$ is equal to

$$\mathrm{N}\sqrt{\frac{hm}{\pi}}e^{-hmu^2}du, \qquad . \quad . \quad (11)$$

where m is the mass of an electron and $h=1/2k\mathrm{T}$. Similar statements can be made about the velocity components parallel to the y and z axes.

When a hot body is in equilibrium with the electron atmosphere surrounding it, not only will the number emitted per unit area per second be equal to the number returning, but the law of distribution of the velocities will be the same, and this will be true, at any rate approximately, even if equilibrium conditions do not exist.

If the principles of the dynamical theory of gases do hold for the emitted electrons, we can deduce from the expression (11) the number of electrons which cross per second unit area of a plane perpendicular to the axis of x, and the average kinetic energy of each. The number of molecules whose velocity lies between u and $u+du$ is given by (11). The number of these which will reach the plane normal to x per unit of time will be found by substituting for N, the whole number of molecules in the enclosure, n the number per unit volume, and multiplying the expression by u. The whole number N′ reaching the plane for all values of u will be given by integrating this expression for values of u from 0 to ∞. We thus obtain

$$\begin{aligned}\mathrm{N}'&=n\sqrt{\frac{hm}{\pi}}\int_0^{\infty}e^{-hmu^2}udu\\&=-n\sqrt{\frac{hm}{\pi}}\left.\frac{e^{-hmu^2}}{2hm}\right|_0^{\infty}\\&=\frac{n}{2}\sqrt{\frac{1}{\pi hm}}=n\sqrt{\frac{k\mathrm{T}}{2m\pi}},\end{aligned}$$

substituting for h its value $1/2k$T. And this is the expression quoted in (1) above.

The kinetic energy parallel to x

$$\begin{aligned}&=\tfrac{1}{2}nm\sqrt{\frac{hm}{\pi}}\int_{-\infty}^{+\infty}e^{-hmu^2}u^2du\\&=mn\sqrt{\frac{hm}{\pi}}\cdot\frac{1}{hm}\cdot\frac{1}{\sqrt{hm}}\int_0^{\infty}e^{-x^2}x^2dx\\&=\frac{n}{4h}.\end{aligned}$$

Thus dividing by n, the number of molecules the average kinetic energy parallel to $x=1/4h=\frac{1}{2}k\mathrm{T}$, and since the average kinetic energy is the same in all directions the total kinetic energy is $\frac{3}{2}k$T.

(ii.) *Maxwell's Distribution Law, Experimental.*—The applicability of Maxwell's law to electron atmospheres has been tested both as regards the component of velocity normal to the emitting surface and those parallel to it.[2] It will suffice to describe the investigation of the normal component. The type of apparatus used is illustrated in *Fig.* 8. The emitting surface was that of a small piece of thin platinum foil H heated electrically. The foil nearly filled a small hole at the centre of the metal plate L, the upper surfaces of L and H being flush with one another. The heating current was let in through t_1 t_2, which were connected by a high-resistance shunt not shown in the figure. The shunt was provided with a sliding contact which could be connected through the metal base B to L. In this way the middle of the strip H could be kept at the same potential as the surrounding plate L. Opposite L is a parallel plate U covered with platinum, to avoid effects arising

[1] J. C. Maxwell, *Collected Works*, i. 380. See also "Observations, Combination of," Vol. III.

[2] Richardson and Brown, *Phil. Mag.*, 1908, xvi. 353.

from contact difference of potential, and provided with a guard ring G and electrostatic shield S. U is connected to the insulated

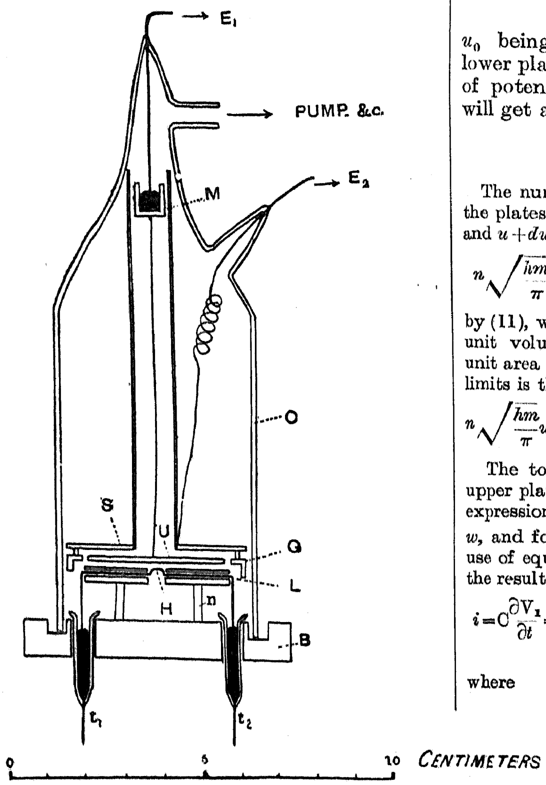


FIG. 8.

quadrants of a sensitive electrometer, whose time rate of deflection measured the number of electrons passing from H to U. The temperature of H was controlled, and estimated, by measuring its resistance in the manner already described. The electron currents from H to U were measured, when different potentials were applied so as to oppose their passage.

Let us suppose the planes U and L are infinite in extent and that they are maintained at fixed potentials, so that the electric force is normal to them and, we shall suppose, parallel to the x axis.

We have the equations:

$$m\frac{d^2x}{dt^2} = -\epsilon\frac{\partial V}{\partial x} = m\frac{du}{dt},$$

$$m\frac{d^2y}{dt^2} = m\frac{dv}{dt} = 0,$$

$$m\frac{d^2z}{dt^2} = m\frac{dw}{dt} = 0.$$

The electric field does not affect v and w, and we get, on integrating the first equation,

$$u^2 = u_0{}^2 - \frac{2\epsilon V}{m},$$

u_0 being the velocity of emission at the lower plate where $V=0$. If V_1 is the difference of potential between the plates, an electron will get as far as the upper plate if

$$u_0{}^2 \geqq \frac{2\epsilon V_1}{m}.$$

The number of electrons per unit volume, between the plates, whose velocity components lie between u and $u+du$, v and $v+dv$, w and $w+dw$ is equal to

$$n\sqrt{\frac{hm}{\pi}}e^{-hmu^2}du\sqrt{\frac{hm}{\pi}}e^{-hmv^2}dv\sqrt{\frac{hm}{\pi}}e^{-hmw^2}dw$$

by (11), where n is the total number of electrons per unit volume. The number passing vertically per unit area per second whose velocities lie within these limits is therefore equal to

$$n\sqrt{\frac{hm}{\pi}}ue^{-hmu^2}du\sqrt{\frac{hm}{\pi}}e^{-hmv^2}dv\sqrt{\frac{hm}{\pi}}e^{-hmw^2}dw.$$

The total number reaching the unit area of the upper plate is therefore obtained by integrating this expression between the limits $-\infty$ and $+\infty$ for v and w, and for u from $\sqrt{2\epsilon V_1/m}$ to $+\infty$. By making use of equation (1) and remembering that $h=1/2kT$, the result can be written in the following form:

$$i = C\frac{\partial V_1}{\partial t} = N\epsilon\int_{\sqrt{\frac{2\epsilon V_1}{m}}}^{+\infty} F(u)du\int_{-\infty}^{+\infty} f(v)dv\int_{-}^{+\infty} f(w)dw,$$

where $\quad F(u) = 2hmue^{-hmu^2},$

$$f(v) = \sqrt{\frac{hm}{\pi}}e^{-hmv^2},$$

N = number of electrons emitted per second,
i = current,
C = capacity of electrometer and connections.

The integration gives

$$i = N\epsilon e^{-2hV_1\epsilon} = i_0e^{-2hV_1\epsilon},$$

where i_0 is the current when $V_1=0$.

Therefore $\quad \log\frac{i}{i_0} = -2hV_1\epsilon,$

or $$\log\frac{i}{i_0} = -\frac{\epsilon V_1}{kT} = \frac{\nu\epsilon V_1}{RT}, \quad . \quad . \quad (12)$$

where ν is the number of molecules in 1 c.c. of an ideal gas at 0° C. and 760 mm., and R is the "gas constant" for this quantity of gas. Both $\nu\epsilon$ and R are well-known physical quantities.

$$\nu\epsilon = 0{\cdot}4327 \text{ em. units.}$$

$$R = 3{\cdot}711 \times 10^3 \text{ erg/° C.}$$

The results of one series of measurements are shown in *Fig.* 9. The points shown thus ⊙ give the current i as ordinates, and the points shown thus X the values of log i.

The abscissae are the values of the corresponding potentials in each case. The points marked X are seen to be situated exceedingly close to a straight line, thus confirming equation (12). From the slope of the line, and the known values of $v\epsilon$ and T, the value of R can be deduced. These ranged from $3{\cdot}61 \times 10^3$ to $4{\cdot}36 \times 10^3$. When the extraordinary difficulties associated with such experiments are taken into account it must be said that the agreement with the known value of R, viz. $3{\cdot}711 \times 10^3$, is highly satisfactory. Moreover, the mean of the different values for R mentioned above is $3{\cdot}719 \times 10^3$. These experiments show conclusively that, so far as the velocity component perpendicular to the emitting surface is concerned, Maxwell's law of distribution holds for the emitted electrons. By means of a modified form of the apparatus described above, the validity of Maxwell's law for the tangential velocity components has also been established.

It should be added, however, that recent experiments made in the Wheatstone Laboratory show that whilst, as a rule, the kinetic energy of the electrons is distributed in accordance with Maxwell's law for some temperature, this temperature for the electrons may in certain cases be much higher than that of the hot body. The conditions which determine these discrepancies are not yet understood and their cause is still under investigation.

(iii.) *Latent Heat of Emission.*—When an electron escapes from the surface of a hot body the latter will lose an amount of energy equal to $w + 2kT$, where w represents the work which must be done against the forces tending to keep the electron in the interior of the hot body, and $2kT$ is the average kinetic energy of the electrons emitted in a given time. The loss of energy per second will therefore be expressed by the equation

$$U = i\left(\psi + \frac{2k}{\epsilon}T\right);$$

where ψ is the P.D. through which an electron has to fall to acquire the energy w, ϵ is

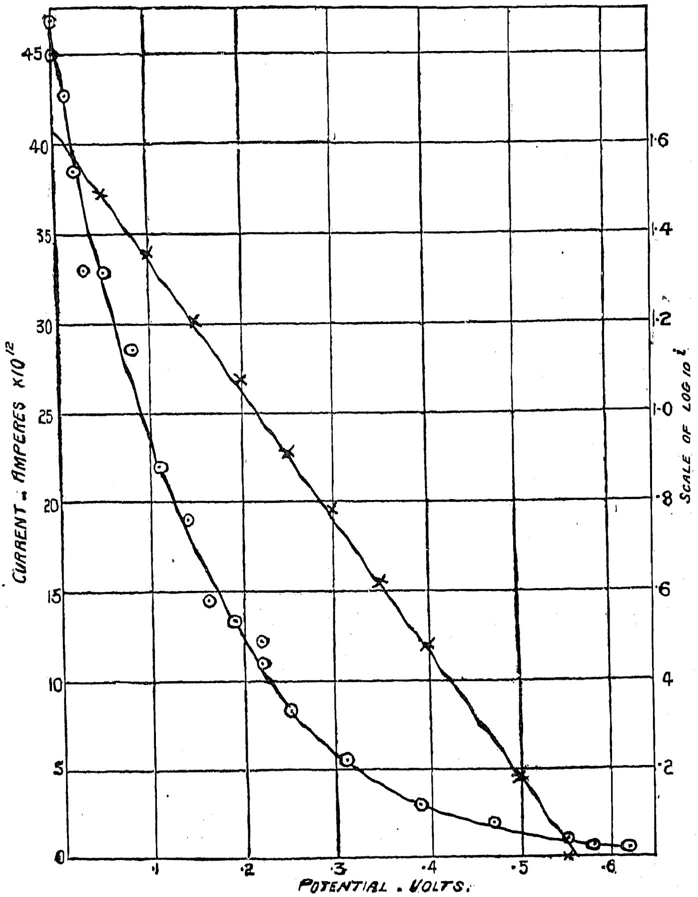


FIG. 9.

the charge on an electron, and i is the thermionic emission current. A hot filament can be prevented from emitting electrons if it is surrounded by an electrode at a sufficiently high negative potential, and the foregoing considerations lead to the expectation that under these circumstances a smaller rate of supply of energy will be needed to keep the temperature of the filament constant than is necessary when electrons are permitted to leave it, and therefore a drop in the temperature of the filament is to be expected when

the potential opposing the emission of electrons is replaced by one which facilitates their emission. Such an effect has been observed and measured by several investigators.[1] From these measurements ψ has been determined, and the quantity b of equation (4′) has been deduced. The values obtained in this way for b agree satisfactorily with those deduced from measurement of the temperature variation of the emission.

When electrons are absorbed by a metal there is a liberation of heat which is the converse effect to that described above. Naturally, if the electrons fall through a considerable potential difference in reaching the metal there will be a development of heat corresponding to the kinetic energy they have acquired. The effect under consideration, however, occurs even if the electrons reach the absorbing metal with zero kinetic energy, and is caused by the work done on them as they cross the surface layer of the metal. This effect has also been detected and measured,[2] and the value of b (equation (4′)) deduced. The measurements are attended with great difficulties and the results are not quite so consistent as those of other thermionic experiments, but the calculated values of b in the case of platinum, for which metal the present data are the most reliable, agree with the best values of that quantity as deduced from the temperature variation of the emission.

§ (6) Current-voltage Characteristics, Space Charge. Influence of Initial Velocities and Contact Electromotive Force. Three Electrode Tubes. (i.) *Current-voltage Characteristics.*—In measurements of the thermionic emission from hot bodies it is essential that the currents should be saturated. It is only then that we can be sure we are measuring the actual rate of emission of electrons. If the enclosure surrounding the hot body is sufficiently thoroughly evacuated impact ionisation hardly occurs, and for the purpose of the measurements the applied potential may have any value greater than that necessary to secure saturation. When the charge density in the space between the electrodes is zero the saturation value of the current is reached with zero potential difference between the electrodes, and the extreme left-hand portion of the characteristic current voltage curve (*e.g. Fig.* 5) is limited to the falling off with increasing potential differences of the current flowing against the field due to the initial velocities of the electrons. In those cases, however, in which we are chiefly interested, the charge density in the space between the electrodes is never quite zero and may have quite appreciable values. The type of curve connecting current and potential difference which is then obtained under the best vacuum conditions is shown in *Fig.* 10. The flat right-hand

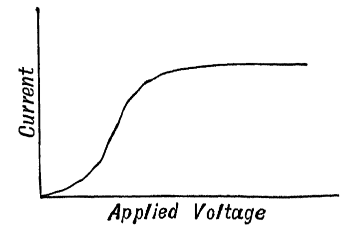


Fig. 10.

part corresponds to complete saturation. The extreme left-hand part approaches horizontality more closely the greater the "space charge" between the electrodes. Under these circumstances, too, the applied voltage necessary to secure saturation also increases.

(ii.) *The Space Charge.*—It is only when the thermionic current densities are rather small that saturation is attainable with potential differences of 20 volts or thereabouts, with devices of the dimensions most commonly used. This is due to the mutual repulsion of the electrons in the space between the electrodes, a phenomenon which has long been recognised.[3] Owing to this effect, if a *given* potential difference is maintained between the electrodes, the current will not increase beyond a certain value no matter how much the supply of electrons at the cathode is increased by raising its temperature. This was established by the experiments of Lilienfeld in 1910,[4] who showed that the shape of the current-voltage curves was independent of the cathode temperature, if this were sufficiently high. He stated later that it was a consequence of these experiments that over a considerable range of the infra-saturation part of the curve the current varied as the 3/2 power of the voltage. A theoretical demonstration of this result was given by C. D. Child[5] in 1911 along the following lines:

Considering the simple case in which the emitting surface is an infinite plane opposite an infinite parallel conducting plane repre-

[1] Wehnelt and Jentzsch, *Verh. der Deutsch. Phys. Ges.*, 1908, 10 Jahrg. p. 610; *Ann. d. Phys.*, 1909, xxviii. 537; Richardson and Cooke, *Phil. Mag.*, 1913, xxv. 624; *Phil. Mag.*, 1913, xxvi. 472.

[2] Richardson and Cooke, *Phil. Mag.*, 1910, xx. 173; *Phil. Mag.*, 1911, xxi. 404.

[3] Cf. Richardson, *Phil. Trans.* A, 1903, cci. 504; and J. J. Thomson, *Conduction of Electricity through Gases*, 1903, 1st ed. 187, 1906, 2nd. ed. 223, 225, 261, 267.

[4] Lilienfeld, *Ann. der Physik*, 1901, xxxii. 673.

[5] *Phys. Rev.*, 1911, xxxii. 492. The result was also reached independently by Langmuir in 1913.

senting the receiving electrode, take the axis of x perpendicular to the planes. Poisson's equation [1] reduces in this case to

$$\frac{d^2V}{dx^2} = -4\pi\rho,$$

where V is the potential at the distance x and ρ represents the volume density of the charge at this point. When no electrons are emitted ρ is equal to zero everywhere, and therefore dV/dx=constant, and may be represented by the straight line PQT in *Fig.* 11. If a small number of electrons are being emitted the charge density will be negative, so that d^2V/dx^2 will be a positive quantity. This requires that the curve relating V and x should be convex downwards, as shown in the lower curve of *Fig.* 11. As the rate of emission of electrons rises, the quantity of negative electricity near the plate increases and the force tending to draw the electrons away decreases, the tangent to the curve at P will become more and more nearly horizontal. Any further increase in the supply of electrons will now have very little influence on the distribution of potential between the electrodes, since there is no force dragging them away from AP, and would have no influence at all but for the fact that the emitted electrons have an initial velocity.

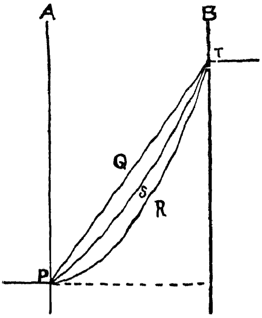


Fig. 11.

The increase in the current which accompanies the increase in the rate of emission of the electrons will go on till dV/dx becomes zero at the hot-plate, or, if we take into account the fact that the electrons are emitted with a certain initial velocity, attains a small negative value. After this the current will remain constant, however much the rate of emission of the electrons may be increased, *i.e.* the current will be independent of the temperature. In order to find out the relation between the current and the applied potential when dV/dx vanishes at the hot-plate, let $+e_1$ denote the numerical value of the negative charge of an electron and $+\rho_1$ the numerical value of the negative charge density at any point. Suppose V=0 at the hot-plate, then an electron at a point where the potential is V has the kinetic energy given by the equation

$$\tfrac{1}{2}mv^2 = Ve_1.$$

The current per unit area carried by the electrons at this point will be

$$i = v\rho_1,$$

and we have

$$\frac{d^2V}{dx^2} = 4\pi\rho_1.$$

Eliminating ρ_1 and v from these equations we get

$$\frac{d^2V}{dx^2} = 4\pi i\sqrt{\frac{m}{2Ve_1}}.$$

Integrating, subject to $dV/dx=0$ when V=0,

$$\left(\frac{dV}{dx}\right)^2 = 8\pi i\sqrt{\frac{2mV}{e_1}}.$$

By integrating again and solving for i we get

$$i = \frac{\sqrt{2}}{9\pi}\sqrt{\frac{e_1}{m}}\frac{V^{\frac{3}{2}}}{x^2}.$$

These effects have been investigated in detail by Langmuir.[2] He has shown that a similar calculation to Child's when applied to a thin hot wire, radius a, surrounded by a coaxial cylinder, radius b, leads to

$$i = \frac{2\sqrt{2}}{9}\sqrt{\frac{e_1}{m}}\frac{V^{\frac{3}{2}}}{r\beta^2},$$

where

$$\beta = \log\frac{r}{a} - \frac{2}{3}\left(\log\frac{r}{a}\right)^2 + \frac{11}{120}\left(\log\frac{r}{a}\right)^3 - \frac{47}{3300}\left(\log\frac{r}{a}\right)^4 + \ldots,$$

i being the current per unit length of the cylinder. He has also adduced arguments in favour of the view that the infra-saturation part of the current will vary as the 3/2 power of the applied voltage whatever the shape of the electrodes, and has supported this conclusion by experimental evidence. Some of his data are shown in *Fig.* 12, which represents the current between two hairpin-shaped tungsten filaments, under fixed differences of potential, at various temperatures. The experimental values lie along the broken curves, the continuous curve representing the saturation curve at different temperatures. At sufficiently high temperatures it will be seen that the curve becomes independent of the temperature.

The electrons, as we have seen, are emitted, not with zero velocity, but with various velocities. Consequently the electric field is not zero at the cathode surface until the current becomes saturated, and in the infra-saturation region there is a minimum potential at a place between the electrodes. This region of minimum potential approaches closer and closer to the cathode as the currents approach the saturation value.

[1] See "Potential," equation (14).

[2] *Phys. Rev.*, 1913, ii. 453.

At low temperatures when the emission and, consequently, the repulsion due to the space charge

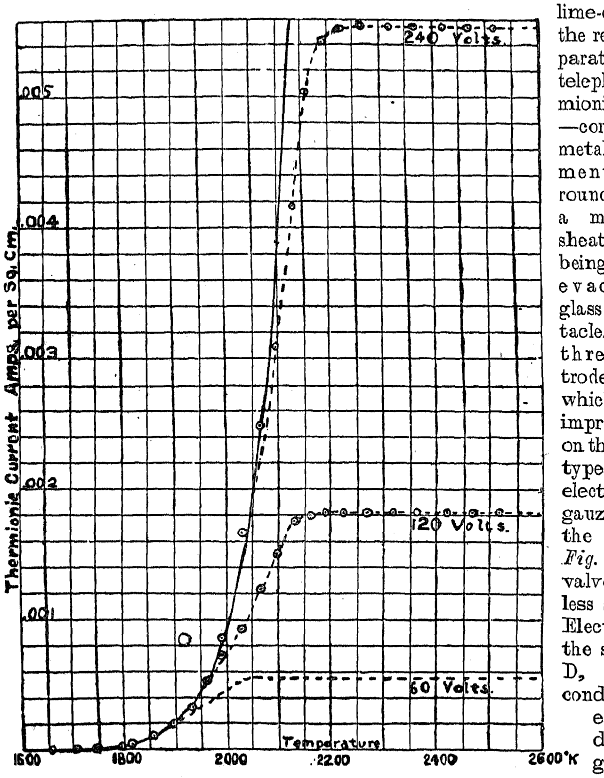


FIG. 12.

is very small we should expect saturation to be attained without the application of any potential difference; since all the electrons are ejected with some velocity. This expectation is not realised, however, even when the emission is very small. No doubt one important cause hindering the attainment of saturation is the effect of the magnetic field, due to the heating current, on the motion of the electrons. In general some of the electrons from the hot filament reach the electrode even when there is no applied potential difference so that the characteristic curve (*Fig.* 10) does not strictly start from the origin of co-ordinates, but from a point a little to the left of that. Other effects which in practice tend to modify the shapes of the current-voltage curves, or characteristics as they are often called, are the drop of potential down the electrically heated cathode caused by the heating current, the presence of conductors other than the two electrodes so far considered and the effect of contact potential differences (volta effect) between the electrodes. All these effects are most important with small accelerating potentials. As a result of their combined influence it is found in many practical three-electrode devices such as are used in the arts that the currents vary more nearly as the square of the applied voltage than as the 3/2 power.

(iii.) *Three-electrode Tubes.*—The electronic emission from a filament of hot tungsten or lime-coated platinum is utilised in the receiving and transmitting apparatus of wireless telegraphy and telephony. The earliest type of thermionic valve[1]—the Fleming valve—consisted of a glowing carbon or metallic filament surrounded by a metallic sheath, both being in an evacuated glass receptacle. The three-electrode valve, which is an improvement on this earlier type, possesses in addition a third electrode which has the form of a gauze or grid and is placed between the filament and metallic sheath, *Fig.* 13. The way in which such a valve can be used to receive wireless signals is illustrated in *Fig.* 14. Electric oscillations are set up in the system consisting of the aerial, D, the inductance, I, and the condenser, K, one side of which is earthed. The ends of the inductance are connected to the grid, G, and the filament, F, which is heated by the current from the battery, E. There will therefore be an alternating small potential difference between F and G. A battery, B, of from 20 to 150 volts, has its positive terminal connected to the sheath,

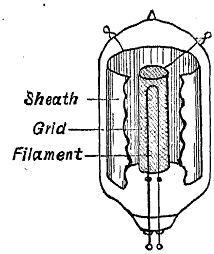


FIG. 13

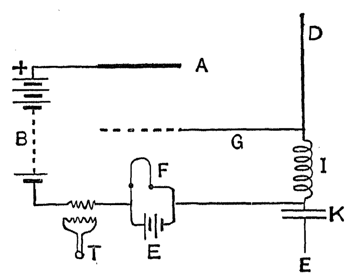


FIG. 14.

A, and its negative one to the filament. A telephone receiver, T, is coupled inductively with this circuit. It is clear that a current will flow in the circuit of the battery,

[1] See "Thermionic Valves," § (1).

B, and its fluctuations will actuate the telephone whenever F has a negative potential relatively to G, since the field between F and G will cause the emitted electrons to move in the direction of G and pass through its meshes. The superiority of this type of valve over the simpler Fleming valve is owing to the fact that the energy used to work the telephone is mainly derived from the battery, B, so that it is possible for the telephone to be actuated even when the oscillations in the aerial are very weak. Devices similar to the one described are employed as amplifiers and telephone repeaters and also for transmitting purposes.

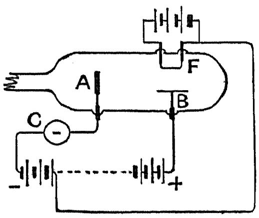


FIG. 15.

A three-electrode device furnishes a convenient means of measuring or comparing very low gas pressures. The three electrodes A, B, and F, *Fig.* 15, are sealed in a glass vessel, the pressure in which it is desired to measure. The electrode, F, consists of a carbon or tungsten filament, which can be heated by a battery as shown in the *figure*. The electrode, B, is in the neighbourhood of F and is maintained at a positive potential of say 100 volts relatively to F, while the remaining electrode, A, is maintained at a negative potential of about 10 volts relatively to F. A galvanometer, C, is employed to measure the current flowing through A. The hot filament, F, emits a stream of electrons which travel towards B and produce ions by colliding with the gas molecules. The positive ions formed are attracted to the electrode, A, and the current in the galvanometer gives a measure of the number of them reaching A per second. Since this is proportional to the rate at which positive ions are produced, and therefore also proportional to the number of molecules per unit volume, it gives a measure of the pressure of the gas.

§ (7) THE EMISSION OF POSITIVE IONS BY HEATED METALS OR SALTS. (i.) *Emission from Metals.*—If the hot metal filament (*Fig.* 1) is maintained at a positive potential relatively to the surrounding electrode a current passes from it to the latter, at any rate if the metal has not been heated too long. This happens even when the space between them is thoroughly exhausted. This and other experiments indicate that there is an emission of positive ions by freshly heated metals. One of the main features of the phenomenon is its transient character. The general way in which the saturation current varies with the time is shown in *Fig.* 16. It can be represented by the equation

$$i - i_0 = Ae^{-at},$$

where t is the time and A and a are constants. This formula can be deduced from the assumption that the ions which carry the part $i - i_0$ of the current are pro-

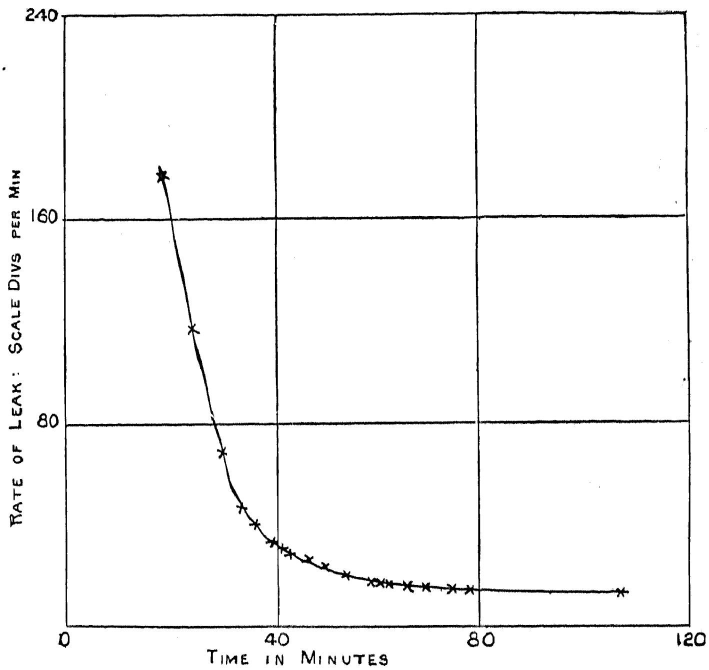


FIG. 16.

duced by the decomposition or evaporation of some substance present in the wire and that the rate of decomposition or evaporation is proportional to the quantity of the substance present. More complete investigation shows that i_0 is not strictly constant, but

varies slowly with the time in an exponential manner.

A wire which has lost the power of emitting positive ions through continued heating in a vacuum can have this power revived in the following ways:

(*a*) *By Distillation.*[1]—If an old wire, A, is mounted near a fresh wire, B, and B is heated and charged positively, A being cold, the passage of the thermionic current from B to A causes A to regain the power of emitting positive ions when heated again. The same thing occurs to a smaller extent when B is negative with respect to A or at the same potential. These experiments indicate that the emission is due, at least in part, to a substance which may be distilled from one metal to another.

(*b*) *By the Effect of a Luminous Discharge.*[2]—When an old wire is placed in a tube through which a luminous discharge is caused to pass in various gases at low pressure its power of emitting positive ions is revived. The effect is greatest if the wire is close to the cathode. If the wire is shielded from the cathode by a solid obstacle it disappears, showing that the revival is caused by something projected from the cathode.

(*c*) An old wire can be revived by heating the walls of the glass vessel in which it is mounted.[3]

(*d*) Klemensiewicz[4] found that an old wire is revived by exposure to atmospheres of hydrogen, nitrogen, or oxygen at pressures of 50 to 100 atmospheres at a temperature of about 200° C.

(*e*) Old wires are revived if heated for a short time in various gases or in a Bunsen burner.

(*f*) *By Straining.*[5]—A manganin wire has been observed to be revived when subjected to the strain caused by passing a current through it in a varying magnetic field.

(ii.) *The Effect of Temperature.*—The temperature variation of the emission has been investigated at low temperatures for which the rate of decay is small enough to make this possible.[6] The currents are found to follow the law which holds for the negative emission, namely,

$$i = \mathrm{A}\ \mathrm{T}^{\frac{1}{2}} e^{-\frac{b}{\mathrm{T}}}.$$

The constant b is, however, very much smaller than the corresponding constant of the negative emission. The value of b for silver has been found to be $1 \cdot 34 \times 10^4$ degrees C., less than one-fourth the value of the corresponding quantity for the electronic emission for most of the metals investigated.

The distribution of kinetic energy among the positive ions emitted from a platinum strip has been investigated by methods similar to those employed in connection with the corresponding problem for negative electrons. The results of such investigations have established the validity of Maxwell's law of distribution of velocities for the positive ions.

Measurements of the ratio of charge to mass indicate that the *initial* positive ionisation from hot metals consists of charged atoms of the alkali metals, chiefly atoms of potassium.[7]

(iii.) *Emission from Salts.*—Heated salts emit positive ions which are charged atoms of metals, not necessarily only metals which are constituents of the salts, but also metals which are constituents of some adventitious impurity.[8]

Measurements of the ratio of charge to mass of the ions emitted by the following salts: Li_2SO_4, Na_2SO_4, K_2SO_4, Rb_2SO_4, and Cs_2SO_4 indicate that they are in each case atoms of the constituent metal which have lost one electron. In the case of the lithium salt the ions emitted in the earliest stages of the heating had an atomic weight in the neighbourhood of 40 and were probably potassium ions originating from impurities in the lithium sulphate.

O. W. R.
W. W.

THERMIONICS, methods for the experimental investigation of. See "Thermionics," § (2).

THERMIONIC TUBES, use of, as detectors in radio-telegraphy. See "Wireless Telegraphy," § (22).

THERMIONIC VALVES

I. CONSTRUCTION AND PRINCIPLES OF ACTION

§ (1) CONSTRUCTION.—A thermionic valve, as originally described by Fleming[9] in 1904, consists essentially of two electrodes insulated from one another and mounted in a vacuum containing vessel. The function of the first electrode—usually known as the Cathode or Filament—is to provide a supply of free electrons. The second electrode—usually known as the Anode—is maintained at a positive potential with respect to the cathode, and its function is to attract the free electrons to itself; so establishing a current of electricity across the vacuum within the valve. With the ordinary convention this current is regarded as one of +ve electricity flowing from anode to cathode. In the majority of valves ionisation plays a relatively small part,

[1] O. W. Richardson, *Phil. Mag.*, 1903, vi. 86.
[2] *Ibid.*, 1904, viii. 400.
[3] O. W. Richardson, *Phil. Trans.* A, 1906, ccvii. 19.
[4] Klemensiewicz, *Ann. d. Phys.*, 1911, xxxvi. 796.
[5] O. W. Richardson, *Roy. Soc. Proc.* A, 1914, lxxxix. 521.
[6] Strutt, *Phil. Mag.*, 1902, iv. 98; O. W. Richardson, *B.A. Reports*, Cambridge, 1904, p. 473.
[7] O. W. Richardson, *Roy. Soc. Proc.* A, 1914, lxxxix. 521.
[8] O. W. Richardson, *Phil. Mag.*, 1910, xx. 981, 999.
[9] Fleming, *Roy. Soc. Proc.*, 1905, lxxiv. 488.

and in reality the current consists almost entirely of a flow of negatively charged electrons from cathode to anode. Except in valves intended only for rectifying, there are in addition to these two principal electrodes, one or more subsidiary electrodes by means of which the current from anode to cathode can be controlled with a relatively small expenditure of power. These subsidiary electrodes are usually in the form of open screens placed between the anode and cathode. From the actual shape taken by the third electrode of the early de Forest valves, the name "Grid" is given to the subsidiary electrodes.

In all the practical forms of valve that have been used up to the time of writing, the cathodes consist of metallic plates or filaments raised to a sufficiently high temperature for the emission of free electrons (see "Thermionics"). In the majority of valves filaments are used which are heated by the passage of currents from supplies external to the valves. For the other electrodes, nickel or molybdenum or other conductors having high melting-points are employed. The containing vessels are generally glass bulbs except in the case of the largest transmitting valves. For both the electrodes and the containing vessels the materials used must be such that the evolution of occluded gas can be reduced to a negligible amount by the treatment given during the pumping-out process.

Fig. 1 shows diagrammatically[1] the construction of two large valves. The cathodes are in the form of hairpin-shaped tungsten wires. The anodes consist of nickel cylinders surrounding the filaments and mounted on long supporting arms well clear of the glass bulbs. Between these two electrodes are the spiral grids serving as the control electrodes. The smaller receiving valves are usually of the same general construction, but in some American valves the anodes consist of two parallel sheets of metal, one on each side of filaments having the shape of an inverted W. In these latter valves the control electrodes consist of a pair of wire grids connected together and mounted one on either side between the anode plates and the filaments.

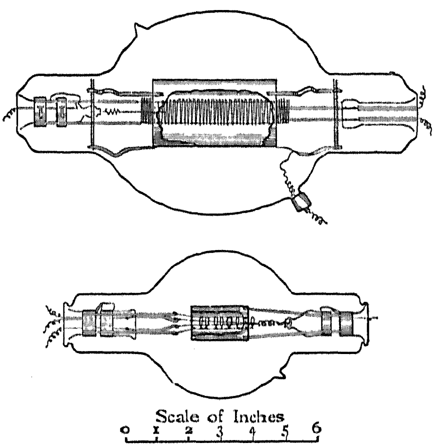


FIG. 1.

(By permission from *Inst. of El. Eng. J.*)

[1] Gossling, *Journal I.E.E.*, 1920, lviii. 678.

§ (2) ACTION OF THE CONTROL ELECTRODE.—The anode is usually strongly positive with respect to the filament, but the grid may be either positive or negative. Under working conditions the combined field due to the two electrodes is one which tends to attract away from the filament the free electrons emitted as the result of the thermal agitation. A stream of negative charges is thus set up away from the zone just outside the filament. The negative charges which are—at any instant—between the electrodes contribute a third component to the electric field at the filament; this component tends to repel electrons back into the filament again and to neutralise the field due to the electrodes. For any given adjustment of the electrode potentials, the stream of electrons can thus rise to some steady value that just reduces the field at the filament to zero. So long as the anode is not too strongly positive this rate at which electrons can be drawn across is less than the rate at which the electrons are emitted, the excess electrons merely falling back again into the filament. But as the anode becomes more positive a field strength is ultimately reached at which the electrons are attracted away as rapidly as they are emitted. The current thus set up is known as the saturation current, and if the emission is constant it is the greatest current that can be obtained however strongly positive the electrodes may be made. The value of the emission current depends on the temperature of the filament and increases very rapidly as this is raised.

Since the grid is nearer to the filament than the anode, a change of potential difference between it and the filament produces a greater change of field strength at the filament than would an equal change of the potential between the anode and filament. Thus a relatively small change of the grid-filament potential difference will cause a relatively large change of the current between anode and filament. As the electrons constituting this current reach the neighbourhood of the grid wires they are attracted both by the grid, if it is at a positive potential with respect to the filament, and by the anode. The attraction of the strongly positive anode tends to predominate, with the result that relatively few of the electrons actually fall upon the grid. The currents reaching the grid are thus small. In

those cases—frequently met with in practice—where the grid is actually negative to the filament, the grid currents are so small that they can be entirely neglected.

The grid thus provides a means of controlling the anode-filament current by small potential changes and very small current changes. That is to say, the control is exercised with the expenditure of but a very small amount of power.

In the foregoing account of the action of the valve it has been assumed that no part is played by the gas in the valve. The ionisation in the great majority of "hard" valves is so small that this assumption is justified. A few valves, however, have a small amount of gas admitted. If the electrode potentials are high enough, the stream of flying electrons ionises this gas, so producing positive ions. These positive ions move towards the filament comparatively slowly owing to their greater mass, and although produced at a relatively low rate the density of the charge carried by them may be comparable with the density of the charge carried by the electrons. In the neighbourhood of the filament the field from these positive charges neutralises the field of the negative electrons farther away from the filament, so enabling a greater current to be maintained for a given potential difference between anode and filament than is the case in a "hard" valve where the number of positive ions is negligible. If the grid is at a negative potential with respect to the filament, it attracts the positive ions and the grid current is either reduced or reversed. This reverse grid current is sometimes made use of as a means of estimating the pressure of the gas in the valve after it has been sealed off from the vacuum pumps.

§ (3) The Characteristic Curves and Contours.—The practical application of the valves depends upon properties that are usually delineated by certain characteristic curves.

For the two-electrode valve only one curve is necessary, viz. a curve of the current passing between the anode and the filament in terms of the applied P.D. Such a curve is shown in *Fig.* 2. This curve is the "$\frac{3}{2}$ power" curve of Langmuir,[1] allowance being made for the variation of the initial velocities of the electrons, for the fall of potential down the filament and for the variation of the temperature over the length of the filament. An increase of heating current in the filament does not change the shape of this curve appreciably but raises the saturation value on account of the increase of temperature throughout the whole length of the filament.

For the three-electrode valve two additional variables are introduced, viz. the P.D. between the grid and the filament, and the current entering the valve at the grid. One method

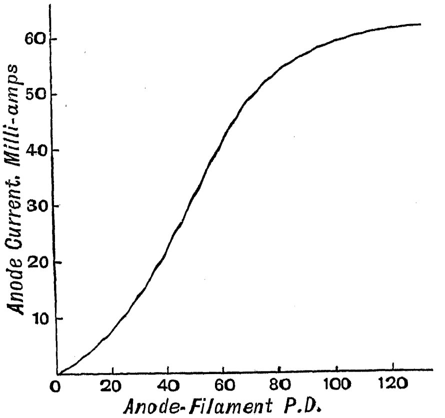


Fig. 2.—Observed Characteristic of Valve with Grid and Plate connected together and used as a Single Positive Electrode. Fil. Cur. = 1·4 Amps.

of showing graphically the properties of the three-electrode valve is that of *Figs.* 3 and 4. Here the anode and grid currents respectively are plotted in terms of the grid-filament P.D., each curve corresponding to a particular value of the anode-filament P.D.

Electrostatic considerations lead to the conclusion that the combined effect of the two electrodes at potentials V_a and V_g with respect

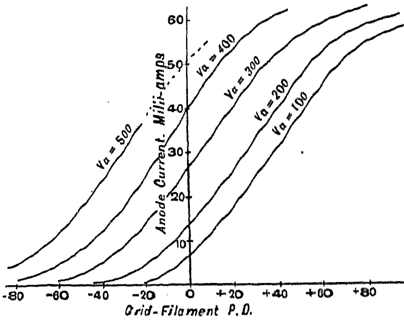


Fig. 3.—Observed Characteristic of 30-watt Transmitting Valve. Fil. Cur. = 1·4 Amps.

to the filament, is equivalent to replacing the grid wires by a hypothetical single electrode maintained at a potential V_h; where[2]

$$V_h = \frac{(V_g + (1/\mu)V_a)}{(1 + (1/\mu))}, \quad . \quad . \quad (3.1)$$

μ being a constant of the valve dependent upon the spacing of the grid wires and the distance apart of the electrodes. The current I_h at this hypothetical electrode will then have

[1] Langmuir, *Phys. Rev.*, 1913, ii. 450. See also "Thermionics," § (6) (ii.).

[2] Gossling, *Journal I.E.E.*, 1920, lviii. 675.

the same form as the characteristic of the two-electrode valve given in *Fig.* 2, and is given approximately by

$$I_h = \text{const.}\left\{\frac{(V_g + (1/\mu)V_a)}{(1+(1/\mu))}\right\}^{\frac{3}{2}} \quad . \quad (3.2)$$

so long as saturation is not reached. The ultimate destination of the electrons constitut-

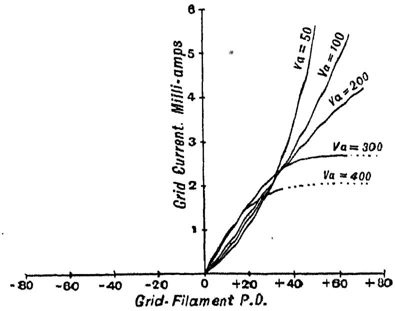


FIG. 4.—Observed Characteristic of 30-watt Transmitting Valve. Fil. Cur. = 1·4 Amps.

ing the hypothetical current I_h will depend upon the relative values of V_a and V_g. In general V_g is comparatively small and the projected area of the grid wires on the anode is a low percentage of the area of the anode. Hence under most conditions the anode current, I_a, constitutes almost the whole of I_h. Thus the curves of I_a in terms of V_g should be parallel for various values of V_a, as is found in practice and shown in *Fig.* 3. Though normally a small percentage of I_h, the grid current I_g is not entirely negligible, and when V_g is positive it becomes appreciable as shown in *Fig.* 4. The tendency for I_g to fall with values of V_g greater than about 50 volts and with high values for V_a is due to the liberation of secondary electrons at the grid which are drawn away to the strongly positive anode behind. When V_g exceeds V_a, these secondary electrons are drawn back again to the grid and I_g becomes large, even larger than I_a in some cases.

An alternative method of delineating the characteristics,[1] which is specially applicable to transmitting valves, is that shown in *Figs.* 5 and 6. Here the variables are V_a and V_g, and the lines are contours of constant grid or anode currents. The advantages of this form of characteristic are that the most important lines, viz. the lines of anode current, are the straight lines

$$V_g + \frac{1}{\mu}V_a = \text{const.},$$

and that in practice V_a and V_g are usually ascertainable from the circuit conditions, so

[1] Fortescue, *Electrician*, Sept. 1919, lxxxiii.

giving immediately the required variation of the currents. For the transmitting valves the

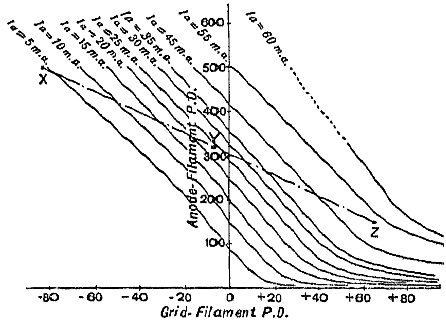


FIG. 5.—Observed Contours of 30-watt Transmitting Valve. Fil. Cur. = 1·4 Amps.

trace of the combined variation of V_a and V_g is approximately a straight line drawn across the diagram, as at XYZ and X′Y′Z′.

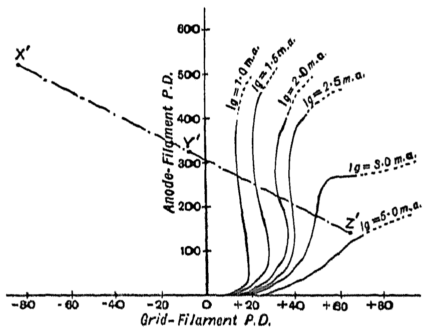


FIG. 6.—Observed Contours of 30-watt Transmitting Valve. Fil. Cur. = 1·4 Amps.

§ (4) THE RELAY ACTION.—Valves are usually working with steady filament currents —and therefore steady saturation currents— and certain definite initial adjustments of the steady potential differences V_a and V_g. Superimposed upon these initial potential differences are small variations v_a and v_g, usually practically sinusoidal in form. In considering the relay action the rate of change of each current with respect to both V_a and V_g is required.

Let

$\frac{\partial I_a}{\partial V_g}$, V_a being constant, be denoted by k_1,

$\frac{\partial I_a}{\partial V_a}$, V_g ,, ,, ,, ,, k_2,

$\frac{\partial I_g}{\partial V_g}$, V_a ,, ,, ,, ,, k_3,

$\frac{\partial I_g}{\partial V_a}$, V_g ,, ,, ,, ,, k_4.

For given initial adjustments, these rates of change are constants of the valve subject only to very gradual changes arising from such causes as the running down of the batteries.

Suppose in the first instance that v_a and v_g are of small amplitude and of sinusoidal form.

Let

$$v_g = \alpha_g + j\beta_g \text{ and } v_a = \alpha_a + j\beta_a, \text{ where } j = \sqrt{-1}.$$

The corresponding current variations are then

$$\left.\begin{aligned} i_a &= k_1 v_g + k_2 v_a = k_1\alpha_g + k_2\alpha_a + j(k_1\beta_g + k_2\beta_a) \\ i_g &= k_3 v_g + k_4 v_a = k_3\alpha_g + k_4\alpha_a + j(k_3\beta_g + k_4\beta_a) \end{aligned}\right\} \quad (4.1)$$

The power supplied *to* the valve at the grid is

$$P_g = \tfrac{1}{2}\{\alpha_g(k_3\alpha_g + k_4\alpha_a) + \beta_g(k_3\beta_g + k_4\beta_a)\}. \quad (4.2)$$

The power *given out by the valve* at the anode is

$$P_a = -\tfrac{1}{2}{}_a(k_1\alpha_g + k_2\alpha_a\ ) + \beta_a(k_1\beta_g + k_2\beta_a)\}. \quad (4.3)$$

Either the difference $P_a - P_g$ or the ratio P_a/P_g may be taken as a measure of the relay action. Often the two potential differences are practically opposite in phase and the k_3 and k_4 terms can be neglected, with the result that the expressions (4.2) and (4.3) are very much simplified. Neglecting the k_3 and k_4 terms is tantamount to assuming that the power absorbed at the grid is negligible compared to the power expended elsewhere.

If the potential variations v_a and v_g are not sinusoidal or are of large amplitude the extent of the relay action can only be found by graphical methods, *i.e.* by plotting out the curves of instantaneous power and finding mean values from the areas.

II. Transmitting

§ (5) General.—The valves used for the generation of the high-frequency currents necessary for Radio-telegraphic and Telephonic transmission are of the ordinary form but designed for the large amount of power required.

Tungsten is generally used for the emitting electrode, though in low-power transmitting valves oxide-coated platinum has been employed to a large extent in America. The tungsten is usually in the form of a straight or hairpin-shaped wire, heated by a current from an external battery. The addition of from 1 per cent to 1·5 per cent of thoria during the course of manufacture leads to improved mechanical properties and increases the electron emission [1] per unit area at a given temperature. Both the electron emission and the rate of radiation of energy increase rapidly with increase of temperature, the former more rapidly than the latter for normal filaments. The efficiency of the filament thus increases with increase of temperature. This increase of temperature is, however, accompanied by a corresponding decrease of effective life, and a compromise has to be struck depending upon the nature of the service for which the valve is to be used. A power expenditure of 150 to 200 watts per ampere of electron emission appears to correspond with a life of about 1000 hours under the manufacturing conditions prevailing at the time of writing.

The grid and anode are usually made of molybdenum or nickel on account of the high melting-points of these metals. The power radiated per unit area by the electrodes varies as a power of the absolute temperature between the fourth and the fifth, and it is found easier to remove occluded gases from a valve having small electrodes operating at high temperatures than from a valve having large electrodes operating at low temperatures.

For a tungsten electrode Langmuir [2] has given the following relation between watts per sq. cm. radiated ($\equiv$Ws) and the absolute temperature ($\equiv$T):

$$Ws = 12.54\left(\frac{T}{1703}\right)^{4\cdot 74}.$$

Provision has to be made in the arrangement of the electrodes for the high potential differences existing between them when in use. This is principally a question of the length of the surface over the insulation. Inside the valve a length of from 0·5 to 1·0 cm. per 1000 volts is satisfactory so long as the vacuum is good and so long as the surface is free from sputtered metal from the electrodes. Outside the valve, however, much longer leakage paths are necessary, with the result that the external form of the valves is, in many cases, similar to that of X-ray bulbs. Of the valves shown in *Fig.* 1, the larger is designed for a continuous dissipation from the anode of 400 watts and for a normal steady P.D. between anode and filament of 5000 volts, and the smaller for 150 watts at 2000 volts. In actual service the potential difference is oscillating at the radio-frequency between double the normal value and zero.

In the course of manufacture, the technique of building up the electrodes is closely similar to that of building up ordinary incandescent lamps. The exhaustion has, however, to be far more complete. Either rotary molecular or diffusion pumps are essential. After thoroughly exhausting at the highest temperature that the bulb will withstand, the electrodes are gradually heated up by electron bombardment until the power expended is from 50 per cent to 100 per cent in excess of the power that

[1] Langmuir, *General Electric Review*, 1920, xxiii. 6.

[2] Langmuir, *Phys. Rev.*, 1912, xxxiv. 401.

will be expended when the valve is working under normal conditions. The permanence of the vacuum depends upon the duration of the last stage of the exhaust, and it is largely a matter of experience to decide when it has been carried far enough.

§ (6) THE USE OF THE VALVE AS A HIGH-FREQUENCY GENERATOR.[1]—For this purpose the valve is invariably associated with an inductance-capacity circuit, or combination of such circuits, in such a way that the relay action renders the whole system unstable. The oscillatory current in the circuit is made to act on the grid so that the power output to the circuit from the anode of the valve exceeds the power input at the grid by an amount which is greater than the other losses in the circuit. The result is that any incipient oscillation is increased until limits imposed by the saturation current of the valve are reached.

The frequency of the oscillations is the natural frequency of the circuit, due allowance being made for the effective resistance and for the capacities of the valve and the connecting leads. Some variation of the interlinking of the valve with the circuit is generally provided, and change of frequency is accomplished by variation of either the capacity or the inductance of the circuit, or of both of them.

Many different circuit arrangements have been used and proposed. *Fig.* 7 represents

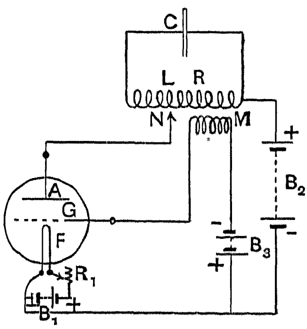


FIG. 7.

diagrammatically one such circuit. L, C, and R constitute the main oscillatory circuit linked to the anode of the valve through the variable contact N, to the grid by the mutual inductance M, and to the filament through the battery B_2 which constitutes the steady supply to the anode. With this circuit the potential variation between grid and filament may be either approximately in phase with, or opposite in phase to, that between the anode and filament, depending upon the connections of the mutual inductance M. The opposite phase connection is required for the maintenance of oscillations. The conditions necessary can be calculated exactly, but the results are cumbrous and the following approximation is sufficient for most practical purposes. Let $i=\sqrt{2}I\sin\omega t$ be a small alternating current in the main circuit, I being the R.M.S. value in amperes. The circuit losses are then approximately I^2R watts.

The variation of the grid-filament P.D.

$$v_g=\omega M\sqrt{2}I\cos\omega t,$$

and the variation of the anode-filament P.D. is

$$v_a=-b\omega L\sqrt{2}I\cos\omega t,$$

where $\omega=1/\sqrt{LC}$, and b is a fraction depending upon the position of N. The positive sign is taken for v_g because the mutual inductance M is so arranged that the grid and anode potentials are of opposite phase. The corresponding anode current is

$$i_a=k_1\omega M\sqrt{2}I\cos\omega t-k_2b\omega L\sqrt{2}I\cos\omega t,$$

and the power supplied *to* the circuit is

$$\begin{aligned}P_a=-\{&-b\omega L\sqrt{2}I\cos\omega t\\&(k_1\omega M\sqrt{2}I\cos\omega t-k_2b\omega L\sqrt{2}I\cos\omega t)\}\\=\omega^2L^2&\left(k_1b\frac{M}{L}-k_2b^2\right)2I^2\cos^2\omega t,\end{aligned}$$

the mean value of which is

$$\omega^2L^2\left(k_1b\frac{M}{L}-k_2b^2\right)I^2=I^2\frac{L}{C}\left(k_1b\frac{M}{L}-k_2b^2\right).$$

Neglecting losses at the grid, the conditions for oscillations are thus

$$I^2\frac{L}{C}\left(k_1b\frac{M}{L}-k_2b^2\right)>I^2R$$

or

$$b\frac{M}{L}>\frac{1}{k_1}\frac{RC}{L}+b^2\frac{k_2}{k_1}. \qquad (6.1)$$

Similar approximations may be applied to any other of the circuits used and similar results obtained.

Since v_a and v_g are practically opposite in phase, the trace of the potential variations on the contours of *Fig.* 5 becomes simply an inclined straight line which crosses the contours in such a manner that as v_a *falls*, v_g *increases*, this condition implying a power output *from* the valve *to* the circuit.

§ (7) THE LIMITATIONS OF THE AMPLITUDE OF THE OSCILLATIONS.—If the initial adjustments are such that an incipient oscillation is built up, the values of v_a and v_g increase correspondingly with I, and owing to the curvature of the characteristics the assumption that k_1,

[1] Fortescue, *Electrician*, Sept. 1919, lxxxiii.; Hazeltine, *Inst. Radio Eng. Proc.*, April 1918.

k_2, k_3, and k_4 can be regarded as constants is no longer valid. The variations of the anode current are no longer sinusoidal and the power output to the circuit can only be found by graphic methods. If this is done it is found that as I increases P_a also increases, but at a rate less than I^2. Hence although initially the combination of valve and circuit is unstable, the tendency is for it to become less and less unstable as the amplitude increases. The falling off in the rate at which P_a increases with increase of I becomes very marked indeed when the maximum value of v_a approaches the steady value V_a. It thus happens with even a wide range of adjustments that the stable balance between P_a and I^2R is reached when the amplitude of the anode-filament potential variation is approximately equal to the P.D. maintained by the battery B_2 (*Fig.* 7) or its equivalent.

Referring to the circuit of *Fig.* 7, this occurs at a value of I such that

$$b\omega L\sqrt{2}I=V_a. \quad . \quad . \quad (7.1)$$

§ (8) The Efficiency of Conversion at the Anode.—This is usually defined as the ratio of P_a to the total power taken from the steady source, the power supply to the filament being omitted. The efficiency varies from about 75 per cent downwards to 20 per cent or even lower in badly designed circuits.

The efficiency can be calculated approximately for the particular case in which the initial steady adjustments of V_a and V_g make I_a equal to one half of the saturation current I_e, and in which the amplitude of the oscillation is such that the anode-filament potential just falls to zero. The R.M.S. voltage variation at the anode is then approximately $V_a/\sqrt{2}$ and the current variation $I_e/2\sqrt{2}$, making the power output $I_eV_a/4$. But the mean power taken from the source will be $I_eV_a/2$, from which it follows that the efficiency is 50 per cent.

Any adjustment which tends to reduce the arithmetic mean current from the steady source *without* reducing the R.M.S. variation of the anode current will lead to an increase of the efficiency. Any choice of the initial values of V_a and V_g giving a *low* value of I_a, especially when V_a is high, gives rise to a tendency of this kind. It is by using high anode-filament potentials and negative values of the steady grid potential that an efficiency as high as 75 per cent can be obtained.

This increase of efficiency is, however, obtained at the expense of the wave form of the anode current variation. As the efficiency rises, the harmonics become more pronounced, and if the main L, C, R circuit has a natural frequency (or false harmonic) corresponding to any of these harmonics it follows that a considerable amount of power will be expended at this frequency instead of at the fundamental frequency.

Reverting to the special condition for 50 per cent efficiency, if I is the R.M.S. oscillatory current, then approximately

$$I^2R=\frac{I_eV_a}{4},$$

and substituting for I in (7.1)

$$b=\left(2\frac{V_a}{I_e}\cdot\frac{RC}{L}\right)^{\frac{1}{2}}. \quad . \quad . \quad (8.1)$$

If the steady grid-filament P.D. V_g is maintained by a battery, the equation (6.1) must also be satisfied with values of the slopes of the characteristics corresponding to this value of V_g. Frequently, however, the rectifying action of the valve is made use of for obtaining a negative value of V_g. In that case V_g is either zero or a small negative value just before the oscillations are started, and the values of k_1 and k_2 chosen for insertion in equation (6.1) must be those corresponding to this initial value of V_g, and not to the steady one reached when the oscillations have set in. Given the value of b from (8.1), the equation (6.1) then gives the value of the mutual inductance M required for the generation and maintenance of the oscillations.

III. Amplifying

§ (9) General.—Amplifying valves differ from those used for transmitting purposes in more than mere dimensions and power. Filaments operating at lower temperatures, so giving longer life, are essential, and the standard of uniformity must be higher since combinations of six or even nine valves may be used together under conditions where a small percentage change in any one valve will lead either to instability or great loss of amplification. The capacity between the electrodes becomes of fundamental importance and the grid currents cannot be neglected. The great difficulty of maintaining a high potential battery in perfect condition makes it imperative to design the valves to operate with low potential differences between anode and filament, and general considerations of the over-all dimensions of multiple valve amplifiers demand that the valve shall be as small as practicable whilst still remaining reasonably easy to manufacture in quantity.

Both pure tungsten and oxide-coated filaments have been used. Thoriated tungsten is at present unsatisfactory in that it has always given an irregular electron emission which, after amplification, leads to the well-known "valve noises." For the same reason with pure tungsten the temperature must be kept

as low as possible consistent with the supply of the electron emission necessary for the required characteristics. In this respect the coated filaments appear to have some slight advantage, which, however, is largely counterbalanced by the fact that in valves so fitted the other electrodes cannot be effectively bombarded during the exhausting process.

The other electrodes are usually of nickel or molybdenum and the method of exhaust is similar to that employed with the transmitting valves, but on a very small scale and continued only just long enough to ensure that no appreciable quantity of occluded gas is evolved when the valves are in use.

The limiting factor in the design of the amplifying vavles of the present cylindrical type is the minimum spacing between the grid and filament that can be employed when the valves are being manufactured in large numbers. To some extent, also, the cooling of the ends of the short filaments and the minimum thickness of wire that can be used safely are controlling factors. The result is that with the present form of valve employing an anode-filament potential difference of about 30 volts, characteristics are obtainable such that the slope k_1 is of the order of 300×10^{-6} amperes per volt and k_2, 40×10^{-6} amperes per volt; the normal saturation current being about 2 milliamperes.

Complete departure from the present type of valve appears to be necessary before any great improvement on these figures can be expected.

§ (10) The Characteristics. — The characteristic curves of amplifying valves are of the same general nature as those of transmitting valves. The combined effect of the grid and anode can be represented by a hypothetical electrode, and the current drawn away from the filament follows the "$\frac{3}{2}$ power" law when due allowance is made for the distribution of temperature and potential along the filament. Owing to the fact that the fall of potential along the filament is relatively much greater with the small valves, the apparent departure from the "$\frac{3}{2}$ power" law is greater than in the case of the large transmitting valves.

The same methods of plotting the characteristic curves are used as for the transmitting valves.

§ (11) The Use of the Relay Action for Amplification.[1]—The amplifying action of a valve is usually more a question of potential step-up than of ratio of power output to power input. This arises from the fact that it is not possible to construct the transformers and similar apparatus used in the complete amplifier in such a manner that the greatest possible power ratio is obtainable. The amplitude of the potential variation to be stepped up is usually very small and of practically sinusoidal form. *Fig.* 8 shows diagrammatically the circuit arrangement for a single valve.

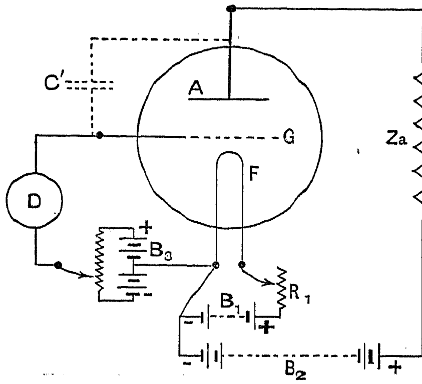


Fig. 8.

The small potential variation from the alternating source D is superimposed on the steady potential difference maintained between the grid and filament by the battery B_3. This results in a corresponding variation of the anode current and a corresponding change of potential across the impedance Z_a in the anode circuit. The ratio of the amplitude of the potential variations across the impedance Z_a to the amplitude of the potential variations applied to the grid is a measure of the potential step-up of the valve and the associated circuits combined.

If, as a first approximation, the capacity between the anode and the grid is neglected, then, using the same notation as for the transmitting valves,

$$i_a = k_1 v_g + k_2 v_a \qquad (11.1)$$

and

$$v_a = -i_a Z_a, \qquad (11.2)$$

whence it follows that

$$i_a = \frac{(k_1/k_2)v_g}{((1/k_2)+Z_a)}. \qquad (11.3)$$

The valve may thus be regarded as a source of alternating E.M.F. of value $(k_1/k_2)v_g$, having an internal resistance $1/k_2$ and acting upon a circuit of impedance Z_a.

Eliminating i_a from (11.1) and (11.2),

$$\frac{v_a}{v_g} = -\frac{k_1}{k_2 + 1/Z_a}. \qquad (11.4)$$

Thus if Z_a is made so large that $1/Z_a$ is negligible compared to k_2, then the voltage step-up attains a maximum value of $-k_1/k_2$. This ratio is often called the amplification factor

[1] Latour, *Electrician*, 1916, lxxviii. 280; Vallauri, *L'elettrotecnica*, January 1917; Latour, *Bul. Soc. Français des Electriciens*, July 1919; Fortescue, *J. Inst. El. Eng.*, 1920, lviii. No. 287.

of the valve. It is independent of the associated circuits and is realised when the impedance of the anode circuit is large compared to the internal resistance of the valve.

Considering the grid current,

$$i_g = k_3 v_g + k_4 v_a.$$

Substituting for v_a from (11.4),

$$i_g = v_g \left\{ k_3 - \frac{k_1 k_4}{k_2 + 1/Z_a} \right\},$$

and the effective impedance of the valve and circuits on the grid side becomes

$$Z_g = \frac{v_g}{i_g} = 1 \Big/ \left\{ k_3 - \frac{k_1 k_4}{k_2 + 1/Z_a} \right\}.$$

Very often k_4 is small, and then

$$Z_g = \frac{1}{k_3},$$

which means that the nature of the anode circuit and the currents flowing there have no appreciable effect on the grid circuit. The valve may thus be used as an amplifier and a coupling between two circuits without introducing the usual complicated effects observed in coupled oscillatory circuits. This, however, is on the assumption that the capacity between anode and grid can be neglected. In many practical cases this is not permissible, a very appreciable capacity coupling between grid and anode circuits arising from this capacity. Making allowance for this capacity, C′, equation (11.3) becomes

$$i_a = v_g \frac{y/k_2}{Z_a y_1 + 1/k_2},$$

where $y = k_1 - j\omega C'$ and $y_1 = 1 + j\omega C'/k_2$. Similarly equation (11.4) becomes

$$\frac{v_a}{v_g} = -\frac{y}{k_2 y_1 + 1/Z_a}.$$

§ (12) THE VARIOUS TYPES OF AMPLIFIER.—The types of amplifier are distinguished by the nature of the anode circuits. In the resistance amplifiers these circuits consist of high resistances having the smallest possible self capacity. With the existing valves it is unfortunately impossible to use resistances of suitable values in which the effects of the capacity are quite negligible when the amplification is at the high frequency of the radiated electromagnetic waves. But for audio-frequencies the effects are negligible, and in that case $Z_a = R_a$ and the voltage step-up is

$$\frac{v_a}{v_g} = -\frac{k_1}{k_2 + 1/R_a}.$$

The external resistance must thus be considerably higher than the internal resistance of the valve, if the maximum amplification is to be realised.

When the capacity across the resistance is not negligible, the effective impedance is

$$Z_a = \frac{1}{j\omega C_a + 1/R_a},$$

C_a being the capacity across the resistance.

In valves of present-day construction k_2 is of the order of 40×10^{-6}; $1/R_a$ should therefore be of the order of 10×10^{-6}. For a wavelength of 1000 metres, $\omega = 1{\cdot}884 \times 10^6$ and the value of C_a making the term $\omega C_a = 1/R_a$ is 4·8 cm. The impossibility of avoiding capacity in the leads and connections to the valve which is at least equal to this very small amount renders the resistance amplifier unsuited for working at radio-frequencies except with relatively long waves.

The variation of amplification of a single valve resistance amplifier as the frequency is changed is a curve as shown in *Fig.* 9

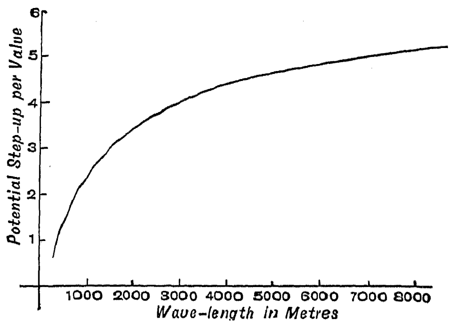


FIG. 9.—Resistance Coupled Amplifier. Amplification Factor $= k_1/k_2 = 5{\cdot}5$.

gradually rising towards the maximum value of k_1/k_2.

The effects of the capacity across the resistance in the anode circuit can be compensated for if the resistance is made inductive and the inductance is given such a value that resonance occurs at the frequency for which the amplifier is to be used. The curve of amplification in terms of the frequency will then have the form shown in *Fig.* 10. The sharpness of this resonance will depend upon the relative values of the capacity, the inductance, and the valve resistance. With only stray capacity, possibly of the order of 10 cm., and working on wave-lengths in the neighbourhood of 2000 metres, the inductance required is about 100 millihenries. If valves of internal resistance of the order of 25,000 ohms are connected across such a circuit the damping will clearly be very heavy and the resonance by no means sharp.

On the other hand, if an adjustable condenser of say 1000 cm. is used, then the necessary inductance is only about 1000 microhenries. This is a low reactance circuit

and the damping caused by a valve of equivalent resistance 25,000 ohms connected in parallel with it is relatively small. The sharpness of the resonance will then be princi-

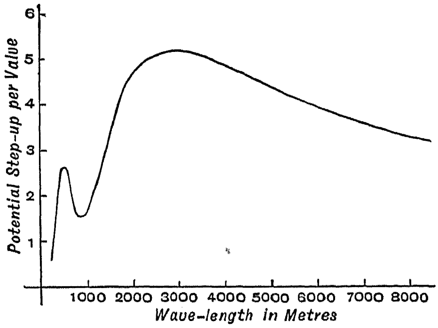


FIG. 10.—Inductive Resistance Coupled Amplifier. Resonance from Stray Capacity only. Amplification Factor $= k_1/k_2 = 5\cdot5$.

pally dependent upon the resistance of the inductance and of the condenser.

In general the effective grid impedance Z_g is a high one. If the valve is used with a small negative potential applied to the grid the value of k_3 may be less than 10^{-6}. It thus becomes possible to apply a relatively high voltage to this impedance before any appreciable power is absorbed. If, for example, the source of the grid variations is a tuned circuit consisting of a condenser of 1000 cm. and an inductance of 1000 microhenries, the power absorbed by the grid when connected across such a circuit will be very small compared to the other losses in the circuit. But if the capacity is reduced to 100 cm. and the inductance is increased to 10,000 microhenries, the relative absorption of power at the grid becomes much greater. But usually it is still only a small proportion of the total power losses in the grid circuit. The ideal condition is one in which one half of the power is expended at the grid and one half in the circuit. With $k_3 = 10^{-6}$ or less, giving an effective grid impedance of a megohm or more, this condition cannot be realised by reducing the tuning capacity. Attempts have been made to realise it by means of step-up transformers having secondaries of very low self capacity. Whether such transformers are of any real value for radio-frequency amplifiers remains, however, a moot point. But for audible frequencies the advantage to be gained by the use of step-up transformers is undoubted.

If ρ is the ratio of secondary to primary turns, the effective grid resistance on the primary side is $1/k_3\rho^2$. This may be in series in the primary circuit or in parallel. In either case the best value of ρ is that which leads to one half of the total power being expended in the circuit and one half in the effective grid resistance. It is found, however, even at audio-frequencies, that this ideal condition is very difficult to realise owing to the capacity effects.

§ (13) REACTION (OR RETRO-ACTION) EFFECTS.—Any relay device in which a direct transfer of power between the output and input sides is possible must necessarily be very sensitive to the extent to which this transfer can take place. A valve relay is no exception to this rule, and the transfer of the power is very difficult to avoid owing to the stray capacity coupling between the grid and anode circuits, either in the valve itself or in the leads. The phase may be such that the transfer may assist or oppose the original oscillation. If it is opposing, then the amplification is reduced; if assisting, it is increased, because for a given power in the output circuit the power input to the input circuit from the original source need not then be so large. This transfer of power is of great value in single-stage amplifiers and may contribute far more to the total amplification than does the valve step-up itself. The limit is obviously reached when the transfer back of power is sufficient for the circuits to be unstable. As this condition is approached the adjustment becomes more and more critical and in practice the degree of amplification obtainable by this means is very largely a question of the skill of the user of the instrument.

The control of this reaction may be obtained in various ways. With a resistance amplifier a part of the resistance in the anode circuit may also form a part of the grid circuit. This is inconvenient to provide for in the case

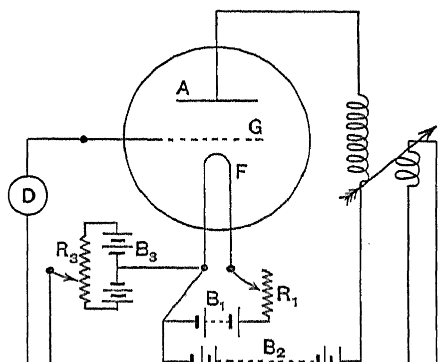


FIG. 11.

of a single valve, but is easy with two valves in cascade.

With an inductive anode circuit the control is very simply provided for as shown in *Fig.* 11. By means of the variometer either positive

or negative values of the mutual inductance are obtainable, so ensuring the possibility of obtaining the correct phase for all possible conditions.

Capacity coupling may be used and has distinct advantages because it is of the same nature as the stray capacity coupling. But capacity coupling, again, is inconvenient with a single valve and is more suited for multiple-stage cascade amplifiers.

§ (14) Cascade Amplifiers.—The use of amplifying valves in cascade, each valve amplifying the output from the preceding one in the series, is a natural development from the use of a single valve, and may be employed with resistance or tuned circuit amplifiers. Various means have been devised for so connecting

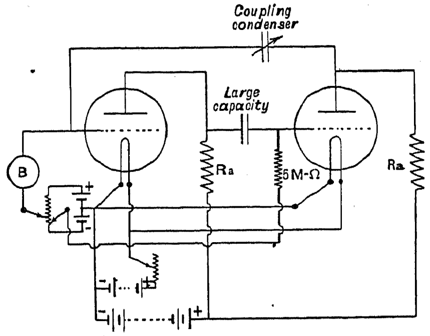


Fig. 12.

together the consecutive valves that the number of batteries and adjustments are reduced to a minimum. Owing to the greater step-up, the effects of reaction become of greater and greater importance as the number of valves in cascade is increased. *Fig.* 12 shows an arrangement of two resistance amplifying valves connected in cascade and having a capacity reaction control. *Fig.* 13 shows a similar arrangement with tuned circuits, transformer grid coupling and control of reaction by means of mutual inductance. The number of valves connected in cascade in these or other similar manners can be extended to four, six, or more valves as

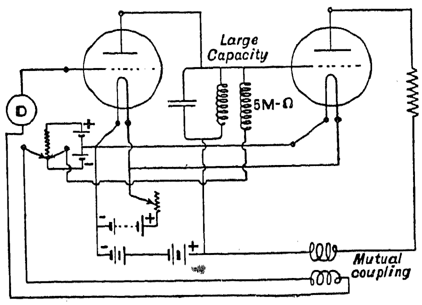


Fig. 13.

desired. Each extra valve contributes approximately its appropriate multiplier. Practical limitations, however, are soon met. As the amplifying power increases the control of the reaction becomes a very delicate balance between the unavoidable coupling and the adjustable coupling. In addition to this, the emission of electrons from the filaments of the valves is slightly irregular and these irregularities are amplified up with the signals and constitute in themselves such loud interference that the signals are either lost or have to be quite unnecessarily loud. An effective limit is thus reached at from four to six valves, beyond which there is nothing to gain with the valves at present obtainable.

§ (15) Multiple-frequency Amplifiers.—After the limit of amplification at a particular frequency has been reached the resulting signals may be rectified and a new series of lower-frequency amplifications started. In general, however, this stage cannot be taken as far as the first one unless some means are devised for sifting out the variations originating in the valves themselves. *Fig.* 14 shows one arrangement of a double-frequency

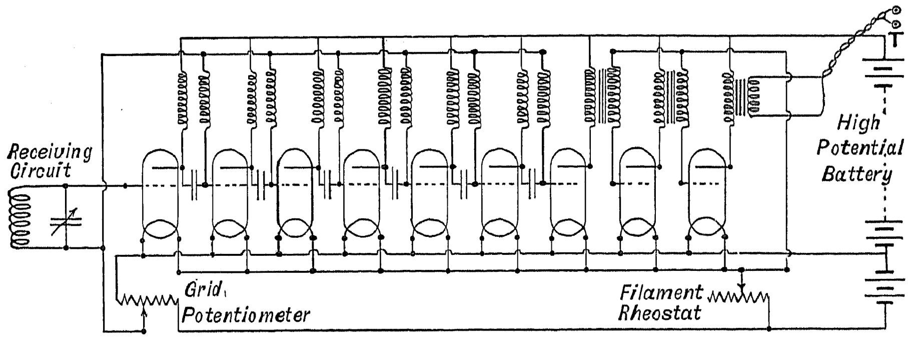


Fig. 14.

amplifier consisting of six high-frequency amplifying valves, one rectifying valve, and two audio-frequency amplifying valves. The cascade connections between both high- and audio-frequency amplifying valves are by transformers. At the high frequency the stray capacity gives a flat resonance at the wave-length for which the instrument is designed. At the audio-frequency, resonance effects are not employed.

C. L. F.

THERMIONIC VALVES, THEIR USE IN RADIO MEASUREMENTS

NOMENCLATURE ADOPTED

Triode = Three-electrode thermionic vacuum tube or valve.

Symbols used

Term.	Filament Circuit.	Anode Circuit.	Grid Circuit.
Battery in circuit .	B_1	B_2	B_3
Ammeters in circuit	A_1	A_2	A_3
Voltmeters in circuit	V_1	V_2	V_3
Steady potential .	..	v_a	v_g
Steady current . .	..	i_a	i_g

Equation of "straight" portion of characteristics:

$$i_a = av_g + bv_a + c.$$

Slopes of characteristics:

$$\frac{\delta i_a}{\delta v_g} = \text{“}a\text{”}; \quad \frac{\delta i_a}{\delta v_a} = \text{“}b.\text{”}$$

Voltage factor of tube:

$$m = \frac{a}{b}.$$

I. METHODS OF DETERMINING CHARACTERISTICS

§ (1) STATIC CHARACTERISTICS.—The chief characteristics of a three-electrode valve are: (α) the relation between anode potential and anode current, when the potential of the grid is maintained constant; and (β) the relation between anode and grid currents and grid potential, when the potential of the anode is maintained constant. A typical set of these characteristic curves corresponding to conditions (α) and (β) is shown in *Figs.* 1 and 2 respectively. These characteristics may be determined directly with continuous current by observing the current flowing between the different electrodes under various differences of potential between them. The necessary apparatus is illustrated by the diagram of connections shown in *Fig.* 3, which is set for the taking of characteristics of either set (α) or (β) above.

Since the filament is the source of the electron emission and the magnitude of the latter varies very rapidly with temperature, the filament conditions must be maintained very constant

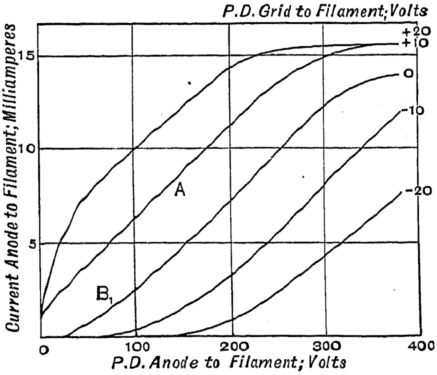


FIG. 1.

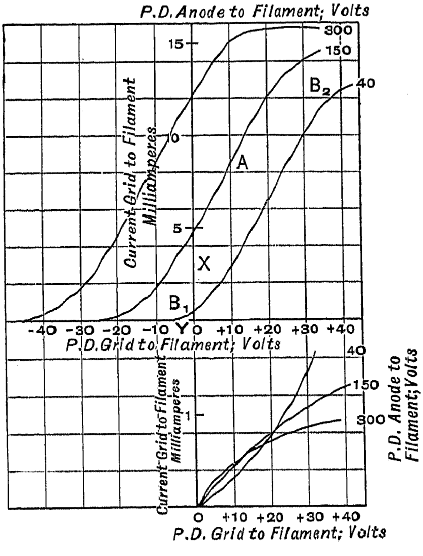


FIG. 2.

for the purpose of making any precision measurements on valves. A very sensitive

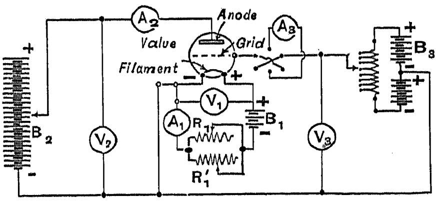


FIG. 3.

ammeter and voltmeter of the requisite range should be connected in the filament circuit,

as shown at A_1 and V_1 in the diagram, for the purpose of maintaining the resistance and hence the temperature of the filament constant throughout the tests. For the control of the filament current under these conditions, it suffices to have in circuit the two variable resistances R_1 and R_1' in parallel, the former being moderately coarse and carrying the majority of the current, and the latter having a much higher resistance and providing a fine adjustment. For very accurate work it is sometimes found necessary to include the filament in one arm of a Kelvin double-bridge with a sensitive galvanometer as an indicator by which the bridge may be kept accurately balanced.

The battery B_2 supplies the necessary positive potential difference between anode and filament, the applied P.D. being variable by means of a tapping from the battery, the best arrangement being to make use of a two-brush selector switch allowing of an increase in voltage in steps without disconnecting the anode circuit. A battery of small accumulators or Leclanché cells of a range of 200 or 300 volts is found convenient for ordinary use, but for large transmitting valves, an anode voltage of anything from 1000 to 10,000 volts may be required. It should be borne in mind that the anode current for small valves is comparatively small, rarely more than 50 milliamps, so that the current capacity of the battery need not be high. The P.D. between anode and filament is observed from the voltmeter V_2 connected as shown. This should be of very high resistance, or preferably of the electrostatic type, as otherwise the current passing through it will be much larger than the anode current, which is observed on the micro-ammeter A_2, connected with suitable shunts directly to the anode.

The potential difference between grid and filament is obtained from a battery similar to that used for the anode, but of smaller range—20 to 50 volts for ordinary purposes. This may be connected in a similar manner to the anode battery, or, as shown in the diagram, a simple potentiometer arrangement may be employed to obtain any desired potential on the grid, positive or negative, with respect to the filament. The P.D. grid to filament and grid current are observed on V_3 and A_3 respectively. In some cases the current is so small as to make a sensitive galvanometer necessary at the position A_3.

With the grid potential set to any definite value, the anode potential is increased in, say, 20 steps, the anode and grid currents being observed at each step, thus obtaining one of the (a) characteristics. Next, with a constant potential applied to the anode, the grid potential is varied in steps of say one volt on either side of the zero value, and the anode and grid currents observed, from which characteristics (β) are obtained. In this case, the direction of the grid current may be found to vary, particularly with soft valves, and such variations should be carefully noted as they afford information with regard to the amount of gas contained in the valve.

In cases where the whole of the characteristic of a valve is not required, the slopes a and b of the curves, at any desired setting, may be determined by an arrangement due to E. V. Appleton[1] and termed a "Slopemeter."

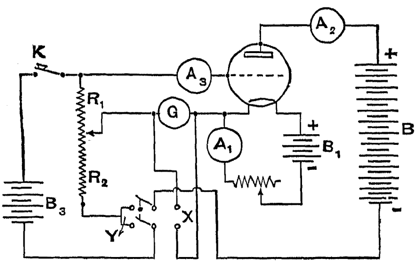


FIG. 4.

The diagram of connections is given in *Fig.* 4. The two measurements are made as follows :

(a) With the change-over switch in position Y, and the key open, G reads the normal anode current. If the ratio R_1 to R_2 is varied so that the deflection is unaltered on pressing K, then

$$\frac{a}{b}=\frac{R_2}{R_1}. \quad . \quad . \quad . \quad . \quad (1)$$

(b) With the change-over switch in position X, and key K open, G reads the normal anode current. If R_1 is adjusted so that there is no change in deflection when K is pressed, then :

$$a=\frac{\delta i_a}{\delta v_g}=\frac{1}{R_1}. \quad . \quad . \quad . \quad (2)$$

The proofs of formulae (1) and (2) are quite straightforward and will be readily seen on redrawing the diagram for the two positions of the switch separately. The galvanometer G should be of low resistance compared with R_1 and R_2.

§ (2) DYNAMIC CHARACTERISTICS. — The Slopemeter arrangement of E. V. Appleton above may also be used with alternating current of audible frequency to determine the slope a of the characteristics under dynamic conditions. To effect this, the circuit arrangement is altered as shown in *Fig.* 4A, corresponding to the position X of the change-over switch. The galvanometer G is replaced by a non-inductive resistance CD, and the alternating voltage supplied to the grid is obtained through a transformer as shown.

To determine the value of the slope

[1] See *Wireless World*, 1918, vi. 458.

$a = \delta i_a / \delta v_g$, the resistance R is adjusted until the alternating potential across CD is zero. The accuracy of this setting may be increased

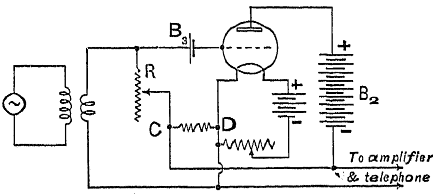


FIG. 4A.

by using a low-frequency valve amplifier in conjunction with the telephone receiver. By this means the applied alternating variations may be made very small indeed, and when the above condition is satisfied, the true tangential slope of the characteristic at the point of working is given by the formula:

$$a = \frac{\delta i_a}{\delta v_g} = \frac{1}{R},$$

providing that the impedance of the transformer winding is small compared with R, a condition easily satisfied. The batteries B_2 and B_3, giving the steady potentials in anode and grid circuits respectively, allow this measurement of the slope to be carried out at any desired point on the characteristic.

A somewhat similar arrangement may be used for the determination of the constants of a valve using alternating current. J. M. Miller has described the arrangement[1] shown in *Fig.* 5. The valve electrodes are brought

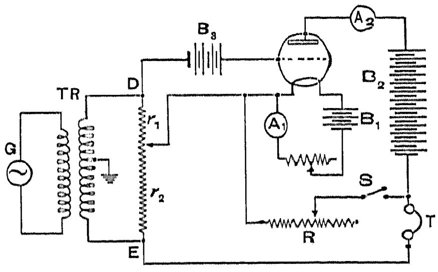


FIG. 5.

to the required steady potentials by the batteries B_2 and B_3, variable alternating E.M.F.'s being superimposed in grid and anode circuits through the potentiometer arrangement shown. G is a generator of alternating current of an audible frequency, which is supplied through the transformer TR to the slide-wire resistance DE, preferably a straight wire of about 7 ohms resistance. R is a dial resistance box reading up to 10,000 ohms, which may be connected in circuit by the switch S.

[1] *Proc. Inst. Radio Engineers*, 1918, vi. 141.

(*a*) For the determination of the amplification constant a/b, S is kept open, and the slider on DE adjusted until there is silence in the telephones T. In this condition, a simple consideration of the E.M.F.'s shows that

$$m = \frac{a}{b} = \frac{r_2}{r_1}. \quad . \quad . \quad . \quad (3)$$

(*b*) By closing S, setting the slider to give a definite ratio of r_1/r_2, less than the value of a/b above, and then varying R to get silence in the telephones, we have:

$$\frac{1}{b} = \left\{ \frac{r_1}{r_2} \cdot \frac{a}{b} - 1 \right\} R. \quad . \quad . \quad (4)$$

From (3) and (4), a can be determined, and hence this arrangement affords a very simple and rapid means of determining the constants of a valve with alternating current. The usual precautions must of course be taken, to avoid inductive and capacity effects, etc., such as shielding all leads and making suitable earth connections on the potentiometer DE.

In both this and the previous method, care must be taken that the E.M.F. applied in the grid circuit is not sufficient to take the anode current off the "straight" portion of the characteristic, as otherwise the results will be vitiated by the curvature at the ends of the latter.

These methods are very convenient and rapid for the determination of the slopes of the valve characteristics, in cases where large numbers of tubes have to be inspected, or where it is required to select valves to operate with given sets of apparatus, such as, for example, in the case of a cascade valve amplifier. The information obtained is perhaps not so complete as the plotting out of the whole characteristic, particularly in the case where experimental valves are being tested and developed, but the method serves well as a standard routine test on a valve before it is put into operation in a set.

In the ordinary way, it has been the practice to test and examine valves by means of their static characteristics, whereas in almost every case the valve is put into operation under alternating current. The above methods may therefore be used to detect any difference that may exist between the static and dynamic characteristics of a valve. Using the modern form of high vacuum tube, with the last described method and a supply of alternating current at an audible frequency, it has been shown that there is no lag of the plate current behind either the grid or anode voltage. In the case of the softer valves, the presence of the gas may bring about a difference of phase, but sufficient data are not yet available as to

the magnitude of this quantity, which in any case will probably be very small.

§ (3) "DERIVED" CHARACTERISTICS.—In the above descriptions of the methods of determining both the static and dynamic characteristics of valves, it was assumed that when the E.M.F. was applied to the grid-filament circuit, the E.M.F. in the anode circuit remained constant, and the variation in anode current was therefore brought about entirely by the control electrode. When the valve is used as a generator of oscillations, however, alternating current in anode and grid circuits brings about an alternating E.M.F. in the anode circuit, as well as in the grid circuit, the magnitude of which depends upon the self-induction in the anode circuit and the mutual inductance between the two circuits. Hence in this case, we have the potential of the grid and anode varying simultaneously, and in order to trace out the operation of the valve as a generator, it is necessary to determine the mode of variation of the anode current under these conditions. The curves which show this variation of the anode current are termed "derived" characteristics.

In the simplest types of valve generator circuits, the ratio of the variation of grid and anode potentials is equal to the ratio of the inductances in the respective circuits, and since the induced E.M.F.'s will be practically 180° out of phase, the variations will be opposite in sign, the potential of the anode decreasing when that of the grid is increasing and *vice versa*.

These derived characteristics may be determined directly from the apparatus depicted in *Fig.* 3. The circuit is first of all set to the steady battery conditions under which the valve is required to oscillate, and then the grid voltage is increased and the anode voltage decreased in the correct ratio, the anode and

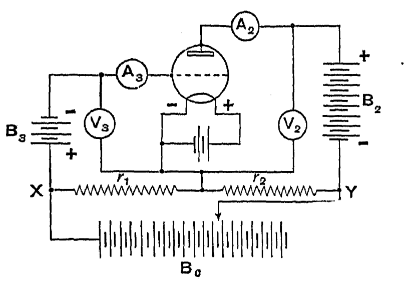


FIG. 6.

grid currents being observed for the successive steps. This method is readily carried out if the complete apparatus is set up as shown in *Fig.* 3, but is otherwise perhaps a little tedious. A simple arrangement is shown in *Fig.* 6, in which, in addition to the steady E.M.F.'s from the batteries B_2 and B_3, a variable E.M.F. may be added in opposite directions in each circuit from the potentiometer arrangement XY. The E.M.F. across this resistance is variable and obtained from the battery B_c, and the ratio in which this is divided $= r_1/r_2$, which is set to the required value. It is then only necessary to move the tapping on B_c and observe the corresponding currents and voltages, from which the derived characteristics may be plotted.

§ (4) "NEGATIVE" CHARACTERISTICS.—When the grid of a hard three-electrode tube is maintained at a relatively high potential (100-400 volts for a small triode tube), the behaviour of the device under steady current conditions becomes somewhat altered. For example, suppose that the grid of a small high-vacuum tube is maintained at 200 volts above the filament and that the anode potential is varied from 0 to 200 volts. The static characteristic obtained will be found to be of the general form shown in the diagram, *Fig.* 7.

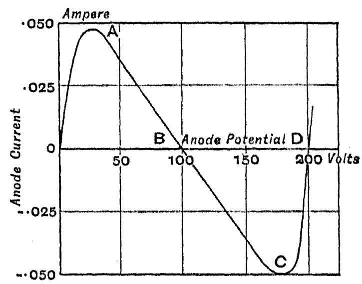


FIG. 7.

In the early stages, comparatively few of the electrons emitted by the cathode reach the anode, for although the electric field between grid and filament is strong, and consequently the acceleration of the electrons great, the field between anode and grid is such as to oppose the motion of these electrons. However, with increasing anode potentials, the anode current will increase until at the point corresponding to A, all the electrons passing through the grid are collected by the anode. At greater anode potentials, the opposing field decreases, and consequently the electrons strike the anode with constantly increasing velocity. Under the conditions assumed, this bombardment will be so great as to liberate secondary electrons from the anode, which moving in the direction of the electric field will be drawn towards the grid. The current in the external anode circuit will now be equal to the difference between the primary electrons striking it and the secondary electrons liberated by it. Hence arise the conditions under which an increase in anode

voltage brings about a decrease in the net anode current, due to the increase in the secondary electrons liberated being greater than the primary electrons absorbed.

The anode current characteristic will therefore follow the path ABC, the anode current attaining a zero value at B, and actually changing its sign over the path BC, when the number of secondary electrons liberated is greater than the number of primary bombarding electrons. As the anode potential increases, the corresponding field between anode and grid decreases; and so the directing force on the secondary electrons decreases, finally failing to send these across the space from anode to grid, after which all the secondary electrons liberated are reabsorbed by the anode. Following these latter conditions, the "negative" anode current attains a maximum value at C and finally decreases to zero at D and then becomes positive again, its value being now solely determined by the primary electrons.

Under the conditions represented by the part AC of the characteristic, the tube has the property of a "negative" resistance device, *i.e.* in which an increase in voltage brings about a decrease in current, and *vice versa*. If the conditions are adjusted to those corresponding to the point B, the part AC, which is practically a straight line, may be represented by the equation: $i = E/R$, in which R has a true negative resistance value.

Triode tubes, in which special consideration has been given to the design of the grid and anode for the production of a useful negative characteristic, have been constructed, and this type has been designated[1] "Dynatron" by A. W. Hull.

The applicability of this arrangement to the generation and amplification of oscillations will be referred to in the succeeding sections.

II. Application of Triodes to Radio Measurements

§ (5) Oscillation Generators. (i.) *Radio-frequency Currents.*—For the purpose of making radio measurements with any approach to accuracy, the first essential is to have a very suitable generator of the high-frequency alternating current to be used. The ideal generator should provide continuous oscillations perfectly sinusoidal and absolutely constant both in frequency and amplitude, combined with ease of control, freedom from noise or other disturbing property, and great flexibility in regard to frequency and strength. It should preferably be very compact in large and small powers, both for portability and purposes of screening.

By a suitable arrangement of circuits, the three-electrode valve may be made to act as a generator of oscillations which approaches more nearly the above ideal conditions than any other type of high-frequency generator. Except in the largest powers, it can be operated from a steady battery supply, and so is free from any difficulties of speed control inherent in rotary generators, whether the latter are used direct or merely as a primary electrical generator. In action the valve generator is perfectly silent, and by careful adjustment of the battery supply voltages the frequency of the alternating current generated remains constant to within a few parts in a million for hours at a time without any attention; and even this small variation seems to be quite regular, thus allowing the exact frequency to be calculated at any given time. The silence of operation is sometimes a most valuable feature of this generator, as it permits of the latter being situated very close to the measuring apparatus and that under test, thus eliminating the necessity of running long supply leads with the accompanying difficulties of screening and compensation of stray fields, both electrostatic and electromagnetic.

The use of the valve as a high-frequency generator, together with the general theoretical conditions involved, has been outlined in another article.[2] The general considerations which are there discussed for the application of a valve generator for radio transmission purposes are equally valid for the use of a valve generator as a source for the carrying out of radio measurements. The chief difference between the two cases is that in the former, the high efficiency is an important feature of the generating set, whereas in the latter, efficiency must nearly always be sacrificed to the obtaining of absolute constancy of output, and of purity of wave-form of the alternating current generated. *Fig.* 8, which is practically a reproduction of *Fig.* 7 in the above article, represents a typical arrangement which may be conveniently used as a source for radio-frequency measurements.

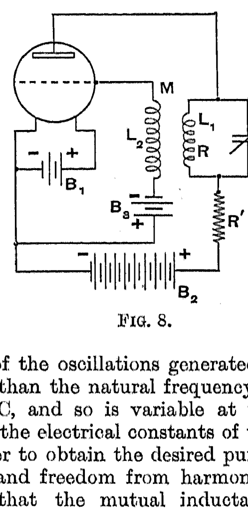


Fig. 8.

The frequency of the oscillations generated is slightly greater than the natural frequency of the circuit L_1RC, and so is variable at will by alteration of the electrical constants of this circuit. In order to obtain the desired purity of wave-form and freedom from harmonics, it is essential that the mutual inductance

[1] *Proc. Inst. Radio Engineers*, 1918, vi. 5.

[2] See "Thermionic Valves," II. §§ (6), (7), and (8).

coupling M between anode and grid circuits should not be too large, even though this tends to a great decrease in amplitude. Care should also be taken that the inductance coil connected in the grid circuit does not possess a natural frequency, due to its self capacity and the capacity of its leads and the valve electrodes, which coincides with the fundamental frequency or that of any of the harmonics of the oscillations being generated. It is preferable to arrange the grid circuit to have a frequency much lower than this fundamental frequency. These latter precautions are especially noteworthy in cases where a valve set is arranged for the generation of current over a large range of frequencies, by using not only a continuously variable condenser but also an inductance variable in steps by means of tappings, as it may easily be possible with certain critical adjustments for the grid circuit to be brought into resonance, resulting in abnormally pronounced harmonics in the current generated.

In view, however, of a possibility of similar occurrences resulting from the overhanging turns on the inductance in the anode circuit, it is far preferable to avoid making any tappings on this inductance, relying upon the variable condenser for making the necessary adjustments for obtaining the radio-frequency required. When necessary, one or more fixed condensers of suitable capacity may be connected in parallel with the variable condenser to increase the total range of frequency adjustment thus obtainable. When it is required to exceed this range, another pair of inductance coils should be substituted in the anode and grid circuits, these being carefully chosen for the particular frequency of range desired.

The circuit in *Fig.* 8 above referred to, may be varied by the introduction of a condenser across the grid inductance, and adjusting it to give resonance conditions. By this means the coupling in may be considerably decreased, while still maintaining the oscillatory condition, but great care is necessary in the tuning adjustment of both circuits to avoid the production of two or more frequencies.

The suppression of harmonics in a radio-frequency generator is important not only from the point of view of the error involved in certain measurements where the result is directly connected with the frequency, such as the high-frequency resistance of wires and coils, but also in those cases where the heterodyne method of detection is utilised in connection with the measurements. In this latter case, a small power auxiliary generator is employed whose oscillations produce a beat tone of audible frequency with the main oscillations, thus rendering the presence of the latter detectable in an ordinary telephone receiver. Beat notes will be produced with the various harmonics which may be present in either generator, and in their presence it may be very difficult to determine which tone refers to the true fundamental frequency required. It is easily possible with harmonics present in both generators to obtain as many as fifty beat-note coincidences within the range of an ordinary variable air-condenser. In such cases, coincidence corresponding to the fundamental frequency is not always readily detected by its prominence, and a somewhat laborious process of tracking it down may be necessary.

The number of ways in which the circuits above described may be varied is very large, and Hazeltine in a single paper[1] considers some eighteen different circuits, classifying them into groups dependent upon the mode of coupling, etc. The arrangement shown in *Fig.* 9 is

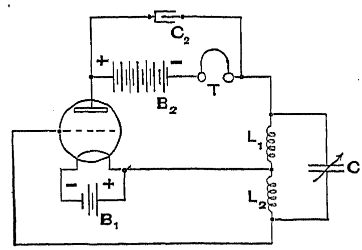


FIG. 9.

sometimes very convenient for use as a small generator for inductance or capacity measurements or for the measurement of frequency of continuous oscillations utilising the heterodyne method of detection. The inductances in the grid and anode circuits are combined in one coil, which is tapped off to the filament at its mid point. A condenser C_1 across the whole coil serves to tune the circuit to the required frequency. A pair of telephones is connected in series with the anode supply, which may be only a six-volt battery for weak oscillations, the telephones and battery being shunted by a condenser C_2 of about 0·003 mfd. capacity to bypass the radio-frequency oscillations. If the coil L_1L_2 be coupled to a coil in which other oscillations are circulating and the condenser C_1 adjusted, an audible beat note will be heard in the telephones when the circuits are nearly in tune, exact synchronism being obtained with moderately good accuracy by adjusting to the silence point. In such a case as this, where a low-power generator is run off a small voltage anode battery, the valve is naturally being operated on the lower bend of the characteristic curve, and the presence of harmonics is difficult to avoid. Hence such an arrangement can only be used when the main generator is giving a pure wave and the presence of harmonics in the auxiliary is not harmful; the beat note due to the fundamental of the latter being easily recognised by its greater intensity.

[1] *Proc. Inst. Radio Engineers*, 1918, vi. 63.

Either of the typical circuit arrangements shown may be used in setting up a generator to develop alternating current at frequencies of from two or three millions down to 10,000 cycles per second for radio measurement purposes, the upper limit being usually set by the difficulty of keeping the distributed inductance and capacity of the circuits very small.

The general principle involved in the design of such a generator is to utilise a valve with ample power supply and to operate the set at a moderately low efficiency in order to obtain great purity of wave-form. With this limitation, it will obviously be advantageous for the best efficiency to reduce the losses of the circuit to a minimum. This implies the use of condensers and inductances suitably designed to give ohmic resistance and distributed capacity at the lowest attainable figure. The condensers, both variable and fixed, should be of the highest quality, and of a type in which the dielectric loss is very small. The inductances, where these are of small values and for high radio frequencies, may conveniently be of the short single-layer type, using the usual forms of well insulated and stranded wire, and with the turns suitably spaced to reduce the losses and self-capacity to a minimum. For the higher values of inductance, in which a single-layer coil would become unduly large and inconvenient, a multi-layer coil may be employed, using, however, some form of "bank" winding, to keep down the capacity. As an alternative, the coils may be built up of the flat "pancake" type, using a basket or honeycomb form of winding. The latter type of coils has an advantage for valve generator sets, in that a much closer degree of coupling can be obtained between two or more of them connected in the grid and anode circuits of a valve, thus satisfying more easily the condition for the production of oscillations.

Instead of using electromagnetic coupling between anode and grid circuits, the latter may be coupled electrostatically by simply connecting a condenser between anode and grid, the remainder of the circuits being essentially the same. This is particularly useful in the case of extremely high frequency, where the distributed capacity of the valve electrodes and inductance of the leads is comparatively large, and any appreciable added inductance is almost prohibitive. Again, where it is desired to obtain oscillations of a very low frequency, when the decrement of the circuits is necessarily fairly large, it is useful to use both electromagnetic and electrostatic coupling to aid in the maintenance of the oscillations. By this means W. C. White has been able to generate[1] currents of frequencies ranging from one up to fifty million cycles per second, and by suitably arranging the circuits for the respective cases, he has obtained currents up to twenty-five amperes at 10^5 or 10^6 cycles per second, and potential differences of 12,000 volts, using only a single triode with an anode supply of 500 volts.

[1] *Gen. Elec. Review*, 1916, xix. 771; 1917, xx. 635.

For the purpose of utilising the current so generated by a triode set, it is sufficient to use a third inductance coil loosely coupled to the anode circuit and with leads taken off to the measuring apparatus at which the current is required.

In the operation of such a set, care should be taken to ensure constancy of conditions everywhere in the circuit. For this reason an accumulator battery supply for the anode is far preferable to a generator, and a separate battery should be utilised for the supply of filament current, which should be maintained as accurately constant as possible. When the valve set is first started the frequency of the oscillations will be found to vary considerably during the first half-hour or so, due to the heating-up of the electrodes and their effect on the filament temperature; but when steady conditions are attained, it will be found that the frequency can be maintained constant except for a small drift of the order of two or three parts in a million per hour. This condition naturally assumes that precautions are taken to avoid any change in capacity effects between the valve set and its leads and external objects, and it is desirable to screen as completely as possible the whole generator set.

(ii.) *Audio-frequency Currents.*—Where a generator is required to develop alternating current of a frequency between the audible limits for use in the usual audio-frequency bridge measurements, using a telephone or vibration galvanometer as detector, the three-electrode valve may conveniently be used in the same way as above described for the higher radio-frequency measurements. It has the advantage, over nearly every other type of audio-frequency generator, of silence and steadiness of operation combined with purity of wave and flexibility of frequency control. The ability to control the frequency very accurately within wide limits is an important asset, as it provides another degree of freedom in a set utilising either a tuned-diaphragm telephone receiver or a vibration galvanometer as the detector in bridge measurements, while the suppression of harmonics is advantageous in the former case in releasing the ear from the necessity of discriminating between the fundamental and the overtones heard in the receiver.

With due allowance made for the difference of frequencies, the principles of the arrangement of an audio-frequency valve generator are identical with those described above for radio-frequency currents. The circuit shown above in *Fig.* 8 is typical of a very convenient

arrangement for general use. In view of the much larger values of inductance and capacity required in the circuits, the self-capacities of the coils become relatively unimportant in giving rise to harmonics, while at audio-frequencies the eddy-current losses are very much reduced, and hence much less precaution is needed in the design of the inductances to be used. At very low frequencies the difficulty is usually met of designing coils of sufficiently high inductance, while keeping the resistance low in apparatus of reasonable dimensions.

The anode circuit inductances of an audio-frequency generator using the circuit of *Fig.* 8 above are conveniently made by winding a multi-layer rectangular-section coil on a short circular bobbin, in two exactly equal halves having an inductance of the order of a hundredth of a henry each, with the resistance of not greater than half an ohm. The grid inductance may be wound with fixed coupling on the same bobbin, to have an inductance of the order of a henry and a resistance of about 100 ohms. The anode coils may be used either in parallel or series, and in conjunction with a good mica condenser variable in steps from about 0·05 to 1 micro-farad, in parallel with a smaller continuously variable condenser, will provide current of any audible frequency usually required. To obtain frequencies of less than 100 or 200 cycles per second, it is usually necessary to use an iron core placed in the inductance, although this naturally introduces distortion effects and possibly harmonics. In cases where it is essential to retain the purity of wave-form at an abnormally low frequency, suitable air-core inductances must be constructed, although these will naturally be somewhat cumbersome.

(iii.) *Diplex Generators.*—In *Fig.* 10 is depicted a circuit arrangement for a triode tube acting as a diplex oscillator, *i.e.* generating oscillatory current of two different frequencies simultaneously. These frequencies may be comparatively close, such as two radio frequencies, but in the typical case illustrated one current is at a radio-frequency superposed on another at an audio-frequency, which arrangement may be found useful in certain types of radio-frequency measurements in which a telephone is used as an indicator.

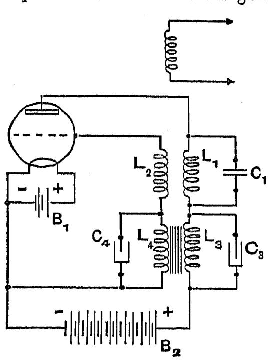


FIG. 10.

The coupled circuit $L_2 - L_1C_1$ acts as the radio-frequency generator in the manner explained above, while the circuit $L_4C_4 - L_3C_3$ acts similarly as the audio-frequency generator. The condenser C_4 is strictly only necessary when any appreciable alternating current of the higher frequency flows in the grid circuit, in order to bypass this current from the coil L_4 which would offer a high impedance to the current. In the anode circuit the radio-frequency current will find a path of low impedance through the condenser C_3, while the coil L_1 will offer a very small impedance to the current of audio-frequency.

The net current flowing in the anode circuit will consist of an audio-frequency oscillation modulated by the radio-frequency, as represented graphically in *Fig.* 10A. This current

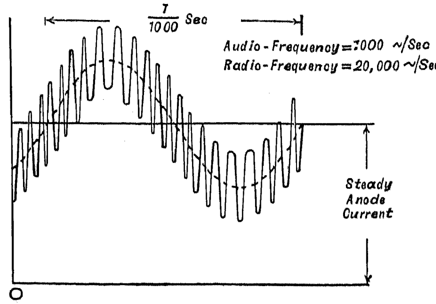


FIG. 10A.

may be used for any radio measurements where constant amplitude is not essential, and when finally rectified will produce audio-frequency pulses through a telephone receiver or other indicating instrument.

(iv.) *The Dynatron Generator.*—With the Dynatron types of triodes, which are specially constructed to give negative characteristics, oscillations may be produced using one oscillatory circuit only, as illustrated in *Fig.* 11, the action being somewhat similar to that

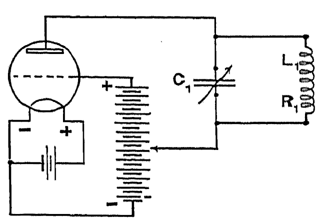


FIG. 11.

of the Poulsen arc. Such a device will always oscillate provided $R_1r < L_1/C_1$, where R_1 is the positive resistance in the circuit and r is the negative resistance of the tube. At low frequencies, and provided that L_1/C_1 is not too great compared with R_1r, the oscillations are practically pure sine waves and free

from harmonics. Although this device has an advantage in having only one inductance and one condenser in circuit, with the consequently greater ease of manipulation and flexibility of control, the high battery potentials necessary are apt to be a disadvantage, and the efficiency is not so great as that of the ordinary triode generator. A. W. Hull has described a dynatron for giving radio-frequency oscillations with an output of 100 watts, at an efficiency less than 50 per cent under the best conditions.

§ (6) Amplifiers.—One of the most important properties of a triode is that of serving as a voltage or current amplifier, and its first application in this respect was to provide greatly increased sensitivity to the ordinary radio-telegraphic receiver. As, however, the valve amplifier can be adapted to the magnification of alternating or interrupted currents of almost any frequency, such an instrument is found very useful in ordinary laboratory work to provide greater sensitivity in bridge measurements at radio or audio frequencies, or for the purpose of increasing the amplitude of various phenomena sufficiently to enable a string galvanometer or other recording instrument to be operated.

The general principles of amplification by means of valves, together with descriptions of the various circuit arrangements which may be used, have been dealt with in some detail elsewhere.[1] In general, the types of amplifiers used in the laboratory are the same as those used for radio-telegraphy with which the above article deals. In some cases, however, the more stringent conditions of laboratory work require a modification in some of the details of the amplifiers.

Where all that is required is to magnify the very small alternating currents which operate the detector in a bridge or other measuring circuit, the problem is a relatively simple one. A compact form of multi-valve cascade amplifier may be used, with either resistance or transformer coupling as described in § (12) of the above article. As is mentioned in the latter, the effective voltage or potential amplification produced by the resistance-coupled amplifier falls off very rapidly at moderately high radio-frequencies due to the stray capacity of the resistances. With transformer coupling circuits, the effective amplification produced attains a maximum value in the neighbourhood of the resonance frequencies of the transformer windings, and any large departure from these frequencies results in a great loss of amplification, particularly at frequencies of 500,000 and upwards.

The resonance curves of such transformers may be somewhat flattened by using high-resistance wire for the windings, thus increasing the resistance relatively to the reactance component of the winding. Thus by using a sectional-wound transformer of resistance wire, with tappings on both primary and secondary windings, several blunt points of resonance may be provided, and the effective amplification may thus be rendered more uniform over a wide range of frequencies. On this principle a compact form of radio-frequency cascade amplifier of two or more valves may be built up with the necessary switches for selecting the transformer tappings, and also common battery leads, filament rheostats, and input and output terminals.

Another manner in which efficient amplification may be carried out over a large range of frequencies is to utilise a similar arrangement of transformers, but to provide also each winding of the transformers with a condenser in parallel, by means of which each circuit may be tuned to resonate with the particular frequency being utilised. The only practical disadvantage of this method is that it requires rather fine tuning of all the circuits, which with a number of valves in cascade may become somewhat laborious.

For audio-frequencies, the problem of obtaining uniform amplification is much simpler. At these lower frequencies the self-capacity of the windings is insufficient to produce any marked resonance effects, although such conditions may be approximately realised by shunting the windings with suitable condensers. In audio-frequency transformers, it is practically essential to use a laminated iron core in order to obtain the requisite impedance in reasonable dimensions. While, as in the case above, the impedance of the transformer windings depends upon the frequency used, the variation is not very marked owing to the high effective resistance, hence the amplification varies comparatively slowly with frequency. One advantage of audio-frequency amplification is that a step-up of voltage may be utilised in the inter-valve transformers, which naturally increases the net magnification produced. In regard to this step-up of the voltage, a limitation is set by the current in the secondary winding due to (1) the self-capacity of the winding, (2) the capacity of the valve electrodes, and (3) direct conduction through the valve.

The effect of (3) may be considerably reduced by employing a steady E.M.F. in the circuit to maintain a negative potential on the grid of the valve, for direct conduction in a hard valve is only appreciable when the grid is positive with respect to the filament. This suppression of the grid current in an amplifier is also necessary to avoid distortion, for since the current only takes place during one half-cycle, the effective E.M.F. is reduced during that period and the whole wave distorted. The current due to (2) is inherent in the

[1] See "Thermionic Valves," III. Amplifying.

construction of the valve and also its supporting socket, but certain types of valves are constructed to reduce the inter-electrode capacity to a minimum. The capacity currents (1) due to the winding itself give rise to a rather difficult problem in inter-valve transformers, for the capacity of such a winding is very difficult to calculate and almost as difficult to measure. The general principle followed in the secondary of the transformer is to reduce the self-capacity as much as possible. In the case of radio-frequency transformers, both primary and secondary windings are wound in sections, placed in alternate slots in an ebonite bobbin. With a view to reducing the amount of wire used to give the requisite reactance, such a transformer may be provided with an iron core of very thin stampings in which the losses at radio-frequencies are not prohibitive to its use. For audio-frequency currents it is practically essential to use an iron core, but here again a sectional winding may be employed if space permits.

At very low frequencies, where the design of suitable circuits becomes somewhat difficult or the transformers too bulky, it is sometimes convenient to use the low-frequency current to modulate the amplitude of a radio oscillation from an independent generator set, and then amplify the latter current to the required extent, using a radio-frequency amplifier, the resulting current being finally rectified to obtain the effect of the amplified low-frequency current. This latter mode of amplification has a great advantage in that one may employ oscillations of constant frequency as the carrier current, and use a suitable amplifier specially designed for this particular frequency. The modulating current is then always amplified under constant conditions. This is an important consideration in certain cases of accurate measurement when it is required to know exactly what ratio of current magnification is produced, or at least to maintain this ratio accurately constant.

While they cannot be considered as precision measurements, methods of determining the absolute magnification produced by an amplifier under working conditions have been developed, and reference to these will be found in the bibliography accompanying this section.

Next to constancy of amplification, absence of distortion is the usual requirement in a valve amplifier for laboratory use. As regards the triode tubes themselves, distortion is avoided by ensuring that there is little or no grid current passing as mentioned above, and also by ensuring that in each tube the current and potential variations are so small that they are determined by the straight portions of the tube characteristics. Usually the conditions can be maintained within these limits, but in certain cases of high magnification, where the amplitude of the alternating current exceeds the limits of the straight portions of the characteristics, it may be necessary to use larger tubes towards the end of the series, which have a greater thermionic emission. Alternatively, two or more small tubes, having practically identical characteristics, may be used at each step, with their corresponding electrodes connected in parallel.

At radio-frequencies and using accurately tuned transformer coupling, little distortion will be produced in a well-arranged amplifier with valves operated under the conditions mentioned above. With audio-frequency transformers, however, considerable distortion may be produced due to eddy-currents and hysteresis losses in the iron core. This may be minimised but not eliminated by the use of very thin stampings or fine wire to form the core. Where distortion must be prevented even at the expense of amplification, a non-inductive resistance-coupled amplifier will be found to have an advantage over the transformer type for audio-frequency work. In cases where the low frequency is impressed upon a high-frequency carrier current, then the distortion involved is determined only by the conditions for the latter.

The Dynatron type of triode tube, which, as mentioned in § (4), possesses the property of a negative characteristic, may be used to give very large amplifications of either current or voltage.

One arrangement in which it can be used as a current amplifier is obtained from *Fig.* 11 if we replace the oscillatory circuit L_1C_1 by a non-inductive resistance R_2 in the anode circuit of the tube. If leads are connected from the terminals of this resistance to an external circuit, then any small alternating current flowing in the latter will result in greatly magnified currents through the resistance and valve respectively. If I is the alternating current in the external circuit and i_1 the alternating component of the current in the resistance R, then

$$\frac{i_1}{I}=\frac{-r}{-r+R},$$

where $-r$ is the value of the negative resistance of the Dynatron. Obviously, the ratio i_1/I increases very rapidly as R approaches the value r.

By slightly rearranging the circuit so that the external E.M.F. may be applied to the triode and the resistance R in series, a very small applied E.M.F. will result in a very large current flow, and hence in a greatly magnified P.D. across the terminals of the resistance.

Various other arrangements of this device may be used in which its negative resistance is

used to neutralise the positive resistance of an external circuit. For further details of these and also of a special type of vacuum tube containing four electrodes, termed a Pliodynatron, with similar properties, reference may be made to the original paper by A. W. Hull.

An arrangement has been described by L. B. Turner, whereby two ordinary triode tubes are interconnected with suitable batteries and resistances to form a combination possessing the property of a negative resistance. The device has been designated the "Kallirotron," and may be used in an analogous way to the Dynatron for purposes of amplification and also for the generation of oscillations.

R. L. S.-R.

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THREE-ELECTRODE THERMIONIC TUBES and their application to wireless telegraphy. See "Thermionics," § (6) (iii.).

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TIMED SPARK, THE: a method of producing undamped electrical oscillations by means of sparks, arranged to occur at certain definite intervals. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (3).

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TONE WHEEL, THE: a device for facilitating the detection of continuous waves in radio-telegraphy. See "Wireless Telegraphy," § (24).

TORQUE OF DYNAMO-ELECTRIC MACHINE, expression for. See "Dynamo Electric Machinery," § (6).

TOTAL FORCE OBSERVATION: determination, with a dip circle, of the value of the resultant or total magnetic force of the earth's field at a required spot. See "Magnetism, Terrestrial, Observational Methods."

TOTAL LOSS: a term used for the energy losses due to hysteresis and eddy currents occurring in magnetic materials. See "Magnetic Measurements and Properties of Materials," § (1).

TRACTIVE FORCE OF MAGNETS: the pull exerted by a magnet, stated by Maxwell's Law to be equal to $B^2A/8\pi$, where B = flux density and A = area of contact surface. For applications see "Electromagnet," § (4).

TRANSFORMER CHARACTERISTICS, DETERMINATION OF. See "Transformers, Static," § (12).

TRANSFORMER CURRENT. See "Transformers, Instrument," § (2).

TRANSFORMER DESIGN, OUTLINE OF. See "Transformers, Static," § (11).

TRANSFORMER EFFICIENCY, COMPUTATION OF. See "Transformers, Static," § (13).

TRANSFORMER GAIN: the gain which would be obtained by inserting an ideal transformer at the junction of telephone circuits. See "Telephony," § (36).

TRANSFORMERS, INSTRUMENT

§ (1).—THE ideal instrument transformer may be defined as a transformer[1] the secondary winding of which furnishes a voltage or a current which is a known fraction of, and in phase with, the primary voltage or current: in practice a departure, large or small, from this ideal is inevitable.

Instrument transformers are employed

(i.) Where the voltages or currents to be measured are too large to be conveniently applied directly to a measuring instrument.

(ii.) To enable the whole of the measuring and protective gear employed on the face of a switchboard to be electrically isolated from a high voltage system.

(iii.) To allow the use of measuring instruments wound for one standard voltage and one standard current for all circuits.

Instrument transformers are divided into two main classes :

(i.) Current or Series Transformers.

(ii.) Potential, Voltage, or Shunt Transformers.

§ (2) CURRENT TRANSFORMERS.—While not differing in principle from power transformers, current transformers are of a somewhat special character, since the primary current is fixed by the load on the line, and not by the apparatus connected to the secondary winding.

[1] For an account of the theory of the transformer see "Transformers, Static."

Current transformers are of two types :

(i.) Wound-primary transformers, in which the primary winding is provided by the manufacturer.

(ii.) Inserted-primary or "straight through" transformers, in which a space is left for the insertion of a cable or bus-bar, forming the primary winding.

The former class is used for moderate currents and voltages: the latter, which is more robust and less difficult to insulate satisfactorily, is universally used where conditions are severe. The British Standard Specification No. 81, 1919, requires that the full load secondary current of current transformers of all classes shall be 5 amperes.

§ (3) GENERAL THEORY OF CURRENT TRANSFORMERS.—A simplified vector diagram for a current transformer of unity ratio is given in *Fig.* 1. In this diagram, which is not to

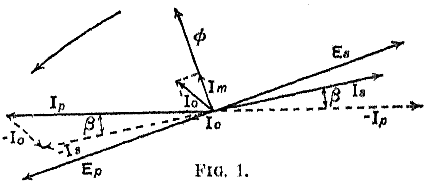

FIG. 1.

scale, I_p is the primary current (the current which the transformer is employed to measure), I_o is the exciting current, made up of a power component I_c (the "core-loss current"), in phase with the primary electromotive force E_p, and a magnetising component I_m, in quadrature with E_p and in phase with the flux ϕ. The secondary current I_s is equal and opposite to the vectorial difference of I_p and I_o.

It will be seen that the effect of the exciting current I_o is to cause the ratio of transformation I_p/I_s to differ from the ratio of the number of turns in the windings, and to introduce an angle β, known as the phase angle, between the primary and the reversed secondary currents. The ratio and phase angle therefore vary with the magnitude and phase of I_o relative to I_p, that is, with the primary current and with the character of the secondary burden.

This variation of ratio makes it impossible to adjust the transformer so that the value coincides with the marked ratio under all conditions, and errors are thereby introduced into measurements both of current and of power.

The effect of the phase angle β is to cause the current in an instrument used with the transformer to differ in phase from the current I_p: an error is thereby introduced into power measurements on inductive circuits

which varies with the current I_p, and with the circuit power factor.

§ (4) Design of Current Transformers.—The principal special consideration in the design of current transformers is the reduction of the no-load current I_0 to the lowest possible value. The principal points requiring attention are:

(i.) Selection of iron having low hysteresis and high permeability at low flux densities.

(ii.) Use of a low flux density, by provision of large cross-section of iron.

(iii.) Large number of turns in the windings, since the magnetising current for a given flux density is inversely proportional to the number of turns.

(iv.) Use of a closed magnetic circuit, carefully built up of well-insulated laminae.

(v.) Reduction of magnetic leakage to the lowest possible value.

(vi.) Liberal design of windings.

§ (5) Selection and Operation.—The same object is to be sought as in design. The secondary burden, or loading, should not be larger than necessary, and the instruments should be of low impedance, so as to keep the necessary secondary voltage low: trip coils and similar apparatus should never be connected to instrument transformers actuating precision instruments.

Single-turn or "straight through" transformers will not give good results when made for low currents.

The secondary winding of a current transformer must never be open-circuited while current is flowing, since the demagnetising effect of the secondary winding is thereby removed, and the flux rises to high values, causing greatly increased iron losses and rapid heating. Further, when the secondary circuit is again closed the iron core may be left in an abnormal magnetic condition, resulting in serious changes in the ratio and phase angle of the transformer. For the same reason direct current should never be allowed to flow through a current transformer winding. The effect of magnetisation due to either cause may be removed by applying an alternating current to the secondary winding, the primary being open, and gradually reducing it to zero. Recent work indicates that this current need not exceed 0·2 ampere: it should be kept low so as not to endanger the secondary winding by the application of a high voltage.

§ (6) Characteristics of Current Transformers.—Current transformers vary greatly in performance, since very great care in design and manufacture is necessary to secure good results. Curves illustrating the characteristics of a high-grade portable transformer are given in *Figs.* 2 and 3. These probably fairly represent the performance of the best transformers at present obtainable, but switchboard transformers, and even some portable types, may have phase angles exceeding 3° at $\frac{1}{10}$ load under the same conditions, and the ratio also may be greatly in error. The vector of secondary current is usually in advance of that of primary current, but the reverse

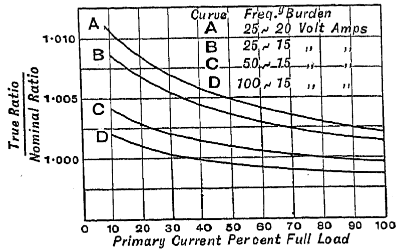

Fig. 2.

condition is sometimes encountered near full-load current with small secondary burdens.

Effect of Frequency.—Since halving the frequency doubles the flux necessary for a given secondary voltage, the effect of frequency

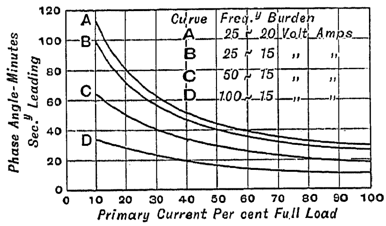

Fig. 3.

variation is considerable, especially upon phase angle. The curves of *Figs.* 2 and 3 show this for the transformer already mentioned. The magnitude of the changes depends upon the grade of transformer. The result of lowering the frequency is in general to increase the ratio and phase angle, and also to make the rate of change of these quantities with primary current greater.

Effect of Secondary Burden.—An increase of secondary burden also involves an increase of flux, and acts in the same way as a decrease of frequency. The relation of ratio and phase angle to magnitude of secondary burden is sensibly linear.

Effect of Wave Form.—The characteristics of current transformers are not affected by wave form, within the limits of commercial practice.

Distortion.—The use of iron in the magnetic circuit causes the secondary wave form to differ from that of the primary. Errors might thereby be introduced, but in practice the effect is quite negligible, due to the low flux density used.

At the same time it is to be noted that the

errors may become appreciable under abnormal conditions, such as the recording of transient phenomena, heavy overloads, etc.

§ (7) CALIBRATION OF CURRENT TRANSFORMERS. GENERAL CONSIDERATIONS.—When a transformer is only to be used with one particular ammeter it is not necessary to test the transformer separately: the combination can best be treated as a unit, and the ammeter scale drawn accordingly, thereby eliminating the ratio error of the transformer. When, however, the same ammeter is to be used with several transformers the scale will not in general be correct for all: it is then necessary either to determine the errors of the instrument when used with each transformer, or else to determine the ratio errors of each transformer and correct the scale reading of the instrument accordingly. With wattmeters both the ratio and phase angle errors of the transformer affect the indication of the instrument, and a scale cannot therefore be drawn which will be correct for loads of the same magnitude but of different power factor. It is therefore frequently necessary to determine the ratio and phase angle errors of current transformers with high accuracy. These determinations present considerable difficulty, and the majority of methods employed are suitable only for laboratories: a number of workshop methods are, however, available.

(i.) *Precision Methods.*—Nearly all precision methods involve the use of the potentiometer principle. In general, non-inductive shunts are placed in the primary and secondary circuits of the transformer, of such values that their voltage drops are equal when the transformer has its nominal ratio. The secondary shunt is then varied for balance, or the out-of-balance voltage is determined by suitable means. It will be noted that a true balance cannot be obtained by resistance adjustment alone, because the primary and secondary currents differ not only in magnitude but also in phase. The out-of-balance voltage may therefore be considered to be composed of two components, one of which is in phase with the primary current, and one in quadrature. This latter component may be balanced by a small quadrature voltage obtained from a variable mutual inductance, as shown in *Fig.* 4. Here CT is the transformer under test, R_p and R_s non-inductive shunts, B the transformer burden, M a mutual inductance, and D a vibration galvanometer or other detector. By simultaneous adjustment of R_s and M the galvanometer may be brought to rest, when it may be shown that

$$\text{Ratio } \frac{I_p}{I_s} = \frac{R_s}{R_p}\sqrt{1+\left(\frac{\omega M}{R_s}\right)^2} = \frac{R_s}{R_p}, \text{ very nearly.}$$

$$\text{Phase Angle} = \tan^{-1}\frac{\omega M}{R_s}.$$

An alternative method involves the use of a selective detector. Thus, suppose a separately excited dynamometer is employed at D, the

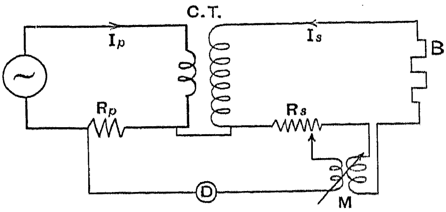

FIG. 4.

excitation current being in phase with I_p. The quadrature component of the unbalanced voltage will not affect such an instrument, and its reading will be proportional to the ratio error of the transformer. If the phase of the excitation current be now adjusted to be in quadrature with I_p the instrument will be sensitive only to the quadrature component, and its reading is then proportional to the tangent of the phase angle.

This system may be used either for a deflectional method, or for the null method already described. With the null method its advantage is that the resistance and mutual inductance adjustments may be made independently.

In actual practice the detector may be a dynamometer, a direct current galvanometer employed with a synchronously driven rectifying commutator (the phase of reversal being variable), or a quadrant electrometer with a step-up transformer. (The results illustrated by the curves of *Figs.* 2 and 3 were obtained with the electrometer.) The independent excitation for the detector may be obtained from a phase-shifting transformer or from a two-phase supply.

(ii.) *Workshop Methods.*—When refined apparatus is not available the best method is to compare the transformer with a standard transformer of the same range, which has been calibrated by a precision method. Such standards are very permanent, and may have several ranges. A convenient method is illustrated in *Fig.* 5. In this diagram CT_x and CT_s are respectively the transformer under test and the standard, with their primary windings connected in series to the supply. The secondary windings are connected in series, so that their voltages assist one another. B is the secondary burden for CT_x, W_1 and W_2 are precision wattmeters, whose potential coils can be connected to either phase of a two-phase supply.

It is evident that if the current coil of W_1 has a sufficiently low impedance the burden of CT_x consists simply of B, and is not

affected by the bridge connection. Also the current through this coil must be equal to the vector difference of the secondary currents of CT_x and CT_a. If the current in the pressure coil of W_1 is in phase with I_a, the reading of W_1 is therefore proportional to the difference in the ratios of the transformers; if it is in quadrature with I_a, the reading is proportional to the tangent of the difference of their phase angles. That is, if D_1 and D_0 be the readings of W_1 for the "in phase"

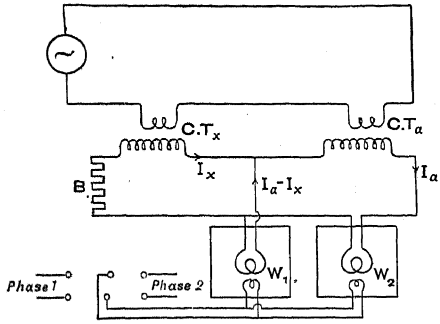

FIG. 5.

and "quadrature" conditions, K its constant in amperes per division, and R_a and R_x and β_a and β_x the ratios and phase angles of the two transformers, we have, very approximately,

$$R_x = \frac{R_a I_a}{I_a - KD_1},$$

$$\beta_x = \beta_a + \tan^{-1}\frac{KD_0}{I_a}.$$

The phase settings are verified by means of the wattmeter W_2. The permissible impedance of the current coil of W_1 varies somewhat for different test conditions; in general a high-grade instrument of 1 ampere range is suitable. A phase shifter may be employed if a two-phase supply is not available.

Specification of Secondary Burden.—The calibration of current transformers with specified burdens is not easy, since the measuring apparatus itself imposes a burden which is seldom negligible. The difficulty may, however, be overcome by making an additional test after adding a burden equal to that of the instruments, and of the same time constant: since the relation of ratio and phase angle to burden is sensibly linear, extrapolation to a zero value is then possible.

It may be said that the complete specification of the test conditions is very important; it is too often assumed that a transformer once calibrated may be used with any instrument without its characteristics being altered. Where the transformer is of low grade or the instruments differ much in impedance this is by no means the case.

§ (8) POTENTIAL TRANSFORMERS.—Potential transformers are less specialised in character than current transformers, and resemble small power transformers. They may be either single phase or three phase with a common magnetic circuit.

The British Standard Specification No. 81, 1919, requires that all potential transformers shall be wound for a secondary voltage of 110 at the rated primary voltage.

For voltages greater than about 3300 the transformer is usually oil immersed.

Potential transformers are usually provided with fuses of the cartridge type, placed in suitable clips mounted on the case.

§ (9) GENERAL THEORY OF POTENTIAL TRANSFORMERS.—The vector diagram of a potential transformer of unity ratio is given in *Fig.* 6 (which is not to scale). Here V_p is the voltage applied to the primary, which produces the primary current I_p. Subtracting vectorially the primary inductance and resistance pressure drops $I_p\omega L_p$ and I_pR_p, we

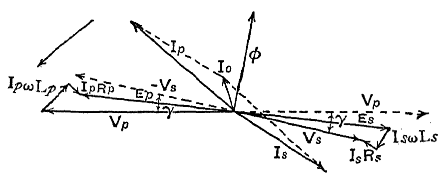

FIG. 6.

obtain E_p the primary electromotive force equal and opposite to the secondary electromotive force E_s. The vectorial subtraction of $I_s\omega L_s$ and I_sR_s, the inductance and resistance drops due to the secondary current I_s, from E_s gives V_s the secondary voltage. I_p is the vector sum of the reversed secondary current $-I_s$ and the no-load current I_0. The ratio of the transformer is V_p/V_s, and its phase angle γ is the angle between the primary and reversed secondary voltages V_p and $-V_s$. The presence of the angle γ, and the variation of V_p/V_s and γ, introduce errors in measurements as in the case of current transformers. It will be seen that both quantities will vary if I_s and I_0 vary.

§ (10) DESIGN OF POTENTIAL TRANSFORMERS.—The objects to be kept in view in design are to keep the value of I_0 and its rate of change with frequency, etc., low, and to reduce the resistance and reactance drops in the windings as much as possible. The following are the chief points requiring attention:

(1) Selection of iron of low hysteresis and high permeability.

(2) Use of a closed magnetic circuit, carefully built up of well-insulated laminae.

(3) Reduction of magnetic leakage to the lowest possible value.

(4) Liberal design of windings.

§ (11) Selection and Operation.—No special precautions are necessary in the use of potential transformers. The secondary burden should be kept as low as possible, and relays, etc., should not be operated from transformers used with precision instruments.

§ (12) Characteristics of Potential Transformers.—Potential transformers are usually employed at constant voltage, and the characteristic of chief interest is the variation of ratio and phase angle with secondary burden: these relations are sensibly linear as in the case of current transformers. *Fig.* 7

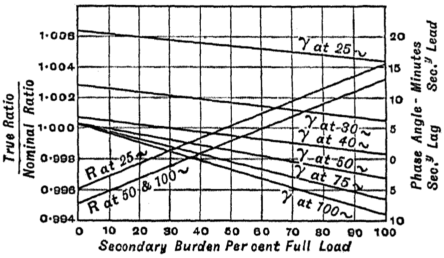

Fig. 7.

shows the values obtained for a high-class precision transformer. Potential transformers vary greatly in performance, and ratio errors of 2 per cent or more and phase angles exceeding 2 degrees are encountered at times.

The secondary voltage is usually in advance of the primary, but may lag behind it, especially with large non-inductive burdens.

Effect of Frequency.—The effect of frequency variation is also illustrated in *Fig.* 7. The results are in general less serious than in current transformers, since the resultant variation of I_0 has only a secondary effect on ratio and phase angle.

Effect of Variation of Voltage.—Good transformers show only small changes of ratio and phase angle with primary voltage.

Effect of Heating.—The effect of self-heating and the consequent change of resistance of the windings is usually quite negligible.

§ (13) Calibration of Potential Transformers. (i.) *Precision Methods.*—The potentiometer principle again affords the most satisfactory test methods. In general a non-inductive potential divider is connected across the primary winding of the transformer, and by means of a variable tapping a portion of the primary voltage is balanced against the secondary voltage. In order to overcome the phase displacement difficulty the circuit shown in *Fig.* 8 may be employed. Here PT is the transformer under test, the desired burden B being connected to the secondary. Across the primary is connected the high resistance R_p+R_s (of which a part r is shunted by a capacity C) in series with a self-inductance L.

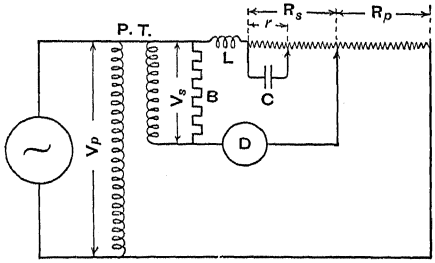

Fig. 8.

The effective inductance of this circuit may be shown to be very approximately $L-Cr^2$: therefore by adjusting the position of the condenser tap point, and so varying the value of r, the phase of the voltage applied to the galvanometer circuit may be matched with that of V_s, when the value of R_s may be altered until balance is obtained. We then have

$$\text{Ratio } \frac{V_p}{V_s}=\frac{R_p+R_s}{R_s}(\cos\gamma)=\frac{R_p+R_s}{R_s},$$

very nearly.

$$\text{Phase Angle } \gamma=\tan^{-1}\omega(L-Cr^2)\left(\frac{1}{R_s}-\frac{1}{R_p+R_s}\right).$$

The detector may be a vibration galvanometer, or preferably one of the discriminating instruments mentioned in the discussion of current transformers.

(ii.) *Workshop Methods.*—The most convenient workshop methods are again those which depend upon comparison with a standard transformer of known characteristics. A suitable method is illustrated in *Fig.* 9. In this

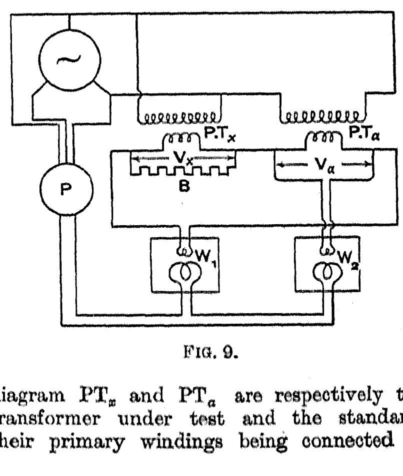

Fig. 9.

diagram PT_x and PT_a are respectively the transformer under test and the standard, their primary windings being connected in

parallel. Their secondary windings are connected, in opposition, to the potential coil of a dynamometer wattmeter W_1. Across the secondary winding of PT_x any desired burden B is connected, and across that of PT_a the potential coil of another wattmeter W_2. The current coils of both wattmeters are excited in series from a phase shifter.

The potential coil of W_1 is evidently excited by the vector difference of the two secondary voltages V_a and V_x. If, therefore, the current coil is excited by a current in phase with V_a the reading of W_1 will be proportional to the difference of the transformer ratios, while if the excitation current is in quadrature with V_a the reading will be proportional to the tangent of the difference of the phase angles. That is, if D_1 and D_0 be the readings of W_1 for the two conditions, K the constant of W_1 in volts per division, and R_a and R_x and γ_a and γ_x the ratios and phase angles of the transformers, we have very approximately

$$R_x = \frac{R_a V_a}{V_a - KD_1},$$

$$\gamma_x = \gamma_a + \tan^{-1}\frac{KD_0}{V_a}.$$

The phase settings are verified by means of the wattmeter W_2. W_1 should be a high-grade instrument of about 30 volts range, but a higher range instrument may be used. A two-phase supply may be used instead of the phase shifter if desired.

§ (14).—For convenience of reference a short bibliography of the more important literature of the subject, grouped under two main heads, is given below. For information as to the standard ratings for instrument transformers, and the limits of error allowed, the British Standard Specification No. 81, 1919, should be consulted.

R. S. J. S.

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TRANSFORMERS, STATIC

§ (1) GENERAL.—A static transformer is a piece of apparatus whereby an alternating current of a definite voltage and frequency can be transformed into any required voltage at the same frequency.

Neglecting losses, the output in K.V.A. of a transformer is the same as the input in K.V.A., *i.e.* the product of the volts and amperes is the same for input and output. From the above it follows that with an increase in voltage on transformation there will be a decrease of current in exactly inverse ratio. A transformer consists of a closed laminated magnetic core, upon which are wound two groups of copper coils. The group of coils, which is connected to the supply, is termed the *primary* winding, while the other group which supplies the transformed voltage is the *secondary* winding.

§ (2) FUNDAMENTAL PRINCIPLES.—An alternating voltage impressed across the primary windings magnetises the magnetic core with an alternating flux of the same periodicity as the supply. The secondary group of coils, being wound on the same magnetic core, is cut by the alternating magnetic field, and hence has a voltage induced. The ratio of the voltages of the primary and secondary windings is practically the ratio of the number of turns in the respective groups.

The section of the copper in the windings is dependent on the load to be taken, and is determined by the current to be carried.

Static transformers have losses which can be defined as those due to the magnetisation of the core, and those due to the current flowing through the copper coils. They can be roughly subdivided as follows:

$$\text{Core losses}\begin{cases}\text{Hysteresis losses,}\\ \text{Eddy current losses,}\\ \text{Building losses.}\end{cases}$$

$$\text{Copper losses}\begin{cases}\text{Resistance loss,}\\ \text{Eddy current loss,}\\ \text{Stray losses.}\end{cases}$$

§ (3) CORE LOSSES.—The hysteresis and eddy current losses are dependent on the quality of the iron and are never separated in commercial tests. These losses are easily obtainable on samples of iron by using the Epstein method of testing. The building losses

are dependent on workmanship in the processes of manufacture, such as in cutting the punchings true to size and without burrs, also in the building of the laminations and the machining of them when required, which operation is necessary in certain types of construction. A percentage is usually added for these building losses, dependent on the type of construction and the experience of the designer.

§ (4) Copper Losses.—Copper losses only occur on load, and are dependent on the load.

The *resistance loss* is equal to the resistance multiplied by the square of the current.

The *eddy current* loss is dependent on the section of copper, and the alternating field set up in the coils by the current.

The *stray losses* are those in the core and attached gear, which comes within the range of the stray leakage fields produced by the current in the windings.

These combined losses heat up the transformer and the heat has to be dissipated, so that the component parts of the transformer do not reach dangerous or destructive temperatures. The usual methods adopted for dispersing this heat are:

(i.) By natural radiation from the surface of the coils and core.

(ii.) By immersing the transformer in a tank of oil, the tank having an extended radiating surface.

(iii.) By means of air-blast through the coils and core or on the surface of the oil tank.

(iv.) By means of water-cooling coils submerged in the oil in the transformer tank.

(v.) By circulating the oil through an external cooler of the surface condenser type, or through cooling coils immersed in water.

§ (5) General Requirements.—The requirements of a successful static transformer are briefly as follows: The magnetic core must be a closed one and formed of laminations of high-resistance and high-permeability iron. The primary and secondary windings must be so arranged relative to one another that their mutual inductance or reactance is within reasonable limits. The windings must be well insulated from each other and from the magnetic core and earth. The arrangement of the windings must be such as to ensure ample ventilation, *i.e.* provision made for free passage of the cooling medium—air or oil.

The losses, *i.e.* the iron loss and copper loss, must be kept within a reasonable figure and have a minimum value at the most general loading, which is usually about 80 per cent of the rated K.V.A. The transformer must be provided with ample radiating surface, to ensure dissipation of the heat generated due to the losses.

This radiating surface in dry, self-cooled transformers is the exposed surface of the coils and laminated core; while in a self-cooled, oil-immersed transformer it is the surface of the tank in which it is mounted; with water-cooled transformers it is the surface of the cooling coils, and with forced cooled transformers the surface of the cooler.

§ (6) Types of Transformers manufactured.—The method of construction varies with the experience of different manufacturers. The most common forms of construction are briefly as follows:

(i.) Shell-type transformers with rectangular coils.

(ii.) Core-type transformers with circular coils.

(iii.) Core-type transformers with rectangular coils.

(iv.) Circular shell-type transformers with circular coils.

There is very little to choose between the various types of construction as regards efficiency and cost. The relative merits of the various types are dependent on the use to which they are to be put—*i.e.* the voltage and service for which they are required—and also upon their useful life and ease of repair.

(i.) *The Shell Type.*—These transformers are manufactured either single or three-phase, and are generally similar to the outline sketch shown in *Figs.* 1 and 2. *Fig.* 1 is a single-

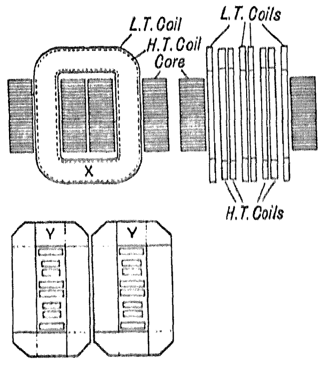

Fig. 1.—Outline of Single-phase Shell-type Transformer.

phase transformer; it has one group of coils X, and two magnetic circuits Y, linking the coils. The coils are generally assembled, together with their insulation, into a box-type construction, and then the magnetic core is built around them. The primary and secondary coils are usually sandwiched to obtain the required reactance and are insulated from one another and from the core with solid barriers of tough insulation, together with numerous oil ducts. The three-phase shell-type transformer is a development of the single phase, having three individual sets

of coils and the three cores arranged to form one composite core, as shown in *Fig.* 2.

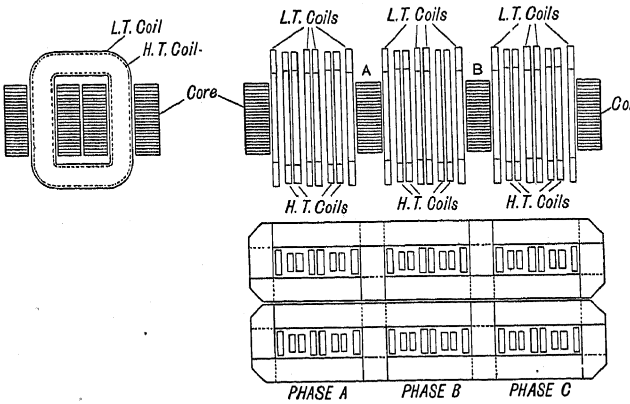

FIG. 2.—Outline of Three-phase Shell-type Transformer.

By arranging the three cores into one composite core there is a saving of approximately 10 per cent of magnetic material, if the winding on the middle limb be reversed and it be arranged for the magnetic fluxes in the inner limbs A and B to be at 120°.

(ii.) *Core-type* transformers having circular coils have usually a magnetic core of cruciform section. The windings are ordinarily arranged in concentric form, having the low-pressure winding near the core and the high-pressure coils surrounding it with the necessary insulating barriers (*Fig.* 3).

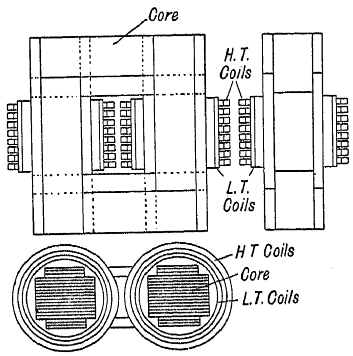

FIG. 3.—Outline of Single-phase Core-type Transformer with Concentric Windings.

This type of transformer in large sizes, say for 5000 K.V.A. and upwards, has the windings in triple concentric form in order to obtain reasonable reactance values and reasonable load losses. By triple concentric is meant that the low-pressure winding is in two portions, part being inside and the other outside the high-pressure winding. Some manufacturers prefer what is known as a sandwich winding for this type. In this case the high-pressure and low-pressure coils are subdivided and distributed over the length of the core (see *Fig.* 4).

(iii.) Core-type transformers with rectangular coils have a core of rectangular section and of greater cross-section than in type (ii.) for the same K.V.A. rating. This type,

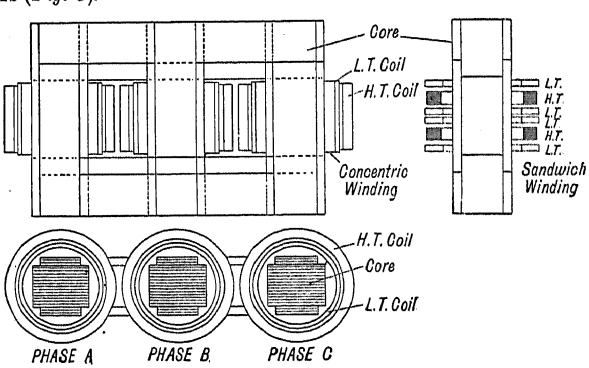

FIG. 4.—Outline of Three-phase Core-type Transformer with Sandwich Windings.

on account of its heavy section, requires numerous ventilating ducts to keep the tem-

perature of the core uniform. The coils are usually wound in sections and assembled in sandwich form on the limbs (see *Fig.* 5).

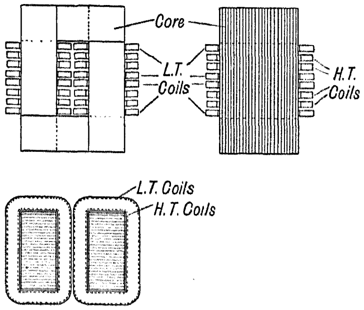

FIG. 5.—Outline of Single-phase Rectangular Core-type Transformer.

(iv.) *Circular shell-type* transformers have a construction peculiarly their own. The coils are usually wound in a concentric or triple concentric manner, and the core is distributed around the coils as shown in *Fig.* 6.

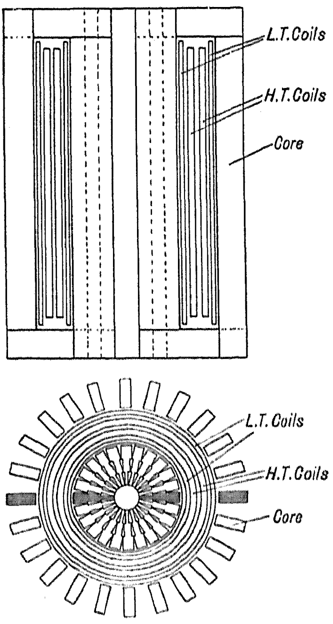

FIG. 6.—Outline of Single-phase Circular Shell-type Transformer.

§ (7) MAGNETIC CORES.—The magnetic cores for the various types above described are all built up of laminated silicon iron having low hysteresis losses, a high permeability and a high ohmic resistance. The lamination varies in thickness from 0·16 to 0·2 millimetres, according to quality. The laminations are insulated from each other by coating one side with a thin paper or thin varnish. This weak insulation prevents eddy currents being set up between the adjacent laminae. In the small transformers, the cores are built of punchings with alternating butt joints, and the punchings are clamped together between end frames, where that is possible, and taped together in other places. In large sizes the punchings are similarly built, but are held together by means of insulated clamping bolts passing right through the core. Shell, core, and circular shell-type transformers have their punchings interleaved throughout, but in large sizes the core type, and sometimes the shell type, have their cores built up of machined blocks of punchings.

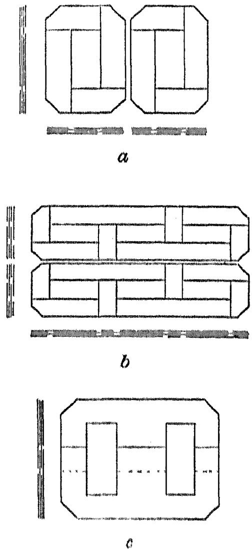

FIG. 7.—Punchings for Shell-type Transformers.

a, single-phase shell-type, above 20 K.V.A. (I plates); *b*, three-phase shell-type; *c*, single-phase shell-type, below 20 K.V.A. (E plates).

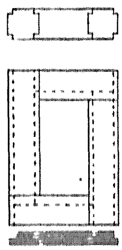

FIG. 8.—Punching for Single-phase Core-type (Interleaved) Transformer.

Figs. 6, 7, and 8 show the interleaving of the shell, core, and circular shell-type punchings,

while *Figs.* 9, 10, and 11 show the butt-type construction.

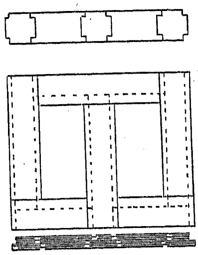

FIG. 9.—Punchings for Three-phase Core-type (Interleaved) Transformer.

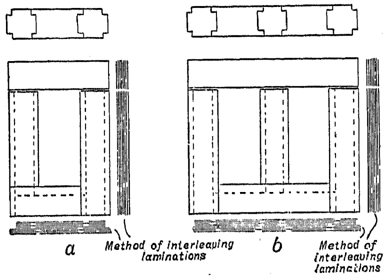

FIG. 10.—Core-type Butt-yoke Core.
a, Single-phase. *b*, Three-phase.

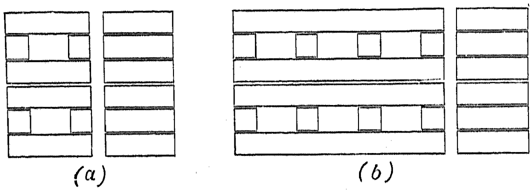

FIG. 11.—Shell-type Butt-jointed Core.
a, Single-phase. *b*, Three-phase.

§ (8) TRANSFORMER WINDINGS.—As previously noted, the coils of the transformer windings may be arranged in various ways. The particular type of construction adopted depends on the type of transformer, voltage, and size. With shell-type transformers the coils are usually wound in sandwich fashion. The individual coils are formed of sections having one turn per layer and include several layers with insulation between the respective turns forming the layers. If the winding is heavy, or can be arranged with a conductor of gross width above 0·5 in., one section usually forms one coil. Often two sections have to be assembled together to form a coil for reason of space and mechanical strength. *Fig.* 12 shows two rectangular section coils wound and assembled together with insulating barrier. Since all the turns are in series and the sections are joined together inside, the two outside turns have a considerable voltage across them, and hence for safety an edgeblock is taped on the last turn of each section, to provide sufficient creepage distance for the voltage. Sometimes three or four sections are used together, when edgeblocks are required inside and outside the section for the

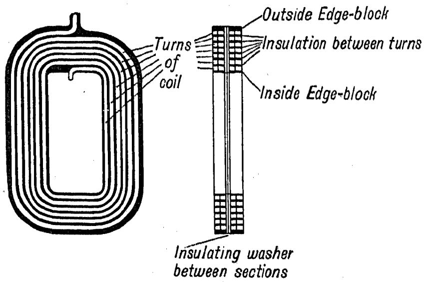

FIG. 12.—Shell-type Transformer Coil in two sections.

same reason. The voltage between turns of a shell-type transformer in large sizes is considerable. For instance, in a 7000 K.V.A. transformer it will be approximately 120 volts at 50 periods, hence the volts per coil will be considerable and possibly reach 3000 volts. With a 500 K.V.A. transformer the voltage per turn will be approximately 20 and per section approximately 400 volts.

In the small transformer the section of the conductor may be small and necessitate the use of a round wire, and in this case cross-over coils are used. This type of coil is shown in *Fig.* 13, having eight turns per layer, and the consecutive layers are wound in opposite directions. The insulating of the layers is obtained by winding in a sleeve of paper or cambric. This sleeving may be single, as shown in *Fig.* 13, or double. A double

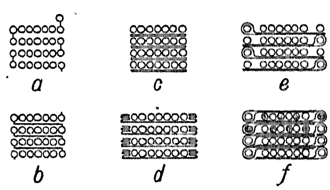

FIG. 13.—Section of Cross-over Coil.

a, layers in opposite directions; *b*, layers in opposite directions with insulation between; *c*, layers in opposite directions with single sleeve of insulation; *d*, layers in same direction; *e*, insulation between layers with crimped ends; *f*, double sleeves of insulation between layers and string between turns of last two layers.

sleeve gives mechanical strength to the coil by holding all turns securely in position, and

is usually adopted if many layers are required with few turns per layer. In core-type transformers the low-pressure coil is usually wound in spiral form, having one or two layers. A single-layer coil is preferable, and the conductors are generally so arranged as to necessitate the use of one layer only. If two layers are used edgeblocks are necessary at the start and finish of the winding. This method is shown in *Fig.* 14. For the H.T. windings cross-over coils with all the layers wound in the same direction may be used, but in this case the wire has to pass across the face of the coil, as shown in *Fig.* 13. This cross-over has to be well insulated and placed in such a position that the distortion of the coil which it causes does not cause inconvenience in assembly. The voltage between adjacent layers of cross-over coils should not exceed 250 volts. This type of winding is

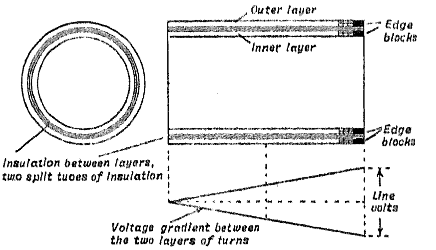

FIG. 14.—Section of Double-layer Spiral Coil for Core-type Transformer.

cheap, but only suitable for small round wires of 0·092 in. diameter or less.

For H.T. coils of larger copper section, section coils are used similar to those already described for shell-type transformers, only in this case edgeblocks are not necessary on the outside turns, for a free extension of the insulating washer used between the sections is available for creeping surface. Edgeblocks are advisable on the inside turns, as usually the inside diameters of washers and coils are flush to permit of packing out solidly to the insulating cylinder (see *Fig.* 15). The H.T. coils are sometimes wound in spiral form, in which case the wire is wound on to a suitable insulating cylinder of approved thickness.

Coils having a voltage of 6000-11,000 volts may be wound in this fashion. The turns are wound on under axial pressure with insulation between turns, and the top and bottom of the coil is packed with a heavy insulating ring which holds the turns in place and provides adequate creeping surface. Such a winding is shown in *Fig.* 16.

The end windings of all transformers above 3000 volts have their end turns reinforced—*i.e.* the insulation between the end turns, for approximately 1 to 5 per cent of the turns according to voltage, is increased in thickness—to stand heavy voltage surges due to switching, lightning, etc.

In cross-over windings this insulation takes the form of string—one or more turns—

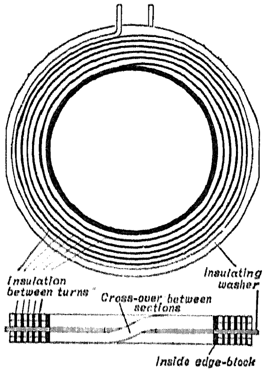

FIG. 15.—Core-type Section Coil.

between the turns of the coil (see *Fig.* 13). In section and spiral windings this reinforcement takes the form of an increased thickness of insulation between turns, and heavier washers between sections, or a sleeving or taping of the end turns with cambric, paper, or empire tape. This is shown in *Fig.* 15.

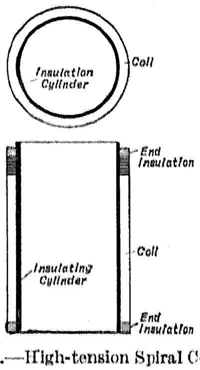

FIG. 16.—High-tension Spiral Coil.

§ (9) INSULATION.—The type of insulation used in transformer manufacture varies with the type of transformer. For dry-type transformers, *i.e.* transformers cooled naturally or by air-blast, micanite insulation can be used for insulating between coils and cores and between primary and secondary coils. Micanite sheets can also be used in oil-insulated transformers where it can be held definitely under pressure, as in the case of washers between sections. This material, however, is only used on small transformers, for while it is an excellent insulation, it is also a heat

insulator and tends to lag the coils and cause overheating.

The usual insulating materials are sheets of fibrous material such as fullerboard and leatheroid, bond paper, bear paper, rope paper, treated and untreated. Treated paper in the form of micarta and bakelite cylinders and sheets are also used, also empire cloth and closely woven linen and silk tapes and treated timber.

The insulation should have a high dielectric strength, and not deteriorate under the influence of heat or oil up to temperatures of 95° C. It should be homogeneous and have no acid tendency; it should be hard and unshrinkable, but not brittle.

(i.) *Shell-type Insulation.*—*Fig.* 17 shows

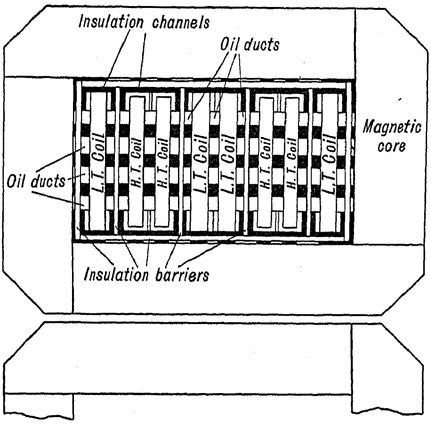

FIG. 17.—Section of Shell-type Transformer Insulation.

the section of a shell-type transformer and its insulation. The coils are wound, as already described, with D.C.C. wire or bare wire sleeved with insulation between turns. The coils are then grouped together with the necessary insulating barriers, and the necessary oil ducts are provided for insulation and ventilating purposes by arranging specially shaped packing strips on to the coils or upon the insulating washers or barriers. The inside and outside edges are insulated from the core by solid insulating barriers, and in the case of high-voltage transformers with oil ducts also.

The necessary creepage surface is obtained between H.T. and L.T. coils by wrapping the group of coils, or by bracing them together in barriers of channel section interleaved with the insulating washers.

The above refers to that portion of the coils inside the core and in line with it. The coil heads are insulated for creepage by washer extensions inside and outside, and in the case of high-voltage work, 50,000 volts or over, by special Vee pieces on the inside to economise space.

The packing strips between coils, which form the oil ducts, should be so designed as to hold in place each wire of the coil so that if subjected to a shock, each wire is firmly held in position. Straight strips are not serviceable, and *Fig.* 18 shows several methods employed to attain the end desired. All the coils should be packed out solidly from the core, and hence all the coils should be packed on the inside to form a solid bed for the coil supports, and of such a strength as to resist shocks in that lateral direction.

Shell-type transformers for air-blast cooling are rarely built for higher voltages than 11,000 volts on account of the difficulty of insulating the coils from earth and from one another. The difficulty lies in the point of cost and efficiency, for higher voltages require excessive creepage distances as well as the use of mica insulation, which in consequence of its heat-insulating properties tends to cause excessive internal heating. The insulating is in general

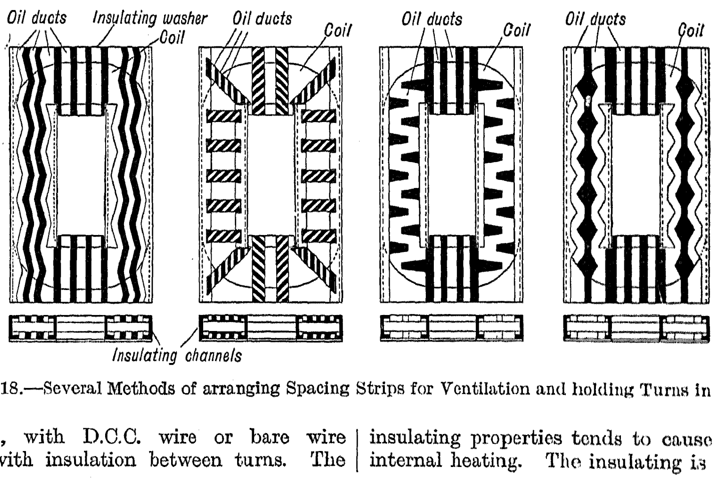

FIG. 18.—Several Methods of arranging Spacing Strips for Ventilation and holding Turns in place.

similar to that already described, except that the coils are individually taped and varnish treated to form a smooth surface. A smooth coil surface reduces the air friction and assists cooling.

(ii.) *Core-type Insulation.*—Core-type transformers of the concentric type, either circular or rectangular, are insulated in a manner shown in *Fig.* 19. The low-pressure coils are placed adjacent to the core and are insulated from it by insulating washers top and bottom, and by means of a tube of insulating material, micarta, leatheroid or fullerboard from the limb. The coil and insulation is packed out from the core with treated timber to form a rigid construction. The low-pressure winding may be a spiral coil or a series of coils as described.

Over the low-pressure winding is placed the main insulating barrier between the high- and low-pressure coils. This takes the form, usually, of a micarta tube, or series of tubes and oil ducts, running the full length of the core. Over this tube are placed the high-pressure coils, with heavy insulating washers top and bottom to insulate the coils from the yokes. The dimensions of the coils and cylinders are so arranged as to allow an oil duct between the low-pressure coils and the cylinder and frequently between the cylinders and high-pressure coils, the latter ducts depending on

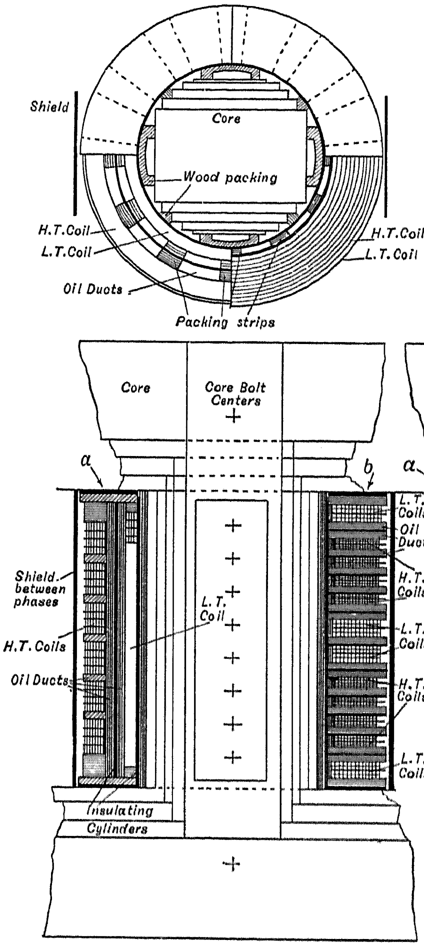

FIG. 19.—Section of Core-type Insulation, Coils and Ventilation.

a, Concentric-type winding.
b, Sandwich-type winding.

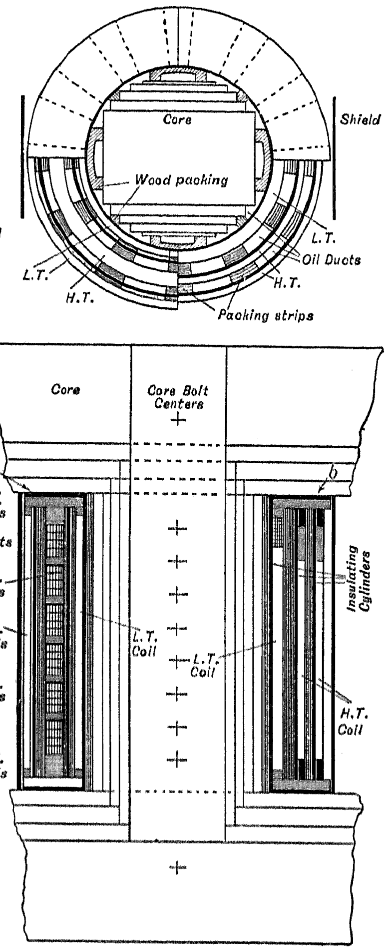

FIG. 20.—Section of Core-type Insulation, Coils and Ventilation.

a, Triple concentric winding.
b, Spiral-type winding.

the depth of the high-pressure windings. The low-pressure coils in large transformers are subdivided into two or more groups, with vertical ducts between each group, to assist ventilation. This method is shown in *Fig.* 20. Core-type transformers wound in spiral fashion are similarly insulated, except that there the high-pressure coils are wound direct on to heavy insulating cylinders.

Sandwich-type windings, which are now used rarely, and only in the cases where primary and secondary coils are of a moderately high and similar voltage, are insulated from one another by barriers of insulation and oil ducts. They are insulated from the core by cylinders of insulation in a similar manner to the spiral and concentric type. A transformer having this type of winding is sometimes mounted with its limbs in a horizontal direction to assist in the ventilation of the coils by bringing the oil ducts partly into a vertical direction. This is wasteful of floor space, and the core limbs tend to vibrate due to beam suspension.

(iii.) *Circular Shell-type Insulation.* — Circular shell-type transformers are wound and insulated in a similar fashion to the above, but the procedure is somewhat different. The centre limb of the magnetic core is assembled in a special mandrel and placed in a lathe. This core is then insulated and the low-pressure coil wound on it direct, generally in spiral form. The main insulating barrier is then placed over this coil and the necessary oil ducts provided by means of spacing strips.

The high-pressure coils are then wound on this insulation or are assembled over this insulation if separate coils are used. Further insulation and oil ducts are then used and the final secondary coil wound over these, and the whole is finished off with insulation which becomes the barrier between it and the core.

§ (10) Methods of Cooling.—The various methods of cooling transformers have already been briefly stated, but the following gives some further details:

(i.) *Dry Air-cooled Transformers.*—In this type the heat is dissipated by natural radiation, dependent on the exposed surfaces of the coils and core, and the ventilating ducts which permit of a free passage for air. This type of transformer cannot be economically built for sizes above 20 K.V.A. due to the fact that for increase of output the losses increase at a greater rate than the cooling surface.

For example, if any transformer be taken and its linear dimension be doubled, then the sectional area of the cores is increased four times and hence the flux permissible and so the voltage.

If the copper section has its linear dimensions doubled, its current-carrying capacity is increased four times and the copper losses sixteen times. Thus for four times the cooling surface there are sixteen times the losses, which necessitates artificial cooling if the original transformer had a maximum temperature rating.

(ii.) *Oil-insulated Air-cooled Transformers.*—A transformer mounted in a tank of oil will transmit its heat to the oil and the hot oil will rise to the surface.

If the tank has sufficient radiating surface, the oil becomes cooled and flows to the bottom of the tank and again rises. The oil acts as a medium and the tank surface radiates the heat. These transformer tanks may be constructed with flutes of various depth, or fins may be welded on to the tank as in *Fig.* 21. Other types of tank are of boiler iron with external cooling tubes as in *Fig.* 22. The tubes may also be internal.

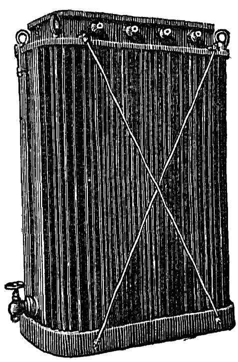

Fig. 21.—Corrugated or Fluted Transformer Tank.

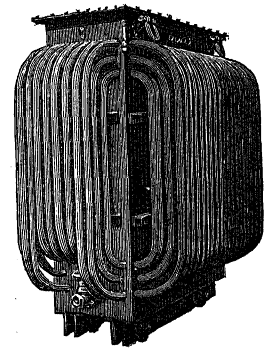

Fig. 22.—Boiler Iron Tank with External Cooling Tubes.

For large sizes up to 5000 K.V.A., self-cooling tanks may be made, in which case radiators are attached to the tanks. The oil, flowing by natural circulation through these radiators, comes into contact with the radiating surfaces and is cooled.

(iii.) *Air-blast transformers* are dry transformers cooled by means of a current of dry air forced through the ventilating ducts in the windings and core.

The transformer, *Fig.* 23, is encased in a sheet-iron casing provided with dampers, by which the air can be regulated as to its direction of flow and quantity. The air is supplied by means of a fan which is connected through an air duct to the base of the transformer.

(iv.) *Oil-immersed air-blast transformers* are mounted in fluted tanks in oil, and around the flutes is arranged a lagging

and the air is directed through the enclosed flutes.

(v.) *Water-cooled transformers* are mounted in plain boiler plate tanks in oil, and have in the upper layers of the oil a metal cooling coil through which water is circulated. The cooling coil is usually made of copper, due to its strength and good conductivity; also they are less liable to corrosion than iron. These cooling coils must have sufficient radiating surface to give required oil temperatures with given quantities of water flowing.

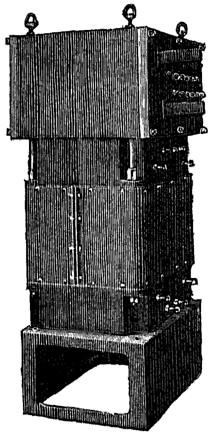

FIG. 23.—Single-phase Air-blast Transformer.

(vi.) *Forced cooled transformers* are mounted in oil in plain tanks (*Fig.* 24), and the oil is circulated through the tank and passed through a cooler, either of the surface condenser pattern or through a copper coil immersed in water such as a river or reservoir. The temperature rise depends on

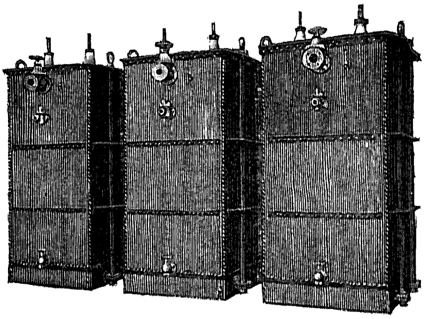

FIG. 24.—Plain Boiler Iron Tanks for O.I.F.C. Transformers.

the relative flow of oil and water through the cooler. The oil in the cooler is maintained at a pressure approximately 30 lbs., so that leakage of water to oil is impossible, and leakage, if any, takes place from oil to water. This safeguards the transformer in the event of leakage. The pressure is obtained by pumping the oil through the cooler, at the exit of which is a spring loaded valve. The oil is drawn from the top of the tank and pumped in at the base (*Fig.* 25—diagram of oil circulation system).

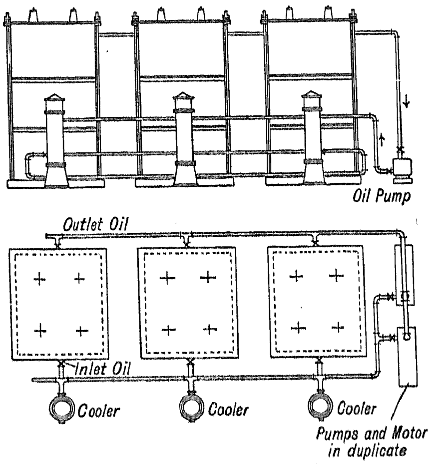

FIG. 25.—Diagram showing Oil Circulation for O.I.F.C. Transformers.

§ (11) OUTLINE OF TRANSFORMER DESIGN.

—(i.) *Ratio of turns primary to secondary* is

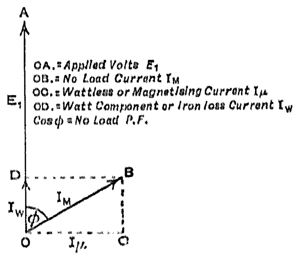

FIG. 26.—No-load diagram.

obtained from *Fig.* 26, a no-load transformer vector diagram. If we put

e_1 = Primary back electromotive force,
e_2 = Secondary back electromotive force,
E_1 = Primary volts,
E_2 = Secondary volts,
N_1 = Primary turns,
N_2 = Secondary turns,
I_M = Magnetising current,
B = Induction in lines per sq. cm.,
A = Core section in sq. cm.,
$\sim$ = Frequency,

then, assuming a sine wave[1]

$$E_1 = 4{\cdot}44 \sim BAN_1 10^{-8}.$$

The back electromotive force e_1 is practically equal to the impressed electromotive force with the secondary open circuit, and so $e_1 = E_1$.

[1] See "Electromagnetic Theory," § (12); "Dynamo Electric Machinery," § (4).

The flux in the core will produce an electromotive force

$$E_2 = 4{\cdot}44 \sim BAN_2 10^{-8},$$

hence

$$\frac{E_2}{\epsilon_1} = \frac{N_2}{N_1},$$

$$\therefore\ E_2 = \frac{\epsilon_1 N_2}{N_1} = \frac{E_1 N_2}{N_1}.$$

From this it is clear that on open circuit the ratio of voltages is equal to the ratio of turns. There is a slight difference between E_1 and ϵ_1 due to the fact that energy is required to excite the magnetic core. The open-circuit current I_M can be divided into two components, I_N the magnetising current in quadrature with E_1, and I_W the watt component which provides the iron loss in phase with E_1.

(ii.) *The Core.*—To find the section of the magnetic core for any given transformer it is necessary to fix the flux density. For general designs, the flux density employed ranges about the figures in the following table:

Type of Transformer.	25 ~.	50 ~.	100 ~.
	lines/sq.cm.	lines/sq.cm.	lines/sq.cm.
Dry air cooled naturally . .	12,000	10,000	6000
Dry air blast .	14,000	12,500	8000
Oil immersed .	14,000	12,500	8000
Water and force cooled . .	15,000	13,500	8500

The voltage $E_1 = 4{\cdot}44 \sim BAN_1 10^{-8}$.

The current as a function of the frame is proportional to the current density employed, the winding space, the copper factor, and inversely as the number of turns.

So that current

$$I_1 = \frac{\Delta \times \text{winding space} \times \text{copper factor}}{\text{No. of turns} \times 2},$$

$$I_1 = \frac{\Delta \times \text{winding space} \times K}{N_1 \times 2}.$$

I = Primary current.
Δ = Current density.
K = Copper factor.
W = Winding space.

Substituting for E_1 and I_1.

$$\therefore\ \text{K.V.A.} = 2{\cdot}22 \sim BA\Delta WK10^{-11}$$

for single-phase transformers of shell type.

For three-phase transformers of core type

$$\text{K.V.A.} = 3{\cdot}33 \sim BA\Delta WK10^{-11}.$$

Here the opening is occupied by the winding sections of two phases.

The current densities usually employed are in the neighbourhood of the following table:

	Amps./sq. in.
Dry air cooled naturally . .	600– 800
Dry air-blast	1200–1500
Oil immersed self-cooled . .	1400–1500
Water cooled	1700–2000
Force cooled	2000–2500

The copper factors for various sizes and voltages are in the neighbourhood of the following table:

K.V.A.	Core Type.		Shell Type.	
	6000 V.	20,000 V.	6000 V.	20,000 V.
10–100	0·2–0·31	0·13–0·19	0·3–0·5	0·2–0·3
100–200	0·31–0·32	0·19–0·25	0·28–0·31	0·16–0·18
200–500	0·32–0·36	0·25–0·31	0·31–0·34	0·18–0·2
500–2,000	0·36–0·4	0·31–0·36	0·34–0·38	0·2–0·24
2,000–10,000	0·4–0·48	0·36–0·4	0·38–0·4	0·24–0·3

As a guide in design, the following table of volts per turn for various types and voltages will serve as a check:

K.V.A. at 50~.	Shell.		Core.	
	Single-Phase.	Three-Phase.	Single-Phase.	Three-Phase.
1,000	35	20	15	10
3,000	60	35	35	25
10,000	100	60	60	50
20,000	140	80	85	70
30,000	..	100	..	85

(iii.) *The Coils.*—The copper section is fixed by the current density employed, and the wire for that section may be chosen. If the copper section is heavy, it is advisable to split it up into several sections in parallel.

Having fixed the core section and opening, the coils must be designed to fit the opening, allowing for the required insulation between turns, between coils, between primary and secondary winding, and from both of these windings to earth. The insulation necessary depends on the type of winding and arrangement of coils as described under methods of winding and insulating.

The figures of safety factor given in the tables will give a guide to the quantity of insulation and the creepage distances necessary.

Voltage and Safety Factor between Adjacent Turns

Normal Volts per Turn.	B.D. Volts between Turns.	Operating Safety Factor.	Testing Safety Factor for Over-potential Test. Voltage=2× normal.
5–10	4,500	900–450	450–225
10–25	7,000	700–280	350–140
25–60	10,000	400–166	200–83

VOLTAGE AND SAFETY FACTOR BETWEEN COIL-SECTIONS AND COILS

Normal Volts between Sections.	B.D. Volts between Turns.	Operating Safety Factor.	Testing Safety Factor for Over-potential Test. Voltage=2× normal.
500	20,000	40	20
1,000	35,000	35	17·5
3,000	45,000	15	7·5
6,000	55,000	9·2	4·6
12,000	70,000	5·8	2·9

VOLTAGE AND SAFETY FACTOR BETWEEN H.T. AND L.T. WINDINGS AND BETWEEN WINDINGS AND FRAME

Kilo Voltage.	B.D. Volts.	Operating Safety Factor.	Test Kilo Volts.	Factory Safety Factor.
2·2	40	18·2	5·4	7·4
6·6	60	9·1	14·2	4·25
11·0	80	7·25	23	3·5
22·0	110	5·0	45	2·45
33·0	140	4·25	67	2·1
55·0	200	3·65	111	1·8

From the foregoing data a transformer design can be figured out, and from it the performance can be obtained. It is probable that several calculations will have to be made to obtain satisfactory dimensions, weights, etc.

§ (12) PERFORMANCE. — To determine the performance we require to know the following:

Iron loss.
Load loss.
Resistance drop in volts.
Reactance drop in volts.
Impedance.
Temperature rise.

(i.) *Iron Loss.*—Knowing the weight of iron and the specific loss of the iron used, as found by Epstein's tests or other suitable means, the total loss is available. The loss is the weight of active iron multiplied by the specific loss per lb. at the required induction and frequency. This loss can be split up into hysteresis and eddy current loss, but this division serves no useful purpose in the design stage. The proportion of losses is, however, useful in determining the quality of iron when specifying requirements for purchase. Beyond the losses determined above, additions have to be made for constructional reasons, such as for butt joints and machined joints in the cores, and also for clamping bolts and plates as well as for errors in building. For butt-jointed cores with machined surfaces, approximately 7 per cent must be added to compensate for the losses in the machined surfaces and in the spreading of the flux. A further 5 per cent must be added to the loss for irregular building and flux spreading at the joints of the laminations. The iron loss is affected only slightly by temperature and is usually neglected. The effect of increase in temperature is to slightly reduce the loss due to the rise in the electrical resistance of the iron with temperature, thus reducing by a small amount the eddy current loss in the iron.

(ii.) *Load Loss or Copper Loss.*—The load loss or copper loss is due to the alternating current flowing through a coil having a definite resistance. In addition to the loss due to the resistance of the copper, there is the eddy current loss in the copper itself, which is dependent on the section and position of the copper and upon the leakage field or reactance flux which cuts the copper. In addition there are stray losses caused by the leakage field cutting other metallic sections of the transformer, such as the core, castings, bolts, etc. The load loss then consists of:

(*a*) Resistance or C^2R loss.
(*b*) Eddy current loss in the copper.
(*c*) Stray losses.

(*a*) The C^2R losses can be easily determined by knowing the resistance of the primary and secondary winding and the current flowing.

$$C^2R \text{ loss} = I_1^2R_1 + I_2^2R_2.$$

I_1 = Primary current.
I_2 = Secondary current.
R_1 = Primary resistance.
R_2 = Secondary resistance.

The resistance loss is affected by temperature, since the resistance of copper increases with temperature at the rate of approximately 0·4 per cent per 1° C. rise in temperature.

The C^2R loss can be determined also from the copper weight, and this method is the one usually adopted when roughing out designs for tender purposes.

C^2R loss hot 60° C. $= 2{\cdot}57 \times \text{cu. wt.} \times \Delta^2 \times 10^{-6}$ watts.

The cu. wt. of a shell-type transformer is
$= 0{\cdot}3215 \times W \times K \times \text{mean turn}$, lbs.
The cu. wt. for a core-type three-phase is
$= \frac{3}{2} \times W \times K \times \text{mean turn} \times 0{\cdot}3215$, lbs.

(*b*) The eddy current loss in the copper is due to the resultant alternating magnetic fields set up by the primary and secondary currents flowing through the windings. The magnitude of this loss is dependent (1) on the value of the leakage field, which is itself dependent on the arrangement of the coils; (2) on the frequency and on a factor which itself depends on the frequency; and (3) on the weight of copper and the section of copper by a plane at right angles to the leakage flux.

We thus arrive at the equation

$$\left.\begin{array}{l}\text{Loss cold at}\\ \text{15° C. at 50}\\ \backsim \text{for a three-}\\ \text{phase core-}\\ \text{type trans-}\\ \text{former}\end{array}\right\} = \frac{1{\cdot}08 \times \backsim^2 \times F_2^2(W_1d_1^2 \times W_2d_2^2\sqrt{2})Z}{2{\cdot}2 \times 10^8}$$
approximately.

$$F_L = F_{max.} \times R$$

$$F_{max.} = \frac{1{\cdot}26 \times I_1N_1 \times \sqrt{2}}{2{\cdot}54 \times l}.$$

$$R = 1 - \frac{d_1 + d_2 + 2d_0}{2\pi l}.$$

F_L = leakage flux lines per cm.
$F_{max.}$ = load flux max.
l = length of leakage path in inches.
d_1 = breadth of H.T. conductor at 90° to flux.
d_2 = breadth of L.T. conductor at 90° to flux.
d_0 = distance between H.T. and L.T. coils.

These eddy current losses in a transformer of normal reactance, say 5 per cent, in which no conductors have a greater breadth at right angles to the leakage flux than 0·2, should not exceed 8 per cent of the C^2R loss at 50 ∿.

If the reactance be 20 per cent the eddy loss would be approximately 30 per cent of the C^2R losses.

The factor Z is constant for the same frequency, but varies with frequency approximately as per table.

∿.	Z.	∿.	Z.
100	0·65	40	1·2
80	0·75	30	1·6
60	0·9	25	2·5
50	1·0	15	4·0

(*c*) Stray losses dependent on load are indeterminate, and have to be approximated from a consideration of type of core and position of coils relative to the metallic constructional parts of the transformers. The value of these stray losses in a well-designed transformer is not more than about 2 per cent, but may reach alarming figures if incorrectly designed. A core-type transformer wound in sandwich fashion, having 20 per cent reactance, will probably have stray losses to the value of 50 per cent of the C^2R loss in the core plates and punchings, due to the leakage flux cutting at right angles. Experience can only guide the designer in estimating these stray losses, which are really eddy current losses in metallic parts other than the copper. With reference to *b* and *c*, these losses decrease with temperature, due to the fact that the resistance of copper and iron increases with temperature and thus reduces the eddy currents circulating in the sections of the material cut by the leakage flux.

§ (13) Transformer Efficiency. — The efficiency of a transformer is computed from a knowledge of the total losses and the output. The total losses are the sum of the iron loss or no-load losses and the load losses, commonly known as the copper loss. The transformer is always rated at its output K.V.A., so that the input is the output plus the losses.

$$\text{Efficiency} = \frac{\text{output} \times 100}{\text{input}} \text{ per cent}$$

$$= \frac{\text{output} \times 100}{\text{output} + \text{losses}} \text{ per cent}$$

$$= \frac{(\text{input} - \text{losses}) + 100}{\text{input}} \text{ per cent.}$$

For transformers that are not continually working at full load, the maximum efficiency should be at the normal load, and to attain this the losses proportioned accordingly. If the transformer is continually in circuit with varying load, such as for lighting circuits or for power plant, running full load in the day and half load at night, an all-day efficiency is important. These conditions are not always known when the transformer is designed, and so the maximum efficiency is arranged about 75 per cent load.

To obtain the efficiency at any power factor other than unity the percentage losses must be taken as a percentage of the kilowatt output and not on the kilovolt-ampere output.

Example: Efficiency of 100 K.V.A. Transformer with No-load Loss of 1·5 per cent and Load Loss of 1·0 per cent

Per cent Load.	Per cent No-load Loss.	Per cent Load Loss.	Per cent Total Loss on Output.	Per cent Total Loss on Intake.	Efficiency.
		Power Factor = 1.			
100	1·5	1·0	2·5	2·44	97·56
75	2·0	0·75	2·75	2·675	97·325
50	3·0	0·5	3·5	3·38	96·62
25	6·0	0·25	6·25	5·875	94·125
		Power Factor = 8.			
100	1·875	1·25	3·125	3·03	96·97
75	2·5	0·937	3·437	3·32	96·68
50	3·75	0·625	4·325	4·14	95·86
25	7·5	0·312	7·812	7·25	92·75

The efficiencies are usually given at 15° C. so that a comparison may be made.

To give the true efficiencies at the different loads, the losses would have to be corrected for the temperature of the transformer windings at those loads. As the temperature varies at different loads, and in different atmospheric temperatures, unreliable figures would result, so that a common temperature is usually used on which to base efficiency figures.

§ (14) Resistance.—The ohmic resistance of any transformer is easily obtained by a

consideration of the copper sections and length of wires used, and from the fact that the resistance between opposite faces of a cube of copper 1 in. in edge is 0·7881 microhms at 60° C.

§ (15) Drop or Regulation at Unity Power Factor.—The resistance voltage drop of a transformer is not due to the ohmic resistance alone, but is dependent on the total losses due to the current flowing, *i.e.* due to the C^2R loss + eddy current loss + stray loss. The apparent resistance drop measured in volts is found by dividing the load loss or copper loss by the current, or simply, the per cent load loss of output = per cent regulation or resistance drop.

§ (16) Reactance Drop.—The reactance drop is the drop in volts on transformation due to the reactance or leakage flux of the transformer windings. When current flows through the primary and secondary coils, a flux termed the load flux is set up inside and around each coil. This load flux is approximately equal and opposite for primary and secondary windings; therefore its value is zero in the magnetic core, but external to the core, due to the fact that the primary and secondary coils cannot both occupy the same position, there is a leakage or resultant flux which flows between and through them. *Figs.* 27 and 28

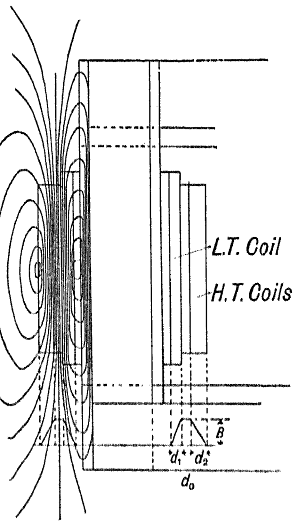

Fig. 27.—Diagram of Leakage Flux for Core-type Transformer with Concentric Coils.

show distribution of leakage flux. This resultant flux induces in the primary and secondary coils a voltage in quadrature to the current which produces it, and with a load at unity power factor this induced flux is in quadrature with the main voltage and current. This reactance voltage can be calculated from a knowledge of the coil dimensions, number of turns, and current flowing.

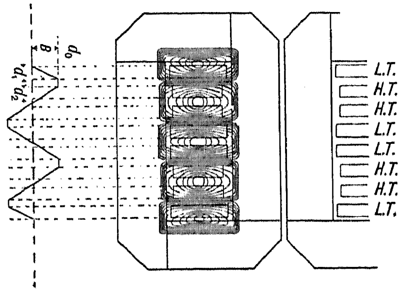

Fig. 28.—Diagram of Leakage Flux for Shell-type Transformer.

For concentric type of winding

Per cent reactance volts

$$=\frac{10{\cdot}2\times I_1N_1\times MT}{l\times E_1}\times\left(d_0+\frac{d_1+d_2}{3}\right)\frac{2\times\sim}{100}\times 10^{-4}$$

approximately.

For sandwich windings

Per cent reactance volts

$$=\frac{8{\cdot}2\times I_1N_1\times MT}{l\times E_1}\times\left(d_0+\frac{d_1+d_2}{3}\right)\frac{2\times\sim}{100}\times 10^{-4}$$

approximately.

Due to the short length of leakage path, the above formula has to be adjusted for sandwich-type windings for values exceeding 5 per cent by taking the calculated percentage as 2·4 (measured percentage 2) derived from tests.

For power transformers of modern design the reactance voltage attained is approximately as in the table given under the section dealing with mechanical stress.

Fig. 31 shows a vector diagram representing a transformer under load. Due to the fact that the magnetising current, resistance drop, and reactance drop are small percentages, it is impossible to draw a diagram to scale, so that by exaggerating or increasing the scale of these items the diagram appears distorted.

OA = applied volts = E_1.
OG = no-load current = I_M.
OH = primary load current = I_1.
GH = secondary load current = I_2.
OB = primary resistance drop = R_1 volts in phase with primary current.
BC = primary reactance drop = S_1 volts in quadrature with primary current.
AC = primary induced volts = ϵ_1.
OD = secondary induced volts = ϵ_2 180° out of phase with ϵ_1.
DE = secondary resistance drop = R_2 volts in phase with secondary current.
EF = secondary reactance drop = S_2 volts in quadrature with secondary current.
OF = secondary terminal volts = E_2.
ϕ_1 = primary phase angle.
ϕ_2 = secondary phase angle.
OC = primary impedance volts.
FD = secondary impedance volts.

§ (17) Regulation at any Power Factor.—In estimating the pressure drop or regulation of a transformer under load at any power factor, it is assumed that in *Fig.* 29 AC = AC′ and GF = GF′.

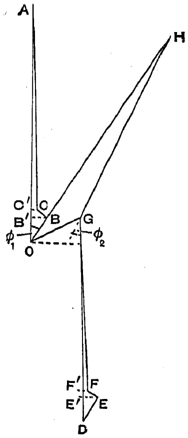

Fig. 29.—Vector Diagram for Single-phase Transformer.

This is not strictly true, since AC′ is the projection of AC on OA, but due to the low values of OB and BC compared with AO the error is negligible.

The error becomes noticeable if CB be greater than 5 per cent, and for these higher values correction should be made.

Primary voltage drop in volts

$$\begin{aligned} OC' &= OA - AC' \\ &= OB' + B'C' \\ &= OB \cos\phi_1 + BC \sin\phi_1 \\ &= R_1 \cos\phi_1 + S_1 \sin\phi_1. \end{aligned}$$

Similarly secondary voltage drop

$$DF' = R_2 \cos\phi_2 + S_2 \sin\phi_2.$$

The total drop on the secondary side

= Primary drop × ratio of turns + secondary drop

$$= (R_1 \cos\phi_1 + S_1 \sin\phi_1)\frac{N_2}{N_1} + R_2 \cos\phi_2 + S_2 \sin\phi_2.$$

Since $\phi_1 = \phi_2$ approximately, *i.e.* P.F. of primary and secondary winding equal

$$\text{Total drop} = \left(R_1\frac{N_2}{N_1} + R_2\right)\cos\phi_2 + \left(S_1\frac{N_2}{N_1} + S_2\right)\sin\phi_2.$$

In common terms

Total drop = resistance drop × P.F. + reactance drop $\sqrt{1 - \text{P.F.}^2}$,

∴ Per cent drop = per cent resistance drop × P.F. + per cent reactance drop $\sqrt{1 - \text{P.F.}^2}$

§ (18) Impedance Volts.—The primary impedance volts

$$OC = \sqrt{\text{resistance drop}^2 + \text{reactance drop}^2} = \sqrt{R_1^2 + S_1^2}.$$

The secondary impedance volts

$$FD = \sqrt{R_2^2 + S_2^2}.$$

Total impedance

$$= \sqrt{\text{Resistance drop}^2 + \text{Reactance drop}^2}.$$

Regulation at any P.F. N.

Therefore total voltage drop may be summarised as follows:

At any P.F. N

$$\%\ \text{drop} = \%\ \text{resistance drop} \times N + \sqrt{\%\ \underset{\text{drop}}{\text{impedance}^2} - \%\ \underset{\text{drop}}{\text{resistance}^2}} \times \sqrt{1 - \text{P.F.}^2}$$

If the P.F. N = 1, *i.e.* non-inductive load,

Per cent drop = Per cent resistance drop × 1
= Regulation at unity P.F.

If the P.F. N = 0, *i.e.* pure inductive load,

Per cent drop

$$\begin{aligned} &= \sqrt{\text{Impedance volts}^2 - \text{Resistance drop}^2} \times 1 \\ &= \text{Per cent reactance voltage} \\ &= \text{Regulation at 0 P.F.} \end{aligned}$$

§ (19) Mechanical Strains and Stress.—The impedance volts determines the total value of the short-circuit current that would flow through the windings, if the secondary terminals were short-circuited and full voltage were applied to the primary terminals, provided the supply voltage is maintained. A short-circuit current with normal applied voltage is an example of a fault in its worst form, but transformers are frequently called upon to stand such strains when in service. This short-circuit current is equal to the full-load current multiplied by the ratio of the impedance volts to full-line volts. If the impedance volts be 5 per cent of the full-line volts, then the short-circuit current will be twenty times full-load current. This is the value of the current if the reactance is obtained by coil grouping. If it is obtained by magnetic shunts, it will exceed this value in consequence of the saturation of the shunts, and their becoming non-effective above this point. Thus transformer coils are subjected to heavy mechanical stresses when under short circuit. In concentric windings the maximum stresses are radial, accompanied, however, by some stress in an axial direction, due to the coil centres of the primary and secondary not being

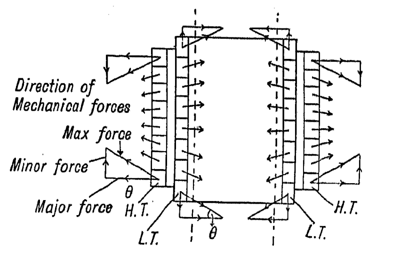

Fig. 30.—Diagram showing direction of Short-circuit Mechanical Stresses in a Core-type Transformer with Concentric Coils.

in line (see *Fig.* 30). In a sandwich-type winding the maximum stresses are in an axial direction with a smaller stress component in a radial direction (see *Fig.* 31).

The forces are due to the well-known fact that two adjacent conductors carrying current in the same direction attract one another, while two conductors carrying current in opposite directions repel each other. These forces are present under normal load conditions but are of small magnitude. They increase

with the square of the current, and so if the short-circuit current be 20 times full load then the forces will be 400 times those under normal

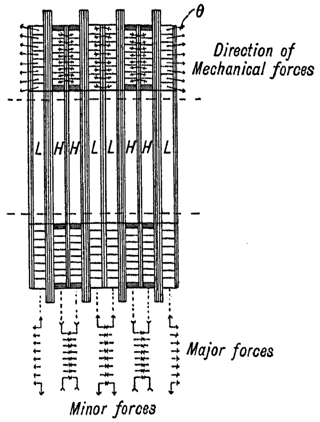

FIG. 31.—Diagram showing direction of Short-circuit Mechanical Stresses on Shell-type Transformer Coils.

working conditions. From a knowledge of the impedance volts, coil dimensions, and frequency these mechanical stresses can be determined. Considering the diagram, *Fig.* 33, the mechanical stress tending and acting along the line of centres to separate the coils is as follows:

Under short-circuit conditions

$$\text{Mechanical stress } F_{max.} = \frac{B_{max.}}{2} \times N_1 \times I_s \times \sqrt{2} \times MT \times \frac{10^{-1}}{981} \text{ grammes.} \quad (1)$$

Primary impedance volts

= Impressed volts at time of short circuits

$$= E_0 = 4 \cdot 44 B_{max.} \left(d_0 + \frac{d_1 + d_2}{3}\right) \omega \times N_1 \times MT \times 10^{-8}.$$

Substituting in (1),

$$\therefore F_{max.} = \frac{I_s E_0 1620}{\omega(d_0 + (d_1 + d_2/3))} \text{ grammes}$$

$$= \frac{I_s E_0 1620}{\omega(d_0 + (d_1 + d_2/3))10^6} \text{ tons.}$$

This stress acts on the total coil surface in the direction of the coil centres.

If the centres do not lie on a straight line parallel to the axis, then this stress has both an axial and a radial component given respectively by $F_{max.} \cos \theta$ and $F_{max.} \sin \theta$, where θ is the angle between the line of coil centres and the axis.

The above is for a shell-type sandwich-type winding, but the same is true for a core-type concentric winding, except that the maximum stresses are now radial. These stresses are directly proportional to the square of the short-circuit current and can only be limited by a limitation of short-circuit current obtained by increasing the impedance drop of the transformers, or by the addition of external reactances in series. For reasonable safety in service, the impedance values given in table are recommended:

Up to 500 K.V.A.	.	4	per cent	impedance.
500 to 1000 ,,	. .	5	,,	,,
1000 to 2000 ,,	. .	6	,,	,,
2000 to 4000 ,,	. .	8	,,	,,
4000 to 8000 ,,	. .	10	,,	,,

The stresses are those obtained when the short-circuit current reaches its final value. These stresses may be quadrupled, due to the doubling of the current during the first cycle, depending on the point of the voltage wave at which the short circuit occurs.

§ (20) TEMPERATURE RISE.—All transformers rise in temperature when switched on to a circuit, and when under load due to the loss of energy in the iron and copper, which is dissipated in the form of heat. The method of determining the temperature rise of a transformer from its losses and radiating surfaces varies with different types of winding and methods of cooling.

(i.) *Air-cooled Dry Transformers.*—For dry transformers, naturally cooled, the total free or exposed surfaces of core and coils are required. Cooling constants for various parts of the windings are known from tests and are approximately as follows:

Outside surfaces of coils: 10 watts per sq. metre per 1° C. rise.

Internal surfaces of coils with free air ducts: 2·5 watts per sq. metre per 1° C. rise.

Outside surfaces of core: 13 watts per sq. metre per 1° C. rise.

Inside surfaces of core with free air ducts: 5 watts per sq. metre per 1° C. rise.

(ii.) *Air-cooled Oil-immersed Transformers.*—Transformers immersed in oil and cooled by radiation from the surface of the tank have a temperature rise depending on the total surface and, if fluted, on the ratio of developed length to outside surface. This method gives the average temperature of the oil in the tank. The maximum temperature, that at the surface of the oil, is greater to an amount depending on the height of the tank. When calculating the temperature rise it must not be overlooked that the temperature varies at different heights as shown in *Fig.* 32. This is due to the hot oil rising in the centre of the tank flowing to the radiating surface and then down along the radiating surface to the bottom of the tank again. The ratio of the maximum to the average temperature is known and depends on the height of the tank, and the maximum temperature of the oil is obtained by multiplying average temperature by this ratio. The cooling constant, *i.e.* the watts

radiated per unit surface per 1° C. rise in temperature, varies with the depth of the flutes. For a plain tank it has the value 13 watts per

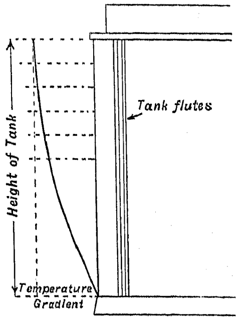

FIG. 32.—Temperature Gradient of O.I.S.C. Tank.

1° C. rise in average temperature for each square metre of surface. The cooling constant varies from 13 watts for a plain tank to 2·85 watts for a fluted tank with ratio of developed surface to plain surface of 7 : 1. The ratio of maximum temperature to average temperature varies from 1·8/1 to 1·5/1 with height of tank.

(iii.) *Air-blast Transformers.*—This type of transformer is mounted above a large air duct up which air is forced under pressure and circulated through the coils and core. The pressure of air required varies from $\frac{3}{4}$ inch of water for a 600 K.V.A. transformer to $1\frac{1}{2}$ inch for 3000 K.V.A. transformer. The quantity of air per kw. for 50° C. rise should be in the neighbourhood of 150 cub. ft. per minute. The coils are provided with numerous and ample ventilating ducts in a vertical direction. The temperature rise above that of the entering air can be computed from an assumed cooling coefficient of 30 watts per square metre of exposed coil surface per 1° C. rise in temperature, provided the full quantity of air per minute is provided. An air-blast transformer starting cold cannot be run without air except for approximately one hour at half load.

(iv.) *Water-cooled Transformers.*—Water-cooled transformers are provided with a cooling coil usually of plain copper, suspended in the tank at the top of the oil. Approximately 0·21 gallons of water per minute per kw. loss for 15° C. temperature rise in the water is required. For a temperature rise of the oil of 1° C. above the average water temperature, the energy dissipated varies from 60 to 75 watts per square metre of surface. The temperature rise in the water can be taken as proportional to the length of pipe, so that the average water temperature will be half the final temperature rise of the water (see *Fig.* 33).

The cooling coils are usually made of drawn copper tube, and of such a strength as to sustain a pressure of 120 lbs. per square inch. It is essential that the cooling coil be sound,

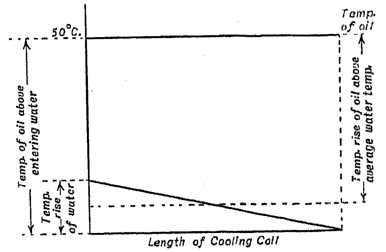

FIG. 33.—Temperature Diagram for Water-cooled Transformers.

as any leakage of water into the transformer oil would endanger the transformer and soon cause a break-down of its insulation. To increase the radiating surface of the cooling tube per foot run, cooling fins of copper are threaded over the tube and soldered in position about every $\frac{5}{8}$ or $\frac{3}{4}$ inch. These fins have an outside diameter approximately three times that of the tube. Lead pipes are frequently used if the cooling water has an acid tendency. These, however, are not so reliable as copper, as the core of the lead pipe cannot be guaranteed uniform. Iron pipes are also used, but are unreliable due to rusting internally. The pipe cannot be satisfactorily galvanised inside, and so it deteriorates, and its length of life is short compared with that of a copper pipe.

(v.) *Forced-cooled Transformers.*—Transformers cooled by the forced circulation of oil through the windings, which is then cooled in an external cooler, are known as forced-cooled transformers. Here the temperature rise depends on the quantity of water and oil flowing through the cooler and the surface of the cooler in contact with the water and oil. It also depends on the relative speed of flow of water to oil. The diagram (*Fig.* 34) shows

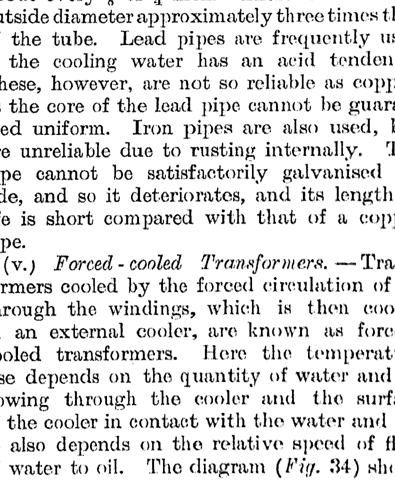

FIG. 34.—Temperature Diagram for Forced-cooled Transformers.

the temperature relations of water and oil in the cooler. The temperature rise of the oil

above that of the entering water is the figure required for the temperature rise. A cooling constant of approximately 150 watts per square metre per 1° C. rise in temperature will give the temperature rise of oil above average water temperature. As the specific heat of water to oil is 3 : 1, the temperature drop of the oil entering and leaving the cooler can be obtained if the quantity flowing is known.

§ (21) General Remarks on Temperature Rise.—When measuring the temperature of any part of a transformer core or winding, mercury thermometers should not be used while the transformer is energised, or errors may possibly be made. This is due to the fact that alternating magnetic stray fields will cut the mercury in the thermo bulb and thus cause local heating, resulting in high and erroneous readings. Spirit thermos or thermocouples should be used when actual temperature of winding and core are required while energised, or the mercury thermometer should be so placed as to be in a neutral zone.

(i.) *Temperature of the Coils.*—The temperature in the oil does not always indicate the actual temperature rise of the transformer coils. Heat is passing from the coils to the oil, and they are therefore at the higher temperature. This actual temperature can be obtained either by the increase of resistance method or by the insertion of thermocouples in contact with the windings, or by the increase in resistance of a special resistance coil placed adjacent to the coils at the supposed hottest point. The resistance is measured before and after the load run, and the increase of resistance gives a measure of temperature rise. Since approximately 0·4 per cent increase in resistance of copper corresponds to 1° C. temperature rise, the temperature rise can be obtained. This temperature obtained from the coil resistance gives the average temperature of the coils, since the resistance is that of the whole winding. The hot resistance should be taken immediately the load is shut down, as for every minute lost the temperature falls. The rate at which the temperature falls depends on the rating of the copper and the ventilation afforded.

(ii.) *Atmospheric Conditions.*—Temperature rise must be corrected for working conditions if the test is taken in an atmospheric temperature differing from that which will prevail on site. The atmospheric temperature during a test may be 20° C., while on site it may be 40° C., so that for the 20° C. difference in temperature a correcting factor must be used to bring the test temperature rise to an equivalent temperature rise on site. A figure of 0·18 per cent for every 1° C. difference in air temperature is an approximate figure which can be used. Thus for 20° C. difference the test temperature rise must be increased 3·6 per cent to give equivalent temperature rise on site. *Altitude* also has an effect on temperature rise. Tests of temperature rise taken at sea-level should be corrected by increasing the temperature rise $2\frac{1}{2}$ per cent on every 1000 feet to obtain the equivalent temperature. The reverse of this does not apply below sea-level, such as underground in mines, for there the atmospheric pressure is artificial due to forced ventilation. If a transformer is to have a 40° C. rise at an altitude of 3000 feet, its temperature rise at sea-level should be 37° C.

§ (22) Transformer Oil.—The oil used for transformers has to serve two purposes: first, it must be capable of absorbing and transmitting heat since it is used as a cooling medium, and, secondly, it must have a high insulating value or dielectric strength. There are many other requirements that a transformer oil has to comply with, such as high flash-point, viscosity, etc., and it must not disintegrate at high temperatures. A transformer oil is a mineral oil obtained from crude oil by distillation at a certain temperature. The quality or suitability of the oil for transformer use depends on the temperature of distillation. The general requirements of a transformer oil are as follows :

(i.) *Dielectric Strength.*—The oil when free from moisture and dirt should have a high dielectric strength. The break-down voltage between $\frac{1}{2}$-in. spheres, $\frac{1}{8}$-in. gap, should be about 25,000 volts for dry oil of reasonable viscosity. The presence of small quantities of moisture in suspension in the oil has a serious effect on its break-down voltage. The presence of ·025 per cent of moisture will lower its dielectric strength 70 per cent approximately. As a precaution against excessive moisture care should be taken in storage. The drum in which the oil is stored should be of steel and dry, and fitted with a screw bung and a lead washer. The drums should be stored under cover and not exposed to the elements, also they should be stored on their sides and not on end. The transformer oil should, before being used, be tested for dielectric strength, and if this shows a low break-down voltage, it should be dried and filtered. Methods of drying will be found under the remarks regarding erection.

(ii.) *Flash-point.*—From the point of view of safety it is essential that the oil should have a flash-point appreciably higher than the maximum temperature that it will attain in use.

As a transformer may obtain a temperature of 90° C., which is the maximum temperature fixed by the B.E.S.C. rules, a flash-point of about 145° C. for the oil vapour and 180° C. for the oil itself can be taken as a safe value.

(iii.) *Viscosity.*—The rate of transmission of heat depends on the viscosity of the oil, and the lower the viscosity the more rapid the circulation of the oil, and hence the greater amount of heat transmitted to the radiating surfaces in a given time. Unfortunately, the lower the viscosity the lower the flash-point, and as this is limited by reasons of safety, so also is the viscosity limited. An oil having a viscosity lower than 30 at 15° C. when measured with a Redwood viscometer, taking rape oil as 100, is not suitable as a transformer oil.

(iv.) *Sludging.*—Most oils when subjected to high temperatures in contact with air and metal begin to disintegrate and deposit hydrocarbons in the form of sludge. This sludge, if allowed to accumulate, will choke up ventilating ducts and insulate radiating surfaces and further aggravate the trouble by causing still higher temperatures. A special treatment of the oil by exposing it to strong sunlight, or by a special treatment with acids, produces what are generally known as non-sludging oils. This treated oil has a more stable chemical composition, and withstands heat better than untreated oils. The chief causes of sludging are: high temperature for long periods, dust or dirt in suspension in the oil, and the contact of free air with the oil.

Various schemes and devices are used to prevent sludging, such as limiting the quantity of oil in contact with the air by the use of expansion chambers as shown in *Fig.* 35, the

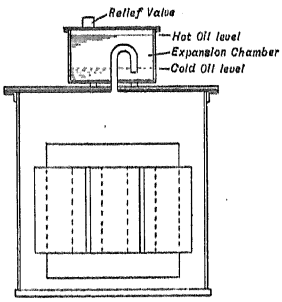

FIG. 35.—Transformer Tank with Expansion Chamber.

use of oil traps, or by using a float on the oil surface. Bare copper aggravates sludging, so that all copper should be tinned or taped.

Russian oils are less liable to sludge than American oils; but they have the disadvantage of a lower flash-point due to a lower viscosity.

(v.) *Acidity.*—Transformer oils must be free from acid, or the metal and insulating material of the transformer will be liable to deteriorate and thus reduce considerably the life of the transformer. The acid content should not be more than 0·02 per cent.

(vi.) *Evaporation.*—The loss of weight of oil due to evaporation at 100° C. should be small and not exceed 0·2 per cent after five hours. At the working temperature of the transformer the loss should be negligible.

(vii.) *Solidification.* — The oil should be mobile at 0° F., for transformers are frequently mounted out of doors in exposed positions.

§ (23) TRANSFORMER TESTS.—The commercial tests to which a transformer is subjected are known as:

1. Ratio of windings and tappings.
2. Polarity of windings.
3. Load or copper loss and impedance.
4. Core loss and magnetising current.
5. Cold resistance.
6. Temperature test or load run.
7. Insulation tests.

(i.) *Ratio.*—The voltage ratio of a transformer on open circuit is practically the same as the ratio of turns. Thus, if a known voltage be applied to the low-tension winding, the anticipated voltage can be calculated for the high-tension side and can there be measured on a voltmeter, or, if the voltage be too high, by the use of a potential transformer of known turns. The above is known as the direct method.

The factory or commercial method, which is quicker than the above and more flexible, is known as the ratiometer method. The ratiometer (*Fig.* 36) consists of a single-phase transformer of known ratio, which is provided with

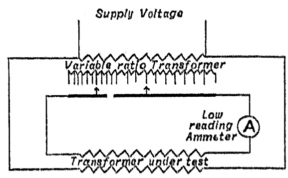

FIG. 36.—Ratiometer and Method of Connection.

numerous tappings connected to dial switches, so that any probable ratio can be obtained. The transformer windings of the ratiometer are adjusted to the expected ratio and are connected in parallel on the H.T. side and in opposition on the L.T., with a low-reading ammeter in circuit. If the ratio is correct, no current will flow through the ammeter.

To ensure that the transformers are excited, the ratiometer ratio is altered by one turn and this will cause a current to flow. If the ratio is wrong a current will immediately flow, and this can be reduced to a minimum by regulating the ratiometer turns. The ratio giving minimum current is the ratio of the winding. The same method is used for the checking of the ratio for the tappings.

(ii.) *Polarity.*—The test for polarity is essential in order to find the winding sense or voltage direction of the primary and secondary windings. This information is required in order to make correct connections for three-phase units and to ensure that banks of transformers will work in parallel with one another, provided other requirements such as ratio and per cent impedance volts are the same. The polarity of transformers in America is standardised in accordance with the standard rules of the Institute of Electrical Engineers. On the Continent the V.E.D. rules are adopted.

In England polarity varies with the different manufacturers, but efforts are being made to standardise and bring all manufacturers into line. The method of taking the test for polarity is simple and resolves itself into connecting the primary and secondary windings in supposed series.

An A.C. voltage is applied across the H.T. winding, and the relation of the voltage across the two windings to that applied determines whether it is standard or non-standard.

With three-phase transformers of the core-type construction, where the three magnetic circuits interlink, the polarity must be taken over two phases in series and so with every pair, *i.e.* phase A and B in series, then B and C in series, and finally A and C in series, and all these must show the same winding sense. Polarity can also be determined by D.C. current. If a D.C. current of known direction be passed through the primary winding and a D.C. voltmeter be connected across the secondary, a sudden break in the D.C. current will induce a voltage in the secondary winding and by its polarity show the sense of winding. This method can also be applied to three-phase core-type transformers where the magnetic circuits are interconnected, but here care must be taken that the D.C. current flows in the opposite direction in one phase to that in the other two.

(iii.) *Load or Copper Loss and Impedance.*—The load or copper loss is that loss which occurs in the transformer windings and core, due to the flow of full-load current through the windings. The voltage required to force this full-load current through the windings is termed the impedance volts. This loss is measured in the following way: For single-phase transformers the connections are as shown in *Fig.* 37. The low-pressure side is short-circuited, and the H.T. side connected to an A.C. supply, having a suitable variable voltage and the required frequency. An ammeter, voltmeter, and wattmeter are connected as shown. The voltage is gradually increased until full-load current is indicated by the ammeter to be flowing. The wattmeter reading at this current is the load or copper loss and the voltage V is the impedance volts, Since the secondary is short-circuited, the voltage is only required to overcome the resistance of the primary and secondary coils, and provide for the leakage flux between them, *i.e.* the reactance voltage. These two voltages are combined geometrically in the voltmeter reading as the impedance volts. From these three measurements of loss, volts, and current the resistance drop and regulation of the transformer at unity power factor can be obtained. In addition the reactance can be found, and hence the regulation at any other power factor can be obtained.

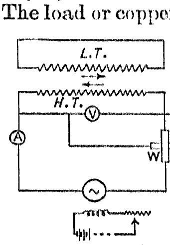

FIG. 37.—Diagram of Copper Loss Connections.

If the copper loss be W_c,
Load current I_1,
Impedance volts V_0,

then $W_c/I_1V_0 = \cos\theta$, where θ = the phase angle between the load current and the impedance volts (*Fig.* 38).

The resistance drop in volts $= V_1 \cos\theta = W_c/I_1$.

The reactance drop in volts $= V_1 \sin\theta$.

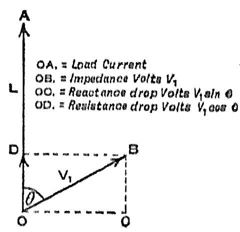

FIG. 38.—Load Loss Diagram.

The copper loss of a three-phase transformer, either of the core or shell type, is measured as above on each phase, and the three readings added together arithmetically.

As the power factor of all these wattmeter readings are low, a wattmeter suitable for low power factor work should be used or erroneous readings will result.

(iv.) *Core Loss and Magnetising Current.*—The core loss or iron loss is measured in a similar way to the copper loss for single-phase transformers, except that the H.T. winding is left open-circuited and the meters are inserted on the L.T. side as in *Fig.* 39.

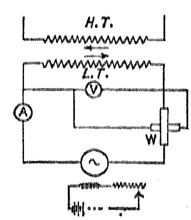

FIG. 39.—Diagram of Core Loss Connections.

The normal voltage is applied at the correct frequency and readings of the ammeter and wattmeter are taken. These measurements of the wattmeter and ammeter are the iron loss and the no-load or magnetising current respectively. From

these can be obtained the phase angle of the no-load current and the applied volts.

$$\text{For } \cos\phi = \frac{\text{iron loss}}{E_1 \times \text{no-load current}} = \frac{W_F}{E_1 \times I_M}.$$

The watt component of the no-load current which supplies the losses and is in phase with E is

$$I_W = \frac{W_F}{E_1} = I_M \cos\phi.$$

The wattless component of the no-load current which energises the core and is in quadrature with E_1 is

$$I\mu = I_M \sin\phi. \quad \text{See } \textit{Fig. } 40.$$

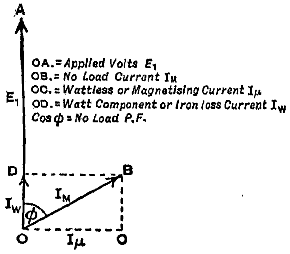

FIG. 40.—Core Loss Diagram.

The core loss of a three-phase transformer cannot be measured either for a shell or a core-type transformer by a measurement of each phase separately.

The three phases of magnetic flux in the core are interlinked in the yokes and hence a three-phase measurement is necessary. In this case the loss may be measured either by the 3-wattmeter or 2-wattmeter method. Both of these methods give accurate results with unbalanced three-phase currents, as is the case with the no-load currents of a three-phase transformer. *Figs.* 41, 42, 43, and 44 show methods of connection for 3- and 2-wattmeter measurements for delta and star connection and for a transformer where the neutral point is unavailable. When the neutral point is unavailable, the neutral of the machine supplying the power is used, or an artificial neutral formed by another transformer or by the three resistances or reactance coils. The single wattmeter method is not suitable for unbalanced loads such as the core loss of a three-phase transformer. In consequence of the unbalanced magnetic circuit of a three-phase transformer the three-phase no-load currents are not equal or at an equal phase angle with the applied voltage. If readings be taken by the 3-wattmeter method on each phase, the no-load phase angle magnetising current, etc., can be plotted in vector form.

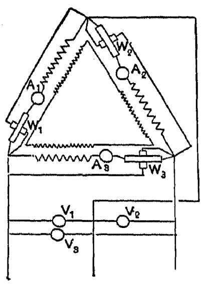

FIG. 41.—Δ/Δ Transformer. Core Loss Measurement. 3-wattmeter. Total Loss $= W_1 + W_2 + W_3$.

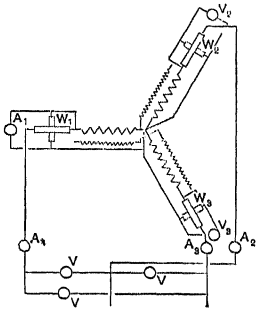

FIG. 42.—λ/λ Transformer. Core Loss Measurement. 3-wattmeter. Total Loss $= W_1 + W_2 + W_3$.

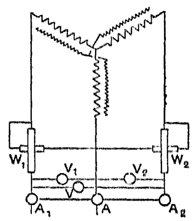

FIG. 43.—λ/λ Transformer. Core Loss Measurement. 2-wattmeter. Total Loss $= W_1 + W_2$.

The no-load current is often termed the magnetising current, but, as shown above, the no-load current is the geometrical sum

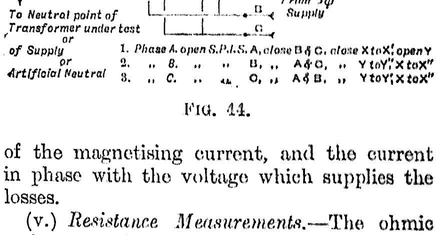

FIG. 44.

of the magnetising current, and the current in phase with the voltage which supplies the losses.

(v.) *Resistance Measurements.*—The ohmic resistance of the primary and secondary windings should be very carefully measured at normal atmospheric temperature. The

transformer should be allowed to stand until atmospheric temperature is assured, whether in oil or air. This is essential, since on this figure and the ohmic resistance figure after a load run is based the temperature rise of the transformer by resistance, as already described above. The resistance is usually measured by passing D.C. current through the windings and noting the voltage drop. To ensure that the D.C. current has reached a maximum value, the other winding is short-circuited, *i.e.* while the H.T. resistance is being measured, the L.T. is short-circuited and *vice versa.* The actual temperature of the winding should be taken by thermometer during the above test.

(vi.) *Temperature Test on Load Run.*—Transformers on completion are usually run on a load test to ensure that the temperature rise is within the required figure. If a test of the radiating capacity of the tank only is required, one transformer only is necessary. If the transformer temperature rise is required by resistance measurements, generally two transformers are necessary. The following briefly describes various methods employed:

(*a*) *Dead load* with resistance or reactance or both as means of loading. The transformer, if it be a single-phase, is placed in its tank and filled with oil and connected up as per *Fig.* 45. The primary is supplied with power from an A.C. supply of correct frequency. The secondary is connected to an adjustable resistance and reactance. The values of resistance and reactance to allow full-load current to flow can be calculated, but usually an excessive value is used for safety and gradually reduced until full-load current flows. This method gives true readings, since the transformer is under exactly the same conditions as it will be in service. It is possible to use this method for small transformers, but for large units it becomes impossible to absorb the enormous amount of K.V.A. and also to supply it in a testing department. Above 10 K.V.A. this method is not recommended on account of the excessive cost and waste of energy.

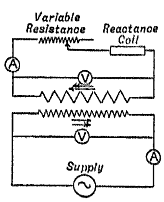

FIG. 45.—Diagram of Connections for Dead-load Run.

→ Voltage direction.
→ Current direction.

(*b*) *The Short-circuit Equivalent Run.*—If the temperature of the transformer is given as that of the oil, the test becomes one of tank surface radiation. The low-pressure side of the transformer is short-circuited and A.C. current at correct frequency passed through the high-pressure winding, until its value is such as to give an energy loss equal to the sum of the known load current loss plus the core loss.

This means the windings are overloaded to such an extent that the overload losses equal the core loss. As the load current loss increases as the square of the current and decreases with temperature, the correct overload can be estimated from the known losses. While this method gives the correct oil temperature and radiating capacity of the tank, it is not recommended, as it subjects the winding to intense internal heating and possible damage unless designed for such overloads. Various diagrams are shown in *Fig.* 46 of the connections for various types of transformer connection, showing the direction of current flow.

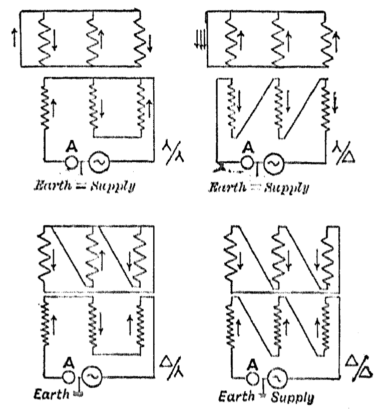

FIG. 46.—Various Short-circuit Connections for Three-phase Transformers.

(*c*) *Full-load Back-to-back Temperature Run.*—Two transformers run back to back, that is, connected in such a way as to be fully excited and allowed full-load current to flow, is the standard method employed. The two transformers in *Fig.* 47 (single-phase for simplicity in explanation) are connected with their windings, primary and secondary, in parallel and excited at the required voltage and frequency. Inserted in the one parallel

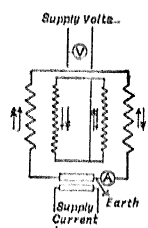

FIG. 47.—Connections for Full-load Back-to-back Run on two Single-phase Transformers.

→ Voltage direction. → Current direction.

connection, usually the low pressure, is an auxiliary transformer which supplies the load current. From the direction of the arrows indicating the voltage, it will be noted that the voltages, primary and secondary, oppose one another and no current flows except the no-load current of the two circuits in parallel.

The current from the auxiliary supply, in this case a transformer, circulates through the two windings and also induces a current in the secondary winding flowing round the parallel connection. In this way the core losses are supplied and the load current losses are supplied, and these comprise the total energy required for the test, except the losses in the supply circuits, which are external to the transformers under test.

In the case of extra H.T. transformers the voltage and current are supplied on the L.T. windings, as in *Fig.* 47, thus leaving the H.T. connection undisturbed, easy to insulate and protect. In this case the magnetic current flows in the same winding as that in which measurement of the load current is taken, and since the two sources of supply are dependent and of same frequency, an error in loading may occur due to the product of these two currents being shown on the meter. This can be overcome by inserting the ammeter in the H.T. circuit and isolating the same or earthing it, or by supplying the load current at a different frequency.

(*d*) *Three-phase transformers* can be run on load in a similar manner to the above, but in this case it is essential that full provision be made for all circulating currents to have a free return path. *Fig.* 48 shows two three-phase transformers connected back to back, and it should be noted that the connection of the neutral points is essential to provide a return path for the load current of one phase.

One transformer L.T. winding has its phases in series and is connected in series with the other transformer similarly connected. The H.T. windings are connected in parallel. The H.T. windings may be open delta connected, as in *Fig.* 48, and series, but in this case the neutral joining the parallel winding must carry three times full-phase load current. The windings may be permanently connected star or delta and connected as in *Fig.* 48, but the impedance of the three phases must be equal for equal distribution of load.

A full-load temperature run can be made on a single three-phase transformer under certain conditions. The winding on the primary and secondary side must be connected in delta and the transformer must not have a high reactance voltage, 5 per cent being the maximum figure for correct results. If the reactance be higher, the leakage flux due to the current in the three phases being in the same phase and same direction causes a choking effect. This choking effect increases the load losses for the same current flow, and hence incorrect and high temperatures result (see

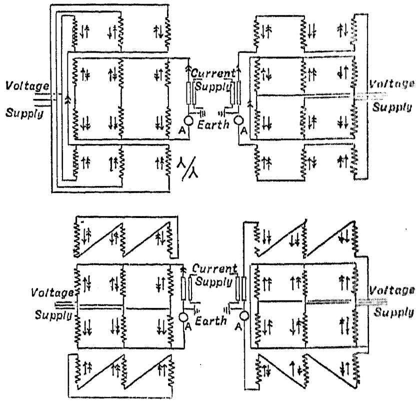

FIG. 48.—Various Connections for 3 ϕ-load Runs.

Fig. 49). If the three-phase unit be three single-phase transformers, *i.e.* with three magnetic independent circuits, this method is quite correct and gives good results.

Auto transformers may be tested in a similar manner to the above, and *Fig.* 50 shows

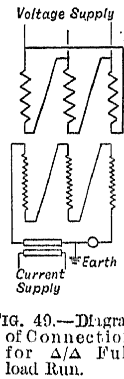

FIG. 49.—Diagram of Connections for Δ/Δ Full-load Run.

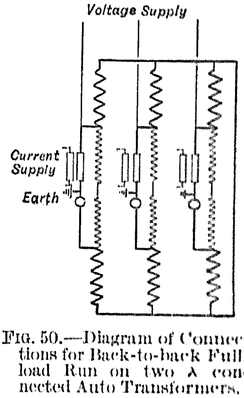

FIG. 50.—Diagram of Connections for Back-to-back Full-load Run on two λ connected Auto Transformers.

two three-phase autos so connected, but in this instance a three-phase auxiliary transformer for supplying current is required. The above do not cover all the possible methods that are available for carrying out full-load runs, but they cover the best-known ones. The length of time required for a temperature run varies with the size of transformer. In the case of a 500 K.V.A. transformer it should reach a constant temperature rise in ten to twelve hours. This period can be reduced by running on overload for the

first three or four hours, and then dropping to the normal load until a steady temperature is maintained. *Fig.* 51 shows typical tempera-

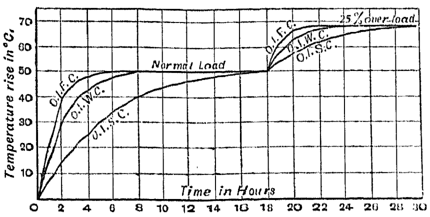

FIG. 51.—Typical Temperature Curves for 4000 K.V.A. Transformers.

O.I.S.C.=oil-immersed self-cooled; O.I.W.C.=oil-immersed water-cooled; O.I.F.C.=oil-immersed forced-cooled. Ratio of copper loss to core loss=2:1.

ture curves for normal load and overload of a 4000 K.V.A. transformer.

(e) *Hot Resistance Measurements.*—Immediately on the shut down of the full-load run the ohmic resistance of the windings must be taken. There should be very little loss of time between shut down and measurement, or the results will give too low a value due to falling temperature. The time between shut down and measurement should be recorded and, if necessary, corrections made for temperature drop during this period.

(vii.) *Insulation Tests.* —All transformers should be subjected to a severe insulation test between primary and secondary windings and between the windings and core. This insulation test should be made while the transformer is hot, *i.e.* at its normal working temperature. The pressure applied is usually that specified by the B.E.S.C., which is 1000 volts plus twice normal voltage for one minute. The high-pressure windings are all joined together and connected to the testing transformer and the other pole of the testing transformer is connected to earth. The low-pressure windings are connected to the core and earth. The test pressure is applied gradually, that is, it is run up to the test value in graduated steps. After the test period of one minute, the pressure is gradually reduced to zero.

The low-pressure windings are tested in a similar manner. The transformer windings are subjected to an over-potential test while hot, and the value of this over-potential test is usually $2\frac{1}{2}$ times normal. This voltage is applied at a higher frequency than normal so that the induction in the core is normal and the current not excessive. If the over-potential test is $2\frac{1}{2}$ times, then for a 50-period transformer the frequency must be approximately 125 for the over-potential test.

Fig. 52 shows the pressure tests for various voltages as specified by the V.D.E., A.I.E.E., and B.E.S.C. rules.

If transformers are to be subjected to test pressures beyond one minute's duration, the

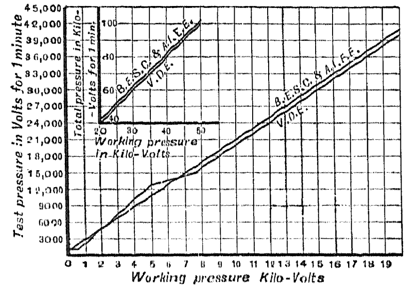

FIG. 52.—Standard Test Pressure Curves.

B.E.S.C.=British Engineering Standards Committee; A.I.E.E.=American Institute of Electrical Engineers; V.D.E.=German Standard Rules 1914.

pressure must be reduced or the insulation used in manufacture must be increased beyond the figures given because the break-down voltage of the insulation material falls with time due to heating, on account of dielectric hysteresis.

Fig. 53 shows the break-down voltage of sheet insulation material with time; the char-

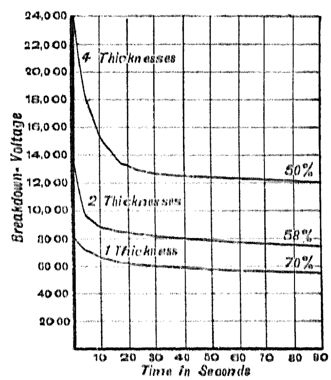

FIG. 53.—Curves showing relation of Puncture Voltage with Time for Sheet Insulating Material.

acteristic it shows is similar to that of other materials.

§ (24) LOCATION, ERECTION AND OPERATION.—Transformers should be carefully examined before putting into operation, and their location should be studied to ensure that the local conditions do not endanger its satisfactory operation or longevity.

The satisfactory location of a transformer is dependent on the method of cooling that has been adopted. An oil-insulated self-cooled transformer should be situated in a well-ventilated chamber and in one free from moisture or dust. There should be approximately two feet clearance between the transformer tank and the sides of the chamber, and if there

be other transformers in the same chamber, a similar distance between them. The ventilation of the chamber should be at floor level and roof, and evenly distributed around the chamber. If the transformer is artificially cooled, the ventilation of the chamber is unimportant, but the same should be free from moisture and dust.

Transformers are shipped to site in line with any one of the following methods, depending on size, handling facilities on site, and destination:

(*a*) Transformer and tank shipped separately.

(*b*) Transformer shipped in its tank.

(*c*) Transformer shipped in its tank complete with oil.

Where possible it is advisable to ship the transformer complete in its tank with oil. This method facilitates erection and often precludes the necessity for drying out the transformer on site. This method of shipment is only possible when the tank is of robust construction, and the handling facilities on site suitable. In the case of large transformers, where the tanks are when upright outside the railway loading gauge, separate shipment of transformer and tank has to be made.

(i.) *Drying.*—Before putting a transformer into service it is necessary to ensure that the transformer and its oil are perfectly dry. The oil can be dried by several methods:

(*a*) By heating it up by means of steam coils or resistance units until all the moisture is driven off. If resistance units be used, care must be taken that the resistances have not too intense a heat or the oil will carbonise and disintegrate and deposit hydrocarbons or sludge.

(*b*) By passing the oil through unslaked lime. This process is rather slow and necessitates the filtering of the oil after passing through the lime to extract all the suspended particles of lime.

(*c*) By passing the oil through a dehydrating plant similar to *Fig.* 54. In this apparatus the oil is passed through a series of blotting-papers under pressure. The blotting-papers are suitably dried and they absorb the moisture.

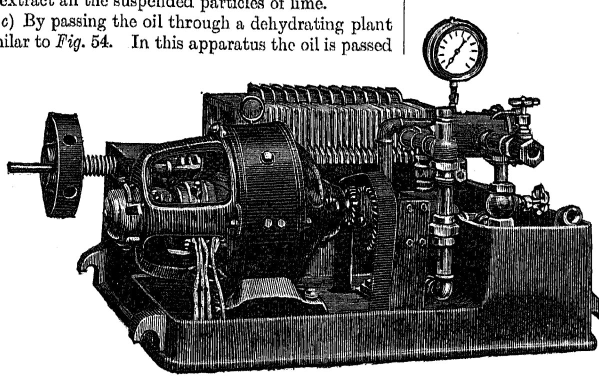

FIG. 54.—Dehydrating Plant.

The oil may be said to be dry if its break-down voltage be 20,000 volts across $\frac{1}{8}$ inch gap needle-points.

The transformer may be dried by several methods:

(*a*) It may be dried in the air by circulating through the primary and secondary windings an alternating current of such a value as to raise its temperature up to 85° C. Great care is necessary in this method, and the temperature and current kept well under control. The approximate time and current required is given in table:

K.V.A. of Transformer.	Full-load Current.	Time approximately.
	per cent.	days.
20–50	100	2
50–200	60	3
200–500	40	4
500–1000	20	5
Above 1000	15	6

(*b*) By immersion in dry oil and heating the transformer and oil up to 105° C. by circulating current through the windings, or by inserting resistances in the oil and heating up the oil by passing current through the resistances. The approximate energy required and length of time is outlined in table:

Oil.	Energy.	Time approximately.
gallons.	kilowatts.	days.
100	5·5	$1\frac{1}{2}$
250	14	$1\frac{1}{2}$
500	28	2
750	42	2
1000	56	$2\frac{1}{2}$
1250	70	$2\frac{1}{2}$
1500	84	$2\frac{1}{2}$

(*c*) By means of hot dry air. Pass through and around the transformer windings and core hot dry air by means of a fan. The temperature of the air should be approximately 85° C.

(*d*) By means of vacuum. The transformer is placed in a special chamber, or if its own tank be strong and has a suitably strong air-tight cover, in its own tank. The transformer is heated up to about 85° C. at normal atmospheric pressure and is then subjected to 26 to 28 inches of vacuum for twelve to thirty-six hours, according to size and quantity of insulation.

The period of time required to dry a transformer varies with its state of dryness, and this can only be roughly estimated by an intimate knowledge

of its history. The insulation resistances of its windings to each other and to earth are no indication of the state of dryness. All transformers for 5000-volt service and over require drying unless they have been shipped in oil. The insulation resistance values taken at different stages of the heating and drying, and plotted against temperature and time, will give a valuable indication as to when the transformer is dry. As the temperature rises the insulation resistance falls, as shown in *Fig.* 55.

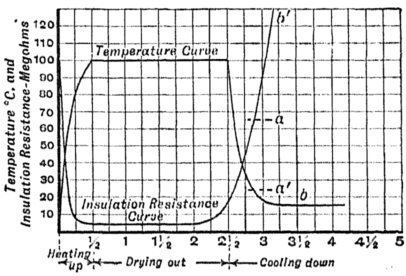

FIG. 55.—Insulation Characteristic with Temperature.

This fall in resistance will continue as the temperature rises. As the temperature becomes steady, so will the insulation resistance, and it will remain at this value for a short or long period according to the quantity of moisture present. As the transformer becomes dry, the insulation resistance will rise but not by a very high figure, but as the temperature is reduced to normal working temperature, the value will rise in most cases to a high and safe value approaching a very large figure.

(ii.) *Switching in.*—Before putting transformers into service the leads and tappings should be checked, and if the transformer has to work in parallel with others, it should be phased out. Transformers should be made alive for a few hours before throwing any load on to the windings to ensure that all air locks are released.

If the transformer is shipped in oil, it does not ensure the same being dry, for in transit moisture may have entered the tank. Before putting such transformers into service a sample of the oil should be drawn from the bottom of the tank and tested. On the result of this test the necessity of drying or otherwise can be decided.

Switching transformers on and off the mains may result in heavy current rushes or sudden high-voltage rises. The heavy current rush is frequently noticed on switching in and is a magnetising current dependent on the point of the voltage at which switching takes place, and is a maximum when the wave is passing through its zero value. Voltage rises of 1·6 to 1·8 times normal may be experienced on switching in and out if at the instant of switching the wave is passing through its zero value. These current rushes and voltage rises are not dangerous, and die down in two or three cycles. They can be reduced by inserting a resistance in series with the circuit through an auxiliary contact on the main switch. Switching in slowly will, in general, reduce these voltage rises and current rushes.

Transformers should be regularly inspected every six to twelve months according to size, voltage, and severity of service. Any auxiliaries such as coolers, pumps, motors, etc., should also be subjected to periodic inspection. The principal points to be watched for in such inspection are shrinkage of windings, slackening of core clamping bolts, moisture and sludge in oil, deterioration of insulation due to heating, freedom of oil or air ducts from dirt or dust. In medium and large transformers the oil should be filtered about every twelve months.

§ (25) PARALLEL OPERATION.—Transformers which are required to run in parallel with one another must share the load evenly, *i.e.* in proportion to their rated capacity, the usual limit of variation from a true proportion being 10 per cent.

For accurate paralleling the transformers should have exactly the same ratio of transformation, the same per cent impedance voltage, the same polarity or winding sense, and the same phase rotation (phase rotation applies only to polyphase transformers).

Before connecting any two polyphase transformers in parallel it is necessary to ensure that the phase rotation is correct, and this can only be done by *phasing out.* The term "phasing out" is applied to the procedure adopted for determining the correct junction of the terminals of two or more transformers.

With single-phase transformers the phasing out is a simple operation. *Fig.* 56 shows two transformers X and Y. Transformer X is connected across the busbars on both primary and secondary side. Transformer Y is connected on the primary side only in a similar manner to X. One pole of the secondary side only is connected to the secondary busbar. If polarity is correct, the voltage between the open end of Y and the remaining bar is zero, and permanent connection can be made.

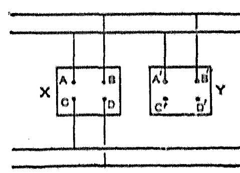

FIG. 56.—Paralleling Single-phase Transformers.

If the voltage is twice the line volts, then the polarity is reversed and the two secondary

leads of Y must be interchanged on the bars. The phasing out of three-phase or polyphase transformers is similar to that described above for single phase, but it is necessarily more complex, and has to be extended to cover the diversity of possible connections. *Fig.* 57 shows the various connections which will parallel together, and those which will not due to the secondary windings being 30° out of phase.

For instance, a delta/delta will parallel with a star/star, but not with a delta/star connection, and similarly a delta/star will parallel with a star/delta but not with a delta/delta or a star/star connection. Phasing out of a polyphase transformer is carried out as follows.

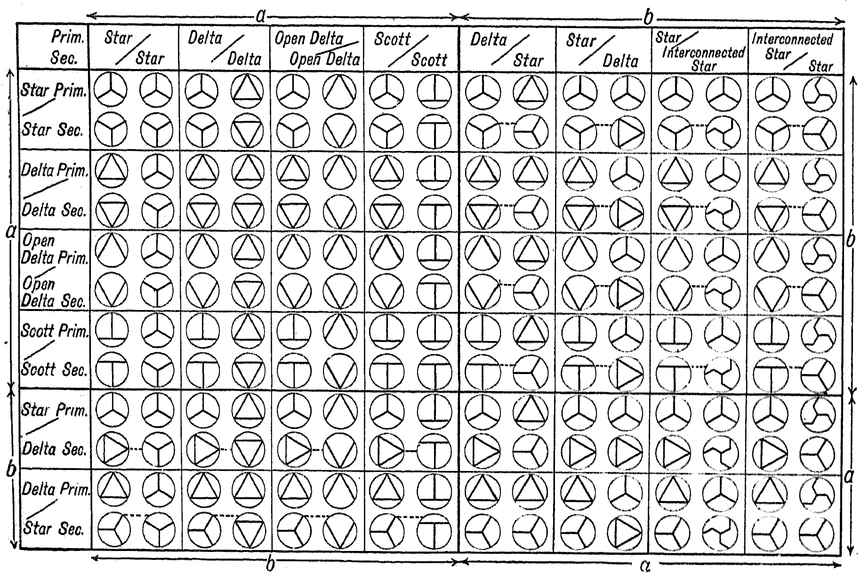

FIG. 57.

a, Various transformer connections that will parallel.
b, Various transformer connections that will not parallel.

Fig. 58 shows two three-phase transformers X and Y. Connect transformer X across the bars on the primary and secondary side and connect in a similar manner the primary of transformer Y. If the two transformers are unearthed on the secondary side, the secondary winding voltage will have no definite relation to the voltage of transformer X, and is said to be floating above or below earth potential. In order to fix some definite relation between the two secondary voltages of X and Y, the neutrals must be connected together if possible, or one phase of the transformer Y must be connected to the bus bars. Only one of these alternative connections must be made, or disaster might result.

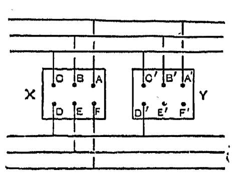

FIG. 58.—Paralleling Three-phase Transformers.

In this instance connect D′ to the same bar as D, and measure the voltage between EE′ and FF′. If these two readings give zero reading, the transformers are correctly phased and may be connected to the line. (In making these measurements, ensure that the voltmeter leads are without a break, and to prove this measure voltage across EF′ and FF′, which should be the line volts.) If readings are obtained across EE′ and FF′ then polarity or phase rotation is incorrect and reconnection must be made.

With the above method of fixing the voltage relation between the two transformers, EE′ and FF′ may read 1·73 times line voltage and E′F and EF′ twice line voltage when the polarity is reversed and the phase rotation is incorrect. To bring this transformer into line, the windings must be connected in reverse polarity and phasing out again determined until EE′ = 0, FF′ = 0, EF′ = line volts, and FE′ = line volts.

This phasing out may be a lengthy operation if not done systematically, for there are thirty-six possible connections of the two transformers which can be made.

Fig. 59 will assist in judging which leads need changing should difficulties arise.

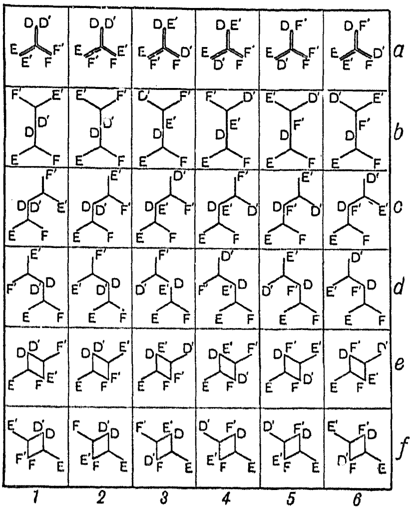

FIG. 59.—Phasing-out Diagram with Line Leads connected.

If the relative voltages of the secondary windings are fixed by joining the neutral points of the two transformers together, as in *Fig.* 60, then twelve possible connections can be made. In this instance the voltage measurements EE′, DD′, FF′ should be taken and zero reading obtained, and as a check ED′, EF′, E′D′, E′F′, F′D′, F′D should be

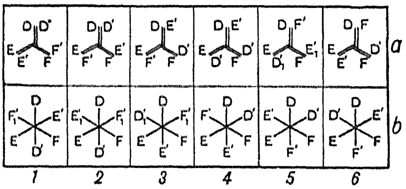

FIG. 60.—Phasing-out Diagram with Star Points connected.

taken and line voltage obtained. This result shows correct polarity and phase rotation, and connection can be made.

If incorrectly phased

EE′	may be	0·578	line volts.
FF′	„	0·578	„
EF′	„	1·15	„
FE′	„	0·578	„
ED′	„	0·578	„
FD′	„	1·15	„

This corresponds to incorrect polarity and correct phase rotation, and from *Fig.* 60 is shown to be in line with *Fig.* 60, 3*b*. A study of these possible connections will assist in obtaining correct connections.

Further points to be taken into consideration when paralleling transformers:

(*a*) That the cables on either side of the main junction are approximately of the same length. When two or more transformers are to be paralleled, whose respective impedances are slightly different, the loading of the two transformers may be made proportional by a judicious selection of the main junction.

(*b*) Transformers should be paralleled at or near the same temperature.

(*c*) The power factor of the load has no effect on the parallel operation of any two transformers.

§ (26) MODERN TRANSFORMERS.—Modern transformers are manufactured on the lines already described under the various previous headings, such as core construction and coil winding, etc. The exacting modern conditions consequent on the ever-increasing power behind the supply systems necessitate special requirements. Modern transformers are subject to severe service conditions on account of the above, and have to withstand severe mechanical strains. On this account the transformer windings must be held rigidly together, so that no movement of the coils is possible under the exacting conditions of short circuit. With this in view, the coils are packed out rigidly from the core and from one another and are held between clamps. These clamps are usually provided with adjusting screws to take up shrinkage, should the same occur. The high-pressure windings of core-type transformers are subject to shrinkage, due to the quantity of fibrous insulation used. This shrinkage is taken up automatically in many designs by means of springs, and in some, to prevent recoil of the spring, dash-pots are provided which enclose the spring (*Fig.* 61). Shell-type transformer windings are not so subject to shrinkage, but the coils are rigidly held in a horizontal and vertical direction by coil supports (*Fig.* 62). Modern transformers are usually provided with tappings to permit of adjusting the voltage ratio, and these tappings are placed at or near the centre of the windings. This position is taken to relieve the end turns of any break in their reinforced insulation and to maintain the magnetic balance of the coils as far as possible. The transformers are usually mounted in strong boiler-plate tanks and so arranged in the tanks by means of stays and supports that movement is not possible during shipment.

Figs. 61 and 62 show typical modern core- and shell-type transformers fitted with adjustable coil supports, etc. *Modern transformers* are used for other services beyond that of stepping up or stepping down on power supply mains. Transformers for feeding rotary converters have specialities that standard power transformers do not require. It is necessary in most cases that the low-pressure windings supply a six-phase voltage in double star or double delta connection when the windings are

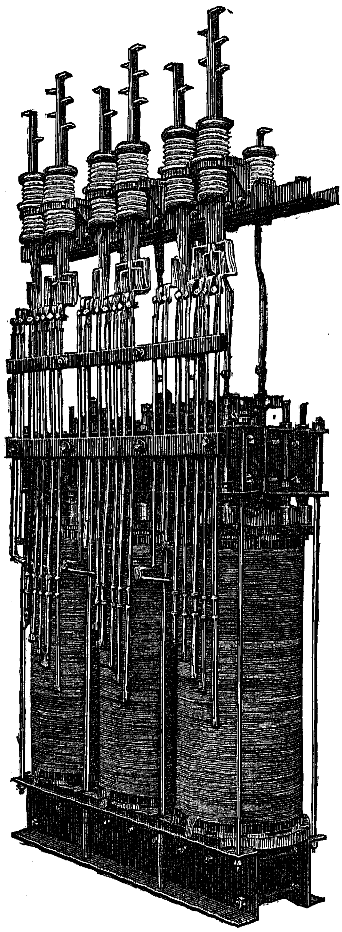

FIG. 61.—Modern Three-phase Core-type Transformer with automatic adjustable coil supports. 900 K.V.A., 40 period, 11,000 Δ/2750 λ volts.

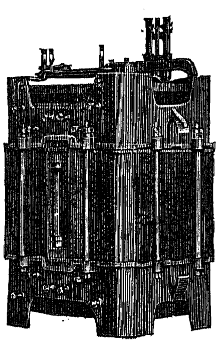

FIG. 62.—Modern Single-phase Shell-type Transformer with adjustable coil supports. 4400 K.V.A., 25 period, 20,000/2200 volts.

arranged as in *Fig.* 63. These transformers also have to have in many instances a high reactance for purposes of voltage regulation on the rotary. This reactance may be obtained

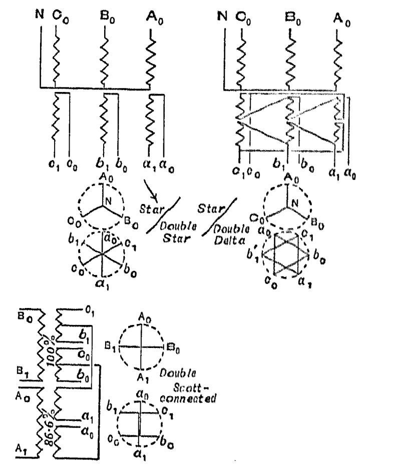

FIG. 63.—Three- and Two-phase Transformer Connections for obtaining Six-phase Supply for Rotary Converters.

by suitably grouping the coils or by means of magnetic shunts situated between primary and secondary windings. The reactance characteristic must, in each case, be a straight line up

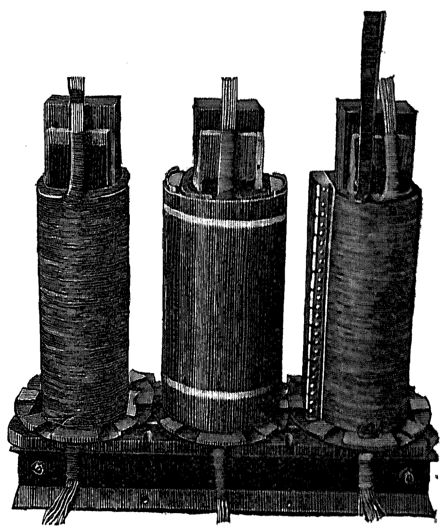

FIG. 64.—Modern High Reactance Three-phase Transformer, partly assembled, showing Magnetic Shunts.

to the maximum load permissible to ensure no distortion of the wave form, which might cause commutation troubles on the rotary; *Fig.* 64,

rotary transformer showing shunts. This requirement is inherent in sandwich-wound transformers having no magnetic shunts, and is obtainable in transformers using magnetic shunts if the flux density in the magnetic shunts is kept within the saturation point. Transformers are also used for supplying electric furnaces, and in this instance the chief requirements over and above that of a standard transformer are high reactance values, rigid construction of coil, and lead supports, as in *Fig.* 65.

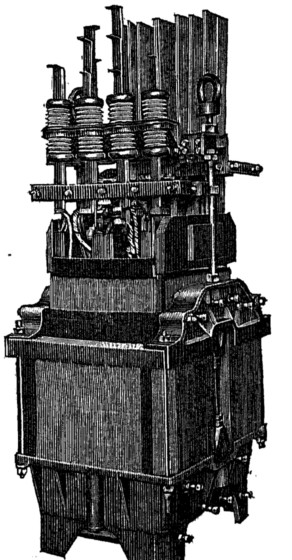

FIG. 65.—Modern Furnace Transformer. 1250 K.V.A., 50 period, 6600/65, 75, 85 volts.

§ (27) TRANSFORMER CONNECTIONS.—There are various methods of connecting the windings of transformers for three-phase supply. The suitability of any one connection is dependent on the position of the supply system and features arising from special service conditions. For instance, step-up transformers are usually connected star/delta, and step-down transformers delta/star. If three single-phase transformers are used, they are frequently connected delta/delta, so that if one fails two may be used in V-connections and so maintain a supply at 57·8 per cent of normal rating. This often obviates the necessity for a spare phase. Three-phase core-type transformers cannot be so connected, due to the interlinkage of the magnetic circuits, but three-phase shell type may be so connected if it be wound delta/delta and one-phase fails, the faulty phase being short-circuited while running. If loading to the neutral point is required, and that loading be unbalanced, the connections should be delta/star, or the regulation will be poor and a heavy voltage drop ensue if out-of-balance load be taken. Instead of a delta/star connection, a star/interconnected star can be used for unbalanced loading should the neutral point be required on the primary side. Transformers can be arranged to transform from two-phase to three-phase or *vice versa* by using the Scott connection. Two distinct transformers are required, one wound for full three-phase line voltage with a mid-point tapping, the other being wound for 86·6 per cent of the line voltage and connected to the mid-point of the 100 per cent transformer. *Fig.* 66 shows a Scott-connected group and position of neutral point.

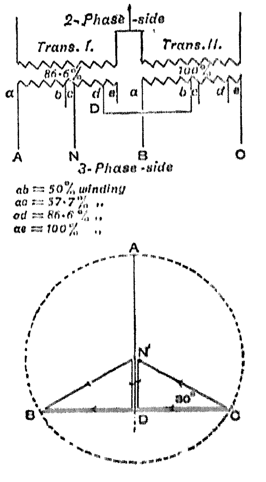

FIG. 66.—Two Transformers Scott-connected with Vector Diagram.

Transformer windings can also be arranged for transformation from two-phase to six-phase rotary converters, each transformer being supplied with two 100 per cent and 86·6 per cent windings respectively and connected to the rotary in inverse polarity, as per *Fig.* 63. Transformers are also used for boosting purposes. This type of transformer is used in series with the supply mains to boost or buck the voltage as required. It is chiefly used in the mains interconnecting two supply stations which have independent control to adjust the load passing from one station to another Regulating transformers are frequently used to feed the booster in order to obtain finer adjustment of voltage. *Fig.* 67 shows the various uses of booster and regulating transformers.

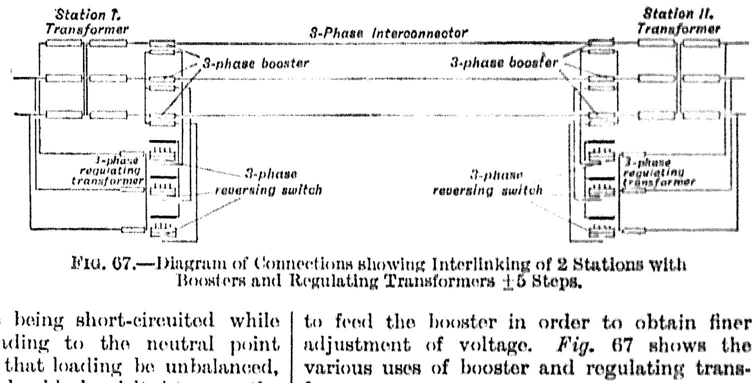

FIG. 67.—Diagram of Connections showing Interlinking of 2 Stations with Boosters and Regulating Transformers ±5 Steps.

§ (28) TRANSFORMER PROGRESS. — Transformers can be manufactured up to very large sizes and for very high voltages. Testing transformers have been manufactured up to 750,000 volts, and higher voltages are contemplated. The maximum size of a transformer group in use in England is approximately 24,000 K.V.A., 25 periods, or an equivalent of 40,000 K.V.A., 50 periods, and has a group weight of 66 tons. The maximum size of transformer yet manufactured is one of 60,000 K.V.A., 110,000 volts to 25,000 volts, manufactured on the Continent. Its weight is 116 tons complete.

There is no limit to the size of transformer which can be manufactured, but its progress in this respect is dependent on the size of generator to which it is connected. Transformers of such large sizes, *i.e.* over 40,000 K.V.A., 50 periods, have to be shipped dismantled and erected on site, due to railway loading and handling conditions.

J. L. T.

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V.D.E., *Verband Deutscher Elektrotechniker.*

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TRANSITION LOSSES: losses in telephone lines at the junctions of circuits. See "Telephony," § (36).

TRANSMISSION EQUIVALENT OF TELEPHONE CIRCUIT: the length of a certain "standard cable," equivalent in transmission loss to one mile of the circuit under consideration. See "Telephony," § (22).

Of Typical Telephone Circuits. See *ibid.* § (23).

TRANSMISSION LOSSES IN TELEPHONE CIRCUITS. See "Telephony," § (38).

TRANSMISSION OF ALTERNATING CURRENT, as applied to telephony. See "Telephony," § (22).

TRANSMITTER, TELEPHONE: Carbon Button type. A type of transmitter depending on the microphonic action of granules of carbon. See "Telephony," § (14).

Consideration of, as an A.C. generator. See *ibid.* § (16).

TRANSMITTING VALVES: thermionic valves used for generating high-frequency current for radio-telegraphy. See "Thermionic Valves," § (5).

TRIODE VACUUM TUBE GENERATOR, use of, as source of audio-frequency current for bridge measurements, etc. See "Inductance, The Measurement of," § (21).

TROWBRIDGE AND DUANE, experiments of, on the measurement of wave-length and frequency for stationary waves on wires. See "Radio-frequency Measurements," § (3).

TUBE OF FORCE. A tube the surface of which is composed of lines of force. Let S_1, S_2 be two sections of such a tube normal to the lines of force, R_1, R_2 the resultant forces at any points of S_1, S_2, and δS_1, δS_2 elements of surface about those points.

Then, if the field is due to forces following the inverse square law and there is no attracting material within the tube between S_1 and S_2,

$$\int R_1 \delta S_1 = \int R_2 \delta S_2,$$

or $\int R\delta S$ is constant over all normal sections of the tube.

See "Units of Electrical Measurement," § (14).

TUBE OF INDUCTION. A tube of which the surface is composed of lines of induction. If B be the induction through δS, an element of a normal section of such a tube, then $\int B\delta S$ is constant over all such sections so long as there is no attracting material within the portion of the tube considered. See "Units of Electrical Measurement," § (14).

TUNED TELEPHONE: a telephone whose natural frequency can be varied so as to give the maximum sensitivity for a range of frequencies.

Use of, as detector in bridge measurements. See "Inductance, The Measurement of," § (32).

TUNGSTEN WIRE, MANUFACTURE OF. See "Incandescence Lamps," § (3).

TUNING-FORK, maintained by triode. See "Inductance, The Measurement of," § (22).

TUNING-FORK INTERRUPTER: an electrically maintained tuning-fork used as a source of alternating or interrupted current for bridge measurements. See "Inductance, The Measurement of," § (13).

— U —

ULTRA-VIOLET LIGHT, effect on a negatively charged body. See "Photoelectricity," § (1).

UNIDIRECTIONAL SYSTEMS: in wireless telegraphy, direction-finders which indicate sense as well as angle. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (11).

UNIFILAR MAGNETOMETER: an instrument for the determination of the declination D and the horizontal force H of the earth's magnetic field. See "Magnetism, Terrestrial, Observational Methods."

UNITS OF ELECTRICAL MEASUREMENT

I. ABSOLUTE UNITS

§ (1) FUNDAMENTAL PRINCIPLES.—The Absolute System of Units on which all electrical measurements are based was originally suggested by Professor Wm. Weber [1] of Göttingen, in connection with the magnetic measurements of Gauss, and brought into general use by the labours of Sir Wm. Thomson (Lord Kelvin) and the British Association Committee on Practical Standards for Electrical Measurements established through his advocacy in 1861.

Appendix C in the Second Report [2] of this Committee, presented at Newcastle in 1863, "On the Elementary Relations between Electrical Measurements," gives a full account of the development of the system.

An absolute system is one which connects all quantities with a limited number of fundamental units, in terms of which they can be expressed, and which have been selected in a manner suited to form a consistent whole and to bring out clearly the inter-relations of the various quantities.

In mechanical science it is found that the quantities with which we have to deal can be expressed in terms of the three fundamental units of Length, Mass, and Time. [3] To define electrical and magnetic quantities completely we have to introduce in addition properties of the medium in which the electrical or magnetic action is taking place; these properties turn out to be very intimately connected with the velocity of light through the medium; they are introduced into the equations expressing the relations between electrical quantities in two different forms, and in consequence we have, as Weber pointed out, two different systems of electrical units, known respectively as the electrostatic and the electromagnetic units. Each involves the fundamental units of length, time, and mass; the first, the electrostatic system, involves also an electric property of the medium known as its electrical inductive capacity or dielectric constant; the second, the electromagnetic, depends on a magnetic property, the magnetic inductive capacity or permeability.

§ (2) ELECTRICAL UNITS OF THE B.A. COMMITTEE.—In constructing their absolute system the B.A. Committee had first to select their fundamental units of Length, Mass, and Time. It was agreed without difficulty that these should depend on the French metric units, the metre and the kilogramme, and after some discussion it was decided to adopt the following:

Unit of Length	.	1 centimetre.
Unit of Mass	.	1 gramme.
Unit of Time	.	1 second of mean solar time.

In this way the C.G.S. (centimetre, gramme, second) system of units came into existence.

For the measurement of electrical and magnetic quantities in terms of these units we have recourse to certain facts established by direct experiment.

Our belief in the facts depends more on the consequences deduced from them than on the experimental evidence by which they were originally established.

The facts in question are expressed in the following statements of the laws to which they lead:

(i.) *Coulomb's Law of Electrostatic Attraction.*—There is a repulsion between two charges e, e' of electricity concentrated at two points at a distance r apart which is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

If F represent the force of repulsion, the law is expressed by the equation

$$F=\frac{ee'}{Kr^2},$$

where K is a quantity which can be shown to depend on the medium in which the charges are, but which is constant for any one medium. It is the inductive capacity already referred to and expresses the property of the medium

[1] W. Weber, *Elektrodyn. Massbestimmungen*, Thl. ii. 259.

[2] See *British Association Reports on Electrical Measurements; A Record of the History of "Absolute Units" and of Lord Kelvin's work in connection with these*. Reprinted by the Association, 1912, Cambridge University Press.

This Appendix, written by Fleeming, Jenkin, and Maxwell, should be consulted by any who are interested in this subject.

[3] See "Dynamical Similarity, Principles of," Vol. I.

which, as stated above, is required to complete the specification of the force.

The quantity $e/\mathrm{K}r^2$ measures the strength of the electric field due to the charge e, or the electrical intensity at the point.

(ii.) *Coulomb's Law of Magnetic Attraction.*—There is a repulsion between two magnetic poles m, m' concentrated at two points at a distance r apart which is directly proportional to the product of the poles and inversely proportional to the square of the distance between them.

If F represent the force of repulsion, the law is expressed by the equation

$$\mathrm{F}=\frac{mm'}{\mu r^2},$$

where μ is a quantity which can be shown to depend on the medium in which the poles are situated, but which is constant, except in cases in which the medium is itself of magnetic material. The quantity μ is the magnetic inductive capacity or permeability of the medium.

The quantity $m/\mu r^2$ measures the strength of the magnetic field due to the pole m.

§ (3) Two Systems of Electrical Units.—The quantities K and μ express two different properties of the medium, and accordingly, as stated above, we have two different systems of measurement based respectively on these two laws.

(i.) *The Electrostatic System.*—The electrostatic system is based on the first law; if we suppose the two charges equal, so that e' is equal to e, the law becomes

$$\mathrm{F}=\frac{e^2}{\mathrm{K}r^2},$$

or

$$e=r\sqrt{(\mathrm{FK})}.$$

We can measure r and F, but we know nothing about K and cannot get further without some assumption.

We assume that for a vacuum the value of K is unity; then we have, if our charges be in a vacuum, the result that $e=r\sqrt{\mathrm{F}}$.

Experiment shows that for our practical purposes there is a very small difference in this respect between air and a vacuum; we may, without serious error, suppose the experiment conducted in air at ordinary atmospheric pressure.

Now consider the case where r is unity, the charges being placed at a distance of 1 centimetre apart, and suppose it is found that F also is unity, being 1 dyne, the C.G.S. unit of force. Then the equation gives us

$$e=1,$$

so that the charge is also unity, or in words:

The unit of electrical quantity on the electrostatic system of measurement is a charge which, when placed in a vacuum—practically in air—at a distance of 1 cm. from an equal charge repels it with a force of 1 dyne.

(ii.) *The Electromagnetic System.*—The electromagnetic system is based in an exactly similar manner on the second law, the law relating to the repulsion between two magnetic poles, but in this case we need to make an assumption as to the value of μ. We assume that in a vacuum μ has the value of unity and then we find, putting m equal to m', that $m=r\sqrt{\mathrm{F}}$ on the electromagnetic system, so that the unit magnetic pole on the electromagnetic system is one which repels an equal pole at 1 cm. distance from it in a vacuum with a force of 1 dyne.

§ (4) Dimensions of the Units.—Again we can from the above equations find the dimensions of both e and m. If we denote by square brackets, thus [ ], the dimensions of the quantity inside them, then the equation for e becomes

$$[e]=[\mathrm{LF}^{\frac{1}{2}}\mathrm{K}^{\frac{1}{2}}],$$

but

$$[\mathrm{F}]=[\mathrm{MLT}^{-2}].$$

Thus

$$[e]=[\mathrm{L}^{\frac{3}{2}}\mathrm{M}^{\frac{1}{2}}\mathrm{T}^{-1}\mathrm{K}^{\frac{1}{2}}], \quad . \quad . \quad (1)$$

and similarly

$$[m]=[\mathrm{L}^{\frac{3}{2}}\mathrm{M}^{\frac{1}{2}}\mathrm{T}^{-1}\mu^{\frac{1}{2}}]. \quad . \quad . \quad (2)$$

We cannot go further without some knowledge as to the manner in which, if at all, K and μ involve the fundamental units, and this we have not got, though, as we shall see shortly, we can prove that

$$[\mathrm{K}\mu]=[\mathrm{T}^2\mathrm{L}^{-2}], \quad . \quad . \quad . \quad (3)$$

or the dimensions of $\mathrm{K}\mu$ are those of the square of the reciprocal of a velocity, and as an experimental result it is found that the value of $1/\sqrt{\mathrm{K}\mu}$ gives the velocity of light in the medium.

On the electrostatic system of units we assume that K is a number without dimensions, which in a vacuum has the value unity; hence

$$[e]=[\mathrm{L}^{\frac{3}{2}}\mathrm{M}^{\frac{1}{2}}\mathrm{T}^{-1}] \text{ electrostatic system,} \quad (4)$$

while on the electromagnetic system the assumption is that μ is a number without dimensions, which is unity for air; hence

$$[m]=[\mathrm{L}^{\frac{3}{2}}\mathrm{M}^{\frac{1}{2}}\mathrm{T}^{-1}] \text{ electromagnetic system.} \quad (5)$$

§ (5) The Connection between Electricity and Magnetism.—But we require to know the connection between these quantities for the next step. This is obtained from Oersted's experiments which showed that an electric current exerts force on a magnet pole, and from Faraday's investigations which give the connection between a quantity of

electricity measured electrostatically and a current.

Faraday proved that the quantity of electricity, as measured by its electrostatic effect, which is conveyed by a current is equal to the strength of the current—assumed to be uniform—multiplied by the time for which it flows. If then we denote by Q the quantity of electricity conveyed by a constant current I in time t, we have

$$Q = It, \quad . \quad . \quad . \quad . \quad (6)$$

and this is true whatever be the units in which Q and I are measured, so long as they are consistent. Ampere's experiments, following out Oersted's discovery, led to the law connecting a current and the magnetic force it exerts, which may be put into the following form :

Let the current I flow in a wire of length l bent into the form of an arc of a circle of radius r, at the centre of which is a magnetic pole of strength m. Then F, the force exerted by the current on the pole, is given by the equation

$$F = \frac{m \cdot I \cdot l}{r^2}. \quad . \quad . \quad . \quad (7)$$

This law may be put in rather more general form thus :

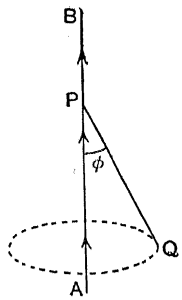

FIG. 1.

Let AB (*Fig.* 1) be a wire carrying a current I, ds an element of the wire at the point P, and suppose it is required to find the force on a unit magnetic pole at Q, where $PQ = r$ and the angle $APQ = \phi$.

Ampere proved this could be done by assuming that the force arising from the element ds was given by the expression $I ds \sin \phi / r^2$. The resultant force due to the circuit of which ds forms part is obtained by finding the sum of all these quantities, *i.e.* by integrating the expression $I ds \sin \phi / r^2$.

The direction of the force is at right angles to the plane containing ds and PQ, *i.e.* in the figure at right angles to the plane of the paper, and with the current flowing as shown it acts downwards through the paper.

If the wire be straight the force will clearly be the same at all points on a circle in a plane at right angles to the wire which passes through Q and has its centre on the wire. This will be a line of magnetic force due to the wire. In the case of a very long straight wire, the value of the integral is $2I/a$, where a is the distance of the pole from the wire.

From equation (6) it follows that a unit current is one which conveys unit quantity round the circuit per second, while equation (7) enables us to obtain the electromagnetic definition of a unit current ; for suppose the length of the circuit and its radius each to be 1 cm., let the pole m be 1 electromagnetic unit and vary the current I until the observed force F is 1 dyne. Then F, m, l, and r are each unity, and equation (7) becomes

$$I = 1,$$

or the current has unit strength on the electromagnetic system.

In one second this current conveys the electromagnetic unit of electricity, so that the electromagnetic unit of electricity on the C.G.S. system is the quantity which, when conveyed in one second through a wire 1 centimetre long bent into the form of the arc of a circle 1 centimetre in radius, exerts a force of 1 dyne on a magnetic pole of unit strength placed at the centre of the circle.

Such a current flowing in a long straight wire would exert a force of 2 dynes on unit pole at a distance of 1 centimetre from the wire.

§ (6) DIMENSIONS OF A QUANTITY OF ELECTRICITY. THE RATIO OF THE UNITS.—The equation (6) also enables us to obtain an expression for the dimensions of the unit quantity of electricity, for we have clearly

$$[I] = [FLm^{-1}] = [QT^{-1}],$$

and this, if for the present we retain the quantity μ, gives

$$[Q] = [L^{\frac{1}{2}} M^{\frac{1}{2}} \mu^{-\frac{1}{2}}]. \quad . \quad . \quad (8)$$

But if in the expressions we retain the value of K, we have already proved that

$$[e] = [L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1} K^{\frac{1}{2}}]. \quad . \quad . \quad (9)$$

If now a quantity of electricity can be expressed in terms of mass, length, and time only, then its dimensions must be the same on either of our two systems, μ and K must have such dimensions as to make them the same, that is, we must have

$$[e] = [Q],$$

and hence

$$[L^{\frac{1}{2}} M^{\frac{1}{2}} \mu^{-\frac{1}{2}}] = [L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1} K^{\frac{1}{2}}],$$

whence $$[K^{-\frac{1}{2}} \mu^{-\frac{1}{2}}] = [LT^{-1}], \quad . \quad . \quad (10)$$

or, as stated previously, the dimensions of $1/\sqrt{K\mu}$ are those of a velocity.

It has been shown that in air the value of this velocity is $2 \cdot 998 \times 10^{10}$ cm. sec.$^{-1}$ to 1 part in 3000, while the best value for the velocity [1]

[1] See "'v,' the Ratio of the Electrical Units," § (6).

of light is 2·9986 cm. sec.$^{-1}$. Thus the two are equal. Returning now to the electromagnetic system, with μ equal to unity, we find

$$[Q]=[L^{\frac{1}{2}}M^{\frac{1}{2}}] \text{ electromagnetic system,} \quad (11)$$

while the dimensions of a current I will be $[QT^{-1}]$.

Thus $$[I]=[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]. \quad . \quad . \quad . \quad (12)$$

§ (7) The Energy of a Current.—But there is another effect of a current which must be considered if we are going to obtain a system of units consistent with mechanical laws.

A current of electricity when traversing a circuit produces heat, and Joule showed that the heat produced is proportional to the square of the current and the time during which it has circulated. Heat is properly measured as work or mechanical energy,[1] and if an amount of heat energy W be generated in time t by a current I, it follows from Joule's experiments that

$$W=RI^2t, \quad . \quad . \quad . \quad . \quad (13)$$

when R is a quantity which depends only on the nature and physical condition of the conductor carrying the current and is constant so long as these remain unchanged.

But the work can be expressed in another manner. If E be the electromotive force or difference of potential between the two points at which the current enters and leaves the conductor, then E measures the loss of energy of a unit of electricity in passing from the one point to the other. Now Q units—equal to It—traverse the conductor in the time considered; the electrical energy dissipated is therefore EIt, and this has manifested itself as heat in the conductor. Hence

$$EIt=RI^2t,$$

or $$E=RI, \quad . \quad . \quad . \quad . \quad (14)$$

$$I=\frac{E}{R}. \quad . \quad . \quad . \quad . \quad (15)$$

R is a constant for the conductor known as its Resistance, and the above equations express Ohm's law which states that the ratio of the electromotive force to the current in a conductor is constant so long as the physical state of the conductor remains unchanged.

§ (8) The Unit of Electromotive Force.—We have already defined the unit of current; the equation W=EIt enables us to define the unit of electromotive force, for clearly E is unity when W, I, and t are each unity, *i.e.* when unit work is done by unit current flowing for unit time, or, to be definite, when one erg of work is performed in a conductor by the C.G.S. unit of current flowing for one second, unit electromotive force exists between the terminals of the conductor. Again, since E is equal to W/It and we know the dimensions of W and I, we can find those of E. We obtain

$$[E]=[L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}]. \quad . \quad . \quad . \quad (16)$$

§ (9) The Unit of Resistance.—By the aid of the equation E=RI we can define the unit of resistance, for R will be unity if E and I are unity, or a conductor has unit resistance if unit E.M.F. applied to its terminals produces unit current. Moreover, since resistance is measured by the ratio of E.M.F. to current, we have for its dimensions

$$[R]=\frac{[L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}]}{[L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]}=[LT^{-1}]. \quad . \quad (17)$$

Thus the dimensions of Resistance are those of length divided by time or velocity. In this manner the fundamental units of electrical measurement are defined.

§ (10) Measurement of Power.—Power or the rate of working is measured by W/t, and since W=EIt the power is measured by EI; if E and I are in C.G.S. units the power is given in ergs per second; for its dimensions we have

$$[E\,.\,I]=[W\,.\,T^{-1}]=[L^2\,.\,M\,.\,T^{-3}]. \quad (18)$$

§ (11) Electrostatic Potential. — The electrical intensity due to a charge e at a distance r is, we have seen, equal to e/Kr^2. The work done in moving a unit charge a distance[2] δr, measured directly towards the charge e, is δv the change in potential, and

$$\delta V=-\frac{e}{Kr^2}\delta r.$$

Thus, if the unit charge be moved from r_1 to r_2 we have

$$V_2-V_1=-\frac{e}{K}\int_{r_1}^{r_2}\frac{1}{r^2}\delta r=\frac{e}{K}\left(\frac{1}{r_2}-\frac{1}{r_1}\right), \quad (19)$$

and clearly when r is very large V is very small, and we have, putting r_1, infinity, and V_1 zero, $V=e/Kr$, thus the potential is inversely proportional to Kr.

Again, the potential due to any system of charges is the sum of the potentials due to each of the charges separately. Hence, if V_A be the potential due any system in air and V that in a medium of specific inductive capacity K, we have

$$V=\frac{V_A}{K}.$$

[1] See "Heat, Mechanical Equivalent of," Vol. I.

[2] Since the motion is towards the charge e, r decreases as the unit charge is moved. Thus δr is negative, hence the negative sign in the value of δV.

§ (12) Strength of the Electric Field.—The force exerted on a unit charge placed at any point of an electric field of force measures the strength of the field or the Electrical Intensity at that point.

§ (13) Electrostatic Induction. Electric Displacement.—When a conducting body is brought into the field of a charged conductor it becomes electrified, and the surface density of the electrification at any point is connected with the electrical intensity at the point and the inductive capacity of the insulating medium separating the two conductors by the equation [1]

$$\sigma=\frac{KR}{4\pi}, \quad . \quad . \quad . \quad . \quad (20)$$

in which R is the electric intensity, K the inductive capacity, and σ the surface density; the charge on the conducting body is positive if R is directed away from its surface. The body is said to be charged by induction, and the induction is measured by the quantity $KR/4\pi$, the surface density of the charge. Faraday visualised induction as a process going on along the lines of force issuing from the charge, a displacement of electricity taking place along these lines, and Maxwell introduced the term electric displacement to represent this.

We have then

$$\text{Electric displacement}=\frac{KR}{4\pi}. \quad . \quad (21)$$

It is part of Maxwell's theory that a change of displacement constitutes a current in a dielectric. Hence,

$$\text{Dielectric current}=\frac{K}{4\pi}\frac{dR}{dt}.$$

On the electrostatic system K is unity for air; for any other dielectric K will be the specific inductive capacity.

§ (14) Lines of Force and of Induction.—A field of electric or magnetic force can be mapped out by drawing the lines of force due to the charges which give rise to it. Consider any small area dS_1 in the field placed at right angles to the lines which pass through it and imagine these lines so drawn that their number per unit area of dS_1 represents the electric intensity at any point of the area; the lines drawn from points on the circumference of the area form a kind of tube known as a tube of force; let dS_2 be a second section of this tube at right angles to its direction, and let R_1, R_2 be the electric intensities over the areas dS_1, dS_2 respectively, N the numbers of lines of force through dS_1. All these lines pass through dS_2, so that N is also the number of lines through dS_2. Then it can be shown,[2] if none of the electricity to which the field is due lies within the tube between dS_1 and dS_2, that $R_1dS_1 = R_2dS_2$.

[1] See "Electrostatic Field," § (4).
[2] See *ibid.* § (3).

Now R_1 is measured by the number of lines crossing unit area of dS_1, and since N lines cross the area dS_1, N/dS_1 cross each unit of area of dS_1. Hence by construction

$$R_1=\frac{N}{dS_1},$$

or

$$N=R_1dS_1=R_2dS_2. \quad . \quad . \quad (22)$$

Therefore $R_2=N/dS_2$=number of lines of force crossing unit area of dS_2.

Thus if the lines of force are so drawn that the number per unit area crossing a small area which cuts them at right angles measures the electric intensity at any point of the area, this also will be true at any other point along these lines, and the number per unit area which cross a small area at that point placed at right angles to the lines will measure the intensity of the field there on the assumption that the tube of force considered does not cut any electrical charges between the two points.

It is convenient, therefore, to consider a field of force to be mapped out thus, and to describe the intensity of the field as measured by the number of lines of force cutting unit area placed so as to be at right angles to the lines of force. Again, since electric displacement or induction is found by multiplying the intensity by $K/4\pi$, similar statements can be made as to lines and tubes of induction.

The property on which these results depend, viz. that $R_1dS_1=R_2dS_2$, is true for any kind of material attracting or repelling according to the inverse square law. It is true, therefore, for magnetic forces and exactly analogous statements can be made as to magnetic force and induction. If lines of force or of induction are drawn as described so that at any point of the field the magnetic intensity is measured by the number crossing unit area, then this will be true for any other section of the tube of force formed by these lines.

§ (15) Strength of a Magnetic Field.—The force on a positive unit magnetic pole placed at a point in a magnetic field measures the strength of the field or the magnetic intensity at the point.

Since the magnetic force due to a pole m at a distance r is m/r^2 we have for the dimension of **H** the magnetic intensity

$$[\mathbf{H}]=[mL^{-2}]=[L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]. \quad . \quad (23)$$

It can be proved [3] that the magnetic potential at any point due to a current in any circuit is equal to the product of the solid angle subtended at the point by the circuit and the strength of the current. If ω represent this solid angle the magnetic potential is $I\omega$ and the intensity **H** measured in any direction ds is given by $\mathbf{H}=-I(d\omega/ds)$, where I is the value

[3] See "Electromagnetic Theory," §

of the current in C.G.S. units. If the circuit take the form of a solenoid[1] of n turns per unit of length, and the force $\mathbf{H}$ parallel to the axis of the solenoid be measured at a point within the solenoid at some distance from the ends, this becomes $\mathbf{H}=4\pi n\mathrm{I}$, where n is the number of turns per unit length of the solenoid.

If i be the current in amperes we have, therefore,

$$\mathbf{H}=\frac{4\pi}{10}ni=\frac{4\pi}{10}\times\text{ampere turns per unit length.}$$

§ (16) Magnetic Induction.—Just as an electric charge produces electrostatic induction in the neighbouring field, so a magnetic pole produces magnetic induction in the magnetic field, and the magnetic induction $\mathbf{B}$ is connected with the magnetic intensity or field strength $\mathbf{H}$ by the equation $\mathbf{B}=\mu\mathbf{H}$, where μ is the permeability or magnetic inductive capacity. On the E.M.S. system the dimensions of $\mathbf{B}$ are the same as those of $\mathbf{H}$. Hence

$$[\mathbf{B}]=[L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}]. \qquad (24)$$

On this system μ is unity for air, so that for any other medium μ represents the value of the permeability relative to air.

Magnetic induction takes place along the lines of magnetic force and may be visualised as a displacement of magnetism along them. There is thus an analogy between the two, electrostatic and magnetic induction, but one important difference should be noted.

The specific inductive capacities of most dielectrics—except gases—differ appreciably from that of air, so that the value of K depends to an important extent on the dielectric. With magnetic induction this is otherwise; except in the case of iron, nickel, cobalt, and one or two other materials the difference between the permeability of the medium and that of air is small. For most substances the value of μ is practically the same as that of air, and is therefore—on the electromagnetic system—treated as unity.

§ (17) Capacity.—The capacity of a conductor measures the charge necessary under specified conditions to increase its potential by unity; it is expressed therefore by the ratio of a quantity of electricity Q to a potential difference E. Thus capacity$=$Q/E, where Q is the charge required to raise the potential by an amount E.

Now we have seen that the potential due to any system depends on the inductive capacity of the dielectric in which the conductor is placed, being inversely proportional to its value. Hence if E be the change of potential produced in a conductor by a charge Q when in air, when the conductor is placed in a medium of specific inductive capacity K the potential difference due to the charge Q will be E/K and the capacity C is now given by

$$C=\frac{Q}{E/K} \quad \text{or} \ \frac{KQ}{E}.$$

Hence the capacity of a conductor is proportional to the inductive capacity of the medium in which it is placed.

The term capacity is generally employed in connection with a conductor known as a condenser, which consists of two conducting surfaces separated by a layer of a dielectric or insulating material; the dielectric may be air, it is more frequently a substance of greater inductive capacity. In this case the capacity is measured by the ratio of the charge to the potential difference between the surfaces. In practice one surface is generally kept at zero potential.

Capacity is measured by the ratio Q/E and hence, denoting it by C, its dimensions are given by the equation

$$[C]=[QE^{-1}]=[L^{-1}T^{2}]. \qquad (25)$$

§ (18) Electromagnetic Inductance.—An electrical current circulating in a wire sets up a magnetic field in its neighbourhood. Lines of magnetic induction are produced which are interlinked with the circuit itself. The total amount of induction through the circuit is measured by the number of lines which thread it, and this number depends on the strength of the current. The ratio of the total induction Φ through the circuit to the current I is defined as the coefficient of self-induction of the circuit and denoted by $\mathfrak{L}$. We have then

$$\mathfrak{L}=\frac{\Phi}{I},$$

or

$$\Phi=\mathfrak{L}I. \qquad (26)$$

Clearly if A be the area of the circuit and the magnetic induction $\mathbf{B}$ is uniformly distributed through it $\Phi=\mathbf{B}A$. If there be another circuit in the field some of the lines of induction will traverse this and the ratio of their number to the current measures $\mathfrak{M}$, the coefficient of mutual inductance between the two circuits. Hence

$$\Phi_{12}=\mathfrak{M}I, \qquad (27)$$

where by Φ_{12} we mean the number of lines of inductance due to unit current in circuit 1 which traverse circuit 2. This, it may be shown, is also the number due to unit current in circuit 2 which traverse circuit 1.

Again since $\Phi=\mathbf{B}A$ we have for the dimensions of Φ

$$[\Phi]=[\mathbf{B}L^{2}]=[L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}], \qquad (28)$$

while since $\mathfrak{L}=\Phi/I$ we find

$$[\mathfrak{L}]=[\Phi I^{-1}]=[L], \qquad (29)$$

and similarly for $[\mathfrak{M}]$.

[1] See "Electromagnetic Theory," § (18).

Thus the dimensions of a coefficient of electromagnetic inductance are those of a line, and the unit of electromagnetic inductance is 1 centimetre.

The value of Φ is connected with that of $\mathbf{H}$ by the equation $\Phi=\mu\mathbf{H}A$. Now the magnetic intensity $\mathbf{H}$ is proportional to I, the current strength to which the field is due.

Thus Φ is proportional to μI. Hence $\mathfrak{L}$ and $\mathfrak{M}$ are both proportional to μ; thus so long as μ is a constant $\mathfrak{L}$ and $\mathfrak{M}$ are constants.

But we have seen that μ is nearly equal to unity—on the electromagnetic system—for all materials except iron, nickel, and cobalt. Thus, except in the case of the so-called magnetic materials, the coefficients of self and mutual induction are constants, and depend merely on the geometrical form and relative position of the circuits concerned.

In this case the numerical values of the magnetic intensity and the magnetic induction are the same. For magnetic materials the value of μ, the permeability, which measures the ratio of $\mathbf{B}$ to $\mathbf{H}$, depends on the current, and therefore on $\mathbf{H}$; though experiment shows that for a considerable range of values of $\mathbf{H}$, so long as they are not close to the saturation value, μ does not vary rapidly with $\mathbf{H}$ and for small variations of the magnetising force may be treated as constant.

§ (19) Electrochemical Equivalent. — There is still another property of a current to be considered. When a current is passed through a liquid containing chemical salts in solution, the salts are decomposed. Faraday showed that the mass of salt decomposed was in all cases proportional to the current, measured either in terms of its magnetic effect or electrostatically.

The mass of salt decomposed by a unit current flowing for a unit of time, *i.e.* by the passage of unit quantity of electricity, is known as the electrochemical equivalent of the substance.

If we denote this by γ, then W, the mass decomposed by a current I flowing for t seconds, is given by the equation

$$W=\gamma It. \qquad . \quad . \quad . \quad (30)$$

Faraday showed further that the electrochemical equivalent of any chemical element is proportional to its combining weight in the solution decomposed by the current.

In certain cases the products of decomposition can be collected at the places where the current either enters or leaves the solution; if the current be measured in some way—say by its magnetic effect—we can find the value of the electrochemical equivalent by weighing these products of decomposition deposited in a given time, or, conversely, if the electrochemical equivalent be known the above equation enables us to calculate the current. The method by which this is done in the case of silver deposited from an aqueous solution of nitrate of silver is described in the article on Electrical Measurements [1] and a similar method can be applied to other metals. Since the electrochemical equivalent is the ratio of a mass to a quantity of electricity we have

$$[\gamma]=[MQ^{-1}]=[L^{-\frac{1}{2}}M^{\frac{1}{2}}]. \qquad . \quad (31)$$

§ (20) Relations of the Units. — The various quantities we have been considering are all connected together as a systematic whole, and form an absolute system of measurement, absolute because through the definitions of current and E.M.F. they can be connected directly with the fundamental units of Space, Mass, and Time. They all depend in some way on current—or quantity, which is the product of current and time—and electromotive force. If we take these two quantities as fundamental units any others can be deduced.

We might choose another pair as our fundamental units; it can be shown that we must know two before we can deduce the others; the two are quite independent and the rest dependent on them.

Now electromotive force and current are connected in a very simple way through Ohm's law by the equation E=IR. Instead of taking E and I as the fundamental units we can choose R and I, replacing E wherever it occurs by the product RI, and this is found to be an advantage; for Resistance is a physical property of a conductor, and we can realise a standard of resistance by selecting a conductor of some definite form and material maintained under definite physical conditions, *e.g.* as to its temperature; a standard of electromotive force is much less easy to realise, and needs complicated appliances. The C.G.S. system of electromagnetic electrical units then is based on Resistance and Current. The units of electromotive force, Capacity, Coefficients of Induction and the rest, are secondary and are deduced from these two.

Definition of Unit Current. — The unit current, when flowing in the arc of a circle one centimetre in length and one centimetre in radius, produces a force of one dyne on a unit magnetic pole placed at its centre.

Definition of Unit Resistance.—A conductor has unit resistance if an amount of energy equal to one erg per second is expended when unit current is traversing it.

II. Practical Units

§ (21) The Units of Resistance, Current, and E.M.F.—The unit of current and of resistance thus defined are found to be inconvenient for practical use; the unit of current, at any

[1] See "Electrical Measurements, Systems of," § (40) *et seq.*

rate in 1865, was somewhat too large, that of resistance much too small, and so a system of practical units was devised based on sub-multiples or multiples of these absolute units. Names—usually those of distinguished electricians—have been given to these practical units; the practical unit of current known as the Ampere is one-tenth—10^{-1}—of the C.G.S. unit just defined, while the practical unit of resistance, called the Ohm, contains a thousand-million or 10^9 C.G.S. units of resistance. The electromotive force required to produce a current of 1 ampere in a resistance of 1 ohm is given by the equation

$$E = RI = 10^9 \times 10^{-1} = 10^8 \text{ C.G.S. units.}$$

This is taken as the practical unit of electromotive force and called a Volt. Thus

1 Ohm $= 10^9$ C.G.S. units.
1 Ampere $= 10^{-1}$ C.G.S. units.
1 Volt $= 10^8$ C.G.S. units.

Before the time of the B.A. Committee the resistance of a column of mercury 1 metre in length and 1 square millimetre in section had been suggested by Siemens as the unit of resistance; now 10^9 C.G.S. units is some 6 per cent greater than this Siemens unit, and it was probably this fact which led to the choice of 10^9 C.G.S. units as the practical standard, while one volt does not differ greatly from the E.M.F. of a Daniell's cell; this led to the choice of 10^8 C.G.S. units for the practical standard of E.M.F., and from these two the value 10^{-1} C.G.S. follows necessarily for the unit of current.

From these the other units of the practical system are derived in the following way.

§ (22) Quantity.—The unit of quantity is the quantity carried by 1 ampere per second or 10^{-1} C.G.S. units. This is known as a Coulomb.

1 Coulomb $= 10^{-1}$ C.G.S. units.

§ (23) Power.—The unit of power or rate of working is the work done per second by 1 ampere flowing between two points between which there is an E.M.F. of 1 volt. It is known as the Watt. Thus

$$\textbf{1 Watt} = 10^8 \times 10^{-1} = 10^7 \text{ C.G.S. units}$$
$$= 10^7 \text{ ergs per second.}$$

§ (24) Work.—The unit of work is the watt second, or work done by 1 watt working for 1 second. It is called a Joule.

Thus **1 Joule** $= 10^7$ ergs.

§ (25) Capacity.—The unit of capacity is the capacity of a condenser which requires a charge of 1 coulomb to increase its potential by 1 volt. It is called a Farad. Thus

$$\textbf{1 Farad} = \frac{10^{-1}}{10^8} = 10^{-9} \text{ C.G.S. units.}$$

A conductor which had a capacity of 1 farad, even though composed of plates very close together, would be very large, and in practice the microfarad or millionth of a farad is used.

1 Microfarad $= 10^{-6} \times 10^{-9} = 10^{-15}$ C.G.S. units.

§ (26) Unit Magnetic Pole.—A unit magnetic pole exerts unit force—in air—on an equal pole at unit distance. Hence

Unit magnetic pole $= 1$ C.G.S. unit.

§ (27) Unit Magnetic Force.—The practical unit of magnetic force was named the Gauss at an International Electrical Conference held in Paris in 1900, and was defined as the C.G.S. unit of magnetic force. Hence

1 Gauss $= 1$ C.G.S. unit.

§ (28) Unit Magnetic Induction and Magnetic Flux.—The practical unit of magnetic induction was named the Maxwell at the same congress, and since on the electromagnetic system the permeability of air is assumed to be unity and to be a pure number the value of the Maxwell is the same as that of the Gauss. Hence

1 Maxwell $= 1$ C.G.S. unit.

§ (29) Electromagnetic Inductance.—The practical unit of electromagnetic inductance was named the *Henry* at the Chicago International Conference in 1893. It is defined as the induction in a circuit when the electromotive force induced is 1 volt and the inducing current varies at the rate of 1 ampere per second. Thus

$$\textbf{1 Henry} = \frac{10^8}{10^{-1}} = 10^9 \text{ C.G.S. units.}$$

Moreover, since the C.G.S. unit [1] of inductance is 1 centimetre, the value of the henry is 10^9 centimetres or 10,000 kilometres.

§ (30) The Realisation of Standards.—Hitherto we have dealt with a system of units. For actual use this has to be realised as a system of standards. The ohm is the resistance of a conductor which has the value of 10^9 C.G.S. units of resistance or 10^9 cm. per sec.

How are we to construct a conductor which shall have this resistance? The methods by which this has been done to a very high order of accuracy are described in the article [2] on "Systems of Electrical Measurements," to which reference should be made. It is there shown how we can determine the absolute resistance of a coil of wire; the intercomparison of resistances is a simple matter, and hence if the resistance of a standard coil has been

[1] See § (18). The C.G.S. unit of electromagnetic inductance is sometimes spoken of as the ab. henry—absolute henry. Thus, the above definition gives 1 ab. henry $= 10^{-9}$ henrys $= 1$ centimetre.

[2] See "Electrical Measurements, Systems of," §§ (9)-(22).

determined absolutely the resistances of other conductors can be found by comparison [1] with it. In this way a series of standard resistances can be set up.

In a somewhat similar manner instruments can be constructed by the aid of which electrical currents or electromotive forces [2] can be measured absolutely, and by comparison with these the readings of ammeters and voltmeters are standardised. In this manner the values of the fundamental units are realised as standards. By a somewhat similar process standards of capacity,[3] inductance [4] and the other quantities which have been discussed in the preceding sections can be constructed.

This work has been carried out in the standardising institutions in various countries, and there is now very close agreement as to the results.

III. International Units

§ (31) The International Conference of 1908.—An International Conference on Electrical Units took place in London [5] in 1908. At that time the close agreement just referred to in the realised values of the fundamental standards had not been reached. The uncertainty as to the absolute resistance of a standard coil was much greater than the errors which occurred in the comparison of two coils; and it was important that the standard should be accurate. The apparatus required for an absolute measurement is expensive and complicated and there was a desire for a system of standards which could, it was hoped, be reproduced with ample accuracy in any well-equipped National Laboratory. An International System of Standards, distinct from but dependent upon the C.G.S. system, had been defined at earlier conferences in America, and the London Conference accepted this international system and arranged for the issue of specifications defining the methods for the construction of the international standards. As has been already stated the resistance of a column of mercury one metre in length and one square millimetre in section differs from one ohm—10^9 C.G.S. units—by a few per cent, and this was made the basis of the international standard of resistance, while the mass of silver deposited per second by an ampere—10^{-1} C.G.S. units—from a solution of nitrate of silver in water was known to be about 0·001118 grammes, and this was made the basis of the international ampere.

[1] See "Resistance, Measurement of."

[2] See "Electrical Measurements, Systems of," §§ (23)-(37).

[3] See "Capacity," §§ (31), (32).

[4] See "Inductance, Measurement of," § (1), and "Inductance, Calculation of Coefficients of," § (7).

[5] See "Electrical Measurements, Systems of," § (38).

The following resolutions were agreed to by the Conference:

(i.) The Conference agrees, that, as heretofore, *the magnitudes of the fundamental electric units shall be determined on the electromagnetic systems of measurement with reference to the centimetre as the unit of length, the gramme as the unit of mass, and the second as the unit of time.*

These fundamental units are (1) the ohm, the unit of electric resistance which has the value of 1,000,000,000 in terms of the centimetre and second; (2) the ampere, the unit of electric current which has the value of one-tenth (0·1) in terms of the centimetre, gramme, and the second; (3) the volt, the unit of electromotive force which has the value 100,000,000 in terms of the centimetre, the gramme, and the second; (4) the watt, the unit of power which has the value 10,000,000 in terms of the centimetre, the gramme, and the second.

(ii.) *As a system of units representing the above, and sufficiently near to them to be adopted for the purpose of electrical measurements and as a basis for legislation, the Conference recommends the adoption of the international ohm, the international ampere, and the international volt defined according to the following definitions:*

(iii.) The ohm is the first primary unit.

(iv.) The **international ohm** is defined as the resistance of a specified column of mercury.

(v.) The **international ohm** is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of a constant cross-sectional area and of a length of 106·300 centimetres.

To determine the resistance of a column of mercury in terms of the international ohm, the procedure to be followed shall be that set out in Specification I. attached to these resolutions.

(vi.) The ampere is the second primary unit.

(vii.) The **international ampere** is the unvarying electric current which, when passed through a solution of nitrate of silver in water, in accordance with Specification II. attached to these resolutions, deposits silver at the rate of 0·00111800 of a gramme per second.

(viii.) The **international volt** is the electric pressure which, when steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere.

(ix.) The **international watt** is the energy expended per second by an unvarying electric current of one international ampere under an electric pressure of one international volt.

Details as to the specifications referred to and the methods of realising these standards will be found in the article on "Electrical Measurements, Systems of." Some of the questions left unsettled by the Conference were

resolved by the work of a small International Technical Committee [1] which met at the Bureau of Standards at Washington in 1911. The London Conference had nominated a standing committee charged with the duty of examining these questions, and specially of determining on a suitable concrete standard of electromotive force for which the Weston normal cell had been suggested.

As the result of the inquiries of this Technical Committee it was found that the value of the E.M.F. of the Weston normal cell at 20° C. is 1·0183 international volts. This cell set up in accordance with the instructions [2] contained in the report of the sub-committee forms thus a convenient concrete standard of E.M.F., consistent with the international definitions of the ohm and ampere, and in practice the ampere is more often measured in terms of the ohm and volt than the volt in terms of the ohm and ampere.

[1] The Report of this Committee, entitled "Report to the International Committee on Electrical Units and Standards of a Special Technical Committee," appointed to investigate and report on the Concrete Standards of the International Electrical Units and to recommend a value for the Weston Normal Cell, was published by the Bureau of Standards at Washington.

[2] See "Electrical Measurements, Systems of," § (48).

The work of the same committee showed that the electrochemical equivalent of silver is 1·11803 milligrammes.

§ (32) Relation between the Absolute and International Standards.—Experimental work carried out since 1908 in various laboratories has enabled us to specify with great exactness the relations between these two systems with the following results.[3]

1 International ohm $= 1{\cdot}0005_2$ ohm
$= 1{\cdot}0005_2 \times 10^9$ C.G.S. units.

1 International ampere $= 0{\cdot}9999_7$ amperes
$= 0{\cdot}9999_7 \times 10^{-1}$ C.G.S. units.

1 International volt $= 1{\cdot}0004_9$ volts
$= 1{\cdot}0004_9 \times 10^8$ C.G.S. units.

International electrochemical equivalent,
1·11800 mgr.

Further details as to these relations will be found in the article on Electrical Measurements already referred to.

[3] See "Electrical Measurements, Systems of," § (39).

— V —

"v," THE RATIO OF THE ELECTROMAGNETIC TO THE ELECTROSTATIC UNIT OF ELECTRICITY, MEASUREMENT OF

§ (1) Definitions.—The *electromagnetic* unit of electric quantity is that quantity which is transferred by the unit current in the unit of time, and its dimensions are $[L^{\frac{1}{2}}M^{\frac{1}{2}}/\mu^{\frac{1}{2}}]$, where μ is the magnetic permeability of vacuous space. The *electrostatic unit* of quantity is that quantity which exerts unit force on another equal quantity at the unit distance and its dimensions are $[L^{\frac{3}{2}}M^{\frac{1}{2}}K^{\frac{1}{2}}/T]$, where K is the dielectric constant of vacuous space. Since the dimensions of both units must be the same, the dimensional expressions must be equal. Hence

$$[L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}] = [L^{\frac{3}{2}}M^{\frac{1}{2}}K^{\frac{1}{2}}T^{-1}]$$

and

$$\left[\frac{1}{\sqrt{K\mu}}\right] = \left[\frac{L}{T}\right].$$

The separate dimensions of K and of μ are not known. The equation tells us, however, that the ratio of the two units is equal to $1/\sqrt{K\mu}$, and that this has the dimensions of a velocity. The velocity itself is usually denoted by "v." To obtain a physical conception of this velocity the following illustration, being a slight modification of one given by Maxwell, is helpful.

Imagine a plane surface charged with electricity to the electrostatic surface density σ, and let it move through space in its own plane with a velocity V. This moving electrified surface will be equivalent to a current sheet, the current passing through a unit breadth of the sheet being σV in electrostatic measure and $\sigma V \sqrt{K\mu}$ in electromagnetic measure. The latter value of the current assumes that $\sqrt{K\mu}$ is the number of electrostatic units in one electromagnetic unit. If another plane surface parallel to the first is electrified to the surface density σ' and moves in the same direction with the same velocity, it will be equivalent to a second current sheet.

The electrostatic repulsion between the two surfaces is $2\pi\sigma\sigma'$ per unit of area, and the electromagnetic attraction between the two current sheets is $2\pi\sigma\sigma'V^2K\mu$. If the velocity V be such that the repulsion is equal to the attraction, then

$$2\pi\sigma\sigma' = 2\pi\sigma\sigma'V^2K\mu,$$

that is
$$V = \frac{1}{\sqrt{K\mu}}.$$

Hence the ratio of the two units may be defined as a velocity such that when two electrified surfaces move parallel to each other with this velocity there is no mutual force between them. The velocity is about 3×10^{10} cm. sec^{-1}.

Methods of determining "v."—In order to determine the value of v it is necessary to measure the ratio of the electrostatic and electromagnetic values of at least one electric or magnetic quantity. By measurements of quantity, current, resistance, electromotive force, and capacity, it appears, therefore, that there may be five main methods by which the value of v can be obtained. There is, however, no way of measuring directly the value in electrostatic units of a steady current, and the number of possible methods is thus reduced to four.

§ (2) THE QUANTITY METHOD.—The first determination of v by any method was carried out by Weber and Kohlrausch in 1856,[1] who chose the quantity method. In this a condenser, the capacity C_s of which is known in electrostatic measure, is charged to a potential E_s which is measured electrostatically by means of an absolute electrometer. The product C_sE_s gives the charge of the jar in electrostatic measure. This quantity of electricity is then measured electromagnetically by discharging the condenser through a ballistic galvanometer, the quantity Q being calculated by the formula

$$Q = \frac{HT}{G\pi} \sin\frac{\theta}{2},$$

where H is the horizontal intensity of the earth's magnetism,
G the principal constant of the galvanometer,
T the period of the magnet, and
θ the kick due to the transient current.

In the experiments made by Weber and Kohlrausch a Leyden jar was used as the condenser, its capacity being obtained by comparison with that of a sphere. The result obtained was $v = 3{\cdot}1074 \times 10^{10}$ cm. sec^{-1}. Maxwell pointed out a probable source of error in these experiments owing to the use of a Leyden jar. A neglect, in the measurement of the capacity of such a condenser, of the phenomenon of "electric absorption," the nature of which was not then well understood, would lead to an overestimation of the electrostatic capacity of the condenser, and consequently to a value of v which would be too great.

[1] *Pogg. Ann.*, 1856, xcix.

Rowland in 1879 made a measurement of v by the quantity method, using two accurately constructed concentric spheres as the standard condenser. The inner sphere was hung concentrically within the outer by a silk cord, and as a check two spheres of different diameters were made, either of which could be used as the inner sphere. When the larger of the two inner spheres was used the capacity was about 50 electrostatic units, and when the smaller sphere was employed the capacity was about 30 electrostatic units. To measure the electrostatic potential a Thomson guard-ring electrometer was used, the surfaces of the guard plates and attracting disc being nickel-plated and accurately worked so that the distance between the two surfaces could be measured with accuracy.

The ballistic galvanometer was a specially constructed instrument, and the horizontal magnetic intensity was independently determined by a Helmholtz - Gaugain electrodynamometer, the constant of which was verified by comparison with a tangent galvanometer having a coil 80 cm. in diameter.

A large number of Leyden jars connected in parallel and charged were used to charge the standard spherical condenser. The Leyden jars had their potential measured by connection with the electrometer, after which the spherical condenser was charged, disconnected from the jars, and discharged through the ballistic galvanometer. This was repeated rapidly four or five times, so that the galvanometer received a series of impulses, and in consequence was deflected, the deflection being corrected for the displacements of the galvanometer needle from zero at the times of the impulses. The reading of the electrometer was taken after the successive discharges, and the mean potential of the standard condenser was calculated from the two electrometer readings. The result obtained by Rowland was

$$v = 2{\cdot}9815 \times 10^{10} \text{ cm. sec.}^{-1}.$$

Rosa and Dorsey[2] regard the method of quantity as being both complicated and difficult. The standard condenser presents no difficulty, but the measurement of potential by means of an attracted disc electrometer cannot, except with extreme difficulty, be made with a probable error less than 1 part in 1000. The force to be measured is very small unless the distance between the plates is reduced unduly, and this renders any uncertainty in the distance to be correspondingly more important. For teaching purposes the method is very attractive.

[2] *Bureau of Standards*, 1907, Bull. 3.

§ (3) The Electromotive Force Method.—The electrostatic potential produced by a battery can be measured by means of an electrometer, and the electromagnetic value can be obtained by measuring the current and the resistance in a circuit containing the battery. Suppose a current I to flow through a resistance R; then if I and R are in absolute measure and are measured electromagnetically, the product IR is the electromagnetic difference of potential. The electrostatic difference of potential V, between the ends of the resistance, can be measured directly by an absolute electrometer. Hence the ratio $v = \mathrm{IR/V}$. This method is due to Lord Kelvin,[1] who, with King, measured I by an electrodynamometer, V by an absolute suspended disc electrometer, and found R by comparison with standard coils.

M'Kichan,[2] Shida,[3] and Lord Kelvin have also made determinations by this method, the electromotive force being measured by a Thomson absolute electrometer and the current by a tangent galvanometer or absolute electrodynamometer. The resistance was deduced from comparisons with standard coils.

In 1897 Pérot and Fabry [4] used an absolute electrometer built up of parallel plates of glass worked to optical surfaces and silvered, the method of optical interference being used to measure the small distances between the plates. The current through a standard resistance was measured by means of a silver voltameter.

The results obtained, corrected when necessary and when possible for the difference between the B.A. unit and the ohm, are

1869	Lord Kelvin and King	$v = 2{\cdot}825$	$\times 10^{10}$ cm. sec.$^{-1}$.
1873	M'Kichan	,, 2·93	,, ,,
1880	Shida	,, 2·955	,, ,,
1889	Lord Kelvin	,, 3·004	,, ,,
1897	Pérot and Fabry	,, 2·9978	,, ,,

Maxwell [5] combined the absolute electrometer and the absolute electrodynamometer in one instrument. The electromagnetic repulsion between two flat parallel coils carrying a current I was balanced by the electrostatic attraction between two discs. One of the discs, with one of the coils at its back, was attached to one arm of a torsion balance, while the other, supporting similarly the other coil, could be moved towards or away from the suspended coil by means of a micrometer screw. The two discs with a guard ring formed an absolute electrometer. A third coil, traversed by the same current in the opposite direction, was attached to the other arm of the torsion balance, so as to eliminate the effect of terrestrial magnetism. A diagrammatic plan of the arrangement is shown in *Fig.* 1, where A represents the suspended

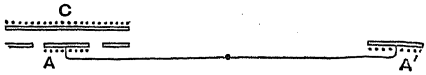

Fig. 1.

coil and disc, C the fixed coil and disc, and A′ the counterpoise coil and disc.

In practice the difference of potential between the discs, and which in the main caused the electrostatic attraction, was distinct from the E.M.F. producing the current through the coils. One electrode of a high potential battery (over 2000 cells) was connected with the fixed disc C and the other electrode was connected with the case of the instrument, the guard ring, and the suspended disc A. A small battery was used to produce a current through the three coils, and this current I passed through one coil G_1 of a standard galvanometer. Through a second coil G_2 (shunted) of the galvanometer a current from the high potential battery passed, the circuit including a high resistance R of

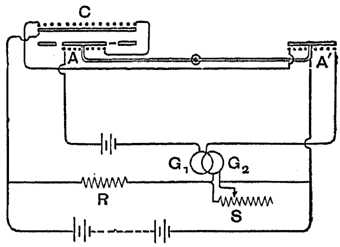

Fig. 2.

about one megohm. The circuits are diagrammatically represented in *Fig.* 2.

Equilibrium was obtained by adjusting the distance between the discs. At the same time equilibrium of the galvanometer was obtained by altering the resistance of the shunt S. The simultaneous values of the micrometer reading and the shunt formed the result of each experiment. It was necessary also to determine the magnetic effects of the two galvanometer coils and so find the ratio of the currents.

If V_S is the difference of potential between the discs A and C in electrostatic units, the attraction between them is

$$\frac{V_S^2 a^2}{8b^2},$$

where a is the radius of the disc A, and b is the distance apart of the discs.

The electromagnetic repulsion between the coils is

$$\frac{I^2 dM}{dx},$$

[1] *Brit. Assoc. Report Elec. Stands. Committee*, 1869.
[2] *Roy. Soc. Phil. Trans.*, 1873, clxiii.
[3] *Phil. Mag.*, 1880.
[4] *Comptes Rendus*, 1897, cxxiv.
[5] *Roy. Soc. Phil. Trans.*, 1868.

where I is the current in the coils, x the distance apart of their mean planes, and M their mutual inductance.

The difference of potential V_m in electromagnetic measure is equal to $(R+r_1)I_1$, where r_1 is the equivalent resistance of S and G_2, *i.e.* r_1 is equal to $G_2S/(G_2+S)$.

The difference of potential V_S in electrostatic measure may be written

$$V_S = I\sqrt{\frac{dM}{dx}}\,\frac{2\sqrt{2b}}{a}.$$

In electromagnetic measure we have seen

$$V_m = \left(R + \frac{G_2S}{G_2+S}\right)I_1.$$

Hence v, the ratio of the two units, is equal to

$$\frac{a}{2\sqrt{2b}(\sqrt{dM/dx})}\,\frac{I_1}{I}\left(R+\frac{G_2S}{G_2+S}\right).$$

In this expression the ratio of the currents I_1 and I is in practice expressed as the ratio of two resistances.

The numbers of turns in the coils were 144 and 121 respectively, and their mean diameter was 3·868 inches. The mean result, after correcting for the B.A. unit of resistance, was

$$v = 2{\cdot}841\times10^{10}\text{ cm. sec.}^{-1}.$$

Hurmuzescu[1] also combined the electrometer and electrodynamometer in one instrument. The difference of potential at the ends of a known resistance was measured by means of an absolute electrometer of cylindrical form, the electrostatic couple being balanced by the forces between the coils of an electrodynamometer, the movable coil of which was attached to the same axis as the electrometer system. The value of v resulting from Hurmuzescu's measurements was found to be $3{\cdot}0010\times10^{10}$ cm. sec.$^{-1}$.

§ (4) The Resistance Method.—This method involves the absolute measurement of a resistance in electromagnetic measure and also in electrostatic measure. The ratio of the two values is equal to v^2. The measurement of resistance in electromagnetic measure is not a very difficult one, and in practice the whole problem lies in measuring the resistance in electrostatic units. This can be done by discharging a condenser of electrostatic capacity C_S through a resistance of electrostatic resistance R_S and measuring the rate of fall of difference of potential.

At any time t the electrostatic value V_S of the difference of potential is such that

$$C_S\frac{dV_S}{dt}+\frac{V_S}{R_S}=0,$$

and hence $\qquad \log V_S + \dfrac{t}{R_SC_S} = \text{constant}.$

If, at a particular time, V_0 is the difference of potential, and t seconds afterwards V_S is the difference, then

$$\frac{t}{C_SR_S} = \log\frac{V_0}{V_S},$$

or

$$R_S = \frac{t}{C_S}\,\frac{1}{\log(V_0/V_S)}.$$

Thus if a condenser of initial potential V_0 discharges itself in t seconds through a resistance R_S to a lower potential V_S, the value of the resistance in electrostatic measure is that given by the preceding equation. Since the ratio of the electromagnetic to the electrostatic unit is v^2 it follows that

$$v^2 = \frac{C_SR_m\log(V_0/V_S)}{t},$$

where R_m is the resistance in electromagnetic units. If it be arranged that $V_S = V_0/2$, then

$$v^2 = \frac{C_SR_m\log_e 2}{t}.$$

It follows, therefore, that to measure v we must know C_S, R_m, and the time in which the condenser discharges to one half its initial potential. To obtain an idea of the magnitudes involved, if $R=1$ megohm and $t=0{\cdot}1$ second the capacity C_S would have to be about 0·14 microfarad. The electrostatic capacity of such a condenser could not be computed from its dimensions, and would have to be obtained by comparison with a smaller condenser the value of which could be calculated. If an attempt were made to dispense with the large condenser and to experiment directly with a small air condenser the capacity of which could be calculated, then the resistance would be of the order of 1000 megohms or more, and the difficulty would not be diminished. Rosa and Dorsey[2] point out that if the electrostatic capacity of the large condenser is obtained by comparing its electromagnetic capacity with that of a small standard condenser, and taking the ratio, then v^2 is immediately obtained from the ratio of the electrostatic to the electromagnetic capacity of one condenser. Hence, the work in getting R is really unnecessary except as affording a value of v by a roundabout way.

§ (5) The Method of Capacities.—This is generally considered to be the best of the methods available. Let C_S be the capacity of a condenser in electrostatic measure and C_m its capacity in electromagnetic units. Then

$$C_S[LK] = C_m[L^{-1}T^2\mu^{-1}],$$

or

$$\left[\frac{1}{\sqrt{K\mu}}\right] = \sqrt{\frac{C_S}{C_m}}[LT^{-1}];$$

hence

$$\frac{1}{\sqrt{K\mu}} = \sqrt{\frac{C_S}{C_m}} = v\text{ cm. per second.}$$

[1] *Ann. Chim. et Phys.*, 1897, x.

[2] *Bureau of Standards*, 1907, Bull. 3.

There are several different ways of finding the ratio C_S/C_m, but in every case the electrostatic capacity C_S must be obtained from the dimensions of a condenser. The differences between the methods arise in the various ways of obtaining C_m, but in all methods the value of a resistance in absolute measure must be known. Rosa and Dorsey[1] divide the methods of finding C_m into eight sections; these are, with slightly different titles from those given by Rosa and Dorsey:

(*a*) By ballistic galvanometer.
(*b*) By steady deflection of a galvanometer.
(*c*) By the differential galvanometer.
(*d*) By the Maxwell bridge.
(*e*) By oscillatory discharges of a condenser.
(*f*) By determination of the product of a capacity and an inductance.
(*g*) By comparing a capacity and a resistance by use of an alternating current.
(*h*) By comparison of a capacity with an inductance.

Of these methods *d* and *c* are considered to be the most direct and capable of the highest accuracy.

(*a*) *By Ballistic Galvanometer.* — In this method the condenser is charged to a high potential E and then discharged through a ballistic galvanometer, the latter being calibrated by establishing a steady deflection by means of a current produced by a known fraction of E and a high resistance. The galvanometer measures, therefore, the ratio Q_m/E_m which is equal to C_m, the quantity desired.

The principle of this method was first used by Ayrton and Perry,[2] who charged a guard-ring condenser DP (*Fig.* 3) to a potential corresponding to that at the extremities of a resistance R of 10,000 ohms in circuit with a battery E of about 420 volts. The contacts at K and K′ were so made that the condenser

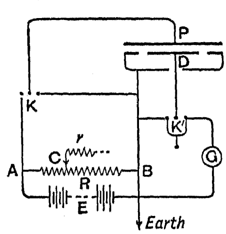

FIG. 3.

was first charged to the difference of potential existing between the extremities of R. Afterwards P was earthed, leaving D charged and insulated: finally D was discharged through the galvanometer G. To obtain the difference of potential between the extremities of R a very high resistance r was included in the galvanometer circuit (making the total effective resistance r'), and a potential difference E/n corresponding to that between two points such as A and C was also included in the galvanometer circuit; the resulting steady deflection α was observed. If θ is the angular deflection given by the transient current and C_m the capacity, in electromagnetic units, of the condenser, then by the formulae for the ballistic and tangent galvanometers we have

$$\frac{C_m E}{(E/n)/r'} = \frac{T \sin \frac{1}{2}\theta}{\pi \tan \alpha},$$

where T is the period of the galvanometer system; hence

$$v^2 = \frac{C_S}{C_m} = C_S \frac{nr'\pi \tan \alpha}{T \sin \frac{1}{2}\theta}.$$

The mean of Ayrton and Perry's experiments, after correction for the resistances used by them, was

$$v = 2{\cdot}995 \times 10^{10} \text{ cm. sec.}^{-1}.$$

(*b*) *By Steady Deflection of a Galvanometer.*— This method is in principle the same as method (*a*), but instead of having a single discharge and a resulting transient current through the galvanometer, a rapid succession of discharges is sent through the galvanometer and a steady deflection produced. The number of charges and discharges per second must be known. This method was used by Klemenčič,[3] who employed a tuning-fork as a means of obtaining a series of charges and discharges. The mean result was $v = 3{\cdot}041 \times 10^{10}$ cm. sec^{-1}. Similar experiments were made by Stoletow,[4] but a revolving commutator was used instead of a tuning-fork. Stoletow's result was

$$v = 2{\cdot}99 \pm 0{\cdot}01 \times 10^{10} \text{ cm. sec.}^{-1}.$$

(*c*) *Differential Galvanometer Method.*—This is a modification of method (*b*), the charging or the discharging current of the condenser

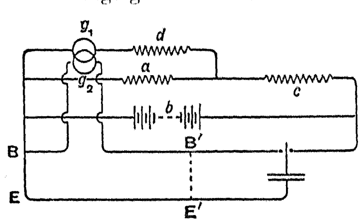

FIG. 4.

being passed through one coil of a differential galvanometer, and through the other coil is passed a steady current shunted off from a portion of a high resistance. The circuit employed by Rosa and Dorsey[5] is shown in *Fig.* 4.

[1] *Bureau of Standards*, 1907, Bull. 3.
[2] *Soc. Tel. Engineers' Journal*, 1879.
[3] *Akad. Wiss. Wien. Ber.*, 1881, lxxxiii.
[4] *Soc. Franc. de Phys.*, 1881.
[5] *Bureau of Standards*, 1907, Bull. 3.

The continuous lines show the connections when the discharge current is sent through the galvanometer. To send the *charge* current through instead, BE is disconnected and connection made through B′E′. When the condenser has a guard ring the connections are those shown in *Fig.* 5, the continuous lines

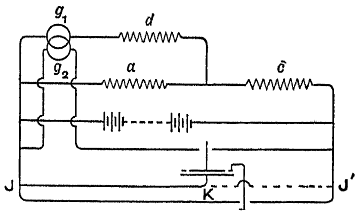

FIG. 5.

showing the connections for comparing the discharge current with the steady current. To measure the charging current, JK is disconnected and connection made through J′K.

When a balance is obtained the electromagnetic capacity C_m of the condenser is given by the expression

$$C_m = \frac{a}{ncd\{1+(g_1/d)+a(1/d+1/c+g_1/cd)\}}.$$

The principle of this method was used by Klemenčič in 1884, by Himstedt,[1] by Abraham at Paris in 1892, and by Rosa and Dorsey (1905–1907). The latter observers also made measurements by Maxwell's bridge method, and the results obtained by them will be given after a description of that method.

The results obtained by Klemenčič, by Himstedt, and by Abraham are

Klemenčič . .	$v=3{\cdot}019$	$\times 10^{10}$ cm. sec.$^{-1}$.
Himstedt . .	$v=3{\cdot}009$	,, ,,
Abraham . .	$v=2{\cdot}9913$	,, ,,

(*d*) *Maxwell Bridge Method.*[2]—This is the best known method, and has an advantage over methods (*a*) and (*b*) of being a null method. *Fig.* 6 shows the bridge connections. The condenser and commutator are placed in one arm of the bridge and the frequency of charging and discharging the condenser is adjusted until a balance is obtained. The relation between the electromagnetic capacity C_m, the frequency n, and the resistances a, c, and d is then

$$C_m = \frac{a}{ncd}\left[\frac{1-(a^2/(a+c+g)(a+b+d))}{\{1+(ab/c(a+b+d))\}\{1+(ag/d(a+c+g))\}}\right].$$

In general the value of C_m differs from a/ncd by a few parts only in 100,000.

The investigation of the relation between the bridge arms given by Maxwell is only an approximation; the above equation was first given by J. J. Thomson.[3]

The changes which occur may be divided into two parts. There is first the steady current which

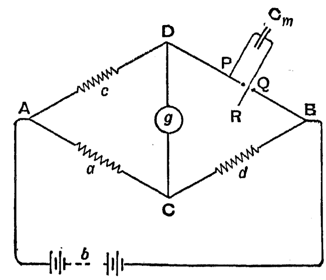

FIG. 6.

flows through g when no current is passing into the condenser. Let this current be $\dot{I}$. In the second part when R and Q are connected there is a transient current which flows while the condenser is being charged. Under these conditions let q be the quantity of electricity which flows through g; this quantity will flow in the opposite direction to $\dot{I}$. Then if n is the frequency of charge and discharge, nq is the quantity per second, and if the period of the galvanometer is long compared with $1/n$ of a second the quantity nq is assumed to produce the same effect as a steady current of value nq. Thus the bridge will be balanced when $\dot{I}=nq$. q is clearly the quantity of electricity which would pass through the galvanometer if the battery were removed and the condenser, after charging, was discharged by connecting R to Q. During such discharge let

$\dot{z}$ be the current along PD,
$\dot{q}$ be the current through the galvanometer,
$\dot{x}$ be the current along CA,

and let the self-inductances between the various points be

L_1 between A and D, *i.e.* that of resistance c.
L_2 ,, A and B through b.
L_3 ,, C and B, *i.e.* of resistance d.
L_4 ,, A and C, *i.e.* of resistance a.
L_5 ,, D and C, *i.e.* that of the galvanometer.

Then from the part DAC we have

$$L_5\frac{d^2q}{dt^2}+\frac{L_1d^2(q-z)}{dt^2}-L_4\frac{d^2x}{dt^2}+g\frac{dq}{dt}$$
$$+c\left\{\frac{dq}{dt}-\frac{dz}{dt}\right\}-a\frac{dx}{dt}=0,$$

where z, y, and x are the quantities of electricity passing during discharge through PD, g, and a respectively.

Initially and finally $\dot{z}$, $\dot{q}$, and $\dot{x}$ vanish. Integrating from just before discharging until after complete discharge we have

$$gq+c(q-z)-ax=0. \quad . . \quad . \ (1)$$

[1] *Wied. Annalen*, 1888.
[2] Maxwell's *Electricity and Magnetism*, § 775. See also "Capacity, Electrical," § (41), for further details.

[3] *Roy. Soc. Phil. Trans.*, 1883.

Similarly from the part CAB we have

$$(a+d+b)x+(d+b)q-bz=0, \qquad (2)$$

and from (1) and (2)

$$q=\frac{z\{c(a+b+d)+ba\}}{(a+g+c)(a+b+d)-a^2}. \qquad (3)$$

Now the current $\dot{q}(=nq)$ in the galvanometer when the condenser is discharged is equal to the steady current $\dot{I}$ passing through the galvanometer when no current is passing into the condenser. Under the same conditions the current through the arm d of the bridge is $\dot{I}+(c+g/a)\dot{I}$ and hence the difference of potential between D and B is

$$\dot{I}g+\left(\dot{I}+\dot{I}\frac{c+g}{a}\right)d$$
$$=\dot{I}\left\{g+d+\frac{d}{a}(c+g)\right\}=\dot{q}\left\{g+\frac{d}{a}(a+c+g)\right\}. \qquad (4)$$

Now when the condenser is fully charged no current will flow into it. Under these circumstances the quantity z will be equal to the product of the capacity C_m of the condenser and the difference of potential between D and B, *i.e.*

$$Z=C_m\dot{q}\left\{g+\frac{d}{a}(a+c+g)\right\}. \qquad (5)$$

Combining this with (3) we have

$$q=C_m\dot{q}\frac{\{c(a+b+d)+ba\}\{g+(d/a)(a+c+g)\}}{(a+g+c)(a+b+d)-a^2}.$$

But $nq=\dot{q}$; hence

$$\frac{1}{nC_m}=\frac{\{c(a+b+d)+ba\}\{g+(d/a)(a+c+g)\}}{(a+g+c)(a+b+d)-a^2},$$

which enables C_m to be expressed in the convenient form

$$C_m=\frac{a\{1-(a^2/(a+c+g)(a+b+d))\}}{ncd\{1+(ab/c(a+b+d))\}\{1+(ag/d(a+c+g))\}}.$$

v is of course obtained from the ratio $\sqrt{C_S/C_m}$.

When guard-ring condensers are used it is necessary for the guard ring to be charged to the same potential as the main condenser. This condition is most readily met by constructing, as suggested by Rosa and Dorsey,[1] a second Wheatstone net exactly duplicating the main set and containing the same battery. Such a double network is shown in *Fig.* 7.

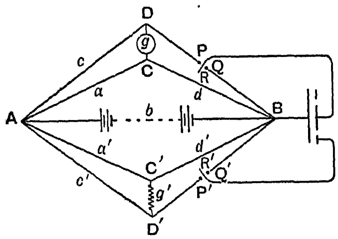

FIG. 7.

The bridge connections used by Thomson and Searle in 1890 are shown in *Fig.* 8.

The Maxwell bridge method of measuring C_m has been used for determinations of v by J. J. Thomson,[2] by Rosa,[3] by Thomson and Searle,[4] and by Rosa and Dorsey.[5]

J. J. Thomson in 1883 used a cylindrical guard-ring condenser designed by Lord Rayleigh, the outer cylinder being about 61 cm. long and 25·4 cm. diameter; the diameter of the inner cylinder was 23·5 cm. The electrostatic capacity of this condenser was calculated and compared with that of another condenser without a guard ring, the electromagnetic capacity of which was subsequently measured in a Maxwell bridge. The main part of the commutator was a strip of brass which vibrated in the field of an electromagnet, the periodicity of the current in the latter being controlled by a tuning-fork. The value of v, resulting from his experiments, was $2{\cdot}963 \times 10^{10}$ cm. sec.$^{-1}$.

Rosa in 1889 used the standard spherical condenser employed by Rowland when measuring v by the quantity method. The advantage of a spherical condenser over a cylindrical one lies in the smaller percentage error associated with inaccurate centring; thus a displacement from the coaxial position of the inner cylinder of a cylindrical condenser such that an error or correction of several parts in a thousand would be introduced would in the case of a spherical condenser of similar capacity have a negligible effect. In these early experiments of Rosa's two tuning-forks were used as standards to control the rate of charge and discharge of the condenser, and it was found that the results obtained when the fast fork was used were slightly lower than those obtained with the slow fork. The results were

$$v=2{\cdot}9994\times10^{10} \text{ cm. sec.}^{-1} \text{ (fast fork)},$$
$$v=3{\cdot}0023\times10^{10} \text{ cm. sec.}^{-1} \text{ (slow fork)}.$$

The results with the fast fork were considered to be the better, and the weighted mean of all the observations was

$$v=3{\cdot}0004\times10^{10} \text{ cm. sec.}^{-1}.$$

In 1890 Thomson and Searle repeated their experiments of 1883, adopting at first the same method. Very consistent results, practically identical with the previous ones, were obtained. As the introduction of the auxiliary condenser increased the possible sources of error it was abolished, and a guard-ring standard condenser was directly measured by the bridge. At first a tuning-fork device for make and break was used, but as the results were irregular it was replaced by a rotating commutator driven by a water motor. The values then

[1] *Bureau of Standards*, 1907, Bull. 3.

[2] *Roy. Soc. Phil. Trans.*, 1883, clxxiv.
[3] *Phil. Mag.*, 1889, xxviii.
[4] *Roy. Soc. Phil. Trans.*, 1890, clxxxi.
[5] *Bureau of Standards*, 1907, Bull. 3.

obtained were distinctly lower than those given by the old method. The cause was found to be due to the guard ring not having its full effect when the capacities of the guard-ring condenser and the auxiliary condenser were compared, thus giving a slightly erroneous value for the latter condenser. In the 1890 determination the auxiliary condenser was therefore dispensed with. The standard guard-ring condenser was a cylindrical one as before, and the guard ring was introduced into the bridge in the manner shown in *Fig.* 8, an

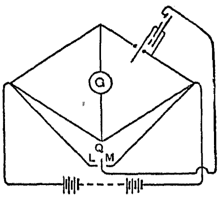

FIG. 8.

additional commutator LMQ being used. The double commutator system was driven by a water motor, the supply of water being at constant pressure. The speed was observed and regulated by a stroboscopic method, a tuning-fork serving as a standard of frequency. The value of v finally obtained by them was $2{\cdot}9955 \times 10^{10}$ cm. sec.$^{-1}$.

Rosa and Dorsey's measurements in 1907 [1] were carried out with extraordinary care, and the probable error of the result obtained by them is less than that associated with any other determination of v. The spherical condenser which was employed in the experiments of 1879 and 1889 was again used, the two balls being repolished and the two halves of the shell reground so that they fitted together water-tight. The capacity of the charging wire touching the inner sphere, and which passed through a small hole at the pole of the outer hollow sphere, was difficult to determine. The difficulty was overcome by obtaining the capacity with two charging wires, one at the pole and one at the equator. First withdrawing the polar charging wire, the change in electromagnetic capacity gave the capacity of the charging wire itself, the ball meanwhile being charged by the wire at the equator. By replacing the polar charging wire and removing that at the equator, the capacity of the latter was obtained. The values so found are free from theoretical objection, and in practice the method gave remarkably accurate results.

Cylindrical condensers were also used by Rosa and Dorsey. These could be measured without the necessity of determining the capacity of the charging wire in a way described by Lord Rayleigh.[2] Lord Rayleigh's arrangement is shown in *Fig.* 9. There are three outer cylinders A, G, D, and two inner cylinders B, F, the components of two pairs being of the same length. One pair A and B are mounted coaxially upon an insulating base and remain undisturbed. The other parts are movable and allow of the formation of two condensers. In the first of these the third outer cylinder D is mounted upon A so that the inner surfaces correspond. Upon the accurately worked top of B is placed a disc C of the same diameter, and D is also closed above by E. The leads make contact with the cylinders A, B at their bases. The capacity of this condenser and its leads are unknown. In the second arrangement the long pair of cylinders F, G are interpolated, and we thus obtain a second condenser of larger capacity than the first. Although the capacity is unknown the *increase* of capacity is accurately that of the intermediate cylinders F, G considered as forming parts of infinitely prolonged wholes. If, of the added parts, l is the length, b the larger radius, and a the smaller radius, the increase of capacity is $l/2 \log (b/a)$.

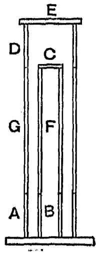

FIG. 9.

Rosa and Dorsey measured first the capacity C_1 of a pair of coaxial cylinders mounted with their axes vertical. A second pair of cylinders of the same radii were then added to the first pair and the new capacity C_2 measured. Then a third pair were added and the total capacity C_3 determined. $C_2 - C_1$ was then the capacity of the second pair of cylinders and $C_3 - C_2$ was the capacity of the third pair. Subsequently a fourth pair of cylinders were added to the set, the two end sections of the inner cylinders being insulated so that they formed guard cylinders after the manner of the guard rings of a plate condenser. Using cylindrical condensers in this way results of great uniformity were obtained.

Rosa and Dorsey also employed a parallel plate guard-ring condenser, but this was found to present greater difficulties than the other forms.

To measure the rate of charge and discharge of a condenser, the commutator was geared to a chronographic drum which was driven at a speed 250 times less than that of the commutator. Once every second a printing magnet printed a dot on the paper on the drum, and from the record the mean speed could be determined within 1 part in 100,000.

Rosa and Dorsey concluded that the results obtained by the use of the plate condensers and of the cylinders without guards were of

[1] *Bureau of Standards*, 1907, Bull. 3.

[2] *Phil. Mag.*, 1906, xii.

little value. They summarise the more reliable of their results as follows:

Date.	Condenser.	v in cm. sec.$^{-1}$.	N.	Δ.	Δ_1.
1905	Spherical (larger ball)	$2{\cdot}99621 \times 10^{10}$	64	$0{\cdot}00018 \times 10^{10}$	$0{\cdot}00018 \times 10^{10}$
1905	Spherical (smaller ball)	2·99637 „	146	0·00011 „	0·00013 „
1906	„	2·99659 „	89	0·00017 „	0·00030 „
1906	Cylinders with guards	2·99624 „	603	0·00027 „	0·00028 „
	Weighted Mean . .	2·99629 „		Mean . .	0·00022 „

In this table N is the number of independent determinations, Δ is the average variation of the members of each group from their mean, and Δ_1 is the corresponding average variation from the weighted mean of all.

The mean value is

$$v = 2{\cdot}9963 \times 10^{10}\left[\frac{\text{cm. int. ohm}}{\text{sec.}}\right]^{\frac{1}{2}}.$$

As the mean temperature at which the work was done was about 20° C., this gives for the value reduced to *vacuo* (assuming the dielectric constant of air at 20°·0 C. and 760 mm. pressure to be 1·00055)

$$v_0 = 2{\cdot}9971 \times 10^{10}\left[\frac{\text{cm. int. ohm}}{\text{sec.}}\right]^{\frac{1}{2}}.$$

The uncertainty is stated to be not more than 1 part in 10,000, and is much less than in any other determination of v. It will be observed that the value was given in terms of the centimetre, the second, and the international ohm, and this was because of the uncertainty in the absolute value of the resistances used. Now according to recent determinations of the international ohm in absolute measure (see article "Electrical Measurements," § (39)), the international ohm equals 1·00052 ohms (10^9 C.G.S. units). If this value is accepted as correct the correction to Rosa and Dorsey's result is + 26 parts in 100,000. The corrected value is

$$v_0 = 2{\cdot}9980 \times 10^{10} \text{ cm. sec.}^{-1},$$

with an uncertainty of not more than 3 in the last place of decimals.

(*e*) *By Oscillatory Discharges of a Condenser.*—A charged air condenser of calculable electrostatic capacity C_S is placed in series with a coil of calculable inductance L_m and caused to discharge across a narrow spark gap, the resistance being small. The period t of the oscillatory discharge is given by

$$t = 2\pi\sqrt{C_m L_m},$$

or

$$C_m = \frac{t^2}{4\pi^2 L_m}.$$

Using this principle, Lodge and Glazebrook [1] made a careful study of the oscillatory discharge of an air condenser with a view to measuring v. The air condenser consisted of eleven flat glass plates entirely covered with tin-foil. They were each about 60 cm. square, and were kept 5 mm. apart by glass distance pieces, 5 in each space, like the pips on a card. In calculating the capacity of this condenser corrections were made for the edges, the thickness of the plates, for the distance pieces, and for the proximity of the floor, walls, and roof. The inductance consisted of 2 coils, each of about 11 inches internal diameter, 4 inches deep, and 2 inches wide; the coils were arranged so that they could be used in series, in parallel, or used separately. The capacity of these coils was considerable, and was one of the chief causes of difficulty in comparing the experimental results with theory.

To determine the period of the oscillatory discharge a photographic plate was whirled by a turbine at from 64 to 85 revolutions per second, and the image of the spark was focussed on this plate. The speed of rotation was controlled and measured by a stroboscopic disc and electromagnetically maintained tuning-fork.

The value of v determined by these experiments was $3{\cdot}009 \times 10^{10}$ cm. sec.$^{-1}$, but the method was not regarded as a very exact one.

(*f*) *By determining the Product of a Capacity and an Inductance.*—This method was suggested by A. Gray,[2] but no experiments have been carried out. If a magnet is rapidly rotated within a coil which is suspended by a bifilar in a vertical plane, the induced current causes the coil to turn through an angle θ. If the coil is in circuit with a self-inductance and capacity, and L_m and C_m denote the total self-inductance and capacity, the angle of deflection θ is dependent on the time of revolution of the magnet, on L_m, and on C_m. The value of the product $C_m L_m$ can be found by observing the deflections θ_1, θ_2, and θ_3 resulting from three different angular velocities n_1, n_2, and n_3 of the magnet. The relation is

$$C^2{}_m L^2{}_m = \frac{1}{n_1{}^2 n_2{}^2 n_3{}^2} \cdot \frac{\frac{n_1{}^3}{\theta_1}(n_2{}^2 - n_3{}^2) + \frac{n_2{}^3}{\theta_2}(n_3{}^2 - n_1{}^2) + \frac{n_3{}^3}{\theta_3}(n_1{}^2 - n_2{}^2)}{\frac{n_1}{\theta_1}(n_2{}^2 - n_3{}^2) + \frac{n_2}{\theta_2}(n_3{}^2 - n_1{}^2) + \frac{n_3}{\theta_3}(n_1{}^2 - n_2{}^2)}.$$

[1] *Cambridge Phil. Trans.*, 1899, xviii.

[2] *Absolute Measurements*, vol. ii.

Rosa and Dorsey have shown that no great precision can be expected from such an experiment, for not only have θ_1, θ_2, θ_3 and n_1, n_2, n_3 to be measured with accuracy, but C_m and L_m must necessarily be large and are not directly calculable. In practice, therefore, the method would prove to be elaborate and the probable error would be large.

(*g*) *By comparing a Capacity and a Resistance by Use of an Alternating Current.*—Let an air condenser of electromagnetic capacity C_m and calculable electrostatic capacity Cs be placed in series with a non-inductive resistance

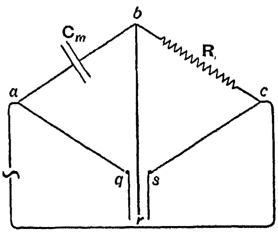

FIG. 10.

R (*Fig.* 10), and let the applied alternating E.M.F. be strictly of sine form. Let *qrs* be an electrometer to indicate when the effective E.M.F. on *ab* is equal to that on *bc*. When equality is obtained

$$R=\frac{1}{2\pi n C_m},$$

where n is the number of alternations per second. A correction for the capacity of the electrometer must be made.

The chief difficulty is to obtain a pure sine wave form E.M.F. M. E. Maltby,[1] who employed the method in 1897, generated the electromotive force by rotating a magnet within a coil and obtained a nearly pure sine wave. At present no attempt at very precise measurements has been made by this method. Maltby obtained the value $v=3{\cdot}001\times10^{10}$ cm. sec.$^{-1}$.

(*h*) *By Comparison of a Capacity with an Inductance.*—The electromagnetic capacity of a condenser can be measured in terms of an inductance by means of bridge methods such as those due to Maxwell, Anderson,[2] etc. If the electromagnetic capacity is thus found, v can be determined if the electrostatic capacity can be calculated or determined by comparison with another condenser. In general the latter course must be adopted, owing to the comparatively large capacity used in the bridge methods.

[1] *Wied. Ann.*, 1897, lxi.
[2] See "Inductance, Measurement of," §§ (106)-(114).

SUMMARY.—Of the methods hitherto used, Rosa and Dorsey are of the opinion that the method of capacities by the Maxwell bridge, and the capacity method by the differential galvanometer, are the simplest and most direct and capable of the highest accuracy. Lord Rayleigh [3] was also disposed to give preference to the method of capacities, but was of opinion that the electromotive force method was worthy of a further trial. The construction of an absolute electrometer presents considerable but not insurmountable difficulties, and there is little doubt that very accurate measurements could now be made. Until there is a satisfactory explanation of the small difference which at present exists between the mean value of v as obtained from the most careful measurements, and the value experimentally determined for the velocity of light, some doubt will exist as to the absolute reliability of the methods used for measuring v. Lord Rayleigh pointed out that in the Maxwell bridge method there is some doubt about the behaviour of the galvanometer. It is assumed that this instrument indicates exactly the *mean* current, whether the current be steady or intermittent. Any error which might be feared due to the oblique position of the needle, and its temporary magnetisation under the currents to or from the condenser, can be eliminated by reversing the battery, but Lord Rayleigh points out that the axial magnetisation may not remain constant even when the axis is strictly perpendicular to the magnetic forces due to the currents, and suggests the use of an Einthoven galvanometer to avoid possible trouble.

Of all the determinations so far made that of Rosa and Dorsey appears to be associated with the least probable error. The value found by them was

$$v=2{\cdot}9971\times10^{10}\text{ cm. sec.}^{-1},$$

but was based on the international ohm. When corrected so as to be based on the ohm (10^9 C.G.S. units) it is

$$v=2{\cdot}9980\times10^{10}\text{ cm. sec.}^{-1},$$

and the uncertainty should not exceed about $0{\cdot}0003\times10^{10}$ cm. sec.$^{-1}$.

The following table gives the results of the principal measurements of v corrected as far as possible for the resistances employed:

1857	Weber and Kohlrausch	$v=3{\cdot}1074\times10^{10}$	cm. sec.$^{-1}$.
1868	Maxwell	„ 2·841	„ „
1869	Thomson and King	„ 2·825	„ „
1873	M'Kichan	„ 2·93	„ „
1879	Rowland	„ 2·9815	„ „

[3] *Phil. Mag.*, 1906, xii.

1879	Ayrton and Perry	$v=2\cdot995$	$\times 10^{10}$ cm. sec.$^{-1}$.
1880	Shida	„ 2·995	„ „
1881	Klemenčič	„ 3·041	„ „
1881	Stoletow	„ 2·99	„ „
1883	J. J. Thomson	„ 2·963	„ „
1884	Klemenčič	„ 3·019	„ „
1888	Himstedt	„ 3·009	„ „
1889	Kelvin	„ 3·004	„ „
1889	Rosa	„ 3·0004	„ „
1890	Thomson and Searle	„ 2·9955	„ „
1891	Pellat	„ 3·0092	„ „
1892	Abraham	„ 2·9913	„ „
1897	Maltby	„ 3·001	„ „
1897	Pérot and Fabry	„ 2·9978	„ „
1897	Hurmuzescu	„ 3·0010	„ „
1899	Lodge and Glazebrook	„ 3·009	„ „
1905–7	Rosa and Dorsey	„ 2·9980	„ „
	Mean	„ 2·987	„ „

In a résumé of all former work upon the subject, H. Abraham [1] stated that he considered the most accurate results to be as follows:

Himstedt	$3\cdot0057\times10^{10}$	cm. sec.$^{-1}$.
Rosa	3·0000	„ „
Thomson and Searle	2·9960	„ „
Abraham	2·9913	„ „
Pellat	3·0092	„ „
Hurmuzescu	3·0010	„ „
Pérot and Fabry	2·9978	„ „

To these should now be added

Rosa and Dorsey . $2\cdot9980\times10^{10}$ cm. sec.$^{-1}$.

The mean of these is $2\cdot9999\times10^{10}$ cm. sec.$^{-1}$. It should be noted, however, that whereas Abraham stated it was probable that the mean of the first seven, which is 3·0001, differs from the true value by not more than 1 part in 1000, Rosa and Dorsey consider their result 2·9980 to be accurate within 1 part in 10,000.

It is of interest to compare the above values of v with the most probable value for the velocity of light. The best determinations of the latter are as follows:

1883 Newcomb	$v=2\cdot99860\pm0\cdot00030\times10^{10}$	cm. sec.$^{-1}$.
1885 Michelson	„ $2\cdot99853\pm0\cdot00060$	„ „
1902 Perrotin	„ $2\cdot99860\pm0\cdot00080$	„ „
1904 Weinberg	„ $2\cdot99852\pm0\cdot00024$	„ „

From these values it appears safe to conclude that within 1 part in 10,000 the velocity of light is

$$2\cdot9986\times10^{10}\text{ cm. sec.}^{-1}.$$

This differs from the mean v of H. Abraham's list by 4 parts in 10,000 and from Rosa and Dorsey's value (corrected) by 2 parts in 10,000. The latter difference is probably within the limits of error of the measurements involved.

Within 1 part in 3000 it may therefore be stated with considerable confidence that

$$v=2\cdot998\times10^{10}\text{ cm. sec.}^{-1}.$$

F. E. S.

[1] *Congrès international de physique*, 1900.

Valency Electrons, Stark's theory of, used in explanation of phosphorescence, fluorescence, and photoelectricity. See "Photoelectricity," § (6).

Valves, Thermionic, use of, as generators of high-frequency electric current. See "Thermionic Valves," § (6); "Wireless Telegraphy," § (17) (iv.).

Van der Pol's Method of determining the Conductivity in Various Parts of Discharge through Gases. See "Electrons and the Discharge Tube," § (5).

Variable and Fluctuating Loads, effect of, on measurement of electrical energy. See "Watt-hour and other Meters for Direct Current. II. Watt-hour Meters," § (39).

Variation of Bearings: variations of the readings of direction-finders used in wireless telegraphy, with natural phenomena, *e.g.* sunset. See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (13).

Vernier Potentiometer, R. W. Paul's, based on an extended form of the Kelvin-Varley slide. See "Potentiometer System of Electrical Measurements," § (3) (v.).

VIBRATION GALVANOMETERS

I. Introductory and Historical

§ (1) Introductory.—A vibration galvanometer is an instrument for measuring alternating current, having a movable part or element which can be so adjusted that it will vibrate in resonance with the frequency of the current that is being measured. Such an instrument is perhaps more properly called a *Resonance Galvanometer*, as the capability of working in resonance with the current is its distinguishing characteristic. Vibration galvanoscopes, such as telephones, which merely indicate current without measuring it, may also be included here, provided they can be tuned to work in dynamical resonance with the current. Electrostatic instruments utilising the same principle are termed vibration (or resonance) electrometers and electroscopes.

§ (2) Damping and Resonance.—In all machines in which motion is maintained there is waste of power due to forces (such as friction) which oppose the motion. When the motion is oscillatory these are called the *damping forces* and their effect is described as *damping*. When the external source of energy

which maintains the oscillatory motion is cut off, the motion gradually dies down to rest owing to the vibratory kinetic energy being gradually used up by the continued action of the damping forces.

When a regularly recurring (or periodic) force is applied to a system free to vibrate, unless the natural (free) frequency of the system is near to the frequency of the applied force, the movement produced will be what is called a *forced vibration* ; but, if the natural frequency is nearly equal to that of the applied force, in general the resulting vibration works itself up to a far greater amplitude, and we have the phenomenon known as *resonance*. When resonance is present, the amplitude of the oscillations goes on increasing until a maximum is reached which is limited by the damping forces. This maximum rises and the point of resonance becomes more and more sharply defined as the damping is reduced. In resonance galvanometers the damping is due to two quite distinct causes, dynamical and electrical, which will be discussed further in III.

The process of "tuning" or adjusting a vibration galvanometer to resonance frequency is usually carried out experimentally. A small alternating current from the given source is sent through the instrument, and the free frequency of the vibrating part is adjusted until a maximum deflection is obtained. If the wave form of the source contains harmonics, care must be taken to make sure that the resonance is with the fundamental component of the current and not with one of the harmonics.

§ (3) Advantages of Resonance Instruments.—When a vibration galvanometer is in tune with the frequency of the current, the resulting resonance vibration is usually enormously greater than the forced vibration which would be produced if resonance did not occur. In virtue of this property of *resonance magnification* vibration galvanometers are *highly sensitive* measurers of alternating current. The fact that they are relatively so insensitive for currents of any other frequency makes them also *highly selective*, for they practically ignore all components of the current which have frequencies not very near the resonance frequency. If tuned to the fundamental frequency of the current wave they are almost unaffected by any harmonic components that may be present.

§ (4) Classification of various Types.—In nearly every instance resonance galvanometers depend for their action upon an electrodynamical effect—a deflecting force is produced directly by the current. A composite type is possible in which the current produces a thermal effect which by causing expansion and contraction gives force (and motion).

Just as with ordinary galvanometers, so resonance instruments may be classified according to the nature of the part which is moved by the current. They thus fall into two main divisions, which may be further subdivided as follows :

Type (i.) Moving-circuit System, which may be—

(*a*) Single wire,
(*b*) Double wire,
(c_1) Moving coil, with unifilar suspension, or
(c_2) Moving coil, with bifilar suspension, or other arrangement.

This class may also be subdivided into two types, (A) in which the current passes directly through the moving-coil circuit, and (B) the induction type in which the main current induces a secondary current in a closed circuit which is movable.

The moving circuit is mounted in a magnetic field produced by a permanent magnet, an electromagnet or another circuit carrying a current, and this field produces the deflecting force or couple.

The control is usually by elastic force (of extension, compression, or torsion).

Type (ii.) Moving-magnetic System, which may be—

(*a*) A single magnet or group of magnets.
(*b*) Of soft iron magnetised only by the action of the main current.
(*c*) Of soft iron polarised, *i.e.* kept magnetised by the action of an auxiliary steady (or even alternating) magnetic field or electric circuit.

The deflecting magnetic field is produced by a coil or coils (with or without iron cores) carrying the current to be measured.

The control may be either by (A) elastic force, (B) magnetic force, or (C) partly by both.

In addition to these two main types there may occur others, perhaps less important, as, for example, a composite type in which a moving coil carries an iron core.

[Thermal types of resonance instrument have not yet been introduced, although the Irwin [1] hot-wire oscillograph points to possibilities of development in this direction.]

§ (5) Historical.—The development of the various types of vibration galvanometer at present in use has been carried out by experimenters in various countries, working in some cases without sufficient knowledge of what had already been done or suggested by others. For this latter reason a good deal of independent reinvention and rediscovery has occurred.

The earliest resonance galvanometer was the Optical Telephone introduced by M. Wien in 1891.[2] Wien demonstrated clearly the great advantages of resonance instruments for null methods of alternating current measure-

[1] J. T. Irwin, *Inst. El. Eng. Proc.*, 1908, xxxix. 617.
[2] M. Wien, *Wied. Ann.*, 1891, xlii. 593, and 1891, xliv. 680 and 689.

ment, and, guided by the theoretical work of Oberbeck [1] and Rayleigh,[2] he applied his optical telephone with great success to a number of bridge methods of measuring inductance and capacity. His work forms the foundation of the best present-day methods of measuring these quantities.

In 1893 Blondel [3] published an investigation on galvanometers of small periodic time (*i.e.* of quick natural frequency). He determined the mathematical conditions for dynamical resonance with alternating current, and showed how such galvanometers can be used as oscillographs for obtaining alternating current wave forms. He suggested galvanometers of moving-circuit type (both moving coil and single bifilar forms) and also with moving magnetic systems as in Type (ii.) § (4) above. His fundamentally important work did not receive adequate notice at the time, and has been perhaps somewhat overlooked by more recent experimenters.

In 1895 Rubens [4] brought out a vibration galvanometer of moving-magnet type. In this instrument the diaphragm and armatures of Wien's optical telephone were replaced by a suspended system of short magnet needles. Six years later M. Wien [5] described a simpler instrument with moving-magnet system, but without polarisation by external permanent magnets.

By this time high-frequency galvanometers had been fully developed as oscillographs by Blondel, who used mainly the soft-iron moving system, and by Duddell, who used the simple bifilar without a coil. In both cases strong damping was obtained by immersing the vibrating part in oil. By removing this damping these oscillographs could be used as resonance galvanometers. They were successfully used in this way by various experimenters,[6] but were only suitable for the higher audio frequencies (*e.g.* 2000 ∿ per sec.).

In 1907 Campbell [7] introduced a resonance galvanometer with a moving coil suspended below by bifilars and above by a single thread, with extensive ranges of frequency (e.g. 50 to

t out a moving

with unifilar strip suspensions. The tuning was adjusted by varying the tension of the strips or by altering the positions of the bridges over which the suspensions were stretched.

Also in 1909 Duddell [9] introduced a resonance galvanometer with a simple bifilar pair as in his oscillograph, but long enough to work with frequencies as low as 100 ∿ per sec.

Some years later Campbell,[10] having discovered that the torsional control exerted by a metal strip can be varied to a large extent by merely altering the tension, improved his galvanometer by employing unifilar suspension and reducing the size of the coil. [This system of tuning had already been used by Hausrath (1909) and much earlier by Blondel.]

C. V. Drysdale [11] in 1910 introduced a galvanometer with moving-iron system, the tuning being done by altering the magnetic field.

A little later Schering and Schmidt [12] used a single-loop bifilar instrument in which the two strips were 100 cm. long, in order to work at frequencies as low as 25 ∿ per sec.

They afterwards [13] brought out an improved form of the Rubens galvanometer, which could be conveniently tuned from a distance by altering the current in the circuit of the polarising electromagnets.

For the purpose of wave form analysis by resonance, Blondel and Carbenay [14] constructed in 1914 a special moving-iron galvanometer with ironless field magnet coils. For the same purpose Blondel [15] has brought out a moving-coil resonance galvanometer with convenient arrangement for tuning from a distance.

Agnew [16] in 1920 described a vibration galvanometer of the moving-iron type, in which a fine steel wire clamped at one end is kept in vibration by an electromagnet excited by the current to be measured.

A vibration galvanometer of the induction type was introduced by Blondel [17] in 1912. The current to be measured passes through the windings of electromagnets which induce a secondary current in a short-circuited aluminium or silver frame carried by bifilar suspension. For the higher frequencies a short-circuited bifilar loop only is used.

II. Descriptions of Actual Instruments

§ (6) In describing the various types of vibration galvanometer it will be convenient to take them mostly in the order of the classification

[1] Oberbeck, *Wied. Ann.*, 1882, xvii. 820.
[2] Lord Rayleigh, *Roy. Soc. Proc.*, 1891, xlix. 203.
[3] A. Blondel, *Comptes Rendus*, 1893, cxvi. 502 and 748.
[4] H. Rubens, *Ann. d. Physik*, 1895, lvi. 27.
[5] M. Wien, *Ann. d. Physik*, 1901, iv. 439.
[6] Mühlenhöver, *Dissertation, Munster-i.-W.*, 1905; E. Giebe and H. Diesselhorst, *Zeits. Instrumentenk.*, 1906, xxvi. 151.
[7] A. Campbell, *Phys. Soc. Proc.*, 1907, xx. 626, and *Phil. Mag.*, 1907, xiv. 494.
[8] H. Hausrath, *Phys. Zeits.*, 1909, x. 750.
[9] W. Duddell, *Phys. Soc. Proc.*, 1909, xxi. 774, and *Phil. Mag.*, 1909, xviii. 168.
[10] A. Campbell, *Phys. Soc. Proc.*, 1913, xxv. 203.
[11] H. Tinsley, *Electrician*, 1913, xxxix. 939.
[12] H. Schering and R. Schmidt, *Archiv f. Elektrotech.*, 1912, i. 254.
[13] Schering and Schmidt, *Zeits. Instrumentenk.*, 1917, xxxvii. 100.
[14] A. Blondel and F. Carbenay, *Ann. de Physique*, 1917, viii. 97.
[15] A. Blondel, *Ann. de Physique*, 1918, x. 195.
[16] P. G. Agnew, *Bureau of Standards Bull.*, 1920, xvi. 37.
[17] A. Blondel, *Lum. élec.*, 1912, xviii. 72.

in § (4), which, on the whole, proceeds from the simpler to the more complicated systems.

§ (7) Einthoven String Galvanometer.—String galvanometers of Einthoven[1] type have sometimes been used as resonance instruments. In *Fig.* 1 are shown the essential parts of a galvanometer of this kind. The

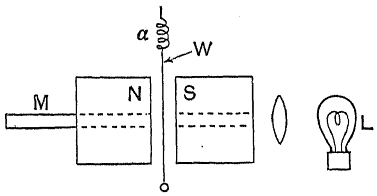

Fig. 1.—String Galvanometer (elementary diagram).

wire W is stretched in a narrow air-gap between the pole-pieces N and S of a magnet, in each of which there is a transverse channel allowing the light from the source L to pass through to the microscope M, by which the motion of the wire is observed. Sometimes an image of the wire is projected on a screen. The frequency can be adjusted by altering the tension of the spring *a*.

According to B. Glatzel[2] a range of frequency from 1000 up to 8000 ∿ per sec. can be obtained with a silver-bronze wire 3·5 cm. long and of 0·05 mm. diameter. The lower limit of the frequency is given practically by the equation[3]

$$n_0 = \left(\frac{22\cdot 4b}{8\pi l^2}\right)\left(\frac{E}{\delta}\right)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad (1)$$

where, for the wire,

E = elasticity (dynes/sq. cm.),
l = length (cm.),
b = diameter (cm.),
and δ = density.

For bronze $E \doteq 8 \times 10^{11}$.

When the deflections are not small the wire may become invisible and special means of observing the motion may have to be used. (As a bright point to be observed by a microscope, Helmholtz used a starch grain stuck to the vibrating body.)

§ (8) Duddell Vibration Galvanometer.—The Duddell galvanometer is of the moving-circuit type. As shown in *Fig.* 2, the current passes through a single loop formed of the two halves of a fine bronze wire, *abcd*, which passes over a small pulley *p*, and is stretched over two bridge pieces B, B, which limit the lengths of the wire free to vibrate. The wire is kept tight by a spiral spring, the tension being capable of variation by means of a screw with a milled head. The wires are placed in a strong magnetic field between the poles N and S of a magnet, and carry at their centre a very minute mirror M. As the current passes up one wire and down the other, it causes the one to move forward and the other back, thus causing the mirror to turn slightly about a vertical axis. As the current alternates the action is reversed, and the mirror is thus kept in angular oscillation, a spot of light reflected from it being drawn out into a band of light on the scale. Owing to the smallness of the mirror (0·02 sq. cm., say) this deflection light-band is rather faint, but it can be intensified by means of a long plano-convex cylindrical lens placed between the instrument and the scale as in *Fig.* 3. The bridge pieces B, B are mounted on two nuts working on left- and right-handed screws on a vertical spindle, by which their distance apart can be varied while keeping the mirror always half-way between them. The galvanometer

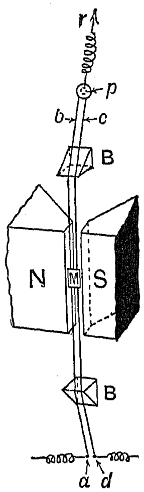

Fig. 2.—Working System of Duddell Vibration Galvanometer.

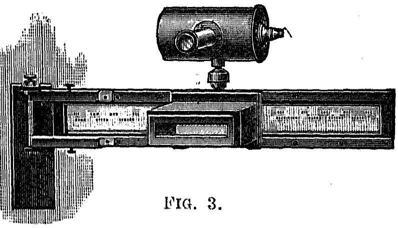

Fig. 3.

can be tuned to any frequency between 100 and 1800 ∿ per sec. by varying the distance apart of the bridgepieces and making a fine adjustment by altering the tension of the wires. The actual tension is indicated approximately on a small scale seen behind the tension milled head in the external view of the instrument shown in *Fig.* 4. The manner in which the natural frequency varies with the free length of the wires and their tension (in grams weight) is shown by the curves in *Fig.* 5, which were experimentally

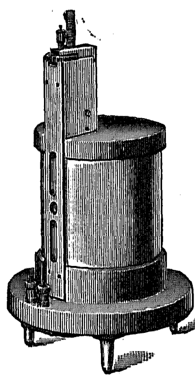

Fig. 4.—Duddell Vibration Galvanometer.

[1] W. Einthoven, *Wied. Ann.*, 1906, xiii. 483.
[2] B. Glatzel, *Elekt. Zeits.*, 1910, xxxi. 1094.
[3] J. K. W. Salamonson, *K. Akad. Amsterdam, Proc.*, 1918, xxi. 235. See also R. Förster, *Elekt. Zeits.*, 1914, xxxvi. 146.

determined by Duddell.[1] The current sensitivity is high, being between 50 and 60 mm.

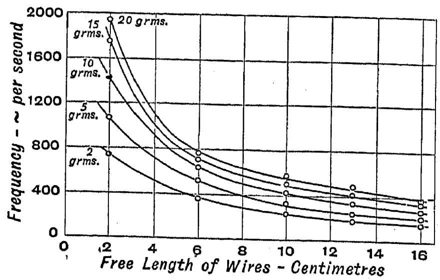

FIG. 5.—Variation of Frequency with Length and Tension of Wires.

deflection at 1 metre (scale distance) per microampere with an effective resistance of about 250 ohms.

The (alternating) current sensitivity falls off as the resonance frequency is raised, as shown, for various free wire lengths, in *Fig.* 6. Duddell proved that the sensitivity is very nearly inversely proportional to the frequency, as can

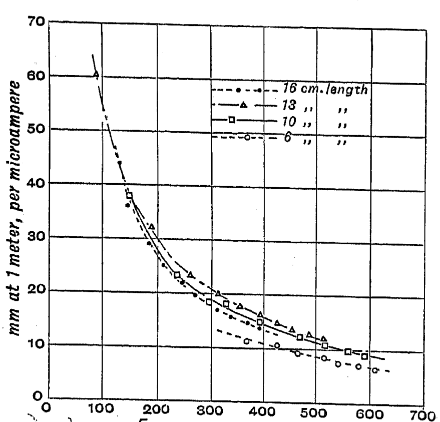

FIG. 6.—Current Sensitivities of Duddell Galvanometer.

be seen from *Fig.* 6, as long as the wire length is not too short. Blondell[2] has shown that, in the case of the moving-coil galvanometer with bifilar suspension, this law holds accurately for any given length of suspension when only the tension is altered. Further data regarding the constants of the galvanometer will be found in III.

The great advantage of this type of galvanometer lies in its high sensitivity at high frequencies, the chief disadvantage being the very small size of mirror which it is necessary to use. A considerable part of the damping is due to air friction. When the galvanometer is used in a vacuum the sensitivity is increased by 30 to 40 per cent.

At the higher frequencies, however, no vibration galvanometer has yet attained the sensitivity of the telephone receiver.

§ (9) SCHERING AND SCHMIDT'S BIFILAR GALVANOMETER.—Schering and Schmidt constructed a galvanometer of the same type as Duddell's, but with much longer bifilar loop in order to work at frequencies as low as 25 ∿ per sec. With copper strips 100 cm. long and 0·2 mm. × 0·02 mm. cross-section, at 50 ∿ per sec. the sensitivity was 10 mm. (at 1 m.) per microampere, with an effective resistance of 90 ohms. About one-seventh of the whole length of the strips was in the magnetic field. With phosphor bronze strips instead of copper the current sensitivity was three times as great, but the direct-current resistance was then 700 ohms.

§ (10) CAMPBELL VIBRATION GALVANOMETER.—The Campbell vibration galvanometer is of the moving-coil type, having bifilar wire suspensions (above and below) in the pattern shown in *Fig.* 7, but unifilar strip suspensions in the

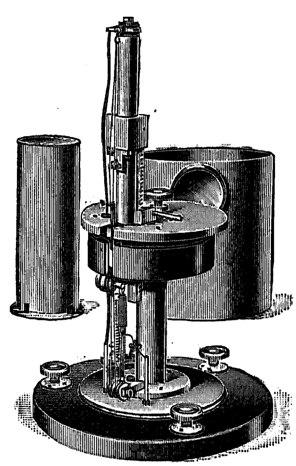

FIG. 7.—Campbell Vibration Galvanometer of Long Range.

most recent types. The lower suspension is brought round a small pulley to a spiral spring, whose tension is adjusted by a screw with milled head working through a lever. The length of the upper suspension is varied by a sliding bridge moved by a rack and pinion. The coil is very light and usually has only a few turns. It is suspended in the air-gap (about 3 mm. wide) of a permanent magnet giving a magnetic field of about 2000. A small mirror is carried by the coil. The current sensitivity attainable depends largely

[1] Duddell, *Phil. Mag.*, 1909, xviii. 168.
[2] A. Blondel, *Ann. de Physique*, 1918, x. 199.

on the size of the mirror. Table 1 shows the kind of values that may be obtained when a good-sized mirror is used.

TABLE 1

Frequency. ∿ per sec.	Sensitivity. mm. at 1 m. per μA.	Effective Resistance. Ohms.
50	60	500
100	30	350
350	3	160
750	0·5	52
1000	0·2	35

It will be noticed that the sensitivity falls off very quickly at the higher frequencies. This is due to the fact that the damping increases very rapidly as the suspension is shortened below a certain length. To obtain the best results the coil and suspensions should be chosen to suit the particular frequency used (and the resistance of the external circuit). Further data are given in III. For very low frequencies very high current sensitivities can easily be obtained. For example, at 10 ∿ per sec. the sensitivity may be 400 mm. at 1 metre distance for 1 microampere.

In observing the band of light on the scale at this frequency the flicker is very evident, but its presence is of advantage in null methods, for the absence of visible flicker is a very sensitive indication of exact balance.[1] The flicker disappears on the scale when the vibration of the light spot is reduced to about 0·1 mm. The sensitivity of indication is almost the same when the light from the mirror is observed directly by the eye without any scale at all. In order to extend the range of any particular coil and suspension to lower frequencies a convenient device is to increase the moment of inertia of the coil by adding to it a small spring clip of suitable size.

A short-range type of the instrument of simpler construction is shown in *Fig.* 8.

The high sensitivity of these instruments is obtained by reducing the damping to a very small amount, and accordingly the resonance is very sharp, as will be seen from the typical resonance curve shown in *Fig.* 9. Hence to employ the sensitivity to the full advantage the frequency should be kept very constant. If the frequency is inconstant it is often better to blunt the resonance curve by adding a shunt of appropriate value.

Vibration galvanometers of this type are very hardy and will bear very large overloads without damage. When using null methods, however, it is nearly always best to begin with the galvanometer heavily shunted, and gradually reduce the shunting to nothing as the condition of balance is approached.

In moving-coil resonance galvanometers, as well as in those of single bifilar loop, for a fixed frequency the deflection is proportional to the current and also to the voltage applied to the terminals.

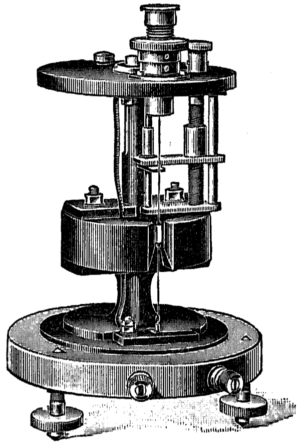

FIG. 8.—Campbell Vibration Galvanometer of Short Range.

In the moving-coil type the damping is usually almost entirely caused by the dynamical hysteresis due to the imperfect elasticity of the suspension wires or strips. Improvement in the elasticity of the material of which the suspension is made gives increase of sensitivity. Phosphor bronze is one of the best materials for the purpose, but tungsten has also been used with success. In the single bifilar loop type a considerable fraction of the damping is due to air friction.

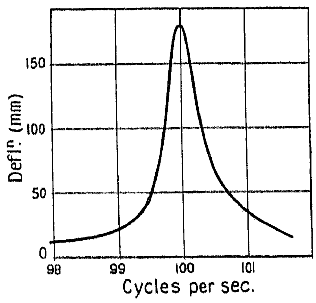

FIG. 9.—Typical Resonance Curve of Campbell Galvanometer.

The use of unifilar instead of bifilar suspensions gives great simplification in construction, but at the same time somewhat shortens the available range of frequency. The bifilar properties of single strips have been investigated by Buckley,[2] who showed that they admit of easy explanation.

[1] A. Campbell, *Phys. Soc. Proc.*, 1919, xxxi. 85.

[2] J. C. Buckley, *Phil. Mag.*, 1914, xxviii. 778; see also H. Pealing, *Phil. Mag.*, 1913, xxv. 418.

§ (11) M. Wien's Optical Telephone.—The working system of Wien's optical telephone is shown in *Fig.* 10, which represents the earlier

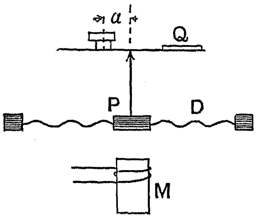

Fig. 10.—Wien's Original Optical Telephone.

pattern. A brass corrugated diaphragm, clamped at the edges, carries at its centre a small piece P of soft iron. The alternating current to be measured passes round the polar winding of the permanent magnet M and the variation of the magnetic field sets P vibrating. P carries a light rod which touches a thin flat spring carrying a light mirror Q, from which a light spot is projected on a scale. Large magnification of the motion of P is thus obtained; it is inversely proportional to a, the working length of the spring. In the later pattern (*Fig.* 11) two horse-shoe magnets are

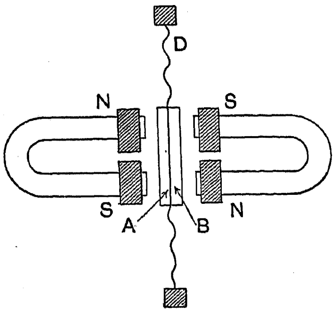

Fig. 11.—Wien's Optical Telephone.

used, being mounted on opposite sides of the diaphragm, which now carries two separate iron armatures A and B with a thin piece of wood between them. The current passes through the polar windings connected either in series or parallel. As the range of tuning adjustment is small, separate diaphragms have to be provided to suit standard frequencies. The final adjustment of the tuning is made by altering the distances of the two magnets from the iron pieces A, B. The vibrations of the iron pieces are strongly damped by magnetic hysteresis and eddy currents due to their motion in the strong magnetic field. Changing the magnetic field thus alters the damping, and hence slightly alters the frequency, which is lowered as the damping is increased. If n is the natural frequency without damping, and n' that with damping, then

$$n' = n/\sqrt{1+(\lambda/\pi)^2}, \quad . \quad . \quad (2)$$

where λ is the logarithmic decrement for $\frac{1}{2}$ period. This adjustment affords a variation of 1 to 2 per cent in the frequency.

The current sensitivity appears to have been of the order of 3 mm. at 1 metre per microampere with a direct-current resistance of 400 ohms (and hence a much higher impedance). The deflection was found to be proportional to the current.

§ (12) Rubens Vibration Galvanometer.—The moving system in the Rubens galvanometer consists of a set of little magnet needles (8 mm. long and 0·35 mm. thick) soldered to a brass strip carried by a vertical brass wire (0·2 mm. in diameter and 10 cm. in length). The ends of the wire are fixed, but the working length can be varied by altering the positions of two clamps, which gives the rough adjustment of the frequency. The magnet system stands in the magnetic field produced by two magnets with polar windings, whose pole pieces face one another as in the Wien optical telephone, and the fine adjustment of the frequency is done by varying the positions of these magnets. The current to be measured is passed through the polar windings, which are so connected that it produces an angular deflection of the magnet system, which is observed in the usual manner by the help of a mirror, light spot, and scale.

The deflection is proportional to the applied terminal voltage, as was experimentally proved by Wells,[1] who also gives a curve showing how the deflection depends on the working length of the suspension wire. According to Wenner,[2] the current sensitivity is 1·5 mm. at 1 metre per microampere, with an effective impedance of 1070 ohms at 100 ∿ per sec.

§ (13).—In Schering and Schmidt's modification of the Rubens galvanometer a single needle of thin sheet iron is used. Two sizes of needle are employed according to the frequency range required:

(*a*) 6 mm. × 4 mm. × 0·18 mm. for 8 to 60 ∿ per sec.; and

(*b*) 4 mm. × 4 mm. × 0·07 mm. for 30 to 140 ∿ per sec.

The suspension is of phosphor bronze wire 4 cm. long, of diameter 0·02 cm.

As shown in *Fig.* 12, the pole pieces c, c, c, c are built up of two sets of silicon iron stampings wound with 4 coils of 800 turns and 23 ohms each. The two pairs are mounted rigidly together and are slid, at the surfaces A and B, between the poles of a direct-current electro-

[1] R. P. Wells, *Phys. Rev.*, 1906, xxiii. 504.
[2] F. Wenner, *Bureau of Standards Bull.*, 1910, vi. 365.

magnet. The needle has the position shown in the figure. The control is due chiefly to the electromagnet, and the tuning is varied by

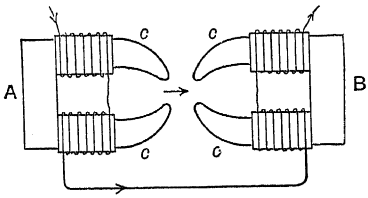

FIG. 12.—Plan of Pole Pieces (Schering and Schmidt).

altering the direct current, which can be done at a distance from the instrument. The sensitivity used is of the order of 10 mm. at 1 m. per μA. The natural damping is small, but artificial damping is sometimes introduced by means of a small copper block at an adjustable distance from the needle.

§ (14) DISADVANTAGE OF MOVING-IRON TYPE. — Direct-current galvanometers with moving-magnet needles are easily affected by stray magnetic fields, while instruments of moving-coil type are almost immune to such disturbances. The same rule applies to the two types of vibration galvanometer. When using one of moving-iron type the greatest care must be taken to keep the galvanometer at a distance from all coils or other circuits carrying alternating current of the resonating frequency. A deflection which persists when the galvanometer circuit is open is often an indication of disturbance due to this cause.

With all kinds of vibration galvanometers external mechanical vibration at the resonance frequency must be avoided.

§ (15) WIEN'S VIBRATION GALVANOMETER. —In Wien's galvanometer a brass wire of moderate thickness (0·1 mm. for 250 ∿ per sec.) is mounted with tuning clamps as in the Rubens type, and carries a mirror of about 3 mm. diameter and a very light system of magnets at most 3 mm. long. These lie parallel to and between the pole faces of a ring electromagnet with a core of well annealed iron wires, round the winding of which the alternating current to be measured passes. No auxiliary polarising current is used. The current sensitivity is inversely as the square of the frequency. At 100 ∿ per sec. it is given as about 70 scale divisions (mm. ?) per microampere. The fine adjustment of the tuning is done by altering the distance between the magnet poles and the needle system.

§ (16) DRYSDALE GALVANOMETER. — Drysdale's vibration galvanometer also belongs to the moving-iron class. The suspension piece carries a light mirror and a piece of thin soft iron, and is suspended between the pole pieces of a permanent magnet as seen in *Fig.* 13. The alternating current to be measured passes through a coil immediately behind the suspension. The tuning is done by means of a magnetic shunt which can be moved along the limbs of the magnet by a long screw underneath the base. The range can be further extended by altering the distance apart of the pole pieces. The total frequency range is from about 20 up to 200 ∿ per sec., and the zero point remains constant during tuning. As the current coils are easily inter-

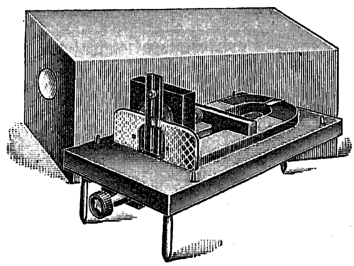

FIG. 13.—Drysdale Vibration Galvanometer (Tinsley).

changeable, any desired number of turns can be used. Table 2, given by Tinsley, shows the values of the sensitivities that can be obtained with coils of various resistances.

TABLE 2

Resistance. Ohms.	Impedance at 50 ~ per sec. Ohms.	Self-inductance. Milli-henries.	Sensitivity, mm. at 1 m., at 50 ~ per sec.	
			Per μV.	Per μA.
0·005	0·005	0·003	16·0	0·08
1·2	1·28	0·98	0·8	1·0
12	12·2	6·9	0·25	3·0
40	41·0	17·0	0·15	6·0
250	285	270	0·06	18·0
1 000	1 050	720	0·023	40
5 800	6 000	2 800	0·012	70
17 000	17 500	5 300	0·005	100

§ (17) AGNEW GALVANOMETER.—The vibration galvanometer of Agnew[1] is another example of the moving-iron type. The vibrating element is a fine steel wire mounted with one end fixed on the pole of a permanent magnet, while the other end is free to vibrate between the poles of an electromagnet excited by the current to be measured. The general arrangement is shown in *Fig.* 14, and *Fig.* 15 gives a plan of the pole pieces and vibrator. As will be seen from *Fig.* 15, the free end of the vibrator lies midway between the tips of the pole pieces of the electromagnet. The core of this magnet is of silicon steel, which

[1] P. G. Agnew, *Bureau of Standards Bull.*, 1920, xvi. 37.

has high initial permeability. The vibrator is usually shielded with a glass tube. Vibrators are made to suit various standard frequencies

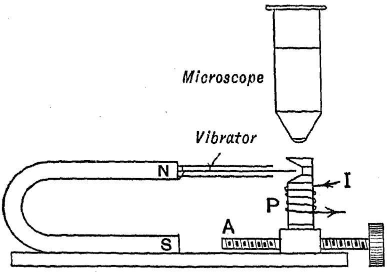

FIG. 14.—Agnew Vibration Galvanometer (working parts).

by using different thicknesses of wire, the final adjustment being made by moving an iron rod A by screw motion towards or away

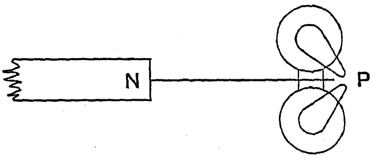

FIG. 15.—Agnew Galvanometer: Pole Pieces and Vibrator in plan.

from the lower pole of the permanent magnet. The vibrator is viewed by a microscope of magnifying power 50 to 100. Owing to the very small mass of the vibrator and the relatively large damping the instrument has very quick responsiveness and is particularly free from disturbance by external vibration. Its sensitivity is not so great as that of some of the other galvanometers already described. With an impedance of 387 ohms in the winding 0·05 microampere can be detected (at 60 ∿ per sec.). For vibrators of mild steel wire the following formula gives the frequency n with fair accuracy:

$$n \doteqdot 65\,000\ (\text{diameter})/(\text{length})^2. \quad . \quad (3)$$

For example, a length of 33 mm. of a 0·1 mm. wire gives about 60 ∿ per sec., and the same length of a 0·04 mm. wire about 25 ∿ per sec.

§ (18) BLONDEL AND CARBENAY'S MOVING-IRON RESONANCE GALVANOMETER. — The moving-iron resonance galvanometer introduced by Blondel and Carbenay for use in the harmonic analysis of alternating-current wave forms has a minute piece of sheet iron (0·8 mm. × 1 mm. × 0·3 mm.) supported by a suspension of silk fibre in a magnetic field produced by a large double solenoid carrying a direct current, as shown in *Fig.* 15A. The tuning is done by the adjustment of this current, and a range of frequency up to 1500 ∿ per sec. can be obtained. A small deflecting coil, with its axis at right angles to that of the solenoid, carries the alternating current to

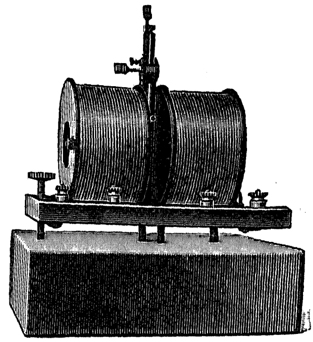

FIG. 15A.—Blondel and Carbenay's Vibration Galvanometer with Soft-iron Needle.

be measured. It was found that the damping is not constant for different frequencies, but increases as the frequency is raised, and hence the current sensitivity is not inversely proportional to the frequency. Also the sensitivity varies somewhat with the amount of the deflection (amplitude of vibration). These effects are due to the magnetic hysteresis in the soft-iron needle.

§ (19) BARUS VIBRATION AMMETER.—Barus[1] has constructed a vibration galvanometer in which the objective of a telescope is mounted on bifilar suspension close to the diaphragm of a telephone. The broadening of an image in the telescope gives a measure of the current in the telephone.

III. MATHEMATICAL THEORY OF VIBRATION GALVANOMETERS

§ (20) INTRODUCTORY AND HISTORICAL.—The main part of the mathematical theory of resonance instruments was known many years ago. For example, Helmholtz[2] gave the fundamental equation and its complete solution [equations (4), (5), (6), (7), (8), and (9) below], mentioning that the damping coefficient included the electrical as well as the dynamical damping. He also points out the condition for current resonance. He applies the theory to a tuning-fork electrically maintained by a periodic current and in resonance with its frequency. A fork of this kind is really a resonance galvanometer, and when carrying a microscope, as in Helmholtz's system, could be used as such. The case of the free oscillations dying down when the

[1] C. Barus, *Nat. Acad. Sci. Proc.*, 1919, v. 211. See also "Vibration Microscope" in Helmholtz's *Sensations of Tone*, § 4, chap. 5.

[2] H. L. F. Helmholtz, *Sensations of Tone*, Appendix 9 (1862).

galvanometer circuit is opened is fully discussed by Maxwell,[1] Gray,[2] and others.

Blondel [3] investigated the practical conditions for short-period galvanometers, particularly for use as oscillographs.

In 1909, when vibration galvanometers were beginning to come into more extensive use, Wenner [4] published a very complete mathematical investigation of their behaviour, not only for a given current, but also when connected in a circuit with a given applied voltage, in which case the electrical damping is explicitly introduced. He also showed how the dynamical and electrical constants of a given instrument can be deduced from experimental data, and he gave the results thus obtained with three different types of galvanometer. The first part of the mathematical theory given below is based upon his work, which deals with galvanometers whose moving systems have only one degree of freedom.

In 1912 Butterworth [5] published a simple solution, by symbolical methods, of the case where the galvanometer is tuned to resonance when connected in a circuit with given applied voltage, and he showed how the maximum voltage sensitivity can be obtained by altering the self inductance of the circuit or one of the constants of the galvanometer. He also extended the theory to include the case of single- or double-string type of galvanometer, in which the moving system has an infinite number of degrees of freedom.

In a later paper [6] he shows how a vibration galvanometer can be replaced by an equivalent branched circuit containing inductance and capacity, and on this equivalence he bases a null method of testing vibration galvanometers.

Zöllich [7] in a paper published in 1915 discusses the problem (for one degree of freedom) by the vectorial method, and points out the interesting analogy between the dynamical and the electrical constants of a vibration galvanometer, illustrating the theory by a number of experimental curves.

In 1912 Kennelly and Pierce [8] applied their interesting motional impedance theory both to telephones and vibration galvanometers.

In 1917 Blondel and Carbenay [9] gave the mathematical theory of their moving iron instrument, and finding that the damping in it is not merely proportional to the angular velocity, they added an investigation of the oscillation of a system in which the damping is proportional to the square of the angular velocity.[10]

In 1918 Blondel [11] published a paper in which he discusses the theory of moving-coil galvanometers with special reference to their use for analysis of alternating-current wave forms. He deals with the general case and uses the symbolical method.

§ (21) General Theory of Vibration Galvanometer with One Degree of Freedom. (i.) *Constants and Nomenclature.*—When an alternating current is sent through a vibration galvanometer it causes oscillatory motion, and this motion can be determined mathematically from a knowledge of the value of the current, its frequency, and a number of constants which depend entirely on the construction of the instrument. These constants are called the *intrinsic* constants of the galvanometer (Wenner) and they are geometrical, dynamical, and electrical (or magnetic).

For a moving-coil system the geometrical constants are N the number of turns, and s the mean area of a turn. The magnetic field B in the magnet air gap is taken as uniform. Thus the flux-turns included by the coil when it is at an angle θ to the direction of the field are BNS $\cos\theta$.

When the moving part is displaced through an angle θ by a current of instantaneous value i, then if the torque due to i is Gi, G is called the *displacement moment.*

If $c\theta$ is the torque due to the control forces (of the suspension, etc.) which tend to bring the moving part back to its zero position, then c is called the *control moment.*

If the moving system is allowed to oscillate freely on open circuit, the damping forces gradually bring it to rest. In general the damping torque may be taken as proportional to the angular velocity $d\theta/dt$. If the damping torque $= b\dot{\theta}$, b may be called the *damping moment.*

K, the moment of inertia of the moving system, is the only other intrinsic constant required to determine the motion when the current is given. When the applied voltage, however, is in question, the resistance r of the galvanometer must also be known.

For clearness of reference most of the symbols used are tabulated as below. They are all expressed in absolute C.G.S. units.

t, time.
θ, angular displacement.
K, moment of inertia.
b, damping moment.
c, control moment.
G, displacement moment.

[1] J. Clerk Maxwell, *Electricity and Magnetism*, chap. 16.
[2] A. Gray, *Absolute Measurements*, ii. part 2, 392.
[3] A. Blondel, *Éclairage él.*, 1902, xxxiii.
[4] F. Wenner, *Bureau of Standards Bull.*, 1909, vi. 347.
[5] S. Butterworth, *Phys. Soc. Proc.*, 1912, xxiv. 75.
[6] *Ibid.*, 1914, xxvi. 264.
[7] H. Zöllich, *Archiv f. Elektrotech.*, 1915, iii. 369.
[8] A. E. Kennelly and G. W. Pierce, *Am. Acad.*, 1912, xlviii. 113.
[9] A. Blondel and F. Carbenay, *Ann. de Physique*, 1917, viii. 97.
[10] See also R. Grammel, *Phys. Zeits.*, Jan. 1, 1913, p. 20.
[11] A. Blondel, *Ann. de Physique*, 1918, x. 195.

n, frequency.
ω, pulsatance ($=2\pi n$).
ϕ, amplitude of steady deflection.
i, instantaneous value of current.
e, instantaneous value of applied voltage.
I and E, corresponding effective values of current and voltage.
$I_{max.}$ and $E_{max.}$, maximum values of same.
$\mathbf{I}$, $\mathbf{E}$, and Θ, vectorial values of i, e, θ.
ψ, angle of lag of motion behind current.
η, angle of lag of current behind impressed voltage.
R, resistance of galvanometer circuit (or sometimes of galvanometer alone).
R′, effective resistance of same (in motion).
L, self inductance of circuit.
L′, effective self inductance.
z, impedance operator.
Z, impedance.
ϵ, base of Napierian logarithms.
j, $=\sqrt{-1}$.

§ (22) Case I. Constant Current through the Galvanometer.—The equation of motion is

$$K\ddot{\theta}+b\dot{\theta}+c\theta=Gi.$$

When i is a constant alternating current of sine wave form and pulsatance ω

$$i=I_{max.}\cos\omega t$$

and the equation becomes

$$K\frac{d^2\theta}{dt^2}+b\frac{d\theta}{dt}+c\theta=GI_{max.}\sin\omega t. \quad . \quad . \quad (4)$$

The auxiliary equation is $Kx^2+bx+c=0$.

When $4Kc>b^2$ the roots of this are imaginary, and the solution [1] of equation (4) is

$$\theta=\frac{I_{max.}G\cos(\omega t-\psi)}{\sqrt{(c-\omega^2K)^2+\omega^2b^2}}-A\epsilon^{-bt/2K}\sin\left[\frac{\sqrt{4Kc-b^2}}{2K}t+A'\right], \quad (5)$$

where A and A′ are constants of integration, and

$$\tan\psi=\omega b/(c-\omega^2K).$$

Equation (5) by its exponential term indicates that immediately after starting the current an unsteady state exists, but usually the term in $\epsilon^{-bt/2K}$ quickly becomes zero, and a steady state of vibration is reached, which persists as long as the current I is unchanged. *Fig.* 16 gives an example of

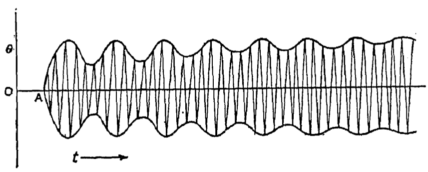

Fig. 16.—Unsteady State from Start of Current (at point A) in Galvanometer with Weak Magnetic Field.

how the oscillations settle down to a constant amplitude; here the current has been switched on at time OA, and the galvanometer has a weak field which exaggerates the unsteady state (Zöllich, *loc. cit.*). Helmholtz [2] observed the beats of this kind when an alternating current was suddenly started in the driving circuit of a tuning-fork.

When the steady state is reached we have

$$\theta=\frac{I_{max.}G\cos(\omega t-\psi)}{\sqrt{(c-\omega^2K)^2+\omega^2b^2}}, \quad . \quad . \quad . \quad (6)$$

where $$\tan\psi=\omega b/(c-\omega^2K). \quad . \quad . \quad . \quad (7)$$

The displacement θ, therefore, follows the simple harmonic law, and the motion lags in phase behind the current by the angle ψ.

If ϕ is the amplitude, 2ϕ being the whole angle of oscillation of the coil, then

$$\phi=\frac{I_{max.}G}{\sqrt{(c-\omega^2K)^2+\omega^2b^2}}=\frac{IG\sqrt{2}}{\sqrt{(c-\omega^2K)^2+\omega^2b^2}}, \quad (8)$$

where I is the effective value of the current.

§ (23) Two Forms of Resonance.—Now, for a given current I, we can obtain a maximum deflection either for alteration of one or more of the intrinsic constants (K, b, c, or G) of the galvanometer or for alteration of the frequency. As M. Wien [3] has pointed out, there are thus two kinds of resonance, one being found by tuning the galvanometer, the other by tuning the frequency of the current. For a moving-coil vibration galvanometer, variation of G gives $\phi_{max.}=\infty$ when $G=\infty$, and so if all the other constants remain unchanged, increasing the magnetic field in which the coil is suspended increases the current sensitivity.

Continuous alteration of K, the moment of inertia, is not as a rule practicable. Sometimes, however, to obtain a longer range the natural frequency of the coil is lowered by adding to it small inertia bars of various sizes. For change of K the maximum deflection occurs when $K=c/\omega^2$, and then

$$\phi_{max.}=IG\sqrt{2}/\omega b. \quad . \quad . \quad . \quad . \quad (9)$$

Intentional variation of the damping constant b is unusual. For variation of b, $\phi_{max.}$ is

$$IG\sqrt{2}/(c-\omega^2K),$$

when b is zero, which gives the limiting case.

§ (24) Control Tuning.—The most important case is when resonance (*i.e.* maximum ϕ) is obtained by altering the control constant c, and this is the adjustment that is nearly always made in practice, n_1, the frequency of the source, being kept constant.

Then, for resonance,

$$\omega_1^2=c/K, \quad . \quad . \quad . \quad . \quad (10)$$

$$\phi=IG\sqrt{2}/\omega_1 b, \quad . \quad . \quad . \quad . \quad (11)$$

and $$\psi=90°, \quad . \quad . \quad . \quad . \quad (12)$$

[1] Helmholtz, *loc. cit.*

[2] Helmholtz, *Sensations of Tone*, Appendix 8.

[3] M. Wien, *Wied. Ann.*, 1896, lviii. 125.

the motion being in phase 90° behind the current i.

§ (25) Frequency Resonance.—If resonance be obtained by altering the frequency of the source while c is kept constant, then if n_2 give maximum ϕ,

$$\omega_2^2 = \frac{c}{K} - \tfrac{1}{2}\left(\frac{b}{K}\right)^2 = \omega_1^2 - \tfrac{1}{2}\left(\frac{b}{K}\right)^2. \quad . \quad (13)$$

§ (26) Free Oscillation.—If the current be cut off, the oscillations of the coil will gradually settle down to zero, the equation of motion now being

$$K\ddot{\theta} + b\dot{\theta} + c\theta = 0. \quad . \quad . \quad . \quad (14)$$

When t is reckoned from the instant of passing through the undisturbed position ($\theta = 0$) the solution of this equation is

$$\theta = A_0\epsilon^{-bt/2K} \sin \omega_0 t, \quad . \quad . \quad . \quad (15)$$

where, corresponding to the free frequency n_0, we have

$$\omega_0^2 = \frac{c}{K} - \tfrac{1}{4}\left(\frac{b}{K}\right)^2 = \omega_1^2 - \tfrac{1}{4}\left(\frac{b}{K}\right)^2, \quad (16)$$

and A_0 is a constant determined by the initial conditions.

Fig. 17 shows how the oscillations die down

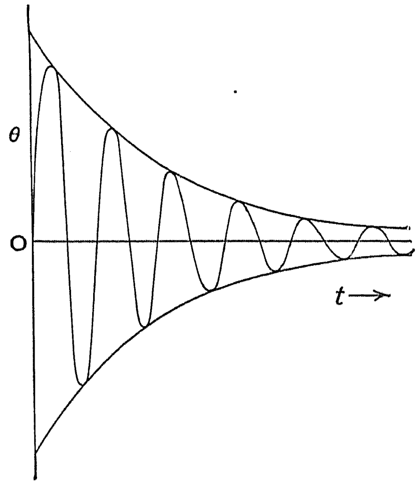

Fig. 17.—Free Oscillation with Damping proportional to Angular Velocity $\dot{\theta}$.

according to equation (15). All the end points of the swings lie on the curves given by

$$\theta = \pm A_0\epsilon^{-bt/2K}, \quad . \quad . \quad . \quad (17)$$

and the logarithmic decrement λ for a complete oscillation is given by the equation

$$\lambda = \pi b/K\omega_0 = b/2Kn_0. \quad . \quad . \quad . \quad (18)$$

If the amplitude ϕ_0 at time $t=0$ falls to ϕ_2 at time t_2, then

$$t_2 = \frac{2K}{b}\log_\epsilon\left(\frac{\phi_0}{\phi_2}\right). \quad . \quad . \quad . \quad (19)$$

When $\phi_0/\phi_2 = \epsilon$, then τ, which may be called the *amplitude time constant* for constant current, is given by

$$\tau = 2K/b. \quad . \quad . \quad . \quad . \quad (20)$$

We also have

$$\omega_0^2 = \omega_1^2 - \frac{1}{\tau^2}, \quad . \quad . \quad . \quad . \quad (21)$$

and

$$\omega_2^2 = \omega_1^2 - \frac{2}{\tau^2}. \quad . \quad . \quad . \quad . \quad (22)$$

As Peirce [1] has pointed out, K/b can be found directly from equation (15) by observing the time taken for the amplitude to fall to a definite fraction (say ½) of its initial value.

In practice the damping is nearly always very small, and hence ω_0, ω_1, and ω_2 are all very nearly equal to one another.

§ (27) Motional Impedance.—If an alternating current is sent through a vibration galvanometer, telephone or other apparatus in which it produces motion, a portion of the power expended in the circuit by the current will be spent in maintaining the motion against the damping forces, and the effective resistance of the apparatus will be greater when the motion is allowed than when it is prevented. Thus if a vibration galvanometer has its effective resistance and self-inductance measured (by a bridge method or otherwise), the values found will be different according as the moving part is free to move or held motionless. If R′, L′ and R, L are the "free" and "held" values respectively, then Kennelly and Pierce [2] call R′ − R the *motional* resistance, (L′ω − Lω) or X′ − X the motional reactance, and (R′ − R) − j(X′ − X) the motional impedance (vectorial).

§ (28) Theory of an Applied Alternating Voltage.—The case of a given applied alternating voltage E in a closed circuit including a vibration galvanometer, or else directly across its terminals, is most simply treated by the symbolical method.

The following very brief discussion of the method may help to illustrate the principle which underlies it.

Let $\mathbf{I} = a + jb$ represent a vector of length $\sqrt{a^2+b^2}$ inclined at an angle $\tan^{-1}(b/a)$ to the horizontal.

Let
$$a = P\cos\omega t,$$
$$b = P\sin\omega t,$$

where t represents time.

Then $$P = \sqrt{a^2+b^2} \text{ and } \tan\omega t = b/a. \quad . \quad . \quad (23)$$

Also $$\mathbf{I} = P(\cos\omega t + j\sin\omega t) \quad . \quad . \quad . \quad . \quad (24)$$

$$= P\epsilon^{j\omega t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

[1] B. O. Peirce, *Am. Acad. Proc.*, 1908, xliv. 63.
[2] A. E. Kennelly and G. W. Pierce, *Am. Acad. Sci.*, 1912, xlviii. 113.

which is now a rotating vector of length P and angular velocity ω.

Its projection on the axis of x is P cos ωt. Now let $R+jX$ operate on the vector $\mathbf{I}$.

Then $$(R+jX)\mathbf{I}=\sqrt{R^2+X^2}Pe^{j\omega t+\alpha}, \quad (26)$$

where $\tan \alpha = X/R$.

The operator has thus altered the length of the vector and advanced its phase by the angle α. The projection of the new vector on the axis of x is

$$P\sqrt{R^2+X^2}\cos(\omega t+\alpha).$$

It will be observed that this is the terminal voltage produced by a current P cos ωt flowing in a circuit of resistance R and reactance X.

Accordingly the sine wave current and terminal voltage may be represented by the vectors $\mathbf{I}$ and $\mathbf{V}$ and the impedance operator z by $R+jX$, where the reactance $X=\omega L-1/\omega C$, L being the self inductance and C the series capacitance of the circuit.

Then we have $$\mathbf{V}=z\mathbf{I}, \quad (27)$$

z having the effect of multiplying the amplitude by $\sqrt{R^2+X^2}$ and advancing the phase by $\tan^{-1}(X/R)$.

Also $$\frac{1}{z}=\frac{1}{R+jX}=\frac{R-jX}{R^2+X^2}. \quad (28)$$

§ (29) Vibration Galvanometer in Circuit with Given Applied Voltage; Impedance Equation.—Let the galvanometer be in a circuit of total resistance R and self-inductance L, in which there is an applied sinoidal voltage $e=E_{max.}\cos \omega t$.

If e_b is the back voltage produced by the motion in the galvanometer, then

$$e_b=Gd\theta/dt. \quad (29)$$

Thus we have the two equations

$$K\ddot{\theta}+b\dot{\theta}+c\theta=Gi$$

and $$e=Ri+L\frac{di}{dt}+G\dot{\theta}.$$

When the steady state has been reached, θ, e, and i are all sinoidal, and we may write

$$\{c-\omega^2K+j\omega b\}\Theta=G\mathbf{I}. \quad (30)$$

and $$\mathbf{E}=(R+j\omega L)\mathbf{I}+j\omega G\Theta, \quad (31)$$

where Θ, $\mathbf{I}$, and $\mathbf{E}$ are vectors.

Eliminating Θ we have

$$\mathbf{E}=\left[R+j\omega L+\frac{j\omega G^2}{c-\omega^2K+j\omega b}\right]\mathbf{I}[1]. \quad (32)$$

$$=\left[R+j\omega L+\frac{j\omega(c-\omega^2K)G^2-\omega^2bG^2}{(c-\omega^2K)^2+\omega^2b^2}\right]\mathbf{I}$$

$$=\left[R+\frac{\omega^2bG^2}{(c-\omega^2K)^2+\omega^2b^2}+j\omega\left\{L-\frac{(c-\omega^2K)G^2}{(c-\omega^2K)^2+\omega^2b^2}\right\}\right]\mathbf{I}. \quad (33)$$

If $$\mathbf{E}=(R'+j\omega L')\mathbf{I}, \quad (33\text{A})$$

then R′ and L′ are respectively the *effective resistance and effective self inductance* of the galvanometer circuit for pulsatance ω.

Thus $$I=E\sqrt{R'^2+\omega^2L'^2} \quad (34)$$

[1] S. Butterworth, *loc. cit.*

and $\tan \eta=\omega L'/R'$ where η is the angle of lag of the current I behind the applied voltage E.

In the above investigation if series capacitance C is present as well as self inductance, $(\omega L-1/C\omega)$ must be written for ωL throughout.

§ (30) Motional Impedance Equations.—Now let the circuit consist of the galvanometer alone, with the applied voltage directly across its terminals. Then the motional resistance is $(R'-R)$ and we have

$$R'-R=\frac{\omega^2bG^2}{(c-\omega^2K)^2+\omega^2b^2}. \quad (35)$$

The motional reactance $\omega(L'-L)$ is given by

$$\omega(L'-L)=-\frac{\omega(c-\omega^2K)G^2}{\omega^2b+(c-\omega^2K)^2}. \quad (36)$$

If the galvanometer control has been tuned to resonance for current (as in § (24)),

we have $$\omega_1{}^2K=c,$$

and hence $$R'=R+G^2/b$$

or $$R'-R=G^2/b, \quad (37)$$

and $$L'=L. \quad (38)$$

Thus for resonance pulsatance ω_1 we have

Motional resistance $=G^2/b$

and Motional reactance $=0$.

If η = angle of lag of I behind E,

$$\tan \eta=\omega L/(R+G^2b). \quad (39)$$

§ (31) Experimental Determination of Galvanometer Constants.—By testing the sensitivity of the galvanometer for direct current, for alternating current, and for alternating voltage, and measuring the direct-current resistance, and the resonance frequency, all the constants in equation (4) can now be determined. Let the following symbols be used:

- h, direct-current sensitivity, mm. at 1 m. per microampere.
- σ, alternating-current sensitivity, mm. at 1 m. per microampere.
- q, alternating voltage sensitivity, mm. at 1 m. per microvolt.
- R, direct-current resistance of coil.
- R′, effective resistance of coil.

For h the ordinary single deflection is here taken, for σ and q the total widening of the light spot. The power sensitivity is taken as the square of the deflection (mm. at 1 m.) divided by the power in micromicrowatts (Wenner).

The source of current is maintained at constant frequency (*e.g.* 100 ∼ per sec.), and by altering the length and tension of the suspension the galvanometer is tuned to resonance. For this tuning the current is kept constant, which is effected, as shown

in *Fig.* 18, by putting the galvanometer in series with a high resistance r across a shunt of low resistance s carrying a constant alternating current. (The added resistance r should be so high that the effective resistance

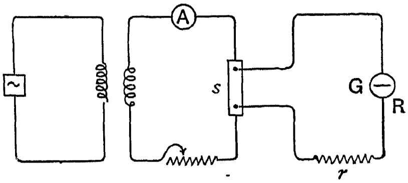

FIG. 18.—Test of Current Sensitivity.

of the galvanometer can be neglected in comparison.) By measuring the current through the shunt by an ammeter (A) the current through the galvanometer is known, and, by observing the corresponding deflection, σ is found. Next the resistance r is cut out, and a much smaller known current sent through s. From the observed deflection and the voltage at the terminals of s the alternating voltage sensitivity q is found. Then the direct-current sensitivity is tested and the resistance R measured. Also the amplitude time constant τ may be found by breaking the current and making observations upon the dying down deflection.

The power sensitivity $=\sigma q$ (when $L=0$).

§ (32) DEDUCTION OF INTRINSIC CONSTANTS.—The constants K, b, c and g may now be obtained from the above observations by the help of equations (10), (11), (34), (37), and (38), which are

$$\omega_1^2=c/K,$$

$$\phi=IG\sqrt{2}/\omega_1 b,$$

$$E=I\sqrt{R'^2+\omega^2L'^2},$$

$$R'-R=G^2/b,$$

$$L'=L.$$

Usually L is very small, so we may here consider only the case in which it is negligible. Hence $R'=E/I$ in absohms $=\sigma/q$ in ohms. It must be remembered that the quantities in these equations are all expressed in absolute C.G.S. units, and now have to be reduced to practical units as used in the actual measurements. The following Table 3 gives the necessary relations:

TABLE 3

1 ampere $=10^{-1}$ absamperes.
1 volt $=10^8$ abvolts.
1 ohm $=10^9$ absohms.
1 microfarad $=10^{-15}$ abfarads.
1 henry $=10^9$ abhenries
1 joule or 1 watt-sec. $=10^7$ ergs.

When the moving system has a vibration amplitude of ϕ radians, the total scale deflection (in mm. at 1 m.) will be 4000ϕ.

The following relations can now be deduced, giving all the intrinsic constants of the galvanometer.

$$R'=\sigma/q, \quad . \quad . \quad . \quad (40)$$

$$G\doteq\sigma/90000(R'-R)\sigma n_1, \quad . \quad . \quad (41)$$

$$c=2G/10000h, \quad . \quad . \quad . \quad (42)$$

$$K=c/\omega_1^2, \quad . \quad . \quad . \quad (43)$$

$$b=\frac{2\sqrt{2}ch}{\omega_1\sigma}\doteq\frac{2\cdot83c}{\omega_1\sigma/h}. \quad . \quad . \quad (44)$$

Also

$$b=2K/\tau. \quad . \quad . \quad . \quad (45)$$

Equation (45) gives a rough check on the value of b found from equation (44). The *resonance magnification* σ/h is given by

$$\sigma/h=\omega_1\tau\sqrt{2}. \quad . \quad . \quad . \quad (46)$$

If the moving coil has N turns of mean area s square cm. and the flux density in the air gap of the magnet is B, then

$$G=BsN. \quad . \quad . \quad . \quad (47)$$

The mean area s, however, cannot be directly measured with accuracy.

§ (33) SELECTIVENESS OF MOVING-COIL GALVANOMETER.—As already pointed out in § (3), one of the great advantages of resonance instruments lies in their selective properties. They are much more sensitive for the frequency to which they are tuned than for any other, and so when harmonics are present in the current wave form their effect is small, and in fact is not usually perceptible. The selective power varies in different instruments. For any particular galvanometer it can be estimated from the intrinsic constants found as above described (see Wenner, *loc. cit.*).

D. W. Dye has shown, however, that the current selectivity is almost entirely determined by the absolute value of the amplitude time constant J. Let n_1 be the resonance frequency and γn_1 any other frequency, then the ratio of the current sensitivities at n_1 and γn_1

$$=\sqrt{(1-\gamma^2)^2\omega_1^2(K/b)^2+\gamma^2}. \quad . \quad (48)$$

Usually the second term (γ^2) can be neglected, and hence current sensitivity ratio

$$\doteq\pm(1-\gamma^2)\omega_1 K/b$$

$$\doteq\pm(1-\gamma^2)\omega_1\tau/2. \quad . \quad . \quad (49)$$

§ (34) RESONANCE RANGE.—The resonance range has been defined by Wenner as the proportional change that must be made in the frequency from its resonance value in order to reduce the deflection by one-half. It has quite different values according as constant current or constant applied voltage is maintained.

Wenner shows that in the first case it is approximately

$$\sqrt{3}b/2K\omega_1 \quad . \quad . \quad . \quad (50)$$

and in the second case approximately

$$\frac{\sqrt{3b^2R^2+8bRG^2+4G^4}}{2KR\omega_1}. \quad . \quad (51)$$

§ (35) Constants of Various Types of Vibration Galvanometer.—In Table 4 are given values of the various constants (obtained by several different observers [1]) for vibration galvanometers of three different types, namely, Rubens (moving iron), Campbell (moving coil), and Duddell (bifilar loop). For the Duddell instrument the values given for K, b, c, and G are only the equivalent constants for a moving coil, as the theory does not hold accurately for this type.

[1] Wenner, Campbell, Duddell (*loc. cit.*).

Table 4

Constants of Actual Vibration Galvanometers

Type.	Rubens Moving Iron.	Campbell Moving Coil.			Duddell Bifilar.
	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
N, turns	..	2·5	10·5	40	1
s, mean area of turn, sq. cm.	..	0·07	0·10	0·07	..
B, magnetic field	..	2500	2500	2700	..
Mirror area, sq. mm.	..	5·2	16	7·2	..
n_1, resonance frequency	100	100	100	100	100
h, dir. current sensitivity	0·018	0·043	0·080	0·165	0·70
σ, alt. current sensitivity	1·5	21·7	50	160	56
q, alt. voltage sensitivity	0·014	1·02	0·285	0·104	0·114
Power sensitivity	0·0021	22·0	14·0	17·0	6·4
R, dir. current resistance	234	6·0	7·0	14·5	135
R′, effective resistance	(1 070)	20·1	175	1540	489
σ/h, resonance magnification	83	505	620	970	80
10^6K, moment of inertia	12 000	6·9	19·2	26	2·3
10^6b, damping moment	120 000	23·2	55	49	50
c, control moment	4 500	2·72	7·6	10·4	0·91
G, displacement moment	200 000	585	3030	8600	3200

It will be noticed that in the sensitive instruments the effective resistance R′ is very considerably greater than R. If R is neglected in comparison with R′, the voltage sensitivity is approximately proportional to $1/Ns$, when the magnetic field and the frequency are fixed; so for high-voltage sensitivity the area × turns of the moving coil should be made very small.

§ (36) Vibration Galvanometer in Circuit with Given Applied Voltage; Equation of Motion.—In § (29) the impedance equation was discussed for the case of a vibration galvanometer in an inductive circuit; by eliminating Θ the general relation connecting E and I was found.

If **I** now be eliminated [2] instead of Θ, the result will give the equation of motion for a given applied voltage.

Let the circuit have self inductance L and series capacitance C, and let

$$b+j(\omega K-c/\omega)=b+jY=\mathbf{Y} \quad . \quad . \quad (52)$$

and

$$R+j(\omega L-1/C\omega)=R+jX=\mathbf{Z}. \quad . \quad . \quad (53)$$

If $\mathbf{E}_b$ be the vectorial back voltage and $\mathbf{E}$ the applied voltage, then

$$\mathbf{I}=\frac{\mathbf{E}-\mathbf{E}_b}{\mathbf{Z}}=\frac{\mathbf{E}-G\dot{\theta}}{\mathbf{Z}} \quad . \quad . \quad . \quad . \quad (54)$$

Hence

$$K\ddot{\theta}+b\dot{\theta}+c\theta=G\mathbf{I}=G\frac{\mathbf{E}-G\dot{\theta}}{\mathbf{Z}}$$

or

$$K\ddot{\theta}+\left(b+\frac{G^2}{\mathbf{Z}}\right)\dot{\theta}+c\theta=\frac{G\mathbf{E}}{\mathbf{Z}}. \quad . \quad . \quad (55)$$

Thus

$$[-K\omega^2+c+j\omega(b+G^2/\mathbf{Z})]\Theta=G\frac{\mathbf{E}}{\mathbf{Z}}$$

or

$$j\omega[b+jY+G^2/\mathbf{Z}]\Theta=G\mathbf{E}/\mathbf{Z}.$$

Hence

$$\Theta=\frac{G\mathbf{E}}{j\omega[\mathbf{YZ}+G^2]} \quad . \quad . \quad . \quad (56)$$

or

$$\Theta=\frac{G\mathbf{E}}{j\omega[bR-XY+G^2+j(RY+BX)]}. \quad . \quad (57)$$

Zöllich points out the close correspondence between the dynamical and the electrical constants of the system; his comparison is given in Table 5.

[2] Butterworth, Zöllich, *loc. cit.*

TABLE 5

Dynamical.		Electrical.
Moment of inertia	K	L Self-inductance.
Damping moment	b	R Resistance.
Control moment	c	$1/C$; C Capacitance.
Deflection	θ	Q Electrical quantity.
	$b/2K$	$R/2L$.
Resonance, $\sqrt{c/K}=\omega_1$		$\omega'=1/\sqrt{CL}$, Resonance.
Dynamical reactance, Y $=\omega K-c/\omega$		X Reactance $=\omega L-1/\omega C$.
Dynamical impedance, $\mathbf{Y}$ $=b+jY$		$\mathbf{Z}$ Impedance (vector) $=R+jX$.

Now from equation (57) we have

$$\phi=\text{ampl.}[\Theta]=\frac{GE_{max.}}{\omega[(RY+bX)^2+(bR-XY+G^2)^2]^{\frac{1}{2}}} \quad (58)$$

$$\text{and} \qquad \tan\chi=\frac{bR-XY+G^2}{-RY-bX}, \quad . \quad . \quad . \quad (59)$$

where χ is the angle of lag of θ behind E.

For greater simplicity let the capacitance $C=0$, and then $X=\omega L$, and

$$\phi=\frac{GE_{max.}}{\{[(c-\omega^2K)R-\omega^2bL]^2+\omega^2[(c-\omega^2K)L+bR+G^2]^2\}^{\frac{1}{2}}}. \quad (60)$$

Case 1.—If by altering K or c we obtain resonance and the amplitude ϕ is a maximum with E constant,

$$\text{then} \qquad \omega^2K-c=\frac{LG\omega^2}{R^2+\omega^2L^2} \quad . \quad . \quad . \quad (61)$$

$$\text{and} \qquad \Theta=\frac{G\mathbf{E}}{\left(b^2+G^2\dfrac{R}{R^2+\omega^2L^2}\right)(-\omega^2L+j\omega R)} \quad (62)$$

$$\text{or} \qquad \theta=\frac{GE_{max.}\cot(\omega t-\chi')}{\omega(b\sqrt{R^2+\omega^2L^2}+RG^2/\sqrt{R^2+\omega^2L^2})}, \quad (63)$$

$$\text{where} \qquad \tan\chi'=R/L\omega \quad . \quad . \quad . \quad . \quad (64)$$

The maximum amplitude now is

$$\phi=\frac{GE_{max.}}{\omega(b\sqrt{R^2+\omega^2L^2}+RG^2/\sqrt{R^2+\omega^2L^2})}. \quad (65)$$

In the case where $L=0$,

$$\phi=\frac{GE_{max.}}{\omega(bR+G^2)} \quad . \quad . \quad . \quad . \quad (66)$$

$$\text{and} \qquad \chi'=\pi/2.$$

Case 2.—If K or c had been adjusted to give resonance for I constant, then $c-\omega^2K=0$, and equations (58) and (59) give

$$\theta=\frac{GE_{max.}\cos(\omega t+\psi')}{\omega\{b^2(R^2+\omega^2L^2)+2bRG^2+G^4\}^{\frac{1}{2}}}, \quad . \quad (67)$$

$$\text{where} \qquad \tan\psi'=\frac{bR+G^2}{\omega bL} \quad . \quad . \quad . \quad . \quad (68)$$

When $L=o$,

$$\theta=-\frac{GE_{max.}\sin\omega t}{\omega(bR+G^2)} \quad . \quad . \quad . \quad (69)$$

$$\text{and} \qquad \phi=\frac{GE_{max.}}{\omega(bR+G^2)} \quad . \quad . \quad . \quad (70)$$

$$\text{and} \qquad \psi'=\pi/2.$$

The identity of equations (66) and (70) shows that when the circuit is non-inductive resonance occurs when $c=\omega^2K$ both when the applied voltage is given or when the current is given.

§ (37) OPTIMUM VALUE OF AMPLITUDE ϕ.[1]—Returning to equation (61), when L is not 0, a further increase in ϕ may still be got by varying L or the displacement constant G. The latter variation can be made by altering the magnetic field in the magnet gap, which may be done by applying a variable magnetic shunt, or by using an electromagnet and varying the magnetising current. By either of these alterations the deflection ϕ can be brought to an absolute maximum value.

$$\text{Thus} \qquad \phi_{max.}=E_{max.}/2\omega\sqrt{bR}, \quad . \quad . \quad . \quad (71)$$

$$\text{when} \qquad G^2/b=(R^2+\omega^2L^2)/R. \quad . \quad . \quad (72)$$

When this condition holds

$$\Theta=\frac{G\mathbf{E}}{2b(-L\omega^2+j\omega R)} \quad . \quad . \quad . \quad (73)$$

$$\text{or} \qquad \theta=\frac{GE_{max.}\cos(\omega t-\chi'')}{2\omega\sqrt{bR}}, \quad . \quad . \quad (74)$$

$$\text{where} \qquad \tan\chi''=R/L\omega.$$

Now also

$$\Theta=\frac{G\mathbf{I}}{j\omega\mathbf{Y}}=\frac{G\mathbf{I}}{j\omega[b+j\omega(K-c/\omega^2)]}. \quad . \quad (75)$$

Hence with constant-voltage resonance by equation (61) we have

$$\Theta=\frac{G\mathbf{I}}{j\omega[b+G^2j\omega L/(R^2+\omega^2L^2)]}. \quad . \quad (76)$$

For optimum amplitude, by equation (72) this becomes

$$\Theta=\frac{RG\mathbf{I}}{b(-L\omega^2+j\omega R)}. \quad . \quad . \quad . \quad (77)$$

Comparing this with equation (73), we have

$$\mathbf{I}=\mathbf{E}/2R, \quad . \quad . \quad . \quad . \quad (78)$$

and I and E are in the same phase.

Thus we see from (73) that the back voltage E_b due to the movement of the coil is equal to half the applied circuit-voltage E, which is the well-known condition for obtaining the maximum dynamical load.

§ (38) MAXIMUM SENSITIVITY IN PRACTICE.—For maximum current sensitivity at given frequency the conditions are simply $c=K\omega^2$. If the magnet field can be altered, the current sensitivity goes on increasing as the field is increased.

For obtaining maximum voltage sensitivity, however, as is shown in § (37), inductance should be inserted in the circuit or the magnet field should be adjusted. Haworth[2] has shown

[1] Wenner, *loc. cit.* p. 376; Butterworth, *loc. cit.*
[2] H. F. Haworth, *Phys. Soc. Proc.*, 1912, xxiv. 230, and 1913, xxv. 264.

experimentally that the latter system gives good results, using a Duddell galvanometer with an electromagnet (*Fig.* 19). But there are practical objections to the use of an electromagnet in this way, as the battery and leads for its exciting current are very liable to introduce disturbing earth capacities, which are a constant source of error in many tests. In any case, if an electromagnet is used, it should be run from a battery of small bulk, placed close to the galvanometer and well insulated. The alteration of the field by a magnetic shunt is not open to this objection.

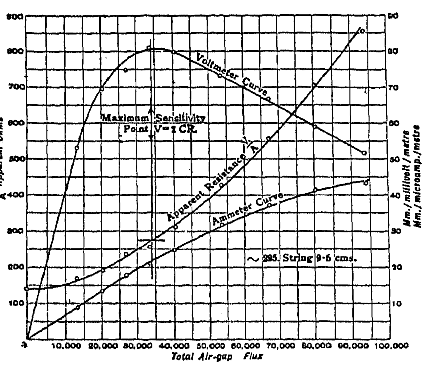

FIG. 19.—Effects of varying the Magnetic Field in Duddell Vibration Galvanometer (Haworth).

§ (39) THEORY OF VIBRATION GALVANOMETER OF THE STRING TYPE.—Butterworth[1] has investigated the theory of the single-loop vibration galvanometer (such as Duddell's), considering for simplicity a single wire stretched between two points with tension T, with a mass M fixed to its middle point, the whole length $2l$ of the wire being in a transverse magnetic field of strength H. Let the mass per unit length of the wire be m, and let ρ and k be the damping constants of the wire and the mass M respectively.

The equation (30) for the moving-coil galvanometer, which is

$$\{c-\omega^2\mathrm{K}+j\omega b\}\Theta=\mathrm{G}\mathbf{I},$$

has now to be replaced by

$$\left[\left(\frac{2\mathrm{T}\beta\cot\beta}{l}-\mathrm{M}\omega^2\right)\mathrm{F}\right.$$

$$\left.+j\mathrm{F}\left(k\omega+\frac{2\rho}{\sqrt{m\mathrm{T}}}\cdot\frac{2\beta-\sin 2\beta}{\beta(1-\cos 2\beta)}\right)\right]\mathbf{U}_0$$

$$=\mathrm{F}\cdot\frac{2\mathrm{H}l(1-\cos\beta)}{\beta\sin\beta}\mathbf{I}, \quad . \quad . \quad . \quad (79)$$

where $$\beta=\omega l\sqrt{m/\mathrm{T}} \quad . \quad . \quad . \quad . \quad (80)$$

and U_0 is the displacement of M.

[1] S. Butterworth, *Phys. Soc. Proc.*, 1912, xxiv. 88.

If the damping is small, resonance practically occurs when

$$2\mathrm{T}\beta\cot\beta-l\mathrm{M}\omega^2=0, \quad . \quad . \quad . \quad (81)$$

and then also $\mathrm{F}=1.$

From (80) and (81) we have

$$\frac{\cot\beta}{\beta}=\frac{\mathrm{M}}{2ml}; \quad . \quad . \quad . \quad . \quad (82)$$

also $$\omega=\frac{\beta}{l}\sqrt{\frac{\mathrm{T}}{m}}, \quad . \quad . \quad . \quad . \quad (83)$$

and equation (79) becomes

$$j\omega\left[k+\frac{\rho l(2\beta-\sin 2\beta)}{\beta(1-\cos 2\beta)}\right]\mathbf{U}_0=\frac{2\mathrm{H}l(1-\cos\beta)}{\beta\sin\beta}\mathbf{I}, \quad (84)$$

say $$j\omega b_1\mathbf{U}_0=\mathrm{A}\mathbf{I},$$

which gives amplitude

$$[\mathrm{U}]=\frac{\mathrm{AI}_{\max.}}{\omega b_1}.$$

To determine the resonance frequency, equation (82) is first solved for β, and then ω is found from (83). Equation (82) has an infinite number of roots, and to each root will correspond a resonance frequency. For each value of β the damping and displacement constants b and A can be calculated. Butterworth gives a table of the values of $(2\phi-\sin 2\phi)/\phi(1-\cos 2\phi)$ for various values of $\mathrm{M}/2ml$, the ratio of the mass of the mirror to that of the wires, when we take the case of the bifilar galvanometer. The table shows that as the mass of the mirror increases, the damping of the harmonics increases rapidly, so that for quite moderate values of M the galvanometer may be treated as having only one degree of freedom. If the mirror is extremely light the selective properties of the instrument may be impaired, and the wires may vibrate with nodes and loops due to the harmonics.

§ (40) BRANCHED CIRCUIT EQUIVALENT TO VIBRATION GALVANOMETER.—Butterworth[2] has shown that the behaviour of a vibration galvanometer can be imitated by an equivalent electrical circuit consisting of a resistance R in series with a parallel combination of a resistance s_v, a capacitance C_v, and a self inductance L_v having no resistance.

If $\mathrm{L}=0$, equation (32) becomes

$$\mathbf{E}=\left[\mathrm{R}+\frac{j\omega\mathrm{G}}{c-\omega^2\mathrm{K}+j\omega b}\right]\mathbf{I}.$$

The vector impedance is equivalent to

$$\mathrm{R}+[1/s_v+j(\omega\mathrm{C}_v-1/\omega\mathrm{L}_v)]^{-1}$$

if $$b/\mathrm{G}^2=1/s_v,\ \mathrm{K}/\mathrm{G}^2=\mathrm{C}_v, \text{ and } \mathrm{G}^2/c=\mathrm{L}_v.$$

Unfortunately the circuit cannot be realised in practice since L_v has no resistance, but Butterworth describes a null bridge method by which the four constants of the equivalent circuit can be

[2] S. Butterworth, *Phys. Soc. Proc.*, 1914, xxvi. 264

directly determined independently of the frequency. This has distinct advantage over tests at exact resonance, if the frequency cannot be held steady. For very sensitive galvanometers, however, the bridge method requires an inductive coil of impracticably large time constant (L/R). Kennelly and Pierce's method (§§ (27) and (39)) is in general simplest and best.

§ (41) MOTIONAL IMPEDANCE CIRCLE.—Kennelly and Pierce (*loc. cit.*) have shown that a very interesting result is got by plotting the motional reactance of a telephone (or similar vibration instrument) against its motional resistance, for various frequencies, as in *Fig.* 20. The curve obtained is a circle passing through the point O where the motional resistance and reactance are both zero. If P be any point on this curve, then OP represents the motional vector impedance $(R'-R)+j\omega(X'-X)$. In Table 6 are given results of observations on a Duddell vibration galvanometer by Kennelly and Taylor.[1] The actual effective resistances and reactances obtained at the various frequencies (by an inductance bridge) are plotted

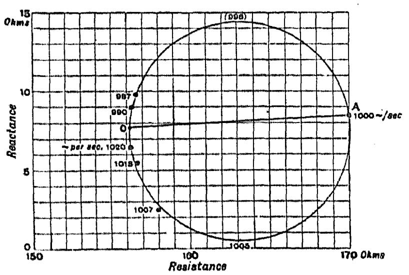

FIG. 20.—Motional Impedance Circle.

against one another in *Fig.* 20, and are seen to lie on a circle, at the point O both the motional resistance and reactance being zero. The line OA represents the maximum motional impedance (13·8 ohms), which occurs at the resonance frequency close to 1000 ∿ per sec. A telephone also gives in the same way a curve which is practically a circle.

TABLE 6

Frequency, n. ∿ per sec.	Effective Resistance. Ohms.	Reactance. Ohms.	Deflection. Radian.
980	156·3	8·95	0·0052
987	156·6	9·88	0·0087
994	157·9	12·38	0·0157
1000	170·0	8·46	0·0611
1007	158·0	2·53	0·0209
1013	156·7	5·48	0·0105
1020	156·3	6·44	0·0044

If the deflection is plotted against the effective resistance an approximate circle is also obtained.

It will be noticed that the motional reactance is zero at the resonance frequency. At frequencies just below resonance it is positive and acts as a self inductance, while at frequencies just above resonance it is negative and acts as a series capacitance. This was pointed out by Butterworth (*loc. cit.*).

[1] A. E. Kennelly and H. O. Taylor, *Am. Phil. Soc. Proc.*, 1916, lv. 415.

IV. APPLICATIONS OF VIBRATION GALVANOMETERS

§ (42) MEASUREMENT OF ALTERNATING CURRENTS AND VOLTAGES.—Vibration galvanometers may be used to measure alternating currents or voltages, and are particularly useful when extremely high sensitivity is required. The preliminary calibrations should be done as described in § (31) (*Fig.* 18). The frequency should be as steady as possible. With unsteady frequency the best plan is to blunt the resonance by applying a suitable shunt. When the wave form is not pure the instrument only measures the sine wave component (fundamental). Vibration galvanometers, as their deflections are in direct proportion to the current or voltage measured, have a great advantage over square-law instruments (thermal, electrodynamic, etc.) whose sensitivity falls off so greatly at the lower readings.

§ (43) EMPLOYMENT IN NULL METHODS.—Perhaps the most important application of vibration galvanometers is their use as detecting instruments in null and other methods of testing inductance, capacitance, effective resistance, and electrolytic resistance. At the lower audio frequencies of the order of 100 ∿ per sec. a vibration galvanometer is very much better than a telephone, but as the frequency is raised the telephone gradually comes up to it in sensitivity, and at 800 ∿ per sec. and the higher frequencies the telephone has the advantage. Future improvements in vibration galvanometers may, however, alter this comparison.

They are also useful in null methods of magnetic testing of iron and also in tests of commercial transformers.

In all cases it is desirable to use with the galvanometer a set of graduated shunts, so as to allow the sensitivity to be suitably reduced while the first adjustments are being made.

§ (44) USE OF A TRANSFORMER WITH VIBRATION GALVANOMETER.—In bridge and other null methods the vibration galvanometer ought to have an effective resistance appropriate to the conditions of the testing circuits. If a galvanometer of suitable resistance is not available the difficulty can often be got over by connecting the galvanometer to the bridge through the medium of a transformer,[2] the primary winding being connected to the bridge and the secondary to the galvanometer. The transformer should be free from magnetic leakage and its windings should be sectioned in order to allow choice of various ratios of transformation. Wenner[3] has shown that if

[2] M. Wien, *Wied. Ann.*, 1891, xlii. 681.
[3] *Loc. cit.*

a be the ratio of transformation (primary to secondary turns), then

$$\phi = \frac{aG\sqrt{2}E}{\omega_1[(r_g + a^2 r_1)b + G^2]}, \quad . \quad (85)$$

where r_g is the resistance of the galvanometer and r_1 that of the transformer primary. Also

$$\text{Voltage sensitivity} = \frac{aG\sqrt{2}}{\omega_1[(r_g + a^2 r_1)b + G^2]}. \quad (86)$$

When $a^2 r_1$ can be neglected, the ratio of increase or decrease of voltage sensitivity $= a$, and the working effective resistance is altered in the ratio $1/a^2$.

Wenner also shows that in certain cases the use of a transformer helps to reduce the relative effects of harmonics in the current wave form.

§ (45) Multiple Coil Vibration Galvanometers.—When the current to be detected comes from a source whose frequency is only known between wide limits, a vibration galvanometer may be constructed with a number of coils covering the range of possible frequency, so that the coil most nearly in tune with the actual frequency shall always give sufficient indication of current. Campbell and Paul have both constructed galvanometers of this type with six coils in one magnet. In Campbell's instrument the suspensions are all similar, while the coils have graduated numbers of turns and can all be tuned by one motion. In Paul's pattern the coils are all identical, but have various lengths of suspension. In both cases all the coils are connected in series. In such a case the coils which are not in motion do not add too much to the effective resistance of the circuit, since this is due mainly to the motion of the coil which is nearly in resonance.

§ (46) Vibration Galvanometers as Relays.—Resonance instruments can be readily adapted to act as relays, and various types of relay of this kind have been used. For example, Lodge [1] introduced a tuned moving-coil instrument in 1898. At the same time Evershed [2] described a highly sensitive relay for low-frequency currents, consisting of two fine wires very close to one another in a magnetic field. When a minute current at their resonance frequency was sent through them the vibration produced brought them into contact and so closed the relay circuit. This relay worked with 0·0001 microwatt. Campbell not long after introducing his vibration galvanometer adapted it to use as a relay by adding to the coil a very short arm working between contacts. With modern moving-coil vibration relays the working power need not be very many micromicrowatts.

[1] O. J. Lodge, *Inst. El. Eng. Proc.*, 1898, xxvii. 851, etc.
[2] S. Evershed, *Inst. El. Eng. Proc.*, 1898, xxvii. 852.

Drysdale [3] has adapted his vibration galvanometer to use as a relay, making a very robust instrument which can work with 0·01 microwatt at a frequency of 10 to 20 ∿ per sec. This has proved very useful in working with currents induced from cables laid at the sea bottom as leader gear for ships. As the contact made by the vibrating tongue of a resonance relay is usually intermittent, both Drysdale and Campbell found it advantageous to pass the local current from the relay through the coil of a strongly damped ordinary relay of slow period. In Drysdale's instrument this second relay utilises the same magnet as the first, and its contact arm, when in oscillation, maintains continuous rubbing contact against two sets of platinum wire brushes.

§ (47) Harmonic Analysis of Alternating Wave Form.—Vibration galvanometers furnish perhaps the best means at present known of carrying out the harmonic analysis of alternating wave forms. Blondel and Carbenay used for this purpose their special moving-iron galvanometer described in § (18), but later Blondel,[4] realising that the moving-coil type was much preferable for the purpose, introduced one of the latter type with a readily tunable system, the tension of the bifilar suspension being alterable by adjusting the current in an electromagnet, to the armature of which the lower ends of the suspensions are fixed. In the latter paper Blondel gives a full discussion of the theory. When the length is not altered, the bifilar suspension gives current sensitivity inversely proportional to the resonance frequency, and when the voltage is applied with a high resistance in circuit, the voltage sensitivity also follows this law. To rectify this falling off in sensitivity Blondel replaces the high added resistance by one or more condensers in series with the galvanometer, as shown in *Fig.* 21. The voltage to be analysed may be applied directly at AB or through a transformer as shown. The resultant series capacity C must be small enough to make the resistance of the galvanometer and the rest of the circuit negligible compared

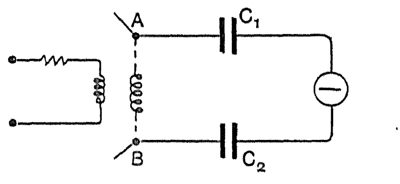

Fig. 21.—Blondel's Series Condenser System for Harmonic Analysis by Resonance Galvanometer.

to the reactance $1/\omega C$. With two condensers as in *Fig.* 21, $1/C = 1/C_1 + 1/C_2$.

[3] C. V. Drysdale, *Inst. El. Engs. Proc.*, 1920, lviii. 582.
[4] A. Blondel, *Ann. de Physique*, 1918, x. 195.

Let the galvanometer be in resonance, then by replacing ωL in equation (65) by $1/\omega C$ we obtain

$$\phi = \frac{GE_{max.}}{\omega(b\sqrt{R^2+1/\omega^2C^2}+RG^2/\sqrt{R^2+1/\omega^2C^2})}. \quad (87)$$

When R is negligible compared with $1/\omega C$,

$$\phi = \frac{GE_{max.}}{b/C+RG^2/C}$$

$$\doteq \frac{GCE_{max.}}{b}.^{1} \quad . \quad . \quad . \quad . \quad (88)$$

Here the voltage sensitivity is inversely proportional to b and not to $\omega(bR+G^2)$ as in equations (66) and (70) when resistance only is in circuit. Also it does not alter with change of ω as long as the damping moment b remains constant.

The use of condensers in this way has very good effect as the galvanometer is tuned to higher and higher harmonics; the undulation due to the presence of the fundamental can never be quite eliminated, but is much more weakened by series capacity than with series resistance. This is well illustrated by Blondel's photographs shown in *Fig.* 22.

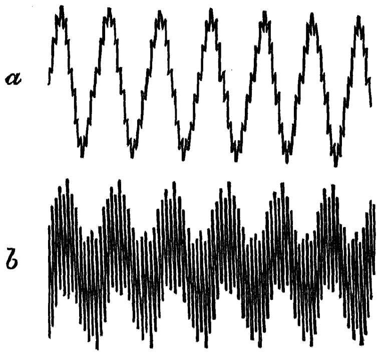

FIG. 22.—Galvanometer tuned to Harmonic 13, with (*a*) Resistance, (*b*) Capacity in Circuit (Blondel).

The observations are made by tuning the galvanometer to resonance for each harmonic and taking on a moving plate photographic records of the respective oscillations.

§ (48) GALVANOMETER SHUNTS WITH CONSTANT DAMPING.—In some measurements it is desirable to be able to shunt the galvanometer to various degrees without affecting its total damping constant $(b+G^2/R)$. Blondel has shown that this can be done with a shunt s and an added resistance r as in *Fig.* 23. Then if the resistance of the source D is negligible, the total resistance R of the galvanometer circuit is equal to r_g in series with r and s in parallel, or

$$r_g + \frac{rs}{r+s} = R,$$

which has to be constant, and hence we find $\frac{rs}{r+s}$ = constant; in other words, the resistance between P and Q when the galvanometer is disconnected is to be constant. Table 7 gives a set of values of r and s calculated by Blondel on this basis.

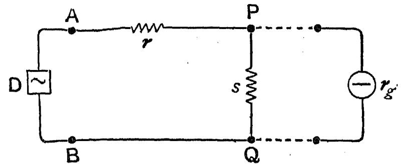

FIG. 23.—Shunt with Constant Damping: ($1/r+1/s$ constant).

[1] A. Blondel, *loc. cit.*

TABLE 7

r Ohms.	s Ohms.	Sensitivity Factor.
100	∞	100
250	166·7	40
500	125·0	20
1 000	111·1	10
2 500	104·2	4
5 000	102·0	2
10 000	101·0	1
∞	101·0	0

A. C.

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VISION, PHOTOELECTRIC THEORY OF. See "Photoelectricity," § (7).

VOLT : the unit of electromotive force on the practical C.G.S. system.

$$1 \text{ Volt} = 10^8 \text{ C.G.S. units of E.M.F.}$$

See "Units of Electrical Measurement," § (21) ; "Electrical Measurements, Systems of," § (7).

VOLT, INTERNATIONAL : the E.M.F. which produces a current of one international ampere in a resistance of one international ohm ; it is equal to $1{\cdot}0064_9$ volts. The E.M.F. of a standard Weston cell at 20° C. is 1·0183 international volts. See "Units of Electrical Measurement," § (31).

VOLT DIVIDING BOX : a subdivided resistance for obtaining a convenient fraction of a high voltage. See "Potentiometer System of Electrical Measurements," § (6).

Testing of. See *ibid.* § (14) (ii.).

VOLTAGE, REVERSIBLE DECOMPOSITION : a term used in electrolysis to denote the theoretical voltage required to bring about the change considered under thermodynamically reversible conditions. See "Electrolysis, Technical Applications of," § (3).

VOLTAGE EFFECT ON RESISTANCE OF DIELECTRICS. See "Resistance, Measurement of Insulation," § (1) (v.).

VOLTAGE REGULATION IN ELECTRICITY DISTRIBUTING NETWORKS. See "Switchgear," § (19).

WATT-HOUR AND OTHER METERS FOR DIRECT CURRENT

§ (1) INTRODUCTORY.—The history of the problem of the correct registration of the electrical energy delivered to a circuit is as old as that of the production of this type of energy, and it is interesting to find that many of those, from Faraday onwards, responsible for the development of methods of generating energy were engaged in devising means for measuring it.

It is required to integrate the energy as found by the equation $E = \int_{t_1}^{t_2} vidt$, where v and i are the instantaneous values of current and pressure, and $t_2 - t_1$ the time during which the energy is supplied, and the term "electricity meter" is by long usage applied to an instrument which integrates and records the total amount of energy supplied. Such instruments may be of two types: (1) a true energy meter in which the record is produced by the action of both the pressure and the current; and (2) a quantity meter which integrates current only, but which, when used on a circuit having a constant pressure, will give an equally accurate measure of the total energy. These two types are generally known as watt-hour meters and ampere-hour meters: the unit, however, universally adopted is 1000 watt-hours, generally known in this country as the "B.O.T. unit," although more lately the term kilowatt-hours (K.W.H.) is becoming more used.

The primary reason for the introduction of the electricity meter was to determine the amount of electrical energy supplied to a consumer; this energy, according to the Electric Lighting Acts, must, unless otherwise agreed, be determined by a meter duly certified and approved by the Board of Trade; but later it was found desirable to keep records of the behaviour of machines, such as electrically driven trains and trams, and meters are used in large quantities for these and other types of machines and apparatus, such

as batteries, where appropriate measurements tend to efficiency of operation.

The use of watt-hour meters and ampere-hour meters varies somewhat in different countries. In Great Britain, France, and Germany the law permits of the use of an ampere-hour meter, but, on the other hand, in America the use of watt-hour meters—instruments which require for their operation both current and pressure circuits and whose record is a product of the two factors—is required.

Some authorities, and notably Ratcliff,[1] have held that the use of a watt-hour meter should be compulsory in England, but for practically all circuits for small currents, say up to 25 or 50 amperes, which are supplied at the normal pressure not exceeding 250 volts, ampere-hour meters are used. The desire for a technically accurate measurement of the actual amount of energy supplied to a consumer is no doubt the reason for the use of watt-hour meters, but against that it is urged that the ampere-hour meter is simpler and cheaper than the watt-hour meter; the energy required to operate the instrument is very much smaller, while the instrument itself is cheaper to instal. The supply authority is required to maintain the pressure (E.M.F.) constant to within ± 4 per cent, and in practice the variation in the great majority of cases is less than this amount. Also, since such variations of pressure are likely to be both above and below the declared value, it will be apparent that the error due to variation of pressure is very small, and that unless and until a watt-hour meter is devised more suitable for use in small power circuits it is advantageous to use an ampere-hour meter. The difference between the nations is interesting, but it would appear probable that the real reason for it is due to the fact that the first types of watt-hour meter were developed in America by Elihu Thomson, while the first ampere-hour meters were produced in England by Ferranti and Hookham. In France and Germany also other types of ampere-hour meters were developed.

This statement of relative merits does not, however, apply to all types of supply; in fact these may be divided into two groups: (1) for house service, where in the great majority of cases the maximum current does not exceed 10 amperes; and (2) a supply of energy to a factory or a workshop. In the first case, the energy is used mainly for lighting, and since the change in candle-power of an incandescent lamp is out of all proportion to the change in E.M.F. the interest of the consumer is best served by maintaining the voltage as nearly as possible constant. Apart from this the energy required for the pressure windings of the watt-hour meters at present in use is from 100 watt-hours to 200 watt-hours per day, which in itself would represent probably 10 per cent or more of the total energy used on a normal small circuit. For the second case, however, where electrical energy is used largely for motive power and where the losses in the meter itself will be inappreciable compared with the total energy used, the case is clearly different; thus, while there is no definite regulation bearing on the matter, in the great majority of cases circuits up to 30 amperes are fitted with ampere-hour meters, and for currents larger than this watt-hour meters are used.

§ (2) General Principles. (i.) *Electro-chemical Meters.*—The meters which depend for their action on the effect of the passage of a current through an electrolyte may be divided into two classes: (1) that in which the effect of the current is to decompose the water constituent of an electrolyte, a measurement being made either of the amount of gas produced or of the decrease in the volume of electrolyte; and (2) that in which the amount of metal transferred from the anode to the cathode of an electrolytic cell with suitable electrodes and electrolyte is measured.

Since the amount of decomposition or deposition will depend solely on (1) the magnitude of the current, (2) the time during which the current flows, and (3) the electro-chemical equivalent of the electrolyte or metal, it is clear that such meters will register ampere-hours only.

Most of the earliest types of meter were based on these principles, the first being probably due to Faraday, who measured the amount of gas liberated from a solution. Later Bunsen suggested that the gas should be allowed to escape and the quantity of electricity be determined by loss of weight of the electrolyte. These ideas, modified somewhat, appear in meters in use to-day. Attempts were also made by Butler,[2] Boucher,[2] and a number of others to make the gas so liberated actuate a counting mechanism. A number of other interesting early suggestions which do not survive in practical form are also mentioned by Gibbings.[2] These referred to attempts to make the change in weight of the electrodes due to the passage of current actuate a counting mechanism, among the most interesting being that due to H. W. Miller, and described as follows:

"Between the electrodes (in an electrolytic bath) is placed a balanced metallic cylindrical wheel. When the current passes it deposits metal upon one side of the cylinder and dissolves it from the other side, thus causing the cylinder to slowly revolve by upsetting its equilibrium. The motion of the cylinder is used to actuate the registering dials."

[1] *Journal I.E.E.* xlvii. 3.

[2] *Journal I.E.E.* xxvii. 551.

The later types of electrolytic meter in use at the present time are described in detail in §§ (13) to (17).

(ii.) *Motor and other Electromagnetic Meters.*—All motor meters are based on the discovery by Faraday, in 1821 onwards, of the principles of electromagnetic action.[1] According to these—

(*a*) When a conductor is moved in a field of magnetic force so as to cut the lines of force, an electromotive force is produced in the conductor which is measured by the rate of decrease of the magnetic flux through the circuit of which the conductor forms part; and (*b*) a conductor carrying a current in a magnetic field is acted on by a force in a direction at right angles to its length and to the direction of the field, which is measured per unit of length by the product of the current and the strength of the field in a direction at right angles to the conductor.

In 1821 Faraday made a machine consisting of a copper disc amalgamated on the edge and at the axle to form good contact with contact brushes which, when rotated between the poles of a magnet, generated a small current. Barlow, as early as 1823, showed that the converse held good, and that when a current was passed through a disc similar to that of Faraday's from the centre to the periphery and in a direction at right angles to the lines of flux of the permanent magnet, the disc would rotate as a consequence of the interaction between the magnetic forces of the permanent magnet and those produced by the current flowing through the disc. And after 1831 several investigators, including Joule, produced motors based on Faraday's laws of electromagnetic action. A meter is necessarily somewhat different from a motor: in the latter the speed can be controlled by the large magnetic forces possible and also by the work which the machine has to do, whereas in the case of a meter the speed must be proportional to the load, and the work reduced to a minimum, and although Ferranti in his first meter successfully made use of a pool of mercury which, supported between the poles of a magnet, revolved at a speed nearly proportional to the magnitude of a current passed through it from the centre to the periphery, it was not until the properties of the eddy-current brake disc, usually called the Foucalt brake, were fully appreciated that a really satisfactory motor-meter was produced.

The action of the eddy-current brake depends on the fact that, when a disc of copper or other good conducting material is rotated in a magnetic field, currents are set up in the disc which are proportional to the speed of rotation. The reaction between these currents and a constant magnetic field produces a force proportional to the speed. This therefore constitutes a brake, the resistance of which is proportional to the speed.

[1] See "Electromagnetic Theory," §§ (9), (11).

In the application of the eddy-current disc to control the speed of a motor-meter two distinct cases arise. In the watt-hour meter,[2] the essential principle of which is shown in *Fig.* 1, the armature or moving element is connected across the pressure circuit with a high-resistance coil S in series. The main current I passes through the field coils F. Under these circumstances, if we suppose all friction to be eliminated, the speed of the armature would rise indefinitely as the effect of the back electromotive force produced would be inappreciable. It is therefore essential that a brake should be applied, and if the rate of rotation of the meter is to be proportional to the current I the controlling forces exerted by this brake must be proportional to the speed. This condition is very closely satisfied by the eddy-current disc, and if this is mechanically connected to the armature the speed of rotation will be proportional to the current.

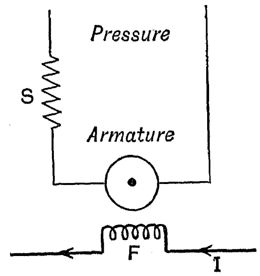


FIG. 1.

In the ampere-hour meter shown in *Fig.* 2 the armature works across the shunt through which the main current to be measured is passed. The speed of the armature is here prevented from rising to an excessive value since the rise of the back electromotive force opposes the volt drop across the shunt. For any given current, therefore, there will be a definite speed at which the motor would run even though friction were completely eliminated. The application of the eddy-current brake disc, though convenient, is not therefore essential to the functioning of the meter.

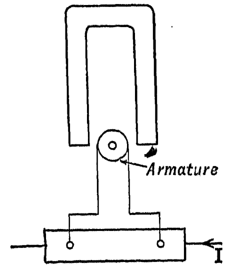


FIG. 2

The same remark applies to the mercury meter, the essential principle of which is shown in *Fig.* 3. Here a pool of mercury contained in a shallow cup carries the current into the outer periphery of a copper disc, which is submerged in the mercury, and out

[2] For further details see "II. Watt-hour Meters."

at the centre of the disc. The element is placed between the poles of a permanent magnet, and is so disposed that the greater part of the current passes through that portion of the combined circuit of mercury and copper disc which is immediately between the poles of the magnet. Owing to the higher conductivity of the copper a large portion of the current traverses the disc. The interaction between the flux due to the permanent magnet and the current causes a rotation of the disc and mercury which is conveyed to a counting mechanism.

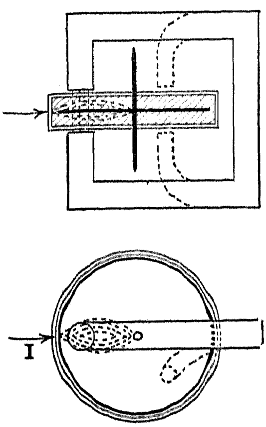


FIG. 3.

The increase of fluid friction with speed and the effect of that portion of the magnetic field which acts on the part of the disc which is not carrying the current, together with the damping, regulate the rate of rotation for a given current.

Here again it is convenient to reduce the speed still further by means of the eddy-current disc, and for this purpose a portion of the magnetic circuit may be diverted so as to pass through the disc and the mercury at a point which is outside the main region of current flow (see *Fig.* 3). Or instead of this a separate disc may be mounted on the same spindle, and a separate magnet may be employed for the purpose of regulating the speed.

The laws of fluid friction in mercury moving at the speed at which the armatures rotate are not well known. It is frequently assumed that, as with water, the fluid friction will increase as the square of the speed, but it is doubtful if this will hold for the low speeds employed. Taking the normal maximum speed of the armature as 100 r.p.m. the speed at the edge of the mercury pool will be only of the order of from 1 foot to 2 feet per second. The curve of typical error (*Fig.* 10) shows clearly that there is a considerable increase at the higher speeds, and it would appear that the highest point on the curve, which occurs at quarter load, represents the critical value of the speed where the change from proportionality to a higher power of the ratio takes place. From a consideration of the viscosity and other properties in which mercury differs from other fluids, it would seem probable that at the higher speeds the fluid friction is more nearly proportional to the 1·5 power of the speed.

In the clock (Aron) meter the same principle of magnetic attraction or repulsion is used to affect the rate of a swinging pendulum. In this case the interaction is due to a magnetic field carried by the pendulum with that set up by the current flowing in a coil placed beneath it, the change of rate as recorded on dials thus becoming a measure of the energy. The principle is explained in greater detail in the section dealing with this meter.[1]

I. Ampere-hour Meters

§ (3) Mercury Rotating Armature Meters.—All direct-current motor meters are based on the fundamental principles of the motor in which the passage of a current through a coil which is in a magnetic field sets up a rotation of the coil. In the mercury motor-meter the arrangement is simplified by the use of a disc of copper instead of a coil in a manner similar to the very early experiment of Barlow, as described in § (2) (ii.) above.

(i.) *Ferranti.*—Ferranti's first meter,[2] which was employed to a large extent in the early days of electricity supply, used a pool of mercury as the moving element. The magnetic field was produced by a coil wound on an iron core, the core being shaped and having a gap in which was fixed the mercury bath. The current passed through the coil and the mercury bath from the outside to the centre, the direction through the latter being at right angles to the flux of the magnetising coil. A light metal fan was carried by the mercury and was connected to a train of wheels which served to record the number of revolutions of the mercury.

(ii.) *Hookham.*—Hookham, after having worked from 1887 onwards with an armature consisting of a coil of wire with a mercury commutator, evolved in 1892 a meter in which a disc of copper was immersed in a pool of mercury, the spindle being weighted so that when the disc was not rotating a pivot fixed under the disc rested in a bearing; the weight was so arranged that only a small portion of the total weight was borne by the bearing. The current entered the mercury at a point on the edge of the pool and passed out at the centre. In consequence of the greater conductivity of the disc, most of the current traversed it from a point on its edge to its centre. Its direction was thus at right angles to the flux due to the magnets, and the disc rotated.

Hookham's meter also replaced the electromagnets by a large permanent magnet, which not only saved the energy required for the electromagnet, but produced a constant magnetic field.

The principle of passing the current through the disc and the use of a permanent magnet

[1] See "II. Watt-hour Meters," § (21).
[2] *Electrician*, xxviii. 358.

is now generally used in mercury motor-meters. There have been differences in the application of the principle, mainly consisting in the use of a cup or drum-shaped armature which, since it could be arranged to cut a greater number of the magnetic lines of force, gave a greater driving torque; difficulties, however, were experienced with this type, and it is now little used.

In 1897 Hookham designed a meter on somewhat similar lines, but with greatly increased working forces. This meter was fitted with a small coil in series with the mercury bath, which by its effect on the magnetic circuit tends to decrease the error at the higher loads due to fluid friction, and also with a second Foucalt brake disc which rotates between the poles of a magnetic circuit magnetically in parallel with the field in which the armature disc operates. The function of this second disc is to provide compensation should the strength of the magnet decrease due to any cause. Also it provides a convenient method of adjustment since, if the distance between the pole pieces is increased or decreased slightly, the speed of the meter will be varied. This pattern, in substantially the same form, is in use at the present time for larger current circuits; the detailed drawing published by the makers in 1897 is reproduced in *Fig.* 4, and the

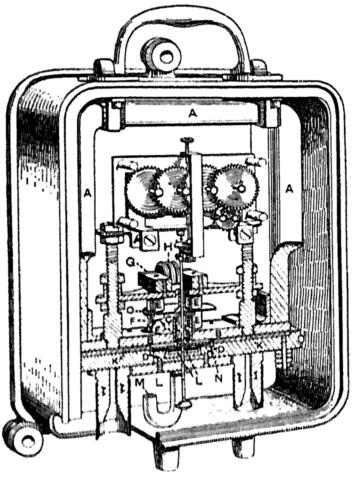


Fig. 4.

description given by Messrs. Chamberlain and Hookham is as follows:

"The magnet consists of a single bent bar AA of tungsten steel. BB are plates of soft iron continuing the magnetic circuit towards the centre, where it is broken by the insertion of a brass piece C. The lines of force pass downwards through the iron bridge-piece DD, being cut by the armature N twice, in opposite senses. They also pass upwards through the brake pole-pieces EE, and the upper iron bridge-piece G. O is the brake-disc; H the correcting coils for fluid friction error; F the reduced saturated neck of one of the brake pole-pieces; KK insulated strips of copper, conducting the current from the terminals II to the mercury cup LL, in which the armature is immersed and partially floated. The mercury is carefully insulated from the containing vessel except the ends of the copper strips KK. The armature is slit radially for about one-third of its diameter all round, leaving a continuous area of copper in the centre.

"The action of the meter is as follows: Owing to the great length of the magnet AA, an intense field is produced at BD, BD. The current flows across the diameter of the disc, being almost entirely confined to the area beneath the pole-pieces, by the radial slits in the armature (which embrace each about one-third of the periphery of the disc).

"The effect of the choking of the brake at F is in this meter very marked. It will be observed that the armature and brake fields are magnetically in parallel, and consequently the armature field is a by-pass to that of the brake. Now, if there were no choking of the brake, the speed of the meter would increase with any falling off in the induction in the steel magnet. But it is obvious that, with the present arrangement, it would be possible, if the saturation of the necks were carried beyond a certain point, to produce the opposite effect. The brake remaining nearly constant, and the driving force of the motor falling off, the speed would decrease with a decrease of field. It is equally obvious that between these extreme effects an intermediate state is possible in which the speed of the meter is, through a considerable range of intensity, independent of the strength of the field. This point has been ascertained by experiment, and realised in practice, and in the present pattern it is possible to apply to the steel bar a demagnetising force of from 200 to 300 ampere turns, without affecting the rate of the meter for practical purposes."

In the later type (1907) of Hookham meter for smaller currents the additional brake disc is eliminated and the meter is of the simple form illustrated by *Figs.* 5 and 6, which show vertical and horizontal sections of the instrument. A is the permanent steel magnet forged in one piece and having wrought-iron pole-pieces B′B′ attached to it. These latter terminate in the circular poles BB, which very closely oppose one another inside the mercury chamber. The mercury chamber is formed of upper and lower brass castings

(EE) of circular form, in which the pole-pieces are embedded. These castings are thickly nickel-plated and insulated with

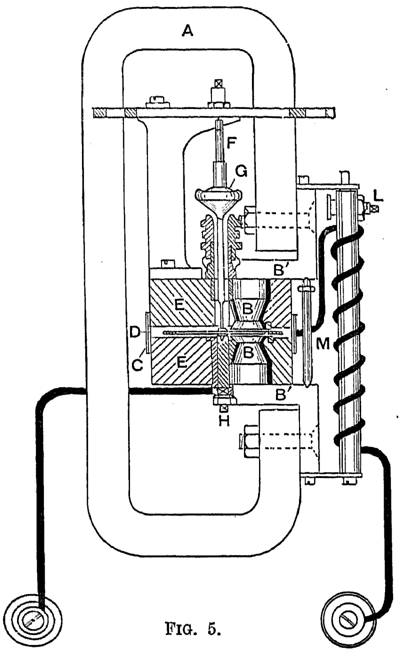


FIG. 5.

japanned linen. The armature D, carried by the spindle F, rotates between the poles BB, and is held in position by pivots at each end of the spindle. The lower pivot rests upon a cup jewel carried by the screw H. The metal ring C, lined with leather and cork, surrounds the two brass castings, and forms the side of the mercury chamber. G

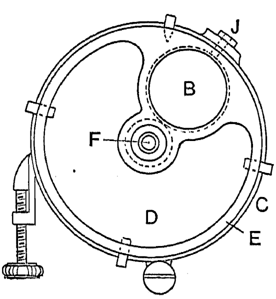


FIG. 6.

is a brass weight which counterbalances the armature to prevent it floating.

The coil M is wound upon an iron core, and serves to correct the slight error caused by fluid friction of the mercury. Iron screws LL, which alter slightly the strength of the field, enable small adjustments of speed to be readily made.

Fig. 6 is a horizontal section showing the position of the pole and armature. It also shows the connection J which conveys the current between the mercury and the correcting coil. The contact J is carried by the band C, but insulated from it.

(iii.) *Ferranti-Hamilton.*—The latest type of Ferranti meter (Hamilton's patent) used the submerged disc as in the Chamberlain and Hookham meter, but the arrangement of the magnetic circuit is different (see *Fig.* 7). Two magnets, M_1 and M_2, are used; the interaction between the flux from M_1 and the current through the disc exerts a driving force, causing the disc to rotate. The additional magnet M_2, between the poles of which that portion of

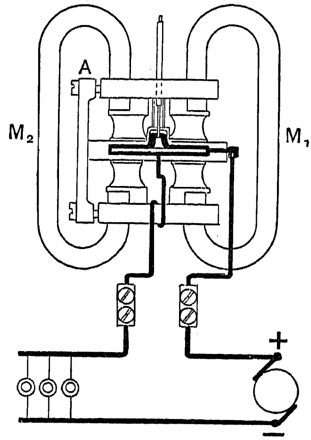


FIG. 7.

the disc rotates which is not carrying any current, exerts a retarding force only by reason of the eddy-currents set up in this portion of the disc. By this means the speed of rotation is kept low while the torque is unchanged.

The secondary magnetic circuit shown in *Fig.* 7 consists of two limbs joined at one side by the adjustable arm A. The current flowing through the bath is taken through one or two turns of wire coiled around the lower limb of the secondary circuit, and thus increases the flux from M_1 and decreases the retarding flux from M_2, the amount of the increased torque being designed to compensate for the fluid friction of the mercury at the higher speeds. The position of the arm A can be adjusted by means of the screws and serves for final adjustment of the meter. The complete meter without the front portion of the case is shown in *Fig.* 8, where C, E, F, D represent the clamping and adjusting screws for the magnetic shunt adjustment K, the screws through which is operated the clamping

device, and A and B the gear wheels which can be changed for calibration.

The "Sangamo" Isaria and British Thomson-Houston mercury ampere-hour meters are

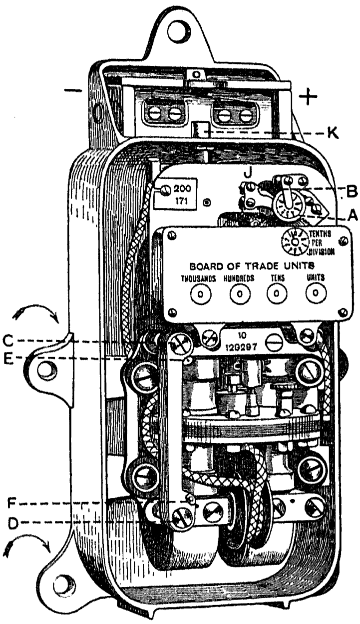


FIG. 8.

based on the same principle as the later Hookham and Ferranti instruments, and differ from them only in detail and arrangements, the B.T.H. having a bell-shaped armature.

(iv.) *Bat Meter.*—The Bat meter, while using the same principle as the Hookham, is fitted with some novel features. The armature is an inverted cylindrical copper bell, the edges of which are amalgamated, the remainder being protected by enamel. *Fig.* 9 shows the construction of the bath and arrangement of the magnetic field. The inverted copper bell C revolves in the annular mercury chamber and is carried on the axle S, the lower pivot of which, J, is centrally guided by the insulating sleeve X. The upper bearing, not shown in the figure, is jewelled and takes the thrust of the armature, which in this case is upwards.

The current is led in to the armature at the terminal *a*, which is a rod embedded in and passing through the insulating material enclosing the mercury; contact is made to the mercury at a position immediately above the side of the bell. The other terminal consists of an arm extending to the mercury bath in the position *b* in the figure, the end of the arm being bent up and passing through the bottom of the container. The position of the arm can be varied, and provides means of adjustment. This lower contact is immediately below the edge of the bell; the maximum amount of current flows through the armature, but a slight displacement results in a distortion of the stream-lines of current with consequent decrease of speed. By this means an adjustment of up to 20 per cent can be made to the rate of the meter. The magnetic field of the magnets PP in *Fig.* 9 is concentrated by means of the pole-pieces F and H at the gap in which the current-carrying portion of the armature rotates.

The temperature coefficient of the meter is greatly reduced by means of a magnetic shunt of nickel steel. This nickel steel, of composition approximately iron 70, nickel 29, manganese 1, has the property that the permeability decreases as the temperature increases, and is so proportioned that the total flux of the magnets effecting the braking is varied by an amount nearly proportional to the increase of resistance of the copper armature due to a rise of temperature.

A further device, which is fitted only in meters of size less than 10 amperes, compensates

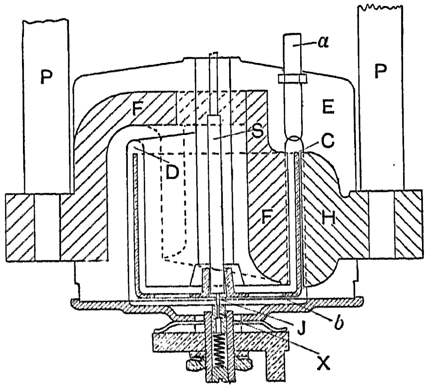


FIG. 9.

for mechanical friction and so renders the meter more accurate at light loads. In this a thermocouple is placed inside a coil which is energised from the supply main, the energy used being about 0·5 watt. The thermocouple is connected directly across the terminals of the mercury bath, and the E.M.F. is sufficient to compensate largely for the friction.

§ (4) CLAMPING DEVICES.—Mercury meters are fitted with a clamping device which lifts the disc off its pivot and seals the mercury chamber. This device is usually effectual, and the instruments are sent from the makers with the baths filled with mercury. In fact, failure of the sealing arrangement to contain the mercury during transit is generally held to be a fair reason for rejection, since the effect of even a small amount of free mercury shaking loose in the case with free access to

the driving train and wheels and to soldered joints, is most serious.

§ (5) WEIGHT OF MOVING SYSTEM.—Owing to the fact that the weight of the disc is borne largely by the mercury the wear of the pivots and jewel is practically negligible. The weight of the moving system in various types of meters may vary: in the Chamberlain and Hookham meter the maximum current passed through the mercury bath is 10 amperes, a shunt being fitted for larger currents, while in the latest type of Ferranti instrument currents up to 40 amperes are taken through the disc, with a consequent increase in the weight of the moving system. In the smaller sizes, say from 2·5 amperes to 10 amperes, the weight of the moving system will be of the order of from 40 to 50 grammes in air, while in the 40-ampere size of Ferranti meter the rotor complete will weigh about 110 grammes. When the disc is immersed in mercury, however, the weight actually resting on the pivot is from 3 grammes to 5 grammes for all sizes of meter, and any consideration of a ratio of torque to weight when compared with other types of meters must be on the basis of the weight actually resting on the pivots.

§ (6) ENERGY LOSSES AND STARTING CURRENT.—Owing to the almost complete elimination of friction the energy losses in mercury meters are very small. The values given for the Chamberlain and Hookham (1907) type are as follows:

1. Size.	2. Starting Current.	3. Pressure Drop across the Terminals of the Meter when Full-load Current is passing.	4. Watt Loss at Full Load.
Amperes.	Amperes.		
2·5	0·05	0·25	0·6
5	0·05	0·20	1
10	0·10	0·20	2
20	0·20	0·10	4

Above 20 amperes a pressure drop of about 0·06 volt, corresponding to 6 watts at 100 amperes, is required with a starting current of one-hundredth of the full-load current.

Ferranti meters require:

1. Size.	2. Starting Current.	3. Pressure Drop across the Terminals of the Meter when Full-load Current is passing.	4. Watt Loss at Full Load.
Amperes.	Amperes.		
2·5	0·05	0·1	0·2
5	0·05	0·1	0·45
10	0·05	0·06	0·6
20	0·125	0·1	2
40	0·25	0·075	3
50	0·3	0·06	3

For sizes above 40 amperes the meters are shunted, the pressure drop across the terminals being of the order of 0·06 volt.

The values stated for the energy losses are well borne out by a large number of tests and those for starting current fairly well, the actual figures for a large number of meters being only slightly higher than the maker's values.

§ (7) TORQUE AND SPEED. — Since the mercury and disc consist essentially of a single turn or half-turn coil in a magnetic field which is itself constant, the torque at full load in a mercury motor meter will vary with the rated size of the meter, but the speed may and does vary with different makers and types definitely in the proportion of the braking effect. A reasonable figure for full load torque for a mercury ampere-hour meter of 10 ampere size is 5 gr-cm.

The unit of torque here taken and generally used in meter practice is that due to a force equal to the weight of one gramme acting tangentially at a distance of one centimetre from the centre of rotation.

The speed in the older types of meter was frequently excessive, but in the more modern types this has been reduced partly owing to the Specification No. 37 of the British Engineering Standards Association, which required that the speed at full load should not exceed 100 revolutions per minute, and partly due to some large users who held that the slower speed meter was more reliable and had a longer life.

§ (8) CALIBRATION.—The methods of adjusting mercury ampere-hour meters vary somewhat with different makers. In the Chamberlain and Hookham meter the adjustment is usually made by means of alteration of the gear wheel connected from the spindle to the recording train.

This method requires that a large number of wheels must be available to cover the complete range of adjustment required, and this plan, although advantageous, since it does not involve alteration of any part of the electrical or magnetic circuits, has the disadvantage (1) that an adjustment made in this way involves a change in the gearing constant of the meter, and (2) that the rotor speed and the gearing cannot be standardised. Small adjustments can, however, be made in the older type by altering the length of the magnetic gap in which the second brake disc works, and in the later instrument by adjustment of the position of the compensating bar.

The Ferranti instrument is adjusted in much the same way as the Hookham, the adjustable shunt bar in the magnetic circuit providing for adjustment of 5 per cent, anything beyond this being carried out by means of change of gearing.

The British Thomson-Houston meter was

set to a given speed (50 r.p.m.) at full load by means of small shunts connected in parallel across the mercury bath; by this means the speed was always set to a standard value and gear wheels and trains were standardised. In the Bat meter the current at the bottom of the mercury bath is led out through an amalgamated arm, the position of which can be varied by means of a milled head, this change in the current path allowing of an adjustment of about 20 per cent.

§ (9) Temperature Coefficient.—For the smaller sizes of mercury ampere-hour meters, where the whole of the current flows through the bath, the temperature coefficient is in most cases large, being of the order of 0·38 per cent for 1° C. This is due to the fact that while the driving torque is proportional to the current, whatever the resistance of the disc, the eddy-currents, and the damping effect, will be inversely proportional to the resistance of the disc, and consequently the meter will record faster with an increase of temperature. In the larger sizes of meter, where a proportion of the current is taken through a shunt, there is a degree of compensation depending almost entirely on the proportion of the current in the shunt to that in the bath. For if the shunt be of a material having negligible temperature coefficient the current flowing through the bath will decrease with increase of resistance of mercury and disc and tend to compensate for the decrease of the eddy-current braking.

Practically the only type of temperature compensation for the small meter is that fitted on the Bat instruments, where use is made of the special magnetic properties of some type of nickel-steel, whose permeability decreases with increase of temperature. A piece of this steel is used to shunt the magnetic circuit operating on the brake portion of the armature, and a fair degree of compensation can be obtained, actual tests showing that the temperature coefficient was not more than 0·1 per cent for 1° C.

§ (10) Accuracy Characteristics.—Owing to the effect of the friction of the armature on its bearing at lower speeds and of the fluid friction of the mercury at high speeds, the mercury meter usually runs slow at light loads, rises to a maximum at about one-quarter of full load, and falls off again at the higher loads. A typical error curve is given in *Fig.* 10. The error is, as will be seen, not unduly large, and there is no difficulty in adjusting so that the various existing specifications are complied with. As has been explained before, the effect of fluid friction is to a fair extent corrected for by the compensating coil: in addition the Bat meter was fitted with a device which enabled the initial friction to be largely corrected. The device, however, required the heating coil to be energised continuously with

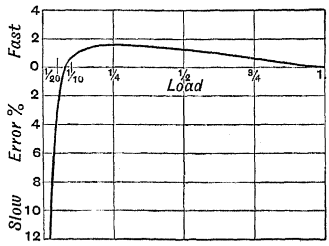


Fig. 10.

consequent loss of energy, and also a third wire had to be run to the meter.

§ (11) Permanence.—In general, mercury ampere-hour meters appear to retain their accuracy for long periods and under severe conditions of use. Difficulties have occurred in practice due to disintegration of the copper disc or fouling of the mercury, especially where the clearance between the disc and the side of the bath is insufficient, but the experience of the engineers who have used such meters for many years would suggest that when all the materials, including the mercury, are quite pure, the meters retain their accuracy for many years. In the early types it was considered necessary to protect the disc from amalgamation, and only the edges and centre were in contact with the mercury, the remainder being coated with varnish. In later meters, however, the whole disc is amalgamated.

§ (12) Range of Sizes and Types of Use.—For ordinary electricity supply mercury ampere-hour meters are generally of size not more than 50 amperes, but for other purposes they are used for much larger currents. For records of charge and discharge of accumulators they are required to meter currents up to several thousand amperes, in which case an external shunt is usually provided, two meters being connected across the shunt but with opposite polarity. Both are fitted with a ratchet-and-pin motion which allows the armature to rotate in one direction only; thus the current on discharge is measured by one meter and the discharge by the other. Two sets of dials have been used, arranged so that motion in one direction operates one set, that in the other direction the second set; this arrangement is still used in the Sangamo meters. This meter also used a device, the principle of which was first introduced on an Aron clock-meter by H. W. Miller, in which the direction of the current operates a device which changes the resistance of the shunt circuit and makes the meter go slow when

the battery is charging, the difference between the charge and discharge readings being adjusted to suit the efficiency of the battery. The reading is on a single dial graduated in ampere-hours, and the position of the pointer provides an indication of the condition of the battery. In the case of the Sangamo meter the change in resistance is effected by an application of the mercury motor principle, as shown in *Fig.* 11, in which a copper bar

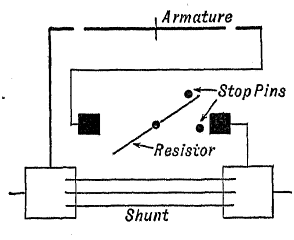


FIG. 11.

is pivoted and floated in mercury, its rotation being limited by two stops. The magnetic field is supplied by the same poles that produce the fields in the motor element. In one position the copper bar is in direct line between the contacts, while in the other position the bar is at an angle to the contacts, and the high resistivity mercury path is longest—that is, the resistance is at a maximum.

§ (13) BASTIAN ELECTROLYTIC METER.—The electro-deposition of a metal or the decomposition of a liquid by the passage of a current would appear to constitute an ideal method of measuring a quantity of electricity, since with such a method there are no rotating or other parts liable to wear, and the principle is founded on the fundamental electro-chemical laws. Moreover, since there is no friction, meters of this type should register accurately a minute current. Many and ingenious have been the attempts to produce a meter of this type, but in most cases it has been found impossible to meet commercial requirements, and there are now only a few types in actual use.

(i.) *Description.*—Among the earliest of these was the meter devised by Bastian about 1898. In this case the action of the meter depended on the decomposition of water, a glass tube in which were fitted platinum electrodes being filled with a solution of dilute sulphuric acid in the ratio of sulphuric acid 1 part, water 10 parts. The electrodes situated at the bottom of the tube were suspended by means of glass rods from a plug which served to close the top of the tube. The passage of the current through the electrolyte decomposed the water into the two component gases, which were allowed to escape through a vent-hole at the top. The tube was necessarily of uniform bore and was graduated in units, the reading being made at the junction of the electrolyte with a thin layer of paraffin oil, which also served to prevent evaporation. This was modified in 1903 by the use of a solution of caustic soda of a specific gravity of about 1·06 and of electrodes made of pure nickel in the form of two concentric cylinders.

Details of this meter are shown in *Fig.* 12, which shows a sectional elevation. The vessel E, containing the electrolyte, is of carefully annealed glass, and is cylindrical in form.

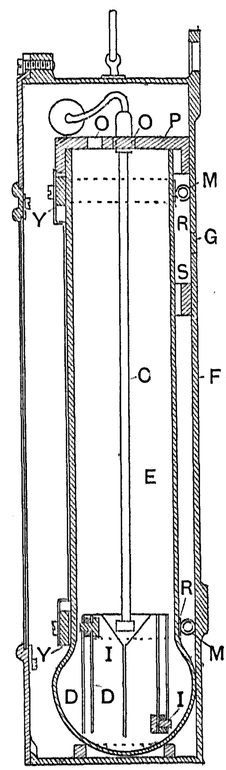


FIG. 12.

Within this vessel are placed the electrodes DD, consisting of two concentric cylinders of nickel (of a purity of 99 per cent) bolted rigidly to each other by means of vulcanite studs with nickel pins.

Stout nickel rods CC are solidly riveted one to each electrode, and are long enough to reach just above the top of the glass vessel, and being uninsulated from the electrolyte, they act as additional electrode surface.

The porcelain lid P slips over the rod by means of two holes OO, and fits loosely on the top of the glass vessel, thus keeping in position the electrodes, which rest on the bottom.

A third hole in the lid enables the gases of decomposition to escape from the glass vessel E and pass into the atmosphere by means of the holes G in the back of the meter-case. This third hole is also used to allow the containing vessel to be filled with electrolyte.

The zinc scale is screwed on to the fibre pieces YY; the latter are attached to the glass-containing vessel by means of the springs MM, which are prevented from scratching the glass by rubber bands RR. To facilitate setting the zero, the scale is provided with slotted holes, so as to be vertically adjustable for about half an inch up or down.

The vessel and scale is securely fixed within

a case F of cast-iron, tin, or other convenient metal by means of a fibre clip S screwed on to the back of the case.

Flexible leading-in wires of copper, soldered to the free end of the rods, pass through rubber washers to the terminal blocks, which are placed in an external chamber, which is then filled with sulphur or other insulating material, thus making the terminals separate and hermetically sealed from the working parts of the meter.

(ii.) *Calibration and Testing.*—The meter was calibrated and frequently tested, first for uniformity of the glass tube by volumetric determination, and for the actual accuracy by a similar method based on a knowledge of the electro-chemical equivalent of the electrolyte. This value is about 0·336 gramme per ampere-hour, but Bastian appears to have found in practice that the value under the conditions of the saturated atmosphere obtaining in his meter was higher than this and worked with 0·346 gramme per ampere-hour.

The Bastian meter required a pressure drop of the order of from 2 to 3 volts at full load, and owing to polarisation was not suitable for use in conjunction with a shunt for measuring large currents. It had the further disadvantage that the tube required to be filled from time to time with water.

§ (14) LONG-SCHATTNER ELECTROLYTIC METER.—About 1900 Schattner produced an electrolytic meter depending for its action on the deposition of copper in a solution of sulphate of copper of specific gravity 1·080, to which was added 1 per cent of sulphuric acid. In this instrument a pivoted arm carries at one end a copper plate which is immersed in the electrolyte and at the other end a counter-weight. The earliest meter was fitted with a graduated arm and counter-weights which were moved along until the loss of copper was balanced, the reading of the arm indicating the actual units, but the type used in practice was the prepayment meter (shown in *Fig.* 13), which shows the

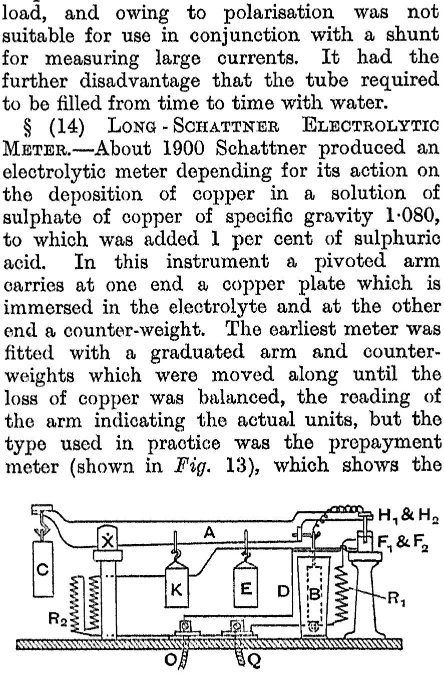


FIG. 13.

details of the construction, the pivoted arm A carrying the copper plate B and two small pails K and E at one side of the pivot X and the weight C at the other side of the pivot. The copper plate is suspended in the electrolyte contained in the copper vessel D.

The resistance R_1 is a permanent shunt across the bath, and thus only a proportion of the current actually flows through the electrolytic cell. F_1 and F_2 are mercury cups, one being full of mercury and the other only half full, H_1 and H_2 being the contact ends of bridging connection. (The arrangement of mercury cup connection is shown more fully in *Fig.* 14.) When first installed

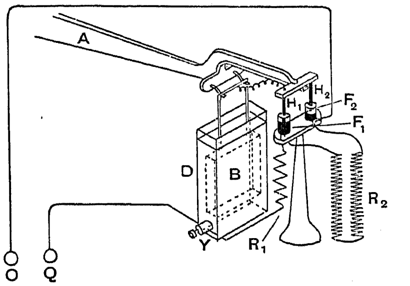


FIG. 14.

the counter-weight was adjusted so that the arm was nearly balanced. Then the insertion of a silver coin in the pail E depressed the arm and made contact between the mercury cup and allowed the current to flow through the meter. When an amount of copper equivalent to the weight of the coin had been transferred from B to D the weight C brought the arm over to an extent sufficient to raise the contact H_2 out of the mercury. The current was not, however, entirely broken but passed through the resistance R_2, which resistance served to dim the light and so give warning that further payment was necessary.

This meter is interesting as one of the earliest types of electrolytic and also pre-payment meters, but it is little, if ever, used at present, owing mainly to difficulties of corrosion.

§ (15) WRIGHT ELECTROLYTIC METER. (i.) *Description.*—The Wright meter, introduced about 1902, employed the principle of the deposition of mercury. An electrolytic cell is employed, containing as an anode a pool of metallic mercury, an electrolyte which is an aqueous solution of a salt of mercury, and a cathode of some suitable metal that will not amalgamate with mercury. The passage of a current through the cell results in a deposition of mercury on to the cathode in an amount almost exactly proportional to the current, and in the Wright meter this falls in small particles from the cathode into a calibrated reading-tube graduated in units. Since the electro-chemical equivalent of mercury is 7·457 grammes per ampere-hour, the weight of mercury required to record an amount of 100 kilowatt-hours at a pressure of 100 volts would be about 7500 grammes:

this, if the whole of the current was passed through the mercury, represents an amount much too bulky and expensive for commercial requirements. Wright therefore arranged his meter so that the greater proportion of the current was taken through a shunt, the actual current passing through the electrolytic cell being only about 0·02 ampere. This involved (1) that the electrolyte and electrode be such that no polarisation or back E.M.F. was set up in the cell, since if it were the meter would not register accurately at all loads, and (2) that compensation should be provided for the effect of temperature changes in the electrolytic cell.

Wright first used a 10 per cent solution of mercurous nitrate, to which was added about 1·25 per cent of free nitric acid, and this solution proved satisfactory for some years, when it was found that, owing to decrease in the acid content with a corresponding increase in mercurous nitrate, there was not sufficient acid present to keep the nitrate in solution, and the latter crystallised out.

Hatfield[1] investigated the cause of the trouble, and finally selected a solution of double iodide of mercury and potassium. The salt in this case is very soluble, the back E.M.F. is low, and Hatfield's experiments, which have been borne out in practice, suggested that a mercury electrolytic cell made with this solution would be stable over a long period. Hatfield discusses a number of other points which may affect the use of his solution, and in particular (*a*) the action of light, which was found to be negligible. (*b*) The action of the electrolyte on glass, when it was found that flint-glass had the slight film of reduced lead, sometimes left by the blower, dissolved off and deposited as a double iodide of lead and mercury in the form of minute crystals. This did not affect the cell in use, but Jena glass, in which the effect was not produced, was finally employed. (*c*) Suitable electrodes. For these he found that platinum, which served perfectly as a cathode in mercurous solutions, became immediately amalgamated when used in a mercuric solution, and in consequence the mercury collected at the cathode and dropped in large drops instead of fine particles. Tantalum was found to serve admirably, but was not easy to use; and finally a cathode of pure iridium, sand-blasted to give it hardness, was selected. (*d*) The circulation of the electrolyte.

In addition to the trouble due to crystallisation the earlier meter had the defect that considerable vibration resulted in a portion of the metallic mercury anode being shaken into the reading-tube with a consequent error in reading, and the construction of the meter was modified to meet this point in the instrument known as the "Wright's Electricity Meter-Hatfield Solution," which was made in 1907, and is substantially the meter in use to-day, although a material which is claimed to be an improvement on the iridium is now used for the cathode. For ordinary supply these meters are supplied in two types—one in which the total record is taken in a single tube, and the other in which the mercury is deposited in a bent tube with a scale of 100 units which, when full, automatically siphons over into a lower receptacle, which is graduated in divisions each representing 100 units, thus providing a total record of several thousand units.

(ii.) *Resetting.*—When, or before, the lower tube is filled with mercury the meter must be reset or it will cease to register. The resetting is effected by tilting the tube and allowing the mercury in the tube to run back into the anode receptacle. Objections have been made to the resetting—first, on the grounds that it is necessary to open the meter in order to actuate the tube, an objection which is met in later types by containing the meter element in a case which is hinged on to a fixed base; and, second, that when the meter is reset the record is destroyed. This latter point is very largely met by the siphon-tube type, which provides for a continuous record extending over several years before it is necessary to reset. Even with the single-tube type on a house, or other small supply where the consumption probably does not exceed 200 units per annum, the use of a meter having a 400 kilowatt-hours scale would involve resetting only once in two years.

(iii.) *Details of Construction.*—*Fig.* 15 shows the details of the electrolytic cell. The anode A of mercury is fed by the additional mercury in the reservoir F, the function of the latter being to maintain a constant level of mercury in the anode chamber. When, in the use of the meter, mercury is taken from the anode the fall of level allows some of the electrolyte to escape into F and a corresponding amount of mercury to flow out. The cathode C is in the form of a short tube, and the initial difficulty with meters of this type, due to mercury being shaken by vibration or shock from the anode into the reading-tube, is eliminated by means of the glass ring or fence G. The fine lines and arrows indicate the stirring action in the electrolyte due to the passage of the mercury, which action, according to Hatfield (*loc. cit.*), was sufficient to ensure that the strength of the electrolyte is uniform. Details of the electrical circuits are shown in *Fig.* 16, where A is the anode, B the glass fence, C cathode, and D and E negative and positive terminals respectively. The main current flows through the shunt K, in parallel with which is a circuit

[1] *Electrician*, lx. 279 and 319.

comprising the electrolytic cell and a resistance H. The resistance of the shunt K for a

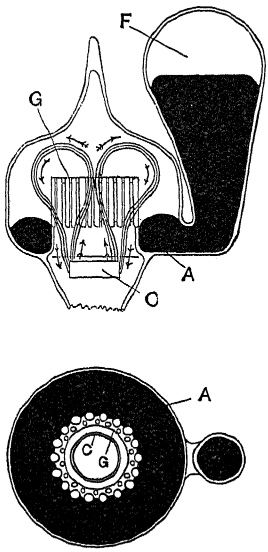


FIG. 15.

10-ampere meter is 0·1 ohm. The resistance H, hermetically sealed in a glass tube, con-

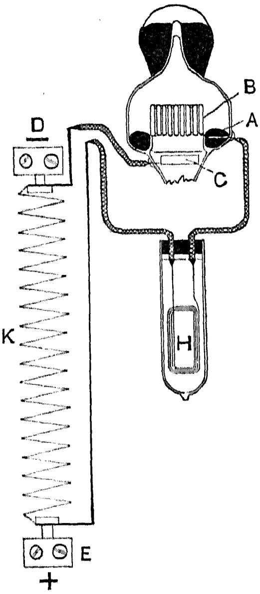


FIG. 16.

sists of a wire having a large positive temperature coefficient so adjusted that its change of resistance with temperature nearly compensates for the negative temperature coefficient of resistance of the electrolytic

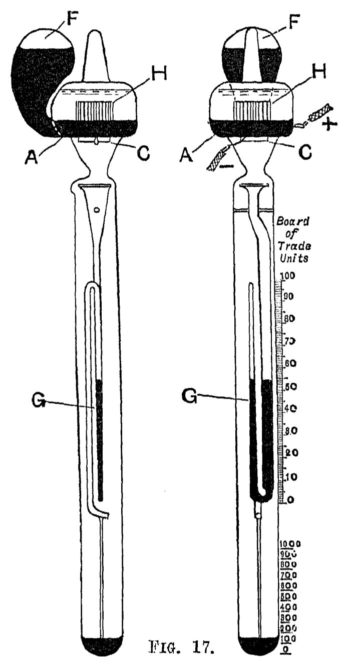


FIG. 17.

cell. *Fig.* 17 shows the complete element of the siphon-tube type, and *Fig.* 18 a side view of a complete meter in position for resetting.

The standard type meter uses a specially selected single tube and is fitted with a mirror-scale.

(iv.) *Temperature Coefficient.*—The temperature coefficient of the electrolyte is probably from 1 to 2 per cent for 1° C., the resistance decreasing with increase of temperature: this is compensated for by making the series resistance H in *Fig.* 15 of iron-wire, which has a temperature coefficient of approximately 0·5 per cent for 1° C. This requires that the compensating resistance must be from two to four times that of the electrolytic cell. The pressure drop therefore must be increased proportionately, and this is generally not less than 1 volt.

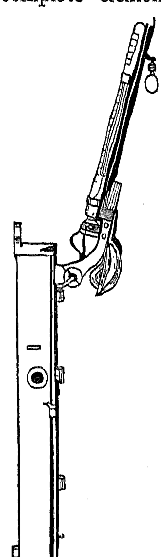


FIG. 18.

(v.) *Calibration and Testing.*—The reading-tubes require to be carefully selected for uniformity and size of bore, but for final calibration and testing there is no alternative to a test carried on sufficiently long to cover the complete range of reading. Instruments of ordinary grade are usually compared with a standard meter, but the better type have to be tested either by maintaining a steady current for the long period required or by comparison with a copper voltameter.

(vi.) *Accuracy Characteristics.*—The accuracy of the meter is very good, it being found in general that the commercial types are within limits of error of ±2 per cent at all loads from one-twentieth to full load, and that the standard type meters are within an error of 1 per cent over the same range of loading. Moreover, the errors are to a fair degree invariable, and with a table of corrections a standard meter of this type can probably be used to an accuracy of ±0·2 per cent of full-scale reading.

§ (16) Mordey-Fricker Electrolytic Meter.—This instrument, introduced about 1907, is a copper-deposition meter suitable only for small-current systems and for pre-payment. The essential feature of the meter is that a roll of thin wide copper ribbon is fed into the electrolytic cell and is the anode of the cell. The cathode consists of a copper plate, the electrolyte being a solution of copper nitrate of specific gravity about 1·3. The cell is contained in a glass jar on which a red line is drawn to denote the correct level of the electrolyte. The working parts of the meter are shown in *Fig.* 19. The roll of copper strip R and A is perforated in the centre with a series of holes which engage in the teeth of a sprocket wheel S. The introduction of a coin makes mechanical connection between the handle H and the spindle geared to the sprocket wheel, and allows the latter to be turned by a definite amount; a given length of the copper strip is thus fed into the electrolyte. When sufficient current has passed to deposit the whole of the piece of strip in the electrolyte the current is automatically broken at the surface. The copper strip is stamped with numbers which denote the number of coins that have been inserted. Insulating rods are fitted to prevent the anode from coming into contact with the cathode, and users are warned not to feed too great a length of the strip into the cell.

The pressure drop across this meter varies somewhat with the amount of copper strip in immersion and will be of the order of from 2 to 3 volts at 3 amperes.

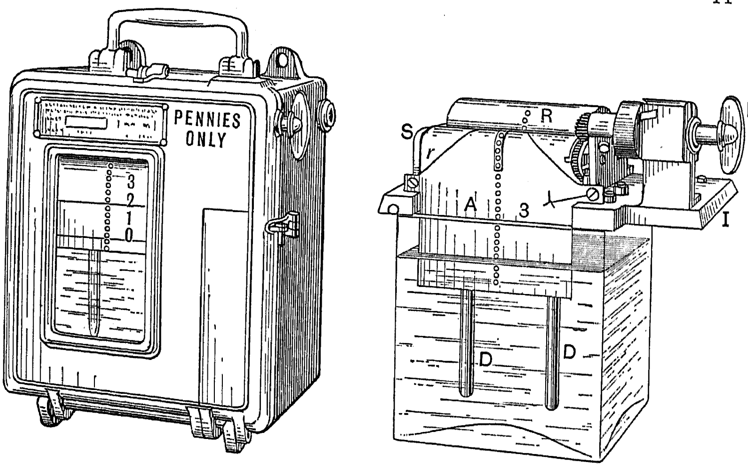


Fig. 19.

§ (17) Holden Electrolytic Meter.—Holden[1] described two meters, in both of which the conversion of water into gas with the passage of a current is employed. Earlier experimenters had measured the volume of the gas produced in order to determine the current, the errors due to temperature and atmospheric pressure being large and difficult to overcome. Bunsen originally suggested that the electrolyte should be weighed and the current determined by the loss of weight, and one of Holden's meters was based on this suggestion (see *Fig.* 20). In this meter the weight of A, the iron vessel filled with a solution of caustic soda and fitted with sheet-nickel electrodes BB, is supported upon four springs, two only of which, marked C_2, are shown in the figure. The rack H is attached to the iron case, and as the weight of the electrolyte decreases

[1] *Journal I.E.E.* xxxvi. 393.

operates the pinion F and indicates the loss of weight in units on the scale. The springs JJ carry the current to the electrodes, the guide lines *aa* being fitted to ensure that the iron vessel will remain central. The gas evolved is allowed to escape at the top of the tube and the tube requires to be filled with water from time to time.

Holden's second meter, while still employing the process of the evolution of gas, provided that the whole of the gas should dissolve at one electrode and reappear at the other.

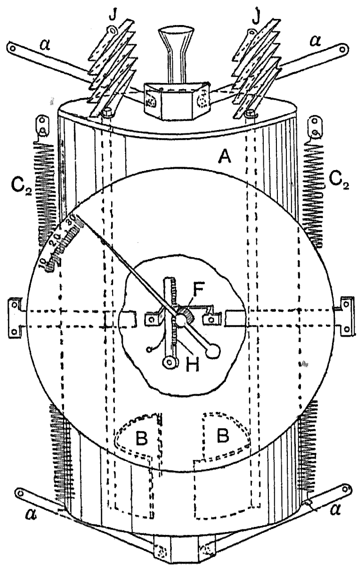


FIG. 20.

This meter is shown in *Fig.* 21, in which the electrolyte of phosphoric acid is contained in a sealed glass tube, the anode A and cathode C being both of platinum covered with platinum black. The anode is only partly immersed in the electrolyte S, the space above being filled with hydrogen. When the current flows hydrogen is evolved at the cathode which escapes up the reading-tube and displaces the electrolyte, and the oxygen which is evolved at the anode immediately combines with the stored hydrogen. The hydrogen first stored at the anode is thus transferred to the reading-tube until it is full, when the operation of tipping the meter allows the gas to return to the space above the cathode.

With this arrangement of electrodes and electrolyte there is little or no depolarisation, and in consequence the meter can be connected across a shunt T, in *Fig.* 20, which drops 1 volt at full load, the resistance R being in series with the tube and serving to compensate for temperature. The resistance of the electrolytic cell and its series resistance is

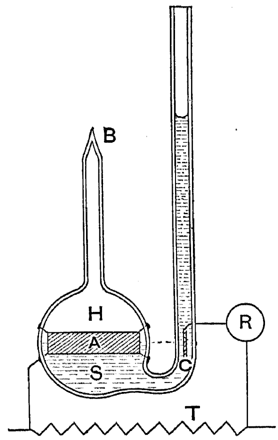


FIG. 21.

of the order of 10,000 ohms, so the current through the cell is very small.

Both of these meters are interesting applications of the principles of electro-chemistry. The second meter has been modified slightly, mainly by different arrangements of electrodes, but neither has yet been used to any extent commercially.

§ (18) SHUNT MOTOR-METER.—Another type of meter in which the principle is of great interest is that generally known as the commutator ampere-hour meter. The principle has been used by a large number of makers both in this country and abroad, but it would appear that it was originally due to Swinburne, who described it in about 1891. In this case a simple motor (see *Fig.* 22), consisting of a

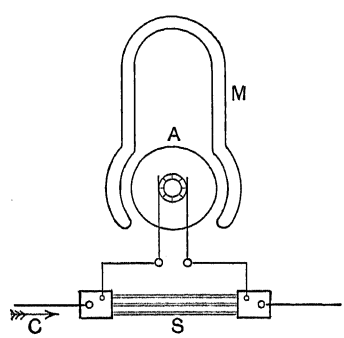


FIG. 22.

three-coil armature A rotating between the poles of a permanent magnet M, is connected across a shunt S, through which flows the main current I. The principle of the meter is

that if a current is passed through the combined circuit of shunt and meter armature the armature rotates and rises to a speed at which the E.M.F. produced in the armature is equal to the E.M.F. across the shunt, and consequently the motor is taking no current and doing no work. This is assuming that the armature has no resistance and that there is no friction in either brushes or bearings. These ideal conditions are of course not attainable, but in the very large number of meters that have been made it has been claimed that the torque is so large that the effect of friction is largely negligible. As Evershed [1] has shown, the law of the meter can be obtained thus:

Let I be the main current, ι and ι' the currents through the armature and shunt respectively, n the number of turns of the armature per second, R the armature resistance, r the shunt resistance, N the number of turns of the armature per parallel, B the flux, and f the moment of friction. Then the back E.M.F. is $4n\mathrm{BN}$.

Since the work done on the armature just balances the friction, we have

$$4n\mathrm{BN}i = 2\pi nf.$$

Also
$$4n\mathrm{BN} + \mathrm{R}i = r\iota',$$

$$\iota + \iota' = \mathrm{I}.$$

Thus
$$4n\mathrm{BN} + \frac{2\pi f(\mathrm{R}+r)}{4\mathrm{BN}} = r\mathrm{I}.$$

Hence
$$n = \frac{r\mathrm{I}}{4\mathrm{BN}} - \frac{2\pi f(\mathrm{R}+r)}{(4\mathrm{BN})^2}.$$

This formula is for a meter without a brake, the addition of a brake lowers the speed and increases the armature current but does not affect the product $2\pi f(\mathrm{R}+r)$. Thus if the product of the friction and the resistance is low the meter will be more accurate, and also if the flux is increased the required speed is decreased. Since with twice the flux the error term will be reduced to one-quarter, the meter would not only run slower but would be more accurate.

The earliest meter based on this principle was that invented by O'Keenan and made in Paris. This instrument (*Fig.* 23) consisted of a drum armature A having four coils rotating between the poles PP of a permanent magnet M, the four-part silver commutator C, and brushes S being above the armature. There is no brake disc, and in consequence the speed is very high, probably about 250 r.p.m., and since the armature is comparatively heavy the wear of the bearings is considerable. The torque of the meter at full load is of the order of 28 gr.-cm., and with this high torque and speed the effects of friction are not appreciable except at loads of about one-tenth maximum. The pressure drop at full load is about 2 volts.

[1] *Journal I.E.E.* xlvii. 69.

A modification of this type made by a number of makers in Germany, and later by

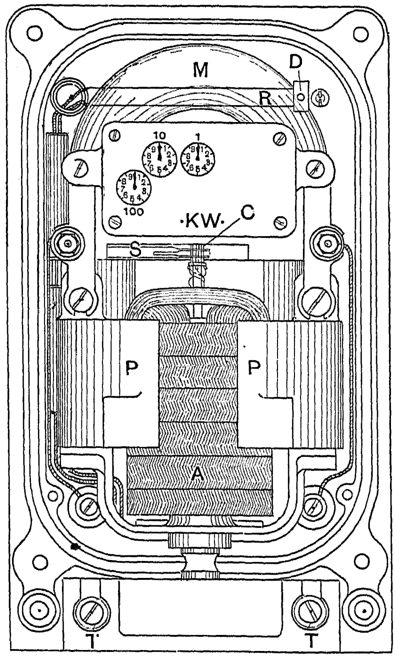


FIG. 23.

several in England and America, consisted in disposing the three armature coils symmetrically about the spindle, as shown in *Fig.* 24. The coils are supported on an

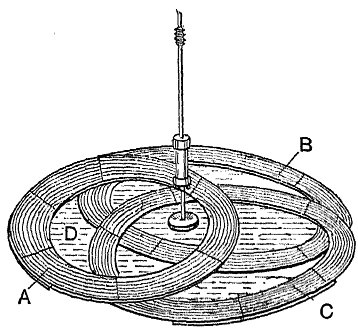


FIG. 24.

aluminium disc and completely enclosed by an aluminium cover. This armature rotates between the fields of a permanent magnet at a speed proportional to the pressure drop across a shunt, and the aluminium disc and cover act as a Foucalt eddy current brake.

Fig. 25 shows the complete meter, in which G is the three-coil armature resting on an aluminium disc and completely enclosed by an aluminium sheet; B is one of two permanent magnets, one being placed on each side of

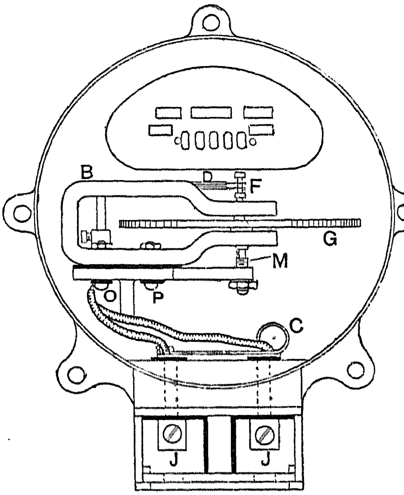


FIG. 25.

the spindle. The three-part commutator F and the brushes D are of 18-carat gold, the commutator being of very small diameter, so that friction is reduced to a minimum. The current is led into the terminals JJ and passes through the shunt C, across which are connected the brushes.

The temperature coefficient of this meter is very small, since the current through the armature is controlled largely by the back E.M.F., which is independent of the resistance of the armature coils.

The principle of this meter is most interesting, and it would appear that within it was contained everything essential for a cheap and accurate instrument. Actually, however, owing probably to the commercial demand for a cheap meter with consequent reduction of size, it has not been very satisfactory in use, although the evidence as to its behaviour is not always consistent. In France, Germany, and Holland, and in other countries, large numbers of the meters are employed and apparently give good service; in Great Britain, however, they are not so satisfactory, and the reason is difficult to find. It is frequently ascribed to the difference in climate, but it is more likely that in these other countries the requirements of accuracy are not so high as in Britain, and also that the more accurate mercury and electrolytic meters are not so freely available abroad for comparison. Great care has, however, been exercised, both in construction and selection of materials, to make the meter as accurate as possible. In order to secure low resistance and eliminate friction the commutator and brush-tips are now usually made of gold, the insulating material between the segments of the commutator being of a high-grade, non-hygroscopic material. To reduce friction to a minimum, jewelled bearings are used and the commutators are reduced to the smallest possible diameter; and to eliminate wear, the speed, which in some meters has been as high as 300 r.p.m., has been reduced to more nearly 100 r.p.m. with a pressure drop of 1 volt across the shunt. There is, however, still some doubt as to the consistency of the meters over a long period of time, and it is possible that this is due to absence of information as to the correct pressure on the brushes to secure (*a*) the minimum contact resistance, (*b*) uniformity of contact resistance at all speeds, and (*c*) minimum wear and consequent increase of friction at the commutator. The latter point is met in part by the use of a device which automatically changes the position of the brushes and so distributes the wear over the commutator.

Further, with the very light brushes used, the meters have been found to be somewhat susceptible to damage when a large momentary current passes, such as occurs in consequence of a short circuit.

A typical accuracy curve given by Ratcliff [1] is shown in *Fig.* 26. Ratcliff states

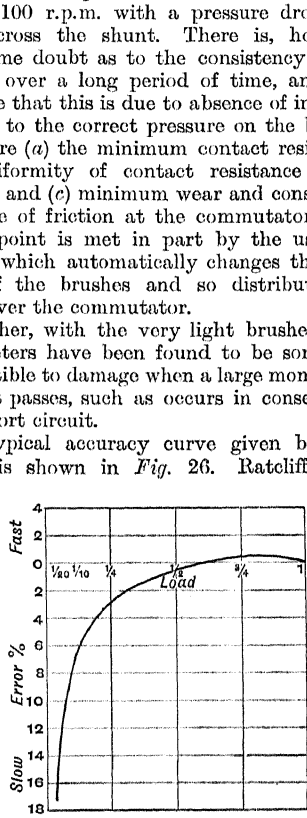


FIG. 26.

that this represents the average performance of a number of meters.

II. WATT-HOUR METERS

§ (1) MOTOR METERS, COMMUTATOR TYPE.—As in the case of motor ampere-hour meters the fundamental principles of motor type watt-hour meters are those of the electric motor. Generally the armature, wound of very fine wire, is connected in series with a

[1] *Journal I.E.E.* xlvii. 3.

resistance across the main conductors of the circuit of which the energy is to be measured. The armature current is thus proportional to the pressure of the supply, and the fixed field coils carry the main current. The torque on the armature, being proportional to the product of the current and the pressure, measures the power supplied, and the meters, arranged to read the product of this and the time, thus give the energy, usually in kilo watt-hours. The direction of the two magnetic fields will be seen from *Fig.* 1, the arrows marked F and

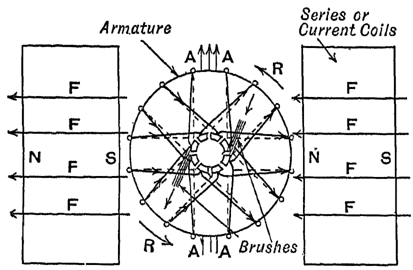


FIG. 1.

A showing the direction of the magnetic fields, N and S denoting the polarity of the field.

The application of the motor principle to energy meters is probably due to Ayrton and Perry, who patented an instrument in 1882, but it was developed and brought into practical use mainly by Elihu Thomson, and is generally known as the Thomson meter. *Fig.* 2 is a

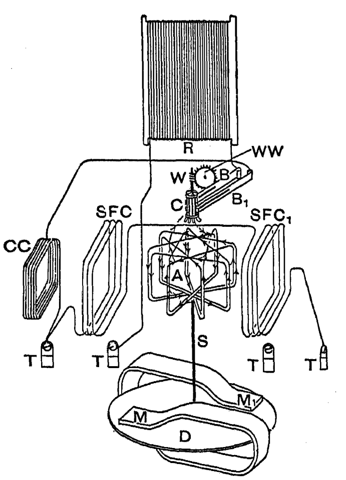


FIG. 2.

diagram of the early Thomson meter, A being the armature, B and B′ brushes, C commutator, D eddy current brake disc, MM permanent magnets for brake system, R resistance in series with the armature, and SFC and SFC_1 the main current coils. The use of the coil CC is a device to compensate for friction, which is discussed later.

Evershed [1] describes methods for calculating the armature torque, brake torque, and frictional resistance for a meter of this type, as follows:

(i.) *Armature Torque.*—The driving torque may be calculated from the formula

$$Q=\frac{HNAI}{10\pi},$$

which gives the torque Q in terms of H, the field of the main-current coils, A the area of the average turn of wire on the armature, N the total number of complete turns in the armature winding, and I the whole current in the pressure circuit from brush to brush.

For the current in each coil is I/2 and the armature is equivalent [2] to a magnet of moment $\frac{1}{2}$NAI/10; the torque on this when the normal to the coil is inclined at an angle θ to the direction of H is $\frac{1}{2}$HNAI sin $\theta/10$; the average value of the torque for all values of θ between 0 and π is therefore HNAI/10π, for the average value of sin θ is $2/\pi$.

A sufficiently close approximation to the true mean value of the field throughout the space occupied by the armature is obtained by calculating H at several points of the axis of the main coils, and taking an average. When the coils are rectangular, the formula

$$H=\frac{8I\sqrt{p^2+q^2}}{pq},$$

which gives the field at the centre of a wire rectangle whose sides are p and q, may be employed. If the coil is square, this reduces to

$$H=\frac{8I\sqrt{2}}{p},$$

p being now the side of the square. The field at the centre of a circular wire is

$$H=\frac{2\pi I}{r}.$$

Thus if $r=\cdot56p$ the circular and square wires will have the same field; hence for purposes of calculation a circular coil of appropriate radius may be substituted for the square coil.

The torque thus calculated is found to agree within 2 or 3 per cent with that obtained directly by measurement.

(ii.) *Brake Torque.*—The difficulty of determining the average path of the eddy current in a given brake conductor prevents the predetermination of the torque unless the effective electrical resistance of the current path in a brake of similar shape has been previously ascertained. There are, however, some general principles applying to the design of Foucault

[1] *Journal I.E.E.* xxix. 771, from which the description is taken.

[2] See "Electromagnetic Theory," § (4); "Dynamo Electric Machinery," § (6).

eddy current brakes which may be shortly stated here.

Consider a cylindrical conductor, of radius r, spinning on its axis between the poles of a magnet. Let the polar gap have a breadth b and axial length l, and let the field within the gap be H. Then if R is the electrical resistance of the current path and ω the angular velocity of the cylinder, the current generated will be

$$I=\frac{Hlr\omega}{R}.$$

Now the force between conductor and magnet is HlI, hence the torque Q will be

$$Q=\frac{H^2l^2r^2\omega}{R}. \quad . \quad . \quad . \quad . \quad (1)$$

When the cylinder extends axially for some distance beyond the poles in both directions, R becomes simply proportional to the resistance of the metal in the gap, and this resistance is $\rho l/bt$, t being the thickness of the metal and ρ its specific resistance. We can therefore write

$$R=k\frac{\rho l}{bt}.$$

The value of k depends on the shape of the polar area, and on the extent of conductor outside that area. Evershed finds k varies from about 3 for a square or circular area of gap up to 6 or 7 with oblong poles, becoming as high as 9 or 10 when there is very little metal beyond the poles. Inserting the value just given for R in equation (1) and multiplying both numerator and denominator by b, we get

$$Q=\frac{H^2l^2b^2r^2\omega t}{k\rho bl},$$

or

$$Q=\frac{B^2r^2\omega t}{k\rho bl}, \quad . \quad . \quad . \quad . \quad (2)$$

where B is the total induction through the brake cylinder, from which we see that for a given value of B the torque increases as the area bl is diminished. This deduction is well borne out in practice, and by giving the brake magnets conical poles ending in a circular gap of small diameter the torque for a given weight of magnet is considerably increased; or, what is generally more important, for a given torque the weight of metal in the brake may be greatly reduced by using magnets with conical poles.

(iii.) *Frictional Resistances.* — When mechanical friction is a fairly large percentage of the full load torque, air friction may be safely neglected; the only retarding forces are then mechanical friction, which is practically constant, and the brake which is proportional to the speed. Hence if s_1 and s_2 are the observed speeds with two torques Q_1 and Q_2, we have

$$Q_1=f+\beta s_1, \quad Q_2=f+\beta s_2,$$

where f is the torque required to overcome friction, and β is the brake torque at unit speed. Hence

$$\beta=\frac{Q_2-Q_1}{s_2-s_1}, \quad \text{and } f=\frac{Q_2s_1-Q_1s_2}{s_2-s_1}.$$

To get accurate results, stray fields must be eliminated by reversing the current in the main coils and taking a second set of readings with the same numerical values for Q_1 and Q_2.

It will be clear that friction alike of air, bearings, brushes, and gearing must be largely eliminated or compensated for to ensure correct registration at light loads.

§ (2) Frictional Defects in Motor Meters. —Schmiedel [1] and Fitch and Huber [2] have investigated the separate sources of friction in meters, the latter giving a number of curves showing that the total friction in different American makes, all of the motor commutator type, varied from 0·7 per cent to 1·5 per cent of the full load torque. Of this total from 55 per cent to 75 per cent was due to friction at the brushes, from 5 per cent to 20 per cent occurred in the bearings, from 3 per cent to 15 per cent in gearing, and from 10 per cent to 15 per cent for air resistance. Details of the constants of the moving elements are also given by Fitch and Huber, and these have been combined in the table on the following page with the values of friction taken from the curves. It should be noted that these particulars are generally the average of the results obtained with three meters of each type, and while they do not necessarily apply to all types and sizes of meters or to conditions after the instruments have been in normal service for some time, they furnish an interesting comparison of the relative effect of the torque, size of commutator, and pressure on the brushes in meters of this type and size.

§ (3) Frictionless Meters. — Evershed [3] investigated the effects of friction on the older type of meter and designed a "frictionless" meter in which the friction at the bearings, and consequently the wear, was eliminated by means of a magnetic suspension and the brush friction by an elastic commutator, details of which are given in his paper as follows:

"The essential working parts of the meter are shown in *Fig.* 3. The armature A, brake disc F, and coils D_1, D_2, which drive the counter, are mounted on a mild steel axle a. The axle has a hard steel point at its lower end resting in a jewel cup J; its upper end has no mechanical support, but is maintained in position by the magnetic attraction of an iron rod R, which is magnetised by the brake magnets MM through an iron yoke Y, and forms the

[1] *Verhandlung des Vereins zur Beförderung des Gewerbfleisses*, lxxxix.-xc.; *The Electrical Review* (London), lxix., December 22.

[2] *Bull. Bureau of Standards*, x. 61. The makers are as follows: A, Columbia Meter Co.; B, Duncan Electric Manufacturing Co.; C, General Electric Co. of America; E, Westinghouse Co.; F, Willis Electric Meter Co.

[3] *Journal I.E.E.* xxix. 771.

DETAILS OF A NUMBER OF TYPICAL METERS

Watt-hour Meter.	A.	B.	C.	E.	F.
Torque in cm.g.	7·47	14·31	16·69	14·92	2·85
Weight in g.	98·4	156·2	101·8	96·1	97·4
Ratio of torque to weight	·076	·092	·164	·155	·029
Diameter of commutator, cm.	·265	·465	·240	·240	·195
Number of commutator segments	3	8	8	8	3
Thickness of disc, cm.	·115	·150	·065	·065	·115
Diameter of disc, cm.	11·40	13·35	12·66	12·70	8·54
Voltage drop across armature with 110 volts on potential circuit	45·1	53·8	37·6	40·1	32·7
Brush pressure, g.	·24	·57	1·5	1·8	·34

Torque.	Unit.	A.	B.	C.	E.	F.
Brush friction	cm.g.	0·038	0·057	0·127	0·090	0·024
	Per cent full load	·50	·40	·76	·60	·84
Gear friction	cm.g.	·010	·006	·010	·004	..
	Per cent full load	·13	·04	·06	·03	..
Bearing friction	cm.g.	·011	·023	·013	·012	·013
	Per cent full load	·15	·16	·08	·08	·44
Air friction	cm.g.	·007	·016	·018	·019	·007
	Per cent full load	·10	·11	·11	·13	·44
Total friction	cm.g.	·066	·102	·168	·125	·044
	Per cent full load	·88	·71	1·01	·84	1·52

supporting pole. The distance between R and the end of the axle is adjusted by screwing R in the yoke Y until the vertical force nearly

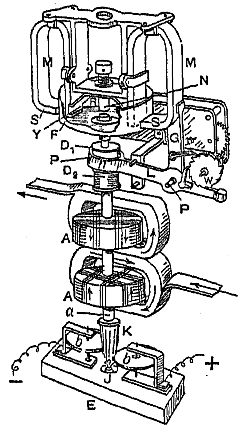


FIG. 3.

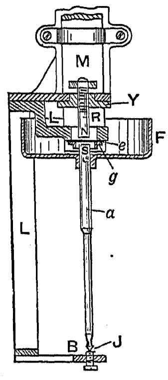


FIG. 4.

suffices to lift the whole weight of the armature, brake, and other parts attached to the axle. The arrangement is more clearly seen in *Fig.* 4, which is a section through the magnetic pivot. A magnetic pivot of this type may easily be made to support a weight of from 100 to 200 grammes.

"The commutator K is placed beneath the armature, and the wheel brushes *bb* are pivoted in frames attached to an ebonite plate E. The commutator and brushes are shown on an enlarged scale in *Fig.* 5. The segments are fine iridio-platinum wires supported at one end in an ivory collet and entirely free at the other end, where they impinge and roll on the brush wheels. The commutator is about 3 mm. in diameter at the rolling circle: the wheels are about 36

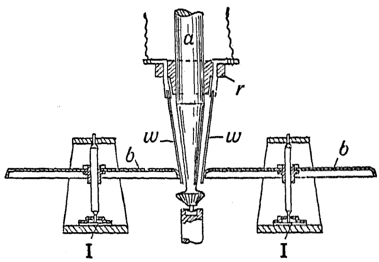


FIG. 5.

mm. diameter, so that they make one revolution to twelve revolutions of the commutator.

"The pressure current is led to the brush wheels through their frames, and to ensure good contact between frame and wheel the step bearing of each wheel is an iridio-platinum pivot resting on a flat plate of the same metal.

"A drum winding is used for the armature, a break being made, in the ordinary course of winding, in each parallel, in order to insert the two train-driving coils D_1, D_2. Thus D_1 is in series with one of the two parallels of the drum winding, and D_2 in series with the other; they are consequently each traversed by one-half of the whole armature current, and

since they are, electrically, a part of the armature circuit, the current in them is reversed twice in each revolution of the axle. D_1 and D_2 are inserted at corresponding points of the drum winding, so that their currents reverse at the same instant, and they are coupled up so that the two currents flow in the same direction and to all intents and purposes D_1 and D_2 behave like one coil."

The Stanley meter (America) also used a magnetic suspension.

The Evershed arrangement worked extremely well in the few instruments that were made, but it was apparently too costly for commercial use, and later developments have been in the direction of reducing friction by making the armature as light as possible and by the use of the compensating coil CC which is shown in *Fig.* 6 and *Fig.* 2. This device consists of a coil connected in series with the pressure circuit, which is placed in such a position relative to the armature that its magnetic field is at right angles to the axis of the rotor. It thus exerts a driving force in the normal direction of rotation, and this by moving the position of the coils can be adjusted so as to compensate for the total friction. In practice it is usual to maintain this compensating force at a slightly lower value than that required to overcome the friction, since an increase of the supply pressure or a small amount of vibration would tend to make the armature rotate when no current was flowing through the main coils. Further protection against the danger of running on no load is frequently secured by means of a small piece of iron fastened to the brake disc, which serves to arrest the movement when the iron comes under the pole of the magnet.

§ (4) Elihu Thomson Meters. (i.) *Ordinary Type.*—In the later types of Thomson meter a circular armature and field coils are used together with an aluminium disc. A diagram of the working parts is shown in *Fig.* 6. The whole of the meter system is borne on a solid frame which carries at the bottom the jewelled bearing and the four separate magnets which provide the flux for the brake disc, at about the centre, the current coils, and at the top, the upper guide bearing and the counting train.

(ii.) *Astatic Type.*—The ordinary type of meter was made in sizes up to 600 amperes and for any normal range of supply pressure. In a later (astatic) type, produced in 1916, obviously to meet difficulties experienced with the older instrument when erected in a position exposed to external stray fields, two sets of current windings and two complete armatures are carried one above the other on the same spindle. The two sets of current coils are in parallel and the two armatures are in series, but in both cases the direction of the current is reversed. Thus, the two armatures exert a force in the same direction, but since the direction of the current in the two coils is opposite in direction, any external stray field, if it affects both armatures to the same extent, would be compensated for. This meter is made for currents up to 600 amperes.

For meters designed for currents up to 15,000 amperes, the current coils are necessarily of much larger sectional area, and the eddy current brake with its magnet is enclosed in an iron box.

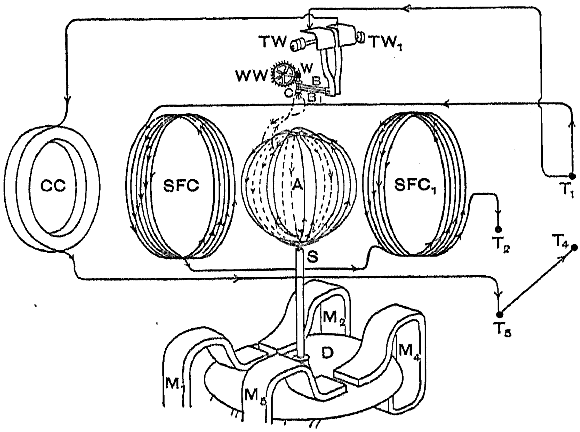


Fig. 6.

§ (5) British Thomson Houston Meters.—The Thomson meter was also developed in England for use on large power circuits. For currents up to 600 amperes, a single armature of the Thomson type was used, the two main-current coils being wound of copper strip. For larger currents than this, a straight length of bar is used. An illustration of a 2000-ampere meter is given in *Fig.* 7. The main current connectors of the meter are made by means of threaded copper stems which are soldered direct into the meter element. These stems project through the base and the main current connections are fixed to them by means of the thin nuts NN (see *Fig.* 8). The whole of the meter system is borne on these two stems SS which allow of the transfer of the meter from its own base B to a position on a switchboard without affecting any of the working parts. Each of the two armatures mounted at right angles to each other on a

common spindle has two coils joining each armature to separate commutators, the brake disc and its magnets are enclosed in an iron shielding box. For sizes up to 3000 amperes the armatures have a core of soft iron, but for currents above this a core is not used.

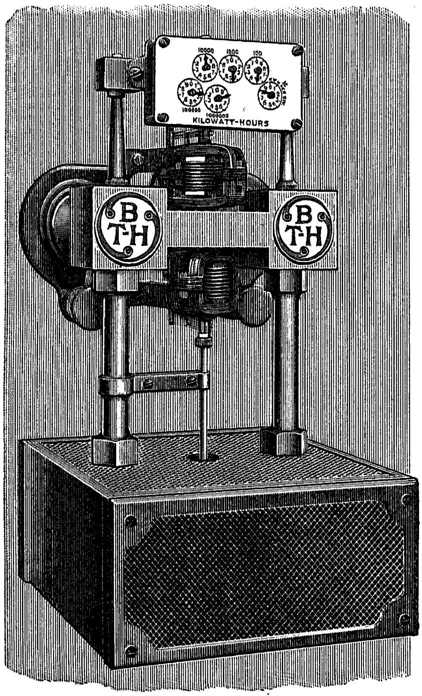


FIG. 7.

The effects of friction are eliminated by means of a small coil somewhat similar to that used in the Elihu Thomson meter. The series resistance, not shown in the *Fig.* 7, was generally mounted in a separate box, but lately, in

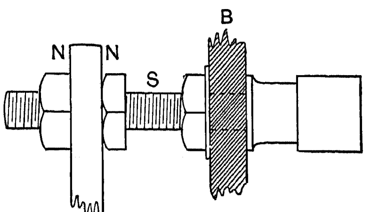


FIG. 8.

order to eliminate temperature effects, is sometimes placed inside the meter.

§ (6) ENERGY LOSSES OF THOMSON METERS.—The energy losses given by Fitch and Huber refer to five watt-hour meters of small size (10 amperes), each made by a different firm, and they may be taken as fairly applicable to all small-size meters of this type. For the larger current astatic meters, the conditions are somewhat different, and the particulars furnished by the makers are as follows:

	Thomson.	B.T.H.
Pressure circuit, watts for each 100 volts .	4 to 7	3·3
Current circuit, watts dissipated (about) .	..	20
Speed at full load, r.p.m.	37	40
Torque at full load, gramme-cm. . . .	15 to 30	20 to 30
Weight of moving system, grammes . . .	175 to 200	280
Ratio of torque to weight (approx.) . .	0·15	0·1
Starting current . .	..	1 per cent of full load current

In the British Thomson Houston meters each of the four armature coils is wound with 1500 turns of 0·004 in. diameter copper wire, the total resistance of the two armatures in series being 1600 ohms. The corresponding coils for compensating for initial friction are wound with 6500 turns of 0·0032 in. diameter copper wire with a total resistance of about 800 ohms. Thus, the complete internal resistance is about 2400 ohms; to this must be added the external resistance, which, for a circuit, whatever the pressure, must be of such value that the current through the armature coils is approximately 0·033 amperes.

§ (7) TEMPERATURE COEFFICIENT AND SELF-HEATING OF THOMSON METERS.—For a 100-volt circuit, the greater part of the resistance would be in the armature and compensating coil circuits, and the changes in the various components of the meter, due to temperature, are nearly as under:

(*a*) the current in the main circuit will not vary;

(*b*) the current in the pressure circuit will decrease with increasing temperature by nearly 0·4 per cent for 1° C;

(*c*) the brake disc will increase in resistance and the braking effect will be less by nearly 0·4 per cent for 1° C.

Since (*b*) and (*c*) nearly balance, a 100-volt meter should not vary appreciably with temperature. The more usual pressure for circuits on which such meters are used in this country is from 450 volts to 600 volts, and for these the temperature coefficient of the meter will depend on the arrangement of the pressure circuit. In the older type of meters, the series resistance was of a material of negligible temperature coefficient, and, in consequence, the decrease in current in the pressure circuit of a 500-volt meter was of the order of only 0·07 per cent for 1° C. Consequently, since the braking effect decreases by 0·4 per cent, the over-all temperature coefficient for a 500-volt meter was

approximately 0·33 per cent, the speed increasing with increase of temperature. Later, however, the series resistance, mounted in an external box, was made of copper, and this served to compensate entirely for all changes of temperature, provided that the box was erected in such a way that any change of external temperature affected both the external series resistance and the brake disc in the meter to the same extent,—a condition which is not always easily satisfied in practice. Also, the external resistance, if of copper, must be of such ample size that it will not heat appreciably with its own current; if the heating is large, the resistance will increase with increase of pressure, and the meter will not register true watt-hours at pressures slightly above or below the normal.

The question of the elimination of the effect of temperature on a meter of this type is closely connected with the self-heating of the meter. When the energy dissipated in the main current coils is of the order of 20 to 30 watts, the brake disc will be heated to an extent depending on its position relative to these coils, and in the older meters of this type the change in rate due to self-heating, after full load current had been flowing for some hours, was frequently from 3 to 5 per cent. This has been reduced in meters of later types, but it is clear that in order to secure satisfactory accuracy under

(*a*) all normal variations of temperature,
(*b*) normal variations of pressure,
(*c*) any conditions of loading up to full load maintained continuously for a long period,

the following conditions must be satisfied:

(*a*) The whole of the pressure circuit must be of copper or other material having a high temperature coefficient, and of such size and construction that the temperature rise, due to the current in the pressure circuit, is small.

(*b*) The energy loss in, and consequent temperature rise of, the current circuit, must be as low as is consistent with satisfactory torque.

(*c*) The series resistance in the pressure circuit must be so disposed that any change of temperature, either ambient or due to heating from the main current coils, will produce an effect equal to that produced in the brake disc.

§ (8) Accuracy Characteristics of Thomson Meters.—When the conditions as regards

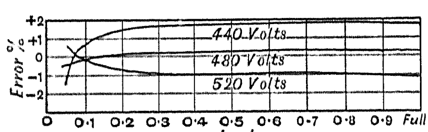


Fig. 9.

temperature coefficient and self-heating are satisfied, the accuracy of this type of meter is good, and it can readily be made to comply with existing requirements. A typical curve showing the errors at normal pressure and at pressures 10 per cent above and below the normal, is given in *Fig.* 9.

§ (9) British Westinghouse Meters. (i.) *Description*.—This meter follows fairly closely the original Thomson principle, but various improvements have been made in design. Particulars of the construction are shown in *Figs.* 10 and 11, and in diagram in *Fig.* 12.

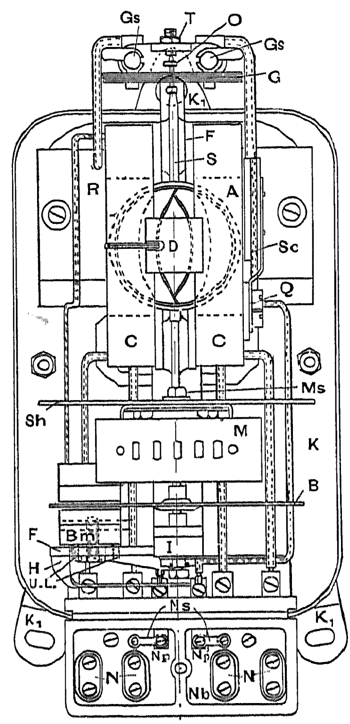


Fig. 10.

The whole of the meter interior, except the resistance R, is carried by the cast frame F, which is fixed to, but insulated from, the outer sheet-metal base K. The main current coils C are fixed to the frame by means of the plates P, which are insulated from, but securely attached to, the coils. A porcelain distance piece D is used to keep the coils rigidly in position. The movement consists of a steel shaft or spindle S carrying an aluminium brake disc B, a pinion or worm W for gearing into the registering mechanism M, and armature coils A. A rounded steel pivot is fitted into the bottom of the shaft in a manner which permits of easy renewal without dismantling the meter. The upper end of the shaft is carried by the top bearing T.

The armature coils A, three in number, are wound approximately 120 degrees apart,

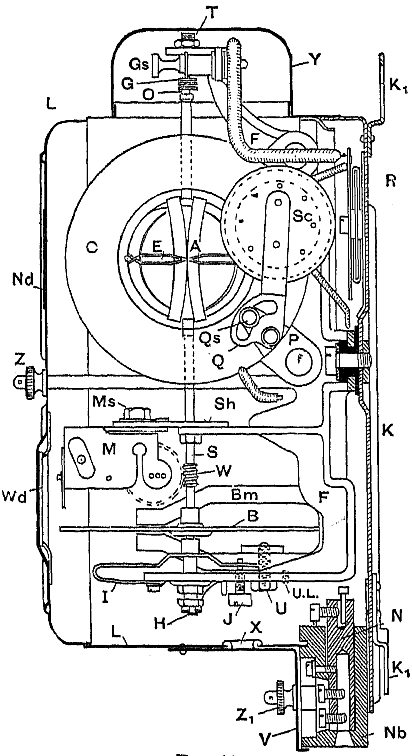


FIG. 11.

and are supported by a specially shaped disc E, mounted on the shaft S. The coils are

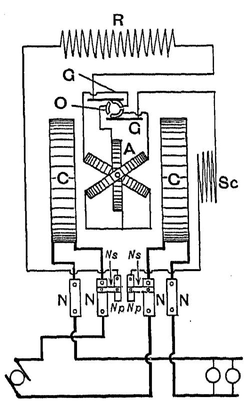


FIG. 12.

connected in star, one end of each being joined to a segment of the commutator O. The current is led to and from the movement by the brushes G, which are secured by the screws G*s*. The lower end of the shaft S is carried by a sapphire jewel bearing, which is supported by a spring H arranged so that the bearing can be quickly removed for examination or replacement. This bearing is also provided with an efficient dust cap.

To prevent the possibility of damage to movement or bearing during transit, a clamping device I is employed, which is manipulated from the outside of the meter by turning the screw J, a hole X being provided in the cover L for this purpose. The necessary resistance for the armature circuit is wound upon the cards R: these are insulated from, but fixed to, the base of the meter. The resistance is of nickel wire in order to compensate for variations in temperature. Also connected into the armature circuit is the compensating coil S*c*, provided to overcome the starting friction of the movement: it is adjustable by means of the slotted quadrant Q. The screw Q*s* securely holds the coil in the desired position. The brake magnet B*m* is supported by three levelling screws UL and secured to the frame F by the screw U. The terminals N are protected by a sheet-metal cover V, which also covers the hole X, and is arranged for sealing, so that the connections to the terminals can be made without interfering with the main seals on the cover. Similarly a separate cover Y is provided to give access to the brushes and commutator without necessitating the removal of the main cover L. Provision is also made for sealing this cover Y to prevent unauthorised removal.

Four main terminals N are provided, and these are embedded in a porcelain block N*b*. Two small pressure terminals N*p*, also let into the porcelain, receive the connection for the pressure, and are connected by links NS to the corresponding main terminals when the meter is in use. By means of these the pressure circuit can be entirely isolated from the series circuit for testing purposes.

(ii.) *Technical Details.*—The technical details of this meter are as follows:

Energy Losses

Pressure circuits watts for each 100 volts	1·5
Current circuits volts at full load .	10
Speed at full load, r.p.m.	40
Torque at full load, gramme-cm. .	6
Weight of moving system, grammes	75
Ratio of torque to weight, approx. .	0·08
Starting current	0·5 per cent of full load current.

The use of nickel for the pressure circuit series resistance and the manner of its disposal provide a very fair degree of compensation for temperature coefficient.

For currents higher than 300 amperes the meters are provided with a shunt, only a small proportion of the current actually flowing through the meter coils. This meter appears to represent an improvement in design and construction on the older types of Thomson meter. It would seem to be most satisfactory for small current circuits up to, say, 200 amperes; for currents much larger than this its use is probably restricted, since the external stray fields set up in circuits of this size might affect the accuracy considerably. It has the advantage, however, that for larger currents a shunt is used, and the meter should prove satisfactory if it can be erected at a sufficient distance from any external magnetic field.

§ (10) SIEMENS-SCHUCKERT METER.—This meter is also of the Thomson type with a single armature, the construction being much the same as in the types before described, with the exception that a flat coiled spring is used to apply the light constant pressure to the brushes. The instrument is, however, particularly susceptible to the effect of external fields, and its use is thus restricted, although in an 8000-ampere meter used with a shunt which carried most of the current a satisfactory record was obtained, when the meter was placed at a distance of 10 feet from the shunt or any other part of the main current circuit.

§ (11) VULCAN METER.—This instrument, made by the Compagnie Anonyme Continentale pour la Fabrication des Compteurs, followed the Thomson type, the only essential difference being that instead of the usual brake disc a cylinder was used which rotated between the poles of a series of magnets supported in a vertical position.

§ (12) OSCILLATING METER. (i.) *Description.*—This meter, introduced by the Allgemeine Elektricitäts-Gesellschaft, Berlin, is of the motor type, but by the use of an ingenious system whereby the current through the armature is reversed by means of a contact arm fixed on the spindle which carries the armature, the use of brushes and commutator is eliminated and the operation of the counting train is effected by a separate relay. The current is normally led into the armature by means of two spirals of silver wire, and when by the action of the current the armature is turned so that the contact arm is brought against a fixed contact, the current is reversed and the armature rotates in the opposite direction until it is arrested by another fixed contact. The spirals of silver wire are long and so proportioned that their torsion is practically negligible over the range of the amplitude of the oscillations. The contact arm which reverses the armature current serves also to operate a relay connected to a counting train. *Fig.* 13 shows the principle of the measuring portion of the meter, M_1 and M_2 being the main current coils, A the armature, C_1 and C_2 the fixed contact points, and

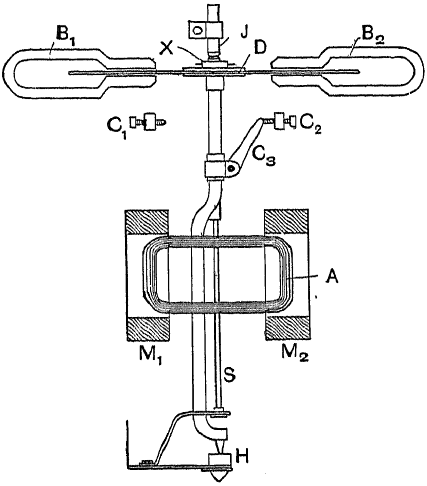


FIG. 13.

C_3 the contact arm fixed to the spindle, and S the silver spirals. The brake disc D and controlling magnets B_1 and B_2 are of the type ordinarily used in motor meters.

Fig. 14 illustrates a case in which the direction of the current is such as to cause the

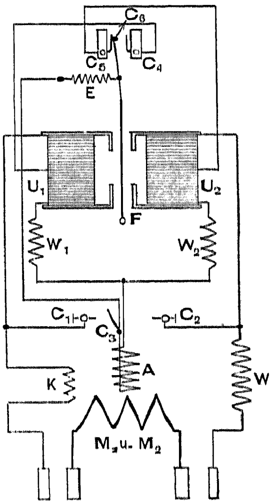


FIG. 14.

armature to move to the contact C_1, when the relay arm C_6 is in parallel with the resistance W_2; when, however, the arm C_3 touches the

contact C_1 the electro-magnet U_1 of the relay will be short-circuited (see *Fig.* 15), and the

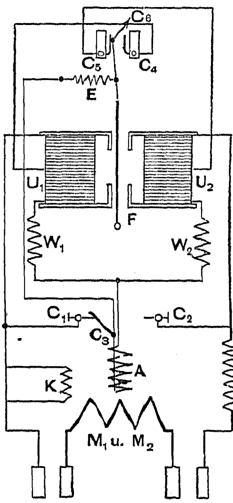


FIG. 15.

relay armature E will be drawn to U_2 (*Fig.* 16), and the armature will be connected in parallel across the resistance W_1, thus effecting the reversal of the current in the armature with the consequent movement in the opposite

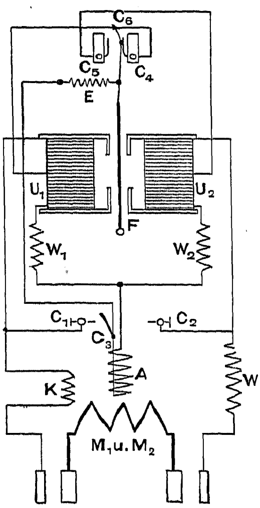


FIG. 16.

direction until the arm C_3 touches contact C_2, as in *Fig.* 17. The same operation will then be repeated by the electro-magnet V_2, and there will be a continuous reciprocating action between the moving system and the relay. It will be seen that the contact operated by the armature does not actually break the current, and therefore the sparking is very small.

For meters of small size a single armature is used, but for large currents two armatures

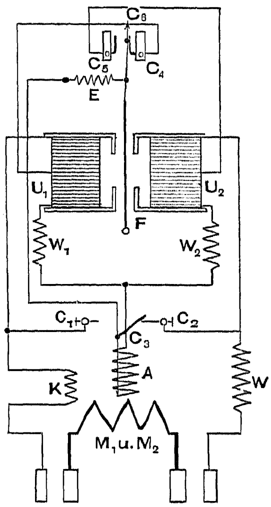


FIG. 17.

mounted on the same spindle are employed, these being mounted astatically one above the other, the illustration given in *Fig.* 18

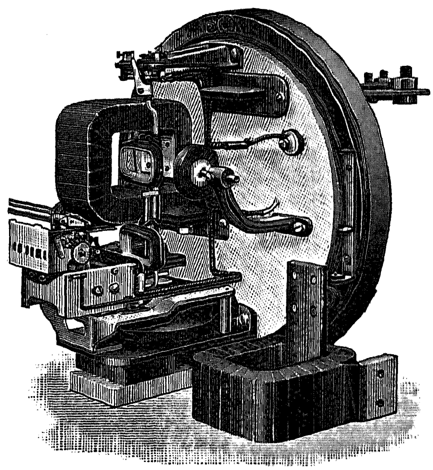


FIG. 18.

being of a 500-ampere 220-volt meter with one main coil removed.

(ii.) *Energy Losses and other Characteristics.*—The energy losses in these meters are generally of the order of:—Pressure circuit

including relays = 1·5 watts per every 100 volts and main current circuit 10 to 15 watts. The other characteristics will be much the same as in the Elihu Thomson and B.T.H. meters, with the difference that owing to elimination of brush and bearing friction a rather better curve of accuracy is obtained, more particularly at light loads. Tests of meters that have been in use for several years show that the system is satisfactory in operation, and it would appear that there is a real advantage in dispensing with the brush friction, wear, and the cleaning of a commutator.

§ (13) Acme Meter. — This motor meter embodied a novel and interesting principle, but is not very much used in this country, and consequently full details of its performance are not available. As with the meter of the Allgemeine Elektricitäts-Gesellschaft, the armature oscillates between contacts which reverse the direction of the current through the pressure coils, but in this case the armature consists of a piece of soft iron bent to suitable shape and energised by a fixed coil connected to the pressure of the supply, the rate of "motion" being controlled by a brake device. Thus there are no connections whatever to the moving part, and the only friction affecting the working parts is that occurring at the bearings.

§ (14) Motor Meters, Mercury Type. (i.) *Description.*—The disc submerged in a pool of mercury used in the Hookham and Ferranti [1] ampere-hour meters is used also with some adaptation as the moving element of a watt-hour meter. In this case the permanent magnet is replaced by an electro-magnet energised by a fine wire coil connected across the pressure of the supply and, since the resultant field is less than that obtained with the powerful permanent magnet employed in the ampere-hour meter, the speed is increased by slotting the armature disc and taking the current from side to side of the mercury bath instead of from the centre to the periphery: this has the effect of increasing the driving forces. Since the intensity of the magnetic field will vary with the applied voltage, a further disc is fitted rotating between the poles of a separate permanent magnet to provide the necessary braking effect. This is also desirable in view of the fact that both sides of the armature disc are used to exert a driving force. The meter is thus nearly the same in principle as a commutator motor watt-hour meter, but it has the important difference that iron is used in the electro-magnets, with the result that, owing to the shape of the normal magnetisation curve, the flux does not vary proportionately with a change in the pressure of the supply and the consequent change in current in the magnetising coils, and therefore, while a fair degree of accuracy is obtained, the meter does not accurately register energy if the voltage varies from that at which it is calibrated.

[1] See Part I. above, "Ampere-hour Meters," § (2) (ii.).

(ii.) *The Hookham Pattern.*—The arrangement of the Hookham watt-hour meter is shown in *Fig.* 19, in which the electro-magnet

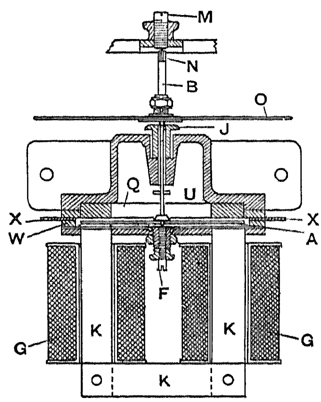


Fig. 19.

KK, wound with the fine wire coils GG, is placed under the mercury bath, the magnetic circuit being completed by the iron ring Q, fixed immediately above the bath. The disc armature A moves through the lines of force which pass from both limbs of K to Q. The current is led into and out of opposite sides of the bath at the points XX. The armature A, the brake disc O, made of aluminium, and the driving pinion N are carried on the spindle B, which rests at the lower end in the jewelled bearing F, and is guided by the upper bearing screw M. The upper and lower plates of the mercury bath are of metal separated by the insulating ring W. The mercury bath is, as will be seen from the figure, constructed on the principle of the unspillable ink-well, but to ensure against mercury being shaken out of the bath during transit, the valve J is lifted by means of a small detachable spring clip which brings the valve into contact with a leather washer fixed on the under side of the brake disc. The permanent magnets used with the brake discs are not shown in the figure. Further complete details are shown in *Figs.* 20 and 21.

(iii.) *The Ferranti Pattern.*—The Ferranti meter differs from the Hookham only in arrangement. Details are shown in *Fig.* 22.

(iv.) *The Sangamo Pattern.*—The Sangamo meter is based on the same principle and the same arrangement of elements is followed. In this meter, however, the weight of the disc armature is balanced so that there

is a slight upward thrust, the upper bearing consisting of a cupped jewel; the lower bearing, serving as a guide only, projects through the bottom of the mercury chamber. This

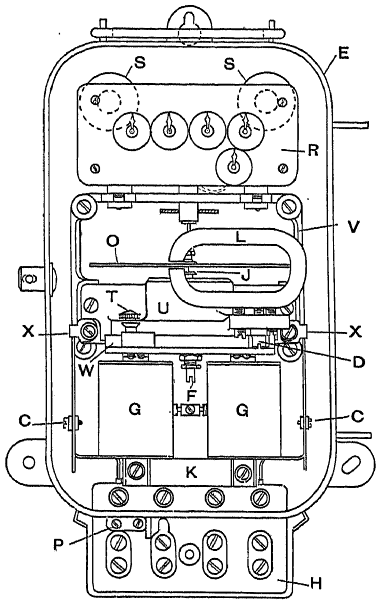


FIG. 20.

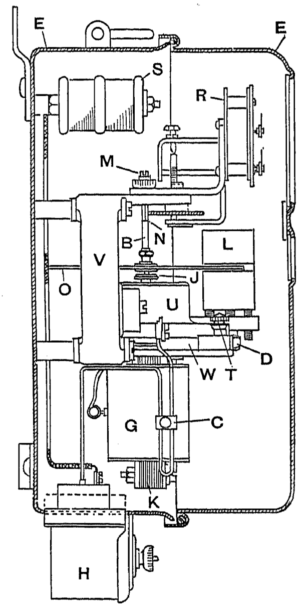


FIG. 21.

meter is also fitted with the thermo-couple device for compensating for friction mentioned in connection with the Bat ampere-hour meter. The general arrangement of the

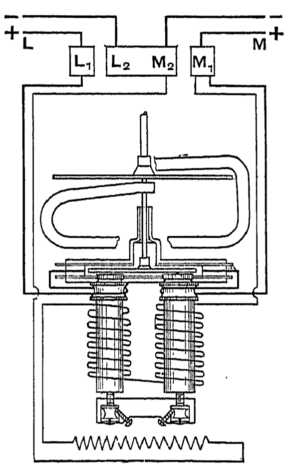


FIG. 22.

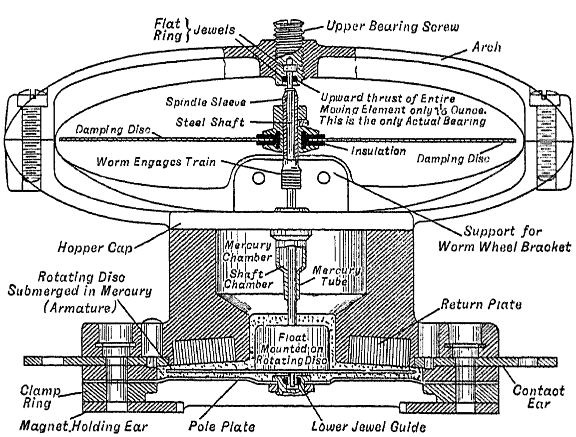


FIG. 23.

Sangamo meter with recording mechanism, field, and brake magnets removed is shown in *Fig.* 23.

(v.) *Fluid Friction.*—To compensate for fluid friction of the mercury all of these meters are fitted with one or more turns of wire in series with the main current circuit, which, being disposed in a manner to increase the driving magnetic flux, partly compensates for the increase of fluid friction at the higher speeds.

§ (15) ENERGY LOSSES IN MERCURY MOTOR METERS. (i.) *Pressure Circuit.*—In the Ferranti and Sangamo meters one standard winding is

apparently used for the electro-magnet for all sizes, and the loss in energy in this circuit is: Ferranti meter, 2 watts per 100 volts; Sangamo meter, 4 watts per 100 volts. In the Hookham meter two standard windings are used, one for 100 volts and the other for 200 volts, with the result that for any circuit of pressure from 100 volts to 240 volts the total energy loss in the pressure circuit is from 3 to 5 watts. For circuits above 240 volts, however, an additional resistance is used, and the loss in the pressure circuit of a 500-volt meter will be of the order of 9 watts.

(ii.) *Current Circuit.*—For currents up to 10 amperes the whole of the current is taken through the mercury bath, but for larger currents a shunt is used. For a 10-ampere meter the energy loss at full load is only from 0·3 watt to 0·6 watt, and for currents up to 100 amperes, where the shunt would be contained in the meter case, the pressure drop across the shunt at full load is of the order of 0·06 volt, corresponding to an energy loss of 6 watts with a current of 100 amperes. For meters of larger sizes it is more usual either to mount the shunt in a separate compartment attached to the case or else to have the shunt quite external, so that it can be connected in the main current circuit and joined by means of flexible leads to the meter. In this case the pressure drop across the shunt will be of the order of 0·1 volt.

§ (16) Temperature Coefficient of Mercury Motor Meters.—The temperature coefficient of this type of meter will vary with range of both pressure and current. Taking the separate components and their change with temperature:

(*a*) The coils of the electro-magnet and the eddy current brake disc will increase in resistance by 0·4 per cent for 1° C.

(*b*) In an unshunted meter the amount of current flowing through the armature disc will not vary with temperature.

Thus, assuming that the whole of the pressure circuit is in the form of the winding around the magnet, the decrease of magnetic flux due to increased temperature should be exactly balanced by the decrease of braking effect. Actually, however, owing to the iron in the electro-magnet, the driving flux and pressure will probably vary more nearly in the ratio of 7 to 10, and the temperature coefficient of such a meter will be made up thus:

Pressure circuit	− 0·3 per cent
Brake disc (pure aluminium) .	+ 0·41 ,,

and, ignoring the magnets, the effect of which is small, the meter will increase in speed by about 0·1 per cent for 1° C. If a portion of the pressure circuit is in the form of an external series resistance of material having a low temperature coefficient, or if the meter is of large size fitted with a shunt, the temperature coefficient will vary according to the proportions of the two parts of the pressure circuit and that of the main current in the shunt. For the more usual size of meter of 500 amperes and 440 volts, one-half of the pressure circuit being in the form of added resistance, the temperature coefficient will be

Pressure circuit	− 0·14 per cent
Brake disc	+ 0·41 ,,
Driving disc (ratio of total current to current in disc = 10/500)	− 0·39 ,,
Temperature coefficient of meter	− 0·12 per cent for 1° C.

§ (17) Speed of Mercury Motor Meters.—The full load speed of these meters is generally low, the figures for the three types described being:

Hookham . . .	20 to 24 r.p.m.
Ferranti . . .	40 r.p.m.
Sangamo . . .	25 ,,

§ (18) Torque and Weight of Mercury Motor Meters.—In considering the torque and weight of the moving element of mercury motor meters due regard must be given to the fact that the greater part of the weight is borne by the mercury. Values of the ratio torque/weight should, therefore, be based on the effective weight on the bearings. In both the Hookham and the Ferranti meters the actual weight on the lower bearing is from 4 to 5 grammes with a torque of about 6 grm. cm. at full load, the ratio torque/weight being about 1·3. In the Sangamo meter, where the upward thrust is borne on the top bearing, the weight (thrust) is stated to be 3 grammes, torque 6 cm. grammes, and ratio torque/weight = 2.

§ (19) Accuracy Characteristics of Mercury Motor Meters.—The accuracy on constant pressure at all currents from one-twentieth load to full load is very good, and providing the iron used for the pressure circuit electro-magnet is suitable for its purpose, meters of this type are constant over a long period of normal use. When the pressure is varied by 10 per cent above or below the value for which the meter is calibrated, the rate of the meter will change by about 8 per cent only or less (this is a fair average figure, the error is sometimes larger but rarely smaller); the error depends on (*a*) the degree of saturation of the iron core of the pressure circuit magnet, and (*b*) on the change in resistance of the copper pressure coils due to the increased or decreased current. Typical error curves are given in *Fig.* 24.

Mercury watt-hour meters have the curious characteristic for a watt-hour meter that the moving element will rotate (but at slower speed) when the pressure circuit is not

energised, in consequence of the small magnetic field due to remanence in the iron cores, or of the field set up by the brake magnets,

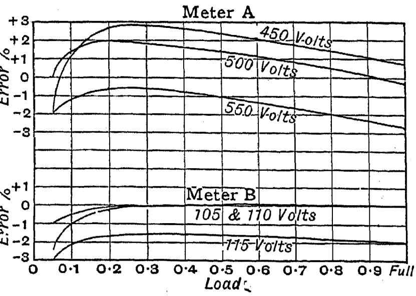


FIG. 24.

or energisation of the pressure circuit magnet by the current flowing through the bath, or these various causes combined; this does not affect the accuracy when both circuits are operating, but rotation is not necessarily evidence that both circuits are in order. Such a meter must, however, be connected with the polarity of the terminals as marked by the makers, since owing to the permanent or semi-permanent magnetic field, reversal of polarity of both pressure and current circuits may result in an error of 30 per cent in the reading of the meter.

§ (20) SIZES OF MERCURY MOTOR METERS.—Meters of this type are supplied in all sizes from 10 amperes to 5000 amperes: for the higher currents an external shunt is provided, connection to the meter being made by means of leads sufficiently long to enable the meter to be erected in such position that the magnetic field due to the current flowing in the shunt does not affect the accuracy of the instrument.

Three-wire Mercury Meters.—For the record of total energy in a three-wire circuit it is more usual to have two meters, one connected in each of the two outer mains. The older Sangamo meter had two elements mounted one above the other, but in the later type there are two complete elements mounted side by side in the same case, the spindle being connected through a differential gear to a single counting train.

§ (21) ARON CLOCK METER. (i.) *Description.*—The principle of a meter in which the electro-magnetic force set up by a current is used to affect the rate of an otherwise free pendulum was introduced by Aron in 1887. The original meter consisted of two clocks, each with its pendulum about 18 in. long, one having the ordinary adjustable brass bob and the other a permanent bar magnet or a coil of wire energised by the pressure of the supply; a coil carrying the main current was placed under the pendulum carrying the magnet. The two pendulums were adjusted when no current was flowing so that their rates were identical, and when the main current was passed through its coil the rate of the one pendulum was accelerated by an amount proportional to the current in the coil. The difference in rate of the pendulums was registered on a counting train by means of a differential gear which is described later. The advantage of a principle of this type, apart from the complexity of the necessary clockwork, will be obvious: the pendulums are kept in motion by a spring which overcomes all friction, and the current can be led into the pendulum by flexible conductors at a point where such connections have little or no effect on the rate. In consequence, the rate of the counting trains should be truly proportional to the energy, and should be invariable whatever the load.

The first Aron meters were largely used on supply circuits in the early days of electric lighting, and were satisfactory for this period. There were, of course, disadvantages, mainly owing to the fact that the clocks required to be wound every week, and also that if one pendulum only were to stop and the other continue to oscillate the meter dials would go forward or backwards by an abnormal amount, and the whole records would be spoiled. In 1897 Aron modified his original instrument, using two pendulums only 5 in. long: and the use of these short pendulums required additional devices to obviate errors that would otherwise be introduced.

The theory of the variation of the rate of a pendulum due to a magnetic effect produced by a current acting on the pendulum in the same direction or against the normal force of gravity is as follows:

Let n be the normal number of oscillations and l the length of the pendulum. Then

$$n=\frac{1}{2\pi}\sqrt{\left(\frac{g}{l}\right)}.$$

Let the acceleration due to the magnetic field be f. Then f will clearly be proportional to the current and may be put equal to $g\mathrm{I}/k$, where I is the current and k a constant depending on the magnetic field produced; moreover, if, with the current in one direction, f acts in the same direction as g, and the pendulum is accelerated when the current is reversed, it will act in the opposite direction, the pendulum is retarded and we have two different frequencies, N_1 and N_2 respectively. Thus

$$\mathrm{N}_1=\frac{1}{2\pi}\sqrt{\frac{g+f}{l}}=\frac{1}{2\pi}\sqrt{\left(\frac{g}{l}\right)}\left(1+\frac{f}{g}\right)^{\frac{1}{2}}$$

$$=n\left(1+\tfrac{1}{2}\frac{f}{g}-\tfrac{1}{8}\frac{f^2}{g^2}\right)+\text{higher terms,}$$

$$\text{and}\quad \mathrm{N}_2=n\left(1-\tfrac{1}{2}\frac{f}{g}-\tfrac{1}{8}\frac{f^2}{g^2}\right)-\text{higher terms.}$$

Hence if f^2/g^2 and the higher terms be neglected,

$$N_1 - N_2 = \frac{nf}{g} = n\frac{I}{k}.$$

Thus $N_1 - N_2$ is proportional to I.

With the introduction of the simpler mercury meter and electrolytic ampere-hour meters for continuous currents, and of the induction type meter for alternating currents, the Aron meter is little used for small-current circuits, but it has been developed and is largely used for large-power circuits, for which it would appear to be most suitable.

(ii.) *Winding Gear.*—In the later meters the original hand-wound clock spring, used to maintain the pendulum in motion, was replaced by a short spring which is wound up periodically by an electro-magnet actuated by

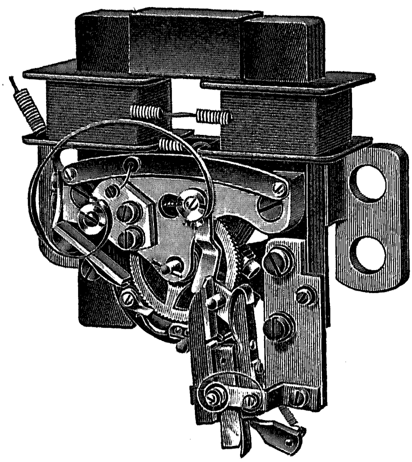


FIG. 25.—Winding Gear.

the supply pressure. *Fig.* 25 shows a general view of the winding gear, and *Figs.* 26, 27, and 28 show details of the working.

The spring which drives the two clocks is marked S in *Fig.* 26, and can be seen clearly in the general view in *Fig.* 25. One end of this spring is fixed to the rectangular magnet M of the winding gear, while the other end is fixed to a Z-shaped rotor N. *Fig.* 26 shows the winding gear in its normal position before the current is switched on, the path of the current being shown by the dotted line A.

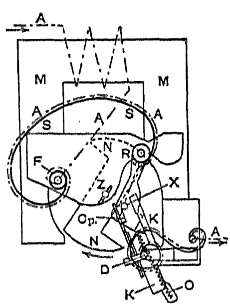


FIG. 26.

As soon as the current is started in the coil the rotor N is turned clock-wise through about a quarter of a revolution, carrying with it the end of the power spring S, which is thus wound up. *Fig.* 27 shows the position when the spring is fully wound: particular attention is called to the mechanism for making and breaking the electrical circuit. This mechanism is marked K in the diagrams, and, being pivoted at D, its upper end is carried over by the pin X, which is fixed to and revolves with the rotor. This pin completes the circuit when the spring has run down by coming into contact with the silver plate *Cp*. The design of the arrangement is such that a rubbing contact is obtained between X and *Cp*, in addition to which a quick make-and-break arrangement is supplied by the spring O, which pulls the switch K sharply on or off, according to whether it is to the right- or left-hand side of the switch-pivot D.

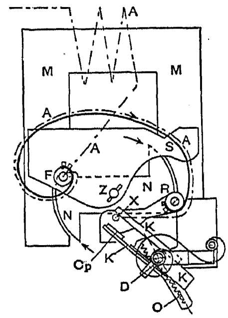


FIG. 27.

Fig. 28 shows the ratchet and pawls by which the power of the spring is transmitted to the driving spindle Z. The pawl *Pl* holds the driving spindle still while the rotor is winding up the spring, and the lower pawl P enables the rotor to carry with it the driving spindle as it is brought back to its original position by the power spring. Under normal working conditions this winding gear comes into action every half minute. In order to enable the two clocks to be driven by one main spring, a second differential gear, exactly similar to that shown in *Fig.* 29, is introduced, and this permits driving to take place when the clocks are going at different speeds.

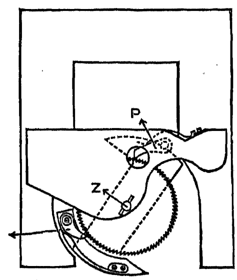


FIG. 28.

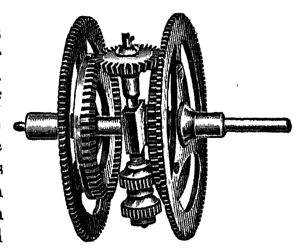


FIG. 29.—Differential Gear.

(iii.) *Pendulums.*—The strength of the driving spring is sufficient to start the pendulums. The pendulums, one of which, with its pallet and escapement is shown in *Fig.* 30, oscillate at a speed of about 12,000 per hour, thus they are so short that it would be most difficult to adjust them to be in complete synchronism or to rely on its maintenance, and to provide complete compensation for any want of synchronism two sets of reversing gear are used, which operate at the same moment. One of these reverses the direction of the current flowing through the coils on the pendulum, and the other reverses the direction of rotation of the motion transmitted from the differential gear to the counting train. This reversing operation takes place about every 10 minutes. *Fig.* 31 is a general view of the commutator which reverses the current in the pendulum coils, and *Fig.* 32 an illustration of the gearing, the upper wheel operated by the pin above it moving from side to side.

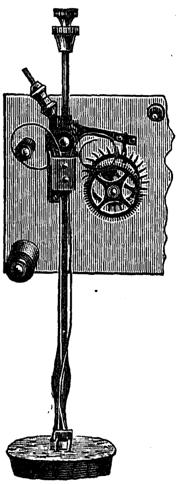


FIG. 30.—Pendulum with Pallet and Escapement Wheel.

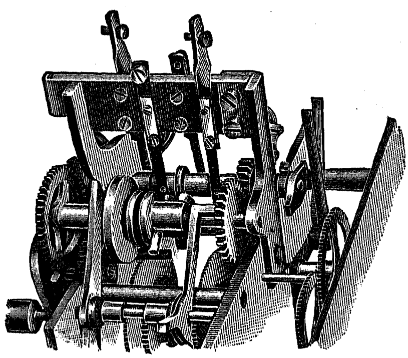


FIG. 31.—The Commutator.

(iv.) *Differential Gear.*—The gear used to transmit the difference in the rate of the two pendulums to the counting train is shown in *Fig.* 29. The spindle, which is connected to the counting train, carries a planet wheel free to rotate on its own axis carried on a main axle shown in the vertical position in the figure; this axle is connected to the spindle. The two large crown wheels which are connected through gearing to the escapement wheels of the clocks are free to revolve on the spindle. Thus if the two pendulums are oscillating at the same speed the movement of the crown wheels results in the planet wheel revolving on its own axis only, but if there is any difference in the rate of the crown wheels the planet wheel and its shaft and the spindle move round. *Fig.* 33 is an illustration of a switchboard meter of

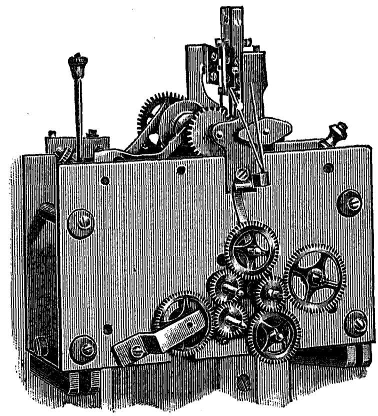


FIG. 32.—Reversing Mechanism.

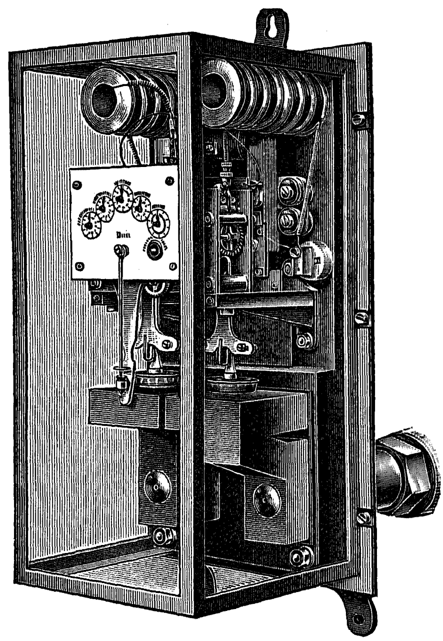


FIG. 33.—1000-Ampere Meter.

this type made for a current of 1000 amperes, complete with series resistances wound on porcelain bobbins and placed at the top of the meter; the main current coils in this case consist of a single turn of heavy copper bar; a plumb bob is fitted to ensure that the meter shall be erected in a vertical position. *Fig.* 34 shows the connections of a two-wire meter, the coils carried by the pendulum being connected in series with the resistances RR across the supply mains with the winding gear with its series resistance in parallel.

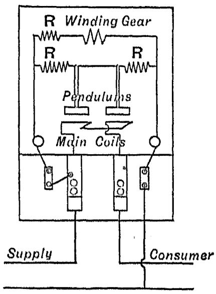


FIG. 34.

The meter can readily be adapted for use on three-wire circuits, being then connected as shown in *Fig.* 35, where one of the main current coils is connected in each outer main of a three-wire circuit, the winding gear and pendulum circuits being connected across the outer wires with an equalising point from the middle or earthed wire running to the centre of the pendulum circuit.

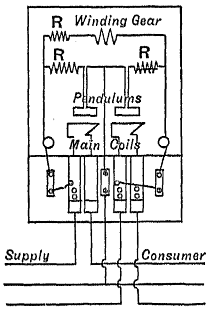


FIG. 35.

§ (22) ARON METERS FOR LARGE CURRENTS.—In the earlier meters the whole of the current was taken through the main coils of the meter, and this type of meter is still made for currents up to 1000 amperes. Above this, however, owing probably to modern switchboard design, it is more usual to use a shunt which can be connected in circuit with the main current bus bars. When used in this way the current coils in the meter take a current of about 6 amperes. Connection from the shunt to the meter is made by means of stout flexible cable, and the contact surfaces are ample in size to ensure that the resistance of such contacts shall not affect the accuracy of the meter. The pressure drop across the shunt at full load is nearly 0·2 volt; about one-half, however, of this is dissipated in a resistance in series with the current coils used to reduce the temperature coefficient.

Since the two pendulums are close to each other any external field, if uniform, will affect both equally, and in consequence the meter is astatic.

§ (23) ENERGY LOSSES OF ARON METERS.—The loss of energy in the pressure circuit is small, the stated value being from 2 to 3 watts for each 100 volts. In the main current circuit for larger currents the energy loss is the drop across the shunt, which is equal to 0·2 volt × the current: in the straight through type the loss is probably nearer one-tenth of this amount.

§ (24) TEMPERATURE COEFFICIENT OF ARON METERS.—In the straight through pattern the temperature coefficient is very small; taking a 240-volt instrument, the resistance of both pendulum coils is of the order of 2000 ohms, and in series with this is a resistance of the order of 10,000 ohms. Thus, since the series resistance is made of an alloy having a negligible temperature coefficient, the temperature coefficient of the meter will be of the order of 0·07 per cent for 1° C.; the meter records low with increased temperature. In the shunted type, however, the drop across the shunt does not vary with temperature, while the resistance will change by 0·4 per cent, and the current and rate of registration would be reduced by this amount for each 1° C. Actually, however, owing to the series resistance mentioned before, the temperature coefficient of a shunted meter of this type is more nearly 0·2 per cent for 1° C., the rate decreasing with increasing temperature.

§ (25) ACCURACY CHARACTERISTICS of ARON METERS.—Theoretically, and in many cases actually, meters of this type should give a record which is in true proportion to the energy. Friction is almost entirely eliminated from the forces set up by the currents, and there is no iron whatever in either of the circuits. In a good meter this ideal condition is nearly realised, the error for all loads from one-hundredth of full load to full load being the same to within ±2 per cent or less; the meter responds to a current very much smaller than 1 per cent of full load. Moreover, this proportion remains true for a fair range of change of pressure also, a change of ±10 per cent in the applied voltage producing no change in the accuracy of the meter. In some cases, however, difficulties have been encountered:

(*a*) Owing apparently to the amplitude of the pendulum swing being too great, with consequent "banking" on the escapement; and

(*b*) Owing to backlash in the clockwork, and slight out-of-balance of some of the wheels; this effect shows itself in a creep, forwards or backwards, of the dials when no current is flowing, with a consequent effect on the accuracy at light loads. This can be cured by fitting a weighted disc on the first spindle.

There is also the possibility that one pendulum or the other, or both, may stop.

This type of meter has been criticised owing to its complexity and the number of moving parts when compared with the more simple motor meter. In answer to this, Professor Ayrton many years ago referred to the case of the simple sun-dial and the modern complicated watch. The instrument is undoubtedly complicated, but modern clockwork is, or can be made to be, thoroughly reliable, and there appears to be no valid reason why such a clock meter should not be made to have an accuracy of operation nearly as good as the modern chronometer.

§ (26) SOME SPECIAL METERS. — Other meters using interesting principles, which gained prizes at a competition held in Paris in 1891 are those of Mares,[1] in which the movement of an arm attached to a movable coil working between two fixed coils, much as the principle of the Kelvin balance, was used to actuate counting mechanism, a conical pendulum being used to supply the integral of time; and of Frager,[2] who used a somewhat similar pendulum supplying the motive power to integrate and record the extent of the deflection of a watt-meter needle.

§ (27) JEWELLED BEARINGS FOR MOTOR METERS. — In nearly all motor meters the lower bearing which carries the weight of the armature consists of a cupped jewel supported on a spring to absorb shock, and so constructed that the jewel can be readily removed for inspection. These bearings are a most important part of the meter since, if as the result of wear or shock the jewel becomes cracked, the accuracy of the meter, more particularly at light loads, is very seriously affected. This is, in fact, a most prolific source of error. In the older meters sapphires were used, but generally these were found to be unsatisfactory, and have been replaced by diamonds or rubies. Sharp[3] considered that the sapphire bearing had a life of not more than one million revolutions, whereas that of the diamond is many millions; he also gives a table of results with a large number of meters showing that the accuracy at light loads with a diamond bearing is much better than with sapphires. It seems clear that the diamond or ruby bearings are much more suitable than sapphires, but further data are required as to the limits of useful life, and in this connection the weight of the armature, the angle of the pivot, and the resultant pressure per unit area (probably of the order of several tons per sq. in.) must be taken into account. Further, it is probable that in some types of meters, even with light armatures, a hammering, due to variation in the horizontal component of the magnetic forces operating the meter, is added to the ordinary grinding of the pivot and bearing; this results ultimately in the cracking of the jewel.

§ (28) REGISTERING MECHANISM (DIALS OR COUNTER TRAINS).—Three types of mechanism are used for registration.

(i.) *Clock or Pointer Type.*—In this a number of axles are continually in gear with each other in the ratio of 10 to 1, each axle carrying a pointer which moves over a figured dial (see *Fig.* 36).

This type is of the pattern originally used as a counter on gas meters, and has become

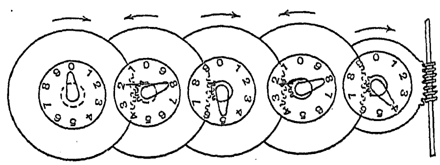


FIG. 36.

established by long use. It has the advantage that if well made the friction is small and continuous, and does not vary in amount, and for these reasons many meter makers recommend it. Against it is urged the possibility of error in reading, particularly by unskilled persons; as will be seen from the figure the pointers rotate in opposite directions, and some confusion may arise on that account.

(ii.) *Counter Types.*—In this type of register the numbers are fixed on discs which rotate behind the dial plate, in which are openings that allow of one figure to be seen. It is here essential, to prevent confusion in reading, that the figures should change quickly, since if two figures are showing on each opening at the same time, as 67, 90, the reading might be either 69, 79, 60, or 70. These gears, therefore, are usually based on the principle of a driving wheel with one tooth engaging once in each revolution with a ten-tooth wheel of the same pitch diameter. The engagement lasts for one-tenth of a revolution, and during the remainder of the interval the driven wheel must be prevented from moving. This is usually effected by means of a moving piece fixed on the driving axle, and so arranged that it occupies a space through which the driven wheel must necessarily pass during rotation. A typical example of such gear is shown in *Fig.* 37, in which the ten-toothed scallop wheel S_1 is locked by the edge of the locking disc L, except when the notch N by passing into the region of engagement permits S_1 to rotate. But the notch is unable by itself to engage with a tooth on the scallop wheel. To effect this a second ten-toothed

[1] *Elec. Review*, xxvii. 546.
[2] *Ibid.* xxviii. 782.
[3] Tema 18. 9, Turin Exhibition, 1911.

wheel S_2 is fixed on the axle B a little distance behind S_1. The teeth of S_1 and S_2 being staggered, the latter will have one tooth in

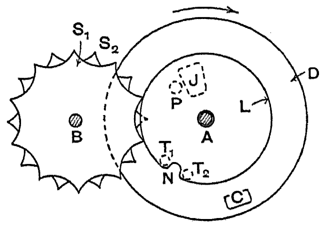


FIG. 37.

the path of the pins T, T, which project from the back of the locking disc; hence at the proper time the leading pin will impinge on this tooth and by moving it on will cause the adjacent following tooth on S_1 to enter the notch. In this way the two axles are brought into gear, and they remain in gear until first the pin and then the notch disengage themselves, and the scallop wheel S_1 is once more locked, axle B having been moved through exactly one-tenth of a revolution. The following pin plays no part in this engagement, but it is brought into use if the direction of rotation is reversed, when it becomes the leading pin.

(iii.) *Jump Device.*—In this counter is also shown a jump device to carry the figures over quickly. The first digit wheel D of the register is fixed on the axle A, which is geared to the reducing gear and moves continuously. The jumper weight J is fixed to the element L, which is loose on the axle and naturally rests with J at the lowest position. A pin P, which projects from the digit wheel D, impinges on the jumper weight and carries it and the element L round with it, gradually raising the weight from its lowest position until after half a revolution of the digit wheel the weight reaches the top. On passing the top the weight overbalances, and in falling to the lowest position it carries the element L rapidly through its engagement with the scallop wheels, driving them forward one step or unit. The weight then remains at rest in the lowest position until the digit wheel has made another half revolution and brought the pin P once more into contact with it. The work done in lifting the jumper weight is spread over a whole revolution of the digit wheel by the addition of a counterweight C, which is fixed on the digit wheel in such a position that during the lifting of the jumper weight, J and C are opposite each other. When the weights are so proportioned that the maximum moment of C is half that of J, the work to be done is equally divided between the two halves of each revolution of the axle A, and the maximum torque required in the process of weight-lifting is reduced to one-half the maximum moment of the jumper weight. In meters of large capacity it is important that the two weights should be carefully adjusted in this respect. This type of counter is used in Ferranti and Westinghouse meters.

In the Aron and Chamberlain and Hookham trains instead of the scalloped wheel a toothed wheel having gaps is used, and instead of the falling weight a spring is used to give the quick motion to the dials. In the Aron, see *Fig.* 38, this consists of a flat coiled hair-spring A, the motion of which is arrested by means of a pin B carried by a sleeve S running freely on the spindle held by a stop P. The spring is wound up for the greater part of the revolution of the driving wheel W, and when the stop is released by means of a cam K, operated by a separate pin F in the driving wheel, the torque of the spring carries the sleeve through a complete revolution which is imparted to the figured disc as one step. In the Chamberlain and Hookham counter one end of a flat spring is carried round by the driving wheel. The free end of this is arrested at one point by a stop; the continual motion of the driving wheel then bends the spring until it is flexed over sufficiently to escape from the stop, the ensuing sudden motion to the normal position of the spring imparting the required quick motion to the first disc, which at this point actuates the following wheel.

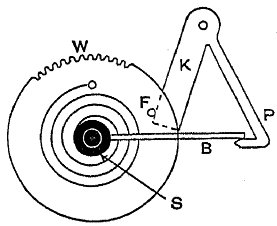


FIG. 38.

(iv.) *Roller Type.*— This type, sometimes called the "cyclometer" or "Harding" counter, is illustrated in *Fig.* 39, taken from the *I.E.E. Journal.* Motion is communicated from A to B by means of a pinion P, which serves at

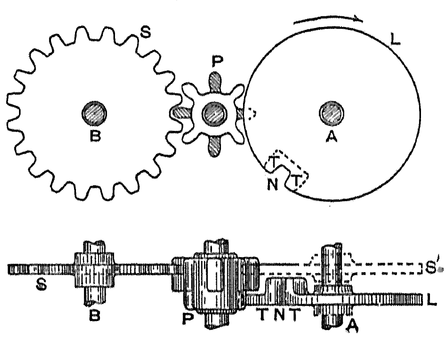


FIG. 39.

one end to interlock with the locking disc L, and at the other end to gear, as an ordinary pinion, with the spur wheel S. As this wheel usually has twenty teeth it is necessary to drive it forward two teeth at a time, and hence there are two teeth T, T, on the driving element to engage with the pinion. Since the pinion is moved by the space of two teeth per step, it is convenient to give it eight teeth, so that it may rotate through one-quarter of a revolution at each engagement. At the interlocking end of the pinion alternate teeth are removed, leaving four teeth at right angles to act as the locking members. For explanatory purposes the driving and driven elements of the Harding counter have been shown on separate axles, but it is obvious that the spur wheel might be arranged in any position round the pinion. In practice it is always placed on the same axis as the driving element, as shown in dotted lines at S′ in the lower part of the figure. Each digit wheel is then made like a flat-faced pulley and carries the driving teeth T, T, and notch N at one edge, and the spur wheel teeth at the other, the digits 0 to 9 being marked round the face of the pulley. A number of complete elements are strung loosely on a fixed axle A, and interconnected by a corresponding series of pinions which are strung on an axle alongside.

This counter is most compact and cheap, all the moving wheels being small and made of die castings of a light alloy, but, as will be seen later, the friction is rather larger than in the other types. Moreover, there is no provision for a jumping device to prevent confusion in reading.

§ (29) Friction in Various Types of Dials.—Evershed[1] investigated the amount of frictional resistance in various types of registering mechanism (parts of his description of the various types have been used above), and found that in a five-dial pointer type register the total friction measured at the rotor shaft was 6·1 dyne cm., of which 5 dyne cm. occurred at the worm reducing gear: with a spur wheel drive from the rotor to the train the total friction was only 1·4 dyne cm., of which 0·3 dyne cm. occurred at the reducing gear. In a counter type mechanism the friction with the first digit wheel engaged was 195 dyne cm., but at the moment when all the wheels were being actuated the total friction was 450 dyne cm. For a roller type counter having five figures when one dial only was being moved the friction was 93 dyne cm., and with four dials 600 dyne cm. Evershed gives tables showing the periodic decrease in speed which occurs in meters of various speeds and different full load torques when any number from one to four of the dials are being operated. Thus in a meter having a full load torque of 5 grammes and full load speed of 50 r.p.m. the speed at one-tenth load will be decreased with a roller mechanism by 0·7 per cent when 2 dials are operating, 1·3 per cent for 3 dials, and 2·2 for 4, which amounts will be increased if the meter is of slower speed or smaller torque.

[1] *Journ. I.E.E.* liii. 498.

The percentage decrease in speed may be calculated by means of the formula

$$100\frac{nWR'}{RaeST},$$

where R is the total friction, R′ the additional friction when the extra dials come into operation, W the capacity of the meter in kilowatts, S the full load speed (r.p.m.), T the full load torque of the rotor in dyne cm., and e the differential coefficient of the reducing gear, $1/n$ the fraction of full load speed at which the meter is running, and a the number of kilowatt hours registered at each step of the first digit wheel.

The differential efficiency, that is, the ratio

$$\frac{\text{Increment of resisting moment on driven axle}}{\text{Corresponding increment of torque on driving axle}}$$

is independent of both load and speed, and is about 0·2 in worm gears and 0·95 in spur gears.

Fitch and Huber[1] gave values for gear friction on six typical American watt-hour meters, five of which were fitted with pointer type dials and one with counter dials, ranging from 0·1 per cent to 0·2 per cent of full load torque, but it is not clear whether this includes the friction when the additional dials on the pointer type counter were in operation.

The question of the friction in dials of the counter type is most important. With a meter having a large torque and high speed, say 100 r.p.m., the effects of friction are probably inappreciable; with an instrument having small torque and slow speed, however, the effect may easily be large enough to stop the meter as the additional dials come into operation, and it is clear that the use of a particular type of dial is conditioned by the torque and speed of the rotor.

§ (30) Registering Mechanism Types of Dials.—Until recent years practice as regards the number and arrangement of dials varied largely, but specification No. 37 of the British Engineering Standards Association issued in 1919 resulted, to a large extent, in the standardising of British practice in this respect. In general, the requirements which are quoted below are based on (*a*) the provision of such a number of dials and of such value that all the indices shall not pass through zero for less than a minimum of 800 hours at full load, and (*b*) the assumption that the dial or opening of the lowest value should move at a rate sufficient to allow of a dial test being made in a suitable

[1] *Bull. Bureau of Standards*, x. 161.

time. The specification is now met by practically all meter manufacturers in this country, in fact some of them provide rather more than the specification requires, especially with regard to testing dials: this is especially the case in the Ferranti meter, where an additional dial, which is most useful in testing, is fitted indicating fractions of units. The main requirements of the specification as affecting the registering mechanism are:

"The registering mechanism of the meter shall be either of the pointer type or of the counter type, and shall comply with the following requirements:

(*a*) In the pointer type the pointers shall indicate on circular scales, each divided into ten equal divisions, and the radius of the scales and pointers shall be not less than 0·28 in. (7·11 mm.).

(*b*) In the counter type all the figures visible within the register, except the first,[1] shall spring quickly into position; the first figure may move continuously. In the case of quickly moving figures each opening shall be sufficiently large to permit clear observation of the figures.

(*c*) There shall be not less than 5 indices which may be circular scales and/or openings. For figures moving continuously, the openings shall be large enough to permit of two consecutive figures being identified at the same time, but a suitable device shall be used to prevent the figures 9 and 0 from being seen simultaneously. Those indices which indicate lower values than one kw.-h. per division shall be made distinctive from the other indices.

(*d*) The circular scales and/or the figure openings shall conform with the requirements of the following table:

Class.	Size of Meter. Full Load in Kilowatts.	Constants for the Openings or for One Division of the Circular Scales.
I.	Up to 1·25 .	100, 10, 1, 1/10, 1/100
II.	Above 1·25 and up to 12·5 .	1,000, 100, 10, 1, 1/10
III.	Above 12·5 and up to 125 .	10,000, 1,000, 100, 10, 1
IV.	Above 125 and up to 1250 .	100,000, 10,000, 1,000, 100, 10
V.	Above 1250 .	In multiples of the above

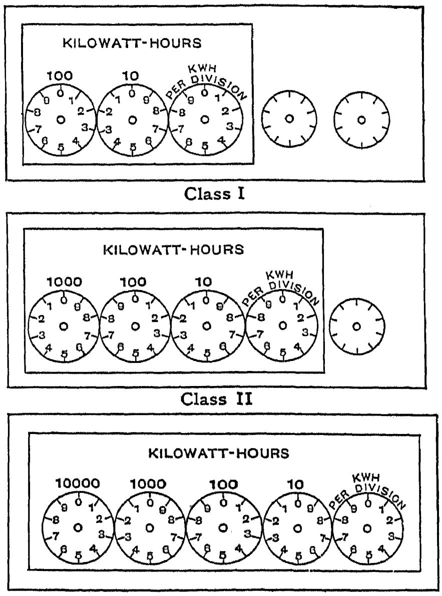
The terms in which the record is made by the register or dial shall be clearly marked as 'kilowatt-hours.'

In the pointer type of register each circular scale shall be marked to indicate the number of kilowatt-hours represented by one division of the scale.

In the counter type one of the openings shall be marked clearly to indicate the value of the indication in that opening.

No other marking of any kind shall be made on the dial plate."

An appendix to the specification gives typical arrangements of meter registers which would comply with the requirements of the clause. *Fig.* 40 shows the pointer types of the three first classes, and *Fig.* 41 the counter types; Classes IV. and V. would be exactly similar to those of Class III., except that the scale or opening marked kw.-h. or kw.-h. per division would be marked 10 kw.-h. for Class IV. and 100 kw.-h. for Class V.



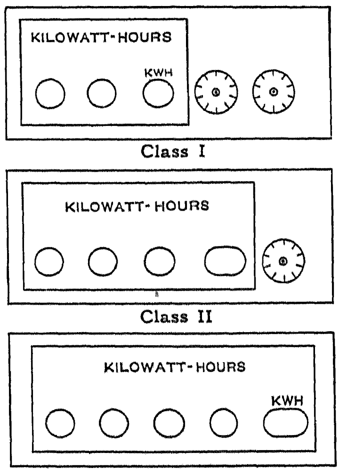
Class I

Class II

Class III

FIG. 40.

It will be seen that for the smaller meters the dials which have to be used to estimate the value of the supply are enclosed by a distinguishing line, those outside the line being testing dials.



Class I

Class II

Class III

FIG. 41.

[1] The "first" figure means the figure having the *lowest* significance in the register.

§ (31) Shunts for Large Meters.—Particulars of the general construction of large shunts have been given in the articles on the potentiometer system of measurement and moving coil instruments, and these generally cover meter shunts except that, in view of the greater accuracy required with a meter, the shunts are more liberally designed than an ordinary ammeter shunt.

§ (32) Variable Tariff Meters.—Although in most cases the charge for a supply of electricity is based on the amount of energy consumed as indicated by the register of a meter, there are two cases where a special type of meter is required to provide the additional information on which to base the charge. This arises when it is required to indicate the maximum demand or to register two alternative rates of supply.

§ (33) Maximum Demand Indicator.—In this case the maximum amount of energy taken by the consumer during any one hour or other nearly similar period is taken as the measure of the proportion of the generating plant and distributive system that has to be maintained to meet his possible demands. On this basis the consumer who takes a supply at a uniform rate is more profitable than one who, while taking the same total amount of energy, makes occasional demands for a heavy rate of supply, and the tariff is so arranged that where the proportion of (total units used)/(maximum demand) is large a substantial rebate is given.

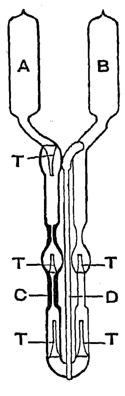


Fig. 42.

The Wright rebate (maximum demand) indicator, the construction of which is shown in *Figs.* 42 and 43, is essentially a differential thermometer. It consists of two bulbs, A and B, connected by a U-tube C, and provided with a reading tube D, to which the scale S is attached. The tube C is filled with a highly hygroscopic liquid, the whole being hermetically sealed. A heating coil H, consisting of a strip of resistance material, is wound round one bulb, and the heat of this strip causes the air to expand and forces the liquid over into the reading tube. The height to which the overflow rises indicates the maximum current which has passed. The scale S indicates both the maximum current and the consumption corresponding to this maximum current which must be used to secure the rebate. The traps shown in the U-tube are to prevent the passage of air from one bulb to the other when the apparatus is inverted. The apparatus is purposely made to be slow-acting so that momentary overloads do not appreciably affect the indicators. The usual time required for the instrument to take up the full reading is about 30 minutes; if the overload lasts only 10 minutes about 95 per cent will be recorded, if 3 minutes only 90 per cent. The loss on the instrument is of the order of 12 watts for all sizes up to 100 amperes.

The instrument is reset quarterly or at any other fixed period by tilting the tube and so allowing the liquid to flow from the index tube back into the U-tube.

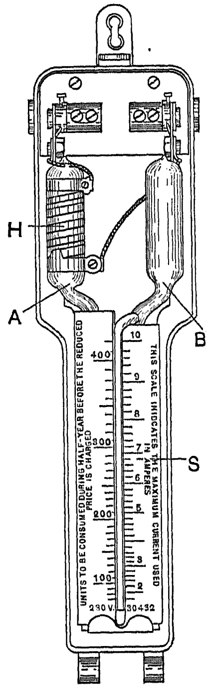


Fig. 43.

§ (34) Atkinson-Schattner Indicator.—This instrument uses the essential principle of a moving iron ammeter in which a soft iron core is drawn into a solenoid, the motion being controlled by gravity. The moving element controls an arm which is a hollow tube, bent into the arc of a circle. The tube contains a number of small balls and is filled with glycerine or thick oil, this being for the purpose of reducing the rate of motion of the balls. The position taken up by the arm depends on the intensity of the current, and a number of the balls, corresponding to the position of the arm when tilted, will fall over into a reading tube, the interval of fall depending on the viscosity of the liquid employed. The principle of the instrument is shown in *Fig.* 44, in which E is the solenoid, A the iron core which is pivoted at C_1 and carries an iron frame H, to which the glass tube D is attached. As the tube is tilted by the action of the solenoid the balls run from the curved portion of the tube into the lower limb, and the number which have fallen over indicate the maximum intensity of the current.

§ (35) Merz-Price Demand Indicator.—This type of indicator as used by several firms consists of an extra separate dial which is put into gear with the ordinary registering mechanism of a meter for a predetermined interval of time. The pointer, therefore, indicates the consumption for this period, usually 30 minutes, the accuracy of the indication being the same as that of the ordinary meter dials. At the end of the period a rocking device operated by a time switch comes into operation and dis-

engages the driving mechanism and allows it to come back to zero, the pointer, however, being left at the deflection to which it has attained. The operation of the time switch is practically instantaneous, the dial being mechanically in periods of 10 minutes of multiples of this. For motor meters, however, the operation of the gear is usually affected by means of a small electro-magnet energised at regular intervals through a time switch

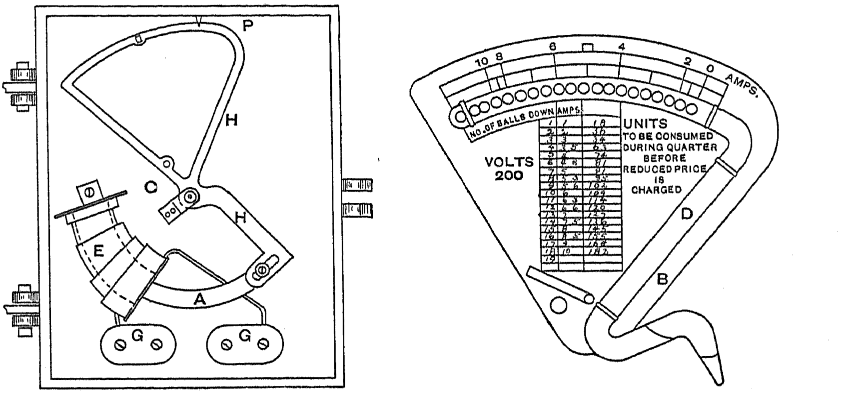


FIG. 44.

switched into gear again immediately. The driving mechanism, however, will not increase the reading of the pointer unless the energy consumed in the second or any further period is greater than that already indicated. A typical mechanism of this type is that made

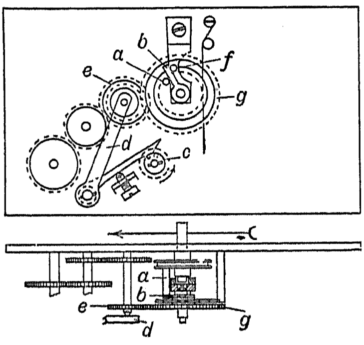


FIG. 45.

by the Aron Co., and is shown in *Fig.* 45, in which a pin *a* is fixed in the arbor carrying the pointer; this pin is moved forward by means of the arm *b* on the indicator wheel *g*, which latter is controlled by a hair-spring. At periodic intervals the wheel *c* lifts the double lever *d* and disengages the wheel *e*, which is connected with the registering mechanism from the indicator wheel, which being left free is carried by the hair-spring back to its original position against the stop *f*.

In the Aron meter the action is simplified, since the clockwork of the meter itself can be used to operate the maximum demand gear described later. In many cases the maximum demand dial is fixed above or below the normal reading dials, but in the Westinghouse dial, shown in *Fig.* 46, the large dial indicating

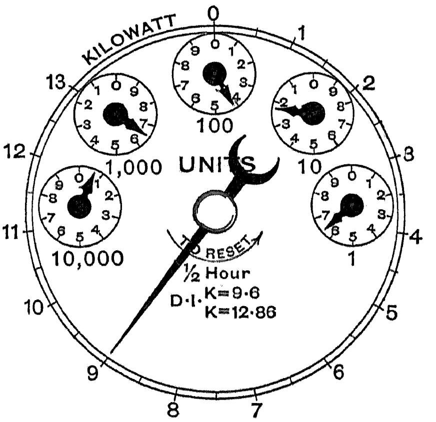


FIG. 46.

the maximum demand is enlarged to enclose the reading dials.

§ (36) TWO-RATE SYSTEMS.—In this system differentiation is made between the energy used during the period at which the maximum load occurs on the supply system and that when the load is much lower, the object being to encourage the use of electrical energy during the periods of light load by means of a lower rate of charge. The differentiation is produced by means of the use of a double set of registering mechanisms, one or the other set of dials

being brought into gear by means of an arm controlled by an electro-magnet operated by a time switch. The arrangement of dials is shown in *Fig.* 47, in which the upper set indicate

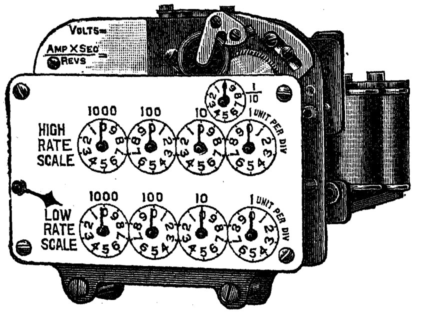


FIG. 47.—Meter Dial showing Electro-magnet.

the consumption during the high-rate period (normally from about 5 P.M. to 10 P.M.), and the lower set the low-rate period (usually about from 10 P.M. to 5 P.M.).

§ (37) TIME SWITCHES.—The switches used for operating the electro-magnetic gear changing devices for two-rate or maximum demand indicators are designed expressly for the purpose, and consist of an electrically or hand-wound clock which makes and breaks a contact at given intervals of time, which interval can be set at any period from 10 minutes to 2 hours for a maximum demand indicator, and for periods of several hours for two-rate meters.

§ (38) THE METERING OF HEAVY CURRENTS.—The measurement of large direct currents presents special difficulties which can generally be overcome by proper precaution either in the construction of the meter or the manner of erection and use. The term " large currents " is usually applied to values of several thousand amperes, but even with a meter of size 500 amperes 500 volts, used on a traction or other large supply where the cost of the energy metered may amount to several thousand pounds per year, the conditions of use are frequently such that large errors may be introduced, and therefore what follows applies to sizes of meters from 500 amperes upwards.

§ (39) VARIABLE AND FLUCTUATING LOADS.—Several investigations have been made to determine the accuracy of direct current watt-hour meters on rapidly fluctuating loads, such as would obtain on a traction circuit. Oerlich and Schultze,[1] in 1909, tested some small meters and also gave a theoretical proof of their experimental results, showing that such loads did not affect the accuracy of the meters.

If K is the moment of inertia of the armature, A the damping couple for unit angular velocity ($\omega=1$), a the angle in circular measure which the armature turns through in t seconds, D the driving couple corresponding to any consumption Q, then the differential equation for the motion of the armature is

$$K\frac{d^2a}{dt^2}+A\frac{da}{dt}=D. \qquad (1)$$

(i.).—Consider first the integral of this equation for two special cases:

(*a*) The case when the maximum load comes on suddenly and then stops suddenly.

(*b*) The case when the rise of current is gradual and the cut-off sudden.

(*a*) At the time $t=0$ suppose the current to take up immediately the maximum value which the meter will register and then remain constant, so that D is constant and equal to D_m, the turning moment corresponding to the maximum load of the meter. The integral of (1) bearing in mind that for

$$t=0, \quad a=0, \quad \text{and } \omega=\frac{da}{dt}=0$$

is

$$a=\frac{D_m \cdot t}{A}-\frac{D_m K}{A^2}\left(1-e^{-\frac{A}{K}t}\right).$$

Also since when there is no acceleration the driving couple must be equal to the brake moment, it follows, if ω_m denote the angular velocity for maximum load, that

$$D_m=A\omega_m.$$

If the armature had no inertia so that $K=0$ the angle moved through would be

$$a_0=\omega_m t=\frac{D_m}{A}t, \qquad (2)$$

therefore

$$a_0-a=\frac{D_m K}{A^2}\left(1-e^{-\frac{A}{K}t}\right) \qquad (3)$$

is the amount by which in consequence of the inertia the record shown is too small.

After a few seconds only, unless A/K is very small, the exponential function becomes vanishingly small, so that the amount of the error becomes

$$a_0-a=\frac{D_m K}{A^2}. \qquad (3\text{A})$$

If the constant maximum load be now reduced to zero, the meter does not stop immediately but while stopping turns on through the same angle (3A). This error and the error which arises in starting up thus cancel each other.

(*b*) The question now arises whether the error due to the inertia of the armature when the current gradually rises from zero to its maximum value is also cancelled when the current is suddenly cut off. Suppose that the turning moment, which is proportional to the load, may be taken to rise from zero to its maximum value D_m according to the law

$$D=D_m(1-e^{-pt}). \qquad (4)$$

If p is large the rise is rapid: if p is small it is slow. It is well known that a current increases according to this law if the circuit contains self-induction.

[1] *Elektrotechnik und Maschinenbau*, xxvii. 801.

If D is given by equation (4), then the integral of the differential equation (1) is

$$\alpha=\frac{D_m}{A}t+\frac{D_m}{p(Kp-A)}(1-e^{-pt})-\frac{D_m pK^2}{A^2(Kp-A)}\left(1-e^{-\frac{A}{K}t}\right),$$

(for $t=0$, $\alpha=0$, $\omega=0$).

After a relatively short time the exponential functions become negligibly small, and we have

$$\alpha=\frac{D_m}{A}t+\frac{D_m}{p(Kp-A)}-\frac{D_m pK^2}{A^2(Kp-A)},$$

or

$$\alpha=\frac{D_m}{A}t-\frac{D_m(A+pK)}{pA^2}. \quad . \quad . \quad . \quad (5)$$

The angle α_0, which gives the motion if the armature has at every moment a speed proportional to the load, is obtained from equation (5) by putting $K=0$. Thus

$$\alpha=\frac{D_m}{A}t-\frac{D_m}{pA},$$

hence the error

$$\alpha_0-\alpha=\frac{D_m K}{A^2}. \quad . \quad . \quad . \quad . \quad (6)$$

If this be compared with (3A), it is seen that the error is of the same magnitude as when the load increases suddenly from zero to its maximum value. Hence if the current increases according to the law of equation (4), is then kept constant for some time and finally suddenly cut off, the inertia of the armature will not introduce any error.

(ii.).—The law found in the special cases (a) and (b) holds generally for motor meters. In general the friction in the meter ought also to be taken into account. According to an experimental investigation of Schmiedel,[1] the friction consists of two parts—the constant part R_0 and the part proportional to the velocity $R_1(d\alpha/dt)$. Suppose now that the load and the turning moment, proportional to it, take from time to time values D_t, which may change in any manner. Suppose the load to be turned on at the time $t=0$, that is to say, at this instant $\alpha=0$, $\omega=0$, then the following equation holds :

$$K\frac{d^2\alpha}{dt^2}+(A+R_1)\frac{d\alpha}{dt}+R_0=D_t. \quad . \quad . \quad (7)$$

If the armature has no inertia the angle α_0 corresponding to this case satisfies the equation derived from this by putting $K=0$, that is to say,

$$(A+R_1)\frac{d\alpha_0}{dt}+R_0=D_t. \quad . \quad . \quad . \quad (8)$$

The last equation shows that even under constant load and constant armature speed the meter will only give readings proportional to the load if the constant term for the friction R_0 is reduced to zero by appropriate means, such as compensating coils. By subtraction we get from equations (7) and (8)

$$(A+R_1)\frac{d(\alpha_0-\alpha)}{dt}-K\frac{d\alpha^2}{dt^2}=0,$$

or bearing in mind the initial conditions

$$(A+R_1)(\alpha_0-\alpha)=K\frac{d\alpha}{dt},$$

$$\alpha_0-\alpha=\frac{K}{A+R_1}\omega_t. \quad . \quad . \quad . \quad (9)$$

[1] *Electrical Review, London*, Dec. 22, lxix.

Equation (9) gives at every instant the magnitude of the error due to the inertia of the armature. The error is at every instant proportional to the speed of the armature at that instant. Since, at a short time after the current is cut off, ω_t becomes equal to 0, it follows that the error due to the variation of the load from the instant when the circuit is made, up to a short time after the circuit is broken, is zero under all the circumstances in meters of motor type. This holds good in whatever manner the load may have varied and at whatever value the moment of inertia, the damping and the friction of the armature may be.

(iii.).—In order to determine how pendulum meters behave under variable load, we have to remember that the period of a single vibration of the pendulum depends only on the forces which are in action during the period considered. The forces acting in the preceding period have no effect since the two periods are divided by intervals of time in which the velocity of the pendulum is zero. The duration of any particular vibration of a pendulum is thus determined by the mean value of the load within the period. Since, for example, in the short pendulum meters of Aron the period only amounts to a fraction of a second in the great majority of the periods, the limits within which the individual mean values can vary are extraordinarily small. It is only when for some considerable time the variations within each period are very great that the mean value is not sufficiently reliable. That is a case which does not arise in practice. We must, therefore, conclude that also for pendulum meters the inconstancy of the load will give rise to no errors.

(iv.) *Experimental Tests.*—Robertson [2] also tested small meters and gave mathematical proof, and Laws and Ingalls, in 1912,[3] dealt with larger meters on similar loads. Melsom and Eastland [4] described experiments made with a number of 200-ampere meters of various types, including Thomson type motor meters, Evershed frictionless meter, Chamberlain and Hookham mercury meter, and Aron clock meter. The measurement of the current here was made with a large copper voltameter, and the results confirmed the theory of Oerlich and Schultze and of Robertson and their experiments with small meters, and thus all available evidence shows that for all practical purposes there is no appreciable difference between the behaviour of any of these types of meters with a steady or a rapidly varying load (current).

§ (40) External Magnetic Fields.—Melsom and Eastland [5] made a number of tests with a field equivalent to that produced in a straight conductor carrying 1000 amperes placed at a distance of 2 feet from a meter, and showed that with some of the meters the errors due to this field were from 10 per cent at one-tenth load to 1 per cent at full load. Actually, however, it would appear that the stray fields met with in practice are

[2] *Journ. I.E.E.* xlix. 489.
[3] *Electrical World*, lix. 1309.
[4] *Journ. I.E.E.* xlix. 465.
[5] *Ibid.*

very much larger than this, particularly where the meter is erected on a switchboard borne on steel supports, which are magnetised by the current in conductors in the immediate vicinity, or where large magnetically operated circuit breakers are erected near the meter. Ratcliffe and Moore [1] mention that "one of the authors has seen a dynamometer watt-hour meter stop at about one-third full load, and actually reverse at one-quarter load, due to the effect of a stray field," and many cases have been shown in which the effect of stray fields is very large. Generally, the mercury watt-hour meter when enclosed in a case of soft iron is comparatively unaffected, as are the Thomson meters of the astatic type and the Aron meter with its astatic pendulum. The freedom, however, depends to a fair extent on the uniformity of the stray field. If it is such as to affect both armatures to the same extent the effect is fully compensated for, but this complete degree of uniformity cannot always be relied upon. Complete immunity can be obtained by removing the meter from the switchboard and erecting it in a position a few feet away, where the stray fields are not effective, running the leads, if the meter coils carry the whole current, close to and touching each other. If the meter is of a shunted type the question of removal to a distance is even easier, since only the comparatively small connecting leads are affected. It will be obvious that difficulties with stray fields are most frequently due to the lay-out of the switchboard where symmetry and arrangement may be considered before accuracy, rather than to inherent faults in the meter, but the importance of the matter is gradually being recognised, and the use of shunted meters operating at a distance from the main current circuits is now more usual.

A simple method which may be used to ensure that stray fields are not affecting a meter is to erect the instrument in its first position and to connect the pressure circuit only, leaving the main current connections disconnected from the meter and short-circuited on themselves. The effects of any stray field should then be shown by a movement of the meter dials.

§ (41) Momentary Excessive Currents.—The effect of the momentary excessive currents which flow when, owing to a short circuit, a fuse or other circuit breaker operates may have a large and permanent effect on the accuracy of the meter. In such a meter as the Aron, having no permanent magnets for brake disc or any iron in the magnetic circuits, there is no appreciable effect, but in the motor meter the effect may be large. In a motor meter where the main current meter circuit, consisting of one or more turns, is in close proximity to the magnets operating the brake disc, if an excessive current flows through the current circuit, the brake magnet may be demagnetised by an amount sufficient to affect the rate of the meter by as much as 5 per cent. This is usually met in practice by the use of a shielding box totally enclosing the brake magnets (a shield consisting of a flat sheet is usually insufficient for the purpose), and if the box be of good quality soft iron it affords complete protection. Cast-iron boxes, are, however, unsatisfactory, since as was shown by Melsom and Eastland,[2] the effect of a short-circuit current equal to thirty times the normal full load of a meter was to magnetise the cast-iron box to an extent sufficient to affect the accuracy of the meter by 9 per cent at one-tenth load, and 2·5 per cent at full load, the additional field making the meter go faster. With a box made of Stalloy iron or of soft wrought iron the differences due to the momentary overloads were less than 1 per cent.

The effects of short circuits are also important on the small size American watt-hour meter, Fitch and Huber,[3] showing changes of from $-2{\cdot}3$ per cent to $+2{\cdot}9$ per cent in a number of 10-ampere meters when short-circuited with a fuse across 240 volts, the maximum peak value of the current being of the order of 600 amperes or only sixty times the normal maximum current of the meter.

§ (42) Error of Level.—This affects the accuracy of motor meters slightly at light loads (the Aron meter is provided with a level whereby the meter may be set), and it is usual in erecting a meter to check the level by means of the well-known method of placing a coin or other small weight on the outer periphery of the brake disc and adjusting the level so that the weighted disc does not rotate when the meter is tapped.

§ (43) Polarity.—It is essential that the polarity marked on the meter terminals should be observed, since the correct direction of rotation does not necessarily indicate that the polarity is correct. This is particularly essential in the case of the mercury meters where, owing to the use of iron in the pressure circuit, the rate of the meter may be 30 per cent in error if the connections are reversed.

In shunted meters the main source of error in connection is due to loose or dirty connections between the shunt and the meter; in meters the coils of which carry the whole of the current the resistance of the main current connections should be reduced to a minimum in order to ensure that there is no appreciable heating which may be conducted into the meter itself. Further, large cables employed must be supported independently of the meter

[1] *Journ. I.E.E.* xlvii. 3.

[2] *Journ. I.E.E.* xlix. 465.
[3] *Bull. Bureau of Standards*, x. 161.

terminals so that there is no possibility of twist or strain on the meter parts.

III. Testing of Meters

§ (44) Meter Tests.—Excellent accounts of methods of testing meters, both watt-hour and ampere-hour, are given by Schmiedel [1] and Fitch and Huber,[2] these referring more particularly to examination of performance of types. Ratcliff,[3] Gerhardi,[4] Solomons,[5] *Electrical Meterman's Handbook*,[6] and Melsom and Eastland [7] furnish among them complete information as to the testing of meters on laboratory or works test-room scale. Other publications of international interest, giving information as to the types of meters in use in various countries and their characteristics, are those of Durand [8] and Sharp.[8]

§ (45) Apparatus. (i.) *Measurement of Energy.*—The instruments generally used are precision-type ammeters and voltmeters which are frequently compared with a potentiometer set. If an ammeter is used for the measurement of current, it must be provided with a number of shunts so that a fair reading on the scale, say not less than one-half of full scale, can be obtained for measurement of any current between one-tenth load and full load of the meter. In many cases a standardised meter is used. For electrolytic meters of small size a copper voltameter has been used with a fair degree of accuracy. For the commercial type meters, however, comparison with a carefully tested standard meter of the same type is probably the most satisfactory method, although the standards themselves will have to be tested either against a voltameter or by means of a measured current maintained at a steady value for the long time required for the tests.

(ii.) *Measurement of Time.*—For short time tests made by counting a given number of revolutions of the rotor, stop-watches or chronographs are generally used, although in some cases the more accurate recording drum chronograph is available. The term stop-watch denotes a watch in which the whole movement is started and stopped by the movement of a lever, and chronograph that in which the movement is running continuously, the gear which operates the seconds and minute hands being put in and out of gear with the movement. The latter type is the most accurate and reliable, but even this has to be exceedingly well constructed to give results consistent to $\pm 0{\cdot}2$ per cent under the somewhat severe conditions of use involved in the testing of a large number of meters.

§ (46) Methods of Test. (i.) *Motor Meters.*—In the case of motor type watt-hour and ampere-hour meters, particularly of small size, satisfactory tests at various loads can be made by counting a number of revolutions of the rotor and timing by means of a chronograph, the loads at which tests are made being full load and $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$th and $\frac{1}{20}$th of full load. To determine the accuracy at the various loads use is made of a "testing constant" marked on the meter, which is based on the ratio of the gearing from the rotor spindle to the first unit dial on the registering mechanism. This constant is sometimes defined as the actual ratio of the gearing, but more generally as the ampere-seconds or watt-seconds for one revolution of the rotor. Confusion existed in older practice, and the specification of the British Engineering Standards Association, Cl. 31 (*h*), requires that the marked constant shall indicate the number of revolutions of the rotor corresponding to one kilowatt-hour. This is generally adopted, but some makers, *e.g.* Chamberlain and Hookham and Ferranti, still give in addition the constant of amperes × seconds/revolutions. Comparison of the actual rate of the meter as determined by the tests with that deduced from the constant gives the amount of error. In many cases the load is set at nearly the required value, but it is much more satisfactory both as regards accuracy and time to set the load at the actual value and to time a number of revolutions in proportion to the load. In addition to these short-time tests, however, it is essential that a long-time test shall be made in order to check the accuracy and operation of the gearing and also to determine the extent of any errors due to self-heating or thermo E.M.F. Thus the ordinary series of tests made on a small size meter would be recorded somewhat as follows:

For a 100-Ampere 240-Volt Watt-hour Meter

(Time deduced from meter constant = 100 secs.)

Volts.	Amperes.	Revs.	Times.	Error.
			secs.	per cent.
240	5	4	103	−3
	10	8	101·2	−1·2
	25	20	99·2	+0·8
	50	40	99·8	+0·2
	75	60	100·4	−0·4
	100	80	100·8	−0·8
After dial test	100	80	99·2	+0·8
Dial test—2 hours at full load			..	+0·2

[1] *Verhandlung des Vereins zur Beförderung des Gewerbfleisses*, lxxxix. 571, and xc. 111.
[2] *Bulletin Bureau of Standards*, x. 161.
[3] *Journ. I.E.E.* xlvii. 3.
[4] *Electricity Meters, their Construction and Management.*
[5] *Electricity Meters.*
[6] *Electrical Meterman's Handbook*, National Electric Light Association, New York.
[7] *Journ. I.E.E.* xlix. 465.
[8] Turin Exhibition, 1911, Tema xviii.

The light-load tests would be repeated after the dial test and, if the meter is fitted with a register of the counter type, would be made in both positions of the weight or spring, operating the jump device.

The short-run test, both before and after the dial test, shows the difference in rate due to self-heating, and comparison of the results of the dial test with a mean value of the two short-run tests shows at once any discrepancy in the counting train or intermediate gearing.

For large current meters a much longer time may be required to obtain the effect of self-heating, and it is usual to extend this test until such time as the meter rate remains constant at full or any load which may be specified by the user. Also, it is usual to determine the pressure drop at the ends of the large shunts where these are used and to measure the amount of thermo E.M.F. produced in the shunt. These measurements are usually made by means of a potentiometer, the pressure drop being measured under three conditions — (1) immediately after full-load current is applied (or at various loads), (2) after the load has been maintained until the shunt has attained to its maximum temperature, and (3) following (2), and immediately after the current has been switched off.

Data of this kind provide the information necessary where meters and shunts may be calibrated or tested separately or be calibrated to a standard pressure drop, and show the change in pressure drop due to heating or thermo E.M.F. in the shunt, with the consequent effect on the accuracy of the meter.

§ (47) Minimum Running Current.—The B.E.S.A. specification requires that the rotor of a motor meter shall start and continue running steadily when a current of one-hundredth of the marked (full load) current of the meter traverses the main circuit, and that when a pressure of up to 10 per cent above the normal voltage is applied to the pressure circuit the rotor shall not rotate. Both of these conditions should be met when the meter is erected in a position free from vibration; in testing for this requirement although much information can be gained from observations extending over a few minutes, it is much more satisfactory to make the test extend over several hours—preferably overnight.

§ (48) Energisation of Pressure Circuit in Watt-hour Meters.—In view of the fact that in some meters the pressure circuit coils are made of copper and will, therefore, take some time to arrive at their full temperatures and resistances, and also that in most other meters the energy dissipated in the pressure circuit heats up the other parts of the meter slightly it is necessary to apply the pressure for several hours before the tests are made.

§ (49) Aron Clock Meters.—In the case of these meters a short-time test is impossible. The reversing period is nearly 10 minutes, and observation at full load for 20 minutes, that is two complete periods, is sometimes regarded as sufficient. To obtain really accurate results, however, the test at full load should be not less than one hour, with a proportionately longer test at the lighter loads. Since, however, the error curve of this meter should be a straight line it is not always necessary to make a greater number of tests than at, say, full load, $\frac{1}{2}$ load and $\frac{1}{10}$th load, with a test for no-load running extending over at least 24 hours; this latter, by indicating a creep of the dials, will frequently explain the reason for a divergence from the line at light loads.

§ (50) Electrolytic Meter.—These meters have to be tested over the whole scale at full load, taking a number of observations at different scale values with a check at a lighter, say, $\frac{1}{4}$ load to ensure that self-heating is not affecting the meter.

§ (51) Temperature. — The temperature must be noted and the observed values corrected to a standard value by means of the known temperature coefficient of the meter. Here, however, unless the test-room temperature has been fairly constant for some time before the tests are made it will not be certain that the temperature of the interior working parts of the meter is the same as the outside air, and in such a case, or where the air temperature fluctuates considerably during the tests, it is essential that precautions should be taken to avoid the errors due to changes of temperature. Where a large number of small meters are being tested a thermometer placed inside a meter which is not connected in the circuit will give a fairly correct value for the other meters, but in the case of large meters it is more usual to enclose them in a lagged enclosure which is maintained at a constant temperature by external heating. This is convenient, since the temperature can be set to be nearly that of the position in which the meter is to be used, such as in a generating station where the normal temperature may be of the order of from 20° to 25° C.

§ (52) Specifications and Requirements for Approval of Meters.—The type of meter which may be used on a public electricity supply is to some extent governed by legal enactment. In Great Britain the Electric Lighting (Clauses) Act of 1899, Cl. 49, requires that "unless otherwise agreed" the supply shall be through an appropriate meter duly certified under the Provisions of the Special Order (*i.e.* the order granted to a Company or Corporation to undertake the supply of electricity), and Cl. 50 defines an appropriate meter as one of a pattern and construction

approved by the Board of Trade and which has been tested and certified to be correct by an electric inspector. Thus, the use of an approved type of meter is not compulsory, and in most cases the Supply Authorities enter into an agreement with the consumer as permitted by the Act. In general, however, the majority of meter types used have been examined and approved, and frequently the Supply Authorities require evidence of such approval before purchasing meters.

The actual examination of meter types is now carried out by the National Physical Laboratory on behalf of the Electricity Commissioners.

In France, according to Decree of the Department of Public Works and Transport of January 1920, the use of an approved meter is compulsory for all public supply, while in America the requirements for meters are those issued in the "Code for Electricity Meters" passed by the Committees of The National Electric Light Association and The Association of Edison Illuminating Companies.

In all the countries the primary function of the inspection and approval of meters was supposed to be that of protecting the interests of the small consumer; thus, the Board of Trade requirements for accuracy limit, in some cases, the amount by which a meter may register fast, but place no restriction on the error in the opposite sense. Actually, however, in the examination of the various types the end in view has been to secure an instrument which can be relied upon to give a fair measure as between supply authority and consumer.

A summary is given below of the requirements of the various countries; with this is given also the requirements of the B.E.S.A. (British Engineering Standards Association), according to the Specification No. 37, 1919.

§ (53) Types of Meter permitted.—In Britain, France and Germany the use of both ampere-hour meters and watt-hour meters is permitted, while in America watt-hour meters only may be used for public supply.

§ (54) General Conditions for Approval. (i.) *Accuracy.*—Requirements for initial accuracy, etc., are laid down in each case, but the actual approval of a type, that is either the principle of operation or details of construction, is left to the judgment of the Approving Authority. Thus, the Board of Trade regulations state—

> A. The construction of the meter should be mechanically suitable for the purpose, and it should fulfil the following conditions . . .

and

> B. The arrangement of the meter electrically should be such as gives a reasonable probability of permanence, and should also fulfil the following conditions. . . .

The French rules say—

> The approval is given if suitable according to the opinion of the Electricity Committee,

and the German—

> A meter is approved when its system has been approved by the Reichsanstalt,

and

> When it satisfies the following conditions. . . .

The conditions applicable are then set out in full.

(ii.) *Duration of Approval.*—In most cases approval of a meter once given is permanent, but in the latest French regulations approval is valid for 10 years, at the end of which time it may be renewed on the demand of the manufacturer.

(iii.) *Particulars to be furnished by the Manufacturer.*—In every case a manufacturer submitting a meter type for approval must submit a number of meters, generally three, but the latest British requirements call for five; of these one is retained permanently, being sealed and kept for reference, and one other may be taken apart and the various components used for separate tests.

Descriptive diagrams in triplicate, illustrating the principle of the meter, together with a detailed plan and description of fundamental working parts, including details of windings, section of wire, etc., are required as is also information as to the torque at full load and the total friction, etc.

Further, the French regulations call for (1) indication of the manner in which the meter is protected from the different causes of error, and for instructions relating to the various regulating devices, and (2) a preliminary certificate from an approved Laboratory, giving the results of tests made on a meter of the type.

(iv.) *Sealing of Case.*—It is required that the meter shall be provided with means for sealing the case so that access cannot be obtained to the working parts without breaking the seals. Generally, in practice, two sets of seals are provided, one for the working parts and the other for the terminal compartment.

(v.) *Labels.*—In all cases it is required that a meter shall bear a label, generally of metal, with the following particulars indelibly engraved thereon:

1. Name of manufacturer.
    - (i.) *British* (Board of Trade).—Name of manufacturer or his agent, or the trade name under which the meter is sold.
      (B.E.S.A.).—Country of origin. Name of manufacturer.
    - (ii.) *French.*—Name of manufacturer.
    - (iii.) *German.*—Name or trade mark of manufacturer.
2. Serial number of the meter.
3. Type of meter whether for continuous or alternating current, and if the latter, the frequency for which it is suitable.
4. Maximum current for which the meter is intended.
5. Pressure of the circuit for which the meter is intended.

6. Testing constant.
   (i.) *B.E.S.A.*—The revolutions of the rotor (if any) corresponding to one kilowatt hour expressed as "Revs. per k.w.h."
   (ii.) *French.*—The time for one revolution of the most rapidly moving part or the energy necessary for one revolution.
   (iii.) *German.*—Usually stated as r.p.m. of the rotor.

§ (55) CERTIFICATE OF APPROVAL. (i.) *British* (Board of Trade).—The meter must *not* bear a statement with the words "approved by the Board of Trade," and in marking the dial plate the words "Board of Trade" or "B.o.T." must not be used in conjunction with the word "Units." "Units" alone is sufficient.

The reason for this apparently is that the use of the words "Board of Trade" on every meter was held to imply that every individual meter issued with the label had been tested and certified by the Board of Trade.

(ii.) *French.* — Every meter used must bear a statement giving the date of the certificate of approval of the type.

§ (56) LIMITS OF ERROR PERMITTED.

(i.) *British* (Board of Trade).

Size up to 3 amperes, from $\frac{1}{10}$th to full load	±3 per cent	
Exceeds 3 amperes but not exceeding 50 amps. from $\frac{1}{10}$th to full load	±2	„
Exceeds 50 amperes from $\frac{1}{10}$th to full load	±2	„
From $\frac{1}{10}$th load to $\frac{1}{20}$th load	+2	„

The rate shall not be appreciably affected by—

(*a*) Variations from the normal pressure of ±10 per cent.

(*b*) For alternating current, by variations from the normal frequency of ±6 per cent.

(*c*) For inductive loads, by variations in the power factor between 0·5 and 1.

In any case given above the term meter includes all or any accessory apparatus, such as separate shunts or transformers.

(ii.) *B.E.S.A.* (*a*) *Permissible limits of error* at the standard or marked temperature, pressure, frequency and at unity power factor.

	Meters without External Shunts or Transformers + or −.	Meters with External Shunts or Transformers + or −.
From full load to $\frac{1}{5}$ load inclusive.	2 per cent	2·5 per cent
From $\frac{1}{5}$ load to $\frac{1}{10}$ load	2·5 „	2·5 „
At $\frac{1}{20}$ full load	4·5 „	5 „

(*b*) *Effect of Variation of Pressure.*—A variation of ±5 per cent in the marked pressure at any load from full to $\frac{1}{10}$th shall not cause a change in the error of more than one per cent. (This is in addition to the error at standard pressure.)

(*c*) *Effect of Heating by Main Current.*—The change in rate due to self-heating by the main current circuit shall not exceed ±2 per cent and the total error shall not exceed the values given above.

(iii.) *French.*—For continuous current meters tested under the following conditions:

	Voltage. (Nominal = 1.)	Current. (Nominal Full Load = 1.)	Temperature.
(1)	1	0·02 to 1·2	Arbitrary between 10° C. and 30° C.
(2)	1·1	0·1 to 1	
(3)	0·9	0·1 to 1	
(4)	1	0·5	θ 0°
(5)	1	0·5	θ 20°

θ 0° and θ 20° are arbitrary temperatures between 0° C. and 40° C.

The limits of error expressed as the space between two parallel lines drawn one above and one below the no error line shall not exceed ±3 per cent for all loads from $\frac{1}{10}$ to full load, for lighter loads between the $\frac{1}{10}$ and the $\frac{1}{20}$ the space between two inclined straight lines allows of an error of ±5 per cent at $\frac{1}{20}$ load and ±10 per cent at $\frac{1}{30}$ load. For loads from full to 1·2 times (20 per cent overload) the lines again are inclined allowing a maximum error of ±4 per cent at 20 per cent overload.

(iv.) *German.*—At a room temperature of from 15° C. to 20° C. the deviation of the reading of a meter from the true reading for loads exceeding 15 watts between marked (full) load and $\frac{1}{20}$ load must not exceed

$$\pm\left\{3+\left(0\cdot3\,\frac{P_n}{P}\right)\right\}\text{ per cent}$$

of the actual true reading, where P_n is the marked (full) load of the meter and P the actual load.

When the marked current is exceeded by x per cent the increase in the error shall not exceed $x/10$ per cent of the error permissible at that load P, which is obtained by replacing the actual current by the marked current, all other conditions remaining unaltered.

This is equivalent to errors as follows:

Current Load.	Error per cent.
Full	±3·3
$\frac{1}{2}$	±3·6
$\frac{1}{4}$	±4·2
$\frac{1}{10}$	± 6
$\frac{1}{20}$	± 9

These limits are much wider than those allowed by other countries.

(v.) *American.*—(Code for Electricity Meters).
(*a*) At normal voltage:

Load.	Permissible Error.
Full	± 2 per cent
$\frac{1}{10}$	± 3 per cent
$\frac{1}{20}$	$\pm 7{\cdot}5$ per cent

(*b*) *Effect of Variation of Voltage.*—A change of ± 10 per cent in the pressure shall not cause an error of more than ± 3 per cent at full load, or 5 per cent at light loads.

§ (57) THREE-WIRE METERS. (i.) *British* (B.E.S.A.).—When the marked current is flowing in one of the two current circuits of a three-wire meter and half the marked current in the other circuit at the marked pressure, the rate of registration of a meter which purports to be a three-wire meter shall not vary by more than 1 per cent from the reading given by an equivalent balanced load.

(ii.) *French.*—Tests are prescribed having one element only energised, but there is no statement as to an additional margin of error.

(iii.) *American.*—The elements may not differ by more than ± 2 per cent.

§ (58) TEMPERATURE COEFFICIENT. (i.) *British* (Board of Trade).—A statement that the record shall not be affected within practical limits by variations of air temperature.

(ii.) *British* (B.E.S.A.).—No limits, but where the temperature coefficient exceeds 0·1 per cent for 1° C. information as to temperature coefficient shall be stated on the label.

(iii.) *French.*—Meters have to be within the standard limits of error over a wide range of temperature, but no separate value is given for this factor alone.

(iv.) *German.*—No limits.

(v.) *American.*—The temperature coefficient shall not exceed 0·2 per cent for 1° C.

§ (59) EFFECT OF EXCESS CURRENT. (i.) *British* (B.E.S.A.).—The accuracy shall not be permanently impaired by a current of 25 per cent in excess of the marked value maintained for one hour, or by a current of thirty times the marked current maintained for one half second.

(ii.) *French.*—The application of a current of ten times the normal, the time being limited by a fuse rated for a current double that of the normal. This operation is repeated five times and the meter error must not exceed the specified limit.

(iii.) *American.*—For meters less than 600 amperes, 400 per cent overload applied three times in periods of two seconds each shall not change the rate of registration by more than ± 5 per cent at light loads and ± 3 per cent at full load.

§ (60) EFFECT OF EXTERNAL FIELDS. (i.) *British* (B.E.S.A.).—A warning that the accuracy may be seriously affected, and an appendix stating precautions necessary in the erection of a meter to ensure that stray fields do not affect the accuracy.

(ii.) *French.*—Specifies tests to be made at different positions in the earth's field.

(iii.) *American.*—A uniform field of 0·1 (c.g.s.) line per square centimetre applied in the direction to have the maximum effect shall not change the rate by more than 2·5 per cent.

§ (61) MINIMUM RUNNING CURRENT.—In all cases this is specified as 1 per cent of full load current, with the further provisions in the B.E.S.A. Specification that when this current is less than one-twentieth of an ampere then one-twentieth of an ampere.

§ (62) ENERGY LOSSES. (i.) *British* (Board of Trade).—No stated limits.

(ii.) *British* (B.E.S.A.).—(*a*) *Pressure Circuit*: For circuits up to 250 volts = 5 watts, for pressures above this 2 watts per 100 volts.

(*b*) *Current Circuit*: 10 watts for meters up to 50 amperes, with the further restriction that the pressure drop at full load shall not exceed two volts. For large meters with external shunts the pressure drop over the shunt shall not exceed 0·25 volt at maximum current.

(iii.) *French.*—*Pressure Circuit*: 4 watts per 100 volts.

Current Circuit: From 5 watts to 50 watts, with a range of sizes of meters from 5 amperes to 100 amperes.

(iv.) *American.*—*Current Circuit*: Pressure drop of 1·5 per cent of the circuit voltage for meters up to 10 amperes and 0·75 per cent for larger meters.

§ (63) INSULATION. (i.) *British* (Board of Trade).—Insulation shall be reasonably good.

(ii.) *British* (B.E.S.A.).—The meter shall comply with the following requirements as regards insulation:

(*a*) The insulation resistance between all the electric circuits of the meter coupled together, and the containing case, or other metal not intended to be insulated when the meter is in use, shall be not less than 5 megohms.

(*b*) The insulation resistance between the main circuit and the pressure circuit, if any, shall be not less than 2 megohms.

(*c*) The insulating material between all the electric circuits and the containing case, or other metal not intended to be insulated when the meter is in use, shall withstand, for one minute, an alternating pressure equal to twice the pressure of the circuit for which the meter is intended, with a minimum of 1000 volts. All the electric circuits shall be coupled together before applying the high pressure.

The insulation resistance shall be measured with a pressure of not less than 200 volts nor more than 500 volts (direct), applied for a sufficient time for the reading of the insulation indicator to become practically steady.

The requirements of (*a*) and (*c*) shall apply

to all auxiliary apparatus, other than pressure and current transformers, used with the meter.

(iii.) *French.*—Values are not specified, but the insulation is one of the factors considered by the Approving Committee.

§ (64) Effect of Inaccuracy of Level.—The B.E.S.A. Specification requires that all meters the accuracy of which is affected by small changes in level shall be provided with means of ascertaining without breaking the seals that it is correctly levelled, and the French requirements state that all meters not provided with a plumb line will be tested both when level and in a position 5° from vertical.

§ (65) Torque.—Both French and American requirements call for a high ratio of torque to weight, but do not give definite values. The French regulations state that the torque will be measured and express the unit as cm.-grammes at full load.

§ (66) General.—In addition to the information quoted here, the B.E.S.A. Specification deals with a number of constructional points and also specifies standard sizes of meters for currents from 2·5 amperes to 5000 amperes.

S. W. M.

WIRELESS TELEGRAPHY

§ (1) ELECTRIC WAVES.—It follows from Maxwell's theory, as developed in his paper on "The Electromagnetic Field" and in his treatise on *Electricity and Magnetism* published in 1873, that the effect of any change in an electromagnetic field is propagated through space with the velocity of light. Hertz's experimental investigation upon very rapid electrical oscillations was published in Wiedemann's *Annalen* for the year 1887. This was supplemented in the year 1889 by his mathematical discussion of the electric and magnetic fields produced in free space by such an oscillation. This discussion, which was based upon Maxwell's Electromagnetic Theory, will be found in chapter ix. of the English translation of Hertz's papers, entitled "Electric Waves." A treatment which involves less mathematics will be found in J. J. Thomson's *Conduction of Electricity through Gases*, at p. 657 of the 2nd edition. Here it is shown[1] that, when a concentrated charge of electricity of magnitude Q, moving with velocity dx/dt, much less than that of free electric waves, is undergoing acceleration, it produces electric waves in the surrounding space. Let r be the distance from the charge to any point in the field of the waves, θ the angle between the radius vector and the axis of x, V the velocity of the waves, and κ the specific inductive capacity of the medium; then the component electric field perpendicular to the radius vector and due to the acceleration of the charge is given by the expression

$$\frac{Q}{\kappa V^2}\frac{\sin\theta}{r}\frac{d^2x}{dt^2}.$$

The simultaneous component of magnetic force perpendicular to the radius vector is given by

$$\frac{Q}{V}\frac{\sin\theta}{r}\frac{d^2x}{dt^2}.$$

These fields exist at the point r, θ at a time r/V later than the instant of the acceleration.

§ (2) ENERGY OF THE WAVES.—In the application of these results to wireless telegraphy, we regard the oscillatory current in a simple form of antenna as a to-and-fro motion of a quantity Q of electricity in a vertical line which we take as the axis of x. Evidently the fields are symmetrically distributed round the axis. In such a case as this, Poynting's theorem of the propagation of energy in the electromagnetic field leads, very simply, to an expression for the rate at which the waves convey energy away from the moving charge. It is shown, by the authors cited, that this rate of radiation of energy may be written[2]

$$\frac{2}{3}\frac{\mu Q^2}{V}\left(\frac{d^2x}{dt^2}\right)^2.$$

This energy passes into free space, does not return, and is the measure of the radiation emitted by the charge; it passes through the sphere of radius r at a time r/V later than the instant of the acceleration.

§ (3) CONTINUOUS WAVES.—The general results now reached can be extended to the case

[1] See also "X-rays," § (2).
[2] See "Poynting's Theorem."

of continuous acceleration occurring according to a given mathematical law. An important case is that of harmonic variation of the acceleration, velocity, and linear displacement of the charge. Let a be the amplitude, ω the pulsatance of the vibration, and x the displacement of the charge from the origin at any time t. Then

$$x = a \cos \omega t,$$
$$x' = a\omega \sin \omega t,$$
$$x'' = a\omega^2 \cos \omega t,$$

where x' and x'' are written for the differential coefficients of x with respect to t.

The radiation fields, electric and magnetic, at distance r are also harmonic but delayed by the time r/V, the tangential electric field being given by

$$-\frac{\mathrm{Q}a\omega^2}{\kappa \mathrm{V}^2}\frac{\sin\theta}{r}\cos\omega\left(t-\frac{r}{\mathrm{V}}\right).$$

The formula already obtained for the total rate of spherical radiation contains the acceleration squared, and since the time average of the squared values of a simple sine function of the time is $\frac{1}{2}$ we have for the mean power radiated

$$\frac{\mathrm{Q}^2a^2\omega^4}{3\kappa \mathrm{V}^3}.$$

Since $$\omega = 2\pi f = \frac{2\pi \mathrm{V}}{\lambda}$$

an equivalent expression is

$$\frac{16\pi^4 \mathrm{V}\mathrm{Q}^2a^2}{3\kappa\lambda^4},$$

where f is the frequency and λ the wave-length. These equations are true for all consistent systems of units.

§ (4) THE DOUBLET.—From these results for a moving charge we may pass to the ideal harmonic doublet of Hertz, which consists of two equal stationary charges of opposite sign varying in magnitude harmonically with the time. The moment of the doublet at the instant when each charge is of value q is defined by ql, where l is the distance between the two stationary charges. Let

$$q = \mathrm{Q}\cos\omega t.$$

Then the moment at any time t is

$$\mathrm{Q}l\cos\omega t,$$

which indicates that the doublet can be replaced by two equal and opposite constant charges Q separated by a distance which varies harmonically so that they move in opposite directions along the same line according to the law

$$x = \tfrac{1}{2}l\cos\omega t.$$

So far as radiation is concerned this is equivalent to a single constant charge Q moving according to the law

$$x = l\cos\omega t.$$

Therefore the mathematical expressions developed above may be converted to Hertz's form by writing l for a.

In the doublet the variation of the charges may be supposed to take place by the flow of electricity from one to the other. In this case the current along the connecting wire is

$$i = \frac{dq}{dt}$$
$$= -\mathrm{Q}\omega\sin\omega t.$$

Hence the amplitude of the current is $\mathrm{I} = \mathrm{Q}\omega$ and the rate of radiation is

$$\frac{(\mathrm{I}l\omega)^2}{3\kappa \mathrm{V}^3}$$

or $$\frac{4}{3}\frac{\pi^2}{\kappa \mathrm{V}}\left(\frac{l\mathrm{I}}{\lambda}\right)^2.$$

These are forms of Hertz's result.

§ (5) FIELD OF AN OSCILLATOR.—A detailed calculation of the field round a simplified Hertz oscillator was given by Hertz, and solutions appropriate to special forms have been made by many others, as, for example, Love. But the physical essence of the process occurring in all cases is most simply seen by aid of Heaviside's graphical treatment of a linear oscillator. In *Fig.* 1 AA indicates an

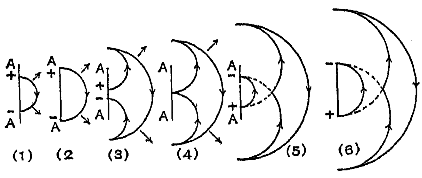


FIG. 1.

insulated wire along which concentrated equal positive and negative charges are moving symmetrically. The Faraday lines joining the charges are lines of longitude on a sphere, but only one of these is shown. Since every element of a Faraday line travels with the velocity of light perpendicular to its length the lines expand circularly until the charges reach the end of the wire. Here the charges are immediately and simultaneously reflected just as is a pulse of rarefaction or condensation at the ends of the air column of an organ pipe, and without loss of time they start again towards the centre. But each original Faraday line, in virtue of its inertia, continues to move outwards and remains a semicircle. A little later, as the third diagram shows, new semicircles are formed within the original one, and when the charges have returned to the centre of the wire a closed loop of electric force is just completed. Further history of this loop is continued in the two succeeding diagrams. The diagrams are drawn at time intervals of $\frac{1}{8}$th of a period.

§ (6) Time Period and Damping of Radiator.—The radiator used by Hertz in his best-known experiments consisted of two metal surfaces joined by a straight rod interrupted centrally by a spark gap, as shown in *Fig.* 2.

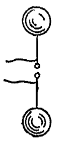


Fig. 2.

When in use each side was joined to a terminal of an inductorium, and on starting this each area became charged until the potential difference broke down the air-gap and filled it with heated ionised air and metal vapour; so long as the gap is ionised the whole apparatus constitutes a single conductor along which the plus and minus charges, to use the language of the two-fluid theory, can rush to and fro symmetrically in opposite directions. The motion of the charges is analogous to the motion of a pendulum; it is periodic, and the energy is alternately all electrokinetic and all electrostatic, and is all ultimately dissipated in radiation and in Joulean losses.

Hertz computed the time period of his radiator by calculating the capacity C of his two end conductors regarded as forming a condenser, the inductance L of the straight rod joining them, and applying Kelvin's formula for the discharge of a Leyden jar through a coil. For approximate purposes this formula is

$$\omega=(LC)^{-\frac{1}{2}},$$

which leads to

$$T=f^{-1}=2\pi\sqrt{(LC)}.$$

Kelvin's analysis gave also an expression for the decay coefficient b of the oscillations so far as Joulean losses were concerned. This may be written

$$b=\frac{R}{2L}.$$

The damped oscillation obeys with sufficient accuracy for our purpose the equation

$$q=Q\epsilon^{-bt}\cos\omega t.$$

Evidently the values of q at two instants separated in time by a complete period bear the ratio ϵ^{bT}. The natural logarithm of this ratio, which is called the logarithmic decrement, is bT, or

$$2\pi R\sqrt{\left(\frac{C}{L}\right)}.$$

The decrement, when dissipation is due to resistance, is thus seen to be larger the greater the capacity and the smaller the inductance.

§ (7) Damping of the Oscillations.—Turning now to the consideration of the damping caused by radiation, let us suppose that at any one of the instants of zero current the charge is Q, then since the energy radiated per second is, as shown in § (3), proportional to Q^2l^2/λ^4 the energy carried away by the next wave emitted is proportional to Q^2l^2/λ^3. But the energy possessed by the radiator at the beginning of this wave is $\frac{1}{2}Q^2/C$; therefore the percentage loss due to radiation is proportional to Cl^2/λ^3. This can be written in the equivalent forms $l^2/\lambda L$ and $l^2/L^{\frac{3}{2}}C^{\frac{1}{2}}$, neglecting constant multipliers. The quantity Q has disappeared during division, indicating that the percentage loss, which is proportional to the logarithmic decrement under the assumptions made, is the same in each oscillation. From this result we see that the decrement due to radiation is greater the smaller the capacity and the smaller the inductance. Thus increase of electric capacity acts in opposite ways while increase of inductance acts in the same way when we compare resistance decrement with radiation decrement.

In a train of oscillations or waves diminishing in amplitude by a given equal percentage at each alternation the number of alternations in the train is easily seen (by summing a geometrical progression) to be inversely proportional to the given percentage; in other words, the "length of a train" is inversely proportional to the logarithmic decrement. For purposes of greater precision than is needed here the length of a train might be defined as the number of waves or oscillations of which the amplitude is greater than, say, 0·2 of the maximum amplitude; which implies that those waves or oscillations whose energy content is less than 4 per cent of the energy content of the greatest do not count.

§ (8) Effect of Inductance.—Let us consider, for simplicity, a Hertzian radiator of very low internal resistance—one in which the damping is due to radiation. We see from § (7) that the number of waves emitted from the radiator in any given discharge is proportional to the product of the inductance and the wave-length, and again, that it is proportional to the square root of the product of the capacity and the cube of the inductance; this is true whatever the voltage to which it is initially charged. An important result here comes to light. Suppose a radiator to consist of two parallel metal plates of variable but always equal area separated by the constant distance l and joined electrically by a wire (and spark) into which inductance coils can be connected. The wave-length is always proportional to the square root of the product of the inductance of the coil and the electrical capacity between the plates. Starting from any medium values, suppose the metal areas each increased fourfold, nothing else being altered; then the wave-length would be doubled and the number of waves emitted (in the sense of § (7)) also doubled. Now suppose the self-inductance of the coil to be increased fourfold instead of the capacity; the wave-length is doubled as before but the number of waves is increased eightfold. Thus the augmentation of the inductance is in this case

four times as effective in prolonging the train of waves as a corresponding augmentation of the capacity. The insertion of an inductance coil into a radiator for the purpose of prolonging the train was first done by Lodge, and this is the fundamental principle of Lodge's important patent No. 11575 of 1897.

§ (9) Resonance. — The principle of resonance is much used in wireless telegraphy. In all applications there is close analogy with resonance in acoustics and the mathematical side is fully discussed in, for instance,[1] Rayleigh's *Theory of Sound.* Without going into the mathematical treatment we may obtain a grasp of the two distinct aspects of the utilisation of resonance in wireless telegraphy: resonance is employed, firstly, for enabling pairs of stations to communicate without disturbing others; and, secondly, for enhancing the accumulation of energy by the receiving antenna and apparatus.

A receiver that possesses a distinct natural period of its own responds most vigorously to forces of that period. The faster the falling off of the response of a receiver to a transmitter as the period of either is altered the greater is said to be the "sharpness of tuning" or the "selectiveness." The word "selectiveness" comes from the idea of being able to select a particular station by tuning to its frequency; and wireless telegraphy carried on with a useful degree of selectiveness has been called "syntonic." Mutual disturbance is less for assigned differences of wave-length the greater the selectiveness or sharpness of tuning. A famous mathematical investigation by O. Bjerknes shows that for a Hertz radiator or resonator the rate of falling away of response as the difference of wave-length increases is greater the smaller the decrements of both radiator and resonator. But it is evident without calculation that if a small departure from the in-tune adjustment is made, the difference of frequency is most easily perceived when both radiator and resonator are capable of prolonged oscillations. This is merely because when there are many oscillations the falling out of step becomes very marked towards the end of the train emitted; and it is no use sending a long train to a receiver unless this can act cumulatively, that is, unless its own energy consumption (or damping) is small.

§ (10) Resonance and Energy. — The energy aspect of resonance takes note of the fact that the electrical work done on a resonator by the electric force in each wave reaching it is calculated by multiplying the electric force by the in-phase current running at the instant in the resonator. To see the consequence of this in a very simple way let us examine the building up of an oscillation in a non-dissipative resonator by means of a train of waves, the receiver and the radiator being of exactly the same frequency. Let i be the maximum value of the current in any alternation, i' in the next one, e the electromotive force induced by the incident waves, T the period, and L the inductance of the resonator. The work done is equivalent to the gain in the electrokinetic energy of the circuit, that is,

$$\tfrac{1}{2}(i+i')e\mathrm{T}=\tfrac{1}{2}\mathrm{L}i'^2-\tfrac{1}{2}\mathrm{L}i^2,$$

$$i'-i=\frac{e\mathrm{T}}{\mathrm{L}}.$$

That is to say, the increase of current per oscillation is proportional to the electric force in each wave. Noticing that the electric force in a wave is proportional to the square root of the energy content of the wave we can now answer the question: Having given a certain stock of energy to a radiator (that supplied just before the spark), is it better to send it to the receiver in two or three big waves or in a long train of small waves? To avoid elaboration, let us assume the respective trains to be uniform, not damped. Suppose that the initial energy of the radiator is 1600 units, and that on one occasion we divide it into four equal waves; each will contain 400 units of the energy and the electric field in each will be 20 units. Let each of these increase the current in the receiver by 2 units, then the four waves give altogether an accumulated current of 8 units, which corresponds to a quantity of energy we may indicate by the number 64. Now, in contrast, divide the initial energy among 16 waves each containing 100 units of energy and having an electric field of 10 units. Each wave will now add 1 unit of current to the receiver, and the 16 waves will constitute a total of 16 units, or an accumulation of energy measured by the square of this, namely, 256. It is therefore profitable merely from the energy point of view to divide a given initial stock of energy among a long train of waves.

The conclusion just reached has to be modified in detail for the practical case in which a damped train of waves excites a damped receiver; but the reasoning holds good throughout a large section of the building-up stage. Later, the dissipation of energy by the resistance of the receivers and possibly by its reradiation increases rapidly with the growth of current, and at a definite instant becomes equal to that imparted by the wave then incident; after this stage, as the incoming waves are continually decreasing in amplitude, the current in the receiver decreases also.

§ (11) Production of Waves on an Engineering Scale.—The radiator of Hertz, as used in the laboratory, was often less than one

[1] See also "Sound," Vol. IV.

metre in length over all, but Marconi in his specification No. 12039, June 1896, and Oliver Lodge in his specification No. 11575, February 1897, describe radiators more than 50 feet long. And besides magnifying the laboratory apparatus both specifications use radiators placed with axes vertical, and Marconi in particular shows cases in which the lower spark-ball is connected to the earth, which amounts in fact to a total suppression of the lower half of the symmetrical radiator. This construction has been followed very widely, though not exclusively, up to the present day.

At first the engineers who were trying to translate the Hertz radiator into practice used elevated metal plates, cones, etc., to which Lodge gave the convenient name of "capacity areas"; but it was soon found that continuous metal surfaces could be replaced by wire networks without loss of effectiveness; indeed Marconi began at an early stage to dispense with capacity areas, and achieved successful signalling by aid of plain vertical wires. For the case of this plain antenna with its lower end connected to earth Heaviside's idealised diagram of the moving lines of force is instructive and is given in *Fig.* 3. The earth is supposed to be a perfectly conducting plane surface and a concentrated positive charge is supposed to start from E at the foot of the wire and to move up the wire with the velocity of waves in free space. At the top the charge is reflected as described in § (6) and travels downwards, ultimately giving rise to the moving field indicated by the circular arcs centred upon A. When the charge reaches the bottom it spreads over the plane conducting earth and leaves a lack of electricity behind it, that is to say, a negative charge; and this in turn starts up the antenna and creates an electric field of circular arcs of negative sign. This diagram is but a skeleton representation of the actual phenomena, because in fact the charge on the moving antenna is distributed and not concentrated. Accurate diagrams of the field round a straight symmetrical radiator, such as those given by Love, need only be bisected by the plane of symmetry of the radiator in order to represent the field of an earthed antenna.

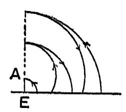


FIG. 3.

§ (12) THE EARTHED ANTENNA. (i.) *Theory.*—This last remark calls attention to the possibility of applying to the earthed antenna the theory of the symmetrical Hertz radiator. If the earth is plane and perfectly conducting, the antenna and its geometrical image in the earth form a symmetrical Hertz radiator and produce the same moving field. But only the field above the earth is physically existent and only to this is Poynting's theorem applicable. Hence the radiation of energy from a plain antenna is half that from the symmetrical radiator of which it is half. The formulae we have already obtained can be applied immediately to what is called a "flat top" antenna, for the horizontal capacity area which forms the flat top is correctly analogous to the capacity area of the laboratory apparatus and the wire to earth carries the same current as an equal symmetrical radiator would; hence if this current be of amplitude I we have for the rate of radiation from a flat top antenna of height h the expression

$$\frac{8}{3}\frac{\pi^2}{\kappa V}\left(\frac{hI}{\lambda}\right)^2.$$

In the practical system of units this is equal to

$$790\frac{h^2I^2}{\lambda^2}\text{ watts}$$

or $$1580\frac{h^2A^2}{\lambda^2}\text{ watts,}$$

where A is the root mean square value of the alternating current. In other words, the resistance that would dissipate energy at the same rate as the antenna radiates it, which is called the radiation resistance, is 1580 h^2/λ^2 ohms, h and λ being in the same units.

(ii.) *Typical Examples.*—In *Figs.* 4 and 5 are shown the two extreme types of earthed antennae, each with its electrical image formed in the surface of the earth, supposed infinitely conductive. If, in *Fig.* 4, C_0 be the electrical capacity between the upper area and the earth, and L_0 the inductance of the down lead, the system of real antenna and image is equivalent to a symmetrical Hertz radiator of capacity $\frac{1}{2}C_0$ (being C_0 and C_0 in series) and inductance $2L_0$. The wave-length is therefore

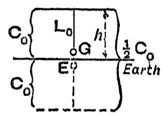


FIG. 4.

$$\lambda = 2\pi V\sqrt{L_0C_0},$$

just as for the complete oscillator of double height.

On the other hand, in the simple antenna of *Fig.* 5 let L_0 be the inductance, C_0 the electrical capacity, of the down lead of length h. The velocity of waves along the wire, which is equal to the velocity of light, is known to be the square root of the product of the inductance per unit length and the electrical capacity per unit length, that is to say,

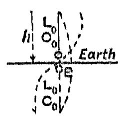


FIG. 5.

$$V = \sqrt{\left(\frac{L_0}{h}\right)\left(\frac{C_0}{h}\right)},$$

hence $$h = V\sqrt{L_0C_0}.$$

A standing wave forms on the simple antenna just as in an organ-pipe with stopped end, there being a node of current at the upper end and a loop at the earthed connection. Thus h is the quarter wave-length, or

$$\lambda = 4h = 4V\sqrt{L_0C_0}.$$

Any consistent system of units may be used in this equation.

(iii.) *Actual Antennae.*—Very many practical antennae rank between these two extremes, which may be called the cases of perfectly concentrated electrical capacity and uniformly distributed electrical capacity. In practice the vertical wire of *Fig.* 5 forks into a harp or a spreading fan or a star of wires in one or several directions, horizontal or inclined. These long wire extensions play a part in the formation of the standing wave on the antenna; but if, as is often the case, they possess great electrical capacity relative to the earth without being very long—as in an umbrella antenna—they approximate in behaviour to *Fig.* 4. In general these practical types lie nearer that of *Fig.* 4 than that of *Fig.* 5, and their natural wave-lengths are covered by the formula

$$\lambda = 4aV\sqrt{L_0C_0},$$

where the factor a ranges from unity for the case of *Fig.* 5 to $\frac{1}{2}\pi$ for the case of *Fig.* 4. The practical types of antenna referred to are, for example, the inverted L antenna, the T antenna, and the umbrella antenna, whose names explain their form sufficiently.

§ (13) Antenna Adjustment. (i.) *Tuning by Inductance.*—Hertz obtained variation of frequency of his laboratory radiators by moving the capacity areas nearer together or altering those areas, but neither of these operations is possible with a large practical antenna. When, however, Lodge proposed in Specification 11575 of 1897 to introduce inductance coils in series with the antenna in order to prolong the radiations and to reduce the damping in the receiver, he pointed out that the coil provided a convenient means for adjusting the frequency. These coils therefore became known as tuning coils, or, since they increase the wave-length of the antenna, as "loading" or "lengthening" coils. As regards the effect of an inductance coil on the decrement due to radiation and to resistance, the discussion in § (7) holds good for every kind of practical antenna whether it is earth-connected or not. As regards the effect of added inductance on the natural frequency or wave-length of the antenna the two extreme forms of antenna shown in *Figs.* 4 and 5 must be considered separately.

In *Fig.* 6 an inductance is shown inserted in the flat top antenna; since the electrical capacity of the flat top is large we neglect that of the down lead. From preceding paragraphs it is obvious that the actual antenna has the same frequency as the symmetrical radiator formed by adding the image in the earth's surface. The capacity of such a radiator is equal to that of two condensers C_0 in series, namely $\frac{1}{2}C_0$, and the inductance is $2(L+L_0)$. Hence by Kelvin's result

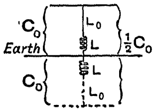


Fig. 6.

$$\lambda' = 2\pi V\sqrt{\{(L+L_0)C_0\}}.$$

Now consider the simple antenna of *Fig.* 5, but with a coil inserted. A standing wave is formed in the manner indicated in *Fig.* 7. It is seen from the diagram that if the tuning coil is of relatively large inductance the tapering of the potential on the straight extended part of the antenna is small, and therefore that it may be represented for approximate calculations as having capacity and inductance ranging between the distributed and the concentrated cases of *Figs.* 5 and 6. To avoid calculation we may appeal to the almost analogous case of a "spring pendulum" consisting of a helical spring supporting a mass; the spring itself has distributed mass, and the lower end nearer the mass vibrates with the full amplitude of the mass, while the upper end is fixed. The familiar rule of the physical laboratory directs that one-third the mass of the spring should be added to that of the suspended object and the total used in the uncorrected formula. This suggestion, applied to the case of the loaded simple antenna, accords well with experiment; it yields the equation

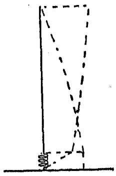


Fig. 7.

$$\lambda' = 4aV\sqrt{\{(L+\tfrac{1}{3}L_0)C_0\}}$$

in the notation of § (12).

This formula is found on trial to be useful for almost all antennae, the value of a ranging in practice from 1·4 to 1·57.

(ii.) *Tuning by Condensers.*—In nearly all stations it is necessary to use a tuning coil in the antenna if only to serve as the primary of the transformer for the detector circuit as described later. The free period of the antenna as thus loaded may be regarded as the normal period, but the antenna may be required to receive on different occasions waves of shorter or longer wave-length than the normal. When it is to be tuned to shorter waves a condenser is inserted in the down lead; and when it is, on the contrary, to be tuned to longer waves additional inductance is first inserted, and if this is not enough a condenser is connected in parallel with the coil, that is, with the antenna

capacity. The two modes of using a condenser are shown in *Figs.* 8 and 9. In *Fig.* 8 it does not matter whether the condenser or the coil is nearer the earth connection. These

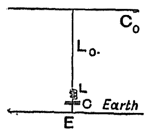


FIG. 8.

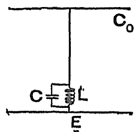


FIG. 9.

modes of using condensers are extremely convenient because it is easy to obtain condensers having a smooth variation through a large range.

When the condenser is in series and the coil is large compared with L_0 the oscillatory circuit starting at E is made up of C, L, L_0, C_0, and back to E again. The acting capacity is made up of C and C_0 in series and is therefore $CC_0/(C+C_0)$. The formula developed in preceding paragraphs now becomes

$$\lambda' = 4aV\sqrt{\frac{(L+\frac{1}{3}L_0)CC_0}{C+C_0}}.$$

When the condenser is, on the contrary, connected in parallel with the antenna capacity we have the total capacity $C+C_0$, and therefore the wave-length is

$$\lambda'' = 4aV\sqrt{\{(L+\tfrac{1}{3}L_0)(C+C_0)\}}.$$

To emphasise the different effects of the two modes of connection let the normal wave-length without condenser be

$$\lambda = 4aV\sqrt{\{(L+\tfrac{1}{3}L_0)C_0\}}.$$

Then we have

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{C}{C+C_0}} \qquad \text{(series connection)}$$

and

$$\frac{\lambda''}{\lambda} = \sqrt{\frac{C+C_0}{C_0}} \qquad \text{(parallel connection)}.$$

These results may be made the bases of methods of determining the electrical capacity and the normal wave-length of a loaded antenna.

In obtaining the formulae we have supposed that the inductance coil in circuit is much greater than the distributed inductance of the antenna, and in practice the formulae are adequate; to treat the matter more fully would take us into the theory of standing waves on antennae and of coupled circuits.

§ (14) COUPLED CIRCUITS.—Let two distinct oscillatory circuits L_1, C_1 and L_2, C_2 be linked together, so that a current in one causes an electromotive force in the other, as, for instance, by mutual inductance between the coils; they are then said to be coupled. Such circuits are frequently used in wireless telegraphy. For discussing the electrical motions in the circuits we need two co-ordinates, and usually the currents i_1, i_2 in the circuits are chosen. The coupled system has thus two degrees of freedom, and therefore it is capable of oscillation in either one of two distinct periodic times; in general, any disturbance of the system will result in a compound oscillation made up of these two possible oscillations. This is easily illustrated by the mechanical analogy suggested in *Fig.* 10. Here two

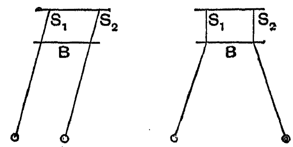


FIG. 10.

simple pendulums supported at S_1, S_2 are coupled by the light bar B. On the left the coupled system is shown oscillating in one possible mode, on the right in the other; no other distinct frequency of oscillation is possible, but in general each bob has both motions and performs an apparently complicated resultant movement.

(i.) *Simple Theory.*—Consider the coupled circuits of *Fig.* 11 and suppose them to be devoid of resistance. In this case the application of the general mathematical process is very simple. For the current in either is equal to the induced electromotive force from the current in the other divided by the reactance. Let ω be either of the unknown frequencies of the system and M the mutual inductance, then

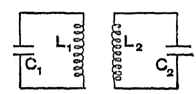


FIG. 11.

$$i_1 = M\omega i_2 \Big/ \left(L_1\omega - \frac{1}{C_1\omega}\right),$$

$$i_2 = M\omega i_1 \Big/ \left(L_2\omega - \frac{1}{C_2\omega}\right).$$

Multiplying these equations together eliminates i_1 and i_2 and gives a quadratic equation in ω^2. But since in practice the circuits are always tuned together we shall, to save algebraical labour, introduce the simplification so suggested and put

$$L_1C_1 = L_2C_2 = \frac{1}{\Omega^2}.$$

We thus obtain, on solving the quadratic equation,

$$\omega^2 = \frac{\Omega^2}{(1 \pm k)},$$

where k is the "coupling coefficient," which is defined by

$$k^2 = \frac{M^2}{L_1L_2}.$$

We may put the solution in another form by writing λ and λ' for the two wave-lengths of the system, and λ_0 for the wave-length of each circuit taken apart; we have

$$\lambda^2 = \lambda_0^2(1+k),$$
$$\lambda'^2 = \lambda_0^2(1-k).$$

If, as usual, the coupling coefficient is small relative to unity, this result states that the wave-lengths of the coupled system are respectively $50k$ per cent above and below the natural period of either circuit taken apart. Values of k commonly occurring are 0·02 to 0·2, which are sometimes spoken of as 2 per cent and 20 per cent coupling. In the former case the wave-lengths of the coupled system are different from each other by 2 per cent, the natural wave-length of the separate circuits falling half-way between them.

(ii.) *Practical Details.*—Tuned coupled circuits were introduced into receiving apparatus by Lodge and into sending apparatus by Marconi, the antenna being one oscillatory circuit in each case. Later, in Marconi's patent No. 7777 of 1900, the sender has two tuned coupled circuits and the receiver two also—that is, all four circuits have the same frequency. The coils between which the mutual inductance functions are called oscillation transformers, or high frequency transformers, or sometimes jiggers.

The use of coupled circuits at the sender gives several advantages. In the first place, by making the antenna the secondary to a primary circuit containing the spark gap, the antenna can have oscillatory energy gradually fed forward to it to be radiated, thus avoiding very high initial voltages on the antenna; and by making the primary condenser big relative to the antenna capacity a larger initial stock of electric energy can be given per spark than if the antenna were charged direct. The primary serves, in fact, as a reservoir and allows the antenna to be smaller than would be otherwise necessary. In the second place, the antenna oscillation is gradually worked up to a maximum, and may continue after the spark in the primary is extinguished, and a prolonged train may thus be obtained. This last, besides carrying the advantages described in § (10) under resonance, eliminates the disturbance often caused to neighbouring stations by charging the antenna itself and suddenly discharging it through its own spark gap.

A disadvantage of the coupled sender is the existence, already discussed, of two wave-lengths. If, however, the coupling is small this is not a serious source of trouble unless very sharp tuning is aimed at. Moreover, the two oscillations are rarely present in equal intensity. Reference to the analogy of the pendulum suggests, and accurate investigation shows, that by charging both primary and antenna simultaneously and discharging them with proper phase relationship it would be possible to make the system oscillate in either one of the two modes. This has not been developed in actual practice, perhaps because the advantages gained would be small.

At the receiving end the antenna is the primary and the indoor circuit the secondary of the coupled system. By adjustment of the coupling the rate at which the energy gathered by the antenna is taken away to the detector can be regulated. In Marconi's early apparatus the coherer was directly in the antenna, and its high resistance while uncohered prevented the antenna oscillation building up and therefore greatly diminished the absorption of energy from the waves. When Lodge introduced the inductance coil into the receiving antenna he made it possible to remove this defect; and in the specification already quoted he showed both direct coupling, that is to say, the coherer connected in shunt to the tuning coil, and indirect coupling, in which the coherer circuit is coupled by mutual inductance to the antenna tuning coil.

§ (15) Spark Methods of Signalling.—A single spark at a transmitting station causes the emission of one train of waves from the antenna; this builds up an oscillation in the receiving antenna, which in turn passes the oscillations to the detector to be rectified. The rectified electricity goes as a unidirectional pulse through a telephone and gives a click at the diaphragm. For example, a train of 30 waves, each of wave-length 2000 metres and period 1/150,000 second, endures 1/5000 second, which indicates the order of duration of the pulse in the telephone. A succession of sparks gives a corresponding succession of taps in the telephone, and a regular succession of sufficient rapidity gives a musical tone. Such musical signals were obtained independently by Marconi and by Fessenden by the use of rotatory dischargers; the Marconi pattern consists of a disc carrying metal knobs which when the disc rotates pass between metal terminals connected to the condenser in the primary of a coupled sender. The supply of high voltage current may be unidirectional current fed through choking coils to the condenser, or it may be alternating current. With the latter type of supply the disc may be rotated so that there is one spark per alternation (synchronous method), or may be asynchronous so that there are sometimes several sparks per alternation of the supply current. The direct current or the synchronous alternating current method gives good musical tones if the time period of the charging circuit be adjusted to suit the rate of sparking. At large power transmitting

stations these discs wear badly at the electrodes on account of insufficient cooling when more than 500 sparks per second are attempted. It is interesting to note that 500 sparks per second with a wave-length of 2000 metres implies that there is room for 300 high frequency oscillations between sparks.

Rapid sparking has several advantages. The energy that can be radiated per spark is fixed by the charge that can be given to the reservoir circuit or antenna before each spark, that is by the capacity of the circuit and the voltage to which it is charged; but the power consumed in radiation, that is to say, the energy radiated per second, is proportional to the rate of sparking. This is one reason for using rapid sparking in long-distance stations. Another reason lies in the greater sensitiveness of the ear to tones of frequency about 400 per second than to low notes of, say, 20 per second.

§ (16) Quenched Sparks.—Still more rapid sparking can be obtained by the introduction of the quenched spark method, though this was not the main consideration in the development of the method. The so-called quenched spark is made to occur between large and very close metal plates kept cool by air or water circulation; the large area and the small thickness of the air space facilitate the de-ionisation of the gap, and therefore very rapid sparking is possible. The quenched gap comprises usually a number of flat plates separated by mica annuli, one gap being allowed for each thousand volts of the supply, and the group of gaps takes the place of the ordinary spark in the primary of a coupled circuit. The primary makes only two or three alternations and is then quenched, and thus the antenna is left free to oscillate with its own frequency and decrement. The design of the circuits and gap should ideally be such that all the energy of the primary is passed to the secondary by the time the gap is quenched. It is easy to use these gaps on powers as great as 50 kilowatts with an alternating supply frequency of 1000 per second. There are many varieties of plant, the earliest being associated with the names of the Lepel Co. and the Telefunken Co. In some form it is clear that even when the supply current is alternating at 1000 cycles per second there are many sparks in each alternation. The form due to Chaffee has been carefully studied in small sizes and shown to be able to spark so fast that there are only 2 or 3 antenna oscillations between successive discharges of the primary condenser.

§ (17) Continuous Wave Telegraphy. (i.) *The Timed Disc.*—The close juxtaposition of spark trains mentioned in the preceding lines has been carried out with powers of 300 kilowatts by Marconi using his "timed disc" plant. In essence this consists of several discs on the same shaft, each carrying metal knobs between fixed electrodes as described in § (28), and staggered so that the rotating discs take turns in producing the discharge of a primary circuit; the heating is thus divided among the discs. In order to obtain extreme precision in the sparking and so ensure accuracy of phase relationship of the successive discharges an ignition disc is used. This disc is a relatively light one with fine teeth; it is required to carry only a small current and is therefore not greatly heated; and its function is to produce at each passage of one of its teeth a small ionising spark across that gap of the main discs just coming into operation. It is found that not only does this preliminary spark ensure precision of timing of the main discharge, but it also reduces the tearing of metal from the knobs during the main discharge. A high-power station transmitting with these machines is in use on one transatlantic route.

(ii.) *Alternators and Frequency Multipliers.*—Another class of methods of generating continuous oscillations bears a closer resemblance to those of ordinary electrical engineering. There are several types of alternator[1] in use, which are named after their inventors: the Alexanderson, the Goldschmidt, the Bethenod-Latour are examples. They may each be used for exciting an antenna direct, as they generate oscillatory current at 10,000 cycles per second and more. In this class we may also place the combination consisting of an alternator and frequency multiplier. The frequency multipliers used at present in wireless telegraphy are all based upon the magnetic properties of iron.

In most of them the iron is brought near to magnetic saturation by a steady current in a winding on the iron; and an alternating current of, say, 10,000 cycles per second is passed through a separate winding on the same core; when the magnetic field of the alternating current and that of the steady current co-operate the flux in the core increases only slightly, but when they are opposed the flux decreases greatly. Hence in a third winding on the same core the electromotive force induced possesses harmonics, that of double frequency being the most prominent; and a current of frequency 20,000 cycles per second can be led away from this third winding by aid of a tuned circuit. By repeating the process once a current of frequency 40,000 can be delivered to the antenna.

(iii.) *The Poulsen Arc.*—The Poulsen arc offers a mode of generating electrical oscillations of very high frequency and high power. Stations of 500 kilowatts and even of 1000 kilowatts are now in operation. The typical

[1] See "Wireless Telegraphy Transmitting and Receiving Apparatus," § (5).

circuit is shown in *Fig.* 12. The condenser C is intended to represent the antenna capacity, and R to represent the same power absorption as is actually occasioned by the antenna resistance and radiation.

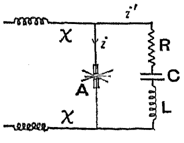


FIG. 12.

The ability of an arc to sustain oscillations arises from its so-called negative resistance, that is to say, its property of offering a smaller back electromotive force in response to the passage of a larger current through it. In the figure the choking coils marked χ are supposed to be so large that the current through them is perfectly constant; therefore when the condenser is taking charging current the arc current is smaller than the supply current, and the high back electromotive force of the arc encourages charging. When the condenser is discharging itself the arc current is greater than the supply current, its voltage is low, thereby again encouraging the process. Oscillations once started are therefore sustained. Let i be the current through the arc at any instant, i' that through the condenser shunt, i' being, of course, purely oscillatory. Let us suppose that oscillations are proceeding with the natural frequency of the condenser and inductance so that the only voltage dropped in the shunt is Ri' and is in phase with the current. When in a small interval of time the shunt current increases by $\delta i'$ the increase of the voltage drop is $R\delta i'$, and the current will not continue to increase unless this is made up by at least an equal increase in the arc terminal voltage. Let this increase be δv; for sustained oscillation we must have $\delta v > R\delta i'$. But $i+i'$ is constant, hence $\delta i = -\delta i'$, and therefore we have as the condition for oscillations

$$-\frac{dv}{di} > R.$$

The value of the differential coefficient can be obtained from a characteristic curve connecting corresponding currents and terminal voltages of the arc. Evidently it is necessary that the gradient of the characteristic be negative, however small the resistance R may be.

The Poulsen arc is formed between a positive copper and a negative carbon electrode in an atmosphere of hydrogenous vapour. The copper is hollow and is cooled by circulating water, and a strong magnetic field is established at the arc perpendicular to its length, the sense of the field being usually such as to drive the arc up from between the electrodes. For regular action not less than about 10 amperes at about 100 volts should be used. When the adjustments are correct the arc is struck on or near the upper edges of the electrodes, is driven outward by the magnetic field, is stretched as its ends travel back along the electrodes, and then suddenly expires. During this process the arc current diminishes and its terminal voltage rises. Later the arc strikes again on the hot carbon near the electrodes, grows rapidly in current and falls in voltage, and then is driven outward as before. These processes may be brought to occur in step with the condenser oscillations, being in fact dictated by them, and when the magnetic field is just right the regularity of the motions of the arc is nearly perfect. If the magnetic field is too weak the arc is moved outward too slowly and is relit at its extinction position instead of between the electrodes, returning to this place, however, after some whole number of oscillations. If it is too strong the arc is moved fast and the voltage rises quickly enough to ignite a new arc inside the first; several concentric expanding arcs may thus simultaneously span the electrodes. These phenomena cause varying periods to arise in the oscillatory current. This explanation of the influence of the magnetic field is due to Pedersen.

(iv.) *Thermionic Method.*—The remaining important method of generating high frequency oscillation is that in which three-electrode vacuum tubes or triode valves are employed. This method is discussed elsewhere.[1]

§ (18) DETECTION OF ELECTRIC WAVES.—The progress of wireless telegraphy has at all times been dependent upon the discovery of improvements in the means of detecting the feeble oscillating currents produced by the signal waves in the receiving antenna and its associated circuits. Instruments for this purpose are called detectors; few if any differ in principle from the instruments familiar in the measurement of alternating currents of the frequencies and amplitudes occurring in alternating current lighting, and many of them are directly derivable from alternating current instruments by the process of carrying some element to the limit of smallness. In general, alternating current instruments make use of (1) the heating effects of currents in conductors, (2) the magnetic effects of currents, (3) rectification, (4) electrodynamic attractions, (5) electrical attractions. For the detection of feeble high frequency currents the first three methods have been much used.

§ (19) THERMAL DETECTORS.—In these it is arranged that the current to be detected is passed through an extremely small mass of matter; the consequent rise in temperature is registered by the change of resistance of the material or by thermo-electric forces. Coherers and barretters and the electrolytic detector are examples of the former class, crystal detectors of the latter. In each class

[1] See "Thermionic Valves," § (5).

the actions may be summarised by saying that a train of oscillations deposits its energy very rapidly in the detector, and the change is chronicled at leisure by telephone or galvanometer.

(i.) *Typical Action.*—The typical coherer consists of two pieces, say galena, held together so that a crystalline corner or edge of the one piece touches a face or edge of the other piece over a very small area. Current passing through the compound conductor converges to cross the joint, and therefore a relatively high current density is attained in a very small mass of matter. The resistance coefficient of galena is negative. Consequently a steadily applied electromotive force in series with the joint produces a larger current immediately after the reception of a train of oscillations. The effect is enhanced by the superaddition of the heating effect of the increased direct current through the joint. In any case the heat developed at the joint is in most practical instruments very quickly conducted away, so that the coherer is "self-restoring." The main result is that a telephone receiver in the direct current circuit gives an audible click on the arrival of each train of waves, and a succession of clicks that builds into a musical note if a regular succession of sparks is employed at the sending station. The self-restoring type of coherer was developed especially for use with the telephone receiver.

(ii.) *Lodge Coherer.*—Other coherers consist of badly conducting films between two conductors; for example, a steel needle resting on a thinly oxidised iron plate is a good coherer. Many oxides and sulphides of the metals have large negative resistance coefficients and make good coherers. The filings coherer consists in its typical form of two metal plugs fixed in a glass tube with a space of a millimetre or two between their confronting plane ends, and with this space nearly filled with metal filings slightly oxidised or otherwise tarnished. The numerous contacts apparently each behave as above described, except that in most forms of instruments which were intended for use with relays and markers the "coherence" is not transient. This happens because, probably, the non-conducting films are exceedingly thin and the heating at the contacts is so considerable as to cause actual welding; calculation of possible temperature rise shows that this is not at all unlikely, and indeed with strong oscillations many experimenters have exhibited the welding together of filings and of single contacts. It was these considerations that led Oliver Lodge to give the single contact instrument, of whose properties he was the discoverer, the name "coherer." E. Branly, who investigated the effects of electrical oscillations on tubes containing filings, called his instruments "radio conductors."

(iii.) *Barretters.*—These are merely fine filaments of matter which become heated when oscillatory current is passed through them; the consequent change of resistance is used as the indication of rise of temperature. R. A. Fessenden used Wollaston wire as a filament, and also used in the same way the thread of liquid in a very fine hole through a diaphragm separating two portions of electrolyte. The liquid filament is the most sensitive form, other things being equal, because its temperature coefficient of resistance is negative, and therefore the increased flow of direct current following any rise of temperature increases and prolongs that rise.

(iv.) *Electrolytic Detectors.*—The electrolytic detector, which is of the same rank of sensitiveness as the best coherer, is made by dipping the end of a piece of Wollaston (platinum) wire into diluted acid in which is immersed another electrode, and passing a current of about a milliampere so as to form and maintain a bubble of gas round the minute electrode. The immersion of the fine wire should be so slight that a small bubble of gas may push the liquid away from its lower end and leave the wire connected to the main body of liquid by the thin film forming the upper surface of the bubble. Any oscillatory current through the instrument heats the minute mass of matter presented by the film, and the resistance of this falls. The device is self-restoring like the single contact detector. It is less sensitive when the point is made negative than when made positive, and it is best at about 3 volts negative to the other electrode.

(v.) *The Crystal Detector.*—This is typified by the very early form which is described by Dunwoody in U.K. Specification 5332, 1907. It is constructed by embedding the base of a crystalline mass of carborundum in solder in a brass cup and clamping it so that a sharp edge or point of the crystal presses firmly on a steel plate. Perhaps the hardness of these two materials ensures smallness of area of contact, with the result that heating by a train of oscillations develops a transient thermo-electric force at the junction. There is no need to have a battery in circuit.

Many crystals possess, like carborundum, very pronounced thermo-electric properties, but as they all apparently possess temperature coefficients of negative sign, they nearly all, including the carborundum detector, prove to be rather more sensitive when a direct current is running through them. The coherer action is thus combined with the thermo-electric in this type of contact detector so used. A favourite combination of crystalline substances is the "perikon" detector introduced by G. W. Pickard and described in U.K. Specification

10772 of 1909, which consists of a piece of zincite in contact with a piece of chalcopyrite. A steel needle touching the fractured surface of a crystal of iron pyrites is also a very sensitive detector.

§ (20) CONNECTIONS FOR THERMAL DETECTORS.—Coherer and crystal detector are both included in the term "Contact Detector," and both of them, it should be noticed, simulate the operation called rectification. For the crystal detector when traversed by alternating current yields a unidirectional current because of the rise of thermo-electric forces, and the coherer, when a battery is in series, gives an additional unidirectional current when an alternating electromotive force is applied in the circuit. Similarly the electrolytic detector is a rectifier. This point of view is useful in explaining the typical reception circuit which is shown in *Fig.* 13. Here LC make the oscillatory circuit, D is the detector, K a blocking condenser, E a battery, T a telephone. The detector is in two branches, namely, DK and DET, which span across the oscillatory circuit LC and tap off a proportion of its energy. High frequency current can pass through the branch DK but not through the branch containing the highly inductive windings of the telephone; unidirectional current can pass through the branch DET but not through the blocking condenser K. Hence, when the oscillations on the antenna induce and build up oscillations in the tuned secondary circuit LC, a small oscillatory current traverses DK and is "rectified," in the sense used above. The consequent unidirectional current is then compelled to pass through the telephone. Each train of damped oscillations therefore produces a click in the telephone.

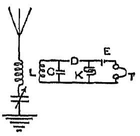


FIG. 13.

§ (21) THE MAGNETIC DETECTOR.—In June 1896 E. Rutherford detected electric waves received from a distance by using the demagnetisation of highly magnetised steel needles by oscillatory currents through a winding round the steel. The demagnetising effect of oscillatory discharges had previously been observed by Henry and studied by Rayleigh.

In 1902 G. Marconi discovered a generalisation of this physical phenomenon and devised a practical form of instrument. Instead of demagnetisation of steel magnetised to saturation, he observed and utilised either magnetisation or demagnetisation of iron at any stage of a slowly performed magnetic cycle. In *Fig.* 14 is shown a magnetic BH curve, and at the points P the effect of superposing an oscillatory magnetic field is roughly indicated in the diagram on one side. An oscillatory current in a winding adds to and subtracts from any value of H attained in the relatively slow progress round the cycle, and carries the representative point from the ordinary curve towards the dotted central curve. This dotted curve represents the behaviour of an ideal substance free from hysteresis, hence the action of oscillatory currents may be summarised by stating that they tend to annul hysteresis. At points such as P and P′ there is a sudden increase of the flux density, at the other points a decrease. Rutherford used the point R or R′; Marconi used the parts of the curve between P and Q or P′ and Q′ where the curve is steep and therefore the sensitiveness great. The differences of sensitiveness have been measured quantitatively, and are very noticeable by aid of a telephone receiver while listening to signals. In the commercial form of the apparatus Marconi uses a moving endless band of stranded iron which passes through a small solenoid while passing also through the magnetic field of a pair of horseshoe magnets. Oscillatory current in the solenoid changes the flux in the iron suddenly, and the change is detected by means of a second solenoid which also surrounds the iron and is connected to a telephone receiver.

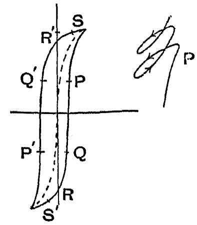


FIG. 14.

The high frequency solenoid of the magnetic detector may be made part of the tuning coil of the antenna, or of the coil in the closed circuit of *Fig.* 13. In the former event the secondary circuit is not needed.

§ (22) IONIC TUBES.—The modes of using thermionic vacuum tubes as detectors or rectifiers is fully treated in another article.[1]

§ (23) DAMPED WAVES AND UNDAMPED.—All the above methods of detection are appropriate for the reception of damped wave signals but not for continuous wave signals. In the latter a dash or dot is rendered as an unbroken unidirectional current by a rectifying detector, and only the beginning and end of the sign are heard in the telephone. By interrupting the continuous oscillations at the transmitter at a rate of more than a hundred times per second musical signals may, however, be transmitted for reception by receiving stations designed for damped waves, but this "tonic train" method, as it is called, loses many of the advantages of true continuous wave telegraphy. For continuous wave reception new methods had to be invented.

§ (24) THE TICKER AND THE TONE WHEEL. (i.) *The Ticker.*—The receiving instrument called a ticker is merely a beating contact

[1] See "Thermionic Valves," § (9).

connected in series with a condenser of about a hundredth of a microfarad, the whole forming a branch across the closed oscillatory circuit at a receiving station. The operator's telephone is connected in parallel with the condenser. The circuit is given in *Fig.* 15. Its mode of operation is as follows: While the contact is open the high-frequency oscillation builds up in the tuned circuit, and when it closes the accumulated energy of this circuit is spent at least in part in charging the condenser; then when the contact opens again the charge remaining on the condenser is sent through the telephone. This charge is sometimes positive, sometimes negative, and therefore the sound in the telephone is not a musical note but rather a rustling noise.

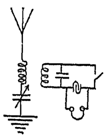


FIG. 15.

(ii.) *The Tone Wheel.*—The tone wheel is a contact device in which the contacts occur at nearly the same frequency as that of the high-frequency oscillations. It consists essentially of a wheel with many teeth and a brush touching the teeth. If the number of contacts is exactly the same as the oscillation frequency, and if they could be adjusted to occur near the maximum positive value of the oscillation, a unidirectional current of constant average value could be led away from the oscillatory circuit by connecting the wheel and brush across that circuit. But if the wheel is run at a slightly different speed the current collected will first be in one direction and then in the other, the rate of alternation being proportional to the difference between the actual speed and the synchronous speed. This difference may be chosen to give current of audible frequency which can be passed to a telephone.

§ (25) HETERODYNE METHODS.—When two pure musical sounds of slightly different pitch fall simultaneously on the ear, a regular throbbing is heard in the combined sound. For example, a note of pitch number 256 and one of 260 produce, if not too different in loudness, 4 easily audible throbs per second. They are called "beats" by musicians and have been utilised for centuries in tuning.

Similarly we may superpose two simultaneous oscillatory electromotive forces on a single circuit and study their relative motion by the simple apparatus of *Fig.* 16. The couplings M_1 and M_2 indicate inductive connection between two sources of oscillatory electromotive force. We shall suppose the oscillatory voltages induced in the detector circuit to be equal. The detector D may conveniently be a crystal detector and the oscillatory path is completed by the block condenser K, which is taken much too large to produce resonance. The rectified current from D necessarily flows through the telephones. This way of detecting the oscillations from circuit 1 was called by Fessenden the heterodyne method because in order to make evident the arrival of oscillations *via* M_1 the other source is invoked to supply auxiliary oscillations *via* M_2. The two oscillatory electromotive forces, if they are not of the same frequency, act on the detector with the beating amplitude already described, and cause correspondingly modulated unidirectional current, which passes through the telephone in a more or less smoothed-out form, the smoothing depending, of course, on the magnitudes of the reactances of K and of the telephone. Taking again the frequencies as 100,000 from circuit 1 and 101,000 from circuit 2 the unidirectional current through the telephone would pulsate at the rate of 1000 times a second and would therefore produce a note of this pitch number in the ear applied to the telephone. The sound is usually a clear flute-like tone possessing some musical quality; its strength depends on the strength of the electrical oscillations.

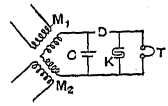


FIG. 16.

Suppose that the frequency of the source in circuit 1 is fixed at 100,000 ~ and that the source in circuit 2 is changeable at will; and suppose that the changes of frequency to be made are such as to leave practically unaltered the equality of the induced electromotive forces in the detector circuit. Then on listening in the telephone while the frequency of circuit 2 is slowly raised to, say, 102,000 ~ the note rises correspondingly to a pitch of 2000 ~, a somewhat shrill tone. As a rule the intensity of the sound will be smaller now than before, principally because the telephone has passed the frequency to which it responds most loudly, but also because the tone may have passed the frequency of the ear's greatest sensitiveness. On raising the radio frequency beyond 102,000 ~ the note becomes shriller, and also fainter for the reasons just explained, till at last no note is audible. On reversing the process the beat note passes down the gamut till when circuit 2 is giving the frequency 100,040 ~ the beat note is the deep bass almost inaudible sound of 40 vibrations per second. Sometimes on lowering the radio frequency further, say to 100,020 per second, a croaking noise is heard, but certainly between that and exact tune with circuit 1 there is silence. We are within the adjustment sometimes called the "pit."

But now on lowering still further the radio frequency till it reaches, say, 99,960 ~, the bass note of pitch number 40 reappears; and on moving further again the beat note

rises. When the frequency reaches 99,000 the loud sound of pitch 1000 ~ is again heard. In fact, on reducing the radio frequency from tune the same sequence of musical sounds is heard as would be gone through on raising it from tune. This is expressed diagrammatically in *Fig.* 17, wherein the assumption is made that the telephone-ear combination is most sensitive at about the pitch of 1000. The curves, it should be mentioned, are not based on actual measurements of intensity.

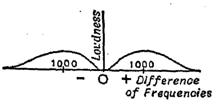


FIG. 17.

§ (26) PROPAGATION OF TELEGRAPHIC WAVES. (i.) *General Consideration.*—In § (4) it has been shown that the energy density of the field of the waves from a Hertz radiator decreases with distance in accordance with the inverse square law. This was found to be the case with telegraphic waves, up to distances of 50 miles, quite early in the history of wireless telegraphy. But in 1902, when crossing the Atlantic Ocean, Marconi discovered unexpected departures from this simple formula. He found, too, that after 500 miles from the sending station great differences appeared in the strength of day and night signals, the day signals being inaudible at 800 miles, while the night signals were readable at 2000 miles. Later observers found that day signals fell off much faster than with the inverse square of the distance, and that night signals, though usually much stronger than day signals, were too variable in intensity to be measured satisfactorily. The measurements made by the American Navy from 1910 to 1915 were reduced by Austin and Cohen, and shown to fit broadly the equation

$$I_2 = \frac{377}{\lambda R_2 x} \frac{h_1 h_2 I_1}{\sqrt{(1+\delta_1/\delta_2)}} \exp. \left(-\frac{0{\cdot}0015}{\sqrt{\lambda}} x\right),$$

where the suffix 1 refers to the sender and 2 to the receiver, and where I_1, I_2 are antenna currents, h_1, h_2 are the effective heights of the antennae measured in kilometres, x is the range, and λ the wave-length measured in kilometres, δ_1 and δ_2 are the logarithmic decrements, and R_2 is the resistance of the receiving antenna in ohms. The decrement correction term was suggested by Barkhausen. Measurements of optimum signals made by the writer on the Pacific Ocean indicate that the connection between field and distance at night is in rough accord with the expression

$$x^{-\frac{1}{2}} \epsilon^{-0{\cdot}0002x},$$

which suggests two-dimensional propagation of energy.

The Austin daylight formula for oceanic transmission shows that at each range a wave-length may be chosen for giving the largest received current. This wave-length varies as the square of the distance, being 562 metres for a range of 1000 kilometres If as a station moves the wave-length transmitted to it be altered so as to be always the best, the received antenna current is inversely proportional to the cube of the distance. That is to say, the energy density in the field of the waves falls off as the sixth power of the distance, as indeed was known or suspected among wireless engineers before the publication of the Austin-Cohen formula.

(ii.) *Effect of Land.*—The propagation of signal-bearing waves is considerably affected by the presence of land, even if the shortest path between the stations merely grazes a coast-line. Mountains and valleys have noticeable influence on cross-country signals. Sometimes stations on opposite sides of a mountain chain can communicate better by night than by day—but this depends upon the heights and distances. For instance, certain experiments by de Groot in the Dutch East Indies between a fixed station and a warship over very hilly country showed that signals over a range of about 200 kilometres were better in the daytime than they were at night.

At night, even on the open sea, signals may vary enormously in strength from hour to hour. For example, ships in the Arabian Sea half-way between Aden and Karachi often receive one station so strongly that the other is "jammed," and first it is one station and then the other that is the stronger, the variations in signal strength within an hour being of order a hundredfold. Of recent years it has been found by those using directional methods of reception that not only the intensity but also the apparent direction of the arrival of waves is subject to great variation at night. This alteration in the direction of arrival has been noticed on ranges as short as 20 miles, and has been observed occasionally to attain an error of 90° on ranges of a few hundred miles. In the daytime, on the contrary, the error in estimation of the azimuth of a sending station is less than the experimental error of the directional apparatus; though it must be noticed that it has been stated by American observers that signals from a distant station are stronger on an inclined antenna than on a vertical one, even in the daytime, the direction of inclination being such as to indicate that the waves received have followed an arched trajectory through the air. As a rule, the night variations in the apparent direction of a distant sending station begin about ten or twenty minutes before sunset and cease about the same time after sunrise.

§ (27) PROPAGATION OF WAVES. (i.) *Theoretical Consideration.* — Various theories are available for the explanation of the variations of intensity and of apparent direction of reception that occur from hour to hour during the night and in the passage from night to day conditions. These are more easily approached by glancing briefly at the general problem of the propagation of electric waves round the globe. Hertz's equations, and also Heaviside's pictorial representation of the wave transmission, are sufficient in respect of transmission over short distances, though the finiteness of the conductivity of the earth's surface is neglected. Sommerfeld showed that the resistivity of a flat earth would tend to increase the intensity of the waves at a distance by causing the wave-front to fall forward slightly as it travelled. But these results leave out of account the sphericity of the earth. Poincaré was the first to obtain a solution of the problem of the diffraction of waves over a perfectly conducting sphere, and a number of other mathematicians obtained solutions for a resisting sphere. References to these workers will be found in a paper by G. N. Watson [1] which contains a complete solution of the problem of propagation over the globe even to the antipodes. This paper has been discussed by B. van der Pol,[2] who shows that the measured signal strength in the daytime at ranges of 2000 miles is thousands of times greater than the strength given by Watson's solution or earlier ones. Another difference between measurement (as expressed by Austin's formula) and the diffraction solution is the presence of $\lambda^{\frac{1}{2}}$ in the index in the former and of $\lambda^{\frac{1}{3}}$ in the latter.

(ii.) *Effect of the Atmosphere.*—Evidently the existence of the atmosphere cannot be ignored. The facts of wireless telegraphy as briefly summarised in preceding paragraphs demand that the atmosphere shall possess a certain number of electrical properties. In the first place, as was suggested by Heaviside in 1900, there must be at a high level a permanently conducting "ceiling" which, by its reflecting properties, gives the two-dimensional propagation sometimes attained on quiet nights over the oceans. Such a ceiling could act as an electrical whispering gallery. It must be at a height far above the troposphere, since cloud formations do not affect the phenomena. In the second place, there must be in the daytime levels in the atmosphere that possess conductivity different in magnitude or even different in nature from that possessed at night. The bulk of our experimental knowledge goes to suggest that the conducting layers of the day lie below those of the night, that they come low in the stratosphere, and that they are daily produced afresh by sunlight as the earth rotates. G. N. Watson,[3] in a later paper than that mentioned above, has obtained a solution for the propagation of waves between a sphere and a deep conducting concentric layer commencing at a definite height. This solution contains $\lambda^{\frac{1}{2}}$ in the index and can be made identical with Austin's empirical formula by assigning a particular value to the product of the square of the height and the conductivity. For instance, if the height of the under surface of the layer be taken as 25 kilometres, the necessary conductivity would be less than a quarter of that of sea water in order to account for the average behaviour of the waves in daylight.

(iii.) *Daylight Effects.*—Other daylight phenomena, such as the propagation over mountain ranges, many of the erratic variations that occur at night, as well as many of the twilight effects, may be explained, at least partially, by supposing that the transiently conducting layers in the atmosphere are due to ionisation of the air by the ultra-violet rays in the sun's light. It has been shown that the presence of ions increases the velocity of electric waves through air, thus causing vertical wave-fronts to bend round the convexity of the earth. At the same time, however, the ions cause absorption of energy, and therefore the strength of signals by the bending downward of rays that would otherwise be lost in space is partially cancelled by increased dissipation in the atmosphere. Experience shows that the day signals are better than diffraction could give, as already stated, and not so intense as the best night signals. Further, it is to be expected that as the great circle of twilight rotates about the globe the recombination of the ions in the stratosphere may give to the twilight region a temporary opacity on account of natural irregularity in the process; and it may also be expected that the final result of recombination is, on occasion, patchy or streaky—that is to say, banks or streaks of ionised air may persist through several hours of darkness. The twilight opacity is well known in practice; and the phenomena of the successive fading and strengthening of signals and those of the alteration of apparent direction as described in § (26) (ii.) may reasonably be attributed to ionic refraction in banks of ionised air at heights above 15 kilometres.

(iv.) *Directional Effects.*—The vagaries in directive telegraphy may arise, it should be mentioned, in two ways. Either there may be a genuine alteration of the path of the rays by refraction in a more or less horizontal plane, or there may be a rotation of the plane of polarisation of waves travelling through a mass of air graduated in its ionisation; in the latter case a ray arriving at a direction-finding

[1] *Proc. R.S.*, 1919, xcv. 83.
[2] *Phil. Mag.*, 1919, xxxviii. 365.
[3] *Proc. R.S.*, 1919, xcv. 546.

station possessing a frame antenna rotatable about a vertical axis will give an erroneous direction reading of amount calculable by the equation

$$\tan \gamma = \sin \alpha \tan \beta,$$

where α is the angle the incoming ray makes with the horizontal and β is the rotation of the plane of polarisation. This is because the plane of the frame must be rotated out of the correct position in order to be in the standard relationship with the non-horizontal magnetic field of the rotated field of the waves.

(v.) *Magnetic Variations.*—The presence of these conducting layers in the atmosphere has recently been discussed by S. Chapman from the point of view of the variations of the magnetic elements. According to his analysis the atmosphere above 100 kilometres is permanently ionised by the incidence of electrified particles from the sun, and in these regions the aurorae appear. Below this, but still perhaps more than 50 kilometres from the earth's surface, is a region ionised by the sun's ultra-violet light. The diurnal magnetic variations appear to show that this ionisation also persists throughout the night. If this is true its lower boundary must be the base of the Heaviside layer, which is responsible for the long-distance propagation at night. Support is lent to this conclusion by the negative results of the inquiries promoted by the British Association Radiotelegraphic Committee into the possible connections between wireless telegraphic phenomena, auroral displays, and magnetic storms. On the other hand the ionised layers that endure only during the day, which are emphatically indicated by wireless telegraphy, do not appear to be disclosed by Chapman's analysis of diurnal or other magnetic variations.

§ (28) Natural Electric Waves.—Electric waves produced by natural causes are continually travelling about the earth and being received on the antennae of wireless stations. The electric oscillations they produce in the apparatus are often more intense than those produced by the strongest signals, and these oscillations appear always to have the natural period of the antenna. It seems probable that they are impulsive in character and that they are produced by electric discharges terrestrial or extra-terrestrial. When reception of signals is being effected on a Morse tape the natural electric waves produce erratic markings that sometimes obliterate all messages; when aural reception is employed they make noises that only too frequently make signals unintelligible. They have been given the name Xs, strays, atmospherics, sturbs, static, parasitic signals, etc.

The British Association Committee for Radiotelegraphic Investigation has analysed a great volume of statistics collected from all sources, and has classified strays into three types, namely, clicks, grinders, and hissing. The hissing noises have long been known to be due to local meteorological phenomena, particularly white squalls, or to discharge of electricity from air, rain, snow, or hail passing the antenna. The other types are not of strictly local origin. In tropical and temperate climates they are louder at night than in the day, but in the polar regions they are reported to be louder in the day. They are stronger and most numerous near tropical mountain ranges and especially on the coasts of mountainous countries. In mid-ocean they are rare and feeble both in the day and at night. C. J. de Groot claims to have shown that in the Dutch East Indies the clicks originate in lightning discharges within 100 miles and that the grinders are due to "cosmic bombardment" of the upper atmosphere. The latter type of strays may therefore arise in the zenith of the station which is disturbed; and acting on this conception R. Weagant has described a type of receiving antenna immune to waves descending more or less vertically, though sensitive to signal waves propagated nearly horizontally. Other observers, especially in temperate latitudes, have found by aid of directive antennae that strays appear to come from the direction of the tropics. In Eastern America C. H. Taylor found that most disturbance arrived from a south-easterly direction plus or minus 20°; H. J. Round recently gave the direction of the strays received in England as 165° east of north. Similar conclusions as regards the tropical origin of strays received in England have been previously reached by observations on a solar eclipse and on twilight phenomena; the probable distant origin of strays was shown by H. Morris Airey and the present writer in 1910 by a process of identifying individual strays received near Newcastle and in London.

What may be called X storms often occur in summer in temperate climates and extend during a few days over large areas; since these periods coincide with the occurrence of meteorological conditions such as usually accompany thunderstorms and other unstable atmospheric conditions we may suppose the strays to be due for the most part to lightning discharges.

W. H. E.

WIRELESS TELEGRAPHY, NAVAL SERVICE

§ (1) Special Conditions.—The instruments and apparatus used for Wireless Telegraphy in the Naval Service differ in no main principle from those used for commercial purposes.

Detail differences are in the direction of providing for a wider range of adjustment of wave-length, greater selectivity, greater efficiency, and greater reliability. These requirements arise from the fact that a fleet needs to have open many separate lines of communication simultaneously. This necessitates, in the first place, a wide range of available wave-lengths, and, in the second place, highly selective receiving instruments together with transmitters designed to enable these selective receivers to be used. The high selectivity, in turn, renders essential a high efficiency in the instruments from beginning to end. A combination of ruggedness and reliability is also essential. An army in the field has various means of rapid communication, but a fleet at sea has wireless only, and this must therefore have the highest possible degree of reliability.

On the side of the personnel, somewhat the same requirements have to be fulfilled. The training of the men responsible for the working of the apparatus, and of the officers in charge of both men and apparatus, must necessarily be more thorough than in the case of the relatively simple commercial working. Even an Ordinary Telegraphist must be capable of using the rather complicated instruments in an intelligent manner. A Fleet Wireless Officer must not only know every detail of the various sets under his control, but must, as well, be a good experimentalist; for to him falls the duty of thoroughly testing all new ideas under sea-going conditions, and progress or stagnation depends to no small degree on the skill with which these initial trials are carried out.

§ (2) The Spark Transmitters.—These differ very widely amongst themselves, as is natural in view of the fact that provision has to be made for all kinds of ships from a battleship to a picket-boat.

The most powerful installation, constituting the main spark transmitting set of the larger ships, is a 14 kilowatt set. The largest possible T- or ⋎-shaped aerials are fitted, and the usual mutually coupled primary and secondary oscillatory circuits are employed. The aerial generally constitutes the limitation of the output, and relatively loose couplings are always used.

The main installation of smaller ships consists of a 1½ kw. set having an asynchronous rotary spark gap and primary and secondary oscillators mutually coupled as in the high-power sets. Similar sets adapted to the smaller aerials are used in destroyers and submarines.

In addition to these main installations, all the larger ships have second spark transmitters intended solely for short range working on short waves.

§ (3) The Receiving Circuit.—The receiving circuit originated practically in the present form in 1905–1906, the Marconi magnetic detector then being used. It consists of a three-circuit arrangement, all three circuits being capacity-coupled across a common condenser. To adapt this circuit for the use first of crystal detectors and then of valves, a fourth tuned circuit, relatively tightly coupled to the third of the existing circuits, is used. An unselective "stand by" condition is provided which is practically two circuits with tight magnetic coupling. The whole circuit is built up of separate components, and, owing to the very wide range of wave-length adjustment for which provision has to be made, has the appearance of being very complicated. It is, however, the result of very lengthy experiments, and, in view of the confidence now placed in it, the introduction of any new design will probably be very gradual. The arrangement of the receiving instruments differs from the usual commercial practice in that the whole of them are enclosed in a thoroughly sound-proof receiving cabinet, so greatly increasing the effective sensitivity.

§ (4) The 1914–1919 Advances.—During the early period of the war, demands for increased range from destroyers and submarines, together with the advent of the valve heterodyne sets, led to the rapid development of the Poulsen arc and to the wide application of continuous wave working. Following on this, the war-time necessity of smaller and lower aerials led to the arc being introduced into the larger ships, generally as an alternative to the high-power spark transmitter. Experience having shown that the arcs were unsuitable for use in ships in company, the high-power transmitting valves are now rapidly replacing them. The valves have the added advantage that with an alternating supply they can be used on the interrupted continuous wave system instead of the spark transmitter. It seems very probable that in the near future the valves will be sufficiently advanced to replace both spark and arc.

For receiving, the valve was introduced in 1914 for simple heterodyne purposes, to be followed a little later by combined heterodyning and detecting. Note magnification was little used for ordinary reception, as the advantages to be gained from it are relatively small in a really silent cabinet. In this respect the conditions are very different from those experienced in the Army or Air Force, where extraneous noises are unavoidable. Low-power high-frequency amplification with valve detector was introduced in 1917 and is practically universal. High-power amplification, however, developed slowly owing to the great difficulty of rendering the instruments sufficiently robust for use in the Naval Service.

§ (5) The Naval Shore Stations.—The equipment of these stations has been, on the

whole, similar to that of the ships. In many cases, identically the same transmitting and receiving apparatus has been used. In the high-power stations the general arrangement is similar, but of course on a larger scale.

§ (6) The Personnel.—The officers specialising on the technical side of wireless have been chosen in the past from the Lieutenants R.N. who have qualified for Torpedo Duties, and from the Royal Marines: a special qualifying course of practical and theoretical wireless being prescribed in the latter case. During the time that these officers are specialising in wireless, a part of their time is spent as Fleet Wireless Officers at sea and a part at the Signal School at Portsmouth. Whilst at sea they are responsible for the technical side of the wireless installations of the ships of the fleet to which they are attached. At Portsmouth their duties are experimental or instructional or, in some cases, largely administrative. A few Lieutenants R.N. qualified for Signal Duties have also specialised in the technical side. Normally the signal officer's responsibility is for the use of wireless for actual communication purposes only, but in these special cases technical responsibility has been added.

The Telegraphist Ratings are selected from boys recruited at the age of about sixteen, and have a preliminary training of about nine months' duration at the Boys' Training Establishment at Shotley. After this period they are well drilled in Morse signalling and learn the principles of the apparatus they will meet with at sea. On completion of this course they are drafted to sea for service in the larger ships under the supervision of Warrant Telegraphists or of the more experienced Petty Officer Telegraphists. In due course promotion to Ordinary Telegraphist, Petty Officer Telegraphist, and in the case of the best men, to Warrant Telegraphist follows, each advancement requiring the satisfactory passing of a combined practical and theoretical examination. For a few of the very best men, further promotion to Commissioned Telegraphist is possible, carrying with it the equivalent rank of Lieutenant R.N.

C. L. F.

WIRELESS TELEGRAPHY TRANSMITTING AND RECEIVING APPARATUS

I. Transmission

§ (1) General Considerations.—It is essential that a system designed for the transmission of wireless waves [1] should not only oscillate at a high frequency but that it should also radiate the maximum possible amount of energy. The simple Hertzian oscillator fulfils these properties, but it is useful only for short distances and for short wave-lengths. It has a high frequency and its capacity is small. In 1896 Marconi first introduced the idea of using the earth as one of the arms of the oscillator, thereby obtaining longer ranges, and in the following year the use of coupled circuits was discovered by Lodge.[2]

For efficient transmission the aerial should have the following properties:

(a) The capacity to be as large as possible; (b) the height to be as great as possible, taking into account the wave-length to be radiated; (c) the damping to be low; (d) the natural wave-length to be about 0·67 of the wave-length to be radiated.

In addition directive properties may be introduced by making the horizontal portion of the aerial long compared with its height. The aerial may take many forms, the most usual of which are the "L," "T," umbrella and fan, the names being a sufficient description of the various types. The capacity of aerials can be calculated approximately by a method given by Howe.[3]

For maximum efficiency the resistance losses must be kept as low as possible; the most important of these are the losses in the earth system. This usually consists of a number of metal plates buried in the earth either vertically or horizontally, good connections being made to a common earth terminal. The plates should be buried to a depth that will ensure a good conducting layer. The resistance is a function of the wave-length, the relation being complicated.

In some cases it is advantageous to use a "capacity earth." This system, which was first employed by Lodge [4] in 1897, consists of a network of wires arranged parallel and close to the ground but insulated from it, so that it forms in reality one of the plates of a condenser. The network must not be raised too high above the ground, as the effective height of the aerial and consequently the radiation will be reduced. If it is installed well this system gives better results than the ordinary earth, as the distribution of current in the earth is more uniform, but it has the disadvantage that insulation difficulties are serious and the losses produced may be severe.

The earliest form of transmitter for distance was a spark transmitter with the spark gap in the aerial system. An induction coil was used to charge the aerial. This system had many drawbacks, *e.g.* (a) the type of note from an induction coil was bad, (b) the damping of the aerial system was large, (c) the power which could be employed was small.

[1] See "Wireless Telegraphy," §§ (11), (12), (13).

[2] British Patent Specification, No. 11575 of 1897.
[3] *Electrician*, lxxiii. 859.
[4] See "Wireless Telegraphy, § (11).

By the use of coupled circuits [1] with the spark gap in the primary, the power can be increased, the damping of the aerial circuit can be kept much lower, and by tuning the primary and secondary circuits, more efficient radiation can be obtained. The amount of energy radiated depends on the sharpness of the tuning and on the degree of coupling between the two circuits. Too close a coupling introduces two distinct frequencies,[2] but too loose a coupling reduces the transfer of energy to the aerial system. Auto coupling systems in which the aerial and earth are directly connected to the circuit containing the spark gap have also been used.

In order to prevent the formation of an arc across the spark gap after it has been made conducting by the spark and the high-frequency oscillation, different means have been adopted; a strong air blast may be used or the spark gap may be made rotary, the separation of the electrodes and the cooling produced preventing any arc from forming. In the quenched gap system use is made of very close coupling to get as much energy as possible into the aerial system, and the damping of the primary circuit is made high by quenching the spark very suddenly, so that a single frequency only is obtained. This is effected by making the spark gap of a number of metal plates separated by thin mica washers and kept cool by air or water circulation.

Spark transmitters have been designed in which both continuous and alternating current have been used for charging the primary condenser. As it is not satisfactory to generate continuous current at high voltages it is usual to connect several machines in series in order to get the necessary voltage. By the use of an alternator, however, the current may be generated at low voltage and transformed to the required value without difficulty or fear of breakdown.

§ (2) Spark Transmission.—An installation in which this system has been employed is the 5 kw. transmitter made by Marconi's Wireless Telegraph Company, Ltd. It is designed to have a working range under normal conditions of 400 nautical miles over water, and it can be used on wave-lengths from 300 to 1200 metres. The diagram of connections is shown in *Fig.* 1.

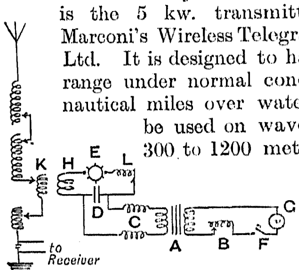


Fig. 1.

(i.) *Power Unit.*—The power unit of the transmitter is a motor-alternator set consisting of a direct-current motor directly connected to a single-phase alternator G and through an insulated coupling to a disc discharger E. If public service mains are not available the set may be driven by belting from an engine, the excitation current for the alternator field being supplied by the motor, or the motor may take its current either from a battery of accumulators or from a generator driven by an oil or petrol engine. At a speed of 2100 revolutions per minute the alternator is capable of delivering 5 kw. at 300 volts and 70 cycles.

(ii.) *Transformer.*—The transformer A is of the single-phase iron core type and is oil-cooled. It is capable of delivering 5 k.v.a. at either 10,000 or 20,000 volts when supplied with alternating current at 300 volts and 70 cycles.

The secondary winding is divided into two equal parts, so that these can be put either in parallel or in series for transmitting on wave-lengths ranging from 300 to 1200 metres. An air-cooled iron-core low-frequency inductance B is connected in series with the primary winding of the transformer, so that this circuit may be brought into resonance with the main oscillating circuit. This inductance is fitted with a controller switch, so that the number of turns of the winding may be adjusted. To protect the windings of the transformer from high-frequency currents, air-core choke coils C are inserted in the secondary leads.

(iii.) *Condenser Battery.* — The condenser battery D consists of eight units, the units being contained in galvanised iron tanks filled with insulating oil. Each unit is built up of zinc sheets and glass plates, supported in a cradle, so that if a plate should break down it may be replaced without delay. Ebonite-bushed brass terminals are fitted to each tank and protective spark gaps are added.

The units are mounted on wooden stands insulated from the floor by porcelain insulators. The transmitting primary and secondary coupling coils H and K and adjustable inductances are also fitted to these stands with the controller by means of which the capacity of the battery of condensers can be varied, so that the primary high-frequency circuit is self-contained. The controller consists of a Swiss commutator, with massive plates and plugs capable of carrying without heating the currents oscillating in the circuit. The adjustable inductance L is built in the form of a heavy copper spiral with square cross-section. A flexible copper contact can be attached to the latter throughout its length by means of a screw of similar pitch to that of the spiral. By this means the wave-lengths between the steps of the condenser combinations can be obtained.

[1] See "Wireless Telegraphy," § (14).
[2] *Ibid.* § (14) (ii.).

(iv.) *The Disc Discharger.*—The disc discharger E is designed to produce a musical note in transmission. It consists of a steel disc about 10 inches in diameter driven off an extension of the motor alternator shaft through an insulated coupling. The disc has sixteen copper studs fixed near the periphery and transverse to the plane of the disc, and these studs rotate between stationary adjustable copper electrodes. The object of the studs is to break up the condenser discharge into a number of definitely spaced discharges, having a particular frequency and so producing a distinctive tone in the sound of the spark when heard in the telephones of a receiving station. In order to get the best results from this discharger, the spark discharge must take place at a definite phase relation to the alternator voltage, and as this phase displacement alters with the wave-length transmitted, the instant at which the discharge takes place relative to the phase position of the alternator is adjustable.

(v.) *Transmitting Primary and Secondary Coupling Coils.*—The primary winding H is composed of independently insulated copper cable arranged so that the current distribution is uniform. The secondary winding K is made of similar cable and consists of a number of turns suitably spaced; tappings are taken at various points to enable the aerial circuit to be tuned to the various waves, the connections being made by plugs and sockets. The secondary slides laterally with respect to the primary, so that the coupling can be varied.

To enable the aerial to be tuned to transmit on longer wave-lengths, special tuning inductances can be inserted in the aerial circuit. These are made of the same cable as the primary and secondary coils, and the requisite amount of inductance can be tapped off. Another separate inductance is provided to obtain a finer variation of inductance.

(vi.) *Method of Signalling.*—The control of the spark discharges is effected by making and breaking the low-tension circuit. As the amount of current in this is considerable, the key F is arranged to operate through a magnetic relay. This relay key is worked by an ordinary hand key placed in the direct-current low-voltage circuit and only allows the alternating current to be broken when it is at or near zero. Speed of operating is thus increased and the sparking at the contacts is reduced.

§ (3) Undamped Waves — The Timed Spark.—A development from the synchronous rotary gap is the "timed spark" system for the production of continuous oscillations.

This consists of several rotary dischargers mounted on the same shaft, so arranged that before the oscillation from one spark dies away, another spark occurs at the right instant, the oscillations produced in the primary circuit being in phase. The current in the aerial produced by a number of these discharges approximates very nearly to a steady oscillation. The means for obtaining accuracy of phase relationship by the use of an "ignition disc" is given in the article on "Wireless Telegraphy."[1] Several high-power transatlantic stations of this type are in existence, *e.g.* at Carnarvon, Stavanger, and Marion.

§ (4) The Poulsen Arc.—In 1900 Duddell discovered that by the addition of a capacity and inductance to the terminals of an ordinary arc, oscillations of musical frequency were set up. In 1903 Poulsen by certain modifications succeeded in producing continuous oscillations of radio frequency. His modifications consisted, firstly, in burning the arc in an atmosphere of hydrogen; secondly, in making the positive electrode of copper and cooling it by means of water circulation; and, thirdly, in placing the arc in a very strong magnetic field. The first two produce a marked increase of slope of the static characteristic of the arc, while the addition of the magnetic field ensures that the arc shall be extinguished at the correct moment. A typical circuit is shown in *Fig.* 2. A current of not less than 10 to 15 amperes should be used to ensure regular action, while the wave-length should be above 1000 metres.

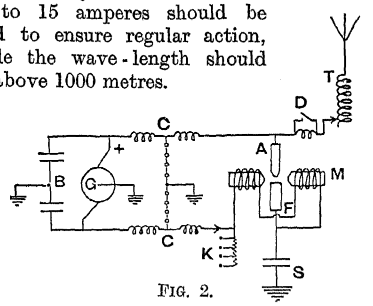


Fig. 2.

The ratio between the direct current and the high-frequency current is between 1·2 to 1·5, and the efficiency usually obtained in practice is between 25 per cent to 35 per cent, although with large stations 38 per cent to 40 per cent may be reached.

The following description may be taken as illustrative of the general design of an arc system.

(i.) *The Arc Chamber.*—The arc consists of a metal chamber of bronze or aluminium into which are fitted the two electrodes and the two poles of a powerful electromagnet. The arc chamber is made with double walls to allow of cooling by the circulation of water, although in some arcs of small power, radiating fins are cast on the arc chamber and a current

[1] See "Wireless Telegraphy," §§ (15), (17).

of air is used for cooling purposes. The electrodes are insulated from the chamber by insulating rings. These are usually of porcelain or quartz, on account of the great heat that they have to withstand.

The positive electrode A, which is round and is usually made of copper, is flattened at the end nearest to the negative electrode F. It is essential that a good conductor be used, and that it should be water-cooled in order that the local heating, produced at the point where the arc strikes, may be limited.

The negative electrode is of carbon and it is raised to incandescence by the arc. In order that the carbon may be used regularly the holder is given a slow rotation by a small motor. To strike the arc a push button on the negative electrode is provided, so that it may be brought into contact with the anode. An adjustment allows the length of the arc to be regulated.

The arc burns usually in an atmosphere of coal-gas; hydrogen is not frequently employed. The rate of flow of the gas is regulated by a tap. If it is not convenient to use coal-gas the vapours produced by alcohol, petroleum, or petrol may be used. In this case a special cup allows the introduction of the liquid into the arc chamber drop by drop, where it at once vaporises. One or more valves are fitted and are kept closed by springs, so that should an explosion of the gas inside occur, the pressure is released. The cover of the arc chamber, which is fitted for cleaning purposes, is also held in place by spring bolts, so it can lift slightly if the pressure becomes excessive. It is usual for the tap controlling the gas or liquid to be operated by an electromagnet which is itself controlled by the master switch. In this way the personal element is eliminated.

(ii.) *Magnetic Field.*—Perpendicular to the electrodes are the two poles of a powerful electromagnet M, which is excited either by means of the current flowing through the arc or direct from the generator in shunt with the arc. The pole pieces are cone-shaped, so that an intense field is produced between them. The axis of the field is slightly above or to one side of the axis of the electrodes, so that the arc, which is curved, is in the strongest field. The windings are arranged so that the arc is displaced upwards or sideways, so reducing the deterioration of the arc from the heat to a minimum. In order to keep the magnetic circuit as short as possible the windings are made very short, so that they generally get hot as the cooling surface is small. In some designs of arcs two windings are employed and these are usually connected in parallel. In another design a horse-shoe-shaped magnetic circuit is used with the pole pieces arranged vertically in the centre. In this case a single exciting coil is fitted and is immersed in oil, a pump maintaining the circulation. One end of the winding is usually connected to the negative pole of the generator and the other to the carbon cathode.

The 500 kw. arcs of the Lafayette radio station near Bordeaux are of this type and the weight of the magnetic system is 60 tons.

For any given set of conditions there is a best value for the magnetic field, and for the best efficiency of the arc. The intensity of this optimum field increases directly with the current and with the resistance of the oscillating circuit, and inversely with the wave-length. It also depends on the density of the gas employed in the arc chamber.

(iii.) *Source of Supply and Protective Devices.*—The current for the arc is supplied by a direct-current generator G, the positive lead of which is connected directly to the copper anode A. The supply voltage varies from 400 volts for a 2 kw. to 1500 volts for a 500 kw. arc.

In order to protect the generator from damage when the arc is first started, *i.e.* by bringing the cathode up to the anode, resistances K are added in series with the arc, and these are cut out step by step until the full voltage is applied to the arc. In the case of small arcs this can be done by hand, but for large power, contactors are used. Chokes, C, are also inserted in both generator leads, partly to maintain the current at a fairly constant value in spite of the fluctuation in the arc, and partly to prevent the high-frequency currents getting back to the generator. A bank of condensers B, with the centre point "earthed," is also connected across the generator as a further protection. In some cases electrolytic condensers are used with the aluminium plates shunted by resistances, these usually being lamps.

In the case of arcs of large power all metal masses of the generator should be "earthed" and a small "earthed" brush should rest on the shaft to prevent any difference of potential. If this is not done sparking may occur between the shaft and the bearing and may give rise to seizing of the shaft. An earth condenser S of large capacity is inserted in the high-frequency circuit between the cathode and earth to prevent the generator being short-circuited should any part of the aerial system become "earthed."

(iv.) *Oscillating Circuit.* — The general arrangement consists in inserting the arc between the aerial and the earth, with an inductance T between the aerial and the arc for regulating the wave-length, and a large condenser S in the earth lead. For the best conditions of working, the wave-length should be two to four times the natural wave-length

of the aerial. The efficiency may be increased by coupling the aerial to the arc or by putting a condenser across it, the best value for the capacity of this being very roughly between that of the aerial and half this value.

The aerial inductance T for large powers consists of coils of either copper tube or copper strip wound on wooden formers supported on porcelain insulators. Wooden bolts or bolts of insulating material must be used instead of metal bolts for holding the framework together to prevent losses. The aerial ammeter is inserted between the arc and earth condenser.

(v.) *Methods of Keying.*—The problem of "keying" with the arc differs considerably from that of damped waves. The production of continuous waves demands definite conditions, and it is necessary that the functioning of the arc should be disturbed as little as possible.

The methods for securing this may be divided into two classes. The first of these modifies slightly the wave-length emitted during the pauses between the signals, the usual change being about 2 per cent. This is effected either by short-circuiting a part of the aerial inductance as shown in *Fig.* 2 at D, or by annulling at the moment of transmission the effect produced by an iron core in a part of the inductance. It is sufficient to saturate the iron by direct current supplied to a winding to obtain this result.

The second class consists in producing at will oscillations either in the aerial or in a local circuit, the transmitting key making the necessary change.

In one method of doing this the local circuit C has values of capacity and inductance as nearly as possible equal to those of the aerial circuit, and the arc is connected to the one or the other; this method is shown in *Fig.* 3.

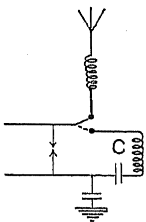


FIG. 3.

For all these methods some means are required for operating the necessary contacts. This may be done by either electromagnetic or pneumatic relays. Under each pair of contacts is fitted a pipe so that compressed air can be supplied to cool the contacts and to extinguish the arc formed between them. It is usual to use several pairs of contacts in parallel so as to reduce the heating.

(vi.) *Accessories.*—*Pump and Motor.*—These are required to keep up the circulation of water in the anode and arc chamber. The water from the pump passes firstly to the anode through a long piece of rubber piping to increase the resistance of the shunt formed by the column of water between anode and cathode; the water then passes to the water jacket of the arc chamber; by this means the coldest water reaches the anode.

Motors for Rotation of Cathode.—Two motors are usually fitted with flexible couplings, so that should one fail, the other can quickly replace it.

Switchboard. — The switchboard carries, besides the usual switches, resistances and instruments for the generator unit, the remote control switch of the contactors for the starting of the arc. It also carries the switches for the pump motor and motor for the rotation of the cathode. A master switch is fitted which, besides breaking or making the main circuit, also closes or opens the water and gas taps. Safety devices are fitted so that should any of the motors, gas or water supply fail, indication is at once given.

§ (5) Radio Frequency Alternators.—From early days attempts have been made to obtain high-frequency currents by the same method as that for producing low frequencies, *i.e.* by alternators.

The chief difficulties met with were, firstly, the high peripheral speed necessary, and, secondly, the losses in the iron cores. The types of machines employed may be divided roughly into two classes, (*a*) Cascade Alternators and (*b*) Homopolar Alternators. The Goldschmidt machine can be classed under the first heading.

The diagram of connections is given in *Fig.* 4.

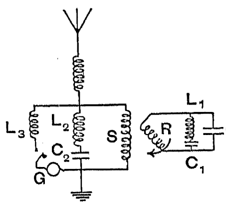


FIG. 4.

It consists of a fixed set of coils S called the stator and a rotating set of coils R called the rotor; connected across the rotor are an inductance L_1 and capacity C_1 of such value that the circuit RL_1C_1 is tuned to a frequency f. A capacity C_3 is also connected to the rotor so that the circuit RC_3 is tuned to a frequency $3f$. An inductance L_2 and capacity C_2 connected to the stator winding are adjusted so that this circuit is tuned to a frequency $2f$. Consider a continuous current from a battery or generator G flowing in the stator, if the rotor is revolving at an angular velocity ω, an alternating current of frequency $f=\omega/2\pi$ will be induced in it. The magnetic field produced by this current is fixed relatively to the rotor but it is pulsating in strength; it can therefore be resolved into two equal and constant fields rotating in opposite directions. With respect to the stator, one of these fields is stationary as it is revolving in the opposite direction to the rotor, and the other is rotating at twice

the angular velocity of the rotor. The latter field will induce in the stator an alternating current of frequency $2f$. Again, the field produced by this current can be resolved into two equal and opposite fields which will induce in the rotor alternating currents of frequency f and $3f$. In the same way the field produced by the current of frequency $3f$ in the rotor can be shown to produce alternating currents of frequency $2f$ and $4f$ in the stator. If an aerial and earth are connected to the stator and tuned to a frequency of $4f$, then currents of this frequency will flow in it and can be used for signalling. It will be seen that the intermediate currents occur in pairs of equal frequency; these are of opposite phase and so tend to nullify each other. Complete neutralisation would occur except for iron, copper, dielectric losses, and magnetic leakage.

Alternators of this type are installed in the Eilvese (Hanover) Station. These deliver about 160 kw. to the aerial with a current of 200 amperes at a wave-length of 7500 metres. The machines carry a winding forming 384 poles which, at a speed of 3130 revolutions per minute, produces in the rotor a current with a frequency of 10,000 cycles per second. The stator is excited by a continuous current generator G (*Fig.* 4), a choking coil L_3 being inserted to protect it from the alternating voltage. Since all the circuits are in resonance, the energy supplied by the exciter is reduced to a minimum and is about 8 kw. The efficiency of the system, on a continuous dash, is about 54 per cent, which rises to about 76 per cent on a signalling load. Signalling is carried out by a key in the exciter circuit; this key also inserts a resistance in the field circuit of the main driving motor so that the drop in speed due to the load is counteracted.

The homopolar machines at present in use are the Latour-Bethenod and the Alexanderson types. The chief losses in high-frequency machines are in air friction and in the iron of the magnetic circuit. To get rid of the heat caused by the friction of the air it is advantageous to adopt cooling by water or oil circulation. In the large power machines made by the Société Alsacienne de Constructions Mécaniques the air friction losses are diminished by running the machine in a reduced air-pressure. Oil circulation is then required since the heat is not withdrawn so quickly.

To diminish the iron losses it is necessary to use very thin laminations, the thickness of these being from 1·5 to 2 mils.

In the Latour-Bethenod alternator the air gap is radial and the field winding consists of a single coil fitted to the stator. The teeth in both stator and rotor are laminated and the armature is wound in open slots.

(i.) *The Alexanderson Machine.*—In the Alexanderson alternator the air gap is axial and the rotor is made of solid chrome-nickel steel, so that high peripheral speeds can be safely employed for large machines (200 kw., 30,000 r.p.m.). The disc has 300 slots set round its edge so as to leave 300 steel teeth. Each of these teeth acts in turn to close the magnetic circuit between two of the poles of the stationary part of the machine. The intervals between the teeth are filled with phosphor bronze; thus the disc presents a smooth surface and losses due to air friction are very much reduced. The alternator is connected to the driving-motor through a double helical gear running in oil with a step-up ratio of 1 to 2·97. By providing the alternators with different numbers of poles and gear ratios, wave-lengths from 6000 to 10,000 metres can be obtained with 25 kw. machines, and 10,000 to 25,000 metres with 200 kw. machines.

In order to equalise the strain, the rotor is made with a thin rim and a thick hub. To avoid large losses through magnetic leakage, the air gap has been reduced to 1 millimetre, and special bearings are fitted to keep the rotor accurately centred. This is attained by the use of thrust bearings which are interconnected by a set of equalising levers. Any tendency towards a change in the air gap is then counteracted by the levers. The main and thrust bearings are lubricated under pressures varying from 5 to 15 pounds per square inch by a pump geared to the main driving shaft. During the periods of stopping and starting and in emergency this circulation is maintained by a special motor-driven pump. An oil-gauge in the main-feed pipe is fitted with a signalling device in case the supply should fail. The main bearings and the armature plate of the alternator are water-cooled by a series of copper pipes.

The frame of the machine (*Fig.* 5) contains two circular field coils A excited by a constant current. The lines of magnetic induction from these pass through the part B and the laminated pole pieces E, being completed across the rotating disc. The reluctance of the circuit will depend on whether a steel tooth or a non-magnetic plug happens to be between the poles; the magnetic flux round the circuit will vary thus periodically going through a complete cycle as each steel tooth passes the pole tips. Wires shown in *Fig.* 5 at F are wound in slots zig-zag fashion round the pole pieces and the varying magnetic field produces an alternating current of the same frequency in these wires which thus form the armature. The exact method of winding

depends on the number of slots. Two methods are illustrated[1] in *Fig.* 6.

The armature winding is divided into sections and is wound in open slots, there being one

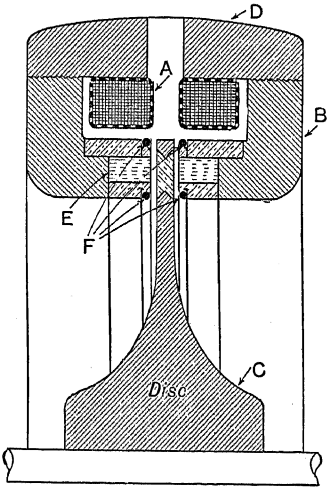


FIG. 5.

conductor in each slot; for a 200 kilowatt machine, there are 64 sections in the armature, 32 on each side of the rotor; each section generates 30 amperes at 100 volts. The sections are connected to independent windings of two air-core transformers, the primary

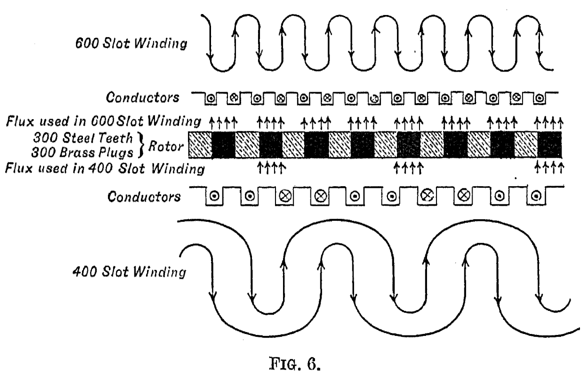


FIG. 6.

windings of which each consist of two turns with 16 separate wires in each turn. The two secondary windings which are connected in parallel consist of 74 turns each and are wound so that their high potential ends are at the centre; in this manner a more uniform potential gradient is obtained. By varying the number of turns of the secondaries of the transformers the alternator can be adapted to the resistance of the aerial system. The voltage at the terminals of the secondary winding of the transformers when the alternator is running at normal speed is about 2000 volts and the normal output current is 100 amperes. The connexions are shown in *Fig.* 7.

[1] Figs. 5 and 6 are taken by permission from a lecture by the late Mr. Duddell, delivered at the Royal Institution on May 17, 1912.

(ii.) *Method of Keying.*—Besides the primary

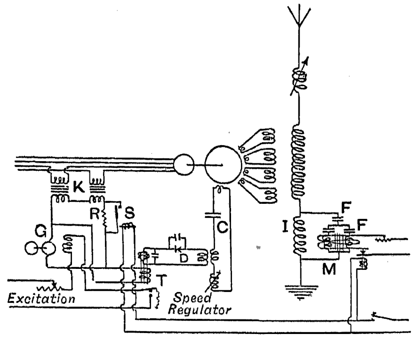


FIG. 7.

and secondary coils, the air-core transformers carry a third winding of 12 turns on each transformer to control the aerial output. These coils are connected in parallel and in series with the aerial circuit; condensers F and an apparatus called a magnetic amplifier are connected in shunt with them. The magnetic amplifier M consists of an iron core with two separate windings, one winding being connected to the intermediate coils of the air-core transformers through condensers, and the other winding to the transmitting key and to a direct-current supply. The impedance of the amplifier depends upon the saturation of the iron core by the direct current, so that when the transmitting key is pressed the impedance is a minimum and the full voltage is applied to the aerial system. When there is no current in the control coil, the impedance is a maximum, the voltage of the alternator is reduced, and the aerial system is detuned.

(iii.) *Speed Regulation.*—It is very important that the speed of the alternator be kept constant during signalling, as not only is the wave-length altered by a variation in speed but the aerial current is also reduced owing to the loss of tuning.

In the Alexanderson system one of the sixty-four sets of alternator windings is connected to a circuit C tuned to a frequency slightly greater than the normal; coupled to this is another circuit D containing a rectifier so that the

radio frequency current is changed into direct current; this rectified current operates the controlling magnet of a regulator T, and the vibrating regulator can be made to control the direct-current driving motor in the usual way.

When an alternating-current supply is used, iron chokes K are inserted between the motor and the power supply.

The vibrating regulator which is controlled by the rectified current is arranged to increase or decrease the voltage of a subsidiary direct-current generator G, which in its turn decreases or increases the impedance of the chokes, so varying the supply voltage to the motor. Additional compensation for the signalling load is provided by a relay S which is connected in series with the transmitting key. When the transmitting key is closed, a resistance R, which is connected in the direct-current winding of the chokes, is short-circuited, so decreasing the impedance and increasing the input to the motor by an amount equal to that imposed by the load. Speed regulation can be obtained at different frequencies by varying the tuning of the closed circuit.

§ (6) Frequency Multipliers.—An alternative method for the production of high-frequency current consists in the use of an alternator combined with a frequency multiplier. By this means the alternator speed can be reduced with a consequent increase in reliability and efficiency. At the Nauen Wireless Station a medium-frequency alternator is installed with an eight-fold frequency multiplier for supplying 400 kw. to the aerial. The alternator is of the homopolar inductor type, giving a voltage of 450 at 1500 r.p.m. and at a frequency of 6000 cycles per second. The high-frequency energy is passed through a step-up transformer before reaching the frequency transformers, which transform the energy to 12,000, 24,000, and 48,000 cycles per second. The connections are shown in *Fig.* 8. The 12,000, 24,000 cycle transformer consists of 36 kg. of stamped iron rings, 0·07 mm. thick, arranged in groups between which oil is circulated, the oil being kept cool by passing through water-cooled pipes.

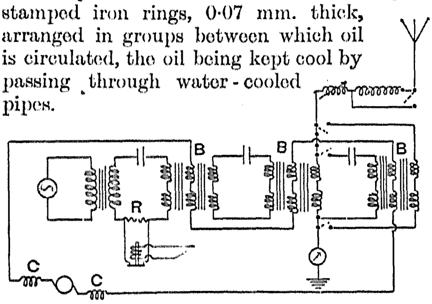


Fig. 8.

The weight of the copper is about 20 kg. A third winding B is provided on the transformers for the saturation of the iron core by direct current, which is supplied by a low-voltage dynamo. Choke coils C are provided to protect the dynamo from the high-frequency current. The efficiency of the frequency transformer at full load is about 90 per cent. All circuits with the exception of the aerial circuit are tuned by capacity.

A resistance R is connected in the first resonance circuit so that signalling may be effected. As long as the resistance is in circuit, frequency multiplication does not occur and there is no current in the aerial. This resistance is short-circuited by the sending key through a relay. A special governor is fitted to maintain the speed constant, and this responds to a change of speed of 0·01 per cent.

§ (7) The Thermionic Valve.—The thermionic valve as a generator of undamped waves is dealt with in a separate article.[1]

II. Receiving Apparatus

The object of all receiving apparatus is the detection of the oscillating currents set up in a receiving aerial by the electromagnetic field produced by a transmitting station. The problem of the design may be divided roughly into three parts: (1) The means for detecting and amplifying the currents produced in the aerial by the signal. (2) The methods for obtaining selectivity, *i.e.* the cutting out of signals which it is not desired to receive. (3) The elimination of atmospheric disturbances.

§ (8) Detectors and Amplifiers.—The early forms of detectors both for damped and for undamped waves have been treated in the article on "Wireless Telegraphy," § (16) to § (19). The use of thermionic valves as detectors and as amplifiers has been discussed in the article dealing with valves.[2]

§ (9) Selectivity of Receiver.—(i.) The simplest form of receiving circuit is that shown in *Fig.* 9. With this type of circuit, however, the effect produced by waves which differ in period from those which it is desired to receive is very serious. This effect can be reduced by the use of a coupled circuit, as shown in *Fig.* 10. In this a tuned secondary

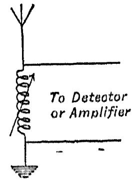


Fig. 9.

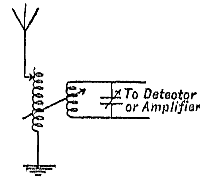


Fig. 10.

circuit is provided, with variable coupling between primary and secondary. The method

[1] See "Thermionic Valves," §§ (5), (6).
[2] *Ibid.* § (9).

of obtaining selectivity was carried still further by Marconi when he designed the multiple tuner (*Fig.* 11). This consists of three inductively coupled circuits A, B, and C, each

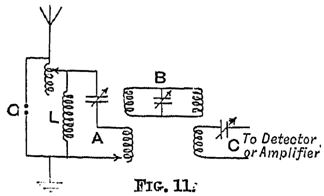


FIG. 11.

of which is adjustable, the first of these being connected to the aerial and the third to the detector or amplifier. By means of adjustments the circuits can be tuned to the desired wave-length, signals of other wave-lengths being eliminated or reduced in strength so that they do not interfere.

To protect the apparatus from electric discharges a spark gap (G) and a shunt of high inductance (L) are provided across the first circuit. The spark gap allows any sudden accumulation of charge such as may be produced by lightning to pass to earth, whilst the high inductance shunt allows charges which are not so violent to leak to earth.

(ii.) *The Rejector.*[1] — Another method for obtaining selectivity consists in inserting between the foot of a tuned aerial and earth two resonating circuits; these circuits are in shunt with each other and each is tuned to the same wave-length as the receiving aerial. One of these circuits, which is called the acceptor (*Fig.* 12), and to which the receiving system is coupled, consists of an inductance and capacity connected in series with each other between the foot of the aerial and earth; the other circuit, which is called the rejector, consists of an inductance and capacity connected in shunt with each other and with the acceptor.

The principle underlying this instrument is that the impedance of a tuned circuit, consisting of a condenser and inductance in parallel, depends on the frequency of the current, being almost infinite for a frequency equal to that to which the circuit is tuned, provided the damping of the circuit is small. To keep the damping low, a large condenser and small inductance are used.

As these two circuits when correctly adjusted are tuned to the same wave-length as the receiving aerial, when an alternating E.M.F. of the same frequency as the waves which are to be received is impressed on these circuits, the whole or by far the greater part of the current flows through the acceptor circuit, the rejector circuit acting as an almost infinite impedance. The receiver being coupled to the acceptor circuit, signals are received in it. If an alternating E.M.F. of any other frequency than that to which the aerial is tuned is impressed on the circuits, the whole or most of the energy will be allowed to pass direct to earth by the rejector circuit, and the acceptor circuit and receiver will be unaffected.

It is possible to obtain increased selectivity by using more than one acceptor circuit or rejector circuit. When more than one acceptor is used these are connected in series with each other; when more than one rejector is used these are connected in shunt with each other across the acceptor circuit, between points which are separated by one or more acceptors. Two forms of the circuit are shown in *Figs.* 12 and 13.

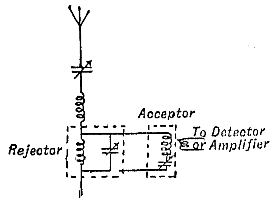


FIG. 12.

(iii.) Another method for obtaining a certain amount of freedom from interfering signals, which has become possible since the construction of multi-valve amplifiers, consists in the use of loop aerials. The strength of signals received on a vertical loop depends on the plane of the loop with regard to the direction of

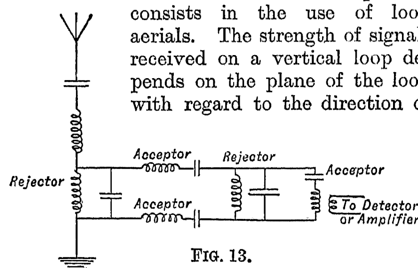


FIG. 13.

the waves, being zero when the direction of propagation is perpendicular to the plane of the loop. Hence, if it is desired to have freedom from interference from any particular station, the loop can be rotated until there is zero signal strength from that station. This will automatically cut down the signal strength from the station whose signals it is desired to receive unless the angle between the two directions is a right angle. This method is naturally not effective in cases where the interfering station and the desired station are on the same great circle passing through the receiving station.

Even in this case it is sometimes possible to eliminate the interfering signal if the receiving station is between the two transmitting stations. This is possible by the use of sense directional systems, *i.e.* systems which

[1] British Patent Specification, No. 17873/05.

give only one zero in the complete 360°. Such systems are described more fully in the sections on directional wireless.

Such methods are particularly effective in allowing duplex working. A loop can be arranged comparatively near to the transmitting station, the loop being directed so as to pick up no signals from the local transmitter. In such a case, as it is necessary to use high amplification, a zero will not be obtained from the local transmitter unless special precautions are taken to prevent the direct influence of the transmitter on the amplifier and leads, and further to eliminate the antenna effect of the loop. (See sections on "Directional Wireless.") Excellent results have been obtained by enclosing the whole of the receiving gear except the loop inside an earthed metal screen.

(iv.) All the circuits described are suitable both for damped and undamped waves, but in the case of the latter special means have to be provided to render the signal audible. This may be done by using a vibrating contact called a ticker, by the tone wheel, or by heterodyne methods. These have been described in a previous article.[1] For the reception by heterodyne methods either an independent oscillator or, if a thermionic valve receiver is used, the reaction principle may be adopted. In the former method the oscillator may consist of a small alternator or of an oscillating thermionic valve coupled loosely to the receiving circuit to obtain the beats. In the reaction method some form of coupling, either magnetic or electrostatic or both, must be used between the grid and anode of the thermionic valve so that local oscillations are set up. This principle is described in the article on "Thermionic Valves."[2] *Fig.* 14 shows a simple form of reactive coupling.

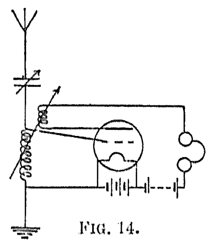


FIG. 14.

§ (10) ELIMINATION OF ATMOSPHERIC DISTURBANCES.[3]—(i.) Owing to the sensitiveness of thermionic valve amplifiers, the range of a transmitting station is determined not by the strength of signals, but by the interference at the receiving station. Various methods have been described by which interference from neighbouring stations can be reduced, but the elimination of those caused by atmospherics is much more difficult. Atmospherics may be classified into two groups—those due to local electrostatic or electromagnetic effects, and those produced at a distance and propagated in the form of waves. In the first of these groups may be considered the disturbances known as "clicks" and "hissing." Clicks are produced by hail-storms occurring at moderate distances from the receiving aerial, and the clicks follow one another in groups. Hissing, which is often accompanied by a reduction in the strength of the received signal due to the overloading of the detector, is probably caused by the variation of the static charge on the aerial by the movement of low clouds; the currents produced in the aerial are usually impulses in one direction. The second group consists of the disturbances known as "grinders." These are the worst kind and produce a continuous rumbling noise in the telephones. Their intensity is greater in the tropics than in the temperate regions, and they have daily and annual variations. Usually the intensity increases with the wave-length to which the aerial is tuned.

In Europe atmospherics are more abundant in summer than in winter. In the Congo they are stronger during the rainy season than in the dry season and the daily variations follow a regular law, whatever the season, the minimum disturbances being just after sunrise and the maximum about midnight. In the tropics usually a day of storm is preceded by some days of bad atmospherics and is followed by some days of a reduced number of atmospherics. It is thought that the majority of the disturbances are due to solar activity, by the emission of ultra-violet rays producing ionised layers in the upper atmosphere, by the convection currents in the air producing variation in the distribution of charge, by the direct emission of charged particles, etc.

Numerous methods have been tried for overcoming the disturbances due to atmospherics, *e.g.* the power of the transmitter can be increased, but this is a costly and inefficient method. Good results have been obtained by raising the spark frequency of the transmitter, the telephone and ear being more sensitive to a frequency of 500 to 1000 cycles per second than to one of 25 to 50.

A reduction can be made to a certain extent by the ordinary tuning arrangements, especially if the signal to be received is from a continuous wave transmitter, as a much looser coupling can be employed, but if the atmospheric is strong, then the receiving aerial system will be set into oscillation and the signal may be lost. Numerous other methods have been tried for the elimination of the disturbances, consisting of filters, limiting and balancing devices. Two systems

[1] See "Wireless Telegraphy," §§ (22), (23).
[2] See "Thermionic Valves," § (15).
[3] See "Wireless Telegraphy," § (28).

which have given satisfactory working consist of the Rogers underground aerial and the Hoyt-Taylor balance of a coil aerial and an underground wire.

(ii.) *The Rogers Underground Aerial.*—This consists of highly insulated wires buried to a depth below the surface of the ground that ensures a permanent water level, the extreme ends being insulated. The general arrangement consists of two wires run in a plane pointing towards the station from which signals are to be received, the inner ends leading to the primary of the receiving apparatus. A condenser is inserted in series with one of the wires, enabling the system to be tuned. *Fig.* 15 shows a schematic diagram of connections. As the surface of the earth

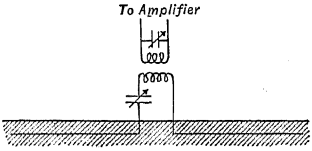


FIG. 15.

is a conductor, atmospheric disturbances do not penetrate through the earth's surface to any great extent, and relief from these is obtained in the receiving apparatus without affecting greatly the desired signals.

It has been stated that with this underground system two-thirds of the atmospheric disturbances can be eliminated when compared with those received on an overhead aerial.

Signal strength as received by the underground system is a function of the wave-length of the received signal, the greater the wave-length the stronger the signal. A signal of approximately 6000 metres wave-length has been received with the same strength on an underground aerial as on a standard 100-foot overhead aerial, while a signal of 600 metres was only one-twelfth of the strength of the signal received on the overhead aerial.

(iii.) *Hoyt-Taylor Balance.*—Another method for the reduction of atmospheric disturbances is the Hoyt-Taylor balance. This system makes use of a loop aerial which is balanced against an underground wire. The loop consists of approximately twenty turns of wire spaced 1 foot apart wound upon a rectangular former 30 feet by 75 feet. One end of the loop is "earthed" and the other is connected through an inductance L, condenser S, and resistance R to the underground wire. The primary circuit of the receiver is connected to the resistance R and to earth, and a variable resistance X is provided between the underground wire and earth to form a path for the atmospheric disturbances. The connections of the system are shown in *Fig.* 16.

The loop and underground wire are affected both by the incoming signal and by the atmo-

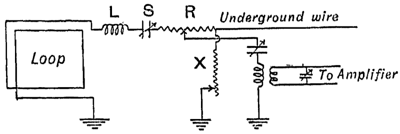


FIG. 16.

spheric disturbances, but the "collecting properties" of the loop for atmospherics are greater than those of the underground wire. By balancing these and adjusting the phase difference, a balance can be obtained so that there is a great reduction in the atmospherics and a signal of fair strength remains.

(iv.) *Effect of Wave-length.*—The disturbing effect due to atmospherics depends on the wave-length of the receiving apparatus. The longer the wave-length the worse is the effect of the atmospherics. Some curves have been recently published by Austin [1] showing the relative order of disturbance for various wave-lengths.

The dependence on wave-length would appear to show that atmospherics have a low frequency. However, it is almost certain that they have no definite frequency, but are in fact practically aperiodic disturbances.

One effect of the influence of the wave-length on the degree of freedom from interference from atmospherics is to modify the hitherto accepted formulae for the optimum wave-length for transmission over fixed distances. This optimum wave-length has hitherto been calculated from the Austin-Cohen formula,[2] which gives the current received in an aerial at various distances from the transmitter. The general result of this was that the optimum wave-length increased as the distance of transmission increased.

However, as atmospherics are worse on longer wave-lengths, this fact must be taken into account, as for efficient communication it is the ratio of signal to atmospheric strength which is important. L. B. Turner [3] has recently taken this into account, and has shown that the optimum wave-lengths as calculated from the Austin-Cohen formula are too high.

Another fact which is of importance in the same connection is that the effect of atmospherics on loops is not so bad as on open aerials,[4] *i.e.* the ratio of signal to atmospheric strength is greater for loops than for

[1] *Proc. Inst. Radio Engineers*, ix. 28.
[2] "Wireless Telegraphy," § (26).
[3] L. B. Turner, *Radio Review*, ii. 524.
[4] Abraham, *Jahrbuch der drahtlosen Telegraphie*, xiv. 259-269.

open aerials. This again makes the optimum wave-lengths different from those given by the Austin-Cohen formula.

It is interesting to note [1] that transmission from Nauen to Togoland was most successful during the war on a wave-length of 4500 metres, which is less than that required by the Austin-Cohen formula, and agrees more nearly with Turner's formula.

(v.) *Limiting Devices.*—Another method for enabling signals to be read through strong interfering signals is by means of limiting devices. The principle underlying such methods is that the sensitiveness of the detector is under control and can be so adjusted that the signal desired gives the maximum effect in the detector. Then any stronger impulse cannot give any stronger effect. One method of achieving this result is to use two crystals in opposition, one of them being less sensitive than the other. If AB and CD (*Fig.* 17) represent the characteristics of the two crystals, then a resultant effect AEF is obtained when they are in opposition. Signal strength can never be greater than that shown by the curve AEF, and by having the crystals more nearly equal the maximum effect can be kept low. In this way the strength of interfering signals with relation to the desired signals can be kept down.

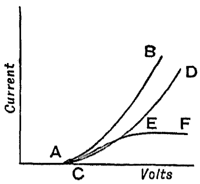


FIG. 17.

A similar effect can be produced by opposing two rectifying valves. Marconi's Wireless Telegraph Co., Ltd., make use of a rectifying valve with a reduced filament current as a limiting device.

As signal strength is usually diminished by such means it is advisable to amplify the remaining signal.

III. DIRECTIONAL WIRELESS

§ (11) (i.) *Loop Receivers.*—In recent years the direction of wireless waves has been measured principally by the use of loops rotating about a vertical axis. The use of a loop for such a purpose can be simply explained by the fact that when the magnetic vector of the waves is parallel to the plane of the loop no E.M.F. is produced, and when perpendicular to the plane of the loop the maximum E.M.F. is obtained.

Rotating a loop about a vertical axis in its plane thus makes the E.M.F. vary from zero to a maximum if the electromagnetic waves are simple. Hence if the plane of the waves be vertical their direction is perpendicular to the plane of the loop when the zero effect is obtained.

The magnitude of the E.M.F. obtained in a loop is smaller than that obtained in a plain aerial.

Formulae [2] for the E.M.F. produced in each case are—

$$\text{E.M.F. (loop)} = \frac{1184 h_s h_r l_r N_r I_s}{\lambda^2 d},$$

$$\text{E.M.F. (aerial)} = \frac{188 h_s h_r I_s}{\lambda d},$$

where λ = wave-length,
I_s = current in transmitting aerial,
d = distance from transmitting aerial,
h_s = height of transmitting aerial,
h_r = height of receiving aerial or loop,
l_r = horizontal length of loop,
N_r = number of turns.

From this it is seen that the E.M.F. in a loop is proportional to the number of turns, and to the product of the linear dimensions of the loop—in other words, to the area turns—or to the summation of the areas of all the turns.

It is advisable to use linear dimensions as large as possible. This, however, limits the number of turns, as for any particular wave-length the total inductance is limited, since there must be some capacity in the circuit and the wave-length depends on the product of the capacity and the inductance.

Comparing the E.M.F. of a 3-metre loop of 5 turns, the wave-length being 1000 metres, with that of a plain aerial 20 metres high, we find the ratio to be 1 : 72. It is thus necessary to use sensitive receivers or amplifiers for loop reception.

A simple method of using a loop is to join a condenser C in parallel with it, and an amplifier across the terminals of the condenser, as shown in *Fig.* 18.

For reasons given below it is sometimes preferable to use coupled circuits, the primary of the coupling in the loop circuit being divided, having the tuning condenser between the parts (*Fig.* 19).

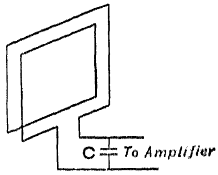


FIG. 18.

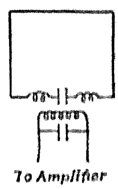


FIG. 19.

Methods using a single rotating loop have had wide application for the determination

[1] Roscher, *Radio Review*, ii. 68.

[2] Dellinger, *Scientific Papers of the Bureau of Standards*, No. 354.

of direction of wireless waves in the British, French, and U.S. Forces during the war.

(ii.) *Bellini-Tosi.*—Before the advent of high amplification, it was impracticable to use single loops of manageable dimensions. Because of the difficulties of rotating a single large loop, an ingenious method of using two large fixed loops at right angles was devised by Artom and made practicable by Bellini and Tosi, later improvements being introduced by Prince and Round.

Two large loops A and B of the same dimensions are erected at right angles to each other (*Fig.* 20).

A special kind of double coupling known as a Radiogoniometer is made use of. This has

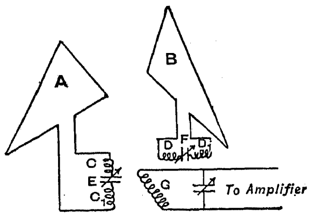


FIG. 20.

two fixed identical coils CC_1, DD_1, at right angles to each other, with a moving coil G arranged to rotate about an axis which is common to all three coils. The fixed coils are joined respectively in series with the aerials. Tuning condensers are provided for the aerials, being connected midway along the fixed inductances.

The couplings from the fixed coils to the moving coil are small. When waves strike both aerials, the moving coil G receives energy from both aerials, and it is possible to find one position where a zero effect is obtained, when the induced E.M.F. in this coil from the two aerials is equal and opposite.

It can easily be shown that the position of the coil G for zero effect bears a definite relation to the orientation of the waves, and it is possible to mark the scale of the moving coil, so that the direction of the waves is immediately read off.

For correct adjustments, an even scale of degrees is used on the radiogoniometer, and the angle of rotation of the moving coil is proportional to the angle of rotation of the waves. As the effects in the moving coil are made up of effects from two aerials, it is essential that the two aerial systems shall be identical in dimensions, inductance, wavelength, and that they shall have no direct mutual effect on one another.

For the reception of continuous waves it is preferable to abolish the aerial tuning condensers and to join the centre of the fixed inductances together and to earth.

Duddell, Glazebrook, and Smith[1] modified the Bellini-Tosi method, so as to use loops of many turns instead of single turns.

The method of observation in both of the methods so far described is preferably the minimum method, for the rate of variation of energy is greatest at the minima. It is customary in such observations to swing through the minimum and to observe the positions on both sides when the signals just become audible, bisecting the angle between these positions.

(iii.) *Robinson Method.*—Instead of using a minimum method, it is often preferable to use a comparison of signals method. A method of doing this which has had considerable application is the Robinson method.[2] Two coils A and B fixed at right angles to each other rotate together (*Fig.* 21). These coils are joined in series with one another and with a tuning condenser E, across the terminals of which is joined the amplifier. A reversing switch D is used to reverse the connections of one coil. The coils are rotated together, the reversing switch being manipulated until the signals are of equal strength. Then it is known that one of the coils is in its maximum and the other in its minimum.

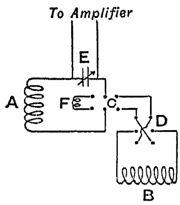


FIG. 21.

Arrangements can be made by means of a change-over switch C to place first of all one of the coils near its maximum, using a balancing inductance F of the same inductance as the coil B. The sensitiveness of this system to direction depends on the ratio of the area turns of the two aerial coils. A good practical ratio is $2\frac{1}{2} : 1$. With such a ratio, an accuracy of direction of 1° or under can easily be obtained. With a ratio of $10 : 1$ an accuracy of $\frac{1}{4}°$ can be obtained.

This method can be adapted to any minimum method of direction finding. In order to apply it to the Bellini-Tosi method it is necessary to construct a radiogoniometer with two movable coils fixed rigidly at right angles to each other, and to use a reversing switch as described above.

(iv.) *Hinton's Method.*—Methods have been devised where fixed aerials are used and where bearings can be obtained to a reasonable accuracy without the use of a radiogoniometer.

Hinton's method[3] is of this nature. This system involves the use of three identical loops at 120° to one another. By comparing the strengths of signals in these loops and in various

[1] British Patent Specification No. 7750/15.
[2] Robinson, *Radio Review*, i. 213-219, and i. 265-275.
[3] British Patent Specification No. 134644.

combinations of them, it is possible to estimate direction to within 15°. This involves the use of a special switching device. Further, by using a special type of radiogoniometer, direction can be measured to as high a degree of accuracy as by other methods.

(v.) *The Telefunken Method.*—This is of a similar nature. Thirty-two open directional aerials are used radiating symmetrically from a centre. The aerials are long compared with their height, and each pair of aerials forms an inverted W. A special switch is arranged so that each pair of aerials can be joined in circuit with the receiving instruments in turn. The angle between the aerials is $11\frac{1}{4}°$, and when two aerials are determined on which the signals are weakest, direction can be determined to about 4° to 5°.

(vi.) *Unidirectional.* — All direction-finding methods whose principle depends on the use of a single rotating loop determine direction only to 180°, but do not determine actual sense, *i.e.* they tell whether the bearing is $\theta°$ or $\theta° + 180°$, but they do not distinguish between these.

In order to determine actual direction it is essential to pay some attention to the phase of the E.M.F. produced in the loop. This is possible by introducing an effect where the phase variations of the E.M.F. are independent of the direction of the wave, *e.g.* a plain aerial. A vertical loop being arranged with its plane in the direction of propagation of the waves will give E.M.F.'s with distinctive phases for the two positions possible to it, the two phases differing by 180°. Combining these effects with those of a plain aerial, the phases will assist in one case and oppose in the other. By a correct adjustment it is possible to obtain the conditions that in the case of opposition a zero is obtained, this being the only zero in 360°. This effect was first pointed out by Bellini and Tosi [1] in connection with their system. It has been used by the U.S. Forces in a system designed by Kolster.[2] In this case a plain aerial is combined with a single rotating loop (*Fig.* 22).

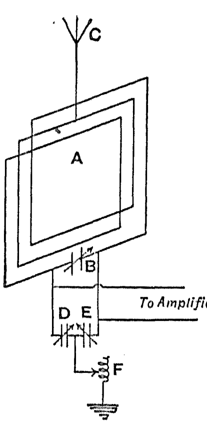


FIG. 22.

The loop is joined in series with the open aerial. The loop A has a tuning condenser B in series with it. The open aerial C is joined to the centre of the upper portion of the loop. Across the tuning condenser B are placed two condensers D and E in series coupled mechanically, the middle point of which is joined through a tuning inductance F to earth. The amplifier is placed across the terminals of condenser B.

(vii.) *Watson Watts' Method.*—An ingenious application of this principle was made by Watson Watts [3] using the Bellini-Tosi aerials. He uses the Bellini-Tosi aerials joined together as the plain aerial, still retaining the directional effects of the separate aerials. His method of connection is for the case of separately tuned aerials. Marconi's Wireless Telegraph Co., Ltd., has recently used this principle in the case of untuned aerials, using an ingenious phasing device in the form of a resistance in the plain aerial circuit.

§ (12) ERRORS. (i.) *The Antenna Effect.* (*a*) *Description.*—There are various sources of error in the use of directional sets. The most important is the antenna effect of loops. This arises from the fact that the loop itself acts as an antenna. It is essential to arrange for certain receiving apparatus which has capacity to earth. From the top of the loop to earth there are two paths, one down each limb of the loop. Unless the receiving apparatus is installed with care, the conductivity to earth down each path will differ and there is left a resultant effect due to the antenna. The ideal method to avoid this is to arrange the receiving gear symmetrically at the centre of the bottom limb of the loop. With valve amplifiers this is not easy, as it is customary to join the filament side of the first valve to one side of the tuning condenser, and the grid of the first valve to the other side. In consequence it is essential to arrange for compensating devices.

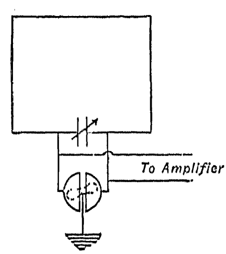


FIG. 23.

(*b*) A simple method of doing this is to arrange a three-plated condenser (*Fig.* 23) in parallel with the tuning condenser, to earth the middle plate, and to adjust it to compensate for the antenna effect. This is achieved when the two zeros are exactly 180° apart and when the zeros are good.

This effect is so important that it is often necessary to compensate for special parts of the receiving apparatus separately. De Belleseize [4] shows how this should be done,

[1] British Patent Specification No. 4801/09.
[2] British Patent Specification No. 138318.
[3] British Patent Specification No. 129336.
[4] British Patent Specification Nos. 132434 and 132935.

even to the extent of correcting for the capacity of the operator.

(c) Another method for correcting for the antenna effect was described by Blatteman,[1] who arranges for a grid of horizontal wires above a receiving loop, the centre of the grid being connected to earth.

The method of connection of the Bellini-Tosi tuned aerials to the radiogoniometer also tends to correct for the antenna effect. The couplings of the aerials to the radiogoniometer are symmetrically broken, and the tuning condensers inserted. In the case of the untuned Bellini-Tosi system the centre points of the lower limbs of the aerials are earthed.

Another effect that is of importance is caused by the amplifier and other receiving apparatus picking up the signals directly. This can be obviated by enclosing all receiving gear as well as the operator in a metal shield.

(ii.) *Quadrantal Errors.*—In applying directional apparatus to mobile bodies such as ships or aircraft, errors are introduced owing to the metal parts of the mobile bodies.

These errors are usually quadrantal in nature both in the case of ships and aeroplanes. In the case of ships, if the loops are installed along the centre line of the ship, bearings are correct along the fore and aft and athwartships direction, but in error in other directions. Mesny [2] has worked out the theory for ships on the assumption that the ship is cylindrical with the lower half in the water, and has shown how the electrical and magnetic vectors are deflected. His conclusions agree with experimental results that for wave-lengths greater than three times the length of the ship the quadrantal errors are independent of wave-length.

In the case of the Bellini-Tosi system applied to ships the best results are obtained by having the aerials fore and aft and athwartships. In this case the size of the aerials can be adjusted to eliminate the quadrantal errors, final adjustments being obtained by adjustment of loading inductances in the aerials.

The principal application of directional systems to aircraft has been by the Robinson system using rotating coils. In this case the coils are placed in the fuselage. Quadrantal errors are produced also in this case, but it appears that the nature of these errors is somewhat different from those on ships. The errors in this case are not independent of wave-length. This appears to be due to the fact that the coils must of necessity be placed in one of the bays of the fuselage and the bracing wires really form an open cage of wires around the coils. A series of closed loops is thus formed around the coils, and the effect of these on the coils will depend on the wave-length.

Quadrantal errors add difficulty to the application of directional systems to mobile bodies. It is sometimes preferable to have fixed coils on the mobile body and to rotate the whole body. This method is particularly useful for aircraft, and the Robinson system is very suitable in this case. The coils are fixed on the wings, the main coil fore and aft and the auxiliary coil athwart-ships.

Methods have been proposed for the elimination of quadrantal errors in the case of coils. One suitable method [3] is to use a subsidiary aerial coil, suitably adjusted as to size, and to gear it up and make it rotate in the opposite direction to the actual direction-finding coils. Other methods have been proposed by Chandler.[4]

§ (13) Variation of Bearings.—The determination of direction by the methods described has been fairly satisfactory. Cases occasionally arise, however, when bearings are in error by amounts which sometimes rise to 90°. Some such variations are permanent, and some are variable. Permanent errors in bearing exist where the direction of propagation is along a coast-line. Eckersley [5] has gone into an explanation of this and has shown that it can be accounted for on the supposition that the velocity of waves over the sea is greater than that over the land in the ratio of 1·02 : 1.

Variable errors occur principally at night, and are of greatest magnitude in mountainous country. Some of the most interesting effects occur at sunset and sunrise.

In attempting to account for such variations an examination of the principles underlying present-day directional methods brings out the following facts:

(a) Present-day methods make use almost universally of loop systems with vertical axes. It can only be expected that such systems will give rise to accurate determinations of direction in case the electric vector of the waves is vertical and the magnetic vector horizontal. The fact that bearings are so often accurate shows that generally this is not far from the truth.

(b) In cases where the electric vectors are not vertical and the magnetic vectors not horizontal, errors in observations must arise. Such cases occur in transmission from aircraft, and also in the neighbourhood of mountains.

(c) Again, in cases where waves arrive at the receiving station from the same transmitting station in more than one direction, errors will arise.

Most cases of variation of bearing are

[1] *Journal of Franklin Institute*, clxxxviii. 289-362.
[2] *Radio Review*, i. 532, and i. 591.
[3] Robinson and Smith, British Patent Specification No. 166780.
[4] British Patent Specification No. 141587.
[5] *Radio Review*, i. 421.

generally supposed to be due to this latter cause. Waves are propagated direct along the surface of the earth. In addition, portions of the waves are reflected or refracted at the Heaviside layer in the upper atmosphere. Hence waves are not necessarily simple at the receiving stations. Again, in cases of such deviations of waves at the ionised layer, the deviations may be of practically any form, and it is probable that the electric and magnetic vectors may be in any planes on reaching the receiving station.

This produces a complicated state of affairs which is difficult to analyse. Even in the simple case where the electric vectors from the two directions are parallel, phase difference may be introduced, and then it is impossible to get a zero on rotating the direction-finding coil.

Eckersley[1] suggests that the portion of the transmission which reaches the upper atmosphere is that from the horizontal portions of the transmitting aerial, and that the electric vector is horizontal. Such horizontal electric vectors will soon vanish in transmission along the surface of the earth. He further assumes that the electric vector of the reflected portion of the waves is horizontal on reaching the receiver and in the direction of propagation. This vector will have an effect on loops and so produce errors. On these assumptions he has proposed that a horizontal loop be used and the effects of this combined with those of the vertical loops. The fact that by such a system the variations are considerably reduced seems to point to the possibility of the assumption being true to some extent.

[1] *Radio Review*, ii. 60, and ii. 231.

It is probable, however, that the electric vector of the reflected waves will not always be horizontal.

In using present-day directional systems it is customary to assume that when zeros are bad the bearings will be bad. This idea is actually used as a guide in giving bearings to ships at sea. It should be remembered, however, that the converse is not necessarily true that when zeros are good the bearings are good, as it is seen from the preceding that even when simple waves arrive at the receiver, errors are obtained when the electric vector is not vertical. Further, in the case of the complication of reflected waves, if the direct and reflected waves happen to be in phase, zeros may be good but the observed direction may be in error.

Variations are usually observed only by night. The reason is that by day the whole atmosphere is ionised and there is more or less of a uniform condition of ionisation in the whole atmosphere. By night the effect of the sun's rays on the lower atmosphere has disappeared and ionisation exists only in the upper atmospheres.

J. R.
R. I. W.

Work done in circling a Current i, n Times with Unit Magnetic Pole. The value of this is $4\pi ni$, or in ampere turns

$$\text{Work} = 4\pi/10 \times \text{ampere turns.}$$

See "Electromagnetic Theory," § (6).

X

X-ray Spectra of the Elements. See "X-rays," § (10).

Spectrometer: an instrument for the determination of the angle at which selective reflection of X-radiation from a crystal face takes place. See *ibid.* § (8).

Tube, history and design of. See *ibid.* § (16).

X-RAYS

§ (1) Production of X-rays.—The discovery of the important series of radiations known as X-rays, or Röntgen rays,[1] was made by Röntgen in 1895. In the course of a research into the production of invisible light rays he enclosed a Crookes tube completely in black paper, in order to shut out all visible light from the discharge, and found that when the tube was excited by a current a barium platinocyanide screen, which was lying some metres away from the tube, fluoresced brightly, just as if it were exposed to ordinary light. The black paper covering of the tube made it quite impossible that the effect should be due to ultra-violet light from the discharge, as black paper is opaque to the ultra-violet rays. The fluorescence must therefore be due to some new type of radiation excited in the tube. This radiation, to which Röntgen gave the name X-radiation pending a further inquiry into its nature, was found to be able to penetrate many substances which are opaque to light. Thus paper, wood, and flesh were found to be comparatively transparent to the rays. Other substances, such as the metals and bone, were found to be comparatively opaque, and cast shadows upon the fluorescent screen when interposed between it and the tube. Thus if the hand was held between the screen and the tube the shadows cast by the relatively opaque bones were distinctly visible upon the screen. The importance of this result to surgery was immediately apparent, and the announcement of the discovery was first made to the Physico-

[1] See also "Radiology," Vol. IV.

Medical Society of Würzburg in November 1895.

By placing metallic obstacles between the tube and a fluorescent screen and producing the lines joining the shadows to the corresponding obstacles backward, Röntgen showed that the rays emanated from the portion of the walls of the tube on which the cathode rays impinged. Further research has shown that X-rays are produced whenever cathode rays encounter a material obstacle. X-rays affect a photographic plate in the same manner as ordinary light. Unlike light, however, they are not reflected by a polished surface or refracted by passing through a prism. They are not deflected by an electric or magnetic field, thus proving that they do not carry an electric charge. They share, however, with cathode rays and the radiations from radioactive substances, the property of imparting a temporary conductivity to any gas through which they pass. This property is generally employed in experimental work as a means of measuring the intensity of the rays.

§ (2) STOKES'S THEORY OF X-RAYS.—The fact that X-rays have their origin at the points struck by the beam of cathode rays led Stokes [1] to suggest that they are electromagnetic waves produced by the sudden stoppage of the negatively charged particles which constitute the cathode beam. He suggested that the absence of optical reflection and refraction was due to the excessive thinness of the pulses produced. The recent measurements of the wave-length of the rays have confirmed the conclusions of Stokes.

The effect of the stoppage of a moving electrified particle can be followed most easily by considering the motion of the Faraday tubes of force attached to the particle. If the velocity of the particle is small compared with that of light its electric field will not be affected by the motion, and the tubes of force will radiate uniformly from the particle, moving with it as if rigidly attached to it.

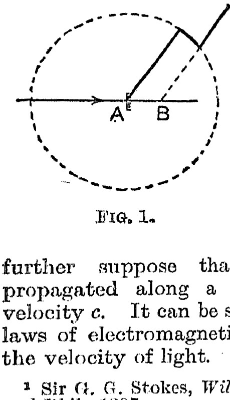


FIG. 1.

Suppose now that such a particle with its attendant tubes of force is suddenly stopped by a solid obstacle at a point A (*Fig.* 1). Let us further suppose that any disturbance is propagated along a Faraday tube with a velocity c. It can be shown from the ordinary laws of electromagnetism that this velocity is the velocity of light. Let t be the time which has elapsed since the particle was stopped, and let us describe round A a sphere of radius ct. The disturbances produced in the Faraday tubes by the stoppage of the particle will obviously lie on the surface of this sphere. Inside the sphere the effects of the disturbances will have passed away and the tubes will be radiating from the stationary position of the particle. Outside the sphere, however, the disturbances travelling outwards along the tubes will not have arrived. The Faraday tubes in this region will not be affected and will thus be travelling forward with the original velocity v of the particle, and will have their apparent origin at the position which the particle would have reached if it had not been stopped; that is to say, they will all diverge from a point B where AB is equal to vt. There will thus be a relative displacement between the two portions of the tubes on opposite sides of the surface of the sphere. As we must regard the tubes as maintaining their continuity, the configuration of the tube must be as indicated by the thick line in *Fig.* 1.

The sphere itself is expanding with a velocity c, and hence the portions of the tube lying along its surface are moving at right angles to their direction with this velocity. It can be shown [2] that the motion of a Faraday tube at right angles to its length produces a magnetic field equal to $4\pi c$, multiplied by the intensity of the tube, where c is the velocity of the tube, the direction of the field being perpendicular both to the direction of the tube and to its velocity; that is to say, the magnetic field will also lie on the surface of the sphere. We thus obtain a sheet of electric and magnetic disturbance spreading outwards from the stopped particle with the velocity of light. On Stokes's theory this constitutes an X-ray.

It will not, of course, be possible to stop the charged particle instantaneously. If τ is the time taken to reduce the particle to rest, the disturbance will clearly be contained in a spherical shell bounded by spheres of radii ct and $c(t+\tau)$ (*Fig.* 1A). The thickness of the pulse will thus be $c\tau$, and will be smaller the more abruptly the particle is stopped. The polarisation in the pulse will be along the thick line in *Fig.* 1A, hence resolving this along and at right angles to

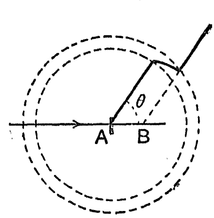


FIG. 1A.

[1] Sir G. G. Stokes, *Wilde Lecture, Manchester, Lit. and Phil.*, 1897.

[2] J. J. Thomson, *Elements of Electricity and Magnetism*, chap. xiii.

the surface of the sphere we have, since $AB = vt$,

$$\frac{\text{Tangential electric polarisation}}{\text{Normal electric polarisation}} = \frac{vt \sin \theta}{\delta},$$

where δ is the thickness of the pulse, and θ the angle between the direction of the tube of force and that of the velocity. But the normal electric polarisation is $e/4\pi r^2$ or $e/4\pi rct$. Hence the value of the tangential electric field is

$$\frac{e}{4\pi r\delta} \frac{v \sin \theta}{c} = \mathbf{E} \text{ say.}$$

The value of the magnetic field, which is also tangential, is found by multiplying this by $4\pi c$. Thus the magnetic field

$$= \frac{e}{r\delta} v \sin \theta = \mathbf{H} \text{ say.}$$

The velocity v cannot be reduced to zero instantaneously; if we suppose the process to last a time τ, during which there is an average retardation f, then $v = f\tau$. Also δ is the thickness of the pulse generated in time τ, thus $\delta = c\tau$. Making these substitutions we have

$$\mathbf{E} = \frac{e}{4\pi rc^2} f \sin \theta, \; \mathbf{H} = \frac{e}{rc} f \sin \theta,$$

where e is the charge on the particle. The tangential force in the wave front thus varies inversely as the distance, while the normal field falls off inversely as the square of the distance. Thus the electric intensity in the pulse will be large compared with that outside it except for points near the particle. The magnetic field in the pulse is, as we have seen, equal to $4\pi c$ times the electric polarisation, that is, $ev \sin \theta / r\delta$. We thus get a pulse of electromagnetic disturbance radiating out from the particle. The energy in the pulse can easily be shown to be equal to $\frac{2}{3}\mu e^2 v^2/\delta$, where μ is the magnetic permeability of the medium. These results were obtained by J. J. Thomson.[1] Larmor[2] had previously shown by an application of Poynting's theorem that any acceleration of a charged particle will result in the emission of radiation, the rate of loss of energy being given by $\frac{2}{3}e^2 f^2/c$, where f is the acceleration.

Since the energy in the pulse is inversely proportional to its thickness, the more quickly the particle is reduced to rest the greater will be the fraction of the energy radiated, in the form of X-rays. In practice only about $\frac{1}{1000}$th of the energy of the cathode beam is transformed into X-rays even under the most favourable circumstances. The remainder is transformed into heat. Since the electric intensity in the pulse varies as $\sin \theta$, where θ is the angle made with the direction in which the particles are moving, the intensity will be zero in this direction and a maximum at right angles to it. In practice, however, the X-rays are found to be emitted fairly uniformly all round the target. This would indicate that the cathode beam is nearly uniformly scattered before the particles make the collisions which result in the emission of an X-ray pulse.

[1] J. J. Thomson, *Phil. Mag.*, 1898 (5), xlv. 172.

[2] J. Larmor, *Phil. Mag.*, 1897 (5), xliv. 503. See also "Poynting's Theorem" and "Radiation from a Moving Charge."

§ (3) The Scattering of X-rays by Matter.—Although X-rays are not optically reflected or refracted, they are scattered to a certain extent in passing through matter. For consider the passage of the pulse through an atom containing electrons. These, during the passage of the pulse, will be subjected to the electric field in the pulse, and will thus be given an acceleration in the direction of the field equal to Xe/m, where X is the field, which has already been calculated, and e and m the charge and mass of the electron. This acceleration will result in the emission of an X-ray pulse by the electron, the thickness of the secondary pulse being equal to that of the primary. Thus part of the energy of the primary beam will be transformed into secondary radiation of the same wave-length as the primary, but radiating out in all directions from the atom. This is known as the scattered radiation to distinguish it from another type of X-radiation which may also be emitted by the radiator, the wave-length of which depends not on that of the primary rays, but on the chemical nature of the radiator. The latter type is called the characteristic radiation, since it is characteristic of the radiator employed. It is sometimes also described as the fluorescent or homogeneous radiation, for reasons which will be apparent later.

The scattering of a beam of X-rays can easily be observed by allowing a narrow beam of rays to pass through a thin plate of aluminium or paper. An electroscope placed near the radiator, but out of the direct line of the pencil, will lose its charge, showing that ionising rays are emitted by the radiator under the action of the primary beam. By using absorbing screens it is easy to show that the quality of the scattered radiation is the same as that of the primary beam from which it is produced.

According to the simple theory of the effect given by J. J. Thomson,[3] and outlined above, the intensity I_θ of the scattered radiation emerging from the radiator at an angle θ with the primary beam should be given by

$$I_\theta = I_{\pi/2}(1 + \cos^2 \theta),$$

[3] Thomson, *Conduction through Gases*, 1906, p. 323.

where $I_{\pi/2}$ is the intensity in a direction at right angles to the primary beam. It should be a minimum in this direction, increasing to twice the minimum value as the direction of the primary beam is approached. This relation is not strictly true. It has been shown [1] that the intensity of the scattered radiation on the side of the radiator from which the primary beam emerges is always greater than that given out at a similar angle from the face by which the primary beam enters. The actual distribution of the scattered radiation depends on the nature of the radiator and the penetrating power of the primary beam. The full curve in *Fig.* 2 represents the distribution of intensity round a thin aluminium radiator R. The dotted curve gives the theoretical distribution according to Thomson's formula. The discrepancy is probably due to the interference produced by neighbouring electrons with each other's motion under the action of the primary pulse, and indicates that the individual electrons are not perfectly independent radiators as assumed by the simple theory.

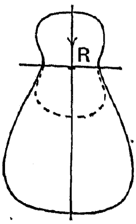


Fig. 2.

§ (4) Energy of the Scattered Radiation.—Assuming, however, that the simple theory of scattering is at any rate approximately correct, a knowledge of the fraction of the energy of the primary beam scattered by unit mass of any substance, or the mass coefficient of scattering, as it is called, enables us to estimate the number of electrons in unit mass of the radiator, and hence the number in the atom. It can be shown [2] that the fraction of the energy of the primary beam scattered by a single electron is $\frac{8}{3}\pi e^4/m^2$. If we can assume that all the electrons radiate independently, the total fraction scattered is thus $\frac{8}{3}\pi Ne^4/m^2$, where N is the total number of electrons affected by the rays.

The mass coefficient of scattering for carbon is about 0·2. Assuming that e is $1{\cdot}57 \times 10^{-20}$ and e/m is $1{\cdot}77 \times 10^7$, the number of electrons in one gramme of carbon is about $3{\cdot}1 \times 10^{23}$. Taking the mass of the hydrogen atom as $1{\cdot}64 \times 10^{-24}$ gm., that of the carbon atom must be $12 \times 1{\cdot}64 \times 10^{-24}$ gm. or 2×10^{-23} gm., and the number of carbon atoms per gm. is therefore 5×10^{22}. This gives a value 6 for the number of electrons in a carbon atom, a value very nearly equal to the atomic number. This result has been confirmed by other lines of approach.

The mass coefficient of scattering is nearly the same for all elements of low atomic weight, thus showing that the number of electrons in the atom is proportional to the atomic weight. The coefficient, however, increases considerably for elements of higher atomic weight, especially if the penetrating power of the primary beam is small.[3] As other lines of reasoning have led us to conclude that the proportionality between the number of electrons in the atom and the atomic number holds throughout the whole range of atomic weights, we must suppose that in the case of the heavier atoms the electrons no longer behave as independent radiators of energy.

§ (5) Interference Phenomena with X-rays.—Attempts to provide some experimental basis for the assumption that X-rays were analogous to light waves of very small wave-length have been numerous. Haga and Wind,[4] on passing a beam of X-rays through a very narrow slit, observed a slight widening of the shadow of the slit in its narrowest part, somewhat analogous to the diffraction effects observed with light. On the assumption that this effect was really due to a diffraction of the rays, they calculated that the wave-length must be of the order of $1{\cdot}3 \times 10^{-8}$ cm. Their results were never accepted as decisive, though it is only fair to say that the value deduced by them has been amply confirmed by recent experiments. Marx [5] endeavoured to show that the velocity of X-rays was equal to that of light, but his experiments were also challenged. Numerous attempts to obtain evidence of regular reflection or refraction of the rays all gave negative results. This, as indicated by Stokes, is due to the extremely short wave-length of the rays.

The shortness of the wave-length also made it obviously impossible to hope to rule a grating which would produce any measurable diffraction in a beam of X-rays. Laue, however, in 1912 conceived the idea that the regular spacing of the atoms in a crystal, as affirmed by modern crystallographers, might provide a natural grating of suitable spacing for the experiment. The regularity of a crystal structure is, of course, a three-dimensional one, and the problem therefore differs considerably from that of the ruled grating in which all the spacings are parallel and in the same plane. Laue was, however, able to show that if a narrow pencil of X-rays was directed symmetrically through a small crystal, diffraction should take place, the diffracted rays emerging from the crystal in various perfectly definite directions with the primary beam. Thus, if a photographic plate were placed at some distance beyond the crystal, it should, on development, show a series of spots arranged

[1] Crowther, *Proc. Roy. Soc.* A, 1912, lxxxv. 41.
[2] J. J. Thomson, *loc. cit.*

[3] Crowther, *Proc. Camb. Phil. Soc.*, 1911, xvi. 356; Barkla and Dunlop, *Phil. Mag.*, 1916 (6), xxxi. 222.
[4] Haga and Wind, *Wied. Ann.*, 1899, lxviii. 884.
[5] Marx, *Ann. der Phys.*, 1906, xx. 677.

according to certain definite laws, surrounding the image of the undeflected beam.

The theory was put to the test by Friedrich and Knipping.[1] The primary beam was produced at the anticathode F of a large focus tube, and was limited to a narrow pencil by passing through a series of circular stops, A, B, C (*Fig.* 3). They then passed sym-

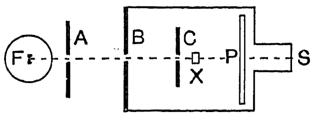


Fig. 3.

metrically through a small crystal placed at X, adjustment being made by a sighting screen S. A photographic plate P was then inserted and an exposure made, lasting for several hours. On developing the plate it was found that the central black patch due to the undeflected beam of rays was surrounded by a symmetrical pattern of small elliptical spots, some of which were deviated through an angle of nearly 40° from the direction of the primary beam. The pattern was found to agree closely with the predictions of Laue. If the distance between the crystal planes had been known, the actual wave-lengths corresponding to the different spots could have been calculated.

§ (6) Theory of the Diffraction of X-rays by Crystals.—In modern crystallography the symmetrical geometric forms exhibited by crystals are supposed to indicate a similar symmetry in the arrangement of the atoms within the crystals. It is supposed that each set of atoms forms a regular system of points in space, the symmetry of the arrangement being the same as that of the crystal. Such a set of points is known as a space lattice. If the crystal contains atoms of more than one element, the space lattices for the different kind of atoms will, of course, be interpenetrating.

In order that a set of points shall form a space lattice they must, of course, fulfil certain geometrical conditions, which can be deduced from the general laws to which all regular patterns in space must conform. It can be shown that the only method of dividing up space which will satisfy the necessary conditions is the following. Three sets of parallel planes are taken intersecting each other at any angle. All the planes in each set must be equally spaced, but the spacing in the different sets need not be the same. The space is thus divided up into a number of parallelepipeds (*Fig.* 4). The corners of these form a space lattice.

These parallelepipeds form the elements from which the crystal is built up, and their symmetry is the same as that of the resulting crystal. Thus in the case of the cubic system

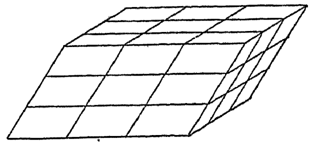


Fig. 4.

all the planes intersect at right angles, and are all equally spaced. The elementary volumes are thus cubes. If the planes intersect at right angles, but the spacings are different for the three sets, the orthorhombic system is developed, and so on.

For simplicity we will consider the case of a crystalline with cubic symmetry. In this case there will only be one space lattice, and the atoms will occupy the corners of a system of cubes, one of which is shown in *Fig.* 5. Suppose, now, that a Röntgen or X-ray pulse traverses the element in the direction OZ. Each of the atoms, under the action of the pulse, will become a centre of secondary disturbances spreading out through space. We may, in fact, regard each atom as forming a kind of isolated Huyghens element.

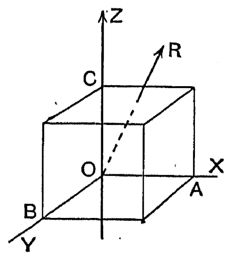


Fig. 5.

Let us consider the atoms at O, A, B, and C. In general the secondary wavelets from the four atoms will interfere, but there will be certain definite directions fixed relative to the sides of the cube in which they will co-operate. The intensity of the scattered radiation will therefore be a maximum along these directions. For the secondary wavelets to co-operate in some direction OR, they must arrive at a plane perpendicular to OR in phase with each other; that is to say, if we draw our plane, for convenience, through the point C, the perpendiculars from the points O, A, and B must each contain a whole number of wave-lengths. Taking O as the origin of co-ordinates, and the three edges of the cube as axes, these distances are respectively $f(1-\gamma)$, $f\alpha$, and $f\beta$, where f is the length of the side of the cube, and α, β, γ are the direction cosines of OR. Thus if λ is the wave-length of the X-radiation, we must have, for a maximum,

$$\left.\begin{aligned} f\alpha &= p\lambda \\ f\beta &= q\lambda \\ f(1-\gamma) &= r\lambda \end{aligned}\right\}$$

[1] Friedrich and Knipping, *Ann. der Phys.*, 1913 (4), xli. 971.

where p, q, r are whole numbers, representing the number of complete wave-lengths in the given distances. Thus

$$\frac{\alpha}{p}=\frac{\beta}{q}=\frac{1-\gamma}{r}=\frac{\lambda}{f},$$

or since p, q, r are integers, α, β, and $(1-\gamma)$ must bear to each other a simple ratio. When this condition is fulfilled the secondary wavelets will reinforce each other and a dark spot will appear on the photographic plate where the line OR meets it. On applying this condition to the photographs of Friedrich and Knipping, it was found that in no case was it necessary to assign values greater than 10 to any of the parameters p, q, r in order to give to the quantities α, β, and $(1-\gamma)$ an integral ratio. The theory thus accounted satisfactorily for all the spots observed on the plates.

The problem is, however, really more complex than has been indicated above. The intensity of the different spots was by no means what was to be expected on the simple theory, and certain others corresponding to quite small values of the parameters were absent altogether although their appearance was to be expected. Since α, β, γ have to fulfil the further relation

$$\alpha^2+\beta^2+\gamma^2=1,$$

it follows that for each spot there is only one value of λ/f which satisfies the conditions, and thus each spot corresponds to X-rays of definite wave-length. Laue proposed to overcome the difficulties by supposing that certain wave-lengths were absent from the primary beam and that thus the corresponding spots could not be formed. This assumption is in itself very improbable. Fortunately a slight modification of the experimental conditions can be made which gives a very much simpler and more certain method of dealing with the phenomena than the original method of Laue.

§ (7) Reflection of X-rays by Crystal Planes.—If we return to our crystal lattice, it will be noticed that along certain sets of parallel planes, the construction planes for example, the atoms are very thickly studded. In fact along any set of planes having some obvious connection with the symmetry of the lattice, say, for example, those passing through opposite edges of the parallelepipeds, the atoms will be comparatively numerous, while along planes having no close connection with the structure they will be very sparse. The former planes bear a close connection with the properties of the crystal. They are all parallel to possible faces on the crystal, and in general represent planes along which the crystal will cleave with comparative ease. It is thus quite easy to identify them when the symmetry of the crystal has been determined.

Suppose that a parallel beam of X-rays falls obliquely on one of these thickly studded planes represented in section by the dots in *Fig.* 6. Each atom in turn will become a centre of secondary radiation as the wave-front passes over it, and the separate secondary pulses will combine to form a new wave-front which, at any considerable distance from the crystal, will be plane and inclined at an angle to the reflecting plane equal to that of the incident radiation. The construction is, in fact, identical with that of Huyghens for the reflection of light, with the exception that the elements are now discrete instead of being continuous. The energy reflected will be proportional to the number of elements, that is of atoms, and will only be appreciable from planes on which the atoms are thickly studded.

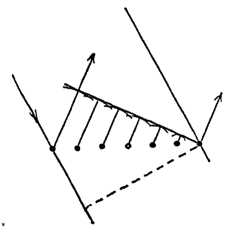


Fig. 6.

The energy scattered by a single plane will, however, in any case be extremely small, and would be quite beyond the powers of experiment to detect. The given plane, however, is in the case of any real crystal only one of a very large number of similar parallel planes in the crystal, each of which will reflect the primary wave in a similar manner. If the secondary rays from the whole set of planes arrive at any point in phase with each other they will reinforce each other at that point, and an appreciable effect will be produced. In other directions they will be out of phase, and the resultant intensity will be negligible.

Let pp, qq, rr, . . . (*Fig.* 7) be a series of such planes as seen in section, and let PP′ be a

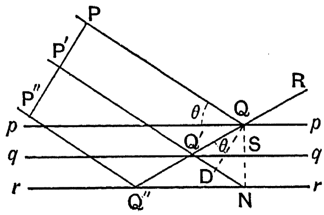


Fig. 7.

primary wave-front advancing in the direction PQ, making some angle θ with the planes. The reflected wave will travel in the direction QR making an angle θ with pp, and in order that the reflected radiation in this direction may be a maximum the secondary waves from the different planes must reach R in the same phase, that is to say, the paths PQR and

$P'Q'R$ must differ by an integral number of wave-lengths. Drawing QSN perpendicular to the planes, and QD perpendicular to the direction of the primary rays, we have QQ' equal to QN, and the path difference between the two rays $= P'Q' + QQ' - PQ$, which is equal to $P'Q' + Q'N - P'D = DN = QN \sin\theta = 2d \sin\theta$, where d is the perpendicular distance between the planes. Hence the maximum reflection will take place at a crystal plane when $2d \sin\theta$ is an integral number of wave-lengths for the radiation employed.

The result is analogous to that for the diffraction grating if the various equally spaced parallel planes are regarded as taking the place of the lines in the grating. Thus, following the ordinary theory of the grating, it can be shown that the maximum is a very sharp one as the number of planes is very large, and that a very slight discrepancy from exact phase agreement will result in the almost complete extinction of the radiation. The reflected image for a given wave-length will therefore be very sharply defined, just as in the case of the grating. It is obvious from the formula that each particular wave-length will be reflected only at certain definite values of θ satisfying the equation $2d \sin\theta = n\lambda$, where n is an integer. Putting n in turn equal to 1, 2, 3, ... we shall obtain reflections corresponding to what may be called the first, second, etc., order spectra. Confining our attention to the first order spectra which give the minimum value for θ (that is, working nearly at grazing incidence), we see that if the incident beam consists of a mixture of two definite wave-lengths λ_1 and λ_2, there will be reflection at two definite angles θ_1 and θ_2 given by $2d \sin\theta_1 = \lambda_1$ and $2d \sin\theta_2 = \lambda_2$. Thus for the same crystal planes and the same order spectrum

$$\frac{\lambda_1}{\lambda_2} = \frac{\sin\theta_1}{\sin\theta_2}.$$

We can thus compare the wave-lengths of different beams of X-rays. If the distance d between successive planes can be calculated the actual wave-lengths of the rays can be determined. Again, by using incident radiation of constant wave-length and measuring the angle of reflection from different sets of planes, we can determine the relative distance apart of these various sets of crystal planes and thus obtain much light on crystal structure. This method of dealing with the problem of crystalline diffraction is due to W. L. Bragg,[1] and experiments on these lines were immediately carried out by W. H. and W. L. Bragg.[2] An interesting summary of the work is given in *X-rays and Crystal Structure* by W. H. and W. L. Bragg, 1915.

§ (8) X-RAY SPECTROMETER.—The determination of the angle θ at which this selective reflection takes place can be made by the use of what may be called an X-ray spectrometer. The principle of the instrument is quite simple, although, as is always the case, if a high degree of precision is desired, numerous modifications and additions are required. The beam of X-rays to be investigated is formed into a narrow pencil by passing it through two or more narrow slits A and B cut in plates of lead or gold (*Fig.* 8).

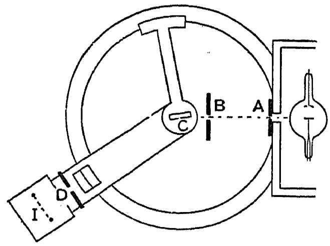


FIG. 8.

This arrangement may be regarded as the collimator of our spectroscope. The emerging pencil then falls on the crystal C mounted on the axis of the rotating table of the spectroscope. Its orientation may be read by means of a circular scale and vernier in the usual way. The reflected beam may be detected either by an ionisation method, or by use of a photographic plate. The former method was the one adopted by W. H. and W. L. Bragg (*loc. cit.*), who were the pioneers of X-ray spectroscopy. The photographic method, however, which was introduced by Moseley,[3] seems capable of greater accuracy and is now generally adopted when results of the highest precision are required. In either case the detecting apparatus is carried by a rotating arm pivoted about the same axis as that of the table carrying the crystal, and its position is read by means of suitable scales and verniers. This part of the apparatus, therefore, may be compared to the telescope of an ordinary spectrometer. In the figure, which represents Bragg's arrangement, I is the ionisation chamber the entrance to which is closed by a lead plate having a narrow slit D, which limits the rays admitted to the chamber. It is clear that with this arrangement there will only be an ionisation current through the chamber I if the rays passing through B fall upon the crystal at the correct glancing angle θ for selective reflection, and if, further, the reflected ray passes through the slit D.

[1] W. L. Bragg, *Proc. Camb. Phil. Soc.*, 1912, xvii. 43.

[2] W. H. Bragg, *Proc. Roy. Soc.* A, 1913, lxxxviii. 428; W. H. and W. L. Bragg, *Proc. Roy. Soc.* A, 1913, lxxxix. 246, 248, 430, 575.

[3] Moseley, *Phil. Mag.*, 1914 (6), xxvii. 703.

The apparatus is first adjusted so that the slit D is immediately opposite the collimating slits, and the crystal is set so that the reflecting planes are parallel to the incident rays. Although it is convenient to speak of the reflection of the rays, it must be remembered that the reflection is a volume effect and not a surface reflection. It is not, therefore, essential that the crystal shall actually exhibit a face parallel to the reflecting planes, though as the depth to which the X-rays penetrate the crystal is generally very small it will usually be convenient to use the planes parallel to one of the actual faces of the crystal. The crystal is then turned through some small angle, the ionisation chamber at the same time being turned through twice this angle so that the slit D will be on the path of the reflected ray. This process is continued until a deflection in the electroscope attached to the ionisation chamber indicates that selective reflection is taking place. The angle θ through which the crystal has been turned, or, more accurately, half the angle through which the ionisation chamber has been displaced, measures the critical angle for the incident radiation. Thus if the incident rays are homogeneous there will only be a current in the ionisation chamber at certain definite angles corresponding to the different order spectra for the wave-length employed. Thus the curve between the current and the glancing angle will show definite sharp peaks or lines (*Fig.* 9) which can be identified as being due to the same radiation by the fact that $\sin\theta_1 : \sin\theta_2 : \sin\theta_3$ as $1:2:3$. The intensity of the spectra of higher order is generally much less than that of the first order spectrum, but as the corresponding angles are much larger they can be measured with considerably greater accuracy. If the ionisation chamber is replaced by a photographic plate the different peaks will be represented by dark lines on the plate, corresponding to the lines observed under similar circumstances in a luminous spectrum.

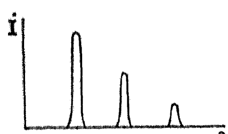


FIG. 9.

If the radiation from the anticathode of an ordinary X-ray tube is examined it is found that reflection takes place at practically all angles although the intensity of the reflected beam increases markedly at certain definite angles. This is shown in *Fig.* 10, which represents Bragg's results for X-rays from a tube with a platinum anticathode. The rays from a platinum anticathode therefore consist of a mixture of radiations of continuous wave-lengths, analogous to white light in the case of the luminous radiation, superimposed upon which are certain radiations of definite wave-length indicated by the peaks on the curve. The radiation from the platinum anticathode

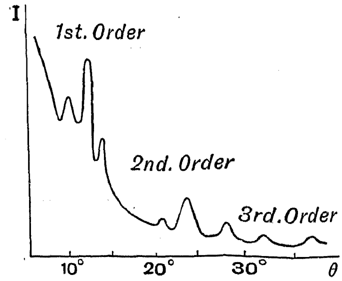


FIG. 10.

shows three of these peaks in the first order spectrum, which reappear in the same order in the second and third order spectra, as indicated in the figure. Bragg showed that the position of the peaks is independent of the nature of the crystal used for the analysis, rock salt, fluor, iron pyrites, etc., all giving the same result. They are, in fact, characteristic of the substance used as the anticathode of the X-ray tube, in this case platinum. Thus the spectrum obtained from an ordinary X-ray tube consists of a large quantity of general or "white" radiation on which is superposed certain definite spectral lines characteristic of the metal of the anticathode.

The relative amount of the general and the characteristic radiations depends very largely on the potential used to excite the tube. With high potentials, such as are employed in radiography, the intensity of the characteristic radiations is generally only a small fraction of the total radiation. In the case of very soft tubes the characteristic radiation may easily account for by far the greater part of the energy radiated, and the general radiation may be quite small.

§ (9) DETERMINATION OF THE WAVE-LENGTH OF X-RAYS.—The equation $2d \sin\theta = n\lambda$ enables us at once to compare the wave-lengths of different kinds of X-radiation by means of the X-ray spectrometer. We can, for example, find the ratio of the wave-lengths of the three different types of platinum characteristic radiation from the curve of *Fig.* 10. To determine the absolute value of the wave-length, however, we require to know the distance d between the planes of our crystal lattice. The clue to this problem has been provided by the experiments of W. L. Bragg [1] on the crystal form of the halogen salts of the alkali metals. These substances are all isomorphic, crystallising in the same form and belonging to one class of the cubic system.

[1] W. L. Bragg, *Proc. Roy. Soc.* A, 1913, lxxxix. 248.

They are known as the sylvine group, from their most important member, sylvine (KCl).

Crystallographers recognise[1] three classes of cubic symmetry:

(1) The simple cube arising from a simple cube lattice such as has already been described.

(2) A cube with a single particle at the centre, known as a cube centred lattice.

(3) A cube with a particle at the centre of each face, known as a face centred lattice.

These particles will, of course, form part of a second cubic lattice interpenetrating the first. Now the most important planes in the cubic system are (1) the faces of the cube itself, denoted in crystallographic nomenclature as $\{100\}$, (2) the planes passing through opposite edges of the cube $\{110\}$, and (3) the planes passing through one corner of the cube and the diagonal of the opposite face. This plane gives rise to the octahedral faces on a cubic crystal and is denoted by $\{111\}$. Now it is easy to see that the distances between two successive planes in the three systems of planes corresponding to $\{100\}$, $\{110\}$, $\{111\}$ will not be the same for the three kinds of cubic lattices. The application of simple geometry shows that

$$\frac{1}{d_{100}} : \frac{1}{d_{110}} : \frac{1}{d_{111}}$$

$$=1 : \sqrt{2} : \sqrt{3} \text{ for a simple cube lattice}$$

$$=1 : \frac{1}{\sqrt{2}} : \sqrt{3} \text{ for a cube centred lattice}$$

$$=1 : \sqrt{2} : \frac{\sqrt{3}}{2} \text{ for a face centred lattice.}$$

Now the ratios $d_{100} : d_{110} : d_{111}$ can be determined by the X-ray spectrometer itself by measuring the glancing angle from the different planes for incident radiation of the same wavelength. In this way we can identify the particular type of cubic symmetry exhibited by a given crystal.

On applying the method to the sylvine group some curious discrepancies were observed. Sylvine itself (KCl) had apparently the simple cubic structure, and potassium iodide that of a face centred lattice. Rock salt (NaCl), although chemically and crystallographically very similar to sylvine, gave quite unusual results. Using the reflections from the octahedral faces $\{111\}$ W. L. Bragg obtained a very weak first order spectrum, a strong second, a weak third, and so on. Judging by the strong lines only, rock salt would appear to be a simple cube lattice. Judging by the first weak spectrum, it would appear to be a face centred lattice. Thus in one case it would be assigned to the same class as sylvine, in the other to that of potassium iodide, while all other chemical and crystallographic phenomena indicate that all three crystals belong to the same class.

[1] See "Crystallography," § (7), Vol. IV.

W. L. Bragg was able to reconcile the phenomena by assuming that the points making up the crystal lattice were not molecules of the salt but atoms of the different elements in the salt. Now the intensity of the scattered radiation from a given atom is proportional to the atomic number. Thus in the case of sylvine the reflections from the planes containing potassium atoms only (K = 19) will be approximately equal to that from the chlorine planes (Cl = 17). In the case of potassium iodide, however, the reflection from the iodine planes (I = 53) will be so great as to obscure that from the potassium planes. In the case of sodium chloride, however, the reflection from the sodium planes (Na = 11), though less than that from the chlorine planes, will be comparable with it, and will thus give reflections which are measurable, though weak. Bragg showed that the whole of the results could be explained on these lines if each of the crystals had the structure indicated in *Fig.* 11, where the metallic atoms are represented by dots and the halogen atoms by circles. If the dots and the circles produce identical effects, as is practically the case for sylvine, the system obviously reduces to a simple cube lattice. If the relative effect produced by the dots is so small as to be inappreciable, which is the case with potassium iodide, we may regard the dots as vanishing from the structure, which then becomes a face centred cube. In the case of rock salt, the two sets of particles are dissimilar, but neither of them is negligible. It will be seen from the figure that all the $\{100\}$ planes and the $\{110\}$ planes are similar, being made up of alternate atoms of sodium and chlorine. The octahedral planes, however, $\{111\}$, are alternately composed entirely of atoms of chlorine or wholly of atoms of sodium. This case corresponds closely to that of a ruled optical grating in which every odd ruling is made wider than the even ones. In the optical case this alternation of wide and narrow rulings results in the production of alternate strong and weak spectra, the latter corresponding to a grating with twice the actual grating space, that is to say, the deflection of the first weak spectrum is approximately half that of the normal first order spectrum. This is what occurs in the case of the X-ray spectrum of rock salt.

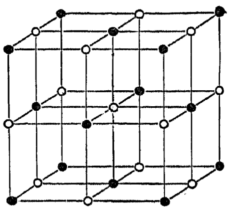


FIG. 11.

Accepting the structure of *Fig.* 11 as representing that of rock salt, we see that each sodium atom is at the junction of eight of

the small cubes into which the figure is divided. Assuming that its mass is equally divided among these elementary cubes, each cube will contain one-eighth of the mass. But there are four sodium atoms associated with each cube, so that each of the elementary cubes includes the mass of one-half an atom of sodium, and consequently one-half a molecule of sodium chloride. The structure in *Fig.* 11 thus represents four molecules.

Now the distance apart of the planes bounding the small cubes is obviously d_{100}, since all these planes are similar. The volume of each elementary cube is thus $(d_{100})^3$. The mass of each cube is that of half a molecule of the salt, that is, $\frac{1}{2}(23+35 \cdot 5) \times$ (the mass of a hydrogen atom). The latter value can be deduced from a knowledge of the unit electronic charge, and is approximately $1 \cdot 64 \times 10^{-24}$ gramme. Now if ρ is the density of rock salt the mass of the elementary cube is also equal to $\rho(d_{100})^3$. Hence

$$\rho(d_{100})^3 = 29 \cdot 3 \times 1 \cdot 64 \times 10^{-24},$$
$$d_{100} = 2 \cdot 80 \times 10^{-8} \text{ cm.}$$

Substituting this value in the equation $2d \sin \theta = n\lambda$, we can determine the absolute value of λ for any given radiation. For the most pronounced radiation from platinum the glancing angle for the first order spectrum is about $11 \cdot 4^\circ$ for the $\{100\}$ planes of rock salt. Hence the wave-length of this particular radiation is about $1 \cdot 10 \times 10^{-8}$ cm.

It may be pointed out that the absolute determination of the wave-length depends, as do many of our atomic constants, on the value assumed for the elementary electronic charge. In spite of much experimental work it is doubtful if this is known with certainty to more than 1 per cent. Recent advances in X-ray spectroscopy due to Siegbahn [1] make it possible to determine the relative wave-lengths with an accuracy approximating to one part in ten thousand. The relative values are therefore known to a much higher order of accuracy than the absolute values.

§ (10) X-RAY SPECTRA OF THE ELEMENTS.—It has already been mentioned that an element, when used as the target in an X-ray tube, emits in addition to the general radiation certain radiations of definite wave-length characteristic of the element, in exactly the same way that a sodium salt when vaporised in a bunsen flame emits the characteristic luminous spectrum of sodium. The X-ray spectrum of an element, as investigated by the X-ray spectrometer, is just as characteristic of the element as the luminous spectrum, while the relation between the spectra of the different elements is much simpler in the case of X-rays than in the case of the luminous rays.

It was found that the characteristic radiation from an element was in general of two distinct types, one of which was of much shorter wave-length than the other. The radiation of shorter wave-length is called the characteristic K-radiation, that of longer wave-length the L-radiation of the element. The existence of these radiations had been recognised before the development of X-ray spectroscopy. The more accurate analysis introduced by the spectroscope showed that each of these radiations was, in fact, complex. The K-radiation for a given element consists of two main lines of not very different frequency known as K_α and K_β, and thus forms what would in optics be described as a doublet. The component of wave-length, K_α, is always considerably more intense than the other. Similarly the L-radiation is also found to be complex, consisting of a doublet, or perhaps a triplet, of which the component of greater wave-length is again the more intense. Thus the spectrum of antimony, for example, consists of lines of approximate wave-lengths $K_\alpha = 0 \cdot 48 \times 10^{-8}$, $K_\beta = 0 \cdot 41 \times 10^{-8}$, $L_\alpha = 3 \cdot 458 \times 10^{-8}$, $L_\beta = 3 \cdot 245 \times 10^{-8}$. Other elements give X-ray spectra of exactly similar type.

As we proceed from elements of lower to elements of higher atomic weight the wave-length of a line of given type, say K_α, for example, decreases with increasing atomic weight, and Moseley,[2] to whom the first extensive survey of the subject was due, showed that there was a very simple and important relation between the frequency of the radiation for a given element and the atomic number of the element (that is to say, its number in a table of the elements arranged in the order of ascending atomic weights). He showed that for a given type of line the frequency ν could be expressed by the formula

$$\nu = a(\mathrm{N} - b)^2,$$

where N is the atomic number, and a and b are constants for a given type of line. This result applied, with a change in the value of the constants, to all the principal lines both in the K and in the L series. It has been confirmed to a very considerable degree of accuracy by later observers.

The two series of radiations have not yet been observed for all elements. This is in part due to experimental difficulties. The K-radiation of elements of high atomic weight is difficult to excite, because, as we shall see later, the velocity of cathode rays necessary to excite a given radiation increases with the frequency. On the other hand, the K-radiation from elements of very low atomic weight, and the L-radiation from elements of quite

[1] Siegbahn and Strenstom, *Phys. Zeit.*, 1916, xvii. 48; Siegbahn, *Phil. Mag.*, 1919 (6), xxxvii. 601; *ibid.*, 1919 (6), xxxviii. 639.

[2] Moseley, *Phil. Mag.*, 1914 (6), xxvii. 703.

moderate atomic weight, is completely absorbed in a thickness of only a few millimetres of air, and is therefore very hard to work with. By working in a vacuum, Siegbahn (*loc. cit.*) has been able to extend the K series down to sodium, and the L series down to zinc. There is, however, some evidence that L-radiation would not be emitted by elements of atomic weight less than 48. Since the K- and L-radiations of an element correspond to the lines in an optical spectrum, we should expect that there would be some numerical relation between them, and it has been pointed out by Whiddington [1] that if an element of atomic weight A_K emits K-radiation of the same wave-length as the L-radiation from some element of atomic weight A_L, then

$$A_K = \tfrac{1}{2}(A_L - 48).$$

If this empirical relation is universally true it follows that an element of atomic weight lower than 48 cannot give out radiation of the L type.

It is supposed that the K- and L-radiations are due to the vibrations of different rings of electrons within the atom, known respectively as the K- and the L-ring, from their supposed connection with these particular types of radiation. Neither the K- nor the L-radiation is quite so simple as has been described. The accurate analysis of Siegbahn [2] shows that each of the two K lines is in reality a very close doublet. The L-radiation of an element seems to consist of three principal lines accompanied by other fainter lines of slightly different wave-length. For a discussion of these and their bearing on atomic structure the work by

Wave-lengths of Characteristic X-radiation

Element.	K Series.		L Series.	
	K_α.	K_β.	L_α.	L_β.
	$\times 10^{-8}$ cm.	$\times 10^{-8}$ cm.	$\times 10^{-8}$.	$\times 10^{-8}$
Al . .	8·364	7·912	..	..
Si . .	7·142	6·729	..	..
Fe . .	1·946	1·765	..	..
Ni . .	1·662	1·506	..	..
Cu . .	1·549	1·402	..	..
Zn . .	1·445	1·306	..	..
As . .	1·170	1·052	..	..
Zr . .	·794	..	6·091	..
Pd . .	·584	..	4·385	4·168
Ag . .	·562	·501	4·170	..
Sn . .	·50	·43	3·691	..
Sb . .	·48	·41	3·46	3·24
W . .	·203	·177	1·486	..
Pt . .	..	..	1·316	1·121
Au . .	..	..	1·287	1·092

[1] *Nature*, 1911.
[2] *Loc. cit.*, and *Ann. der Phys.*, 1916, xlix. 611.

Sommerfeld,[3] or original papers by Sommerfeld,[4] Debye,[5] and others may be consulted. The approximate wave-lengths of some of the more important lines are given in the foregoing table, taken mainly from Moseley's work (*loc. cit.*). For more accurate values the papers by Siegbahn should be consulted.

It is found that a given spectral line is only excited if the cathode rays impinging on the anticathode reach a definite critical velocity characteristic of that line. Whiddington,[6] by allowing cathode rays of different known velocities to impinge on radiators of different kinds, showed that the K-radiation characteristic of the element was only emitted when the velocity of the rays reached a value which was given approximately by $2A \times 10^8$, where A is the atomic number of the element. The energy which the electron must acquire to excite the radiation is therefore proportional to the square of the atomic number. It is thus by Moseley's results also proportional to the frequency of the radiation excited, a result which is in accordance with Planck's quantum theory of radiation. The numerical agreement with the quantum theory is also very satisfactory. The critical velocity required to excite the K-radiation of nickel, for example (atomic number, 28), is 56×10^8 cm. per sec., or, since the mass of an electron is about 9×10^{-28} gramme, the energy of each of the exciting particles is $\frac{1}{2}(9 \times 10^{-28}) \times (56 \times 10^8)^2$ or about $1{\cdot}4 \times 10^{-8}$ ergs. The frequency of the nickel K-radiation is $1{\cdot}8 \times 10^{18}$. On Planck's theory, therefore, the energy necessary to excite it should be $h\nu$, where ν is the frequency and h is Planck's constant, which has the value $6{\cdot}55 \times 10^{-27}$. Substituting in this expression, the quantum of energy for the nickel radiation should be $(6{\cdot}55 \times 10^{-27}) \times (1{\cdot}8 \times 10^{18})$ or about $1{\cdot}2 \times 10^{-8}$ ergs. This agrees sufficiently closely with the critical energy of the cathode particle as deduced from Whiddington's experiments.

§ (11) Characteristic Secondary Radiation.—The characteristic X-ray spectrum of an element may be excited not only by the impact of a beam of cathode rays, but also by the action of a beam of X-rays. When so produced it is known as the characteristic secondary radiation of the element. In order to excite it, it is necessary that the incident rays should be of shorter wave-length than those which it is desired to excite. Thus the characteristic K-radiation of nickel is not excited by the K-radiation from iron or by the K-radiation from nickel itself, or even by that of copper. It is, however, excited by

[3] Sommerfeld, *Atombau und Spektrallinien*, Braunschweg, 1921.
[4] Sommerfeld, *Ann. der Phys.*, 1916, li.; *Phys. Zeit.*, 1918, p. 297.
[5] *Phys. Zeit.*, 1917, p. 276.
[6] R. Whiddington, *Proc. Roy. Soc.* A, 1911, lxxxv. 323.

the K-radiation of zinc. In this respect the phenomenon resembles that of fluorescence, and the radiation emitted is sometimes known as the fluorescent radiation.

It is found that a beam of homogeneous X-rays is absorbed by matter in accordance with an exponential law. Thus if I_0 is the initial intensity, and I the intensity after passing through an absorbing screen of thickness d, $I = I_0\epsilon^{-\lambda d}$, where λ is a constant for a given wave-length and a given absorbing substance. Either the K- or the L-radiations of an element, though really complex, are sufficiently homogeneous to give an exponential law of absorption within the limits of experimental error; and the existence of these groups of characteristic secondary radiations was first recognised by Barkla[1] from a study of the absorption of the secondary rays by various absorbing screens. In this way the K-radiations of the elements from sulphur to barium, and the L-radiations of the elements from silver to bismuth, were identified and their absorption coefficients in various materials measured. It was shown by Owen[2] that the coefficient of absorption of the K-radiation from a given element in a light substance such as aluminium was inversely proportional to the fifth power of the atomic weight of the radiator. This result is found to hold approximately for any absorbing material providing that the incident radiation does not excite an appreciable amount of the characteristic radiation of the absorbing material. Any substance is peculiarly transparent to its own characteristic radiation, and exceptionally opaque to those radiations of slightly shorter wave-length which strongly excite its characteristic radiation. This is, of course, to be expected from energy considerations, as the energy of the excited secondary radiation must be drawn from that of the primary beam, which must therefore be strongly absorbed. A fuller account of the effect is given in a paper by Barkla and Collier,[3] and the more recent work of Auren.[4] A resurvey of the whole subject, however, by the more powerful and precise methods afforded by the X-ray spectroscope is much to be desired.

In any substance in which the characteristic radiation is not excited the coefficient of absorption decreases with decreasing wave-length. It has been shown that under these circumstances the coefficient of absorption in a given substance $\propto$ (wave-length)$^{\frac{5}{2}}$. For this reason X-rays of long wave-length, and therefore great absorbability, are often called "soft" X-rays; while the penetrating rays of short wave-length are termed "hard." The absorption coefficients of some of the characteristic radiations are given in the appended table.

Mass Coefficient of Absorption of Characteristic Radiations

Element emitting the Radiation.	$\frac{\lambda}{\rho}$ (K Series).			
	In Al.	In Cu.	In Zn.	In Sn.
Cr . .	136	143	170·5	713·7
Fe . .	88·5	95·1	112·5	472
Co . .	71·6	75·3	91·5	392
Ni . .	59·1	61·8	74·4	328
Cu . .	47·7	53·0	60·9	272
Zn . .	39·4	55·5	50·1	225
As . .	22·5	176	203·5	131·5
Se . .	18·9	149·8	174·6	112
Ag . .	2·5	24·3	27·1	16·5

§ (12) Secondary Corpuscular Radiation.—In addition to the scattered and the characteristic radiations, which are themselves X-radiations, any material substance when subjected to the action of X-rays emits a radiation of small penetrating power which can be deflected by a magnet and is therefore corpuscular. The particles carry a negative charge, and a determination of the ratio of e/m shows that they are negative electrons. The corpuscular radiation has been studied by Beatty[5] and Sadler,[6] among others. It is found that the velocity with which the electrons are ejected is practically independent of the material from which they are produced, but depends on the wave-length of the exciting radiation. Working with homogeneous radiation, it is found that the velocity of the secondary electrons is, within the limits of experimental error, equal to the critical electron velocity necessary to excite the primary X-radiation employed. Thus it would appear that when an X-ray is produced by the impact of an electron, the whole of the energy of the electron is transformed into that of the X-ray pulse, while if, after travelling any distance, the latter falls on an atom in such a way as to cause the ejection of an electron, the whole of the energy of the pulse is carried away by this electron. This result is, of course, in good agreement with the "quantum" theory, but it is difficult to reconcile with the ordinary theories of light radiation.

The number of electrons emitted increases with the atomic weight of the radiator. It is also very much increased if the incident radiation is capable of exciting the characteristic radiation of the radiator. The cor-

[1] Barkla and Sadler, *Phil. Mag.*, 1908 (6), xvi. 550.
[2] E. A. Owen, *Proc. Roy. Soc.* A, 1912, lxxxvi. 426.
[3] Barkla and Collier, *Phil. Mag.*, 1912 (6), xxiii. 987.
[4] Auren, *Phil. Mag.*, 1919 (6), xxxvii. 165.

[5] Beatty, *Phil. Mag.*, 1910 (6), xx. 320.
[6] Sadler, *Phil. Mag.*, 1910 (6), xix. 337.

puscular radiation is practically completely absorbed in a few millimetres of air at normal pressure. Owing to this great absorbability the ionising effect is very large. It also has a strong though superficial effect on animal tissues. Hence metal screens used in radiography should be covered with pads of paper or wash-leather if they are likely to come into contact with the skin of the patient, in order to reduce the effect as much as possible, and so to avoid the possibility of unpleasant burns.

§ (13) Ionisation by X-rays.—X-rays in passing through a gas produce ionisation in it, and thus render the gas a partial conductor of electricity. The ionisation is not due to the direct action of the rays, but to the production from some of the atoms of the gas of the corpuscular radiation already described. These corpuscular rays which, as we have seen, are projected with velocities equal to those of the particles in the cathode rays, then ionise the molecules of the surrounding gas by collision. The whole process is shown very clearly in the cloud photographs of the passage of a beam of X-rays through a gas, taken by C. T. R. Wilson.[1]

If the primary beam is strong the conductivity produced is, for a gas, very considerable. It is worth recalling, however, that the number of molecules per unit volume which are ionised at one and the same time is relatively very small, amounting, even in favourable cases, to only about one in a billion. Thus a given molecule of gas would only become ionised once in a hundred years. The phenomenon is of importance as it affords the most accurate and convenient way of measuring the intensity of a beam of X-rays.

The relative ionisation produced in different gases has been studied by numerous observers, including Crowther,[2] Beatty,[3] and Barkla and Philpot.[4] It is found that the amount of ionisation produced is directly proportional to the intensity of the radiation and to the mass of gas through which the radiation passes. It is thus directly proportional to the pressure of the gas at constant temperature, and is independent of the temperature so long as the mass of gas is kept constant. The intensity of the ionisation depends on the nature of the gas, and is very much greater for gases or vapours containing elements of medium or high atomic weight than for those containing only the lighter elements. Thus for equal masses of gas the ionisation in ethyl bromide vapour is about 20 times and that in mercury methyl about 50 times as intense as that in air. The ionisation in a given gas relative to that in air seems to be independent of the wave-length of the X-radiation, provided that the characteristic secondary radiation of the gas is not excited. If the secondary radiation is excited the ratio is largely increased.

It seems probable that for a given intensity of the incident beam the ionisation produced would decrease with decreasing wave-length, as a relatively smaller fraction of the energy of the beam would be absorbed in the gas. Owing to the absence of any reliable method of measuring the energy in a beam of X-rays there seems to be no experimental evidence on the question. The intensity of the ionisation produced, therefore, does not afford any certain indication of the relative intensities of two beams of X-rays which differ in wave-length.

§ (14) Methods of Estimating the Intensity of the Rays.—No method seems to have been devised for comparing accurately the intensities of two beams of X-rays of different wave-length. The intensity of a beam of X-rays is always estimated in practice either by the ionisation it produces in a given mass of gas, or by the chemical change it produces in some given substance. The ionisation method is much the more accurate and is always employed in scientific work. The chemical methods, as requiring less apparatus, are generally employed in medical radiology.

It can be shown that if two plates are immersed in ionised gas and charged to a sufficiently high difference of potential the current between the plates is directly proportional to the rate of production of ions in the gas between the plates. The current is usually far too small to be measurable on a galvanometer. It can, however, easily be measured by an electroscope or electrometer. Suppose that one of the plates is insulated and connected to an electroscope or electrometer, and that this system is originally at zero potential, the other plate being kept charged to some constant potential sufficiently high to produce the saturation current through the gas. The insulated system will begin to charge up, and after an interval t, its potential, will have risen to some value, V, which can be measured on the electroscope or electrometer scale. If i is the current flowing into the electrode the quantity of electricity reaching the electrode in the interval t will be it. Thus if C is the electrical capacity of the insulated system we have $it = CV$. For a given system the time taken to charge up to some definite potential will, therefore, be inversely proportional to the current through the gas, and therefore inversely proportional to the rate of production of ions in the gas by the ionising agent. The intensity of the rays is thus

[1] C. T. R. Wilson, *Proc. Roy. Soc.* A, 1912, lxxxv. 285.
[2] Crowther, *Proc. Roy. Soc.* A, 1909, lxxxii. 103.
[3] Beatty, *Proc. Roy. Soc.* A, 1911, lxxxv. 230.
[4] Barkla and Philpot, *Phil. Mag.*, 1913 (6), xxv. 832.

inversely proportional to the rate at which the insulated system charges up. Further, the total change in potential of the insulated system will be proportional to the total energy of the rays which have passed through the apparatus.

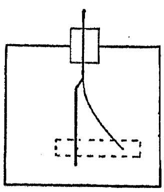


Fig. 12.

The method can be applied most simply by the use of a simple electroscope (*Fig.* 12), one side of which is covered with thin aluminium foil to admit the rays. The gold-leaf system is charged by means of an ebonite rod, the rays are passed into the electroscope, and the time taken by the leaf to fall from one fixed division on the scale to another is measured. If the same two divisions are always employed it is not necessary to know the corresponding voltages. The intensity of the rays will be inversely proportional to the time taken.

For more accurate work it is generally preferable to use a separate ionisation chamber containing an insulated electrode connected to an electrometer. A convenient form is shown in *Fig.* 13. It consists of a shallow

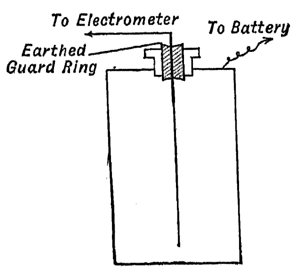


Fig. 13.

cylindrical box the two ends of which are closed by thin aluminium foil to allow of the passage of the rays. Midway between these is mounted a thin insulated aluminium sheet connected to the electrometer, and serving as the insulated electrode. The outer case is connected to a battery of cells and serves as the charged electrode. Currents as small as 10^{-15} ampere can easily be measured with this arrangement, using a Dolezalek electrometer or a Wilson electroscope, and by placing additional capacity in the system currents as large as 10^{-8} amperes can also be measured. The currents produced by X-rays in such an apparatus are generally of the order of 10^{-10} to 10^{-15} ampere. A compact and convenient apparatus working on these principles has been placed on the market under the name of the Ionto-quantimeter.

The methods generally employed by medical radiologists in estimating the quantity of the rays are based on the chemical action of the rays. Various reactions have been suggested and adopted, among them being the action on a photographic plate, the precipitation of calomel from a mixture of mercuric chloride and ammonium oxalate, and the change in colour of pastilles of compressed barium platinocyanide (Sabouraud Pastilles). The latter is the method usually adopted. The colour of the pastilles, originally bright green, changes, under the action of X-rays, first to a lemon yellow and then to a deep orange. The pastille is placed at some specified distance from the anticathode of the tube and the exposure continued until the colour of the pastille matches one or other of a number of standard tints. The method is very easy in practice and fairly reliable, but is not to be compared in precision with the ionisation method.

§ (15) Practical Measurement of the Quality of the Rays.—The quality of the radiation is, of course, most accurately expressed in terms of its wave-length. The X-ray spectrometer, however, is not yet a sufficiently convenient instrument for everyday use, and more rapid methods have to be employed. We have seen that the coefficient of absorption of a homogeneous beam of rays in a substance such as aluminium is directly proportional to (wave-length)^3. The coefficient of absorption can therefore be used as a measure of the quality of the radiation. The coefficient of absorption can easily be determined by interposing sheets of the absorbing substance between the source of the rays and an ionisation chamber. Alternatively the quality of the rays may be expressed by the thickness of some standard material required to cut down the radiation to half value. Distilled water has been suggested as a suitable standard for radiological purposes, as its absorbing powers are very similar to that of animal tissues.

In practice, however, among medical men some form of penetrometer such as that of Wehnelt or Benoist is generally employed. These penetrometers are based on the difference between the absorbing properties of aluminium and those of some element such as silver whose characteristic radiations are strongly excited by the range of wave-lengths generally employed in radiology. The coefficient of absorption of silver in this region decreases less rapidly with decrease in wave-length than that of aluminium, so that as the rays get harder an increasing thickness of aluminium is required to absorb the rays to the same extent as a given thickness of silver. The Benoist radiometer (*Fig.* 14) consists of twelve numbered sectors of aluminium of increasing thickness surrounding a uniform silver plate. The density of the

shadow cast by the rays of the silver disc is matched against those cast by the aluminium sectors by means of a fluorescent screen. The

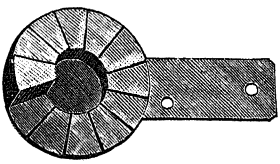


FIG. 14.

number on the disc which makes the best match gives the quality of the rays on the Benoist scale. The Wehnelt penetrometer differs from that of Benoist merely in substituting an aluminium wedge for the set of stepped sectors. The scales of these instruments are, of course, quite arbitrary.

The penetrating power of the rays increases with the potential employed to excite the X-ray tube. The rays from an ordinary X-ray tube are, as we have seen, very complex, consisting of a general radiation of continuous wavelengths, together with some radiation characteristic of the anticathode. The mean wave-length of the general radiation is found to decrease as the potential across the tube is increased. Thus the radiation from a tube requiring a high voltage to excite it has a greater average penetrating power than that from a tube working on a low voltage. The potential across the tube is generally estimated by means of an adjustable spark gap placed in parallel with the tube. The distance apart of the terminals at which sparking begins to take place furnishes a rough estimate of the potential difference. The length of the alternative spark gap thus gives a somewhat crude estimate of the hardness of the rays from the tube. It is, however, not very reliable, as different tubes working at the same alternative spark gap do not invariably give rays of the same quality.

§ (16) THE X-RAY TUBE.—The original tube with which Röntgen discovered X-rays consisted merely of a pear-shaped bulb with a flat disc cathode. The anode was situated in a side tube, and the cathode rays impinged directly upon the walls of the tube. As the X-rays are emitted from all parts struck by the cathode particles, the shadows cast by the rays from such a tube were necessarily indistinct owing to the large area of emission. Moreover, if the current was at all large the cathode rays rapidly fused the glass walls on which they fell. Campbell-Swinton in 1896 improved the design by placing a platinum target obliquely in the path of the cathode beam, and in the same year Jackson employed a concave cathode in order to bring the cathode rays to a focus on the target, a device employed by Crookes in 1874. The cathode rays, being ejected at right angles to the surface of the cathode, are brought to a focus at a point on the axis which, owing to the action of the electric field, is somewhat beyond the geometrical centre of curvature of the surface. In this way the origin of the X-rays is confined to a very small area, and the radiograms produced are very distinct and full of detail.

The design of a modern X-ray tube does not differ materially from that of Jackson, and different patterns of tube differ mainly in their size and strength, and in the devices employed to eliminate the very large quantities of heat generated by the discharge. A modern X-ray tube is required to carry currents of 10 milliamperes or more with a potential difference between the terminals of 100,000 volts. This means the production of heat in the tube at the rate of 240 calories per second or more. The greater part of this heat is liberated at the point on the target struck by the cathode beam. The target has also to withstand the mechanical effects of the cathode stream, which are often by no means inconsiderable.

The cathode of the tube (*Fig.* 15) is made of aluminium, as the disintegration by an

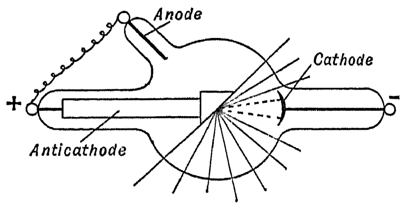


FIG. 15.

electric discharge (cathodic sputtering) is less for aluminium than for other metals. It is mounted just at the entrance to the large bulb, which constitutes the main part of the tube, connection with the outer negative terminal being made by a platinum wire fused through the glass. The anticathode or target is placed in the centre of the bulb, and its surface is inclined at an angle of about 45° to the cathode stream. An auxiliary electrode, known as the anode and connected with the anticathode, is generally inserted. Its function is not easy to explain, but it seems to steady the discharge.

The metals generally employed for the target are platinum and tungsten. The fraction of the energy of the cathode beam which is emitted as X-radiation increases with the atomic weight of the target. Thus platinum transforms four times as much energy as iron,[1] and from this aspect is the most suitable

[1] Kaye, *Phil. Trans.* A, ccix. 123.

metal for the purpose. It is, however, rather soft, its melting-point is rather low for the purpose, and it sputters very badly. Tungsten, although its radiating power is only about 91 per cent of that of platinum, is therefore generally preferred on account of its hardness and infusibility, and also on account of its greater thermal conductivity, which permits of the more rapid diffusion of the heat produced at the focal point of the rays. In tubes designed for heavy discharges the tungsten target is set in a massive block of copper in order to increase the mass of metal, and so decrease the rise of temperature.

If the tube is required for continuous work various cooling devices may be fitted to the anticathode. The copper tube carrying the anticathode may be fused into the glass walls of the tube, so that the interior of the anticathode is open to the air. The copper tube may also be furnished with radiating vanes to increase the rate of cooling. The most efficient device is to cool the back surface of the anticathode with water. In this case (*Fig.* 16) the target is usually of platinum fused

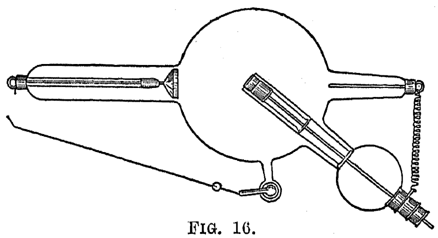


FIG. 16.

directly on to the glass tube carrying the water supply. If necessary a continual stream of cold water can be made to play on the back surface of the platinum target, and overheating is thus completely avoided. The very efficient cooling also allows the cathode beam to be brought to a finer focus without damage to the anticathode than is the case with other types, and the radiograms produced by a well-made, water-cooled tube excel in sharpness and fineness of detail.

§ (17) THE REGULATION OF AN X-RAY TUBE.—The X-ray tube is exhausted to a fairly high vacuum, so that the Crookes' dark space almost completely fills the tube, except for a small region about the anode. In fairly soft tubes there may be sufficient residual gas to render the cathode beam visible as a faint bluish stream, but in the harder tubes the whole of the gas is dark, the only visible effect of the discharge being the vivid green fluorescence excited by the rays in the hemisphere of glass in front of the anticathode. The colour of this fluorescence, which depends on the nature of the glass, changes slightly with the hardness of the tube, becoming greyer and less brilliant as the rays become harder. An expert radiographer can gauge the hardness of his tube with some accuracy by observing the fluorescence. Any fluorescent light behind the anticathode is an indication that part of the discharge is passing through the tube in the wrong direction.

The potential required to excite the discharge, and hence the hardness of the X-rays emitted, increases as the pressure of the residual gas is diminished. The potential difference between the ends of the tube increases with the current passing through it, but only slowly. The hardness of the rays emitted, therefore, depends mainly on the degree of exhaustion of the tube. Unfortunately the vacuum in a discharge tube is by no means a constant quantity. If the tube is a new one gas will be liberated from the metal electrodes by the action of the discharge. This increases the pressure and thus softens the tube. In well-made tubes a considerable portion of this gas is removed in the process of manufacture by heating the tube during exhaustion, and also by passing a current through it, but in spite of this a new tube is always liable to soften from this cause if too great a current is passed through it.

At the same time gas tends to disappear from the tube, and after the supply of gas from the electrodes has ceased a tube will always harden in use. The cause of this disappearance has not yet been satisfactorily explained. Campbell-Swinton [1] produced evidence that molecules of the gas were driven into the walls of the tube by the discharge. Other observers [2] think that the effect is due to chemical action between the gas and the glass of the tube. Possibly both causes are operative. It is obvious that if some means is not provided of introducing fresh gas into the tube it will eventually become much too hard for use.

The method usually employed is to enclose in a side tube communicating with the bulb some absorbent substance, such as asbestos or glass wool, which liberates occluded gas when heated. The tube is generally furnished with auxiliary electrodes (*Fig.* 16), so that when necessary a portion of the electric discharge can be sent through the tube. This liberates the occluded gas and thus increases the gas pressure in the main tube. The supply of occluded gas is of course limited and the regulator eventually ceases to work. The life of an X-ray tube is, however, subject to so many accidents and dangers that in

[1] Campbell-Swinton, *Proc. Roy. Soc.* A, 1908.
[2] Willows and George, *Proc. Phys. Soc. London*, 1916.

practice the regulator generally outlives the tube. By moving the regulating wires with an insulating stick the tube can be regulating while actually running, a great convenience in practice.

Another effective method is to seal into the walls of the bulb a small platinum or palladium tube, the outer end of which is closed while the open end communicates with the bulb. The tube forms a perfect seal when cold, but if heated by a flame hydrogen diffuses slowly through the metal into the bulb and thus softens the tube. The amount of gas which can be introduced in this way is unlimited, but as the method which is known as the osmosis method is not quite as convenient as the former it seems to have fallen into disfavour.

An X-ray tube which is too soft for use can only be hardened by continuous running with small currents, a tedious and not always successful process. If, however, the residual gas in the tube is hydrogen, an osmosis regulator can be used for extracting the gas, by heating the metal tube, not with a flame, but by means of a red-hot spiral wound round it. The hydrogen then diffuses out to the atmosphere, where the pressure of free hydrogen is less than in the bulb. This method is employed in the Snook hydrogen tube.

§ (18) The Coolidge Tube. — In the ordinary X-ray tube the electric discharge is carried by the residual gas, and changes in the pressure of this gas produce variations in the quality of the rays. An ingenious tube has been devised by Coolidge, from which the gas has been eliminated, the current being carried by the electrons emitted under the action of heat on an incandescent metal. The cathode C (*Fig.* 17) consists of a

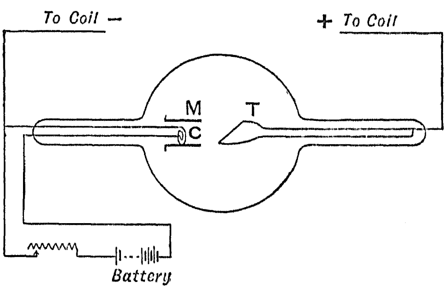


Fig. 17.

spiral of tungsten wire, which can be raised to incandescence by means of an electric current supplied by an insulated battery of accumulator cells. The cathode rays are focussed into a beam by surrounding the spiral with a molybdenum tube, M. The target is a massive block of tungsten T, and no method of cooling is considered necessary, as the tube is said to function well even with the target at red heat. The current through the tube depends only on the temperature to which the spiral is raised, while the hardness of the rays is determined solely by the potential produced by the exciting coil. Thus the two factors—intensity and quality of the rays—can be independently controlled, which is not the case in an ordinary gas tube, and the difficulties introduced by the changes in pressure on the gas tubes are completely eliminated. Some radiographers consider that the radiograms produced by a Coolidge tube are not so well defined as those produced with an ordinary tube. This is possibly due to imperfect focussing of the cathode rays, owing to which X-rays of appreciable intensity seem to be produced in nearly all parts of the tube. This defect will probably be overcome in the near future. For the long exposures required in radiotherapy, the ease of regulation of the tube, the certainty with which previous exposures can be repeated, and the constancy of the quality of the rays render it a most valuable instrument.

§ (19) Excitation of the X-ray Tube—Induction Coils and Transformers.—In practical work a potential difference of from 50,000 to 150,000 volts is employed to excite the X-ray tube. This is supplied either by a large induction coil or by some form of high-tension transformer. In spite of certain obvious disadvantages the induction coil still holds the field for general work. A 16-in. coil, that is to say, one capable of giving a 16-in. spark in air at ordinary pressure, is sufficient for most purposes.

The great disadvantage of the induction coil is that the potential produced is not unidirectional, the induced E.M.F., when the

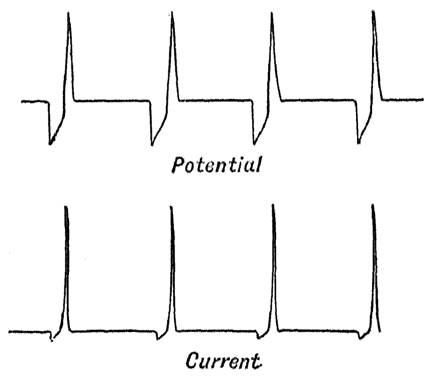


Fig. 18.

primary current is made, being in the opposite direction to that when the current is broken (*Fig.* 18).[1] The latter or direct E.M.F. is, in

[1] Duddell, *J. Rönt. Soc.*, 1908.

a well-designed coil, much higher than the "inverse" E.M.F. at "make." Since the X-ray tube allows no current to pass through it until the potential difference reaches a certain minimum value the inverse E.M.F., if sufficiently low, will be unable to send a discharge through the tube so that the current will be unidirectional. The presence of inverse current is, however, a constant source of trouble to users of induction coils, and special precautions are taken to suppress it. The current supplied by a transformer is an alternating sinusoidal current. The inverse part of the current can, however, be either suppressed or, if desired, rectified by a commutator running in phase with the alternations, so that a unidirectional pulsating current is actually supplied to the tube (*Fig.* 19).

FIG. 19.

It will be seen that in the case of the induction coil the voltage rises almost instantaneously to its maximum value, remains constant for a very short interval of time, and then falls to zero almost as rapidly as it rose. Thus most of the current passes through the tube at the maximum voltage, and the bulk of the electrons are therefore high-speed electrons producing penetrating X-rays. In the case of the transformer the voltage rises and falls gradually, and there is a larger proportion of low-speed electrons producing soft X-rays, many of which cannot even penetrate the walls of the tube. The result of this is that for a given production of penetrating rays there is far more heat produced in the tube when excited by the transformer than when a coil is used. The tubes therefore become over-heated and rapidly deteriorate. The wave form produced by the induction coil is far more suitable for X-ray work than that of the transformer. The transformer, however, has the advantage in power, as it allows of the use of much larger primary currents. For a discussion of the whole problem reference may be made to a discussion at the Institution of Electrical Engineers.[1]

The weakness of the induction coil lies in the difficulty of producing a really sharp interruption in a large current. The large self-induction in the primary circuit of the coil tends to cause the current to arc across the gap and thus to impair that suddenness of break upon which the action of the induction coil depends. The old-fashioned hammer break is practically useless for radiographic purposes, the breaks employed being either some form of mercury interrupter, or an electrolytic break. The former are of two main types, the jet break, and the centrifugal break. In the jet break (*Fig.* 20) a rotating

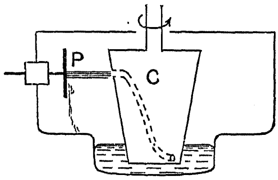


FIG. 20.

cone C, pierced with an oblique channel, driven by an electromotor and dipping into a pool of mercury, acts as a kind of simple centrifugal pump and forms the mercury into a rotating jet. In the course of its rotation the jet impinges on a copper plate P, which is connected to one end of the primary circuit, and so makes metallic contact between it and the pool of mercury, which is connected with the other end of the circuit. The circuit is again broken when the rotation carries the jet past the copper plate. The duration of contact, which should be sufficient to allow the iron core of the coil to reach its maximum intensity of magnetisation, can be adjusted by altering the width of the plate. If more frequent interruptions are required several plates can be placed in the track of the rotating jet. In order to avoid the contamination of the mercury by oxidation the break is generally completely enclosed in an air-tight vessel, which can be filled with hydrogen or coal gas. Occasionally oil or methylated spirit is employed instead. This, however, rapidly emulsifies the mercury, and the process of cleaning is both wasteful and unpleasant.

In the centrifugal type of break the mercury reservoir is whirled round at high speed, so that the mercury rises up the sides in the form of a paraboloid. A smaller insulating cylinder, carrying a copper strip connected to one end of the primary circuit, rotates excentrically inside the mercury chamber in the same direction as the mercury, so that the copper strip is alternately plunged into and withdrawn from the whirling fluid, and thus makes and breaks the circuit. The duration of contact can be altered by raising or lowering the cylinder. It is claimed that these interrupters produce a sharper break in the current than the jet type, and permit of the use of heavier discharges. The many varieties of these two types are amply illustrated in the makers' catalogues.

While mercury breaks are almost invariably employed for ordinary work on account of

[1] R. Morton, C. E. S. Phillips, R. S. Wright, and others, *Journ. Inst. Elect. Eng.*, 1920, lviii. 719.

their uniform working and ease of control, for very intense discharges some form of electrolytic break must be employed. The Wehnelt interrupter consists of a large lead cathode C dipping into dilute sulphuric acid (Sp. gr., 1·2), the anode being a platinum wire A protruding through a small hole in the bottom of a porcelain cylinder P, which also dips into the acid (*Fig.* 21). Interruption takes place at the platinum point, and is probably brought about by the constant formation and collapse of bubbles of gas on the wire. The frequency of the interruptions depends on the area of platinum wire exposed, being greater as the area is reduced, and may rise as high as 2000 per second.

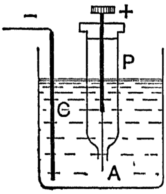


FIG. 21.

If very intense currents are required, three or more anodes, each with its protecting porcelain cylinder and dipping into the same vessel of acid, may be used, connected in parallel. Wehnelt interrupters will not work satisfactorily with currents of less than 10 amperes, and the potential should be between 60 and 80 volts. They give rise to far more inverse current in the secondary circuit than mercury interrupters, but are capable of carrying much heavier currents. In the Caldwell or Simon electrolytic interrupter (*Fig.* 22) the anode is a lead plate enclosed in a porcelain cylinder which communicates with the main reservoir of acid by a small conical hole at its extreme end. The interruptions take place in this hole. The Simon break requires at least 100 volts for its efficient working, and works well on voltages up to 200 volts. This type of break gives rise to less inverse current than the Wehnelt pattern, and can also be used for alternating currents.

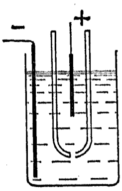


FIG. 22.

§ (20) HIGH-TENSION RECTIFIERS.—The inverse current produced by an induction coil at make is a constant source of trouble to radiographers. It passes through the X-ray tube in the wrong direction, making the target the cathode. As the target is a metal of high atomic weight this gives rise to much cathodic sputtering which rapidly blackens the tube. This blackening makes the inner walls of the tube conducting, an effect which tends to render the discharge unstable. Moreover, the milliammeter used for measuring the secondary current is generally of the moving coil type, and therefore registers the difference between the direct and the inverse current. If there is much inverse the milliammeter readings thus furnish no reliable information either as to the quantity of direct current or the total energy supplied to the tube. This leads not only to serious error in exposure, if a radiogram is being taken, but also to serious risk of overloading the tube.

The best method of eliminating the inverse current, if a mercury interrupter is being used, is to attach a disc of ebonite to the rotating shaft of the break, furnished with a copper strip which completes the secondary circuit at the moment when the primary current is being interrupted in the break, and breaks the secondary circuit while the primary current is being made in the interrupter. A rotating commutator may be employed instead of the simple disc. This method cannot be employed with an electrolytic break.

The method generally employed is to insert in the secondary circuit some unsymmetrical conducting system which allows the current to pass more readily in one direction than the other. Thus a spark gap consisting of a sharp point and a large plane will allow a spark discharge to pass much more readily if the point is positively charged than if it is negative. This device is not particularly efficient and causes a very considerable loss of voltage in the circuit. A discharge tube (*Fig.* 23) furnished with a cathode of large area in the centre of the tube, and a small anode placed in a narrow side tube, and exhausted to a suitable pressure, forms a very efficient rectifier or valve. The tube is exhausted until the Crookes dark space is well developed round the cathode but without approaching the walls of the tube. At this stage the current passes through the tube very readily if the large electrode is the cathode, but only with great difficulty in the opposite direction. These valve tubes, of which there are many patterns, harden rapidly with use, and should be furnished with regulators in the same way as the X-ray tube itself. It is customary to work the valves in pairs, one on each side of the X-ray tube. As the equivalent spark gap is only about $\frac{1}{4}$ inch the loss of potential is not serious.

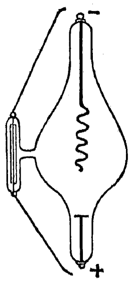


FIG. 23.

§ (21) MEDICAL APPLICATIONS OF THE X-RAYS.—An X-ray installation is an essential part of any modern hospital, the value of the rays, especially for diagnostic purposes, being incalculable. Ordinary animal tissue consists mainly of elements of very low atomic weight and is comparatively transparent to the rays from a tube having a 3-inch or 4-inch alternative spark gap. Bones, however, consisting largely of calcium phosphate, are relatively opaque to these rays, so that if a beam of rays is

passed through the body the bones stand out as clearly defined dark shadows on the fluorescent screen. By substituting a photographic plate for the screen a permanent record, or radiogram, is obtained, which with a well-focussed tube shows the minutest structure of the bones. The position and extent of fractures and the position of the fragments can thus be immediately determined, while the progress and extent of union can also be watched. Diseases of the bone, such as tuberculosis, which cause an absorption of the calcium salts, with a consequent loss of depth in the shadow, and various kinds of tumours which cause swellings or exostoses on the bone, can easily be detected, and from various differences in the details of the shadows their differential diagnosis can be accomplished. The heart, kidneys, and liver, being denser than the surrounding tissues, also cast shadows, from which the size and condition of these organs can be detected. The majority of urinary calculi also are sufficiently dense to cast perceptible shadows on a good radiogram, although skilful technique is required, the density and thickness of the body in the region of the kidneys being considerable. The detection and localisation of foreign bodies was a very important branch of X-ray work during the war. The depth of the foreign body beneath the skin can easily be ascertained by triangulation processes, radiograms being taken with the X-ray tube in two different positions.

The condition and behaviour of the digestive system can be followed by mixing some substance opaque to the rays with the ordinary food. Bismuth carbonate is the substance generally used, though barium sulphate, if pure, can be employed instead with a considerable saving in cost. Using two ounces of bismuth carbonate in a pint of bread and milk or porridge the whole process of digestion can be watched and any abnormalities detected. As the coating of the stomach becomes outlined by the salt the position and size of any ulcers or tumours in the stomach can be detected.

The early detection of tuberculosis of the lungs is one of the most useful achievements of X-ray diagnosis, as the possibility of cure depends so largely on the early recognition of the onset of the disease. Tuberculosis produces local increases in density in the very transparent lung tissues, which give rise to a very characteristic floccular appearance in the radiogram. It is claimed that the disease can be diagnosed by the radiographic method with certainty at a considerably earlier stage than by any other means. The extent to which the tissues are involved can also be clearly ascertained.

The rays are also of value in dentistry, as any abnormality in the teeth, abscesses at the roots, necrosis of the jaw, etc., can be readily detected. The condition of the nasal sinuses can also be investigated, any inflammation giving rise to increased density towards the rays. There is thus a very wide and constantly widening field of usefulness for X-rays as a means of accurate diagnosis.

The application of X-rays to the therapeutic purposes is still in its infancy and the results have, up to the present, not entirely justified the somewhat glowing expectations that were indulged in at one time. This is partly due to want of precise knowledge as to the nature of the action of the rays, and partly to the technical difficulties involved in supplying a sufficiently large dose to the diseased parts without grave injury to the surrounding healthy tissue. X-rays are extensively used in the treatment of ringworm, the effect of the rays in this case being merely to produce a complete epilation of the scalp. The action of the rays appears to be beneficial in certain malignant skin diseases, and various kinds of malignant growths have been successfully treated by the rays, especially when near the surface of the body. Very large doses are required, the application of small quantities of the rays having an adverse effect. This materially increases the difficulty of applying the rays to deep-seated malignant growths owing to the destructive action of the rays on the skin through which the rays have to be passed. It is, however, being overcome by the use of very penetrating rays. The difficulties of estimating the quantity and quality of the rays is also a serious handicap to their application for therapeutic purposes, as, while small doses are inoperative or harmful, an overdose may give rise to disastrous results.

X-rays are also being applied successfully in gynaecology as a means of artificially accelerating the menopause, and for the treatment of excessive haemorrhages, fibroid growths, and myomata. In this way severe abdominal operations can often be avoided, but in the absence of proper methods of measuring the rays considerable skill is required for their application to such cases. For further particulars medical treatises such as that of Knox[1] should be consulted.

§ (22) Physiological Effects of X-rays—Precautions in Use.—X-rays in large quantities have a destructive effect on animal tissues. The effect is a cumulative one, so that exposure even to rays of comparatively feeble intensity may be dangerous if prolonged over days or weeks, while the injury may take several weeks before it becomes

[1] R. Knox, *Radiography and Radiotherapeutics*, 1917.

apparent. It is obvious, therefore, that great care should be exercised by all workers with the rays. The exact cause of the changes produced in the tissues by the rays is not known with certainty. It has been proved that lecithin can be decomposed by the rays, possibly with the production of cholin or other cell poisons. This would account for their action on malignant growths which are known to contain a larger percentage of lecithin than normal tissues.

The action on the skin is to produce first a bronzing effect followed by an erythema, which on further exposure gives rise to the intractable sores, or X-ray dermatitis, which in the past have not infrequently resulted in death. The action on the skin is mainly produced by the easily absorbable rays, or possibly by the secondary corpuscular radiation which they excite. The much more penetrating rays now employed in radiography have far less action on the skin, probably owing to the very small fraction of their energy which is absorbed. X-rays cause degeneration in the testicles and ovaries, and complete sterilisation can easily be produced. In this case the hard rays must be effective, as the parts would be largely protected from the softer rays by the surrounding tissues.

Very little is known as to the general effect of the rays on the system. Several workers with radium have died from profound anaemia produced by the radiations from that substance, and from the general similarity in their properties X-rays might be expected to produce similar effects. A prolonged exposure to strong X-rays produces headache and nausea.

Protection against the action of the rays should begin at the source. The X-ray tube should be completely enclosed in a box covered with not less than 6 millimetres of rubber impregnated with lead salts. This lead rubber is preferable to metallic lead as it is a non-conductor of electricity, and is therefore much less likely to puncture the tube by sparking to it. A considerably greater thickness of it is required for efficient protection than of the pure metal, 2 millimetres of lead forming a fairly efficient screen. An aperture in the box permits the rays to emerge in the required direction, and the emergent beam should be limited by metal diaphragms to the size actually required for the experiment. A lead glass window can be inserted in one side of the box to enable the condition of the tube to be observed. A thickness of from 1 to 2 centimetres of lead glass may be regarded as providing reasonable protection. As a properly protected box is of considerable weight, some makers provide shields which enclose only the active hemisphere of the tube. This method is to be deprecated, as the primary rays falling on the glass of the tube and on the shield itself set up secondary radiation of appreciable intensity which emerges from the unprotected part of the shield. The protective box should be carefully tested before being used, as some of those sold are far from being efficient.

Fluorescent screens should always be covered by thick lead glass, and the X-ray worker will avoid as far as possible any exposure of his person to the direct action of the rays. The fact that a beam of X-rays produces secondary rays from any material through which it passes must not be overlooked, as the cumulative effect of this secondary radiation may be considerable. Aprons, masks, and gauntlets lined with lead rubber may be worn if necessary as additional protection against stray radiation, but protection at the source is far more efficient and convenient. As a test of the efficiency of the protecting devices the worker may carry an unexposed plate in his pocket throughout a normal working day. The extent to which the plate is darkened by the rays can be compared with the darkening produced on a similar plate by some known fraction of the dose required to produce erythema of the skin. In view of the rapid extension of the use of X-rays for the examination of materials, with the possibility of the apparatus being left in charge of persons not well versed in the effect of the rays on the body, the necessity for ample protection cannot be too strongly insisted upon.

J. A. C.

X-RAYS:

Determination of the Wave-length of. See "X-rays," § (9).

Interference Phenomena with, the experimental basis for the assumption that X-rays are analogous to light waves of very small wave-length. See *ibid.* § (5).

Ionisation produced by, when passing through Gases. See *ibid.* § (13).

Medical Applications of. See *ibid.* § (21).

Methods of Estimation of Intensity of. See *ibid.* § (14).

Reflection of, by Crystal Planes. See *ibid.* § (7).

Stokes's Theory of the Nature of. See *ibid.* § (2).

The Scattering of, by Matter. See *ibid.* § (3).

Theory of the Diffraction of, by Crystals. See *ibid.* § (6).

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Printed in Great Britain by R. & R. CLARK, LIMITED, *Edinburgh.*